An application of shell model to low-energy induced fission

fissione di un biscotto nucleare

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NUCLEAR COOKIES SEMINARS

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1. Introduction
2. Shell Model for induced fission
3. Summary

G.F. Bertsch and K.H., Phys. Rev. C107, 044615 (2023).

Nuclear Cookies Seminar at the University of Padova, 2023.7.13

Introduction: particle emission decays of unstable nuclei

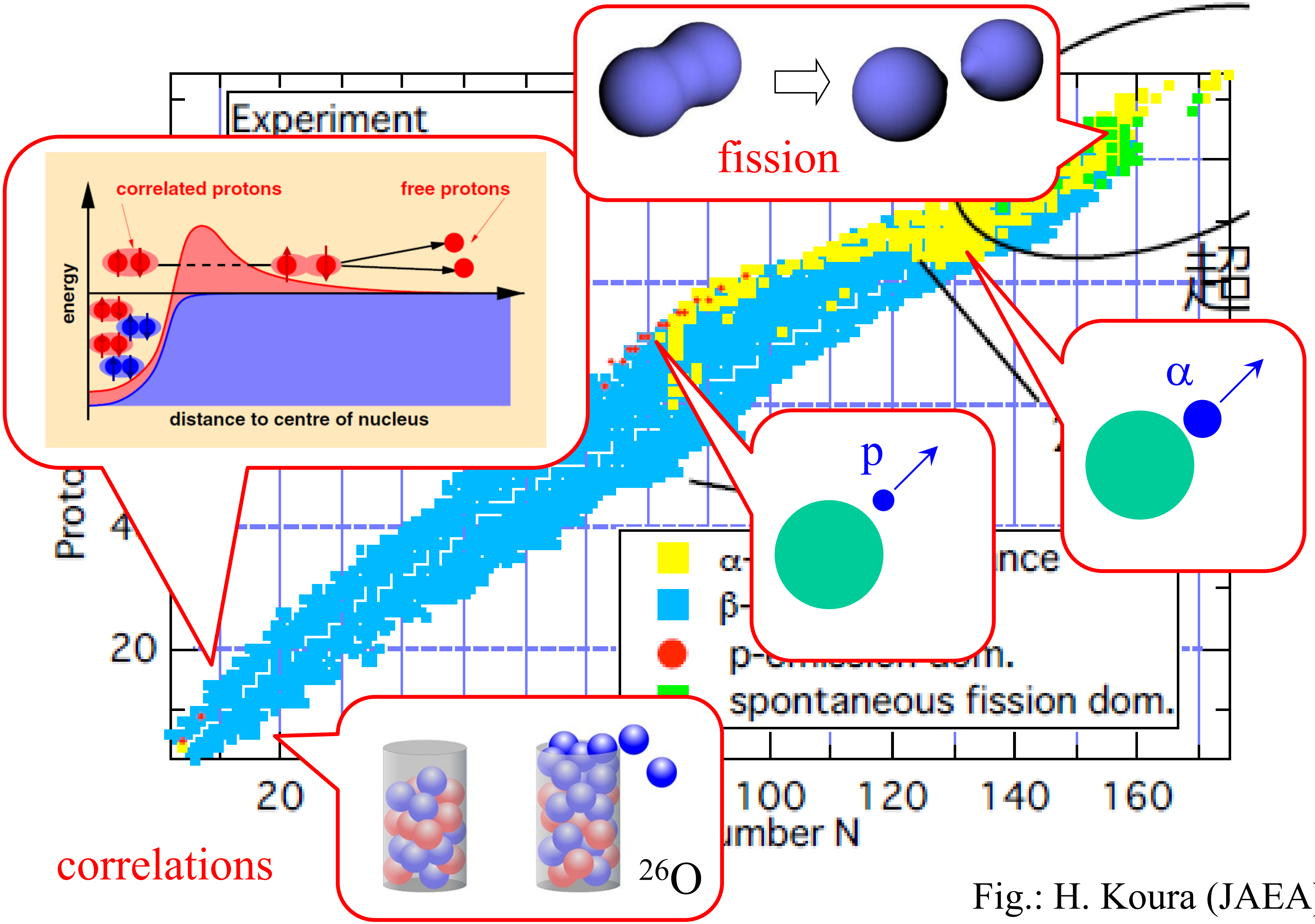
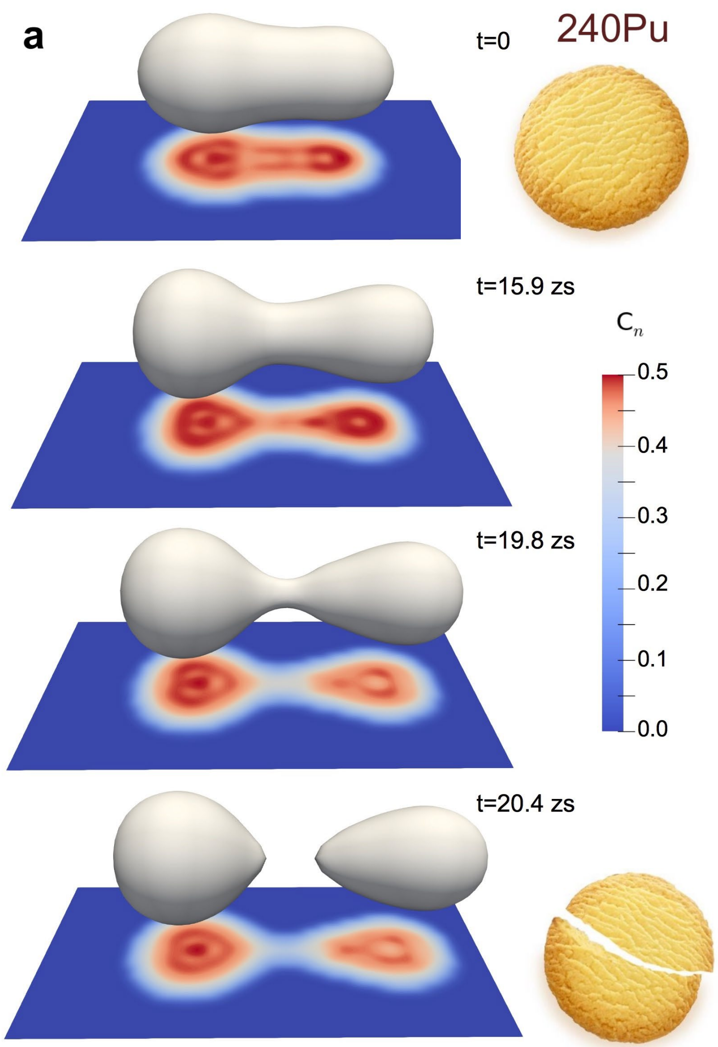


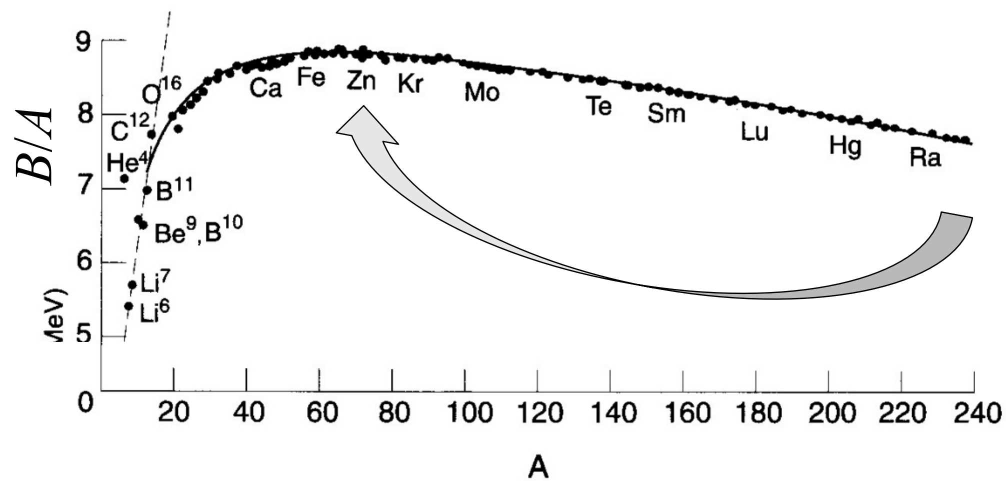
Fig.: H. Koura (JAEA)

Nuclear Fission



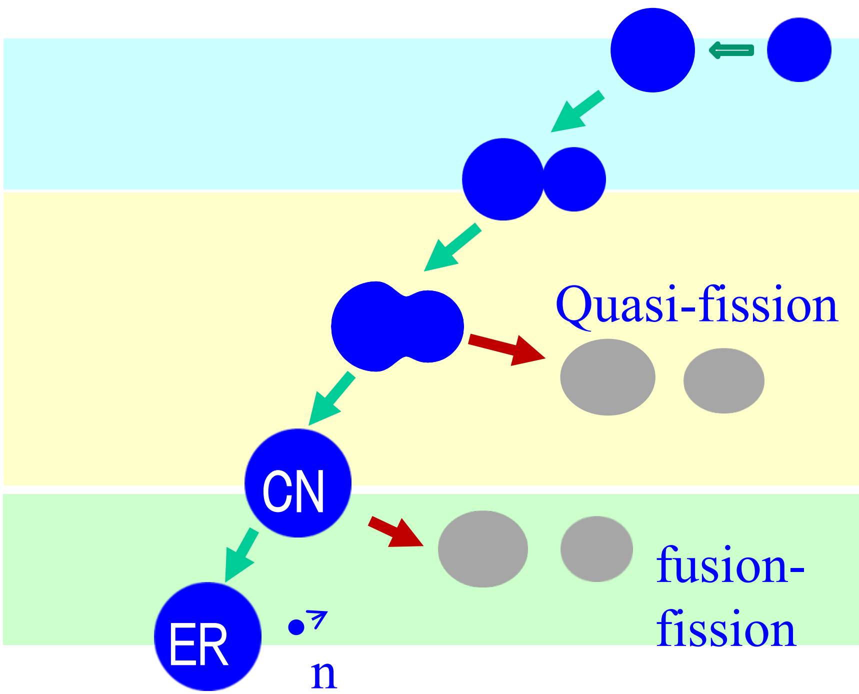
G. Scamps and C. Simenel, Nature 564 (2018) 382

- discovered about 80 years ago (in 1938) by Hahn and Strassmann
- a primary decay mode of heavy nuclei

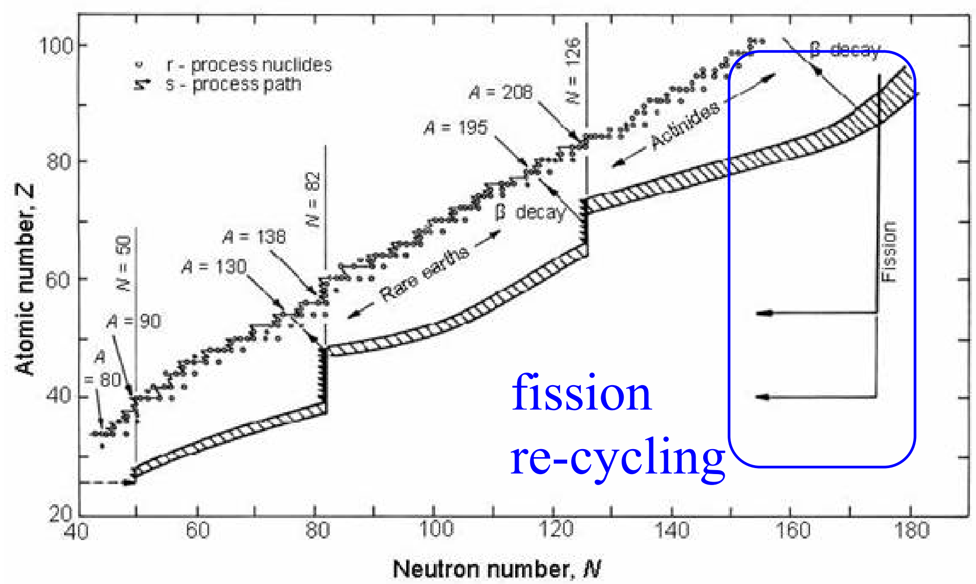


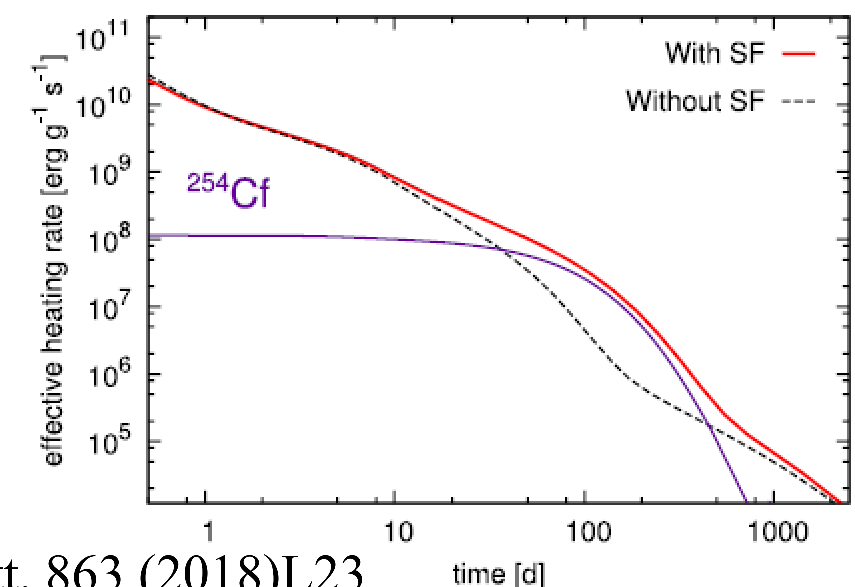
- important role in:
  - energy production
  - superheavy elements
  - r-process nucleosynthesis
  - production of neutron-rich nuclei

Superheavy elements



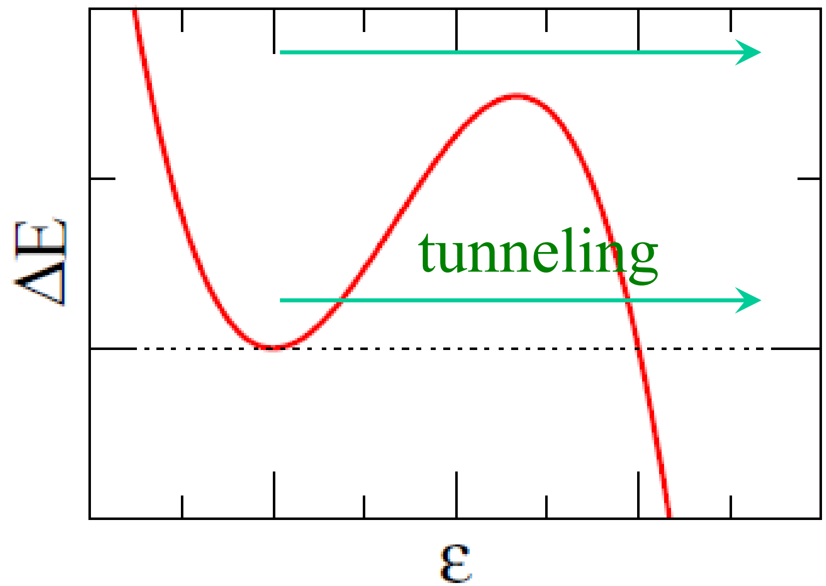
fission in r-process nucleosynthesis



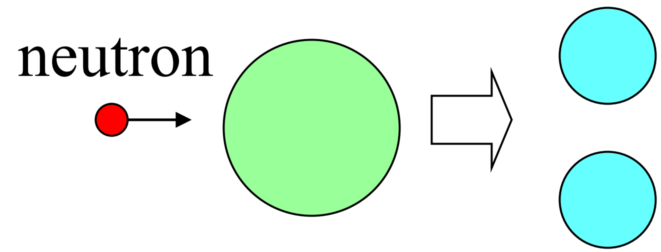


Y. Zhu et al.,
Astrophys. J. Lett. 863 (2018)L23

➢ various fission processes

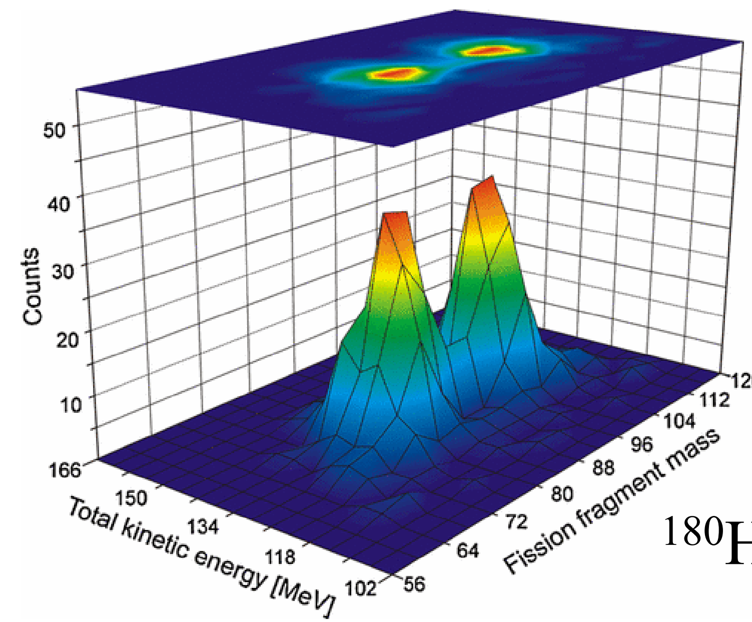


induced fission



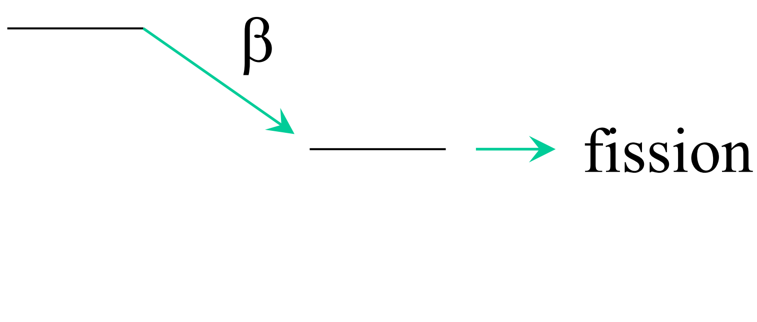
spontaneous fission

asymmetric fission



^{180}Hg

beta-delayed fission

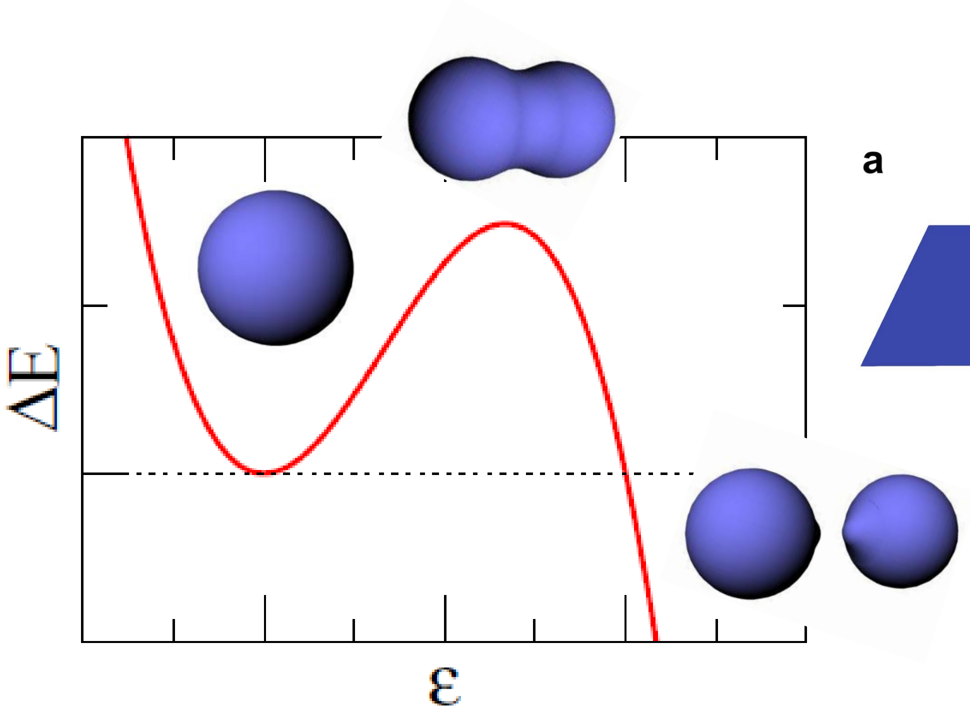


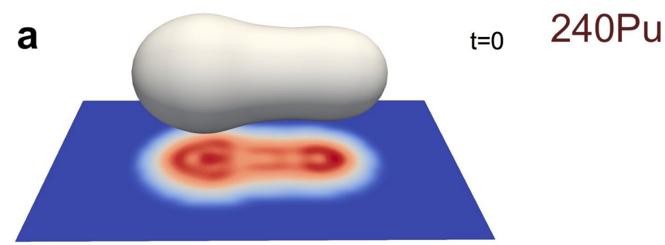
A.N. Andreyev et al., PRL105('10)252502

- macroscopic understanding:

  competition between the surface and the Coulomb energies

  → fission barrier



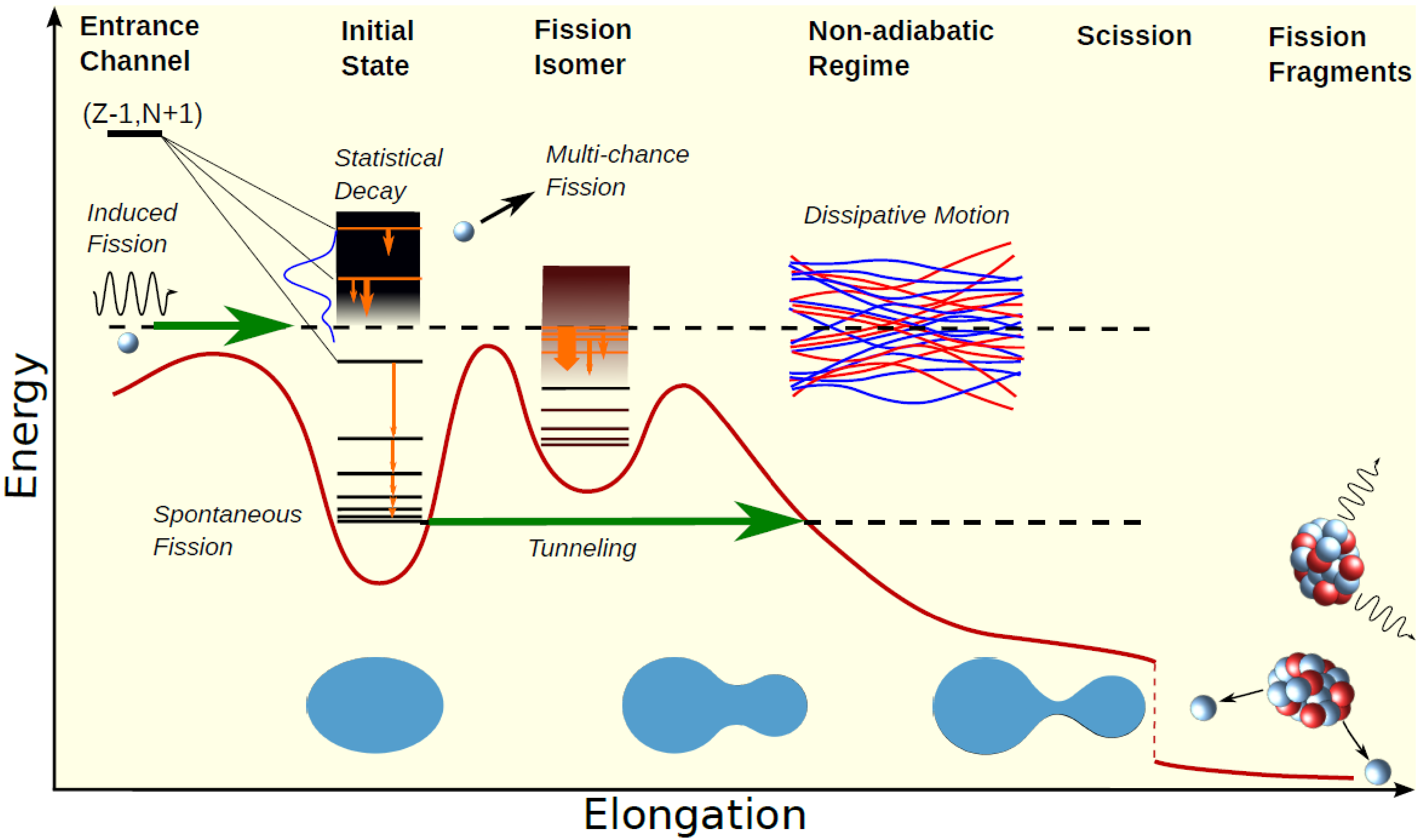


- a microscopic understanding:

  large change of nuclear shape

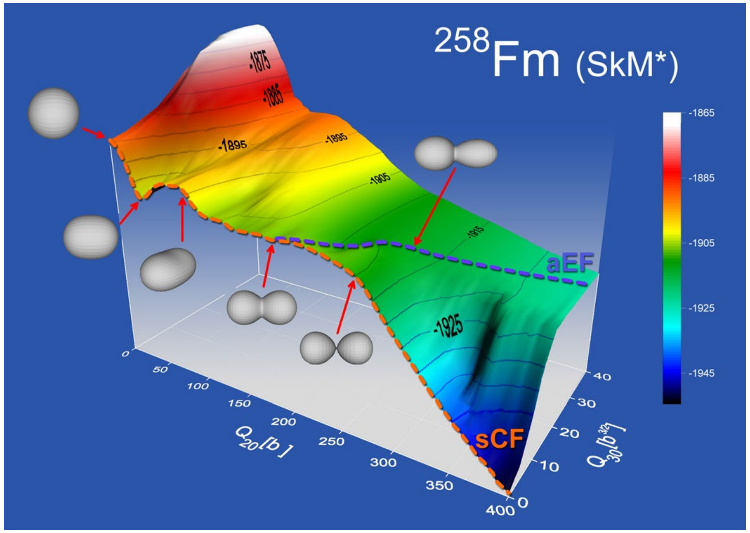
  → microscopic description : far from complete

  an ultimate goal of nuclear physics



"Future of fission theory"
M. Bender et al., J. of Phys. G47, 113002 (2020)

- spontaneous fission



A. Staszczak, A. Baran, J. Dobaczewski, and W. Nazarewicz, PRC80 ('09) 014309

constrained Hartree-Fock (+B) method:

$$\delta\langle\Phi|H-\lambda Q_{20}|\Phi\rangle = 0$$

$$\rightarrow \Phi(Q_{20}),\ E(Q_{20})$$

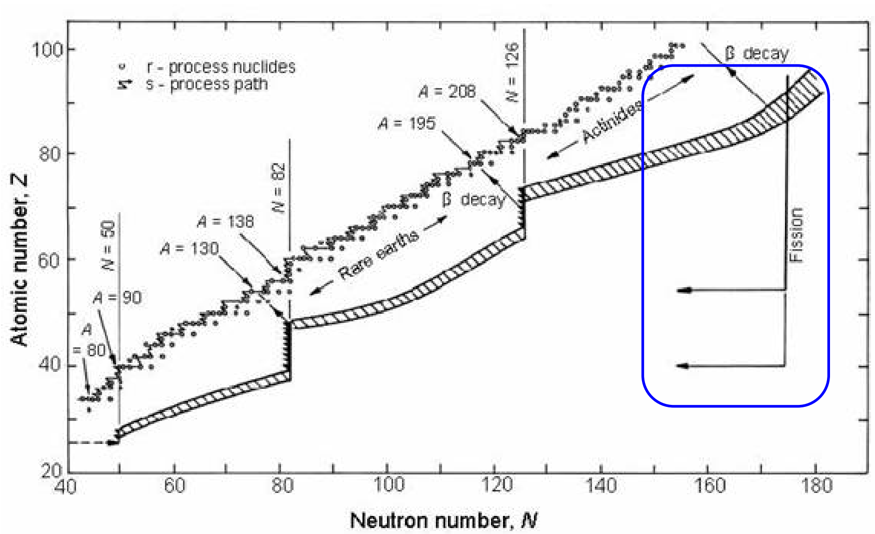
$$\rightarrow P = \exp\left[-2\int dq\sqrt{\frac{2B(q)}{\hbar^2}(V(q)-E)}\right]$$

- induced fission

almost nothing has been developed for a microscopic theory

Importance of a microscopic approach

➢ r-process nucleosynthesis



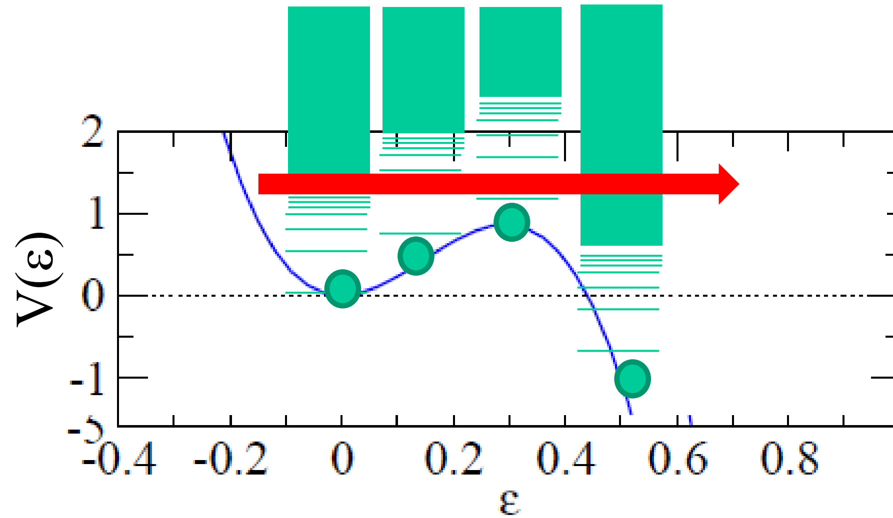
(neutron induced) fission of neutron-rich nuclei

→ low E^* and low $\rho(E^*)$

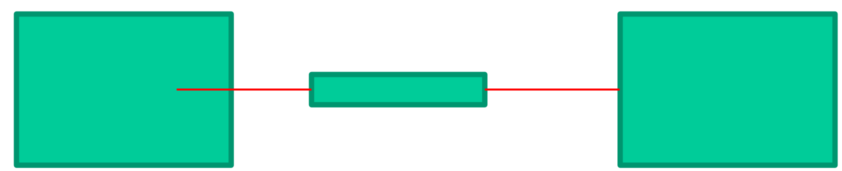
✓ Validity of statistical models?

✓ Validity of the Langevin approach?

➢ barrier-top fission



CN | barrier-top | post barrier



high ρ(E) | low ρ(E) | high ρ(E)

↓

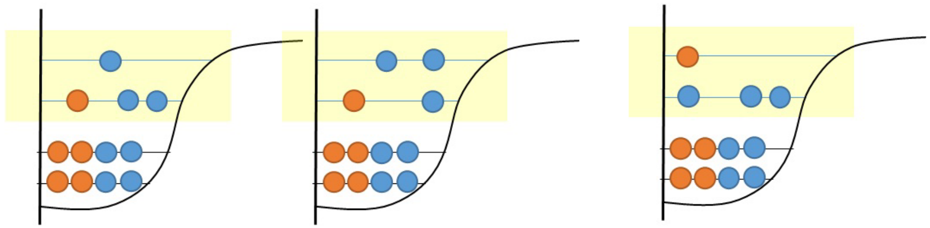
discrete levels

How to connect to a many-body Hamiltonian?

Shell model approach?

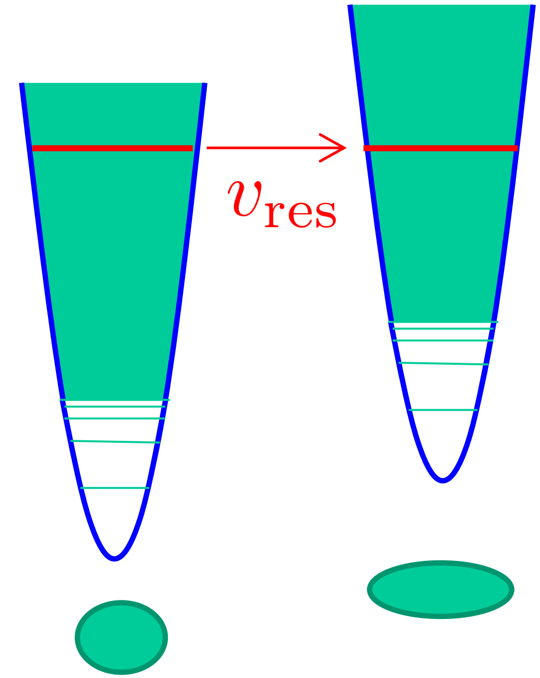
A similar approach for nuclear fission?

Shell model



$$|\Psi\rangle = v_1|m_1\rangle + v_2|m_2\rangle + v_3|m_3\rangle + \cdots$$

Figure: Noritaka Shimizu (Tsukuba)



many-particle many-hole configurations in a mean-field potential

→mixing by <u>residual interactions</u>

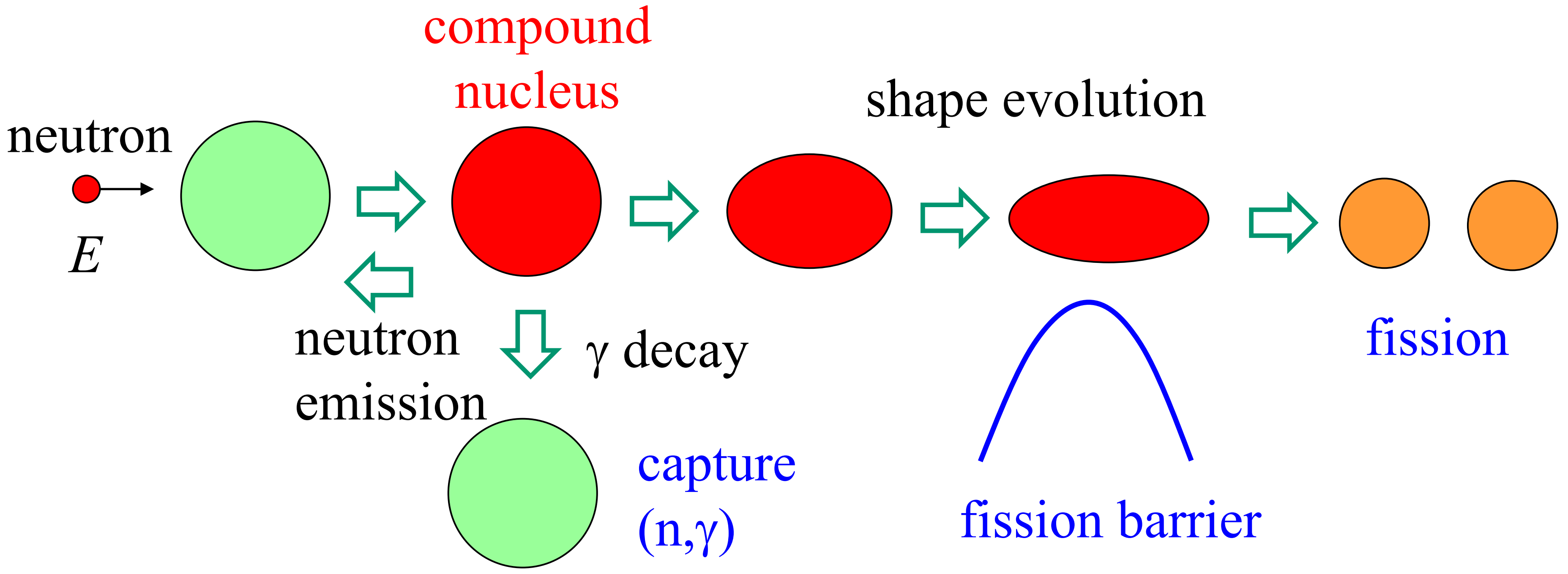
- ➢ Many-body configurations in a MF pot. for each shape
- ➢ hopping due to res. int.

→ shape evolution

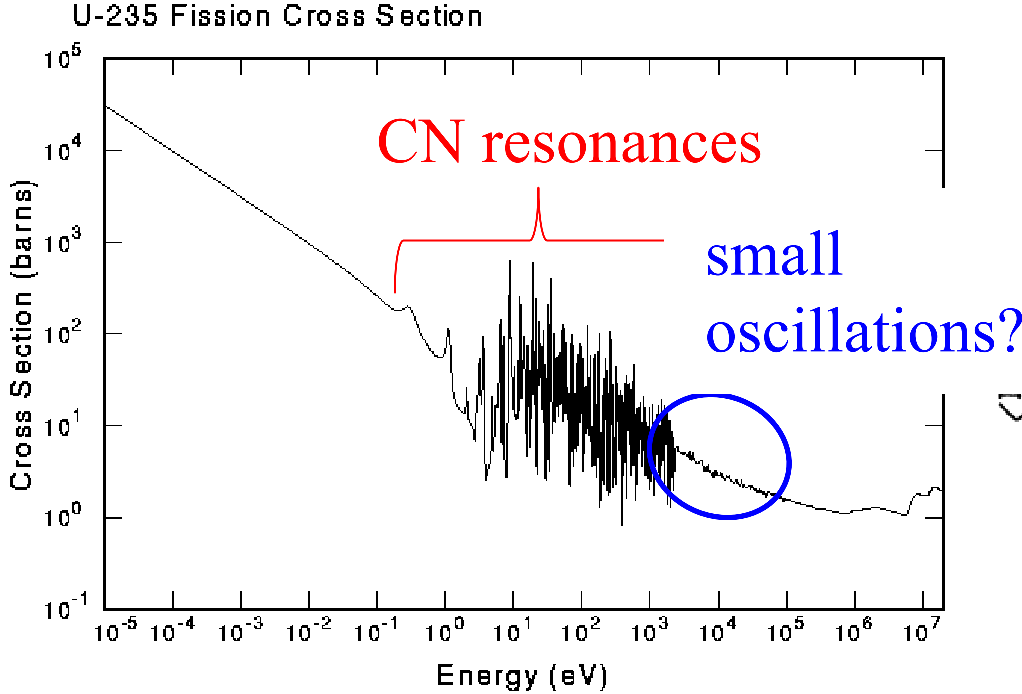
$$|\Psi\rangle = \int dQ \sum_i f_i(Q)|\Phi_Q(i)\rangle$$

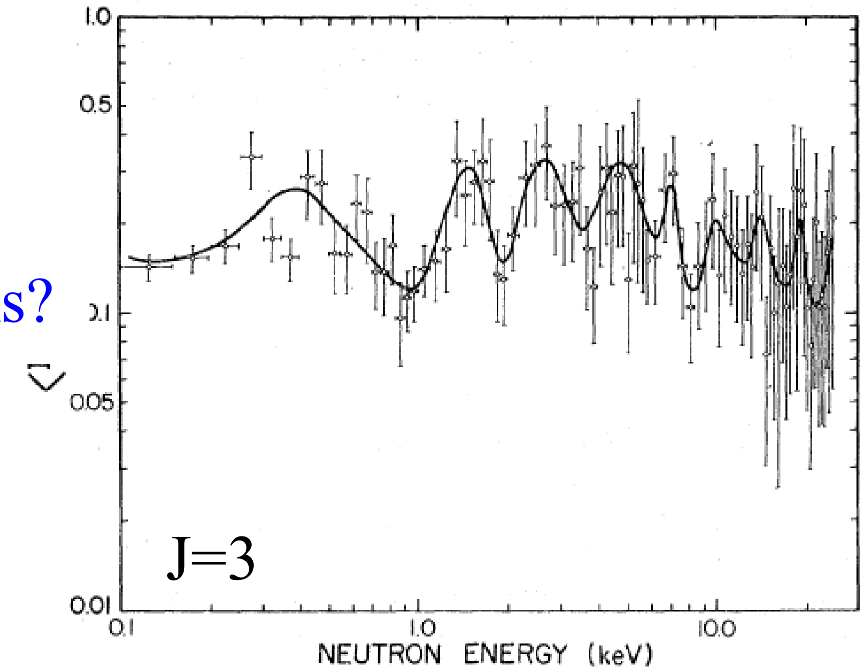
GCM with excited states

a good connection to nuclear reaction theory a process which we would like to dicscuss



$n + {}^{235}U \rightarrow {}^{236}U^* \rightarrow$ fission

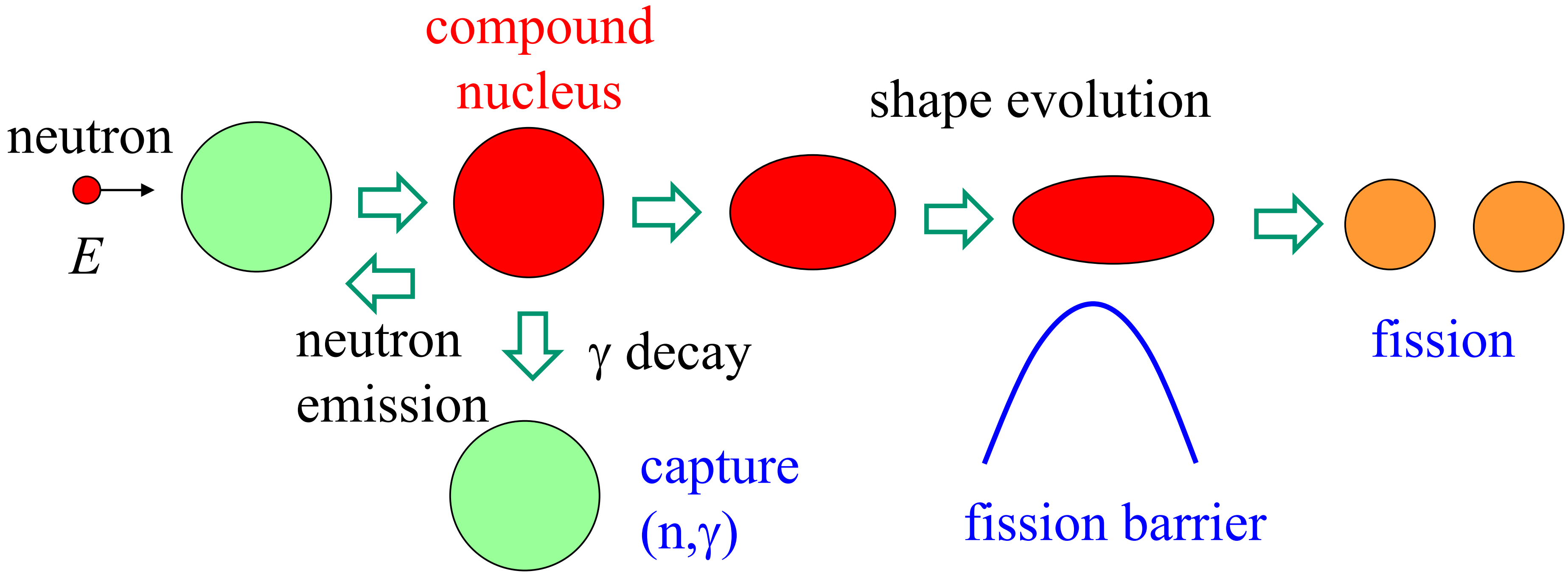


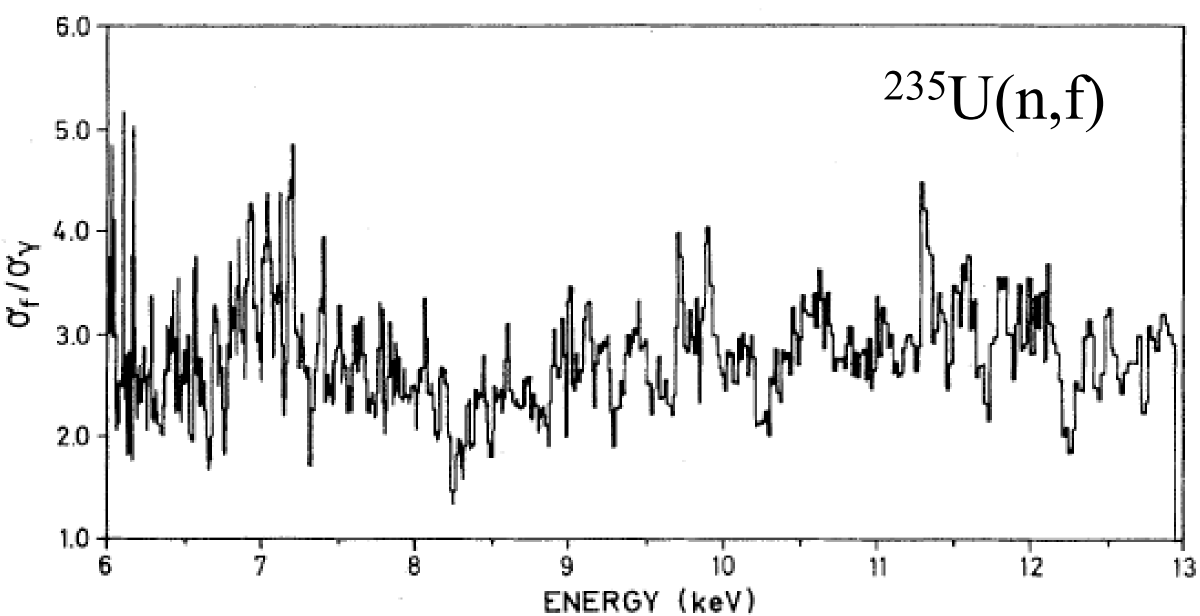


https://t2.lanl.gov/nis/tour
/sch002.html

M.S. Moore et al.,
PRC18 (‘78) 1328

a process which we would like to dicscuss





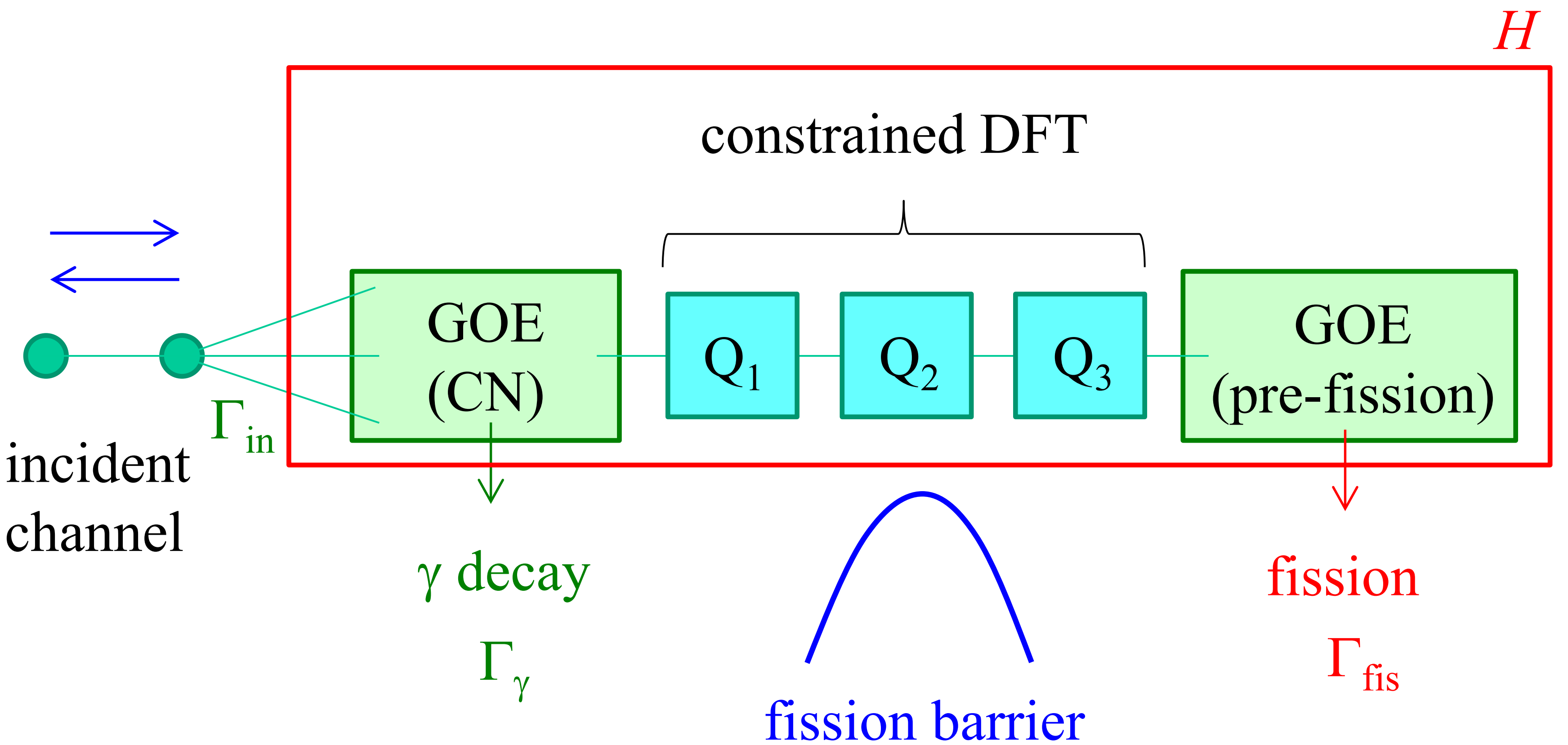
branching ratio

$$\alpha^{-1} = \frac{\sigma_f}{\sigma_\gamma}$$

sensitive to intermediate structure

M.S. Moore et al., PRC30 ('84) 214

a process which we would like to dicscuss



Reaction theory (absorption probability):

$$T_{\text{fis}} = Tr[\Gamma_{\text{in}} G(E) \Gamma_{\text{fis}} G^\dagger(E)]$$

$$T_{\text{cap}} = Tr[\Gamma_{\text{in}} G(E) \Gamma_{\gamma} G^\dagger(E)]$$

"Datta formula"

$$G(E) = [H - i\Gamma/2 - EO]^{-1}$$

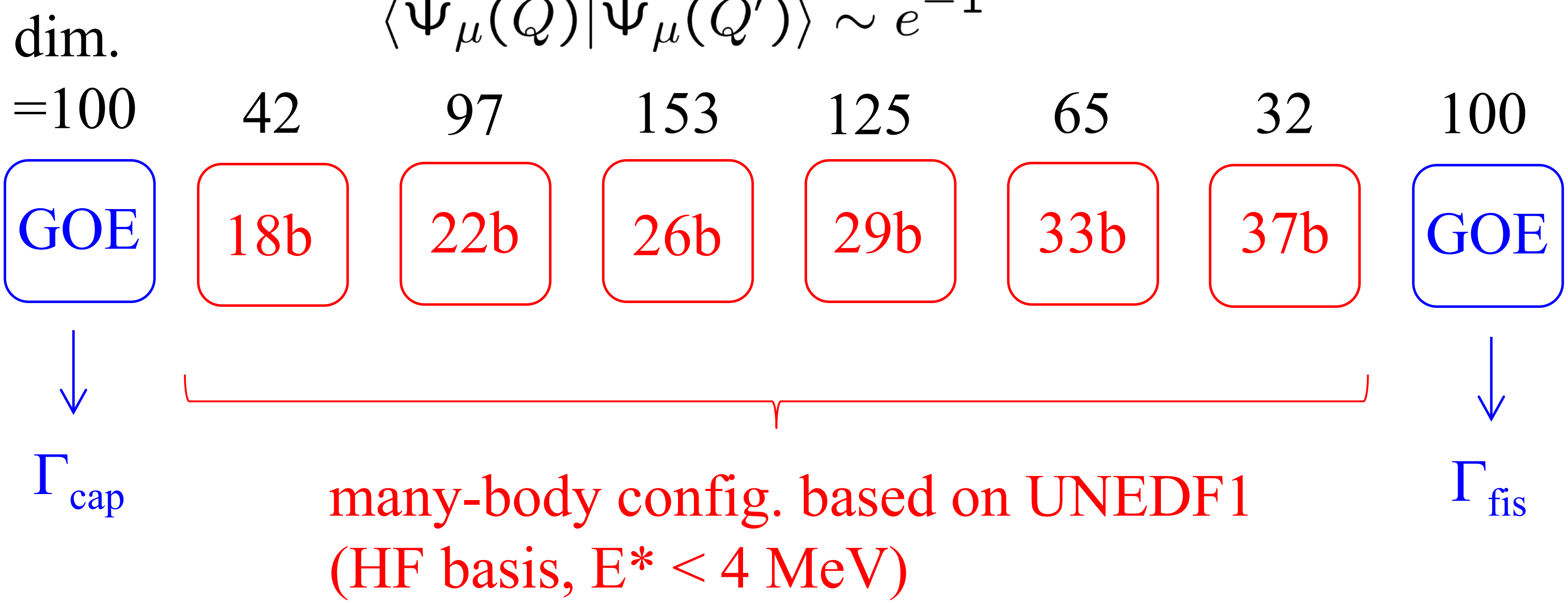
Calculations based on Skyrme Hartree-Fock method

G.F. Bertsch and K.H., Phys. Rev. C107, 044615 (2023).

Simplifications:

- ✓ ^{236}U : only neutron configurations, up to 4 MeV
- ✓ Dynamics of the first barrier: axial symmetry
- ✓ seniority-zero config. only: occupation of (K, -K)
- ✓ a scaled fission barrier with $B_f = 4$ MeV

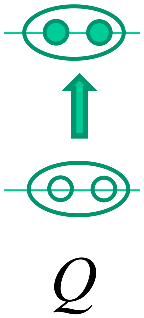
$$\langle \Psi_\mu(Q) | \Psi_\mu(Q') \rangle \sim e^{-1}$$

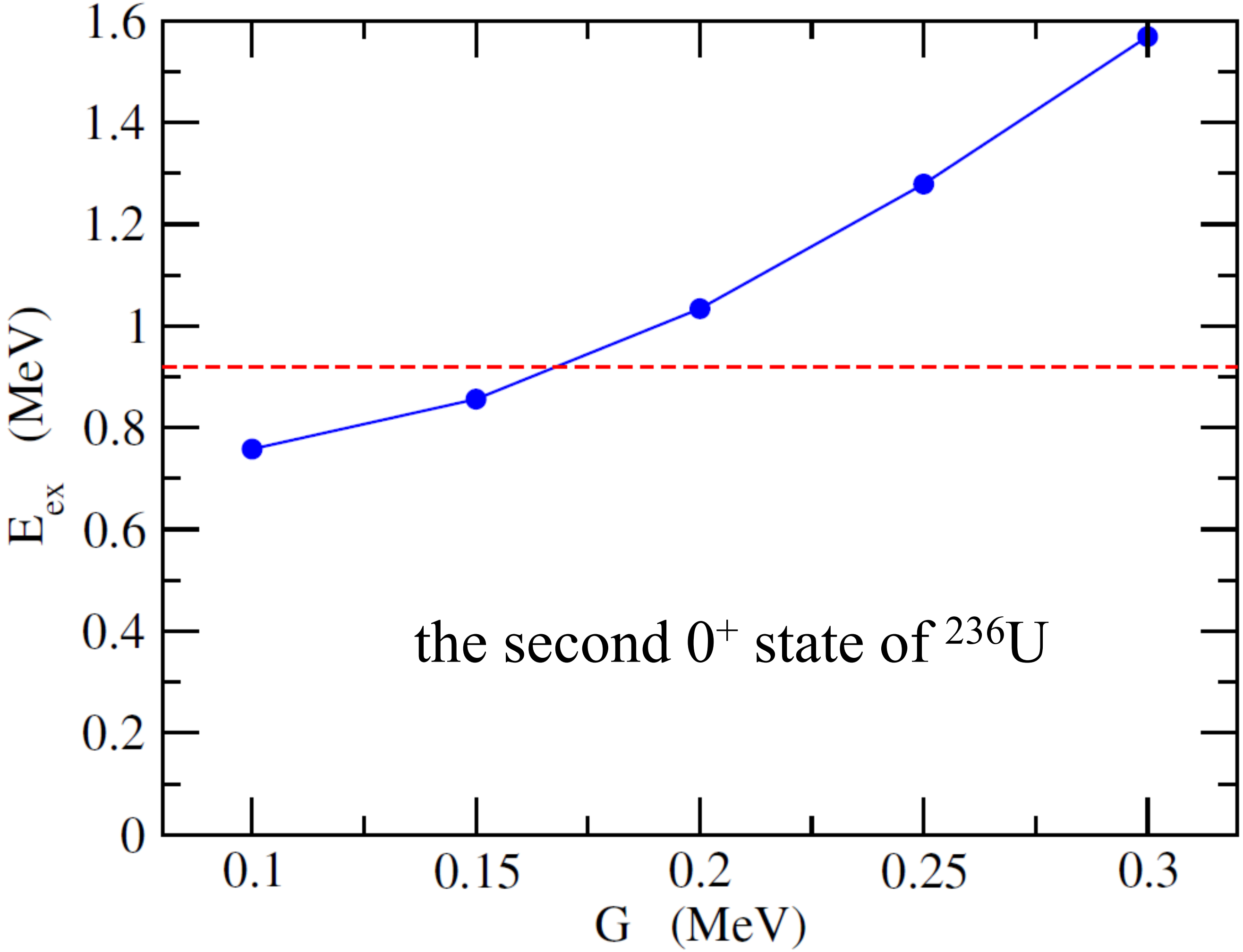


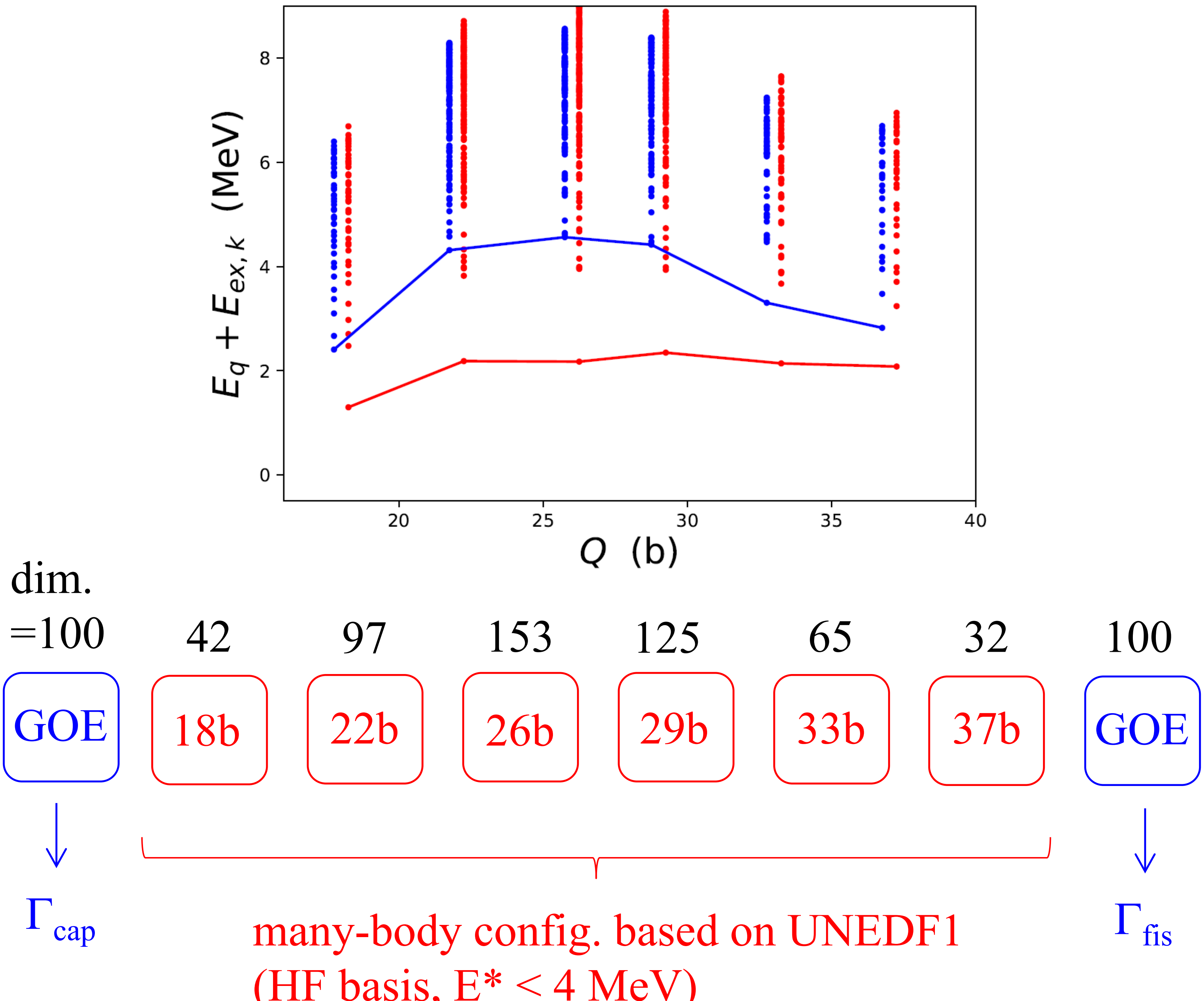
714x714 Hamiltonian matrix

$$H = \sum_k \epsilon_k a_k^\dagger a_k - G P^\dagger P$$

$$P = a_k^\dagger a_{\bar{k}}^\dagger$$







$E_q + E_{ex,k}$ (MeV)
Q (b)
0
2
4
6
8
20
25
30
35
40
dim.
=100
42
97
153
125
65
32
100
GOE
18b
22b
26b
29b
33b
37b
GOE
Γ_{cap}
Γ_{fis}
many-body config. based on UNEDF1
(HF basis, E* < 4 MeV)

Calculations based on Skyrme Hartree-Fock method

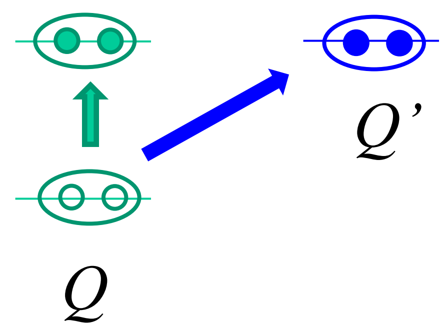
G.F. Bertsch and K.H., Phys. Rev. C107, 044615 (2023).

GOE → Γ_{cap} | 18b | 22b | 26b | 29b | 33b | 37b | GOE → Γ_{fis}

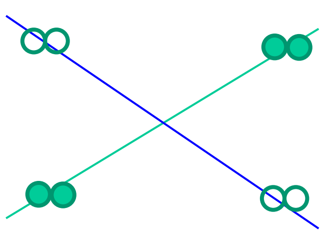
many-body config. based on UNEDF1

✓ overlap: $\langle \Psi_\mu(Q) | \Psi_\mu(Q') \rangle \sim e^{-1}$

✓ pairing: $v_{\text{pair}} = -GP^\dagger P$



✓ diabatic:

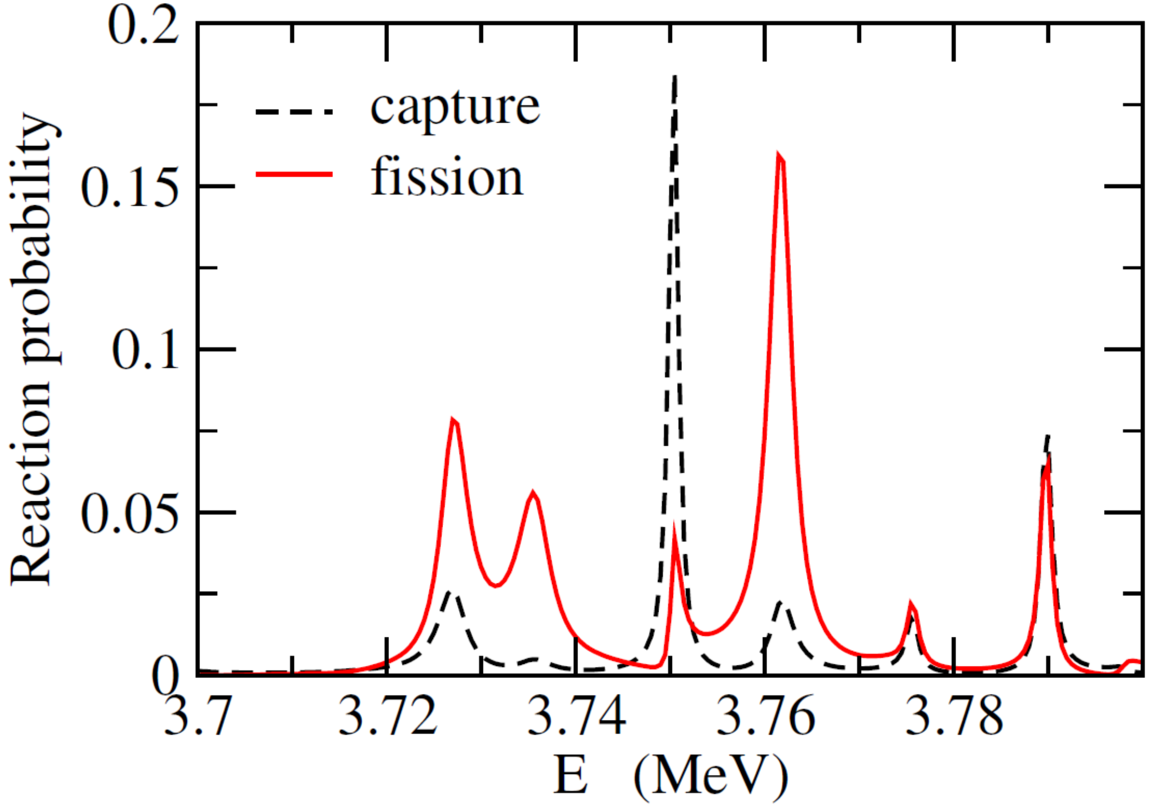


$$\frac{\langle \Psi_\mu(Q) | H | \Psi_\mu(Q') \rangle}{\langle \Psi_\mu(Q) | \Psi_\mu(Q') \rangle} \sim E_\mu(\bar{Q}) - h_2(\Delta Q)^2$$

✓ Γ_{cap}: exp. data (scaled according to N_{GOE}), Γ_{fis}: insensitivity

$$T_{\mathsf{fis}}(E) = Tr[\Gamma_{\mathsf{in}} G(E) \Gamma_{\mathsf{fis}} G^\dagger(E)]$$

$$T_{\mathsf{cap}}(E) = Tr[\Gamma_{\mathsf{in}} G(E) \Gamma_{\gamma} G^\dagger(E)]$$



$\Gamma_{in} = 0.01$ MeV

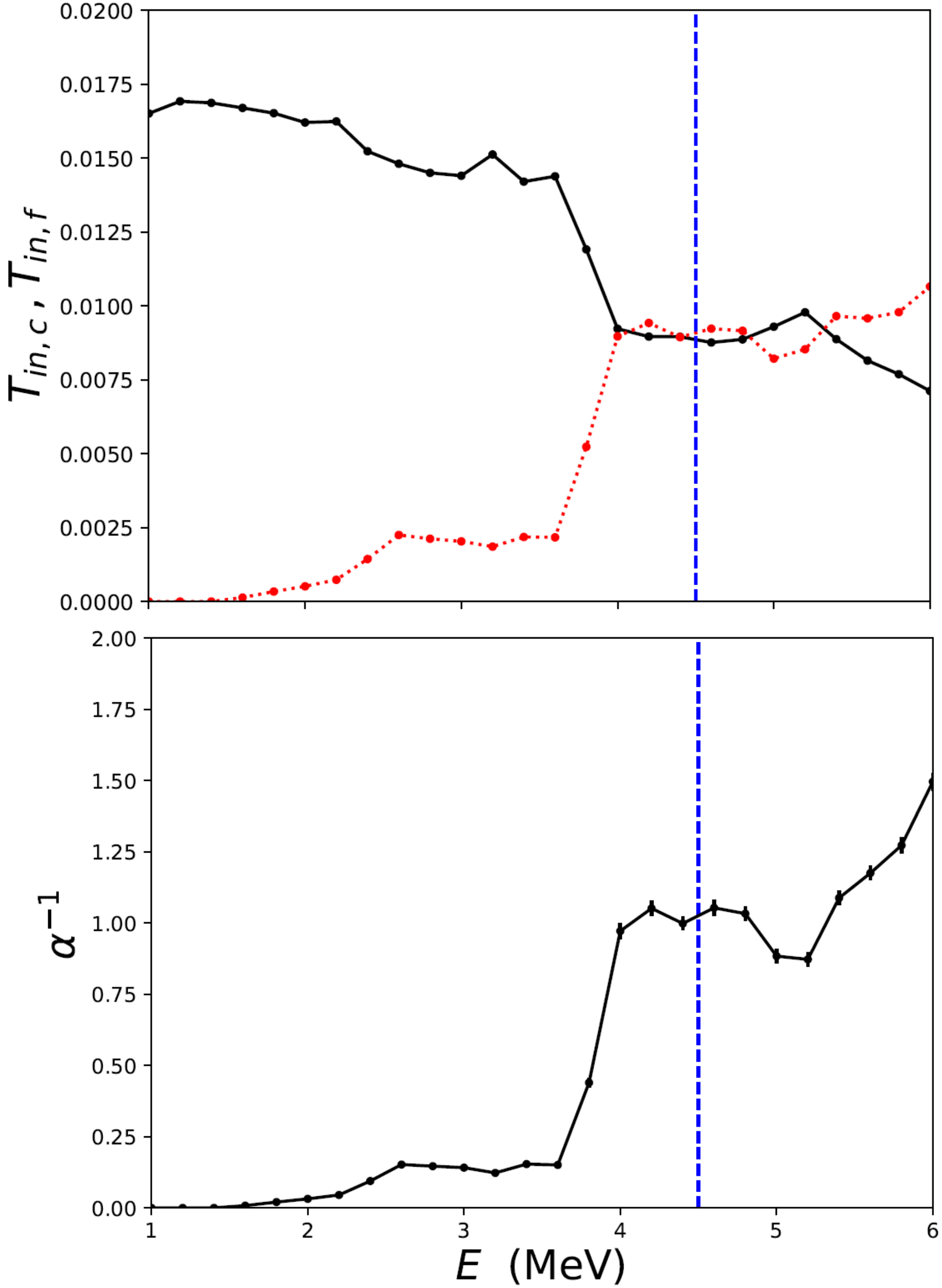
$\Gamma_{cap} = 0.00125$ MeV

$\Gamma_{fis} = 0.015$ MeV

energy average

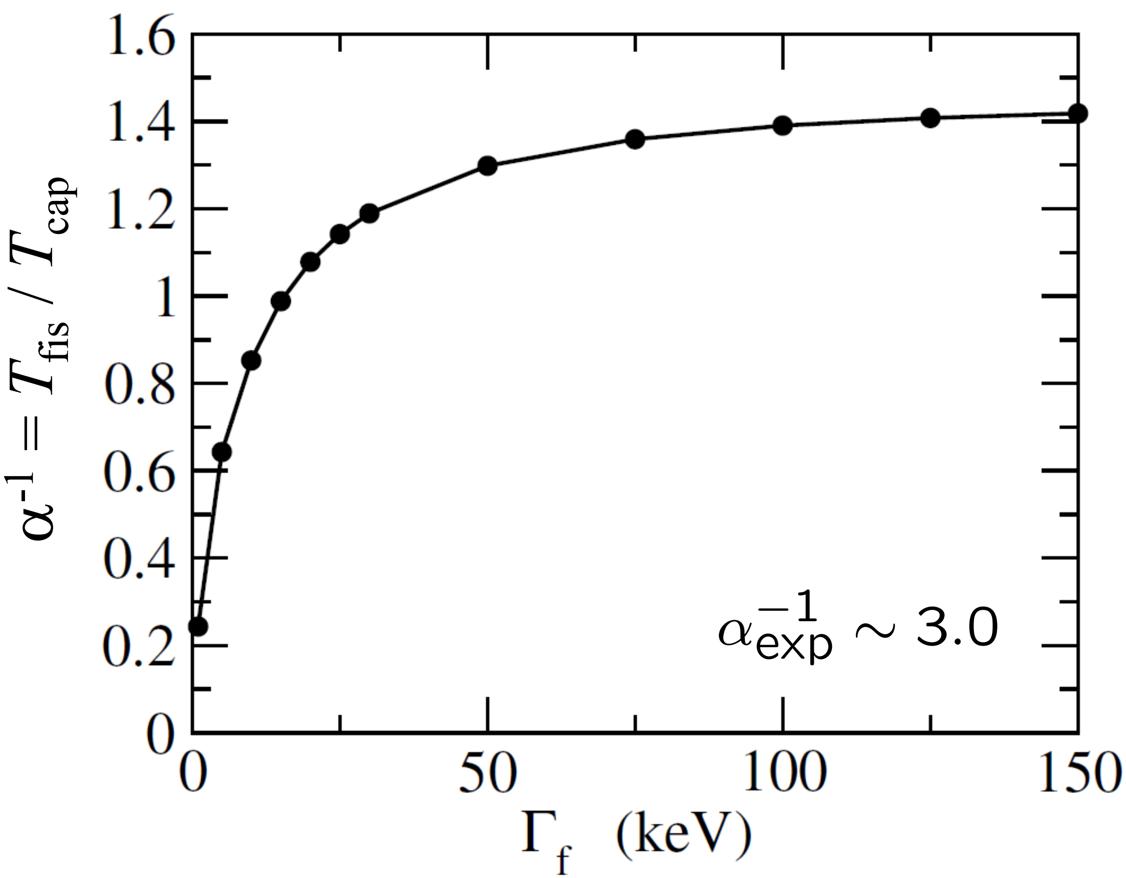
$$\alpha^{-1} = \frac{\int_{\Delta E} T_{\mathsf{fis}}(E') dE'}{\int_{\Delta E} T_{\mathsf{cap}}(E') dE'}$$

$\Delta E = 0.5$ MeV

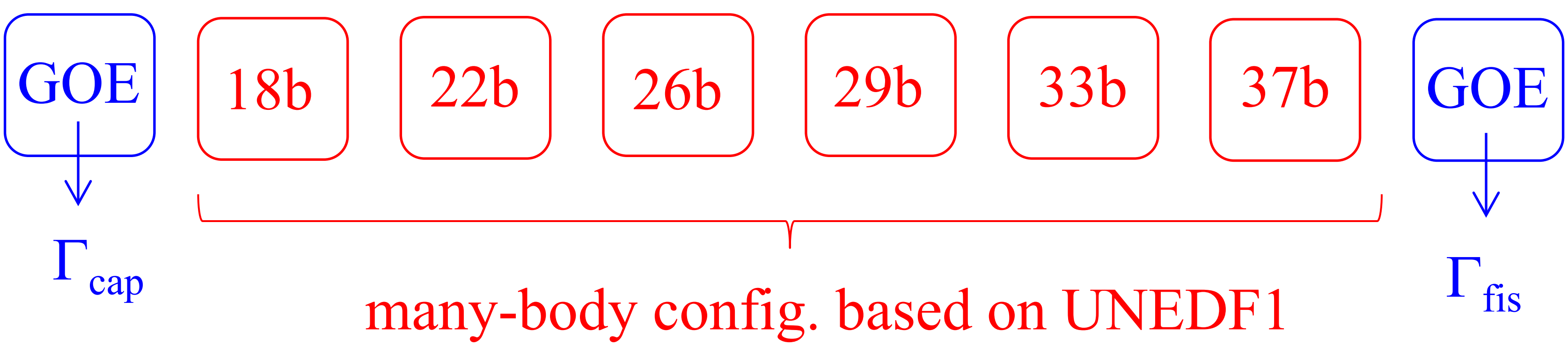


0.0200
0.0175
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
$T_{in,c}$, $T_{in,f}$
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
α^{-1}
1
2
3
4
5
6
E (MeV)

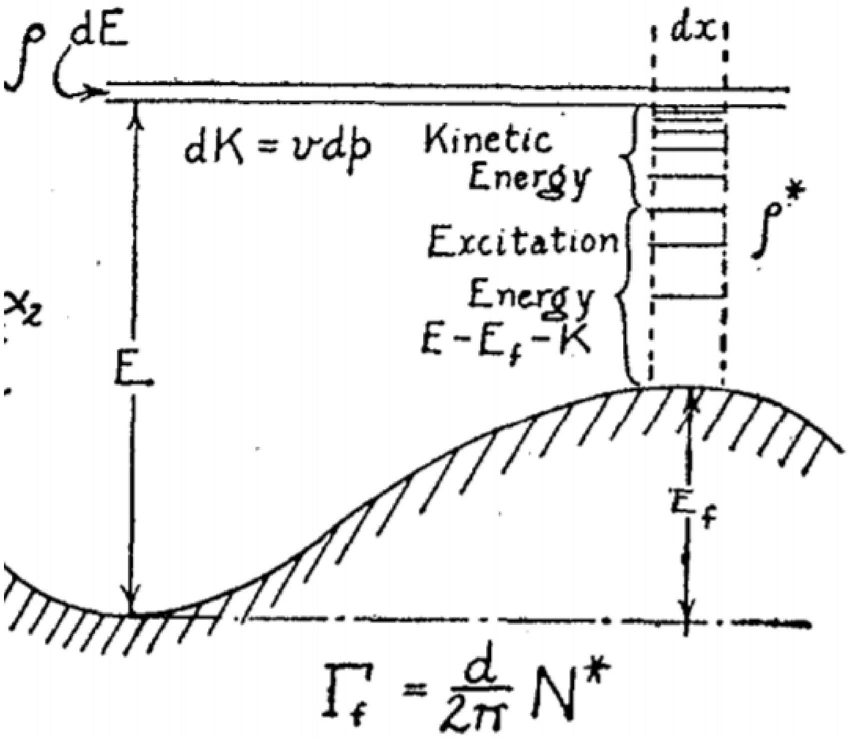
insensitivity property



insensitive to Γ_f
(post-barrier dynamics)
→the main assumption of TST



[the transition state theory]

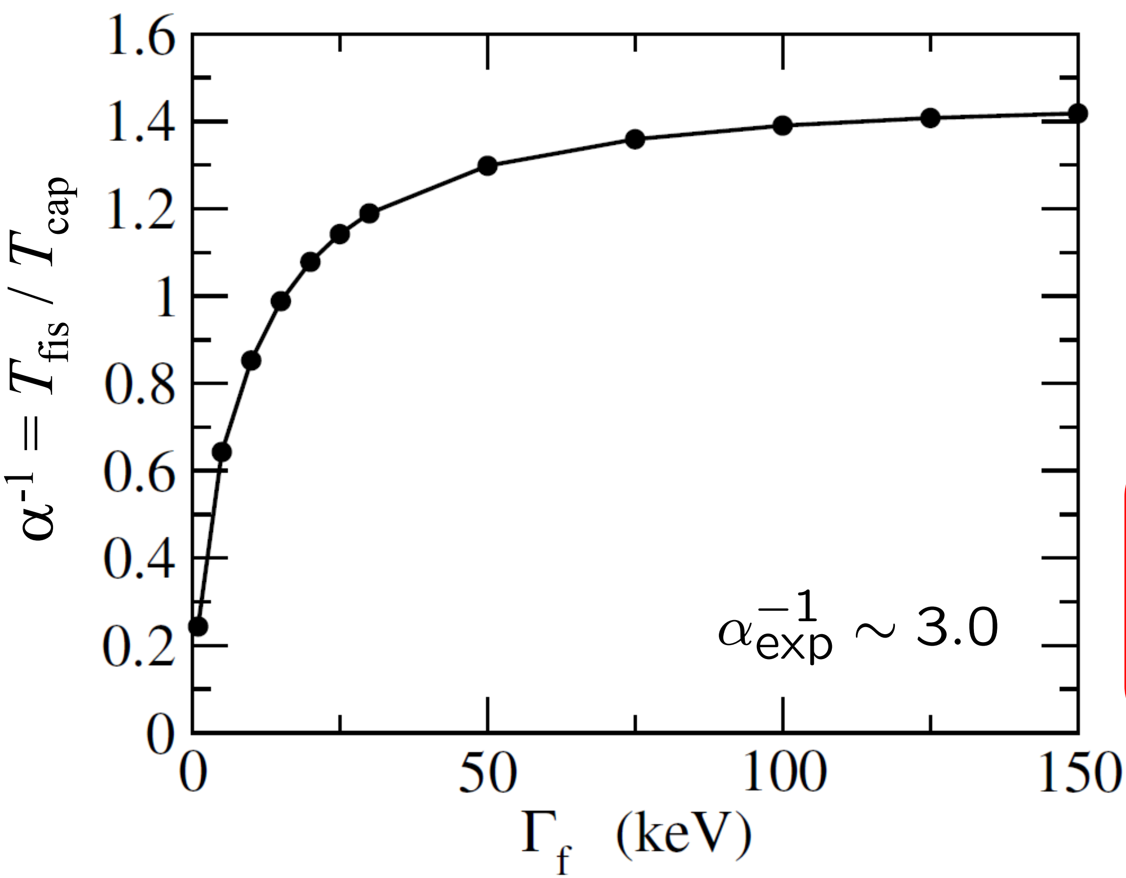


N. Bohr and J.A. Wheeler, Phys. Rev. 56, 426 (1939)

$$\Gamma_f = \frac{1}{2\pi\rho_{\rm gs}(E^*)}\int_0^{E^*-B_f}\rho_{\rm sd}(E^*-B_f-K)dK \rightarrow \frac{1}{2\pi\rho_{\rm gs}(E^*)}\sum_c T_c$$

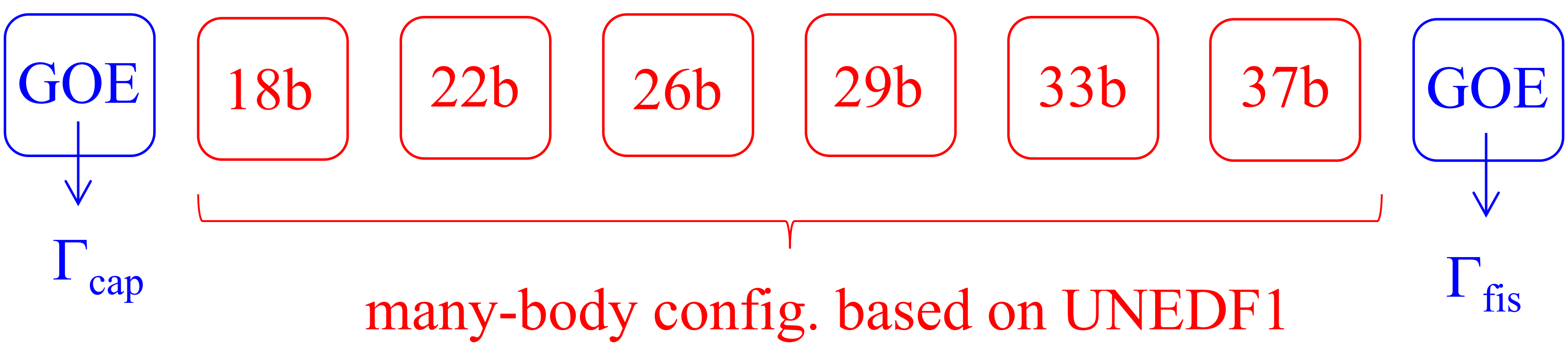
- ✓ decay dynamics: entirely determined at the saddle
- ✓ **does not depend on what will happen after the barrier**

insensitivity property



insensitive to Γ_f
(post-barrier dynamics)
→the main assumption of TST

The main assumption of TST is realized for the first time!



sensitivity test

$$\frac{\langle \Psi_\mu(Q)|H|\Psi_\mu(Q')\rangle}{\langle \Psi_\mu(Q)|\Psi_\mu(Q')\rangle} \sim E_\mu(\bar{Q}) - h_2(\Delta Q)^2$$

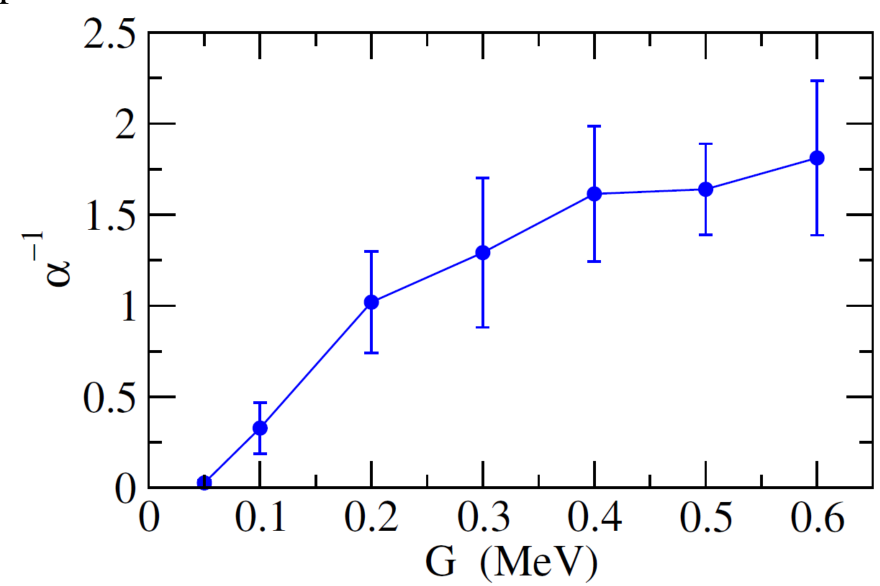
$h_2 \to 2h_2$
$G_{pair} = 0.2$ MeV
$h_2 = 0.3$ MeV
$\to \alpha^{-1} = 1.10$

base set
$G_{pair} = 0.2$ MeV
$h_2 = 0.15$ MeV
$\to \alpha^{-1} = 0.95$

$G_{pair} \to G_{pair}/2$
$G_{pair} = 0.1$ MeV
$h_2 = 0.15$ MeV
$\to \alpha^{-1} = 0.37$

$h_2 \to 0$
$G_{pair} = 0.2$ MeV
$h_2 = 0.0$ MeV
$\to \alpha^{-1} = 0.13$

cf. $\alpha^{-1}_{exp} \sim 3.0$



- sensitive to the pairing, though less than in spontaneous fission
- h_2 effect is not negligible, but insensitive to h_2 when it is large

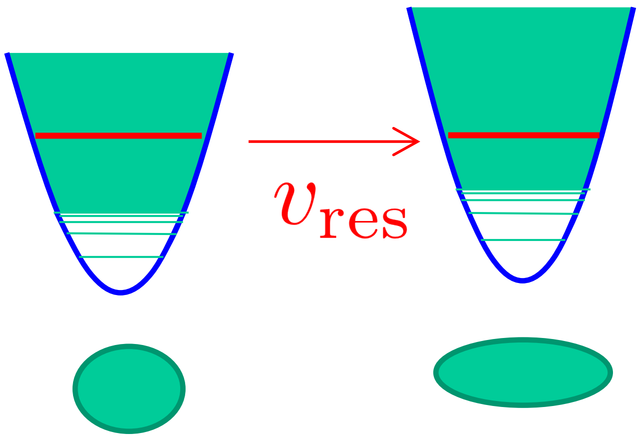
Summary

<u>r-process nucleosynthesis: fission of neutron-rich nuclei</u>

requires a microscopic approach applicable to low E^* and $\rho(E^*)$

also for barrier-top fission

➡ a new approach: shell model + GCM



an application to induced fission of ^{236}U
based on Skyrme EDF

✓ neutron configurations only
✓ pairing and diabatic interactions
✓ truncation at 4 MeV

→an importance of the pairing interaction

<u>Future perspectives:</u> seniority non-zero config. →pn res. interaction

Uzawa, Hagino, Bertsch, arXiv:2303.16488

a large scale calculation (~ 10^6 dim.)