Fine structure and hindrance in the $^{12}C+^{12}C$ fusion

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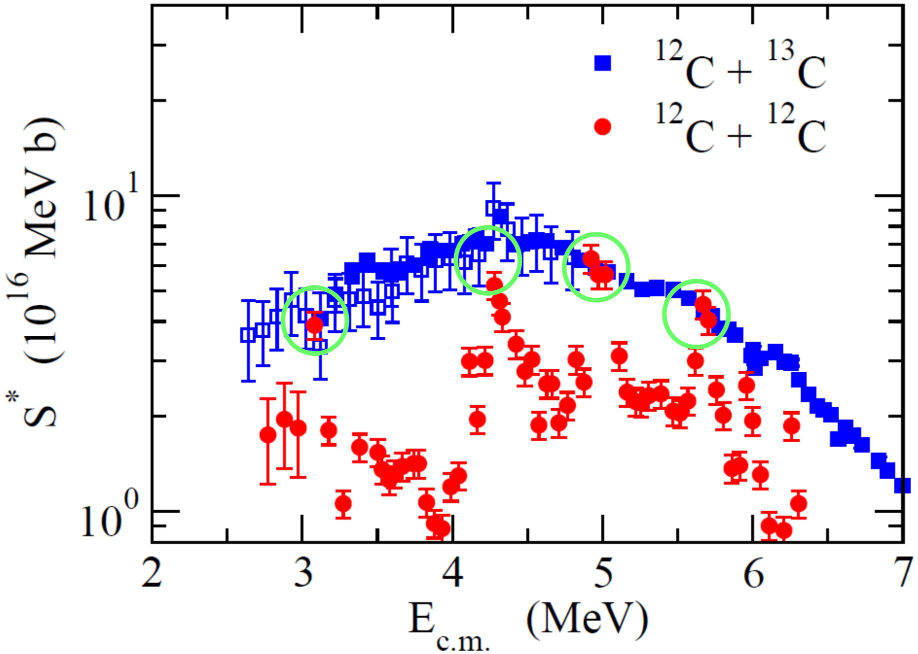
1. Introduction: fine structure and fusion hindrance
2. Deep sub-barrier hindrance
3. Re-analysis of the $^{12}C + ^{12,13}C$ reactions
4. Energy dependence of fusion cross sections
5. Summary

An informal workshop on $^{12}C+^{12}C$ fusion reaction, May 19, 2026, Kyoto University

Introduction: fine structure and fusion hindrance in $^{12}C+^{12}C$ fusion

Fusion cross sections of the $^{12}C+^{12,13}C$ systems

$$S^*(E) = E\sigma(E)\, e^{2\pi(\eta-\eta_0)}$$



prominent resonance structure in $^{12}C+^{12}C$ fusion

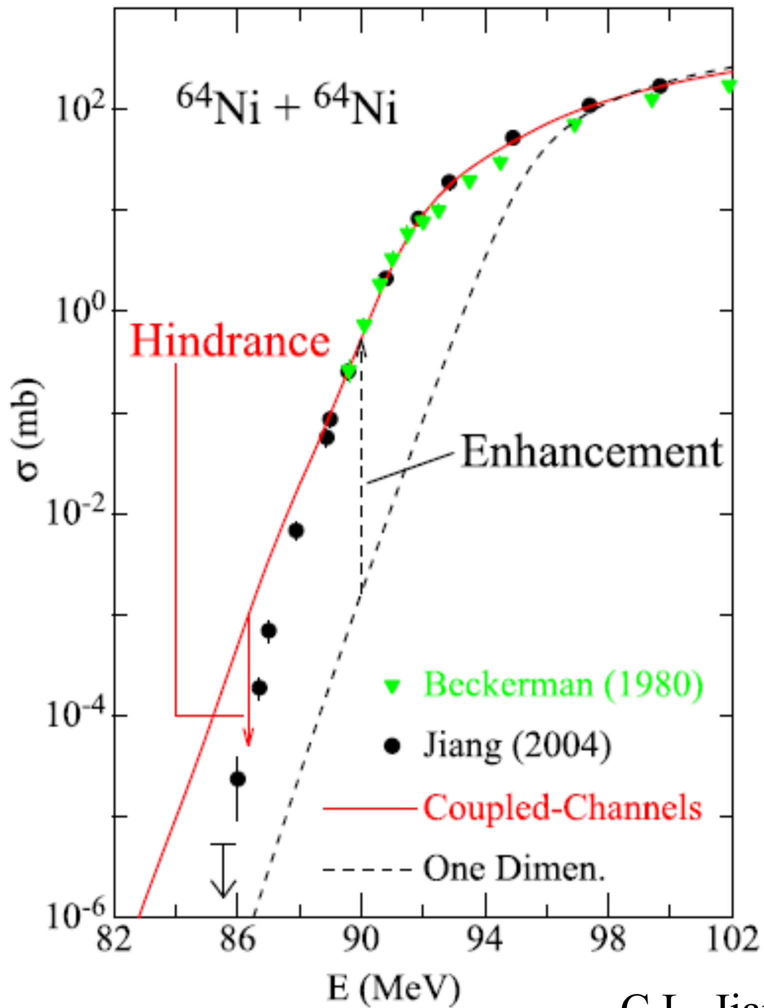
- ✓ off-resonance: fusion inhibition
- ✓ on-resonance: match with $^{12}C+^{13}C$

M. Notani, X.D. Tang et al.,
PRC85 (2012) 014607

Open issues:

- ✓ How can one understand the difference in these two systems?
- ✓ Does $^{12}C+^{13}C$ provide the upper limit of $^{12}C+^{12}C$ cross sections?

Introduction: fine structure and fusion hindrance in $^{12}C + ^{12}C$ fusion

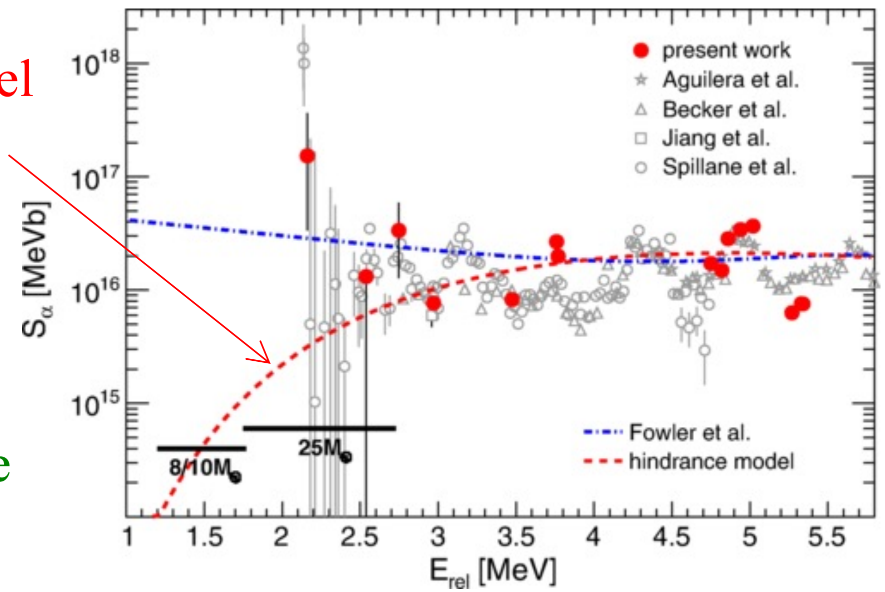


C.L. Jiang et al., Eur. Phys. J. A57 (2021) 235

To what extend does the fusion hindrance affect nuclear astrophysics?

hindrance model (Jiang et al.)

a huge impact if this was true

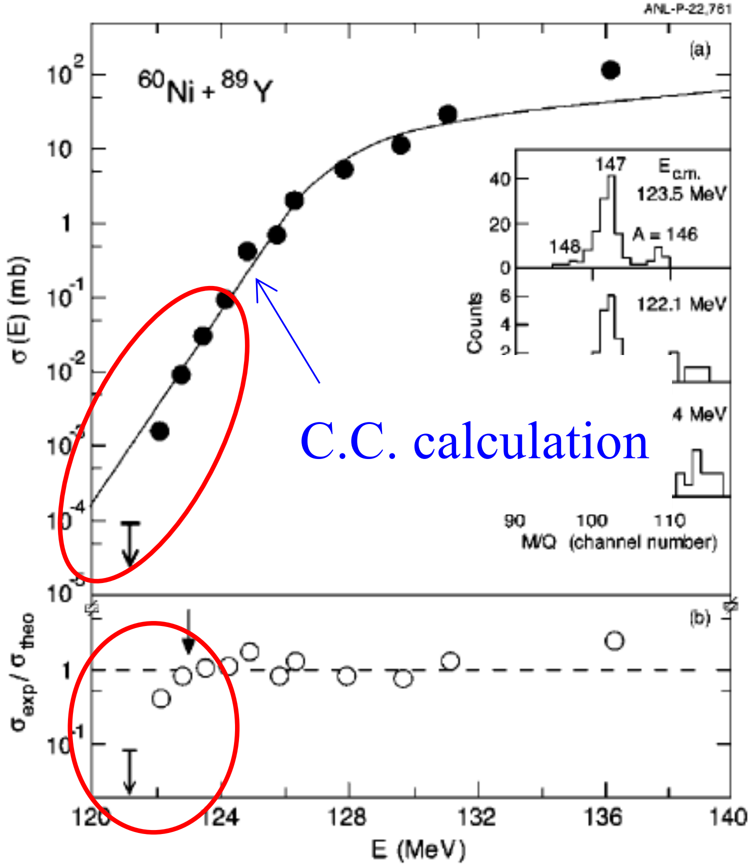


G. Fruet et al., PRL124, 192701 (2020)

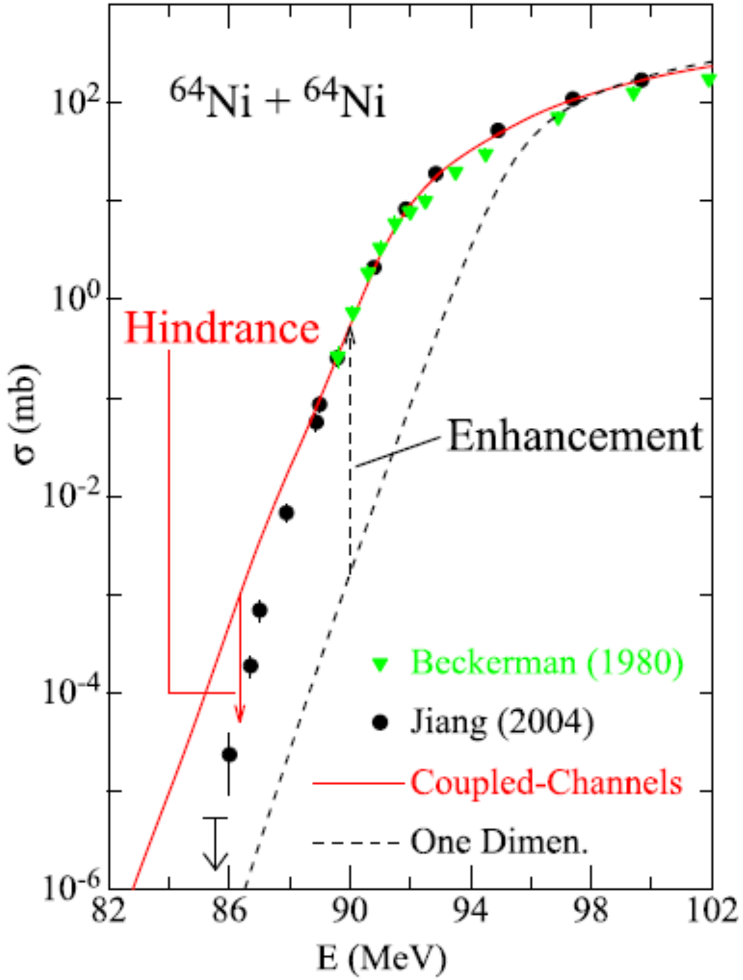
(i) Fusion hindrance

Deep sub-barrier fusion hindrance

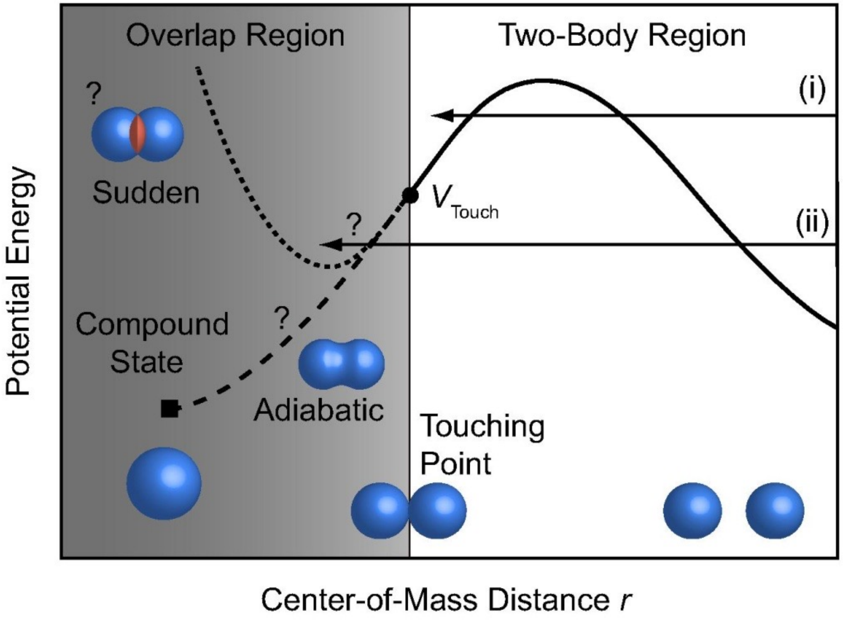
the first measurement at ANL



C.L. Jiang et al., PRL89 (2002) 052701



C.L. Jiang, B.B. Back, K.E. Rehm, K. Hagino, G. Montagnoli, and A.M. Stefanini, Eur. Phys. J. A57 (2021) 235 theoretical approaches



T. Ichikawa, K.H., A. Iwamoto,
PRC75('07) 064612 & 057603

dynamics after the touching

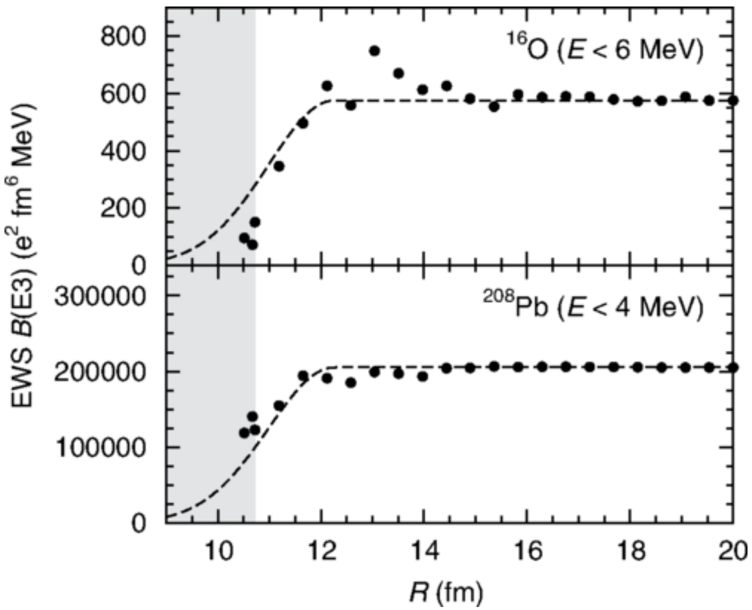
cf. fusion for superheavy nuclei

i) Sudden approach → a repulsive core

S. Misicu and H. Esbensen, PRC75, 034606 (2007)
C. Simenel et al., PRC95, 031601(R) (2017)

ii) Adiabatic approach → neck formation

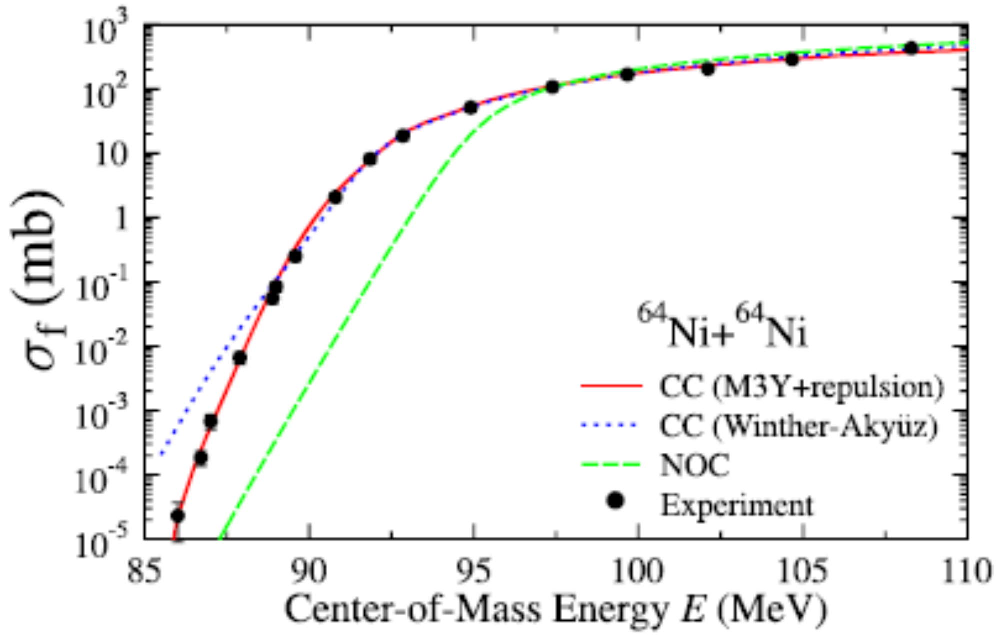
T. Ichikawa, K. Hagino, and A. Iwamoto,
PRL103, 202701 (2009).



T. Ichikawa and K. Matsuyanagi, PRC92, 021602(R) (2015).

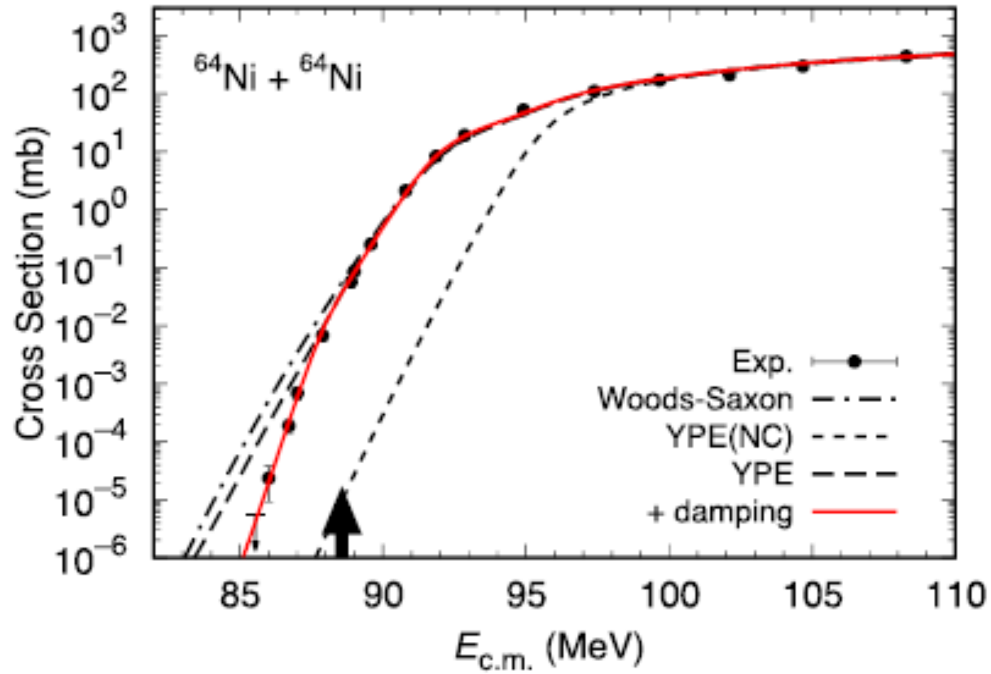
the sudden versus the adiabatic models: ^{64}Ni+^{64}Ni

the sudden model



S. Misicu and H. Esbensen,
PRL96 (‘06) 112701, PRC75 (‘07) 034606

the adiabatic model



T. Ichikawa, PRC92 (‘15) 064606

Both models reproduce the data equally well.

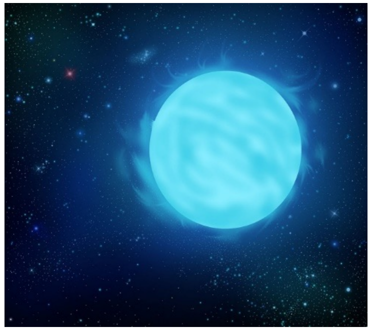
Fusion hindrance in medium-light and light systems

To what extend does the fusion hindrance affect nuclear astrophysics?

hindrance model (Jiang et al.)

a huge impact if this was true

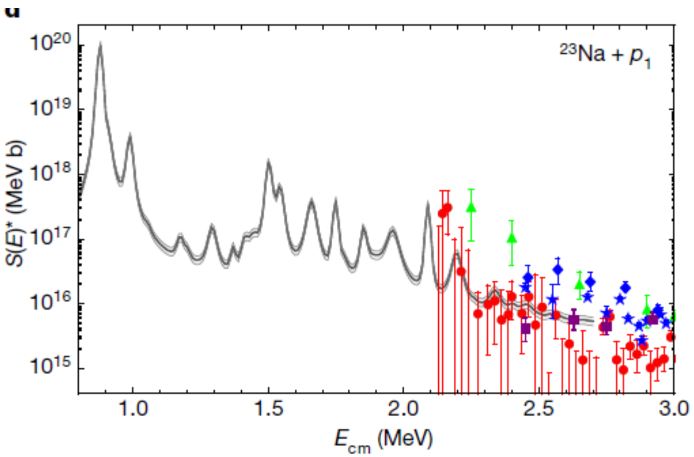
e.g., $^{12}C+^{12}C$ fusion : a key reaction in nuclear astrophysics



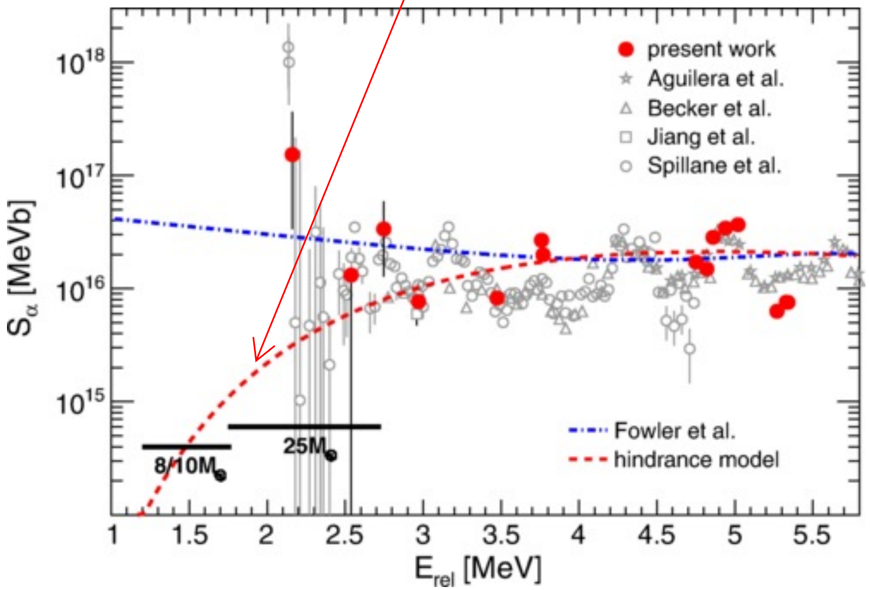
- ✓ Carbon burning in massive stars

$$^{12}C+^{12}C \rightarrow \alpha + ^{20}Ne$$
$$^{12}C+^{12}C \rightarrow p + ^{23}Na$$

- ✓ Type Ia supernovae
- ✓ X-ray superburst



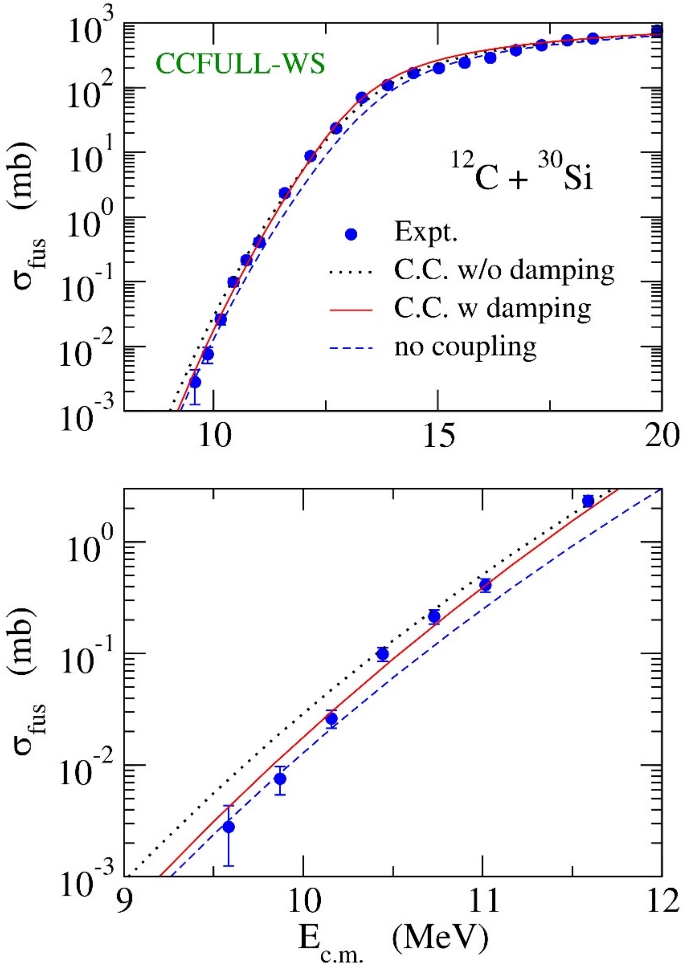
Trojan Horse Method
A. Tumino et al., Nature557 (2018) 687



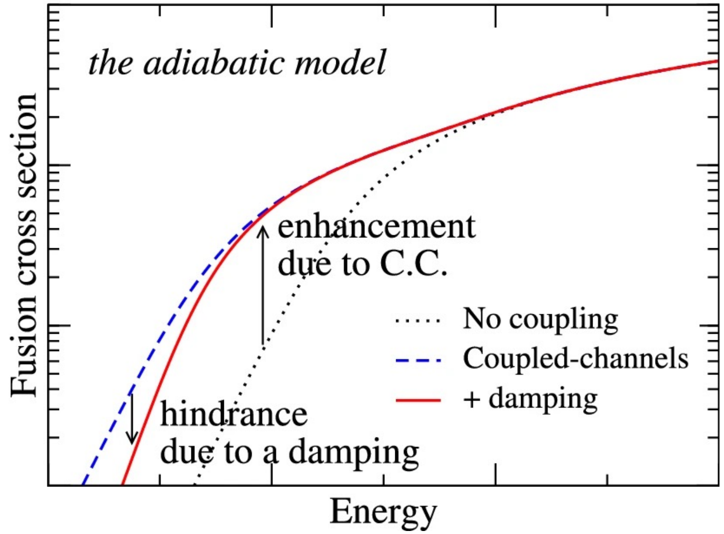
G. Fruet et al., PRL124, 192701 (2020)
← direct measurement

Fusion hindrance in medium-light and light systems

a very small hindrance has been observed in medium-light systems



the adiabatic model: a natural explanation



light systems: small enhancement → small hindrance

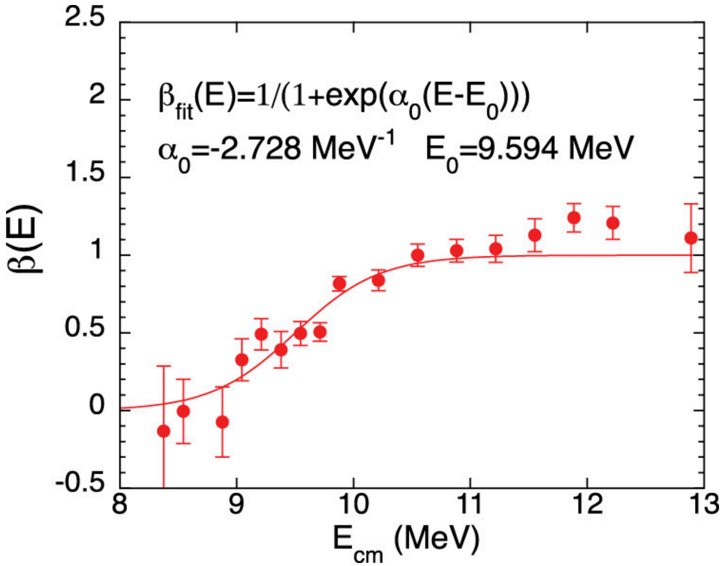
G. Montagnoli, A.M. Stefanini, C.L. Jiang,
K. Hagino et al., PRC97, 024610 (2018).

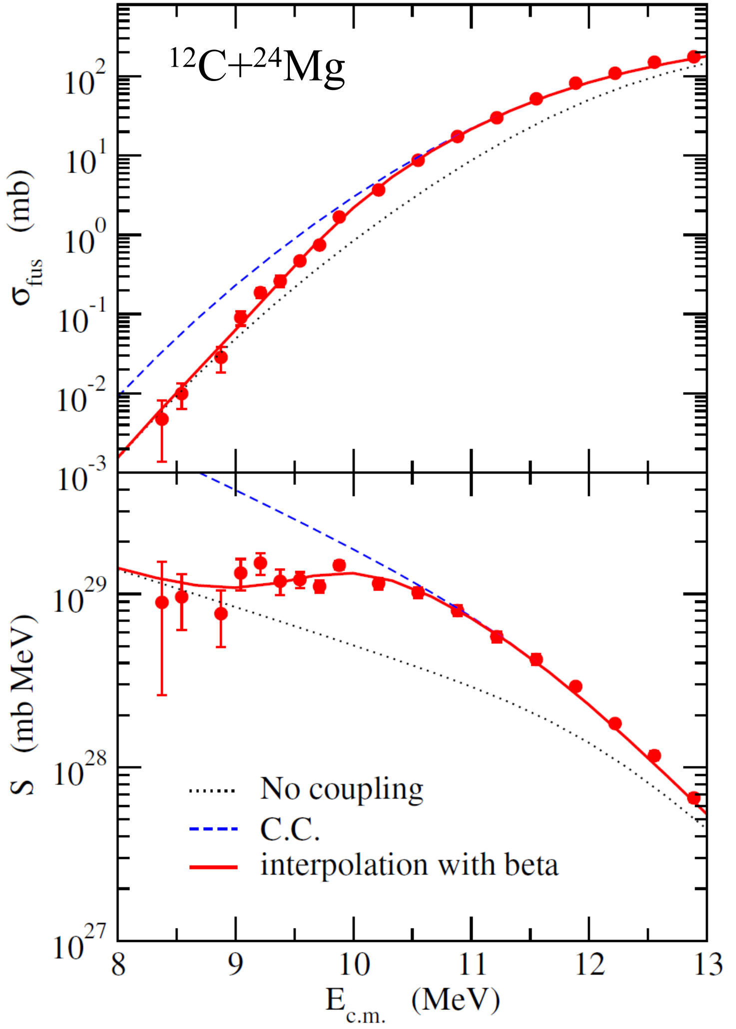
A simplified adiabatic model

a simple interpolation:

$$\ln(\sigma(E)) \sim \beta(E)\ln(\sigma_{\rm CC}(E)) + (1-\beta(E))\ln(\sigma_{\rm noc}(E))$$

$$\beta(E) = \frac{1}{1+e^{\alpha_0(E-E_0)}}$$

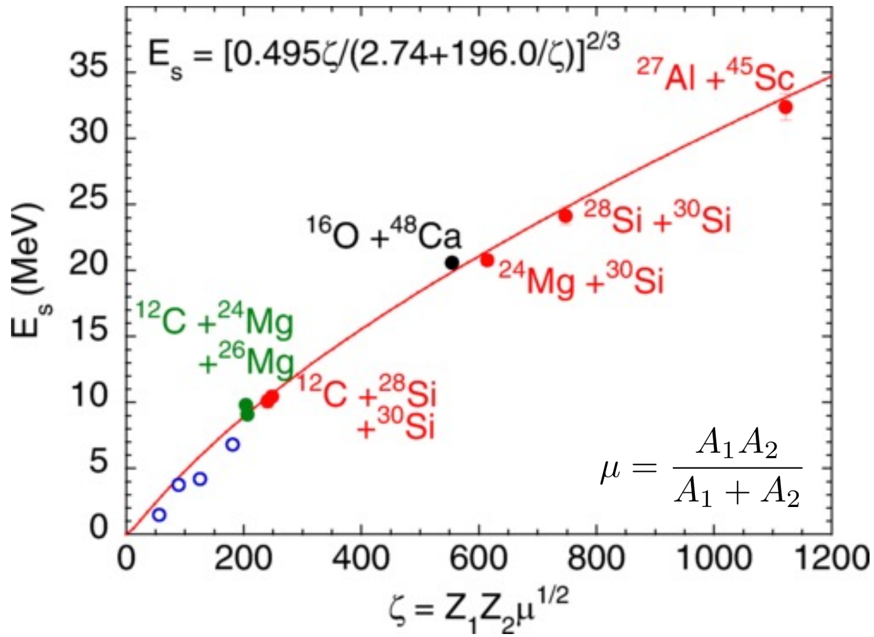




G. Montagnoli, A.M. Stefanini, C.L. Jiang,
K. Hagino et al., J. of Phys. G49, 095101 (2022).

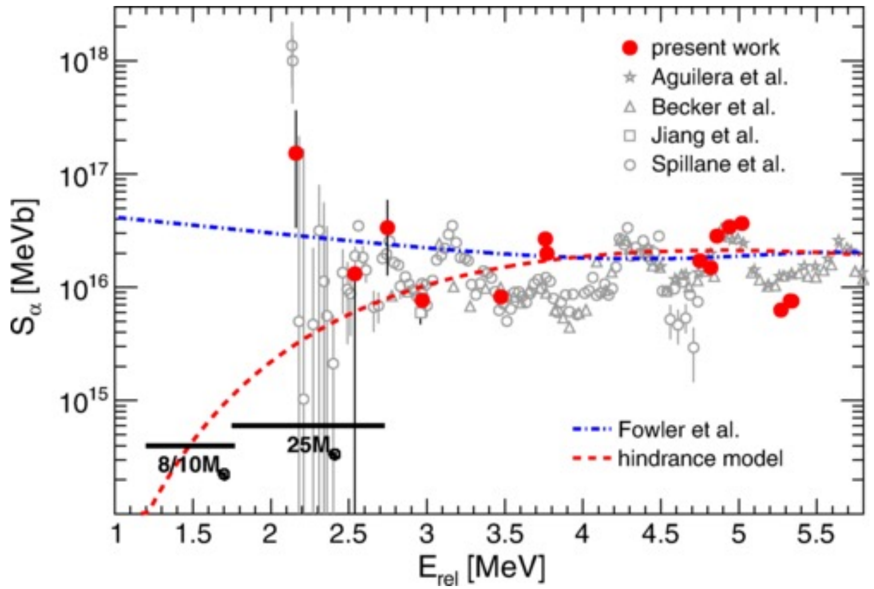
Fusion hindrance in medium-light and light systems

systematics: threshold energy for hindrance



A.M. Stefanini, G. Montagnoli et al.,
PRC111, 064620 (2025)
PLB872, 140084 (2026)

→ ^{12}C+^{12}C: E_s = 4.26 MeV



E_s seems to agree with Jiang et al.
….but how about the extrapolation of cross sections?

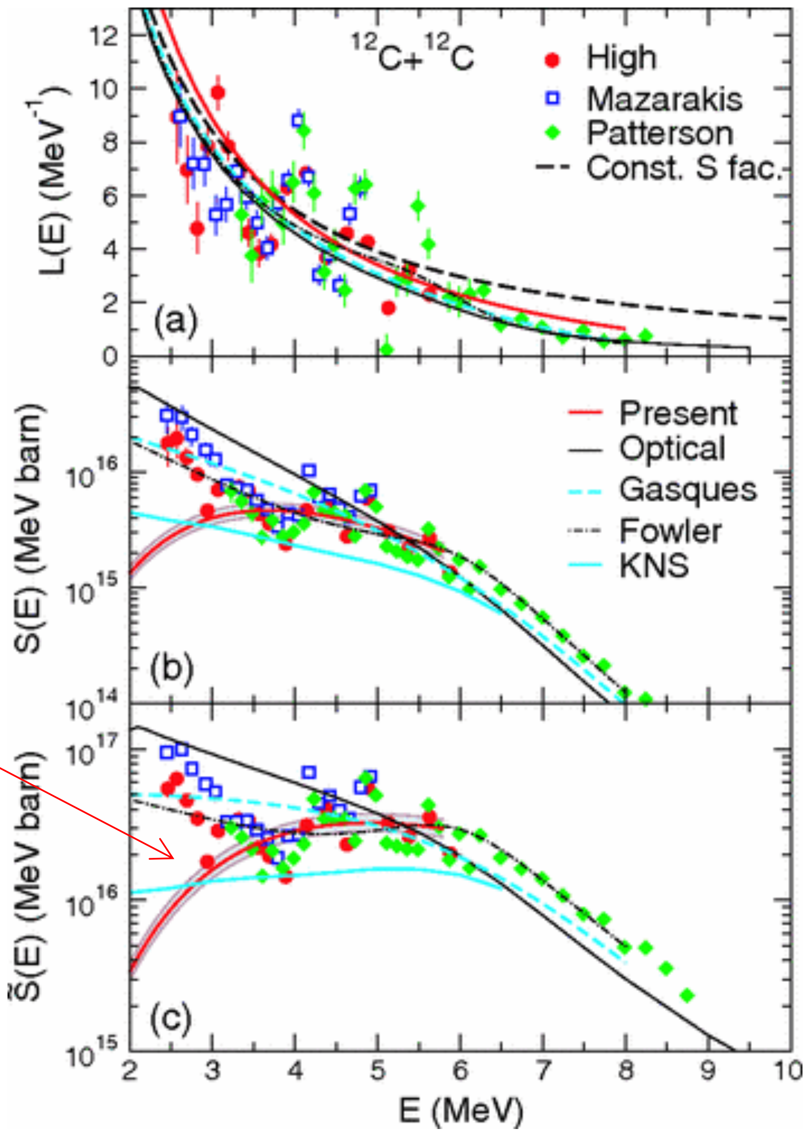
Fusion hindrance in $^{12}C+^{12,13}C$ systems

C.L. Jiang et al., PRC75, 015803 (2007)

$$L(E) = \frac{d}{dE}\ln(E\sigma)$$

fit with: $L(E) = A_0 + B_0/E^n$ (n=1.5)

$\rightarrow$ a large hindrance



(note)

$$\sigma(E) = \frac{S(E)}{E}e^{-2\pi\eta}$$

for S = const. $\rightarrow$ $L(E) \propto \frac{d\eta}{dE} \propto E^{-3/2}$

S can deviate from a constant.
$\rightarrow$ Re-fit with a free *n* ?

Fusion hindrance in $^{12}C+^{12,13}C$ systems

K. Uzawa and K. Hagino, PRC112, L061601 (2025)

$$L(E) = \frac{d}{dE}\ln(E\sigma)$$

fit with: $L(E) = A_0 + B_0/E^n$ (n=1.5 → free)

the ordinary procedure (weighted χ^2 fit):

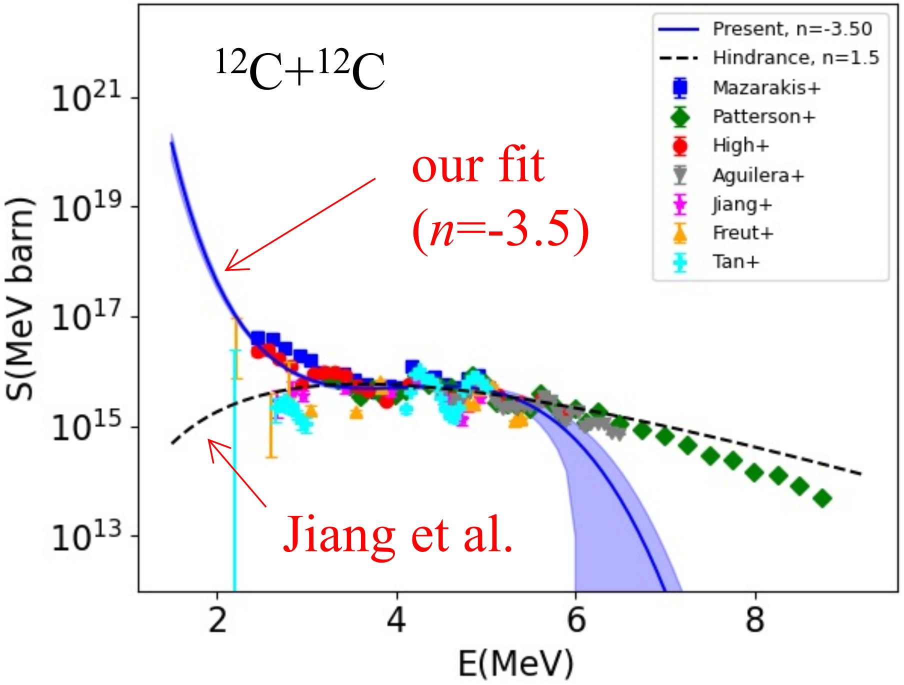
$L(E) \to \sigma(E) \to S(E)$

$$\chi_w^2 \equiv \sum_i \left(\frac{S_i - S(E_i)}{\Delta S_i}\right)^2$$

→ a little emphasis on low-E data (large $\Delta S_i/S_i$)

we take: $$\chi_o^2 \equiv \sum_i \left(\frac{S_i - S(E_i)}{\Delta S_0}\right)^2$$

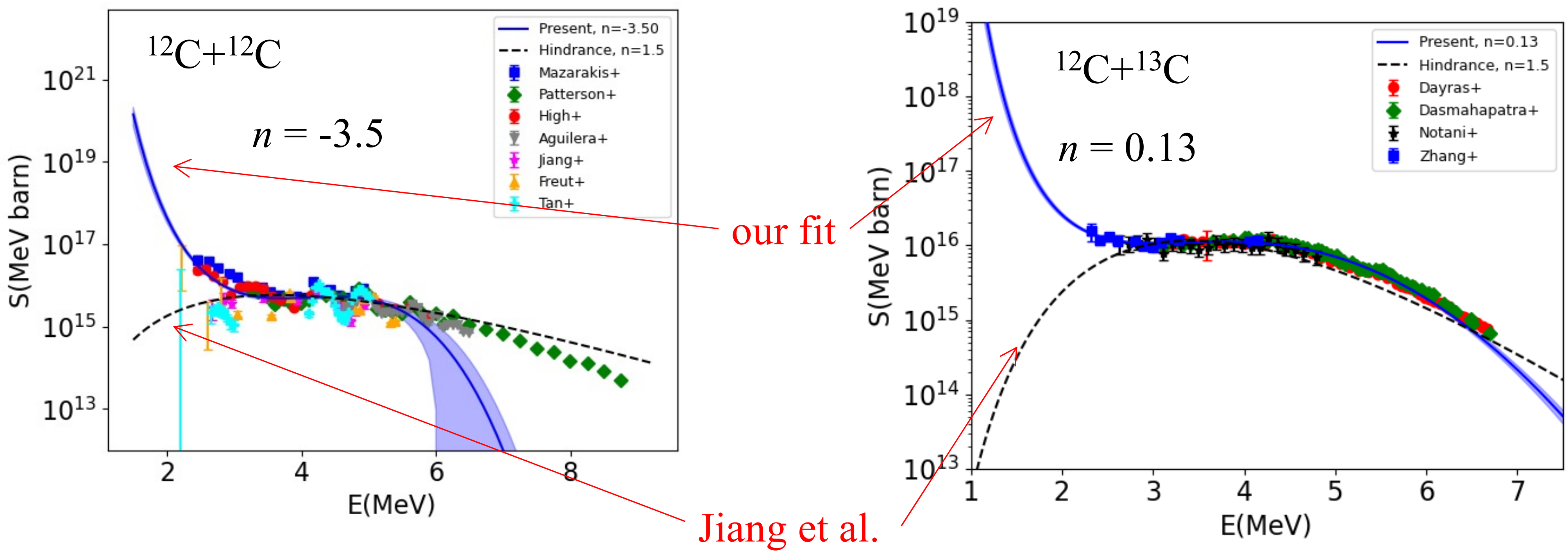
a constant



our fit → no hindrance!

Fusion hindrance in $^{12}C+^{12,13}C$ systems

K. Uzawa and K. Hagino, PRC112, L061601 (2025)



our fit → no hindrance!

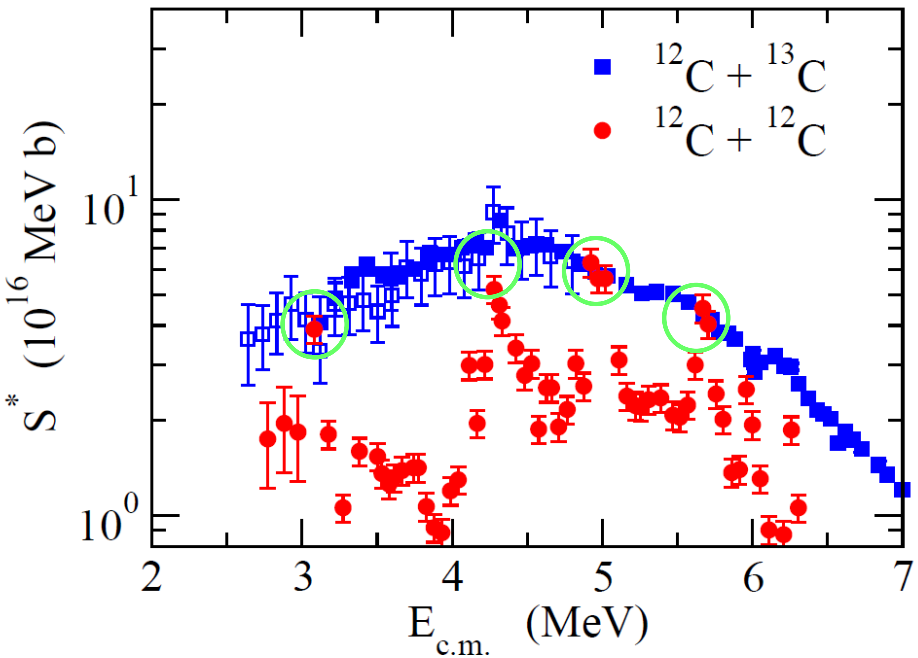
the same conclusion with polynomial fits:

$$\ln(S(E)) = \sum_{m=0}^{6} a_m E^m$$

(ii) Fine structure

Energy dependence of fusion cross sections for the $^{12}C+^{12,13}C$ systems

$$S^*(E) = E\sigma(E)\, e^{2\pi(\eta-\eta_0)}$$



prominent resonance structure in $^{12}C+^{12}C$ fusion

- ✓ off-resonance: fusion inhibition
- ✓ on-resonance: match with $^{12}C+^{13}C$

cf. the next talk by Y. Taniguchi

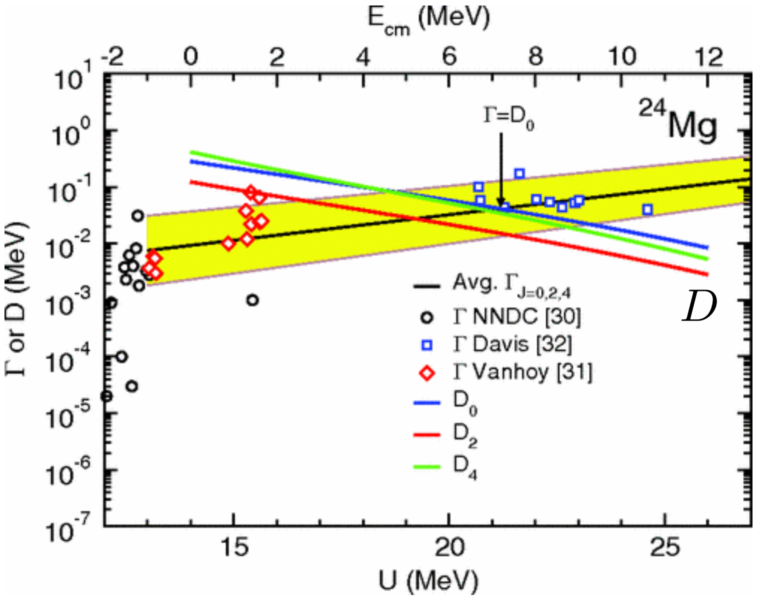
M. Notani, X.D. Tang et al., PRC85 (2012) 014607

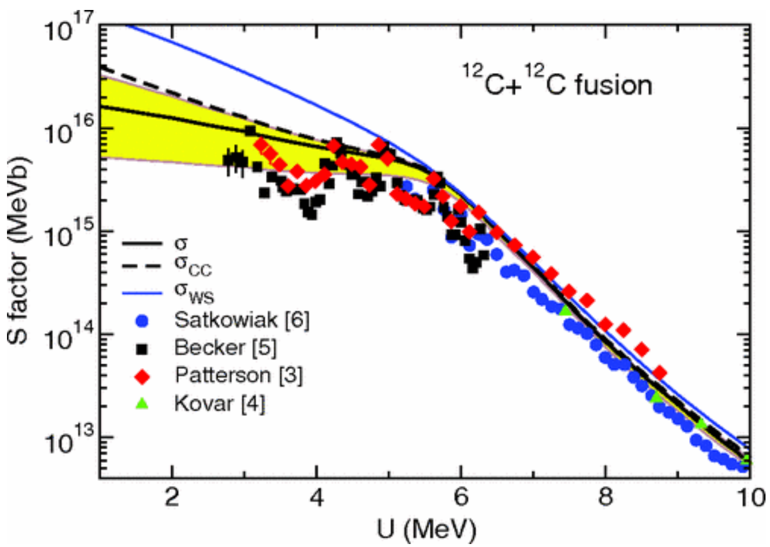
An open issue:
Does $^{12}C+^{13}C$ provide the upper limit of $^{12}C+^{12}C$ cross sections?

cf. C.L. Jiang et al., PRL110, 072701 (2013): Γ/D of the compound nucleus

Effects of isolated CN resonances

C.L. Jiang et al., PRL110, 072701 (2013).





Moldauer factor

$$P_J = 1 - \exp(-2\pi\Gamma_J / D_J)$$

$$\rightarrow \sigma = \sum_J \sigma_{CC}^J P_J$$

E < 7 MeV

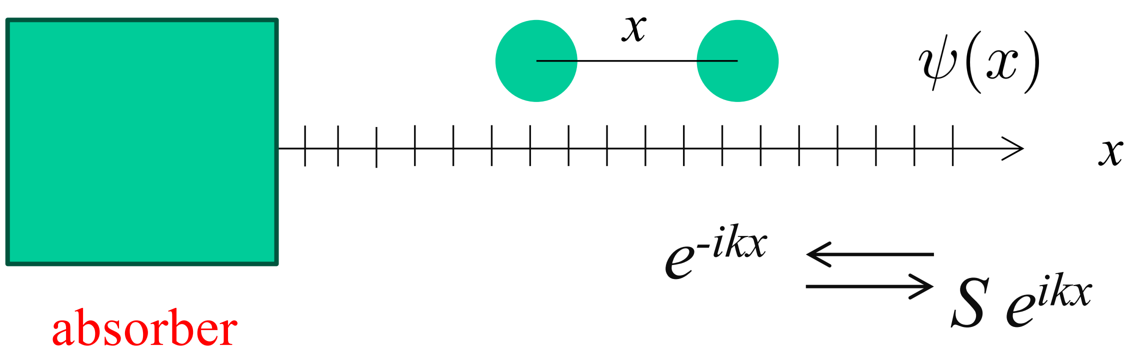
$\Gamma < D \rightarrow$ fusion inhibition

E > 7 MeV

$\Gamma > D \rightarrow P_J \sim 1$

a schematic model for the imaginary part : a microscopic understanding of the optical potential

scattering problem with absorption:



absorption $\rightarrow |S| < 1$

cf. for strong absorption
$S \sim 0$

cf. optical potential

$$V_{\rm opt}(r) = V(r) - iW(r)$$

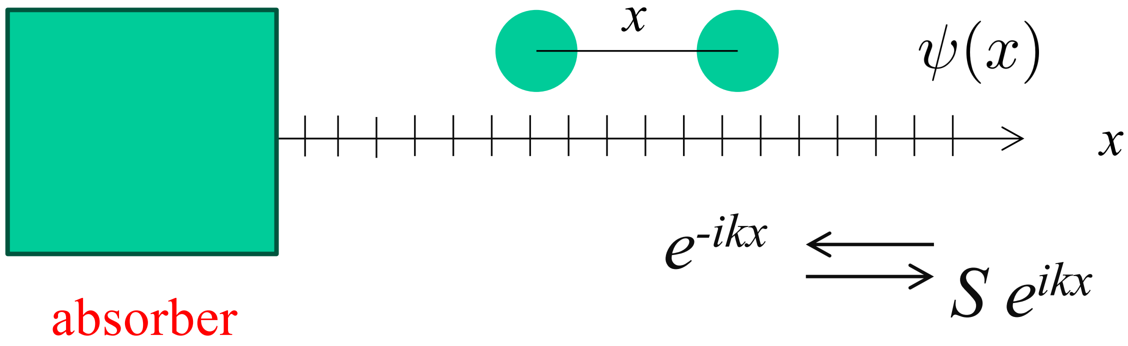
- elastic scattering
- absorption process

$-iW(r)$: a loss of incident flux
- ✓ inelastic scattering
- ✓ transfer reaction
- ✓ fusion

etc.

a schematic model for the imaginary part : a microscopic understanding of the optical potential

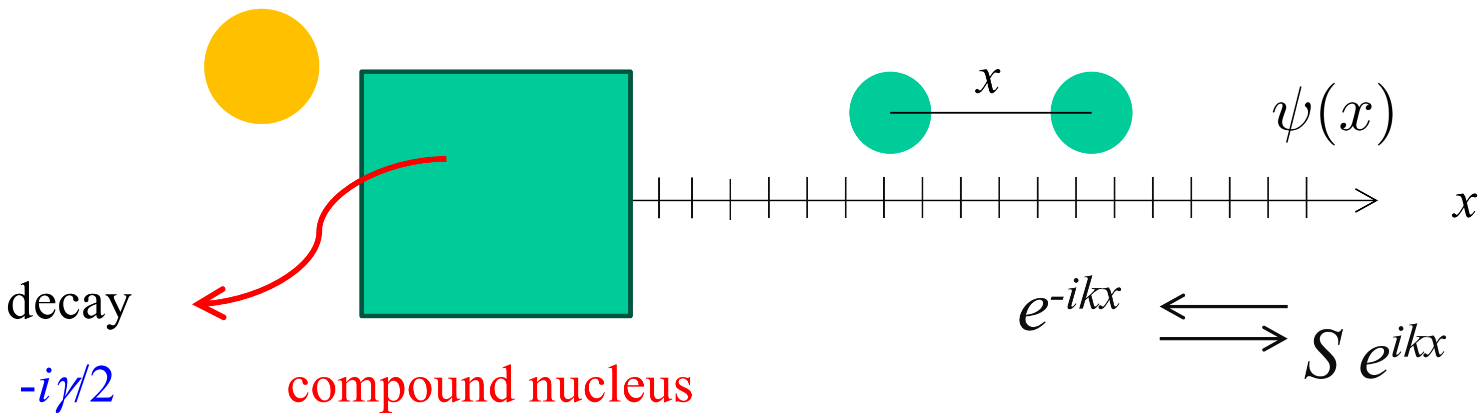
- scattering problem with absorption:



absorption $\to |S| < 1$

cf. for strong absorption
$S \sim 0$

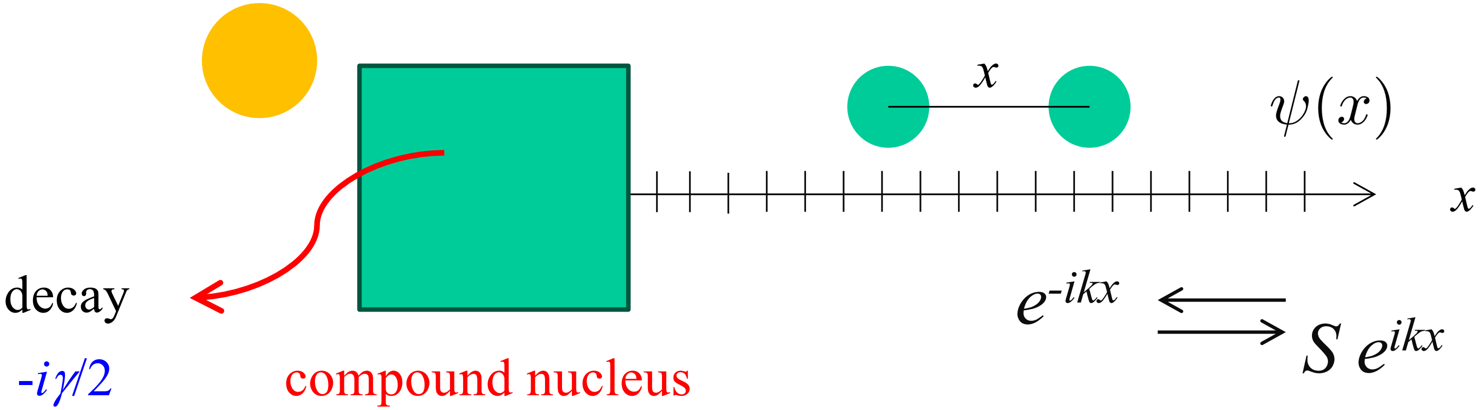
- a microscopic origin for absorption: compound nucleus formation (fusion)



✓ couplings to CN
✓ decay of CN
$\to$ not coming back to the entrance ch.
$\to$ loss of flux (absorption)

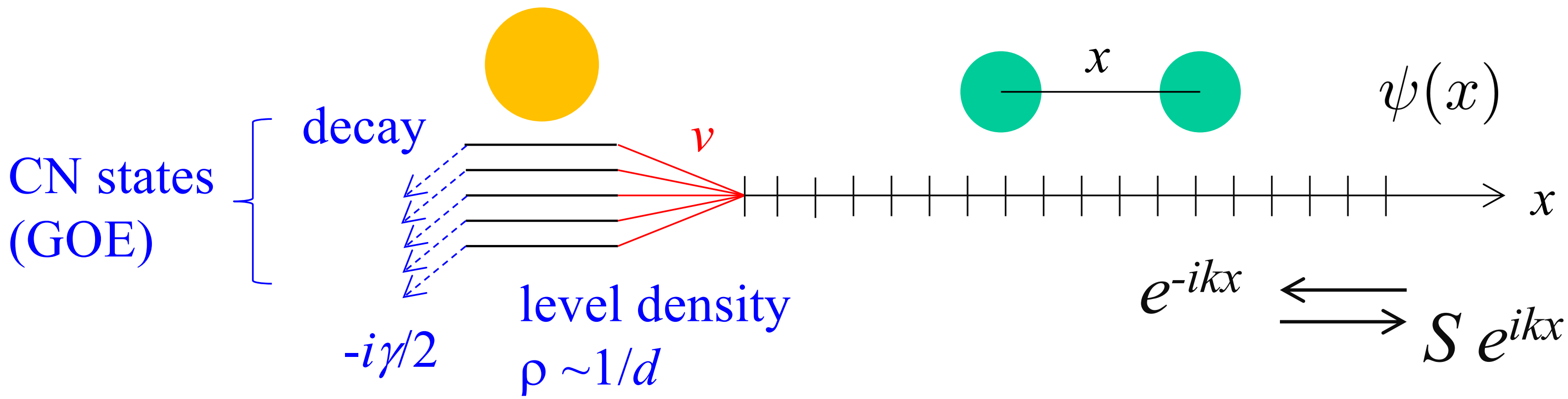
a schematic model for the imaginary part : a microscopic understanding of the optical potential

- a microscopic origin for absorption: compound nucleus formation (fusion)

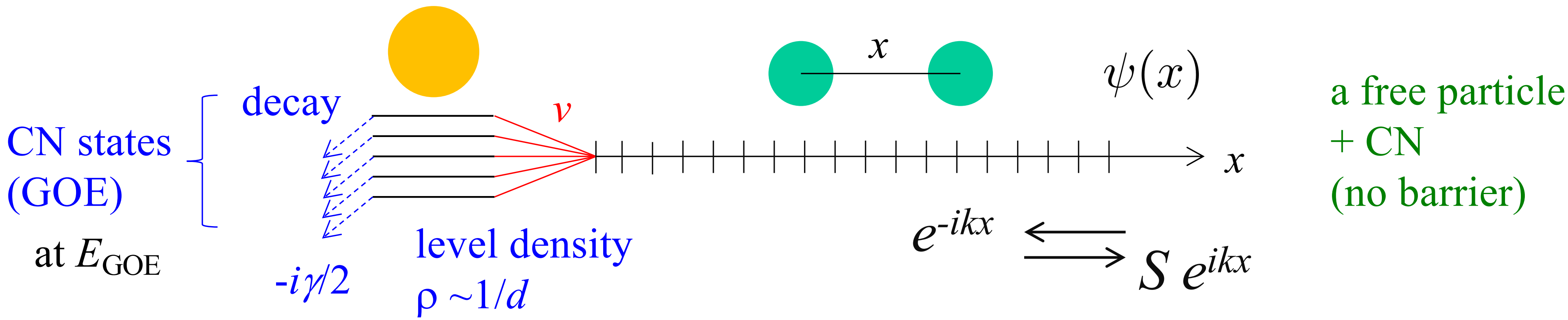


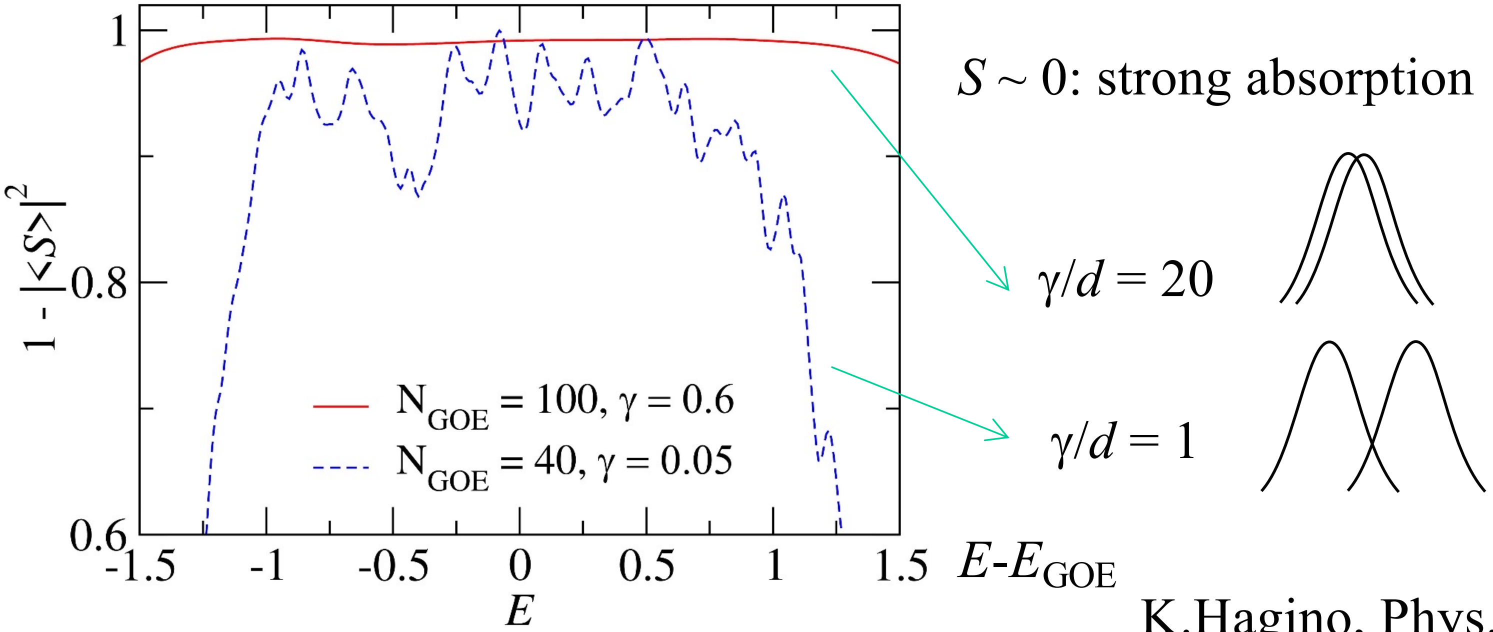
✓ couplings to CN
✓ decay of CN
→ not coming back to the entrance ch.
→ loss of flux (absorption)

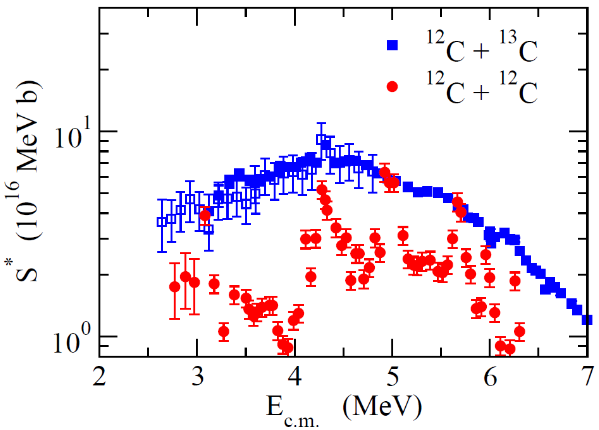
- a modelling:



a schematic model with a random matrix





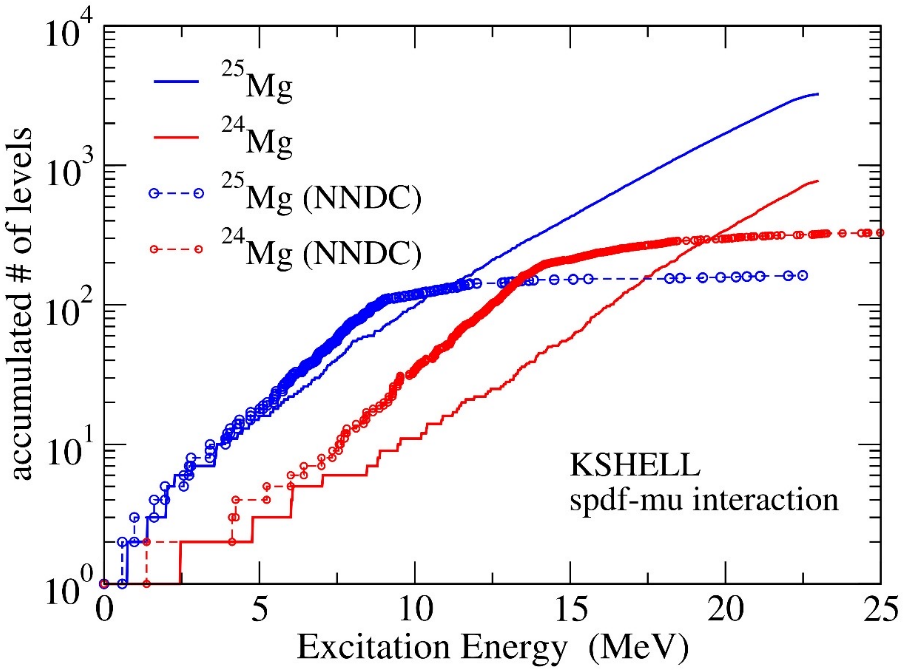


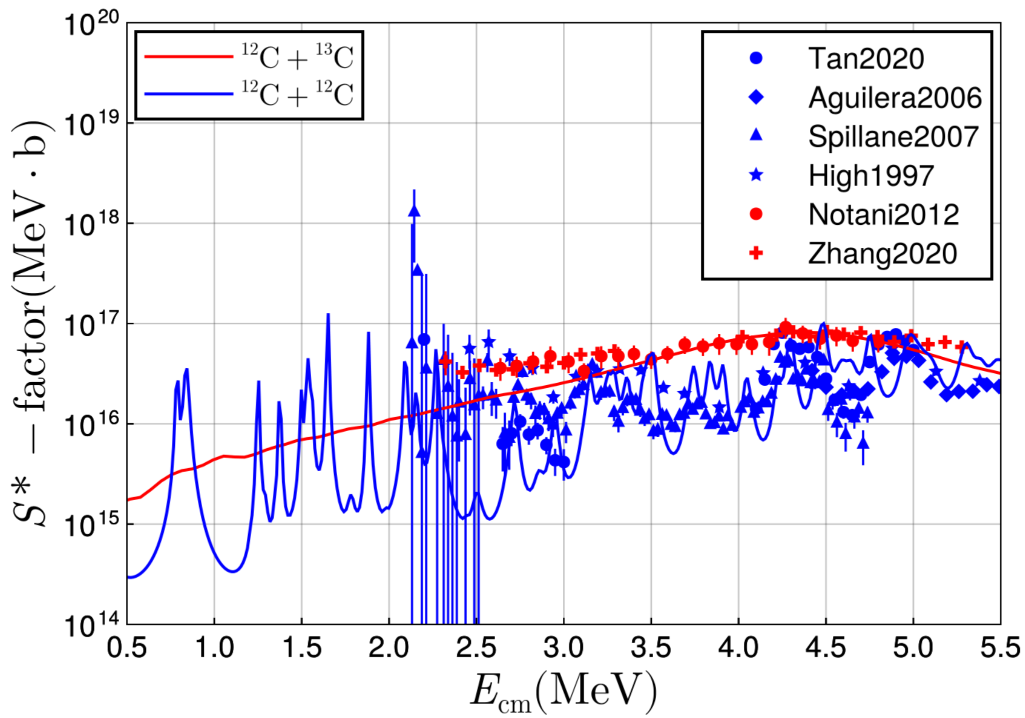
K.Hagino, Phys. Rev. C112, 034611 (2025).

a more realistic model with shell model   K. Nagao, K. Hagino, and K. Uzawa, in preparation.

- ✓ the spectrum of CN states: RMT → shell model (KSHELL)
- ✓ decay widths → statistical model
- ✓ 3D calculations with the Coulomb barrier

next talk by Nagao-kun

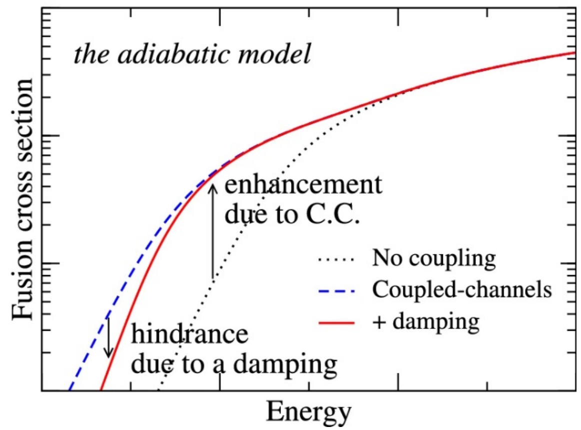




Summary

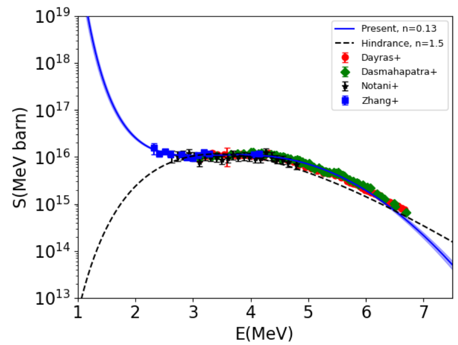
Heavy-ion fusion reactions

- sub-barrier enhancement of cross sections
- deep sub-barrier hindrance



influence on nuclear astrophysical reactions?

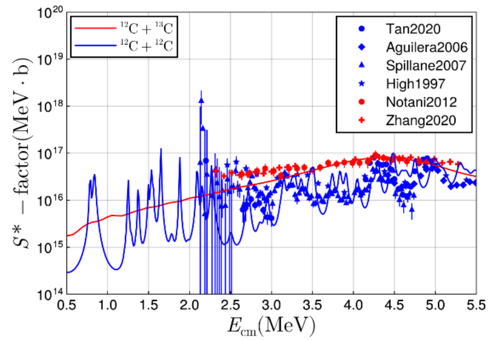
- experimental data for medium-light systems (LNL)
  → only small hindrance
- previous fit by Jiang et al. for ^{12}C+^{12,13}C → a large hindrance
- our re-analysis → no hindrance!



energy dependence of fusion cross sections for ^{12}C+^{12,13}C

^{12}C+^{12}C : prominent resonance peaks
^{12}C+^{13}C : a much smoother energy dependence

a new model with shell model calculations for the CN



future direction: 1-channel → coupled-channels