$$(p_\mu \gamma^\mu - mc + i\varepsilon)(p_\mu \gamma^\mu + mc - i\varepsilon) = p^2 - mc^2 + i\varepsilon$$

▢ 相対論的プロパゲータ

$$\left[\gamma_\mu \left\{ i\hbar \frac{\partial}{\partial x'_\mu} - gA^\mu(x') \right\} - mc \right] S_F(x';x) = \hbar \delta^{(4)}(x-x')$$

・自由粒子の場合

$$\left(i\hbar \gamma_\mu \frac{\partial}{\partial x'_\mu} - mc\right) S_F^{(0)}(x',x) = \hbar \delta^{(4)}(x-x')$$

フーリエ変換： $S_F^{(0)}(x',x) = \int \frac{d^4p}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p(x'-x)} \tilde{S}_F^{(0)}(p)$

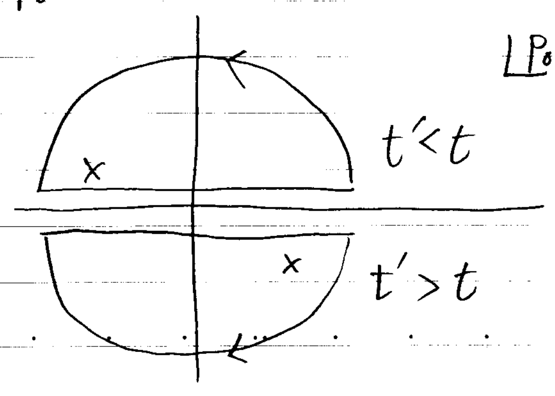
$$\rightarrow \tilde{S}_F^{(0)}(p) = \frac{\hbar}{\underbrace{p^\mu\gamma_\mu}_{= \not{p}} - mc} = \hbar \frac{\not{p} + mc}{\underbrace{p^2}_{= p_0^2 - \mathbb{P}^2} - m^2c^2}$$

特異点， $p^2 = p_0^2 - \mathbb{P}^2 = mc^2$

$$\rightarrow p_0 c = \pm\sqrt{\mathbb{P}^2c^2 + m^2c^4} = \pm E$$

時間に順行する粒子 → $E > 0$
逆行 $E < 0$
} となるように極をずらす

$\int dp_0\, e^{-\frac{i}{\hbar}p_0c(t'-t)} \cdots$



$$S_F^{(0)}(x',x) = \int \frac{d^4p}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p(x'-x)} \times \frac{\hbar(\not{p} + mc)}{p^2 - m^2c^2 + i\varepsilon}$$

とするのと等しい

規格化された平面波解

$$\psi_{ps}^{(+)}(x) = \sqrt{\frac{E+mc^2}{2E}} \begin{pmatrix} \chi_s \\ \frac{c(\sigma\cdot p)}{E+mc^2}\chi_s \end{pmatrix} \frac{e^{-ipx/\hbar}}{(2\pi\hbar)^{3/2}}$$

$$\psi_{ps}^{(-)}(x) = \sqrt{\frac{-E+mc^2}{2E}} \begin{pmatrix} \frac{-c(\sigma\cdot p)}{E-mc^2}\chi_s \\ \chi_s \end{pmatrix} \frac{e^{ipx/\hbar}}{(2\pi\hbar)^{3/2}}$$

を用いて；　$\rightarrow \int d^4x\, \psi_{ps}^{(\pm)}(x)^{\dagger}\, \psi_{p's'}^{(\pm)}(x) = 2\pi\hbar\, \delta_{ss'}\delta(p-p')$

$$S_F^{(0)}(x',x) = -i\theta(t'-t)\int dp \sum_s \psi_{ps}^{(+)}(x')\,\bar{\psi}_{ps}^{(+)}(x)$$
$$+ i\theta(t-t')\int dp \sum_s \psi_{ps}^{(-)}(x')\,\bar{\psi}_{ps}^{(-)}(x)$$

と書くこともできる。

$$\rightarrow \quad \theta(t'-t)\,\psi_{ps}^{(+)}(x') = i\int dr\, S_F^{(0)}(x',x)\,\gamma_0\,\psi_{ps}^{(+)}(x)$$

$$\theta(t-t')\,\psi_{ps}^{(-)}(x') = -i\int dr\, S_F^{(0)}(x',x)\,\gamma_0\,\psi_{ps}^{(-)}(x)$$

「ファインマン伝搬関数」

DEPARTMENT OF PHYSICS
GRADUATE SCHOOL OF SCIENCE
KYOTO UNIVERSITY

No.

• $q \neq 0$ の場合

$$\left(i\hbar\gamma_\mu \frac{\partial}{\partial x'_\mu} - q\gamma_\mu A^\mu(x') - mc\right) S_F(x', x) = \hbar\,\delta^{(4)}(x'-x)$$

$\downarrow$

$$S_F(x', x) = S_F^{(0)}(x', x) + \frac{q}{\hbar}\int d^4y\, S_F^{(0)}(x', y)\not{A}(y) S_F(y, x)$$

(note)

$$\left(i\hbar\gamma_\mu \frac{\partial}{\partial x'_\mu} - mc\right) S_F(x', x)$$

$$= \left(i\hbar\gamma_\mu \frac{\partial}{\partial x'_\mu} - mc\right) S_F^{(0)}(x', x)$$

$$+ \frac{q}{\hbar}\int d^4y\, \underbrace{\left(i\hbar\gamma_\mu \frac{\partial}{\partial x'_\mu} - mc\right) S_F^{(0)}(x', y)}_{=\ \hbar\,\delta^{(4)}(x'-y)} \not{A}(y) S_F(y, x)$$

$$= \hbar\,\delta^{(4)}(x'-x) + q\not{A}(x') S_F(x', x)$$

$$\rightarrow \left(i\hbar\gamma_\mu \frac{\partial}{\partial x'_\mu} - q\not{A}(x') - mc\right) S_F(x', x) = \delta^{(4)}(x'-x)$$

同様に、

$$\Psi(x) = \underbrace{\psi(x)}_{\text{自由粒子}} + \frac{q}{\hbar}\int d^4y\, S_F^{(0)}(x, y)\not{A}(y)\Psi(y)$$

(note)

$$\Psi(x) - \psi(x) = \frac{g}{\hbar}\int d^4y\; S_F^{(0)}(x,y)\not{A}(y)\Psi(y)$$

$$\begin{cases} \xrightarrow{t\to\infty} \displaystyle\int d\mathbf{p}\sum_s \psi^{(+)}_{ps}(x)\left[-i\frac{g}{\hbar}\int d^4y\; \bar{\psi}^{(+)}_{ps}(y)\not{A}(y)\Psi(y)\right] \\ \qquad\qquad \text{：時間に順行する正エネルギー解} \\ \xrightarrow{t\to-\infty} \displaystyle\int d\mathbf{p}\sum_s \psi^{(-)}_{ps}(x)\left[i\frac{g}{\hbar}\int d^4y\; \bar{\psi}^{(-)}_{ps}(y)\not{A}(y)\Psi(y)\right] \\ \qquad\qquad \text{：時間に逆行する負エネルギー解} \end{cases}$$

$$\boxed{S_{fi} = \langle \psi_f | \Psi \rangle = \delta_{f,i} \underset{E<0}{\overset{E>0}{\mp}} i\frac{g}{\hbar}\int d^4y\; \bar{\psi}_f(y)\not{A}(y)\Psi_i(y)}$$

四 クーロン散乱

静止した Ze の電荷がつくる静電ポテンシャルによる
電子 の散乱

$$A_0(x) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{c}\frac{Ze}{r}, \quad \boldsymbol{A}(x)=0$$

↓

$$S_{fi} = i\frac{e}{\hbar}\int d^4x\, \bar{\psi}_f(x) \cancel{A}(x)\, \Psi_i(x) \qquad (i \neq f,\ g \to -e)$$

↓ 摂動の最低次

$\psi_i(x)$

(平面波解)

$$\begin{cases} \psi_i(x) = \psi^{(+)}_{p_i s_i}(x) = \sqrt{\dfrac{mc^2}{E_i}}\cdot\underbrace{\sqrt{\dfrac{E_i+mc^2}{2mc^2}}\begin{pmatrix}\chi_s \\ \dfrac{c\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_i+mc^2}\chi_s\end{pmatrix}}_{\equiv\, u(p_i, s_i)}\dfrac{1}{\sqrt{V}}e^{-ip_i x/\hbar} \\[2em] \bar{\psi}_f(x) = \bar{\psi}^{(+)}_{p_f s_f}(x) = \sqrt{\dfrac{mc^2}{E_f V}}\ \bar{u}(p_f, s_f)\, e^{+ip_f x/\hbar} \end{cases}$$

↓

$$S_{fi} = i\frac{e}{\hbar c}\frac{Ze}{4\pi\varepsilon_0}\cdot\frac{1}{V}\frac{mc^2}{\sqrt{E_i E_f}}\ \bar{u}(p_f s_f)\,\gamma_0\, u(p_i s_i)$$

$$\times \int d^4x\ \frac{1}{r}\, e^{\frac{i}{\hbar}(p_f - p_i)x}$$

$$\int_{-\infty}^{\infty} e^{ikx} dx = 2\pi \delta(k)$$

(note) $\int dx_0\, e^{\frac{i}{\hbar}(p_f^0 - p_i^0)x_0} = c\int_{-\infty}^{\infty} dt\; e^{\frac{i}{\hbar}(E_f - E_i)t}$

$$= 2\pi \hbar c\, \delta(E_i - E_f)$$

$$\int d\mathbf{r}\, \frac{1}{r} e^{-\frac{i}{\hbar}(\mathbf{p}_f - \mathbf{p}_i)\cdot \mathbf{r}} \cdot \underbrace{e^{-r/a}}_{\text{スクリーニング}} = \frac{4\pi\hbar^2}{|\mathbf{p}_f - \mathbf{p}_i|^2 + \frac{\hbar^2}{a^2}} \xrightarrow[a\to\infty]{} \frac{4\pi\hbar^2}{|\mathbf{p}_f - \mathbf{p}_i|^2}$$

$$S_{fi} = (\cdots) \times \frac{\bar{u}(p_f s_f)\gamma_0 u(p_i s_i)}{\underbrace{|\mathbf{p}_f - \mathbf{p}_i|^2}_{= q^2}} 2\pi\hbar\, \delta(E_f - E_i)$$

◀ 遷移確率

$$P_{fi} = |S_{fi}|^2 = (\cdots) \frac{|\bar{u}_f \gamma_0 u_i|^2}{q^4} \cdot \underbrace{(2\pi\hbar\, \delta(E_f - E_i))^2}$$

$$\times \frac{V d\mathbf{p}_f}{(2\pi\hbar)^3}$$

$$\| \; T\to\infty$$

$$2\pi\hbar\, \delta(E_f - E_i) \int_{-T/2}^{T/2} e^{\frac{i}{\hbar}(E_f - E_i)t} \times dt$$

$$= 2\pi\hbar\, \delta(E_f - E_i) \int_{-T/2}^{T/2} dt$$

$$= 2\pi\hbar\, T\, \delta(E_f - E_i)$$

単位時間当たりの遷移確率：

$$R_{fi} = \frac{P_{fi}}{T} = (\cdots) \frac{|\bar{u}_f \gamma_0 u_i|^2}{q^4} d\mathbf{p}_f\, \delta(E_f - E_i)$$

散乱断面積 $d\sigma = \frac{R_{fi}}{J}$ フラックス：

• フラックス $\mathbf{j} = c\psi^\dagger \vec{\alpha}\psi$

$$= c\frac{1}{V}\cdot\frac{E+mc^2}{2E}\left(\chi_s^\dagger \quad \chi_s^\dagger \frac{c\sigma\cdot\mathbf{p}}{E+mc^2}\right)\begin{pmatrix}0 & \sigma\\ \sigma & 0\end{pmatrix}\begin{pmatrix}\chi_s\\ \frac{c\sigma\cdot\mathbf{p}}{E+mc^2}\chi_s\end{pmatrix}$$

$$= c\cdot\frac{1}{V}\frac{E+mc^2}{2E}\left(\chi_s^\dagger\sigma\frac{c\sigma\cdot\mathbf{p}}{E+mc^2}\chi_s + \chi_s^\dagger\frac{c\sigma\cdot\mathbf{p}}{E+mc^2}\sigma\chi_s\right)$$

$\mathbf{p} = p\,\mathbf{e}_z$ とすると

$$\mathbf{j} = c\frac{1}{V}\cdot\frac{E+mc^2}{2E}\cdot\frac{2cp}{E+mc^2}\,\mathbf{e}_z = \frac{pc^2}{E}\cdot\frac{1}{V}\mathbf{e}_z$$

↓

$$\frac{d\sigma}{d\Omega} = 4\cdot\frac{\hbar^2 Z^2\alpha^2 m^2 c^2}{q^4}\left|\bar{u}_f\gamma_0 u_i\right|^2$$

• スピンを区別しないとき (Mott 断面積)

$$\left(\frac{1}{2}\sum_{s_i}\right)\sum_{s_f}\left|\bar{u}(p_f s_f)\gamma_0 u(p_i s_i)\right|^2$$

$$= \cdots = \frac{E^2}{m^2c^4}\left(1-\beta^2\sin^2\frac{\theta}{2}\right) \; ; \; \beta\equiv\frac{pc}{E}$$

θ は $\mathbf{p}_i$ と $\mathbf{p}_f$ の間の角

$$\downarrow \quad \frac{d\sigma}{d\Omega} = \frac{1}{4}\cdot\frac{\hbar^2 Z^2\alpha^2}{p^2\beta^2\sin^4\frac{\theta}{2}}\left(1-\beta^2\sin^2\frac{\theta}{2}\right) \leftarrow \begin{cases} q = 2|\mathbf{p}_i|\sin\frac{\theta}{2} \\ \beta = |\mathbf{p}_i|c/E \end{cases}$$

(note) NR の極限

$$p\to mv_0, \quad \beta = \frac{pc}{E}\to\frac{mv_0c}{mc^2} = \frac{v_0}{c}, \quad 1-\beta^2\sin^2\frac{\theta}{2}\to 1$$

$$\downarrow \quad \frac{d\sigma}{d\Omega} = \frac{1}{4}\left(\frac{Z\alpha c\hbar}{mv_0^2}\right)^2\frac{1}{\sin^4\frac{\theta}{2}}$$: (ラザフォード散乱と一致)