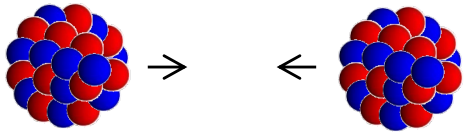
Low-energy dynamics of heavy-ion reactions



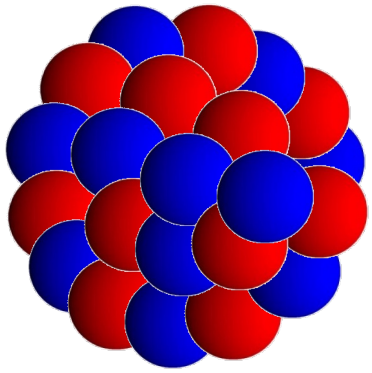
Kouichi Hagino
하기노 코이치
Kyoto University, Kyoto, Japan

1. Introduction : low-energy nuclear reactions
2. Optical potential approaches
3. Imaging of low-energy heavy-ion reactions
4. Energy functional approach to heavy-ion fusion reactions
5. Summary

based on Japan-Korea collaborations

Symposium on Nuclear Structure, Reaction, and Astrophysics (NuSRAP2026), University of Tsukuba, July 13-15, 2026

Low-energy nuclear physics

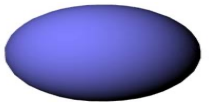


Nuclei as quantum many-body systems of nucleons

← to understand several properties in terms of nucleon d.o.f.

- static properties: nuclear structure

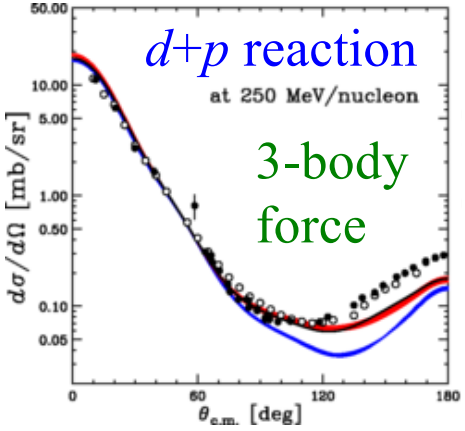
  the mass, the size, the shape....



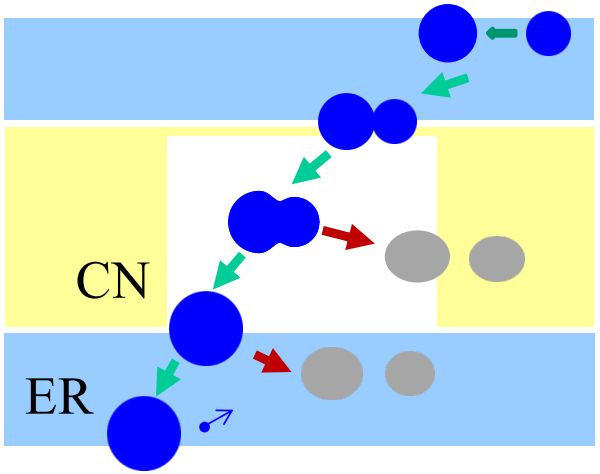
- dynamics: nuclear reactions

✓ Nuclear Reactions as a tool to investigate nuclear structure

knock-out reactions



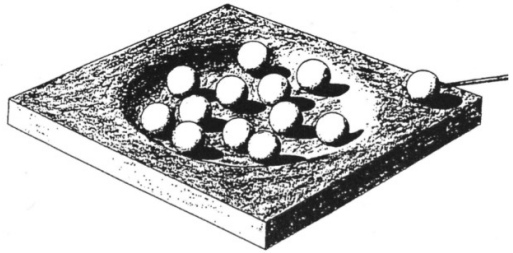
K. Sekiguchi et al.,
PRC89 (‘14) 064007



a synthesis of SHE

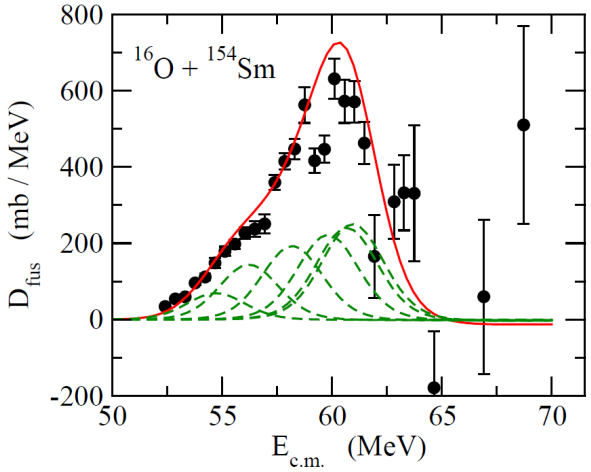
Two aspects of nuclear reactions

- ✓ a tool for nuclear structure
- ✓ reaction dynamics ⟵ this talk



compound nucleus and quantum chaos

fusion barrier distribution



nucleus: a composite system

- ✓ rich reaction processes
- ✓ a rich interplay between nuclear structure and reaction

- ✓ elastic scattering
- ✓ inelastic scattering
- ✓ particle transfer reactions
- ✓ fusion reactions

etc.

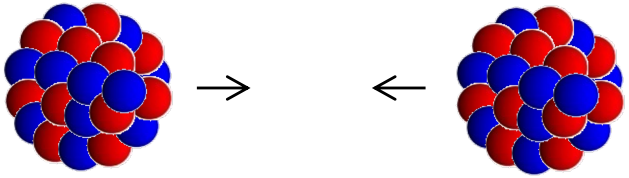
- ✓ g.s. properties (mass, size, shape….)
- ✓ excitations

In this talk:

- $^{12}C+^{12}C$ fusion
- imaging of elastic scattering
- DFT for nuclear reactions

Optical model calculations

elastic scattering



g.s. g.s.

optical potential

$$V_{\rm opt}(r) = V(r) - iW(r)$$

- ✓ inelastic scattering
- ✓ transfer reaction
- ✓ fusion etc.

cf. Dahee Kim's talk in this session

- elastic scattering

$$f(\theta) = \sum_l (2l+1)\frac{S_l - 1}{2ik} P_l(\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

- absorption process

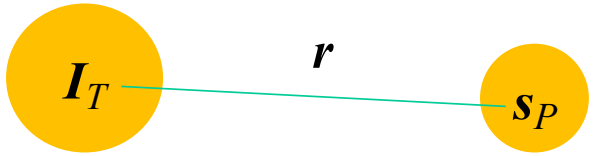
$$\sigma_{\rm abs} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$

cf. for a system with odd-mass nuclei:

$$V_{\rm opt}(r)$$

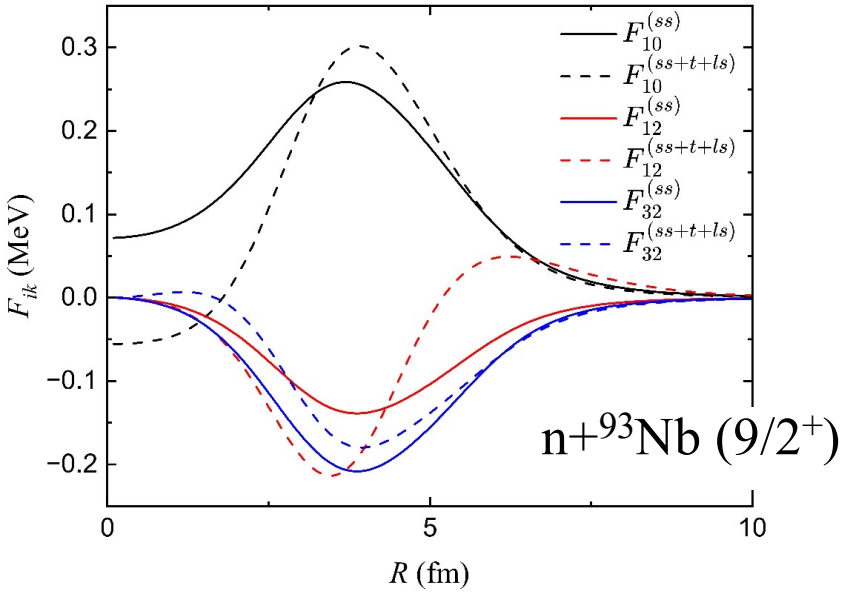
$$\to V_{\rm opt}(r) - \sum_k F_{1k}(r)[\boldsymbol{I}_T \boldsymbol{s}_P]^{(k)} \cdot Y_k(\hat{\boldsymbol{r}}) + \cdots$$

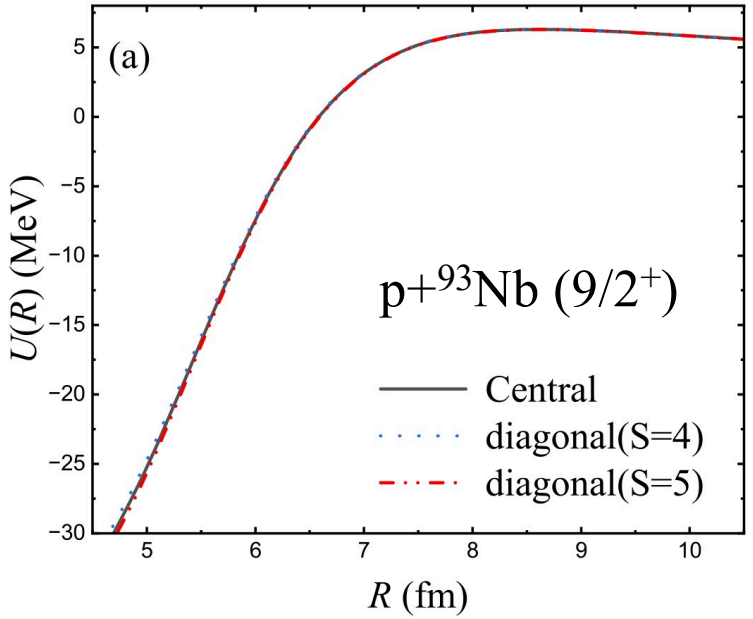
Optical model calculations



for a system with odd-mass nuclei:

$$V_{\rm opt}(r) \to V_{\rm opt}(r) - \sum_k F_{1k}(r)[\boldsymbol{I}_T \boldsymbol{s}_P]^{(k)} \cdot Y_k(\hat{\boldsymbol{r}}) + \cdots$$





$\Delta\sigma_{\rm fus}$~0.01-0.03%

→ does not affect the barrier, thus fusion cross sections

cf. Heo's talk (this afternoon)

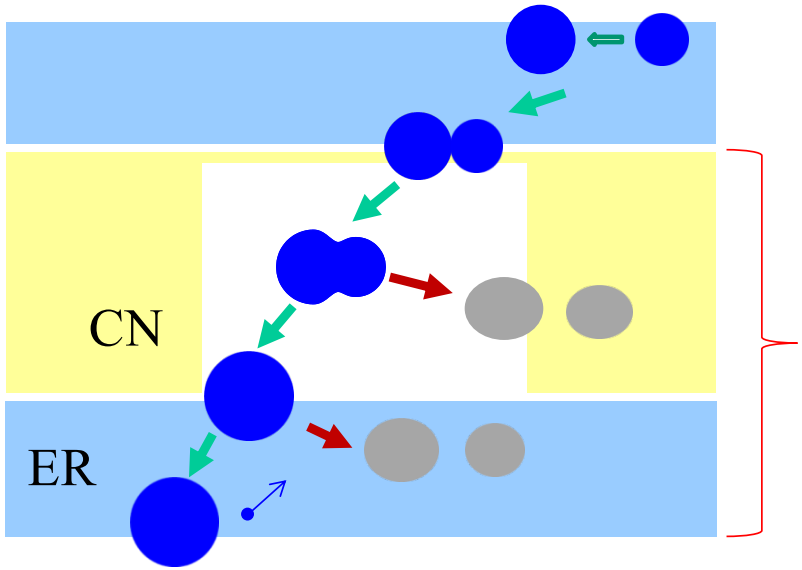
K. Heo, M.-K. Cheoun, and K. Hagino, arXiv: 2605.26561

Optical model calculations

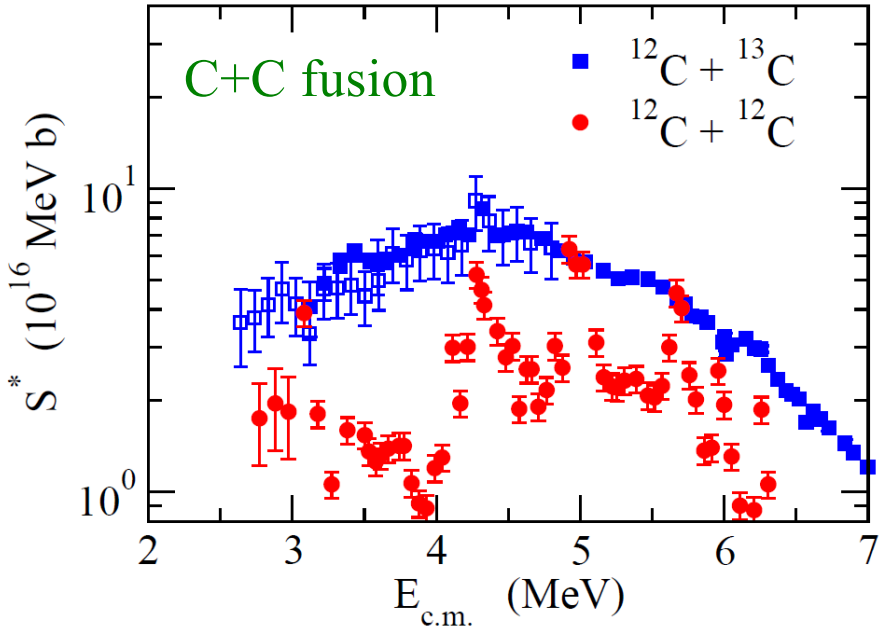
$$V_{\rm opt}(r) = V(r) - iW(r) \;\rightarrow \sigma_{\rm fus} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$

- no dynamics after the capture: only the absorption of elastic flux
- dynamics are important in several cases:

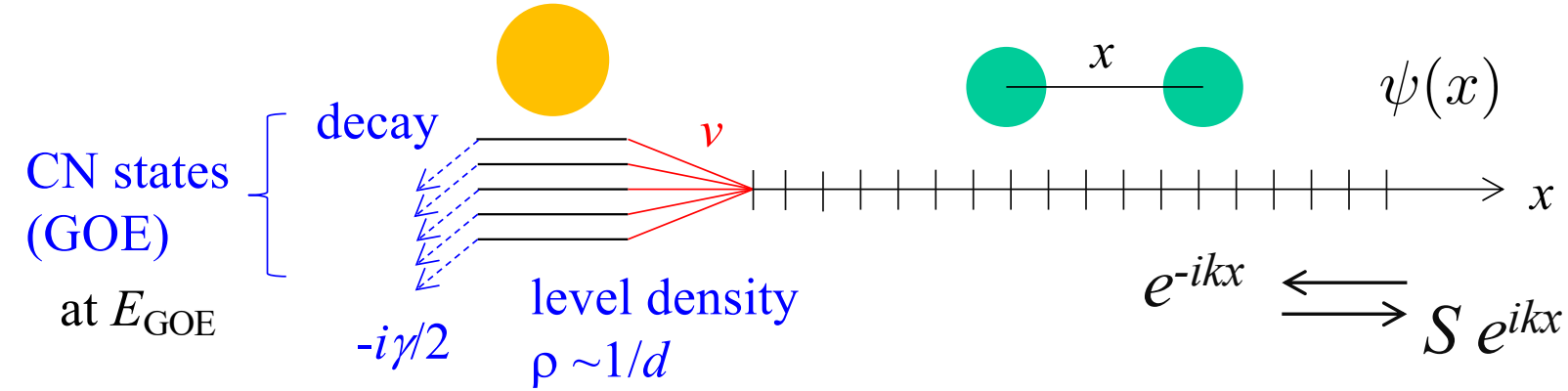
syntheses of superheavy elements



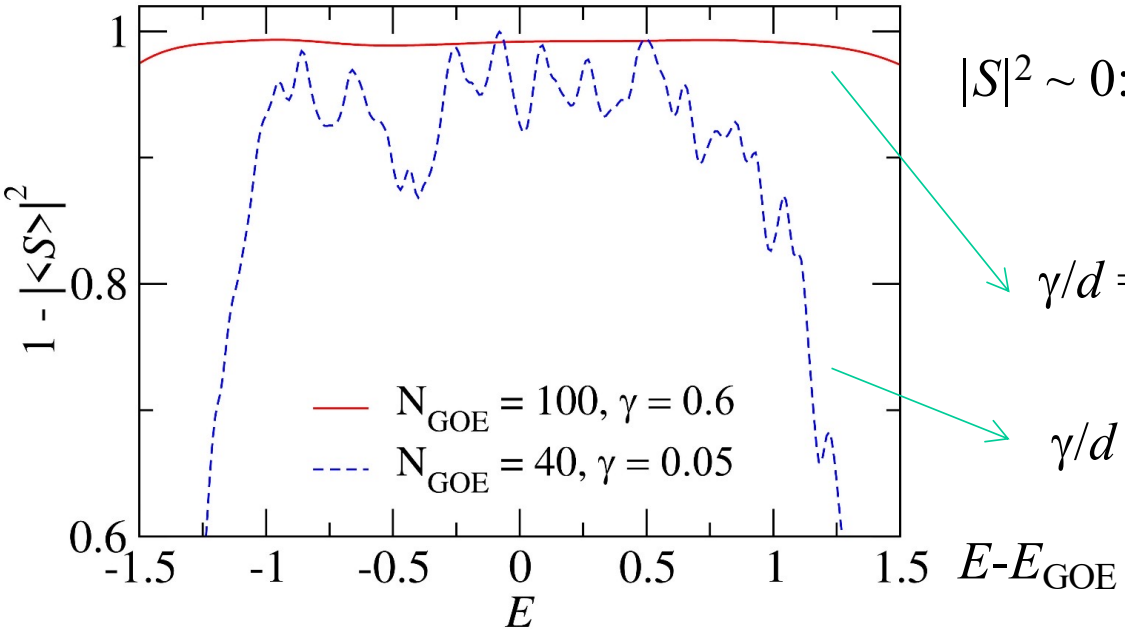
light-ion fusion reactions



a schematic model with a random matrix



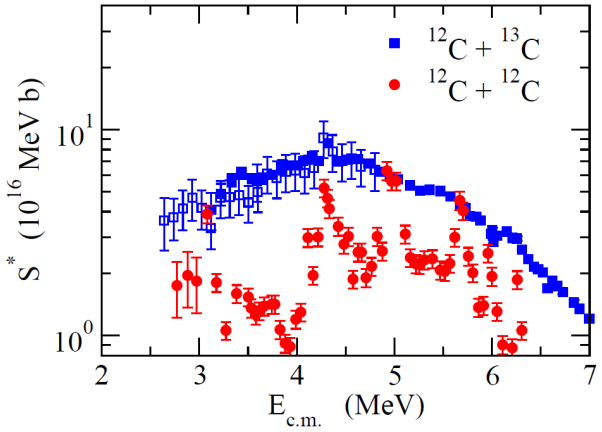
a free particle
+ CN
(no barrier)



$|S|^2 \sim 0$: strong absorption

$\gamma/d = 20$

$\gamma/d = 1$



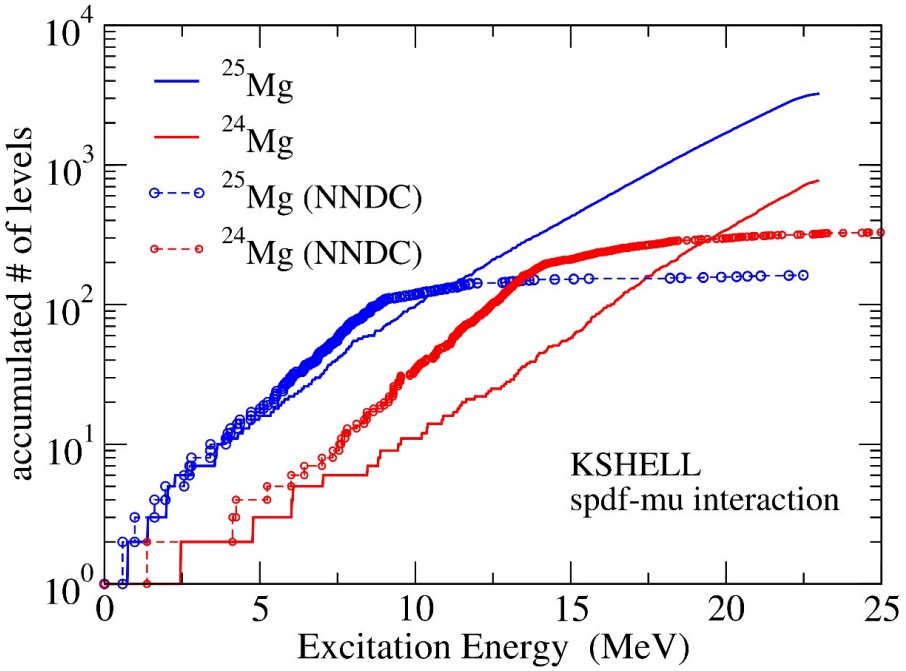
K. Hagino, Phys. Rev. C112, 034611 (2025).

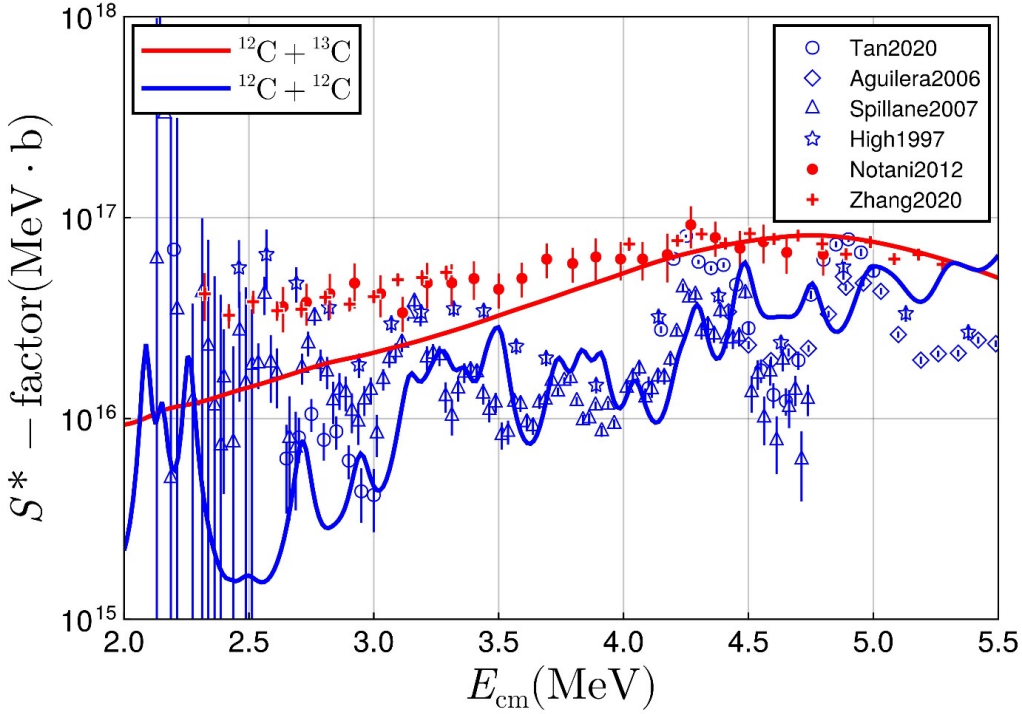
[a more realistic model with shell model]

K. Nagao, K. Hagino, and K. Uzawa, arXiv:2607.08502.

- ✓ the spectrum of CN states: RMT → shell model (KSHELL)
- ✓ decay widths → statistical model
- ✓ 3D calculations with the Coulomb barrier

Please see Nagao's poster

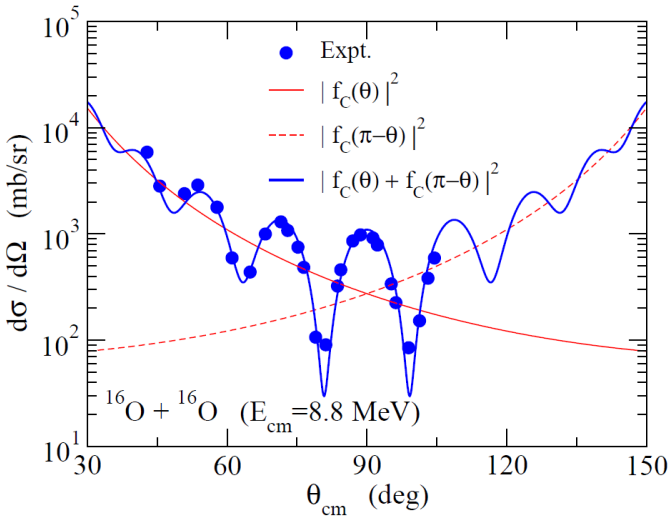




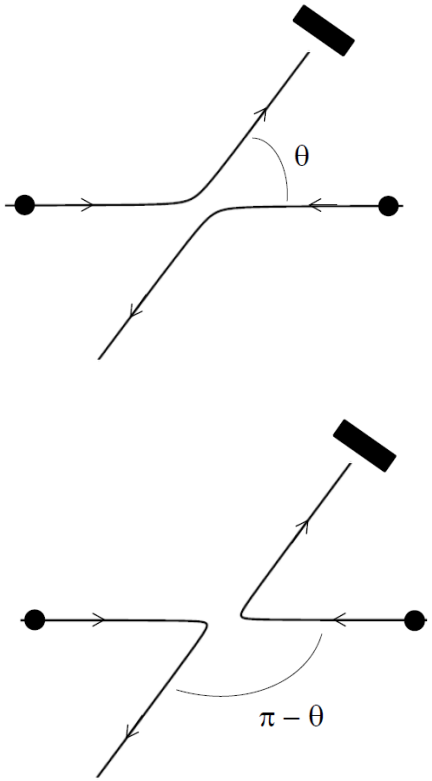
imaging quantum interference phenomena

quantum interference phenomena in heavy-ion reactions

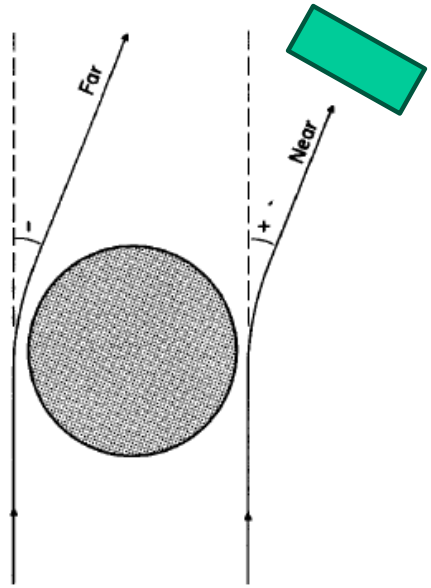
✓ Mott Scattering



expt: D.A. Bromley et al., Phys. Rev. 123 (‘61)878



✓ near side-far side interference

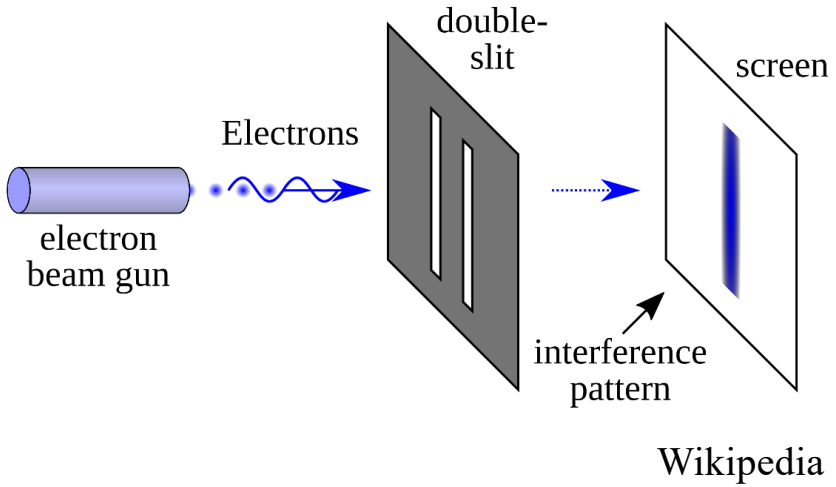


✓ nuclear-Coulomb interference

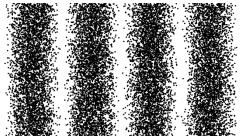
✓ barrier wave-internal wave interference

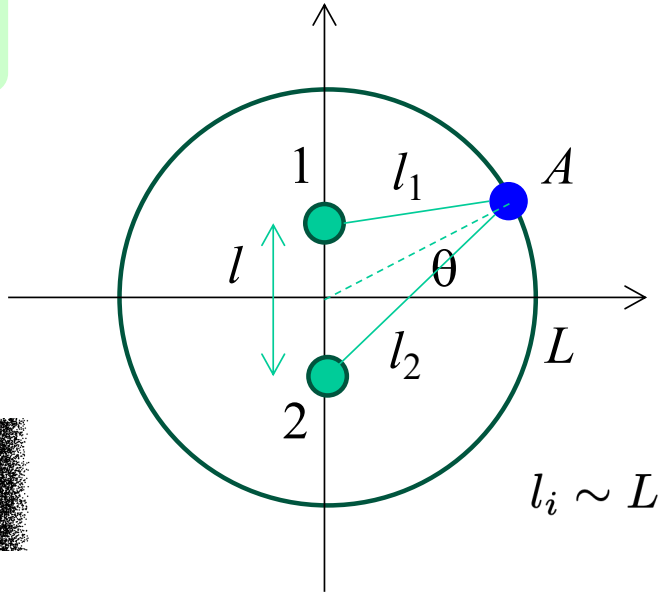
imaging quantum interference phenomena

a double slit problem



Wikipedia





$$l_i \sim L\left(1 \pm \frac{l}{2L}\sin\theta\right)$$

the amplitude at A

$$f(\theta) \quad = \quad f_1(\theta) + f_2(\theta) \rightarrow P = |f_1(\theta) + f_2(\theta)|^2$$

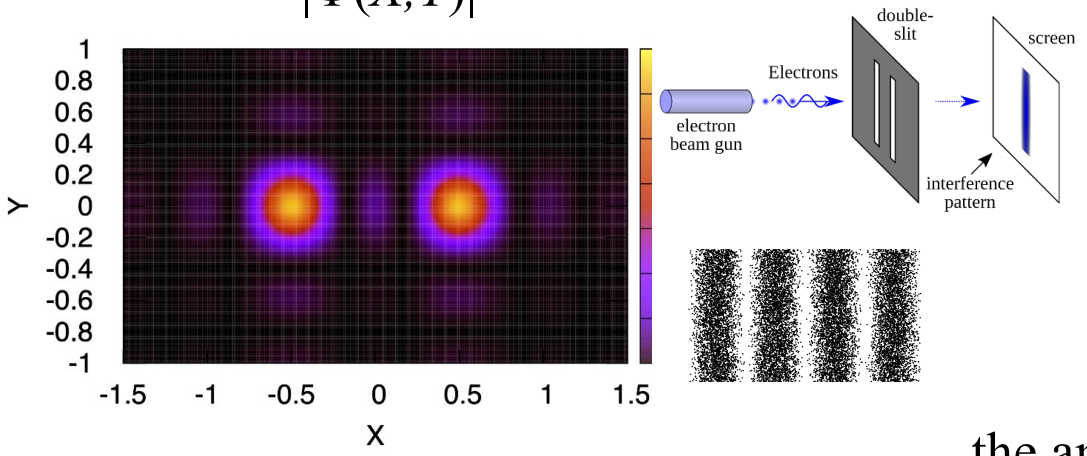
$$f_i(\theta) \quad = \quad A\sin\left(\frac{2\pi}{\lambda}l_i - \omega t\right)$$

the Fourier transform of $f(\theta)$:

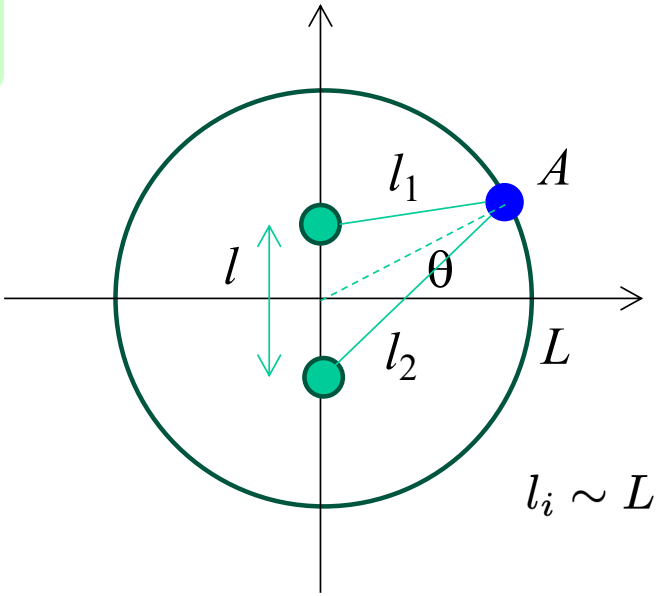
$$\Phi(X,Y) \propto \int_{\theta_2}^{\theta_2} d\theta \int_{\varphi_1}^{\varphi_2} d\varphi\, e^{ik(\theta X + \varphi Y)} f(\theta,\varphi)$$

imaging quantum interference phenomena

$|\Phi(X,Y)|^2$



K. Hashimoto et al.,
PTEP2023, 043B04 (2023)



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imaging quantum interference phenomena

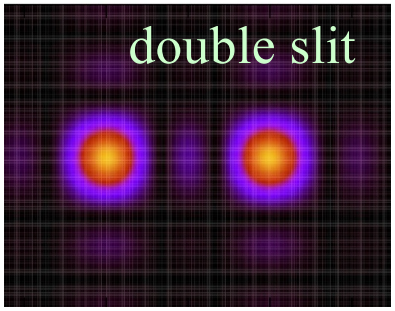
K. Hagino and T. Yoda, PLB848, 138326 (2024).
K. Heo and K.Hagino, PRC111, 034612 (2025).

Application of nuclear reactions:

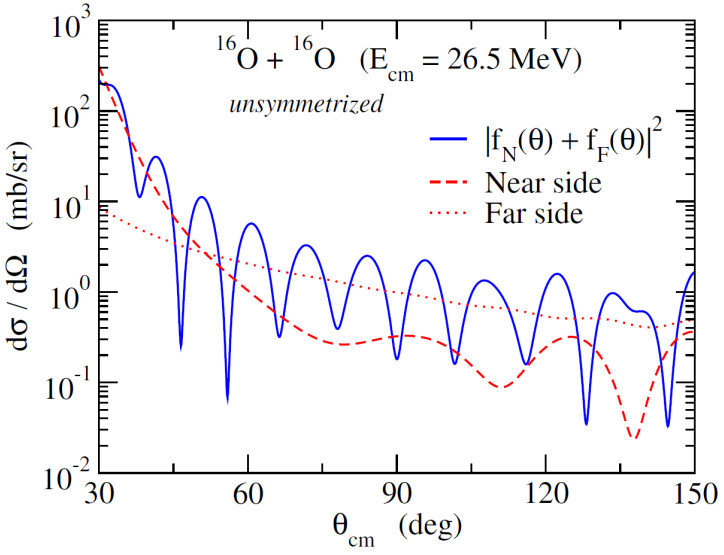
$$V_{\rm opt}(r) = V(r) - iW(r) \longrightarrow f(\theta) \longrightarrow \frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

F.T.

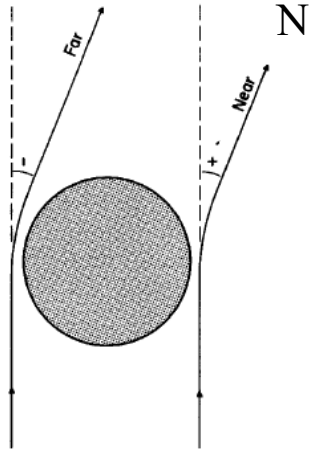
$$\Phi(X,Y) \rightarrow |\Phi(X,Y)|^2$$



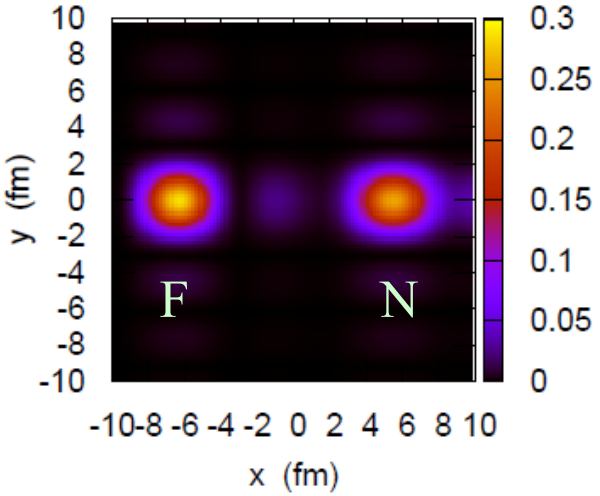
^{16}O+^{16}O at E = 26.5 MeV



F N

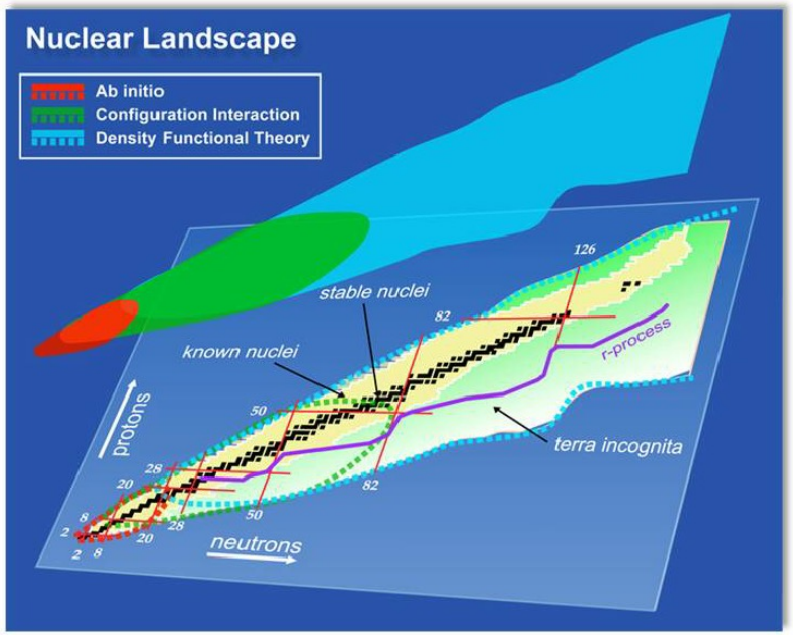


θ$_0$=55 deg., Δθ=15 deg.



Energy functional approach to heavy-ion fusion reactions

Density functional theory (DFT)

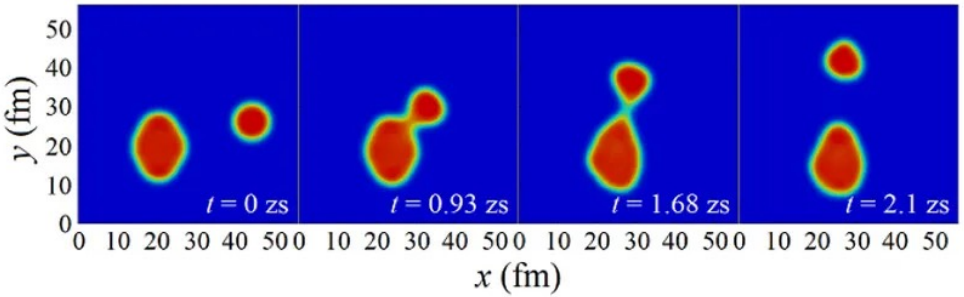


$$E = \int d\boldsymbol{r} E[\rho(\boldsymbol{r})]$$

- ✓ Skyrme
- ✓ KIDS, Fayans
- ✓ Gogny
- ✓ CDFT

For nuclear reactions:

time-dependent Hartree-Fock (TDHF)



multi-neutron transfer reaction of ^{64}Ni+^{238}U
K. Sekizawa, Front. Phys. 7, 20 (2019).

⇒ no tunneling (no subbarrier fusion)
cf. TD-GCM

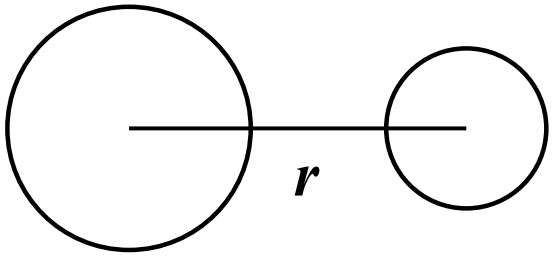
⇒ $V(r)$ based on DFT

- density constrained HF (S. Umar et al.)
- energy functional formalism

Energy functional approach to heavy-ion fusion reactions

Energy functional formalism

D.M. Brink and Fl. Stancu,
Nucl. Phys. A243, 175 (1975), ibid A270, 236 (1976).



$$V(r) = \int d\boldsymbol{r}' E[\rho_P + \rho_T] - \int d\boldsymbol{r}' E[\rho_P] - \int d\boldsymbol{r}' E[\rho_T]$$

$$E_{\rm Sk}[\rho] = \frac{\hbar^2}{2m}\tau + \frac{t_0}{2}\left(1 + \frac{x_0}{2}\right)\rho^2 + \cdots,$$

extended TF (OF-DFT)

$$\tau_{\rm TF} = \frac{3}{5}(3\pi^2)^{2/3}\rho^{5/3} + \frac{1}{36}\frac{(\boldsymbol{\nabla}\rho)^2}{\rho} + \cdots$$

Recent developments in Orbital-free DFT (OF-DFT)

✓ a phenomenological adjustment of τ_2

$$\tau_2 = \frac{1}{36}\frac{(\boldsymbol{\nabla}\rho)^2}{\rho} \rightarrow \frac{1}{20}\frac{(\boldsymbol{\nabla}\rho)^2}{\rho}$$

G. Colo and K. Hagino, PTEP2023, 103D01 (2023).

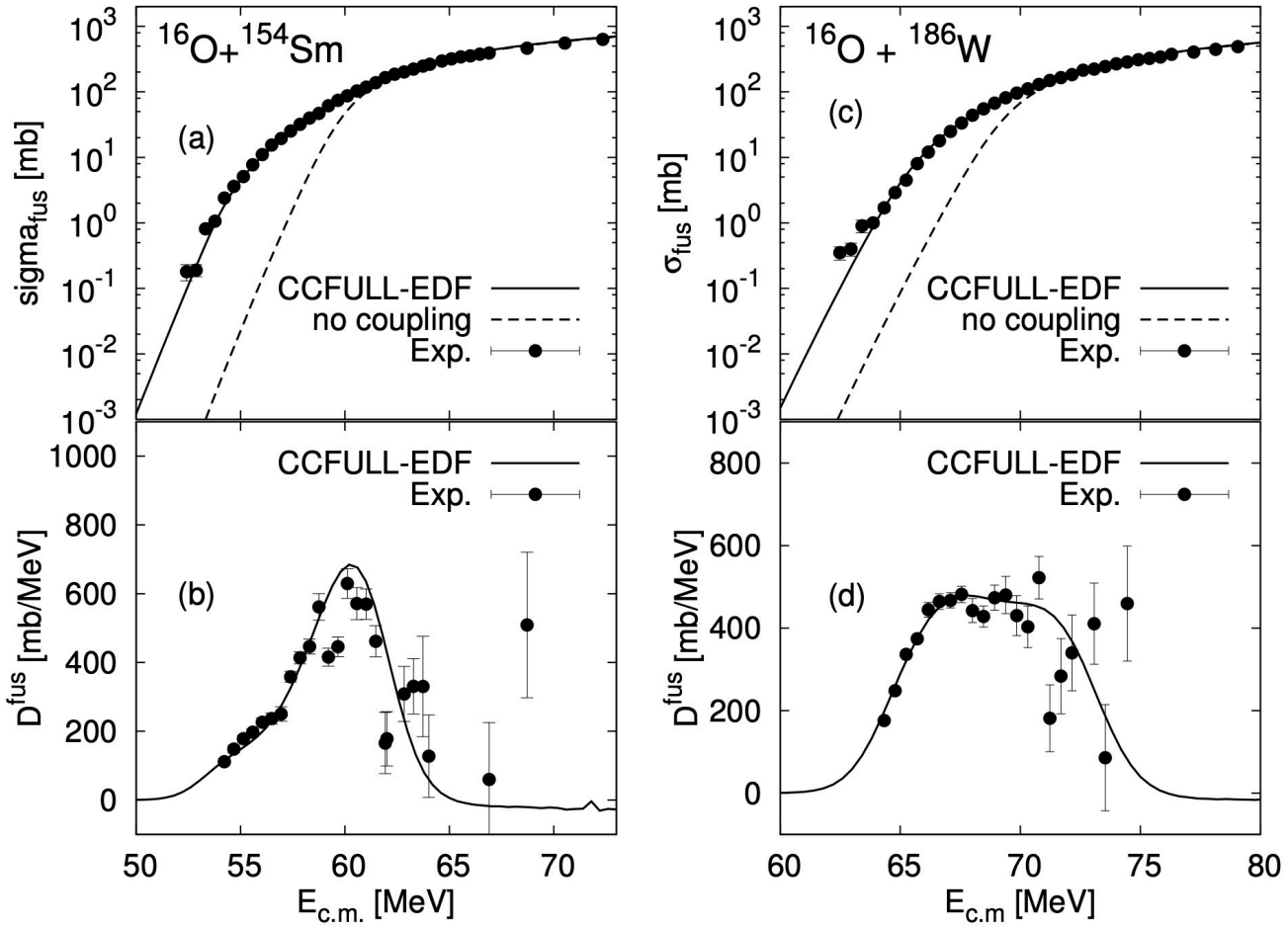
✓ a non-local form of $E_{\rm kin}$ → shell effect

$$E_{\rm kin}[\rho] = \frac{\hbar^2}{2m}\int d\boldsymbol{r}d\boldsymbol{r}'\, \rho^{5/6}(\boldsymbol{r}) f(\boldsymbol{r},\boldsymbol{r}')\rho^{5/6}(\boldsymbol{r}')$$

X. Wu, G. Colo, K. Hagino, and P. Zhao,
PRL136, 092501 (2026).

Energy functional approach to heavy-ion fusion reactions

sub-barrier fusion cross sections



- ✓ SkM* (spherical SHF)
- ✓ $V(r) \rightarrow N\,V(r)$
- ✓ $V(r)$: fit with WS → CC
- ✓ β_λ for CC: adjusted

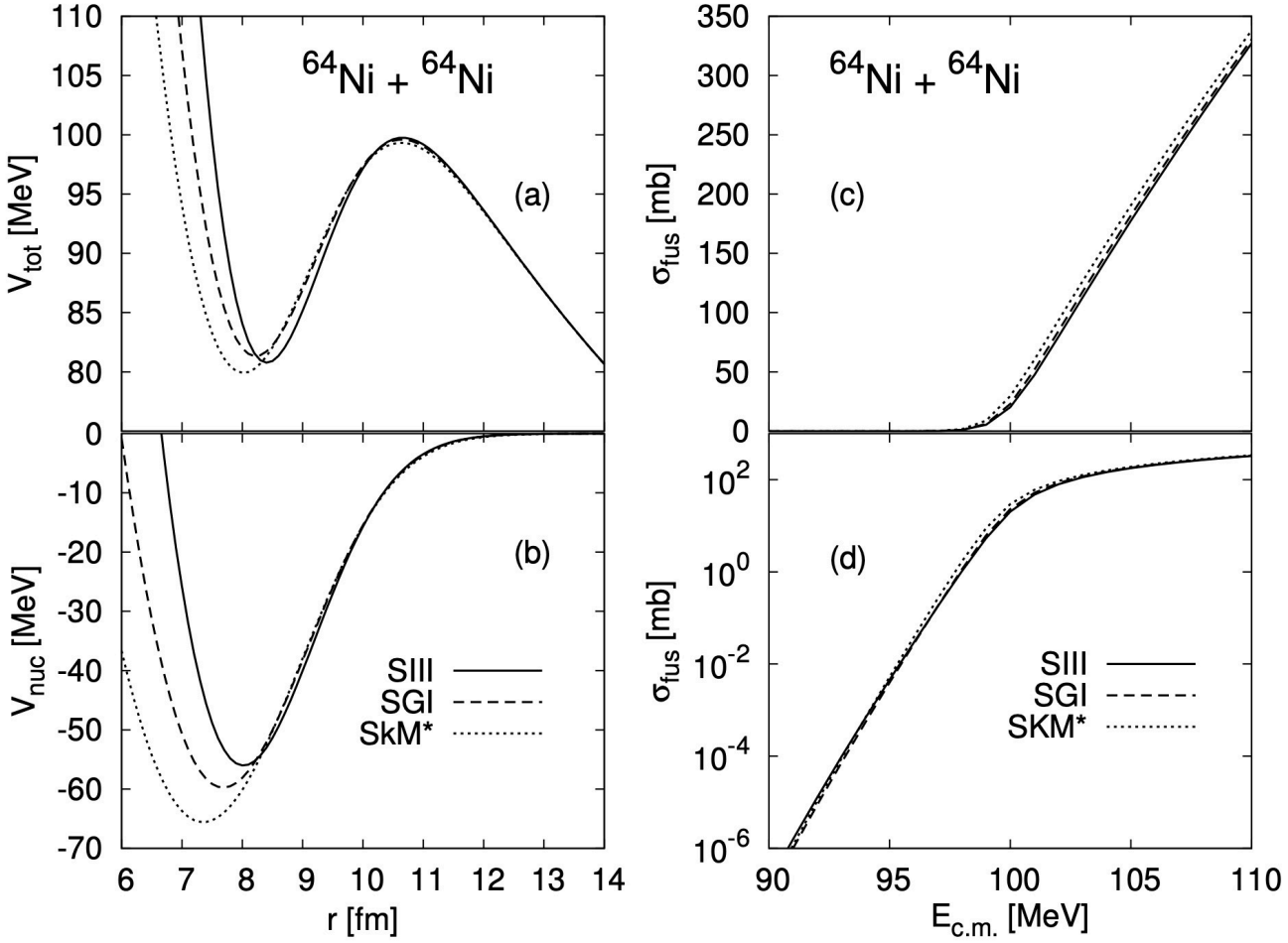
fusion barrier distribution

$$D_{\rm fus} = \frac{d^2(E\sigma)}{dE^2}$$

Muhammad Zamrun F.,
K. Hagino, and N. Takigawa,
AIP Conf. Proc. 853, 309 (2006).

Energy functional approach to heavy-ion fusion reactions

role of incompressibility *K*



incompressibility *K*

SIII: 355.4 MeV

SGI: 269 MeV

SkM* : 216.7 MeV

On going work:

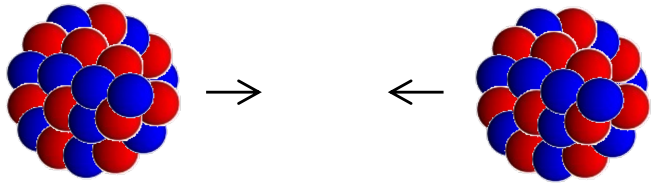
similar studies with KIDS (Hyun, Choi, Hana, So, Kim, Hagino…)

cf. Kyungsik Kim's talk tomorrow

Muhammad Zamrun F., K. Hagino, and N. Takigawa, AIP Conf. Proc. 853, 309 (2006).

Summary

Low-energy Nuclear Reaction Dynamics



- Role of spin-spin interaction

  K. Heo, M.-K. Cheoun, K. Hagino

- imaging elastic scattering ← Fourier transform of $f(\theta)$

  K. Heo and K. Hagino

- $^{12}C+^{12,13}C$ fusion reactions at astrophysical energies

  not a direct collaboration, but C+C is
  a Cheoun-san's favorite system

- DFT approach to nucleus-nucleus potential

  On going: Hyun, Choi, Hana, So, Kim, Hagino…

Chat GPT

KIDS