Microscopic modeling of compound nucleus reactions
: low-energy induced fission and astrophysical nuclear fusion reactions

Chongqing, March 2011

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1. Microscopic calculations for low-energy induced fission
2. $^{12}C+^{12}C$ fusion reactions at astrophysical energies
3. Discussion: the validity of compound nucleus picture at low energies?
4. Summary

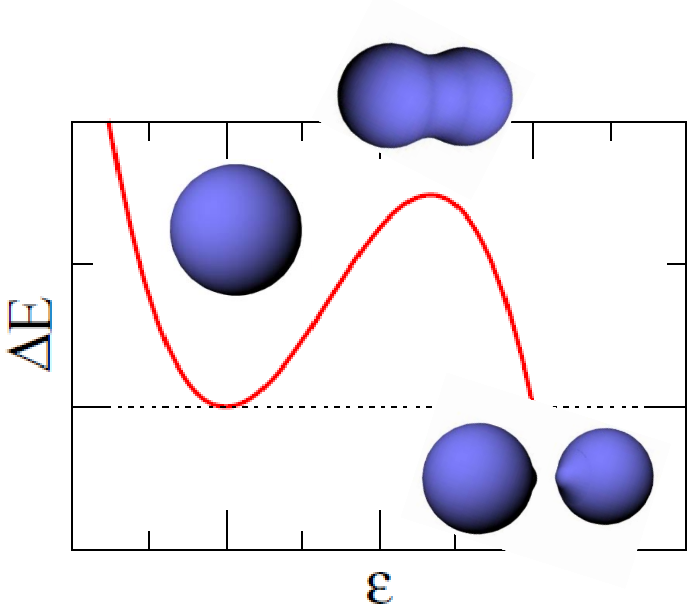
Workshop on Fission Dynamics 2026, May 11-15, 2026, Chongqing Chaotianmen Voco Hotel, Chongqing, China

Microscopic understanding of nuclear fission

How well can one describe nuclear fission microscopically?

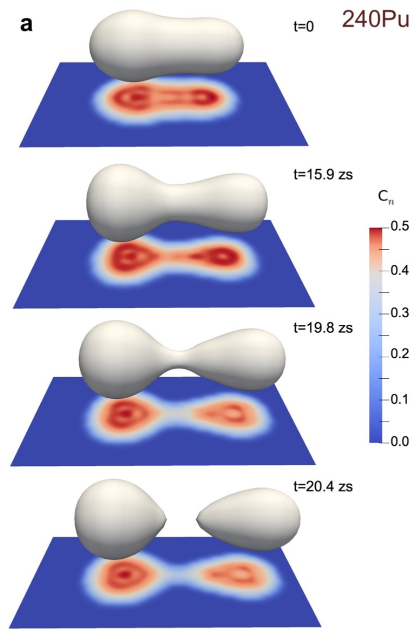
- macroscopic understanding of fission:

  a competition between the surface and the Coulomb energies
  → fission barrier



- a microscopic understanding of fission:

  large change of nuclear shape
  → **far from complete**

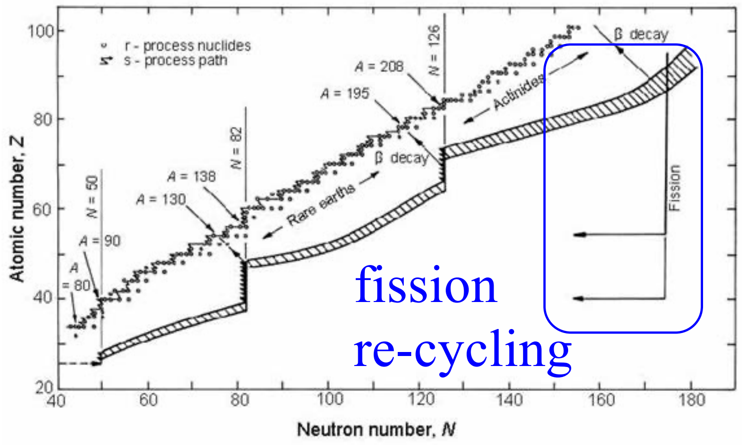


G. Scamps and C. Simenel, Nature 564 (2018) 382

Microscopic calculations for low energy induced fission

Why is a microscopic theory for fission important?

- r-process nucleosynthesis



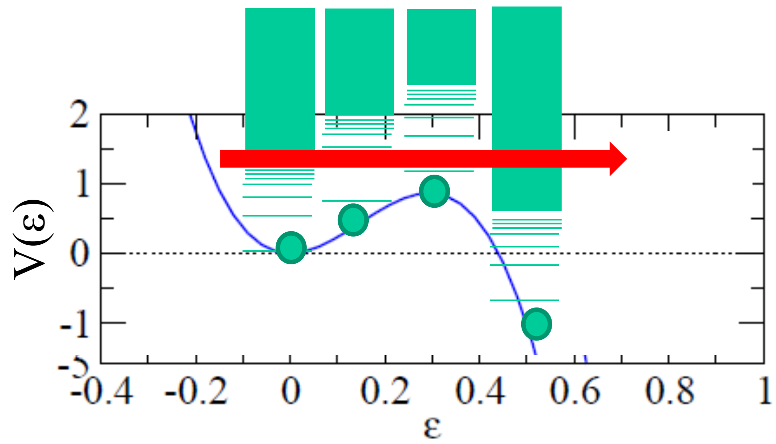
cf. talks by Kajino, Nishimura, and Giuliani tomorrow morning

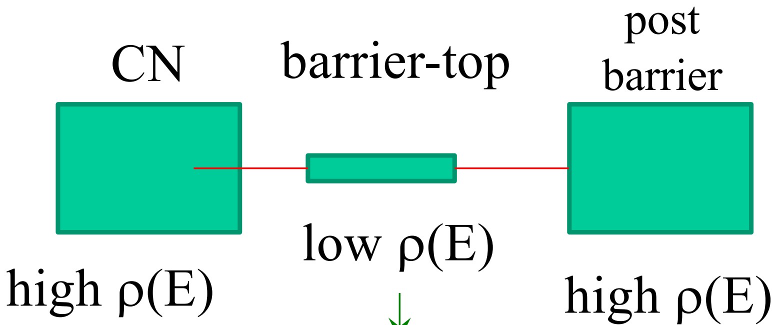
fission of neutron-rich nuclei

→ low E^* and low $\rho(E^*)$

✓ Validity of statistical models?
✓ Validity of the Langevin approach?

- barrier-top fission





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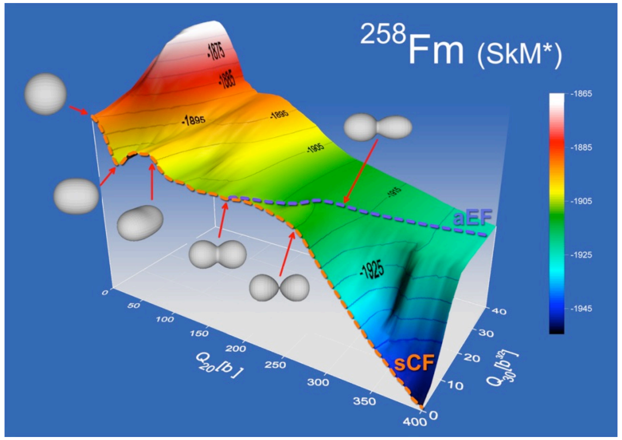
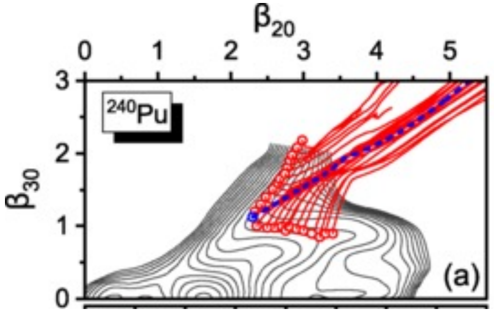
discrete levels

Microscopic approaches to fission

- spontaneous fission





A. Staszczak et al., PRC80 ('09) 014309

constrained Hartree-Fock (+B) method:

$$\delta\langle\Phi|H-\lambda Q_{20}|\Phi\rangle=0$$

$$\rightarrow \Phi(Q_{20}),\ \ E(Q_{20})$$

$$\rightarrow P=\exp\left[-2\int dq\sqrt{\frac{2B(q)}{\hbar^2}(V(q)-E)}\right]$$

cf. the next talk by K. Washiyama

- induced fission: developments → only recently
  - ✓ Time-dependent GCM

$$|\Psi(t)\rangle=\int dq f(q,t)|\Phi_q\rangle$$

D. Regnier et al., PRC93, 054611 (2016)
J. Zhao et al., PRC99, 054613 (2019)
B. Li et al., PRC111, L051302 (2025)

  - ✓ TD-SLDA

A. Bulgac et al., PRL135, 062501 (2025)

  - ✓ Finite *T*-TDHF

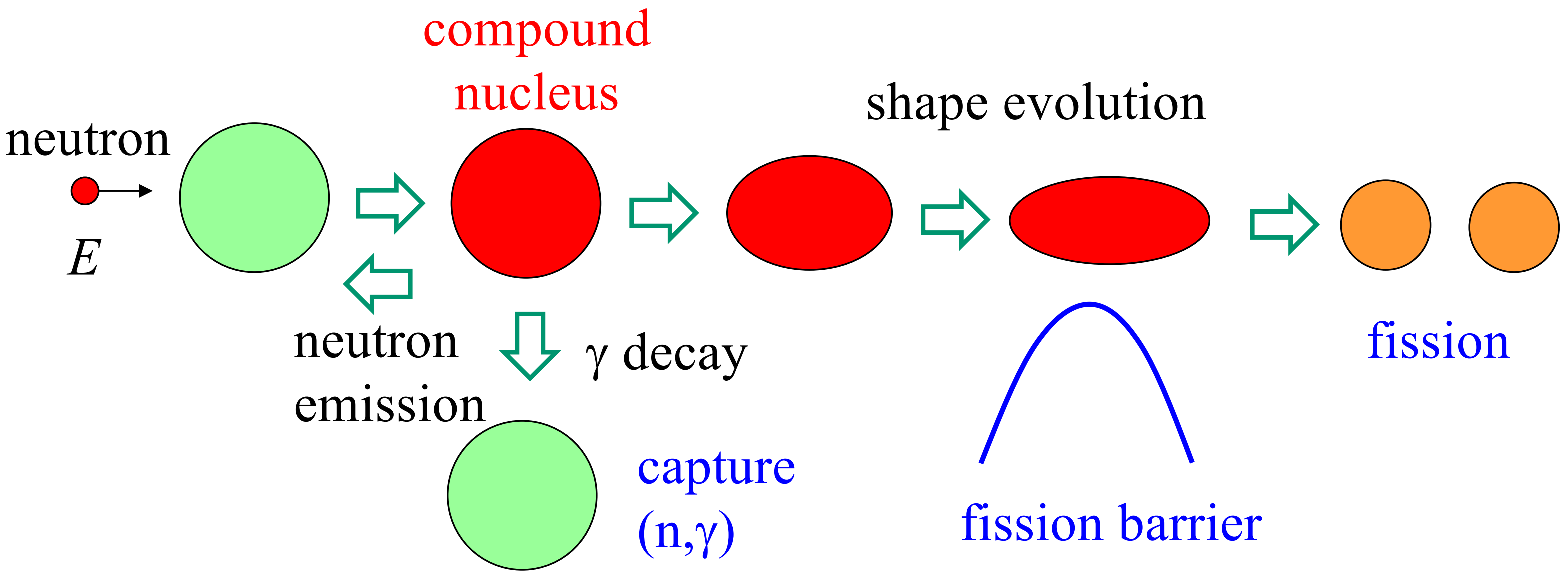
B. Li et al., PRC110, 034302 (2024)

  - ✓ Our approach

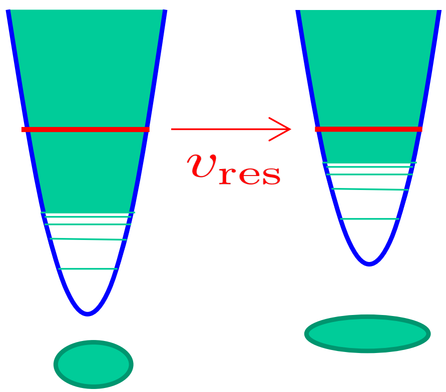
in the same philosophy as TDGCM,
but with a time-*independent* approach using
a Green's function

G.F. Bertsch and K.H., Phys. Rev. C107, 044615 (2023).

Microscopic calculations for low energy induced fission



the basic idea:

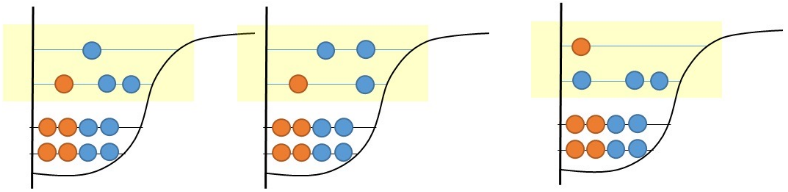


- Many-body configurations in a MF pot. at each shape
- hopping due to residual interactions

→ shape evolution

Microscopic calculations for low energy induced fission

Shell model approach for nuclear structure

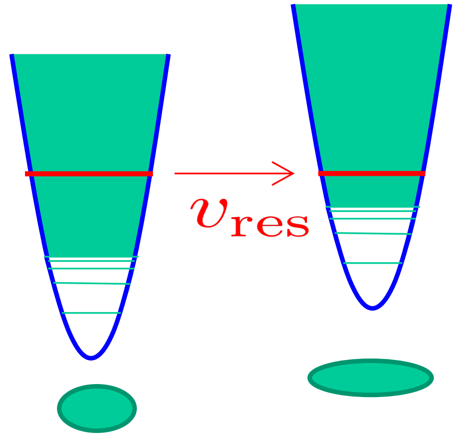


$$|\Psi\rangle = v_1|m_1\rangle + v_2|m_2\rangle + v_3|m_3\rangle + \cdots$$

Figure: Noritaka Shimizu (Tsukuba)

many-particle many-hole configurations
<u>in a given mean-field potential</u>
→mixing by residual interactions

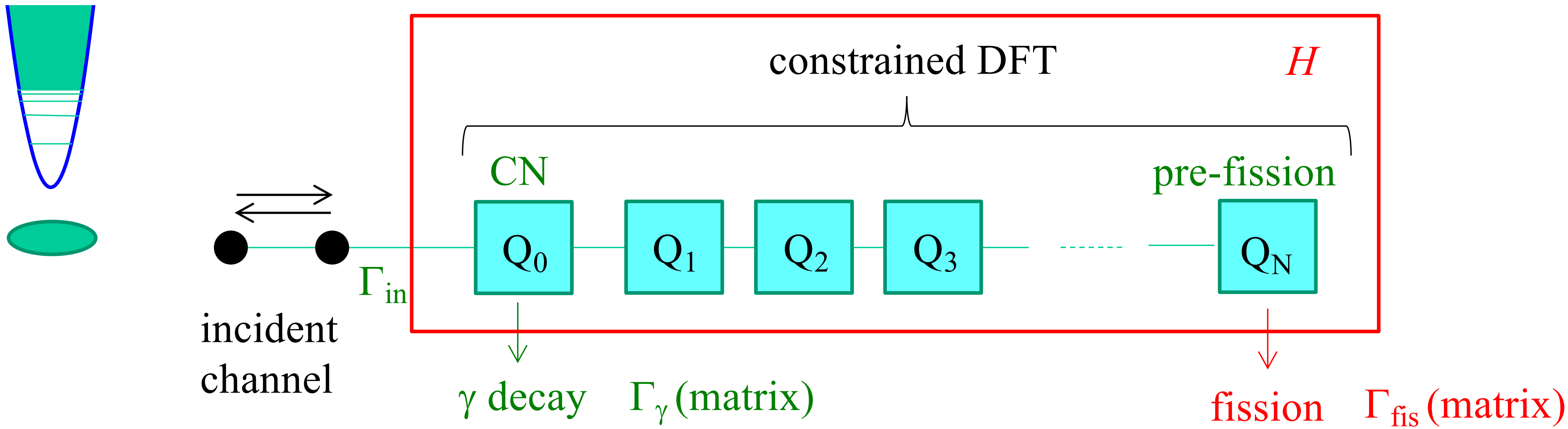
for induced fission

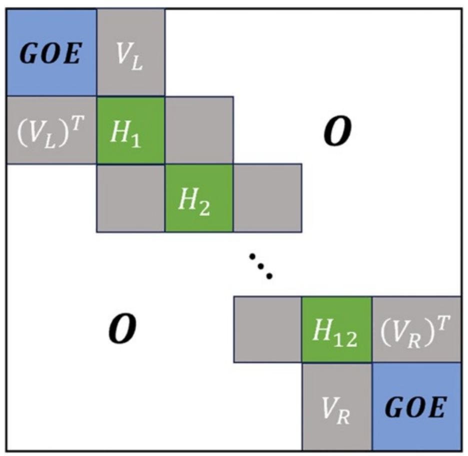


shape dependent mean-field potentials

G.F. Bertsch and K.Hagino, Phys. Rev. C107, 044615 (2023).
K. Uzawa and K. Hagino, Phys. Rev. C110, 014321 (2024).

Microscopic calculations for low energy induced fission





$$H = H_0 - G P^\dagger P + V_{\rm rand}$$

Non-equilibrium Green's function method

$$T_{CN\to{\rm fis}} = {\rm Tr}[\Gamma_n G\, \Gamma_{\rm fis}\, G^\dagger]; \quad G = (H - NE)^{-1}$$

some complications

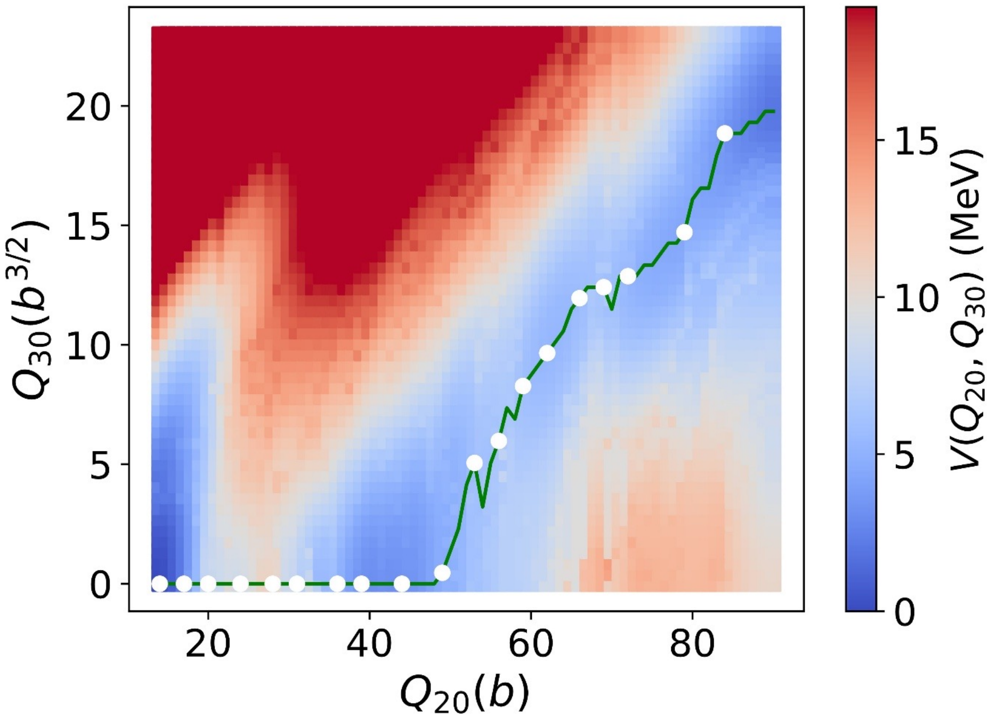
$$\begin{cases} N_{\mu\mu'} = \langle \Psi_\mu(Q) | \Psi_{\mu'}(Q') \rangle \\ \dfrac{\langle \Psi_\mu(Q) | H_0 | \Psi_\mu(Q') \rangle}{\langle \Psi_\mu(Q) | \Psi_\mu(Q') \rangle} \equiv \langle \Psi_\mu(Q) | V_{\rm diabatic} | \Psi_\mu(Q') \rangle \end{cases}$$

Application to low-energy fission of ^{236}U

the talk by K. Uzawa on Friday

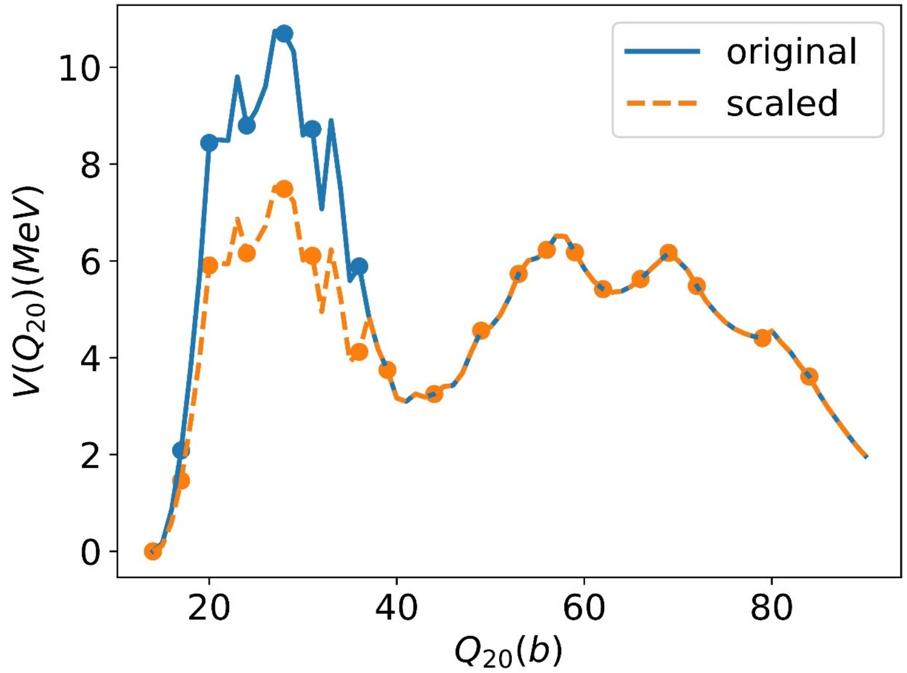
K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).

fission path based on Skyrme DFT with UNEDF1



$\langle \Psi_\mu(Q) | \Psi_\mu(Q') \rangle \sim 0.52$

fission barrier without pairing



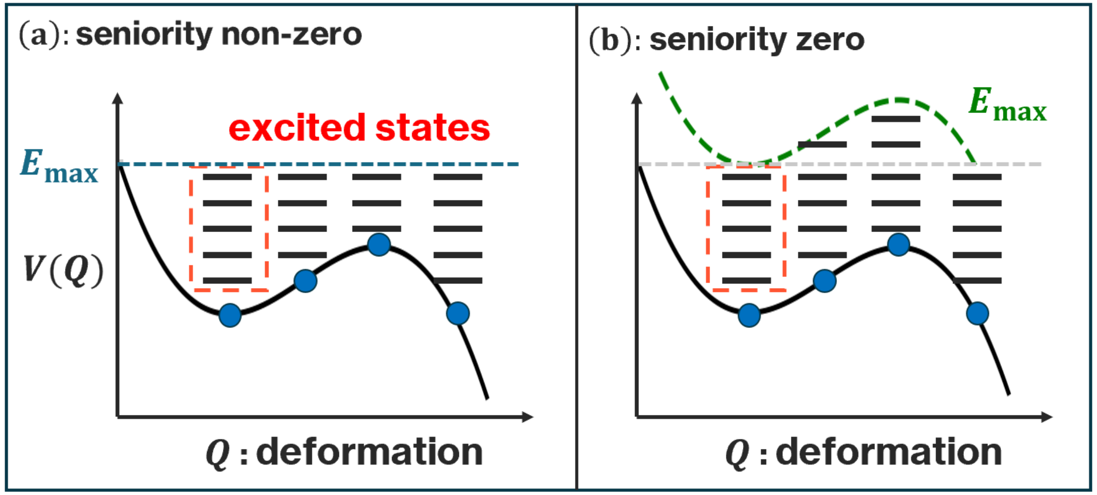
scale the first barrier
$\rightarrow B \sim B_{\rm empirical}$ if with pairing

Application to low-energy fission of ^{236}U

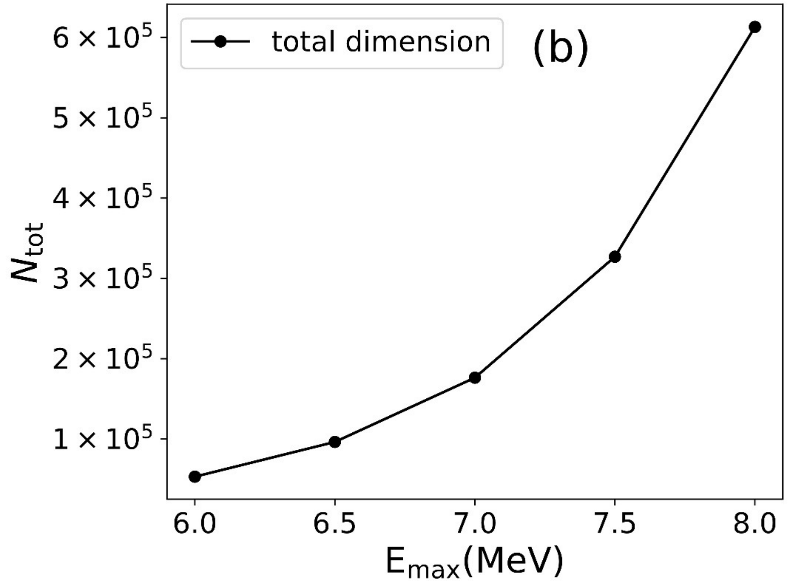
the talk by K. Uzawa on Friday

K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).

np-nh configurations up to E_{max}



dimension of $H \sim O(10^5)$



$$H = H_0 - GP^\dagger P + V_{\rm rand}$$

- $G \leftarrow 0^+_2$ in ^{236}U for a given model space
- $V_{\rm rand} \leftarrow$ systematics
- $\Gamma_{\rm cap}$, $\Gamma_{\rm n} \leftarrow$ empirical
- $\Gamma_{\rm fis}$ for a pre-fission config. $\leftarrow$ insensitivity

inverse of a huge matrix

- LSMR method
- shift-invert Lanczos method

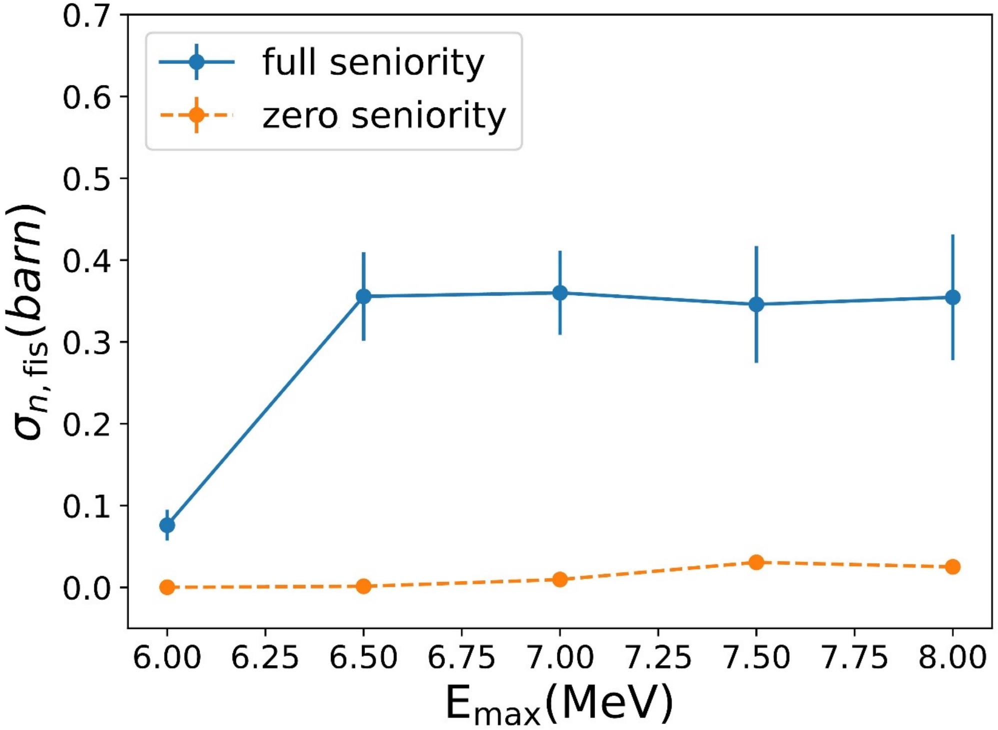
K. Uzawa and K. Hagino
PRE110, 055302 (2024).

Application to low-energy fission of ^{236}U

the talk by K. Uzawa on Friday

K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).

fission cross section (E = 10 keV)



convergence at $E_{max} \sim 6.5$ MeV

the empirical value: $\sigma_{n,fis} = 3.116$ b

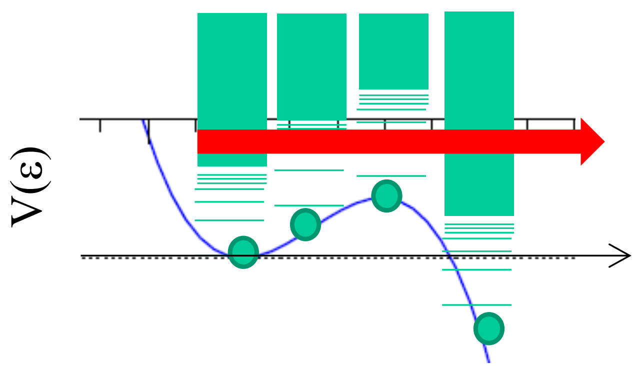
→ reproduced within a factor of 10

possible origins of the discrepancy:

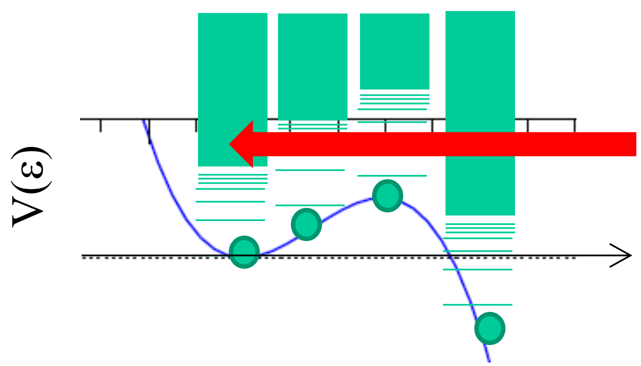
- axial symmetry → triaxiality
- G = const. → $G(\beta)$
- Skyrme UNEDF1 → other functionals?
- collective coordinates: Q_2 and Q_3?
  e.g., the momentum dependence?

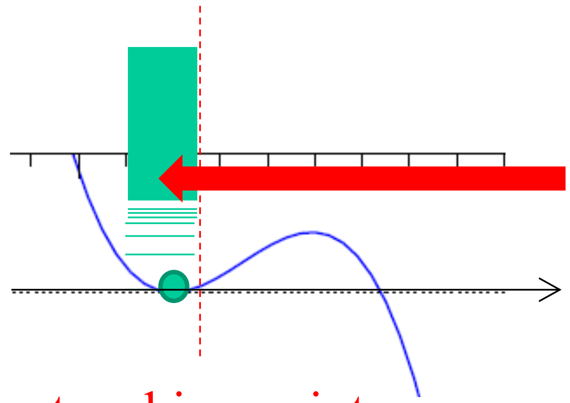
$^{12}C + ^{12}C$ fusion reactions at astrophysical energies

fission



fusion





touching point

fusion for
superheavy elements

↓

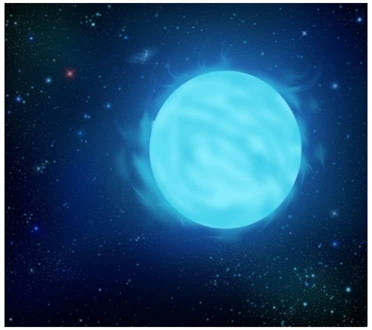
a microscopic description
for shape evolution

fusion of light nuclei
(nuclear astrophysical
reactions)

relative motion
+ compound nucleus

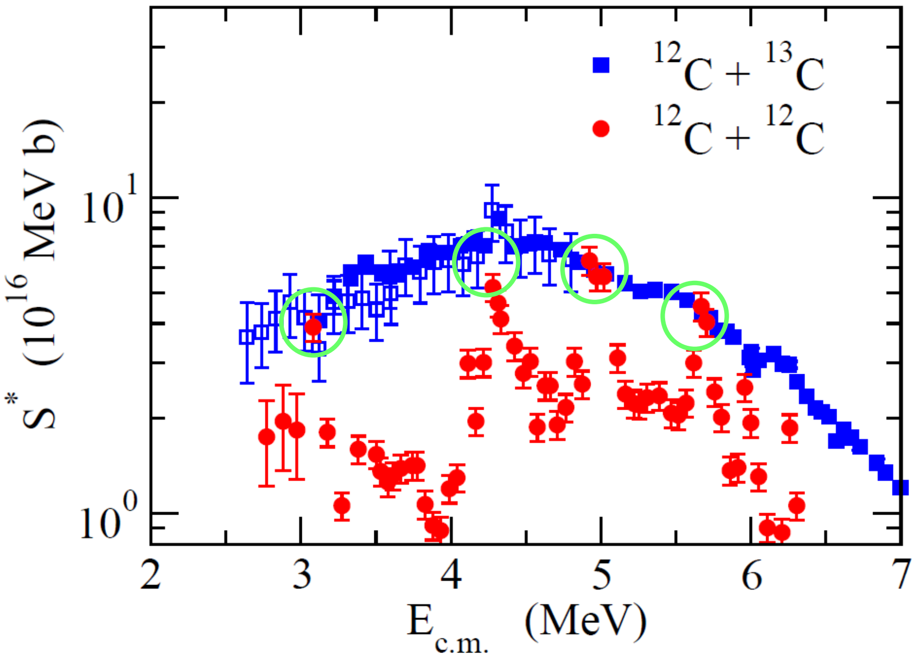
^{12}C+^{12}C fusion reactions at astrophysical energies

^{12}C+^{12}C fusion
: one of the key reactions in nuclear astrophysics



- ✓ Carbon burning in massive stars
- ✓ Type Ia supernovae
- ✓ X-ray superburst

$$S^*(E) = E\sigma(E)\, e^{2\pi(\eta-\eta_0)}$$



C.L. Jiang et al., PRL110, 072701 (2013).

difference between ^{24}Mg and ^{25}Mg

- ^{24}Mg: isolated CN resonances
- ^{25}Mg: overlapping CN resonances

need a new reaction model

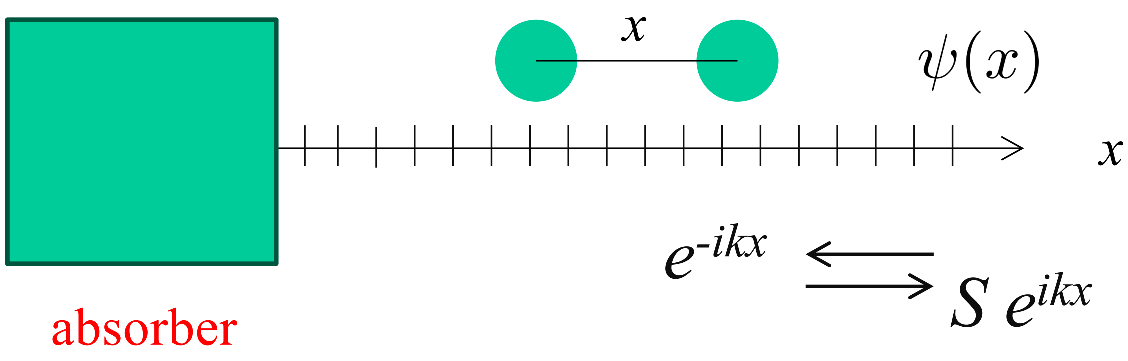
→

prominent resonance structure in ^{12}C+^{12}C fusion

- ✓ off-resonance: fusion inhibition
- ✓ on-resonance: match with ^{12}C+^{13}C

a schematic model for the imaginary part : a microscopic understanding of the optical potential

□ scattering problem with absorption:



absorption $\rightarrow |S| < 1$

cf. for strong absorption

$$S \sim 0$$

cf. optical potential

$$V_{\rm opt}(r) = V(r) - iW(r)$$

- elastic scattering
- absorption process

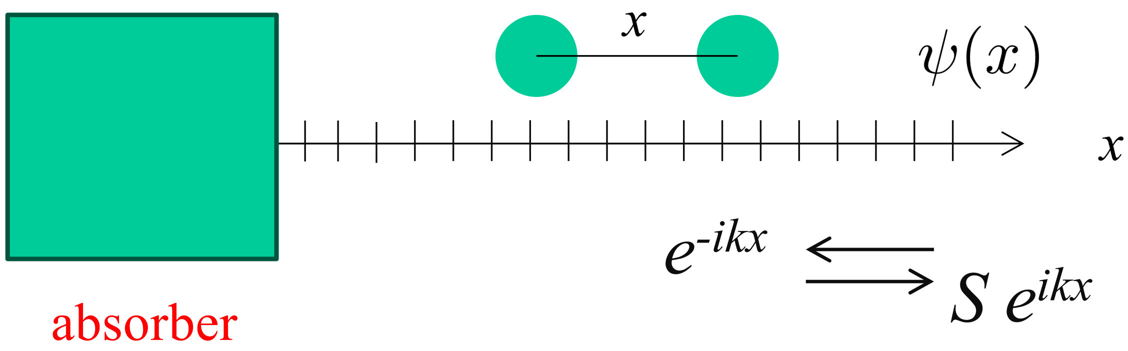
$iW(r)$ ← a loss of incident flux

- ✓ inelastic scattering
- ✓ transfer reaction
- ✓ fusion

etc.

a schematic model for the imaginary part : a microscopic understanding of the optical potential

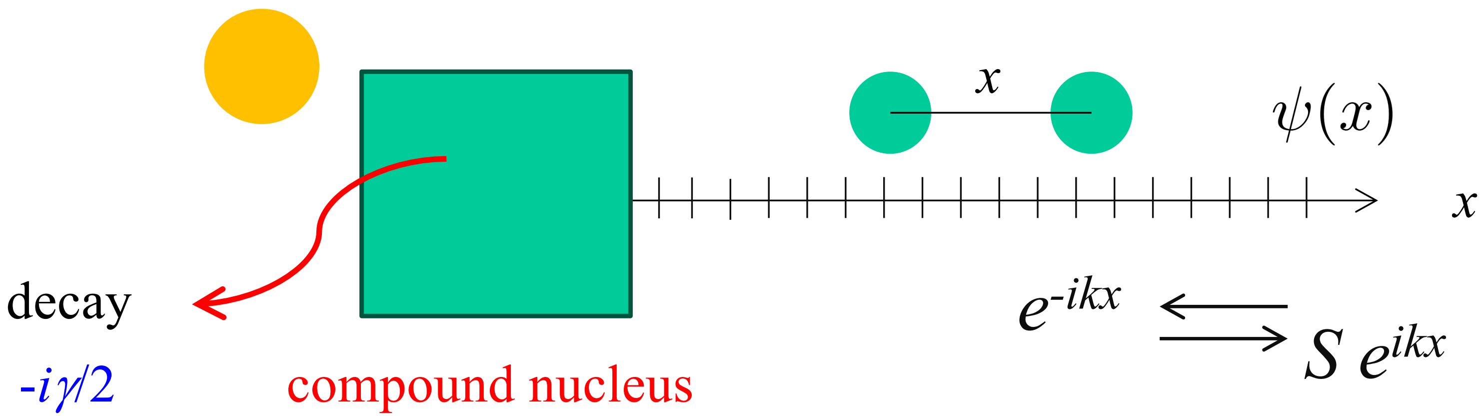
- scattering problem with absorption:



absorption $\rightarrow |S| < 1$

cf. for strong absorption
$S \sim 0$

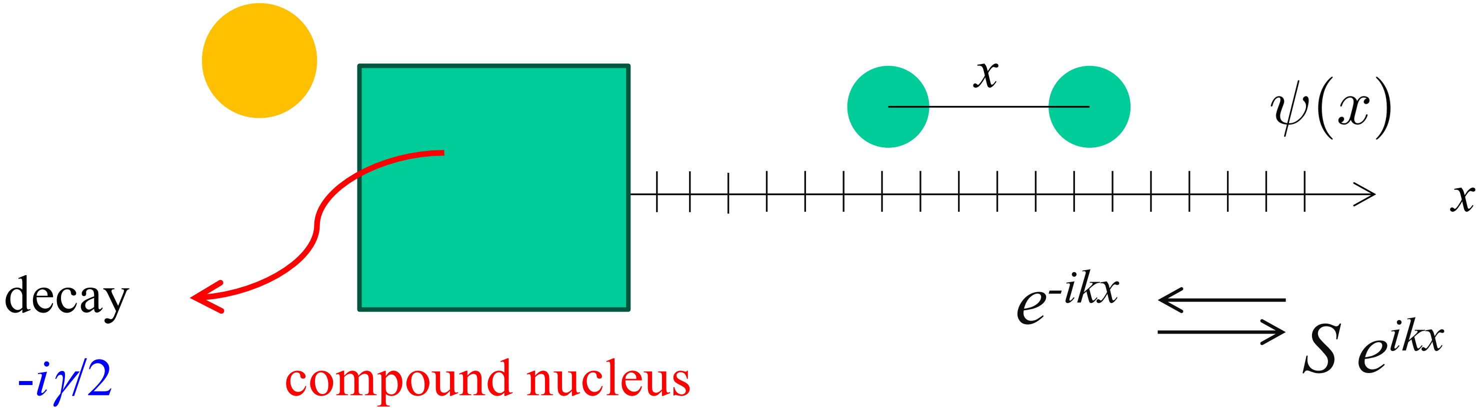
- a microscopic origin for absorption: compound nucleus formation (fusion)



✓ couplings to CN
✓ decay of CN
$\rightarrow$ not coming back to the entrance ch.
$\rightarrow$ loss of flux (absorption)

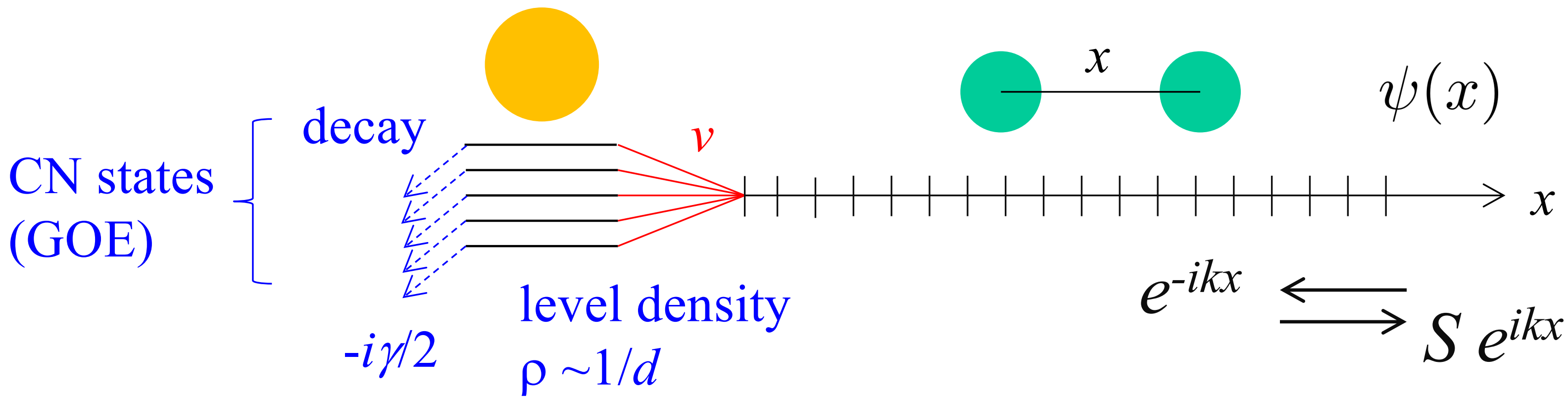
a schematic model for the imaginary part : a microscopic understanding of the optical potential

- a microscopic origin for absorption: compound nucleus formation (fusion)

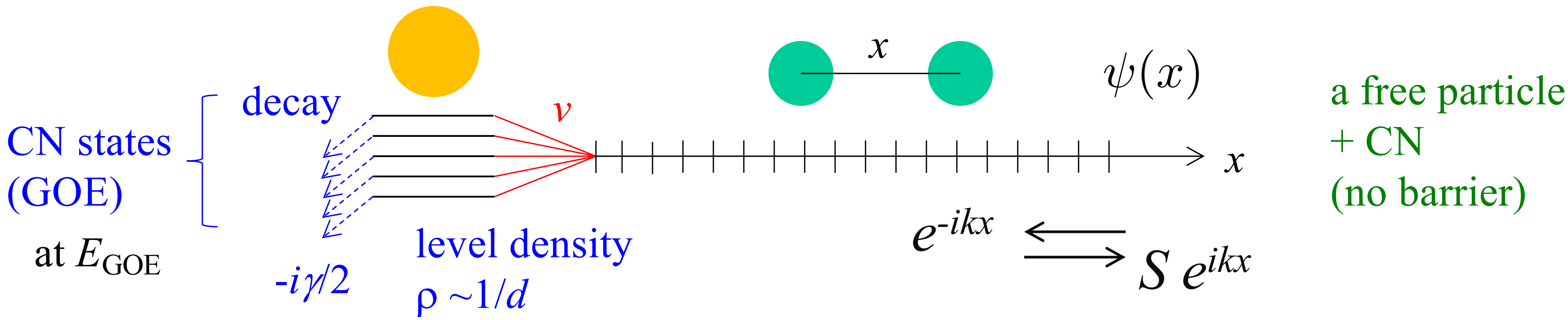


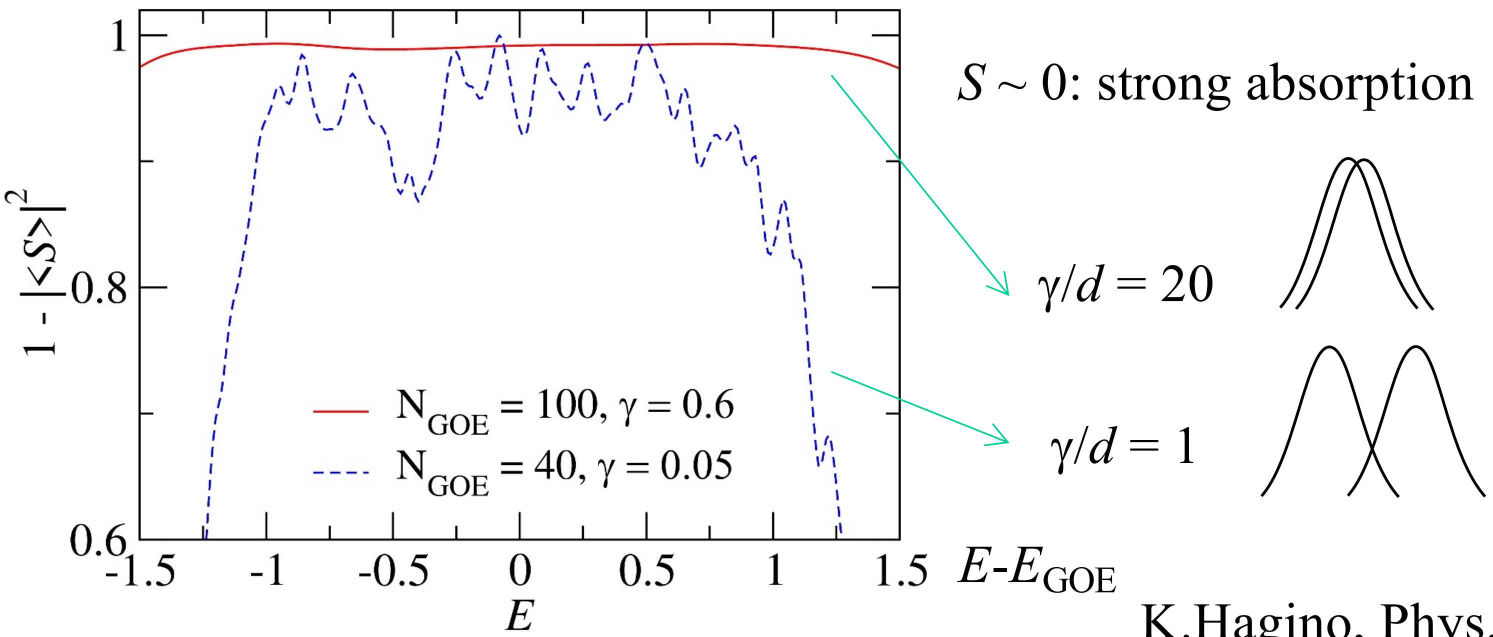
✓ couplings to CN
✓ decay of CN
→ not coming back to the entrance ch.
→ loss of flux (absorption)

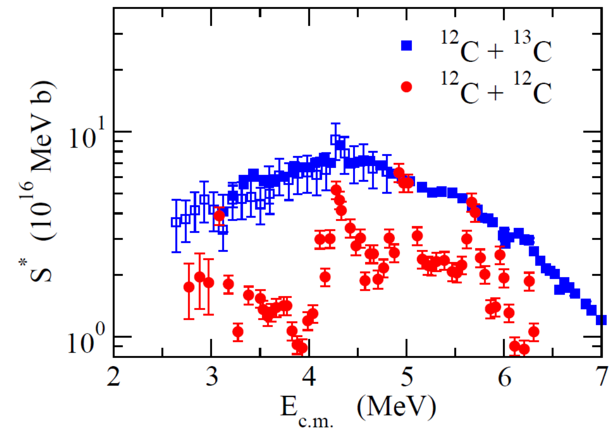
- a modelling:



a schematic model with a random matrix



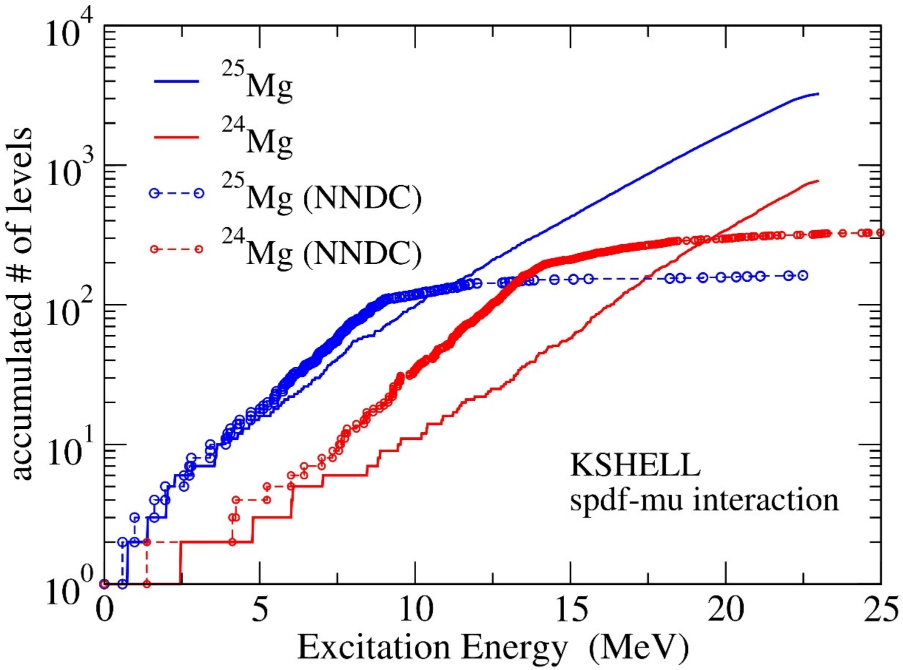


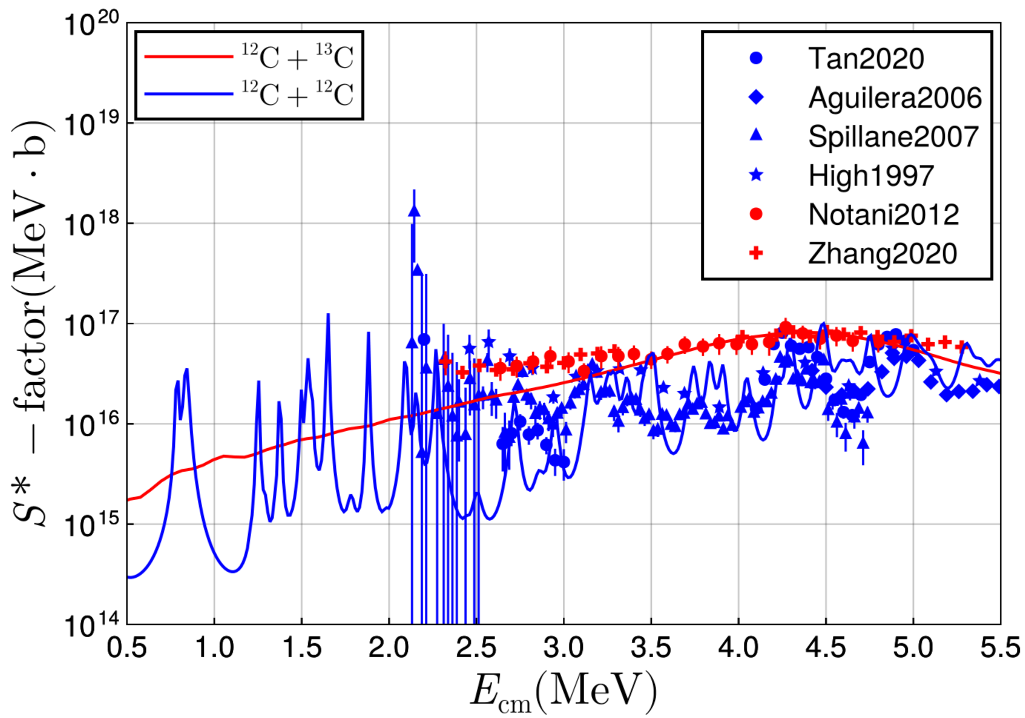


K.Hagino, Phys. Rev. C112, 034611 (2025).

a more realistic model with shell model K. Nagao, K. Hagino, and K. Uzawa, in preparation.

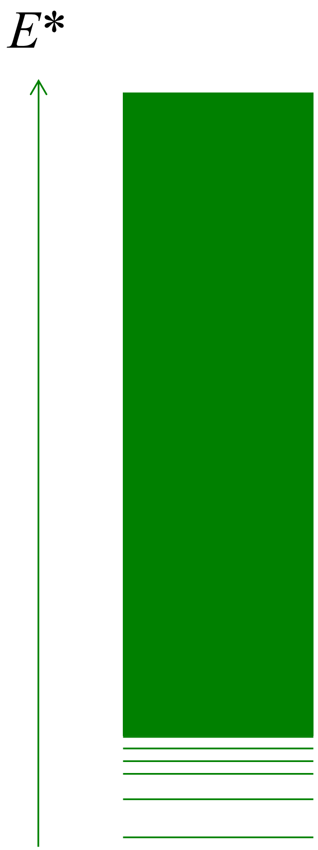
✓ the spectrum of CN states: RMT → shell model (KSHELL)
✓ decay widths → statistical model
✓ 3D calculations with the Coulomb barrier





Discussion: the validity of compound nucleus picture at low energies?

A question: at what E^* does the compound nucleus picture begin to be valid?



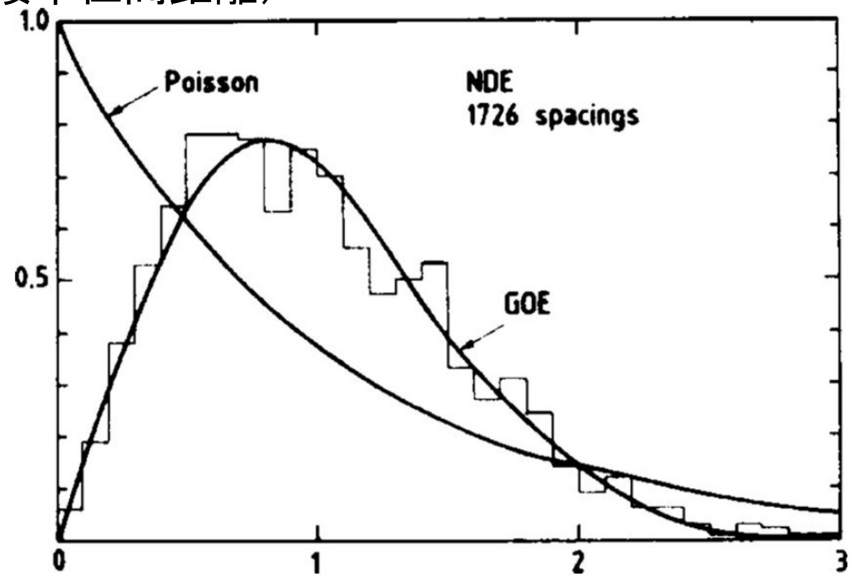
quasi-continuum
spectrum
→ statistical treatment

$$\rho(E) \sim e^{2\sqrt{aE^*}}$$

regular motion

random matrix: provides a good description of CN

the nearest neighboring level spacing distribution
（最近接準位間距離）

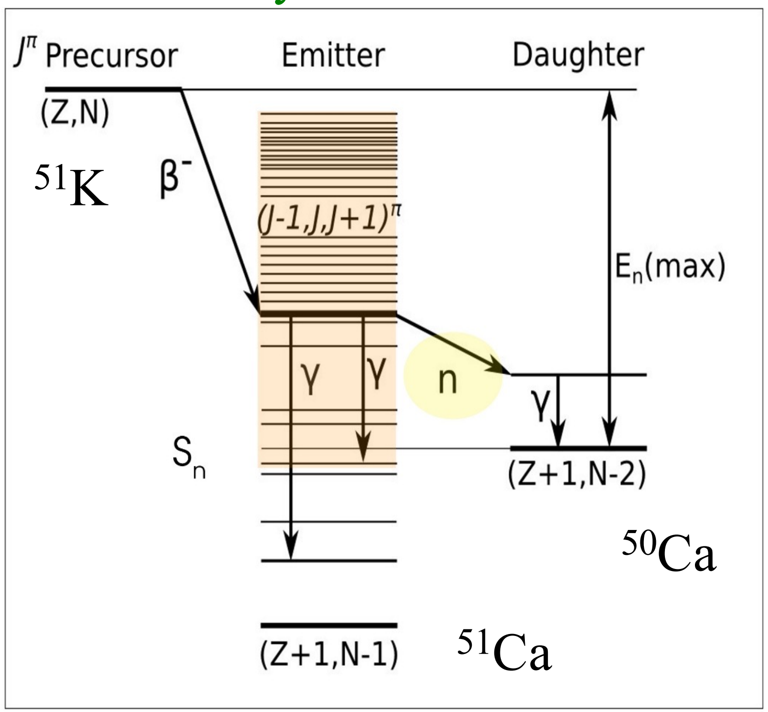


O. Bohigas et al., “Nuclear Data for Science and Technology”,　p. 809.

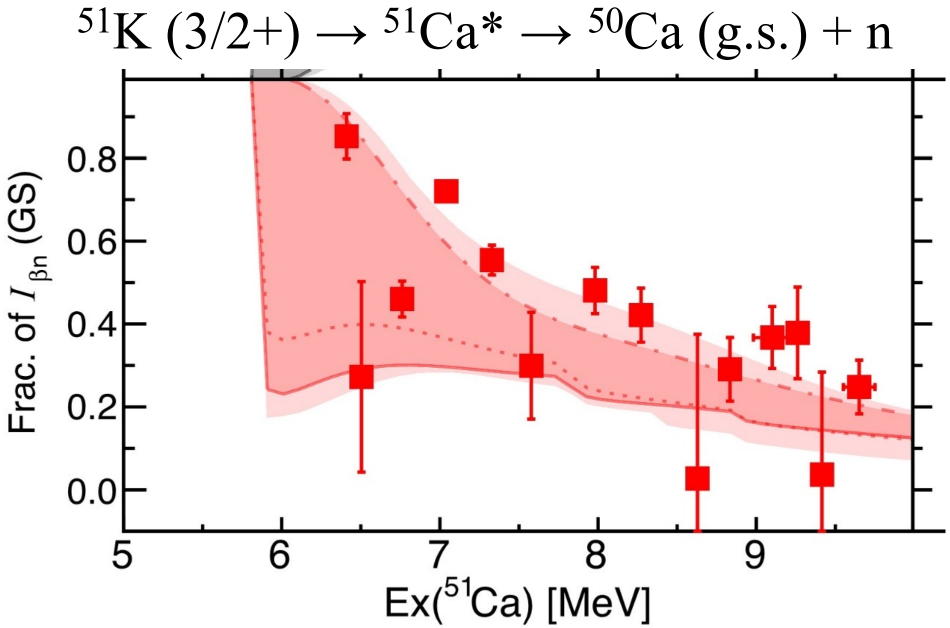
Discussion: the validity of compound nucleus picture at low energies?

A question: at what E^* does the compound nucleus picture begin to be valid?

beta-delayed neutron emission



a figure by R. Grzywacz (Tennessee)



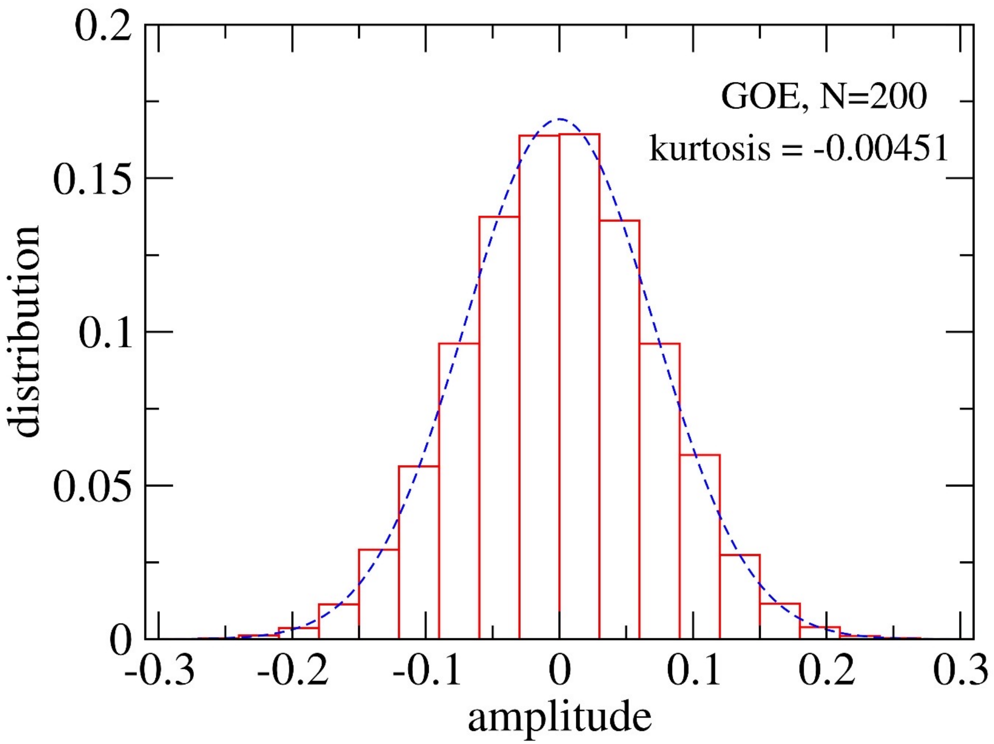
Z.Y. Xu, R. Grzywacz et al., PRL133, 042501 (2024)

Statistical model calculations seem to work well despite that the level density is low (10-20 states/MeV).

Discussion: the validity of compound nucleus picture at low energies?

A question: at what *E** does the compound nucleus picture begin to be valid?

We address this question by investigating the kurtosis（尖度）of a distribution.



$$\psi_k = \sum_\mu C_\mu^{(k)} |\mu\rangle \quad \rightarrow \text{decay width} \sim |C|^2$$

Random Matrix (GOE):

$$P(x) = \sqrt{\frac{N}{2\pi}}\, e^{-Nx^2/2}; \quad \left(x = \{C_\mu^{(k)}\}\right)$$

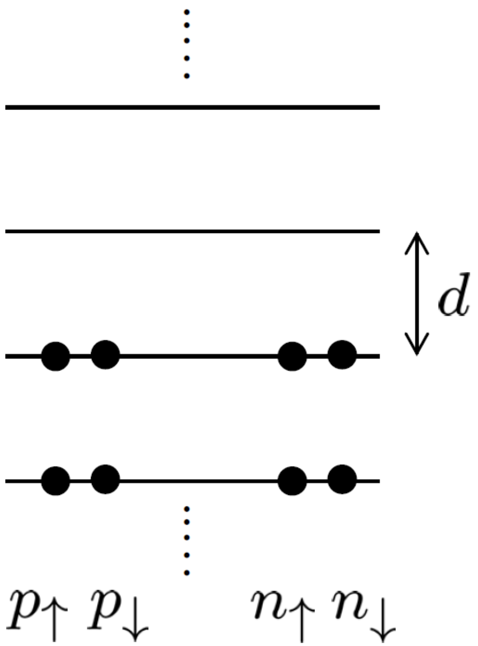
$$\text{kurtosis} \equiv \frac{\langle x^4 \rangle}{\langle x^2 \rangle^2} - 3$$

= 0 for a Gaussian distribution

Discussion: the validity of compound nucleus picture at low energies?

uniform spacing single-particle levels

K. Uzawa and K. Hagino,
Phys. Rev. C108, 024319 (2023).



$$H = H_0 - G\sum_{k,k'} a_k^\dagger a_{\bar{k}}^\dagger a_{\bar{k}'} a_{k'} - v\sum r a^\dagger a^\dagger a a$$

$$G = 0.3d, \quad v_{np} = 0.025d, \; v_{nn} = v_{pp} = 0.02d$$

(note) kurtosis = model space dependent, and basis dependent

$$|\psi_{N,k}\rangle = \sum_\mu C_\mu^{(k)} |\phi_{N,\mu}\rangle$$

$$[H_0 + H_{\rm pair}]\,|\phi_{N,\mu}\rangle = E_\mu|\phi_{N,\mu}\rangle$$

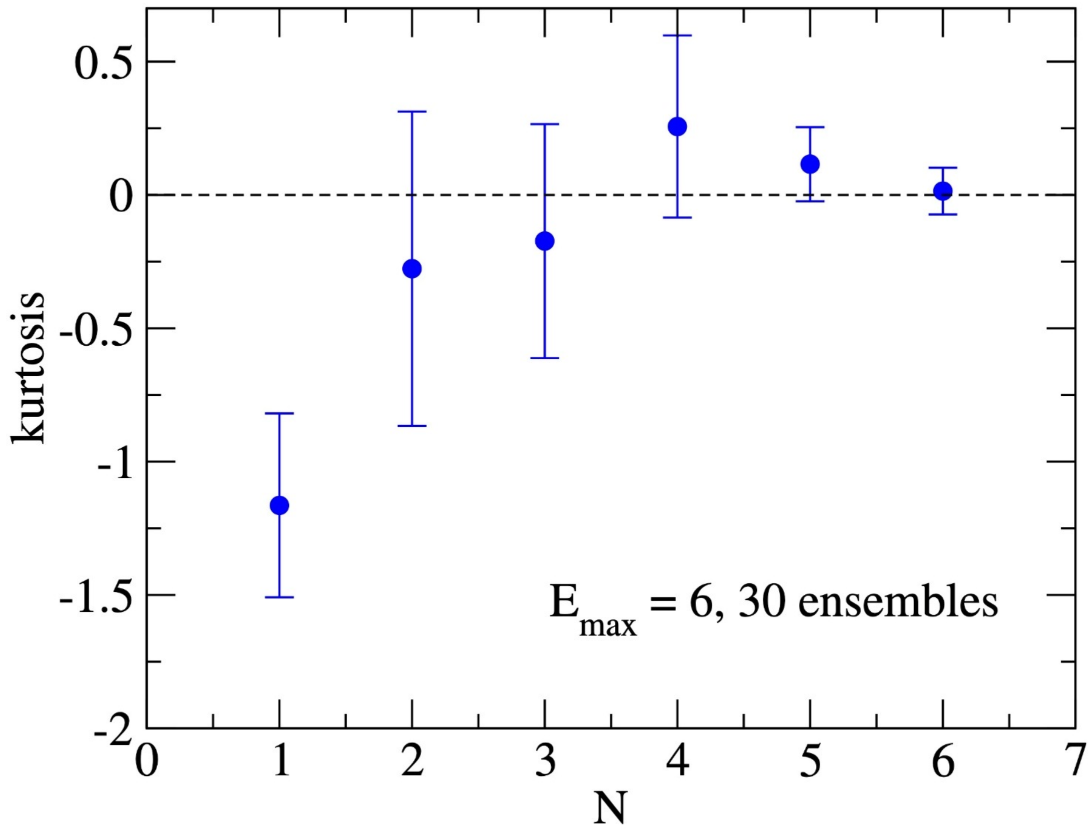
with $E_\mu \sim Nd$

mixing due to the random interaction → kurtosis

Discussion: the validity of compound nucleus picture at low energies?

uniform spacing model

K. Uzawa and K. Hagino, Phys. Rev. C108, 024319 (2023).

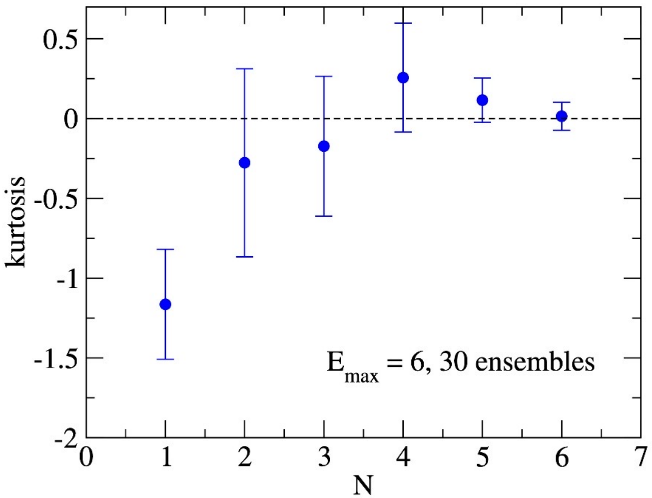


# of levels for K_{tot} = 0:

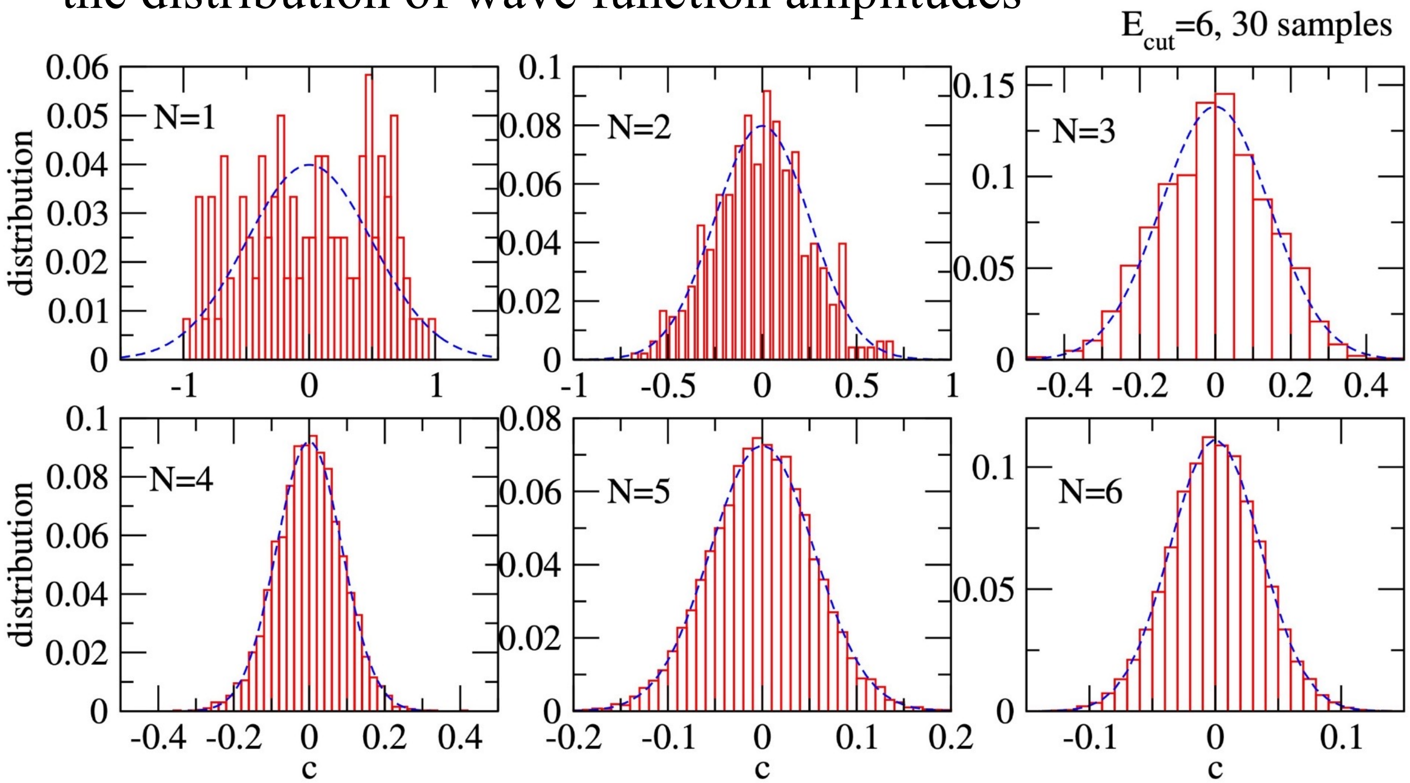
4 (N=1)
16 (N=2)
48 (N=3)
133 (N=4)
332 (N=5)
784 (N=6)

the kurtosis is consistent with 0 even for N = 2.

Discussion: the validity of compound nucleus picture at low energies?



the distribution of wave function amplitudes



$$|\psi_{N,k}\rangle = \sum_\mu C_\mu^{(k)} |\phi_{N,\mu}\rangle$$

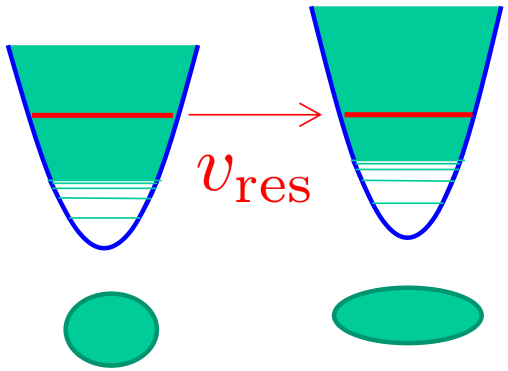
$$[H_0 + H_{\rm pair}]\,|\phi_{N,\mu}\rangle = E_\mu |\phi_{N,\mu}\rangle$$

$N = 2$: approximately Gaussian

$N = 6$: almost complete Gaussian

K. Hagino and K. Uzawa, in preparation.

Summary



- r-process nucleosynthesis: fission of neutron-rich nuclei

  requires a microscopic approach applicable to low E^* and $\rho(E^*)$

  ➡ a new approach: shell model + GCM

  an application to induced fission of ^{236}U based on Skyrme EDF

  → • the insensitive property
  • a small value of d.o.f.
  } ← the transition state theory

- energy dependence of fusion cross sections for ^{12}C+^{12,13}C

  ^{12}C+^{12}C : prominent resonance peaks
  ^{12}C+^{13}C : a much smoother energy dependence

  a new model with shell model calculations for the CN

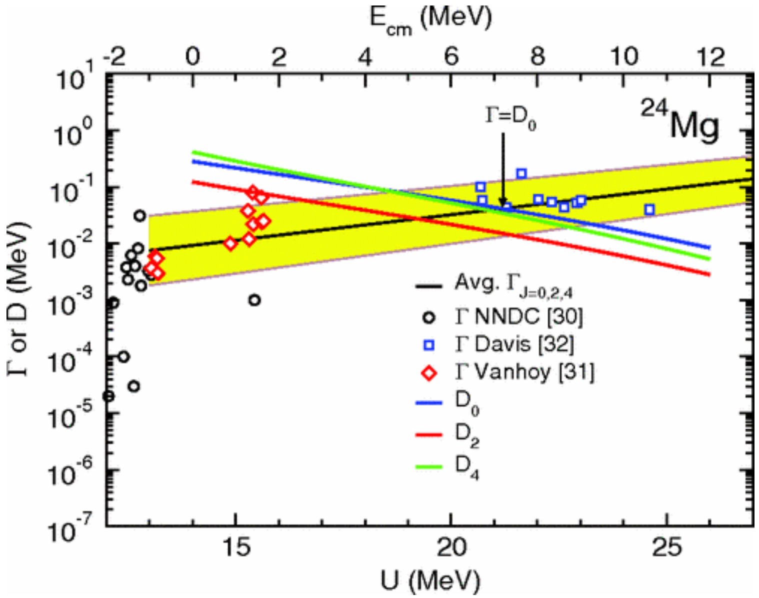
  future direction: 1-channel → coupled-channels

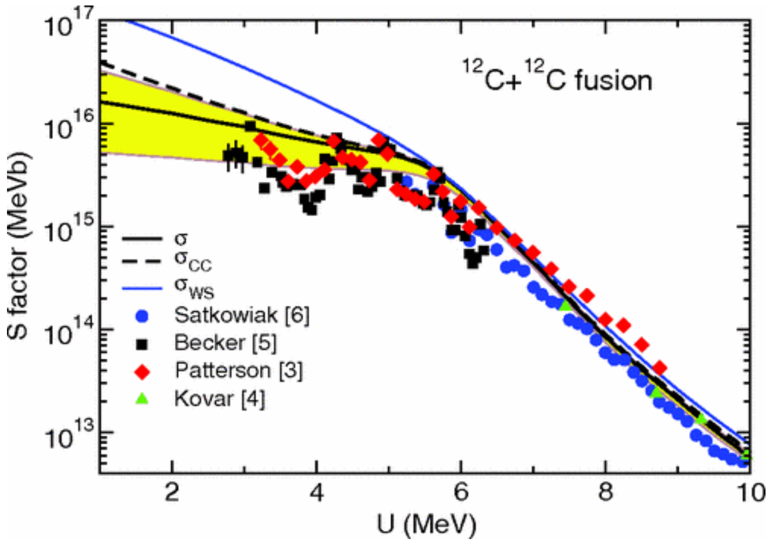
- the validity of the compound nucleus picture at low excitation energies

  ✓ recent measurements of beta-delayed neutron emissions
  ✓ the kurtosis from an uniform spacing model → consistent with kurtosis ~ 0 even for N=2

Effects of isolated CN resonances

C.L. Jiang et al., PRL110, 072701 (2013).





Moldauer factor

$$P_J = 1 - \exp(-2\pi\Gamma_J/D_J)$$

$$\rightarrow \sigma = \sum_J \sigma_{CC}^J P_J$$

E < 7 MeV

$\Gamma < D \rightarrow$ fusion inhibition

E > 7 MeV

$\Gamma > D \rightarrow P_J \sim 1$