Heavy-ion fusion reactions at deep sub-barrier energies

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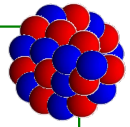
1. Heavy-ion fusion reactions at sub-barrier energies
2. Deep sub-barrier hindrance
3. Re-analysis of the $^{12}C + ^{12,13}C$ reactions
4. Energy depence of fusion cross sections
5. Summary

Italy-Japan Symposium on Joint Activities in Fundamental Physics, March 9-12, 2026, Kore University of Enna

Introduction: low-energy nuclear reactions

nucleus: a composite system

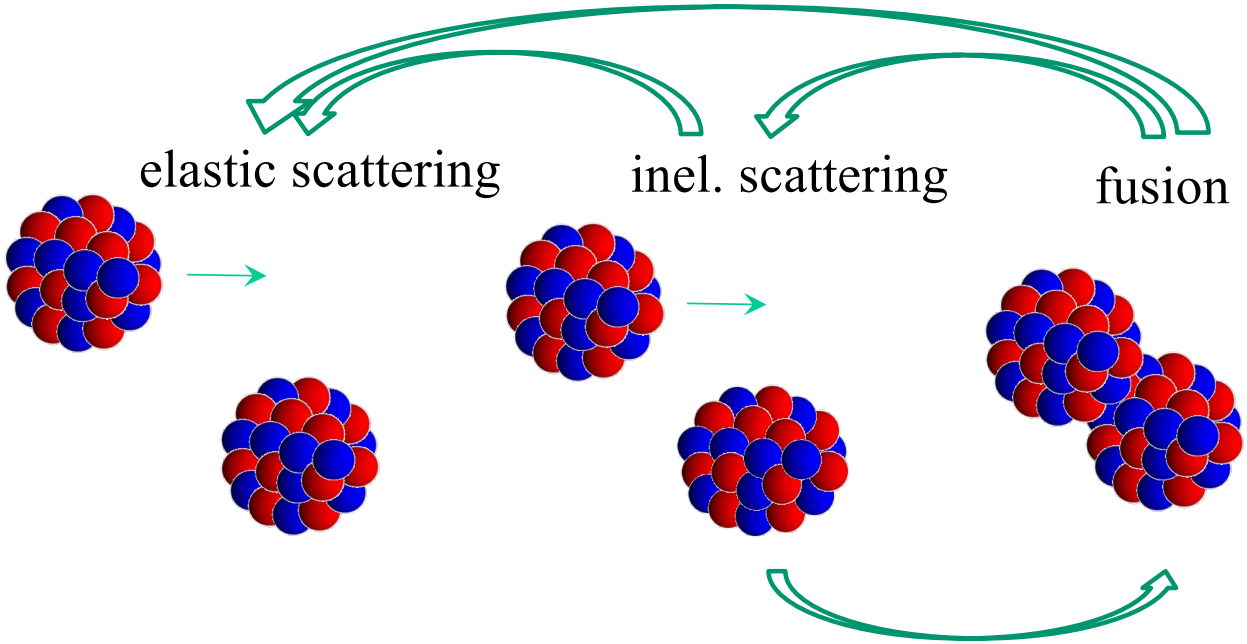


✓ various sort of reactions

✓ an interplay between nuclear structure and reaction

shapes, excitations, …..

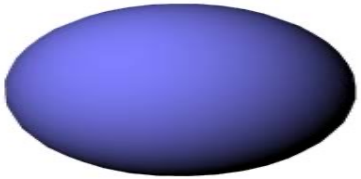
- elastic scattering
- inelastic scattering
- transfer reactions
- breakup reactions
- fusion reactions



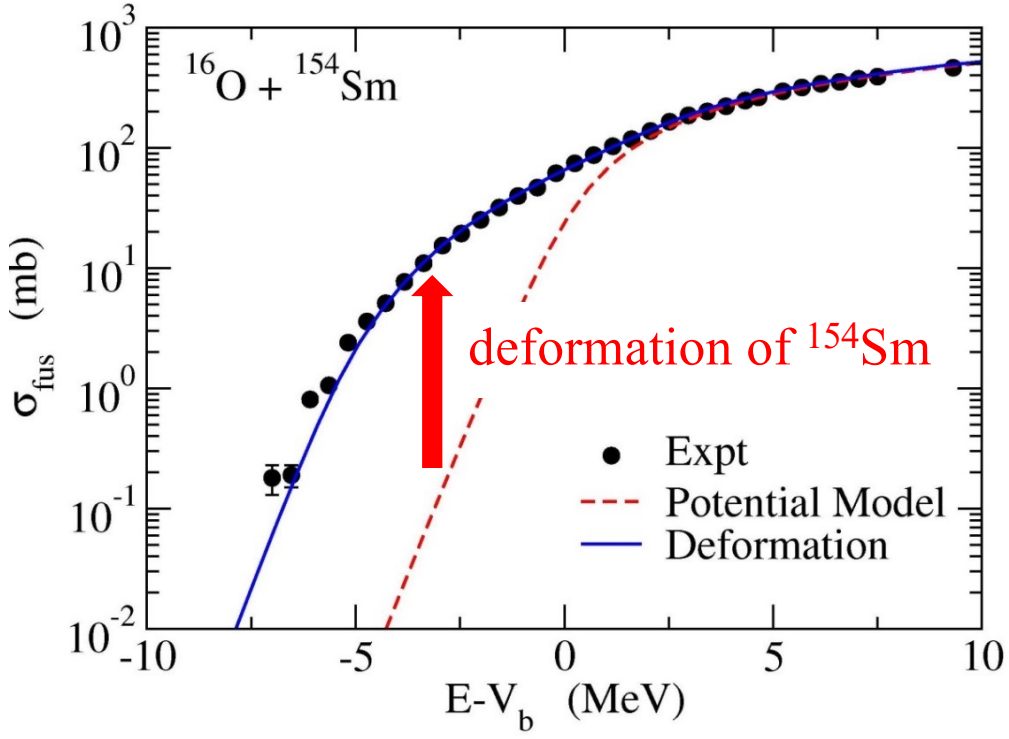
A good example of the interplay between nuclear structure and nuclear reaction

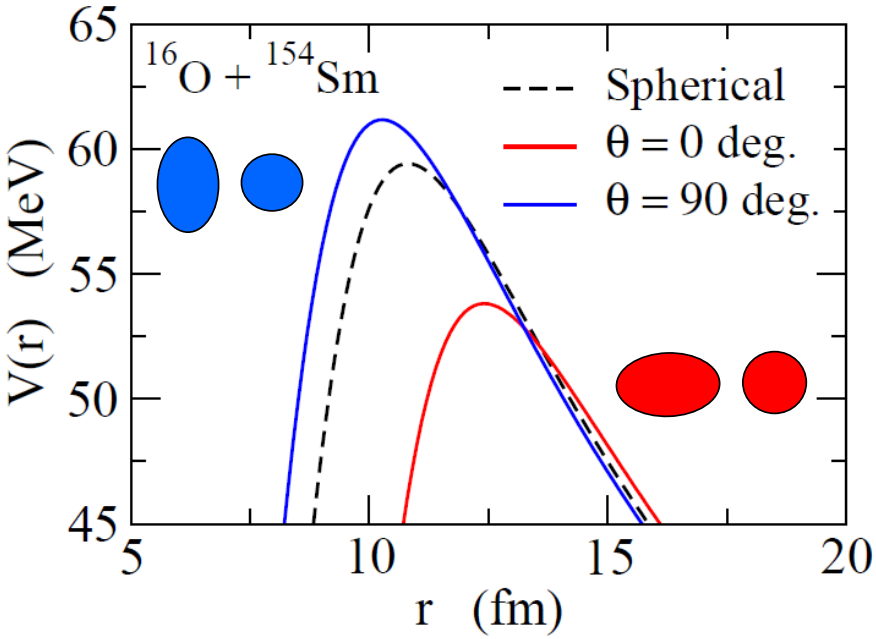
Sub-barrier enhancement of fusion cross sections

: a large impact of inelastic processes on fusion



^{154}Sm





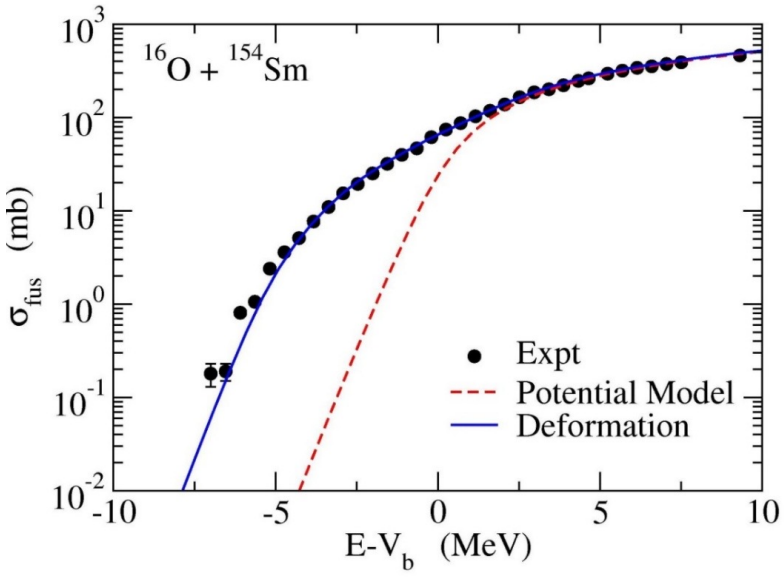
$$\sigma_{\mathrm{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\mathrm{fus}}(E;\theta)$$

K.Hagino and N. Takigawa, PTP128 (2012) 1061

G. Montagnoli and A.M. Stefanini,
EPJA53 (2017) 169

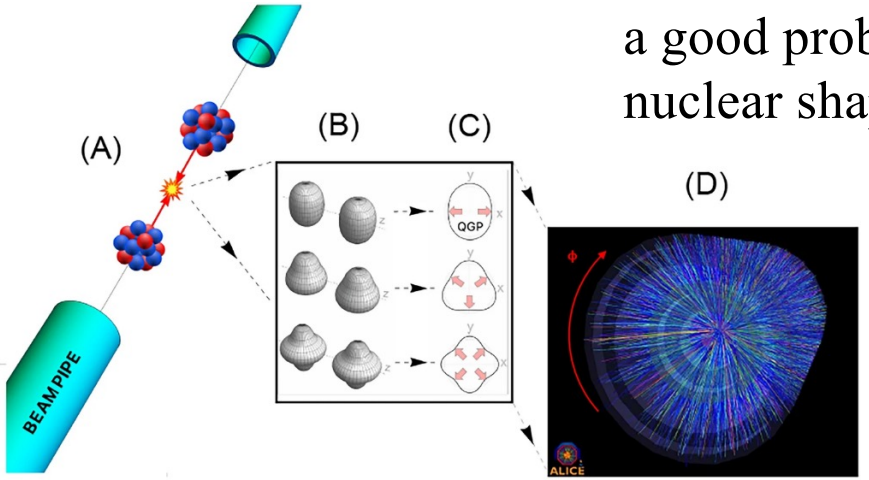
Taking a snapshot of a nucleus with a "fast" nuclear reaction

low-energy H.I. fusion reactions of a deformed nucleus



in recent years increasing interests in:

relativistic H.I. collisions with a deformed nucleus



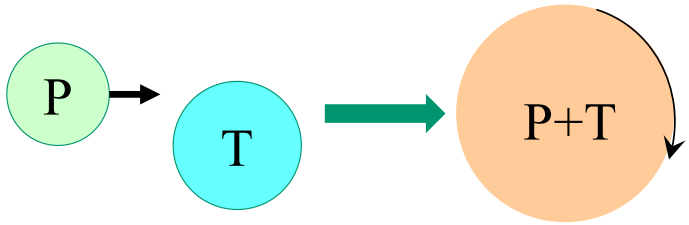
a good probe of nuclear shapes

J. Jia et al., Nucl. Sci. Tech. 35, 220 (2024)

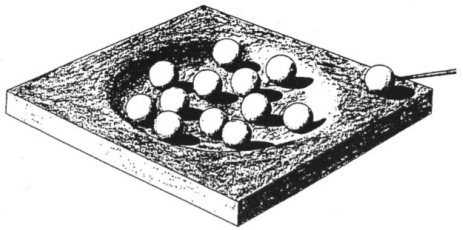
Large similarities → intersection of **High-*E*** H.I. collisions and **Low-*E*** nuclear reactions

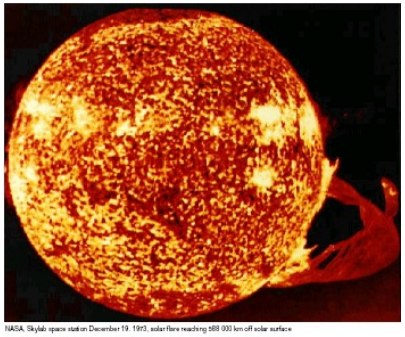
cf. K. Hagino and M. Kitazawa, Phys. Rev. C112, L041901 (2025).

Fusion reactions: compound nucleus formation

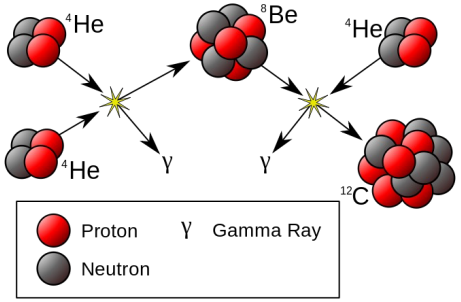


compound nucleus

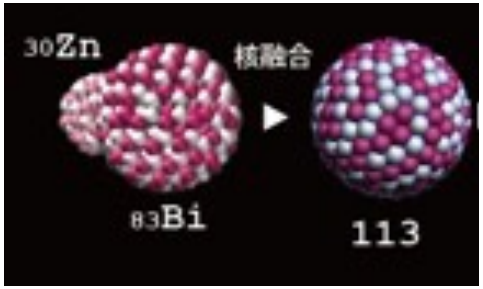




energy production in stars (Bethe '39)



nucleosynthesis



superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems

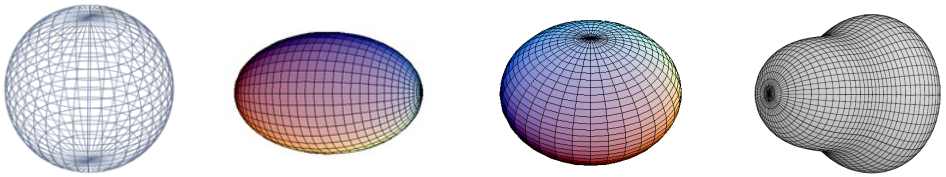
← microscopic understanding: an ultimate goal of nuclear physics

Sub-barrier fusion reactions and quantum tunneling

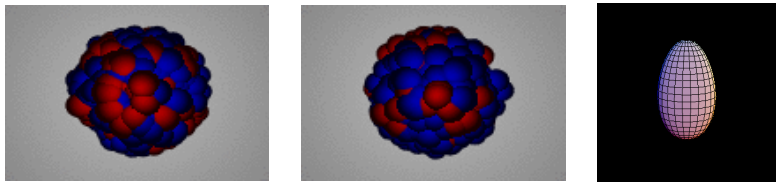
Fusion with quantum tunneling

with many degrees of freedom

- several nuclear shapes



- several surface vibrations

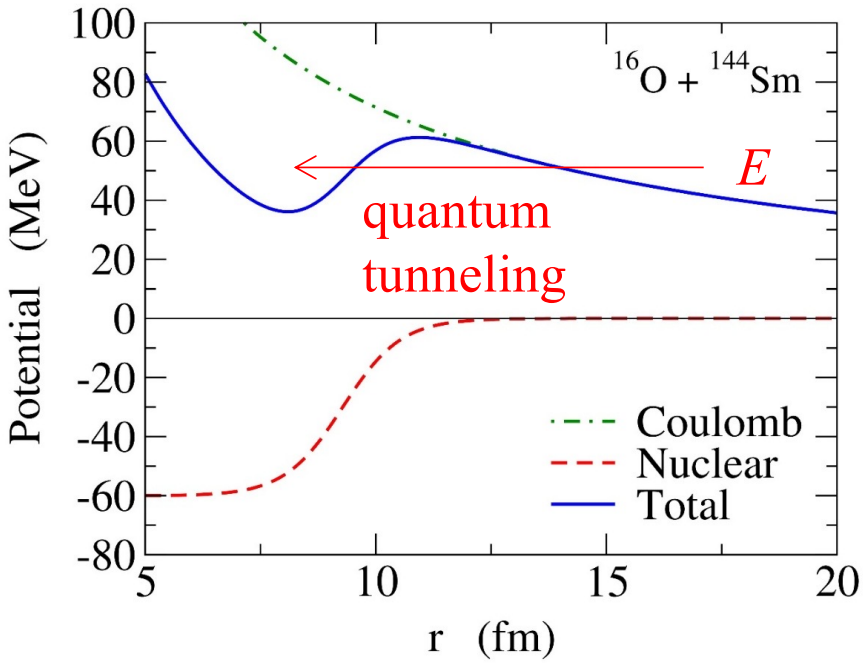


several modes and adiabaticities

- several types of nucleon transfers

Tunneling probabilities: the exponential E dependence
→ nuclear structure effects are amplified

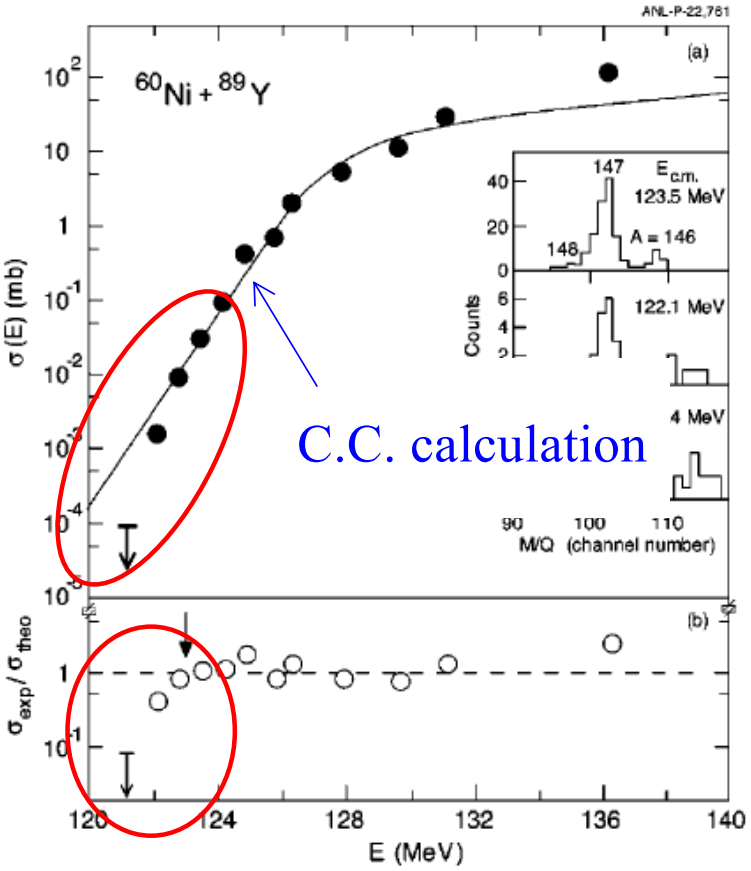
Sub-barrier fusion reactions



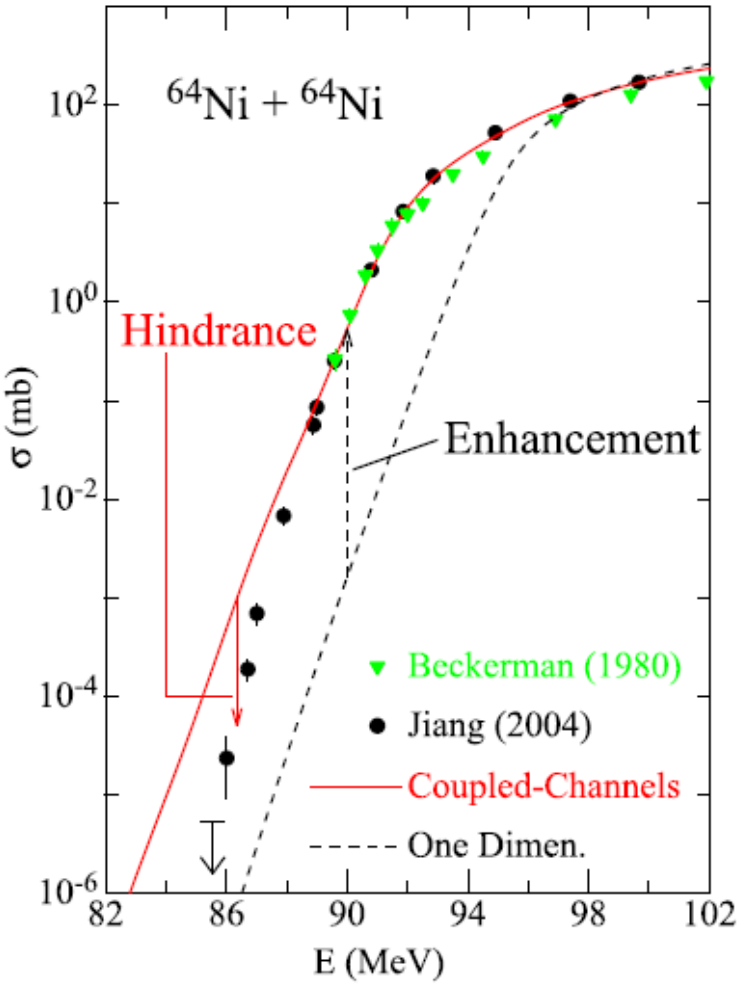
→ subbarrier enhancement
of fusion cross sections

Deep sub-barrier fusion hindrance

the first measurement at ANL

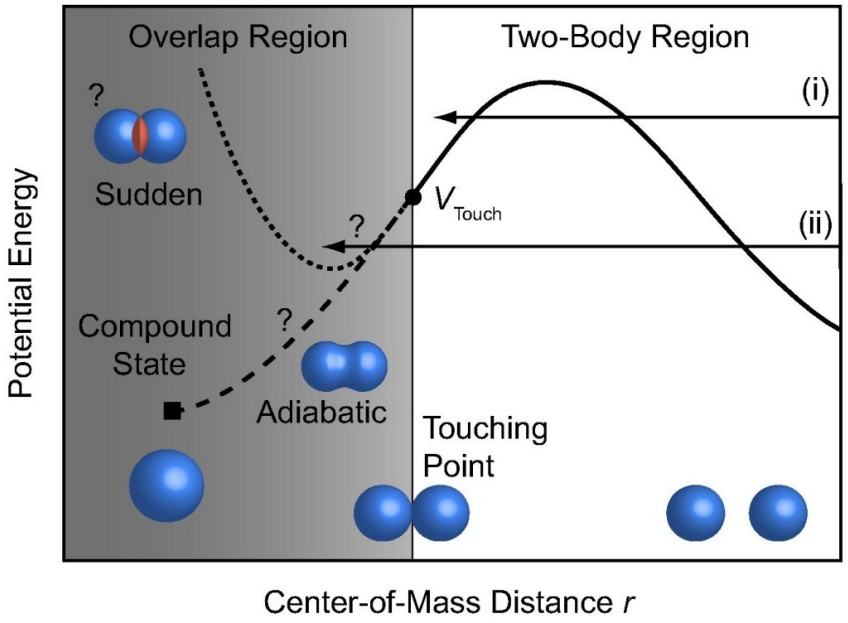


C.L. Jiang et al., PRL89 (2002) 052701



C.L. Jiang, B.B. Back, K.E. Rehm, K. Hagino, G. Montagnoli, and A.M. Stefanini, Eur. Phys. J. A57 (2021) 235

theoretical approaches



T. Ichikawa, K.H., A. Iwamoto,
PRC75('07) 064612 & 057603

dynamics after the touching

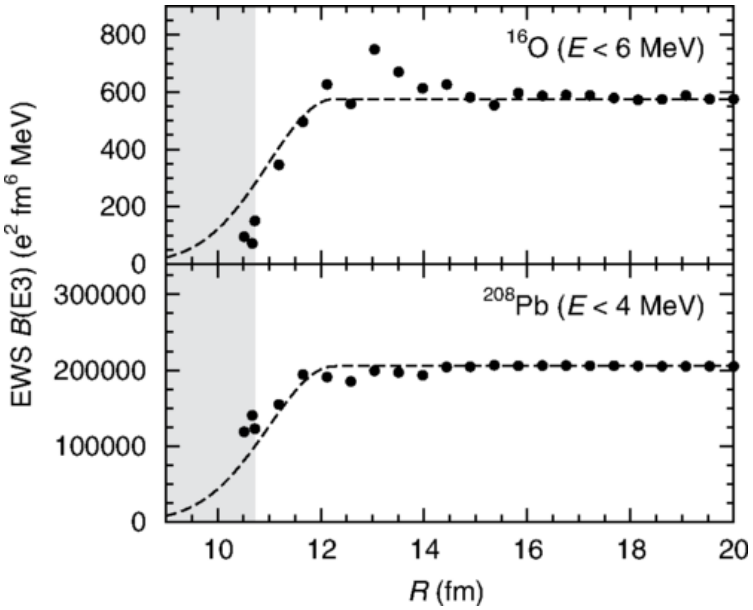
cf. fusion for superheavy nuclei

i) Sudden approach → a repulsive core

S. Misicu and H. Esbensen, PRC75, 034606 (2007)
C. Simenel et al., PRC95, 031601(R) (2017)

ii) Adiabatic approach → neck formation

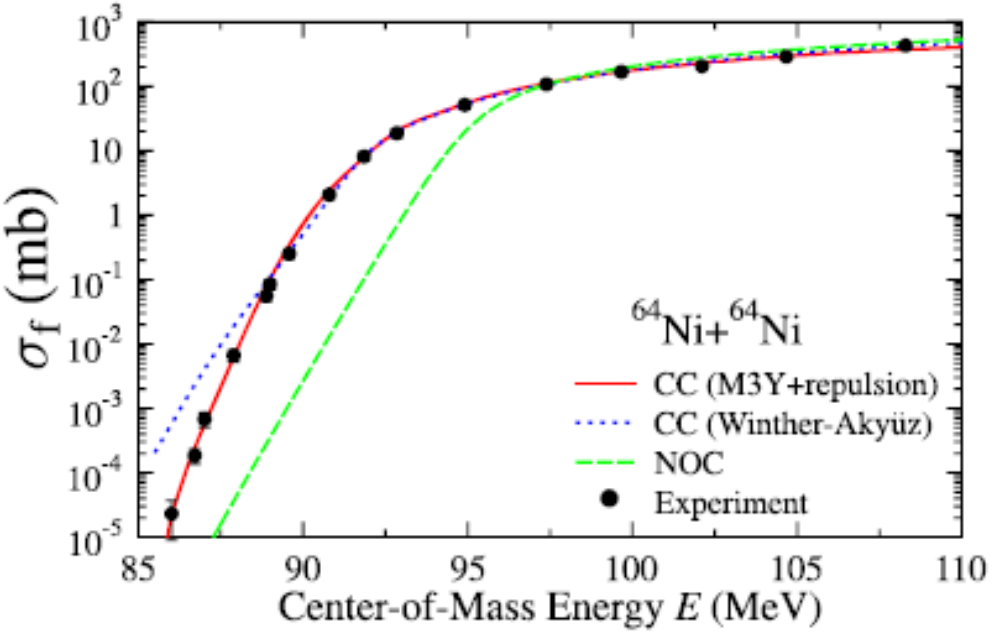
T. Ichikawa, K. Hagino, and A. Iwamoto,
PRL103, 202701 (2009).



T. Ichikawa and
K. Matsuyanagi,
PRC92, 021602(R)
(2015).

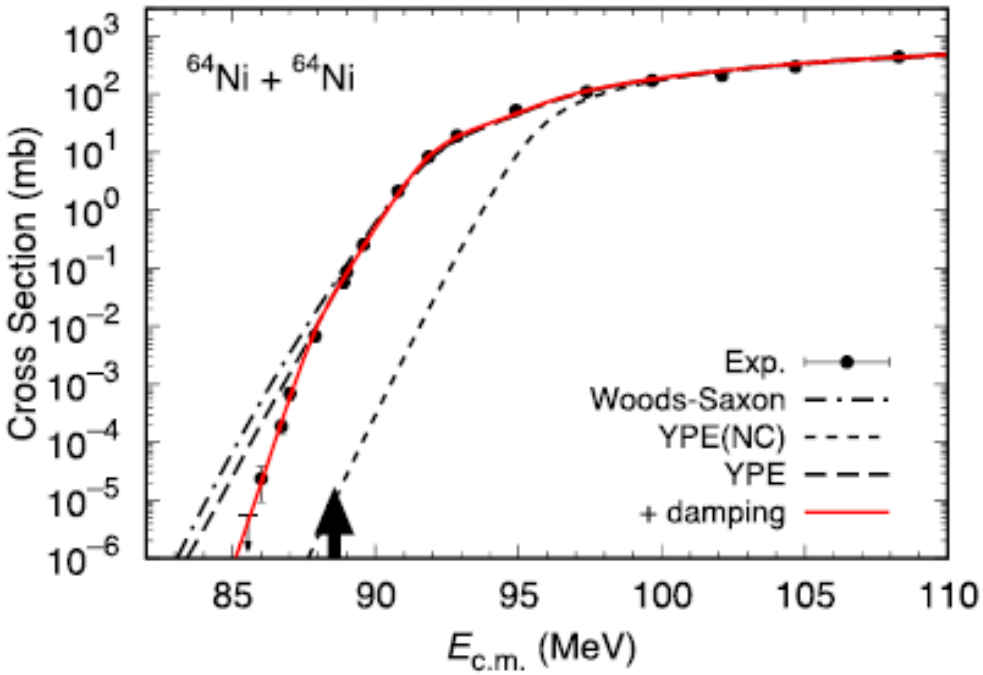
the sudden versus the adiabatic models: ^{64}Ni+^{64}Ni

the sudden model



S. Misicu and H. Esbensen,
PRL96 (‘06) 112701, PRC75 (‘07) 034606

the adiabatic model



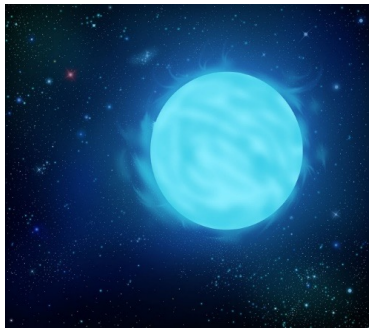
T. Ichikawa, PRC92 (‘15) 064606

Both models reproduce the data equally well.

Fusion hindrance in medium-light and light systems

To what extend does the fusion hindrance affect nuclear astrophysics?

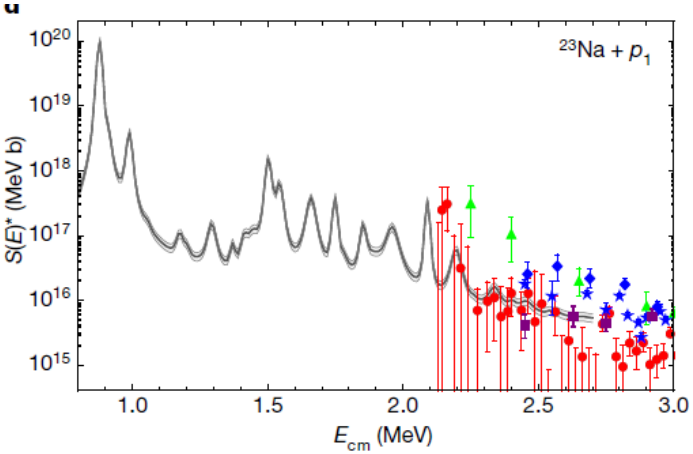
e.g., $^{12}C+^{12}C$ fusion : a key reaction in nuclear astrophysics



- ✓ Carbon burning in massive stars

  $^{12}C+^{12}C \rightarrow \alpha + ^{20}Ne$

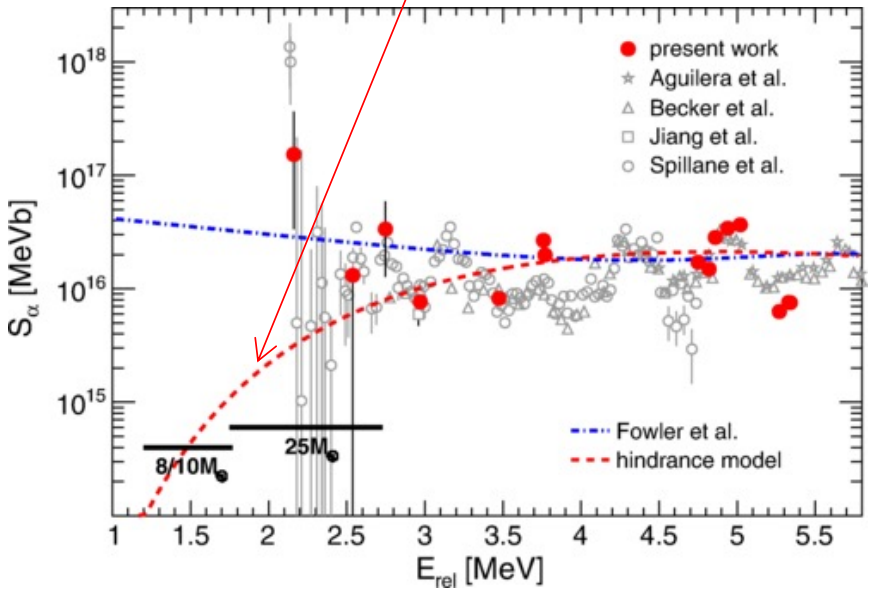
  $^{12}C+^{12}C \rightarrow p + ^{23}Na$
- ✓ Type Ia supernovae
- ✓ X-ray superburst



Trojan Horse Method
A. Tumino et al.,
Nature557 (2018) 687

hindrance model
(Jiang et al.)

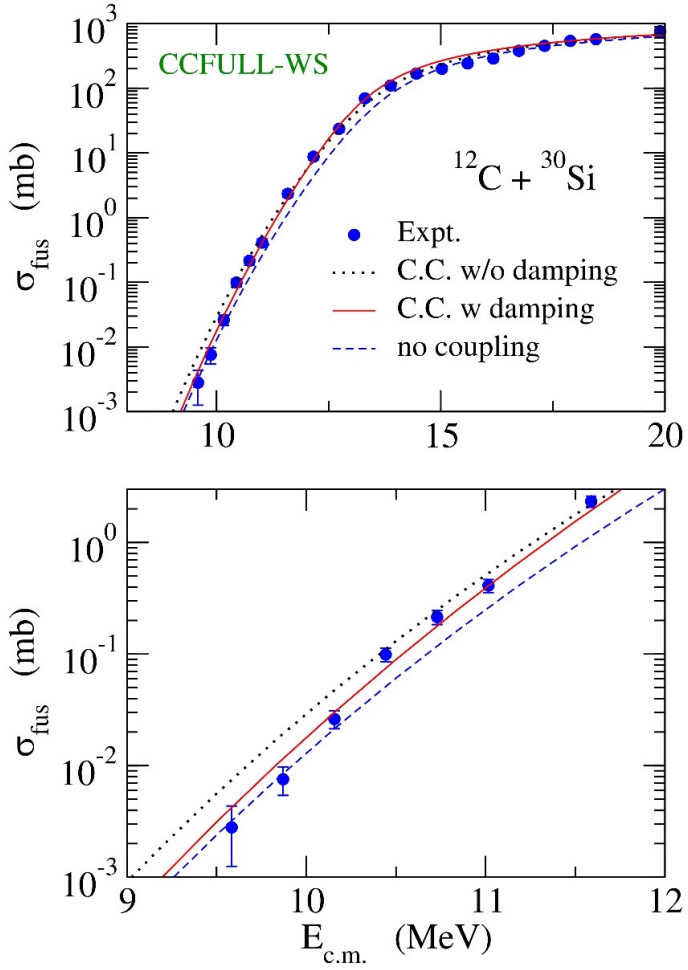
a huge impact
if this was true



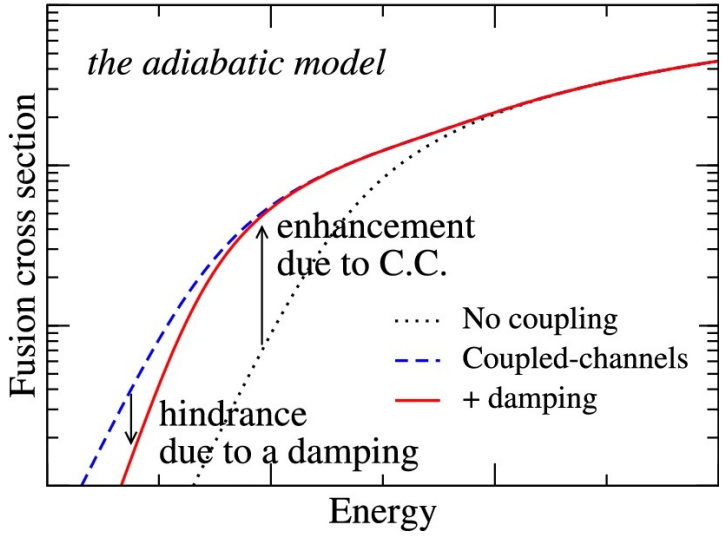
G. Fruet et al., PRL124, 192701 (2020)
← direct measurement

Fusion hindrance in medium-light and light systems

a very small hindrance has been observed in medium-light systems



the adiabatic model: a natural explanation



light systems: small enhancement → small hindrance

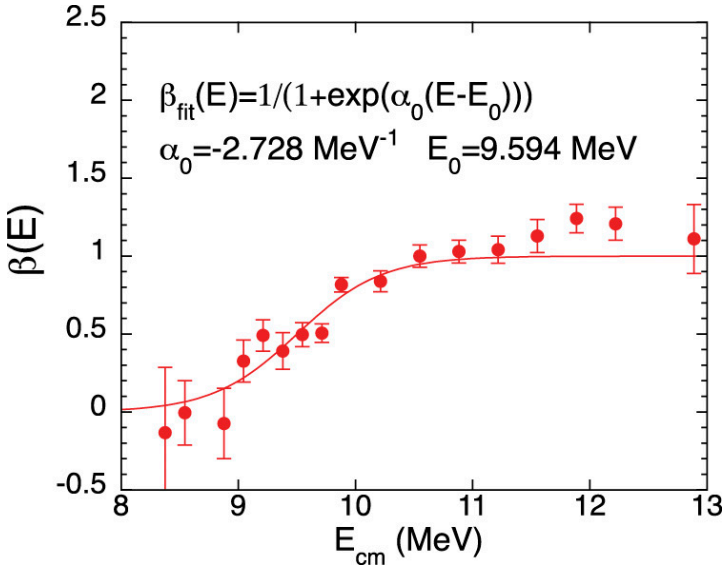
G. Montagnoli, A.M. Stefanini, C.L. Jiang,
K. Hagino et al., PRC97, 024610 (2018).

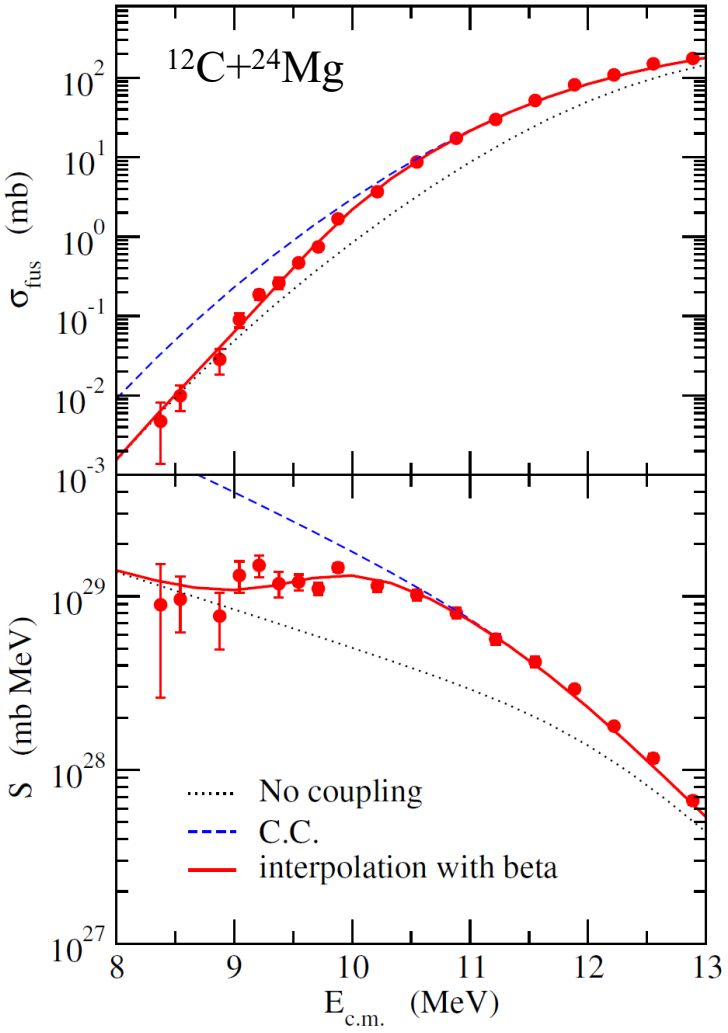
A simplified adiabatic model

a simple interpolation:

$$\ln(\sigma(E)) \sim \beta(E)\ln(\sigma_{cc}(E)) + (1-\beta(E))\ln(\sigma_{noc}(E))$$

$$\beta(E) = \frac{1}{1+e^{\alpha_0(E-E_0)}}$$

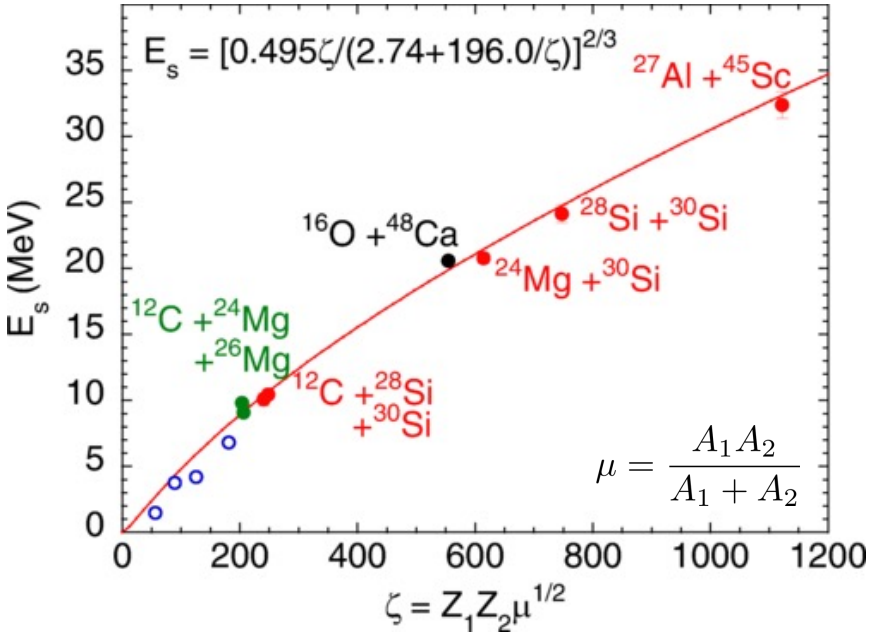




G. Montagnoli, A.M. Stefanini, C.L. Jiang, K. Hagino et al., J. of Phys. G49, 095101 (2022).

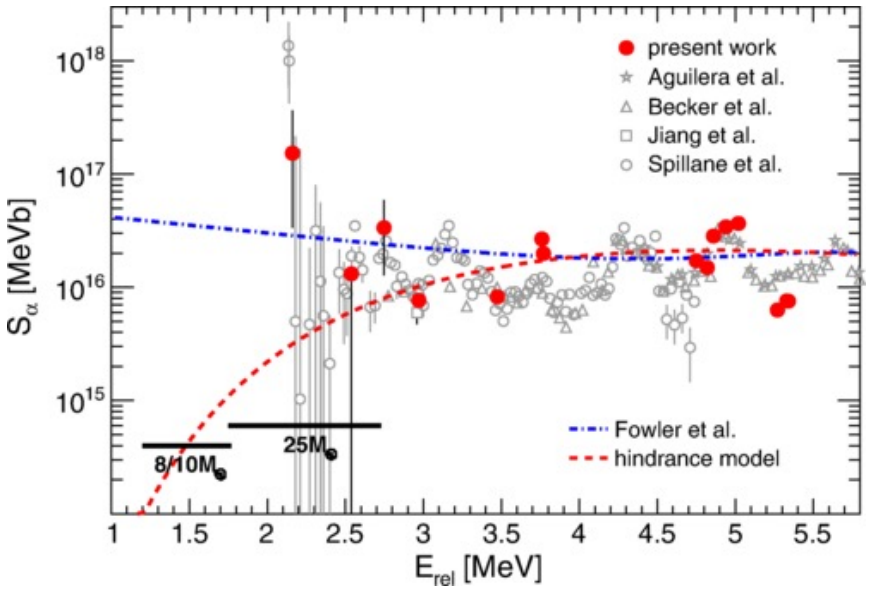
Fusion hindrance in medium-light and light systems

systematics: threshold energy for hindrance



A.M. Stefanini, G. Montagnoli et al.,
PRC111, 064620 (2025)
PLB872, 140084 (2026)

→ ^{12}C+^{12}C: E_s = 4.26 MeV



E_s seems to agree with Jiang et al.
….but how about the extrapolation
of cross sections?

Fusion hindrance in $^{12}C+^{12,13}C$ systems

C.L. Jiang et al., PRC75, 015803 (2007)

$$L(E) = \frac{d}{dE}\ln(E\sigma)$$

fit with: $L(E) = A_0 + B_0/E^n$ (n=1.5)

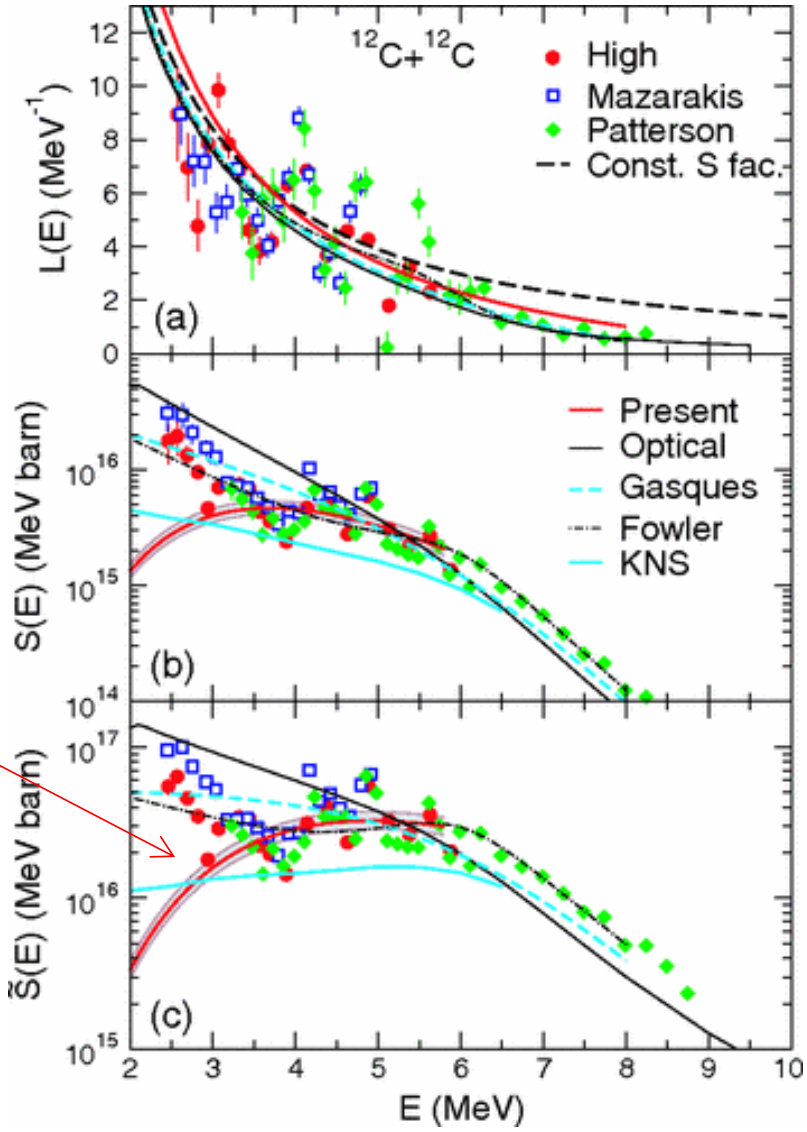
→ a large hindrance

(note)

$$\sigma(E) = \frac{S(E)}{E}e^{-2\pi\eta}$$

for S = const. → $L(E) \propto \frac{d\eta}{dE} \propto E^{-3/2}$

S can deviate from a constant.
→ Re-fit with a free *n* ?



Fusion hindrance in $^{12}C+^{12,13}C$ systems

K. Uzawa and K. Hagino, PRC112, L061601 (2025)

$$L(E) = \frac{d}{dE}\ln(E\sigma)$$

fit with: $L(E) = A_0 + B_0/E^n$ (n=1.5 → free)

the ordinary procedure (weighted χ^2 fit):

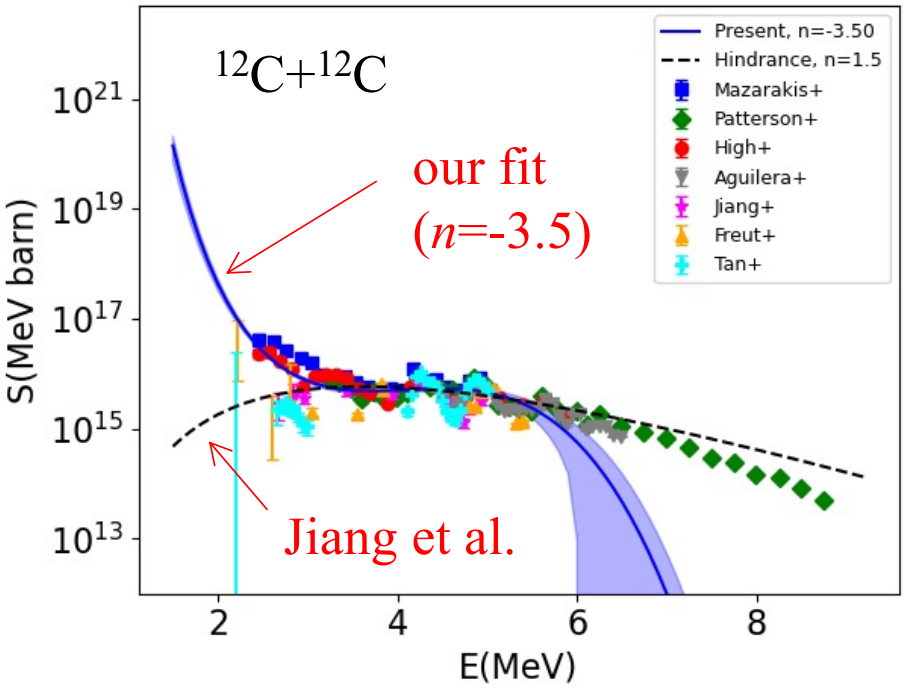
$$L(E) \rightarrow \sigma(E) \rightarrow S(E)$$

$$\chi^2_w \equiv \sum_i \left(\frac{S_i - S(E_i)}{\Delta S_i}\right)^2$$

→ a little emphasis on low-E data (large $\Delta S_i/S_i$)

we take: $$\chi^2_o \equiv \sum_i \left(\frac{S_i - S(E_i)}{\Delta S_0}\right)^2$$

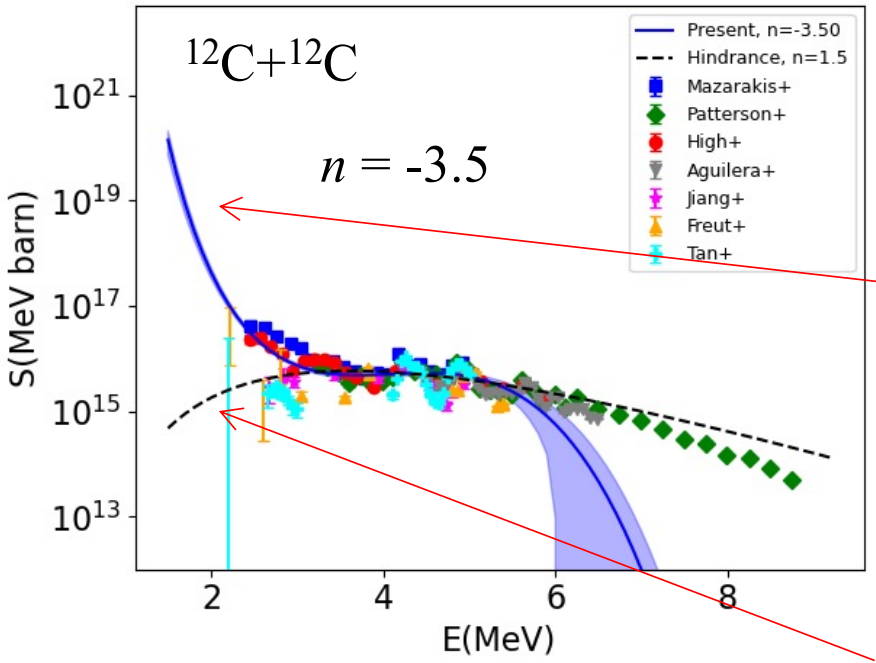
ΔS_0: a constant



our fit → no hindrance!

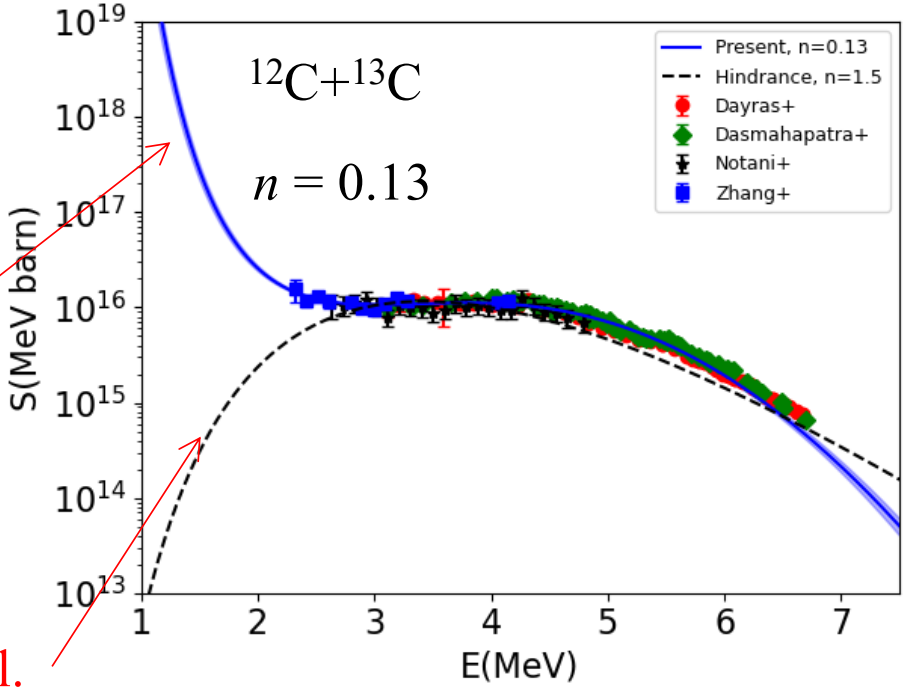
Fusion hindrance in $^{12}C+^{12,13}C$ systems

K. Uzawa and K. Hagino, PRC112, L061601 (2025)



our fit

Jiang et al.



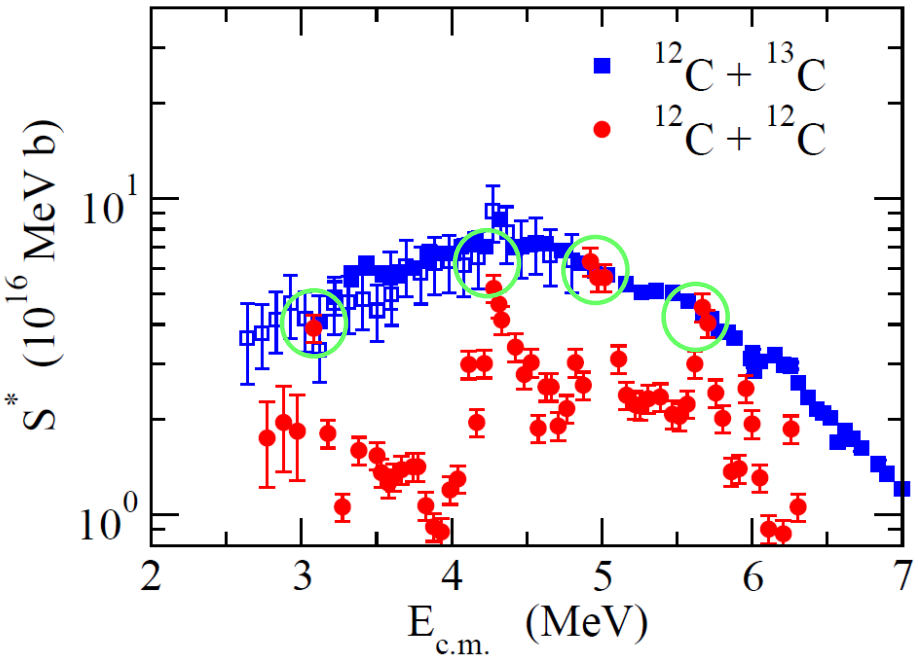
our fit → no hindrance!

the same conclusion with polynomial fits:

$$\ln(S(E)) = \sum_{m=0}^{6} a_m E^m$$

Energy dependence of fusion cross sections for the $^{12}C+^{12,13}C$ systems

$$S^*(E) = E\sigma(E)\, e^{2\pi(\eta-\eta_0)}$$



prominent resonance structure in $^{12}C+^{12}C$ fusion

- ✓ off-resonance: fusion inhibition
- ✓ on-resonance: match with $^{12}C+^{13}C$

cf. the next talk by Y. Taniguchi

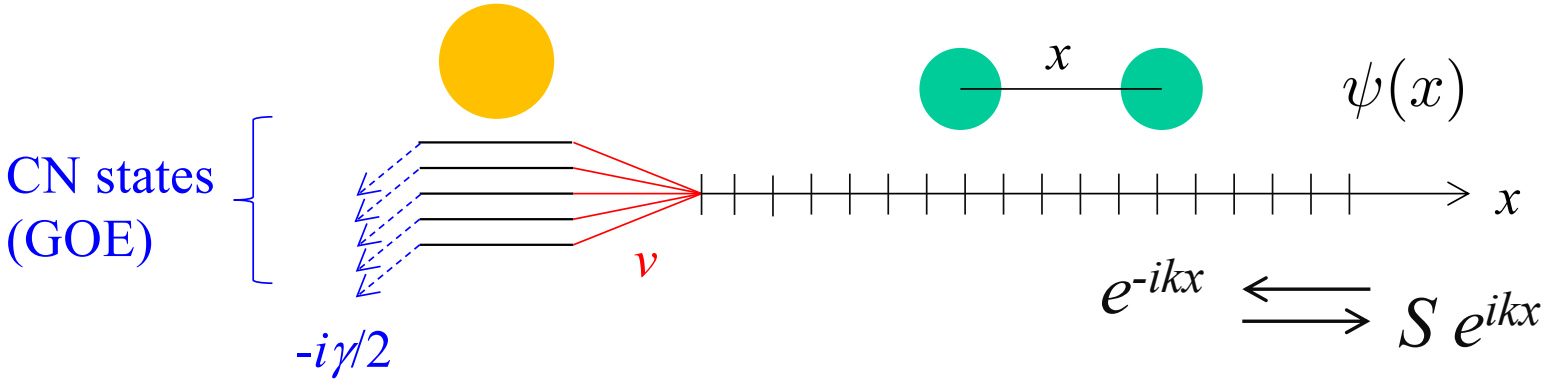
M. Notani, X.D. Tang et al.,
PRC85 (2012) 014607

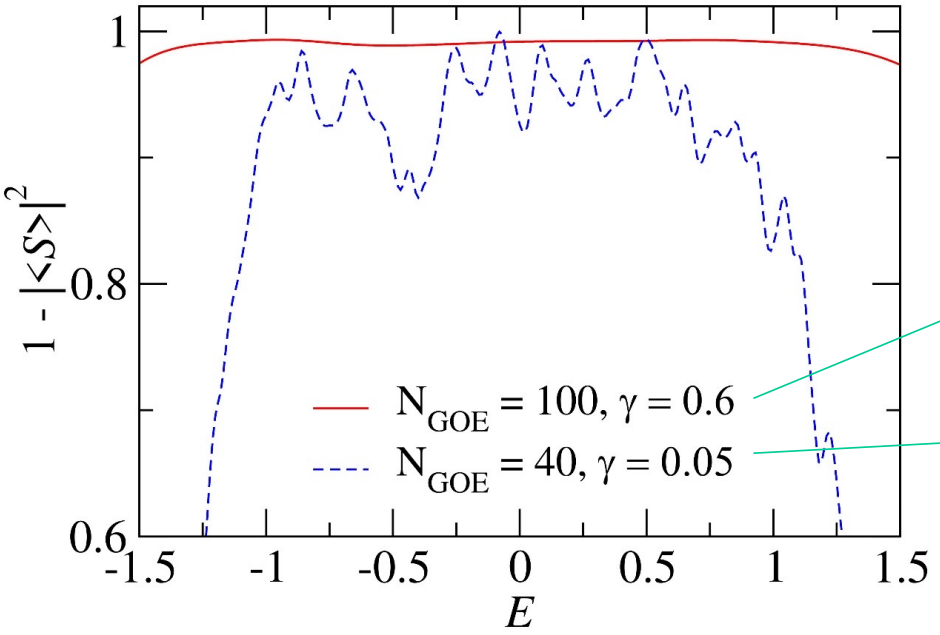
An open issue:

Is it possible to construct a model which explain both the systems?

cf. C.L. Jiang et al., PRL110, 072701 (2013): Γ/D of the compound nucleus

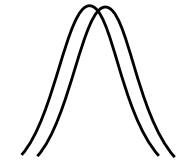
[a schematic model with random matrix]



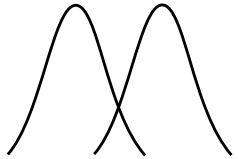


$S \sim 0$: strong absorption

$\gamma/d = 20$



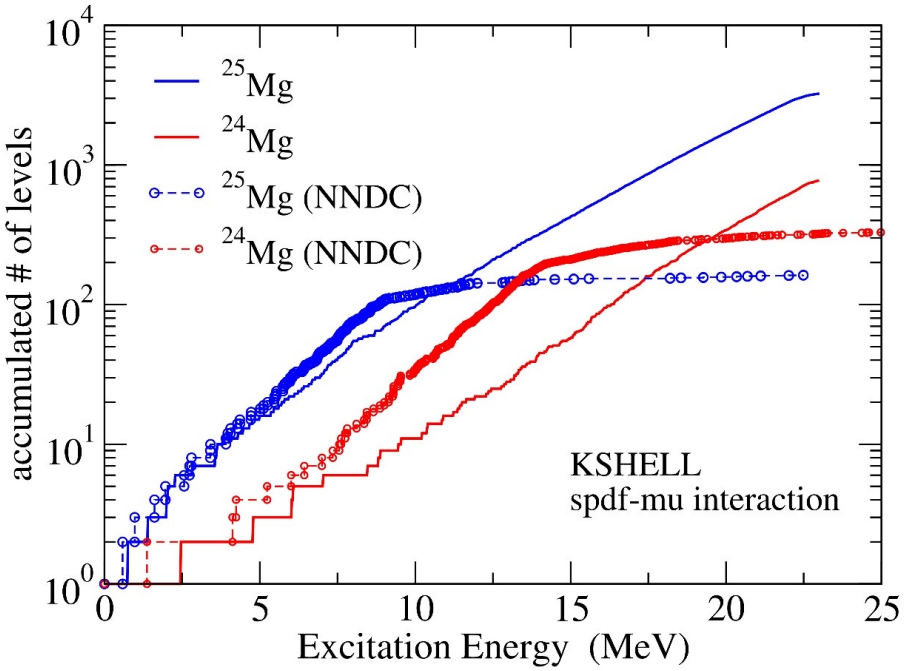
$\gamma/d = 1$

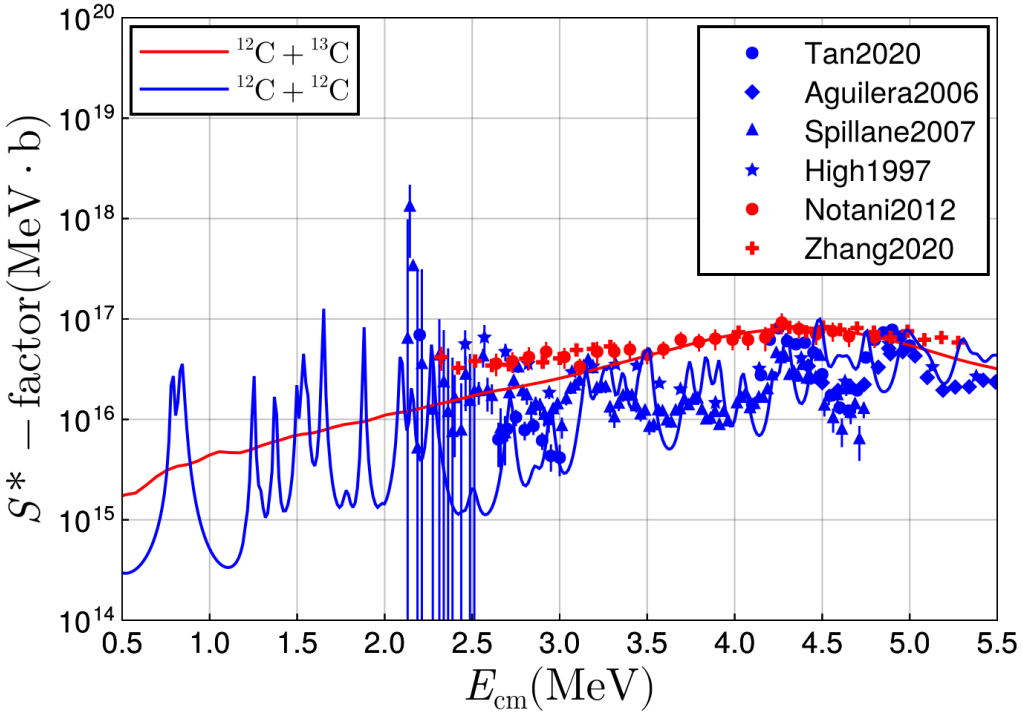


K.Hagino, PRC112, 034611 (2025).

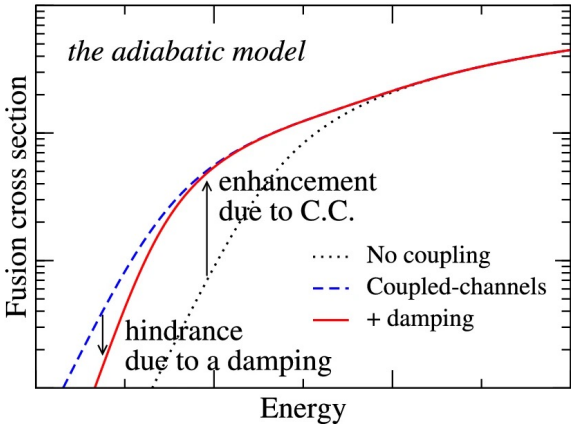
a more realistic model with shell model K. Nagao, K. Hagino, and K. Uzawa, in preparation.

- ✓ the spectrum of CN states: RMT → shell model (KSHELL)
- ✓ decay widths → statistical model
- ✓ 3D calculations with the Coulomb barrier





Summary



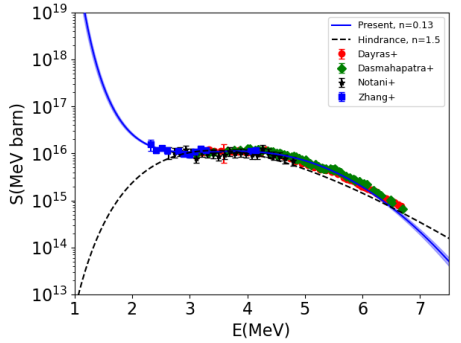
GIAPPONE E ITALIA
160 ANNI E OLTRE!
AMBASCIATA DEL GIAPPONE

Heavy-ion fusion reactions

- sub-barrier enhancement of cross sections
- deep sub-barrier hindrance

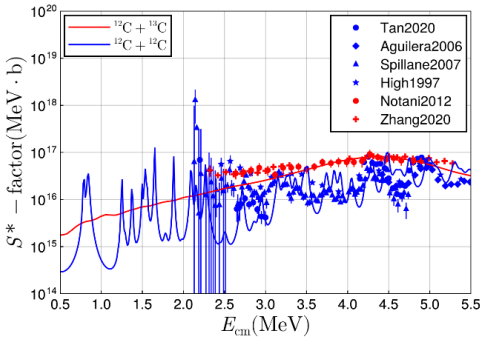
influence on nuclear astrophysical reactions?

- experimental data for medium-light systems (LNL)
  → only small hindrance
- previous fit by Jiang et al. for $^{12}C+^{12,13}C$ → a large hindrance
- our re-analysis → no hindrance!



energy dependence of fusion cross sections for $^{12}C+^{12,13}C$

$^{12}C+^{12}C$: prominent resonance peaks
$^{12}C+^{13}C$: a much smoother energy dependence



a new model with shell model calculations for the CN

future direction: 1-channel → coupled-channels