

Towards a microscopic understanding
of **low-energy induced fission**
and its implication to fission in **r-process nucleosynthesis**

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1. Introduction: nuclear fission
2. **Shell Model for induced fission**
3. Summary

G.F. Bertsch and K.H., Phys. Rev. C107, 044615 (2023).

Introduction: particle emission decays of unstable nuclei

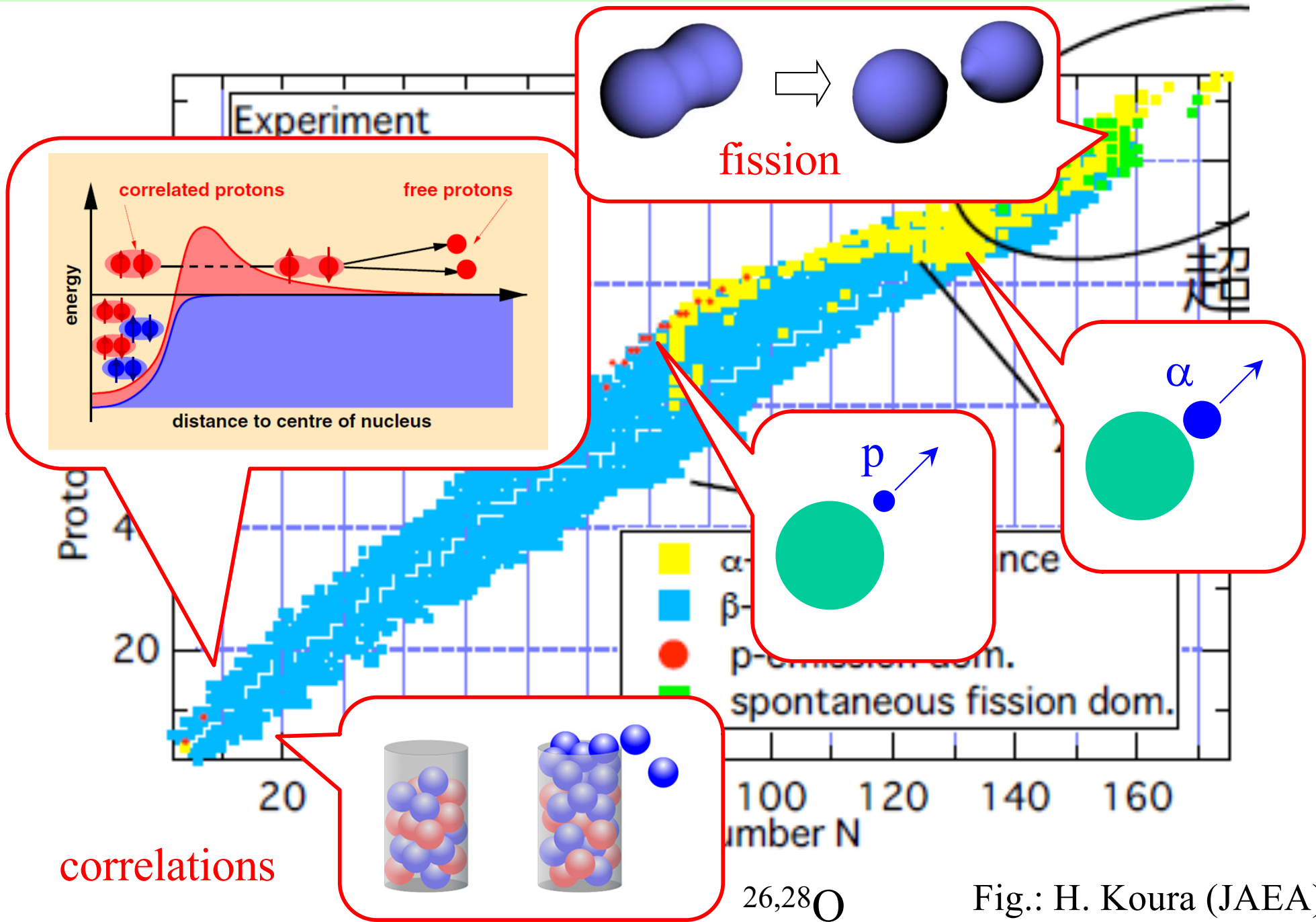
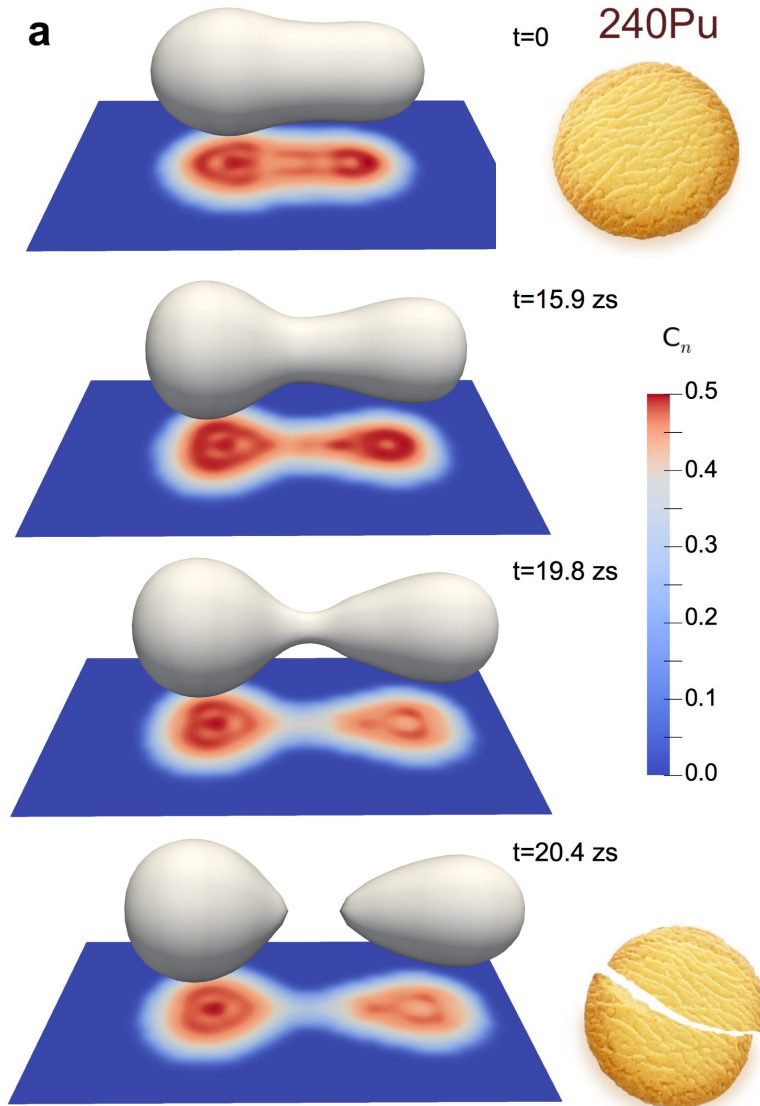


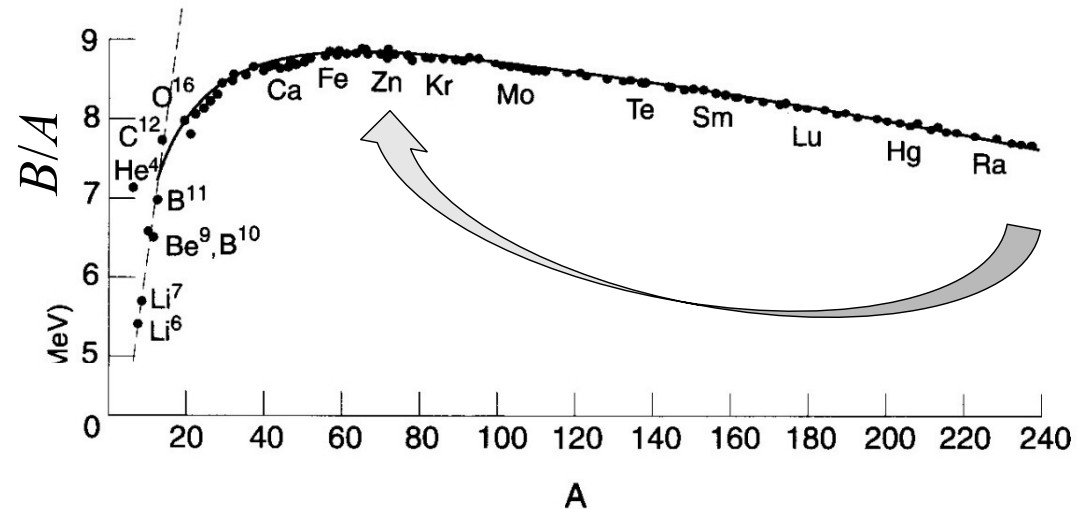
Fig.: H. Koura (JAEA)

Nuclear Fission



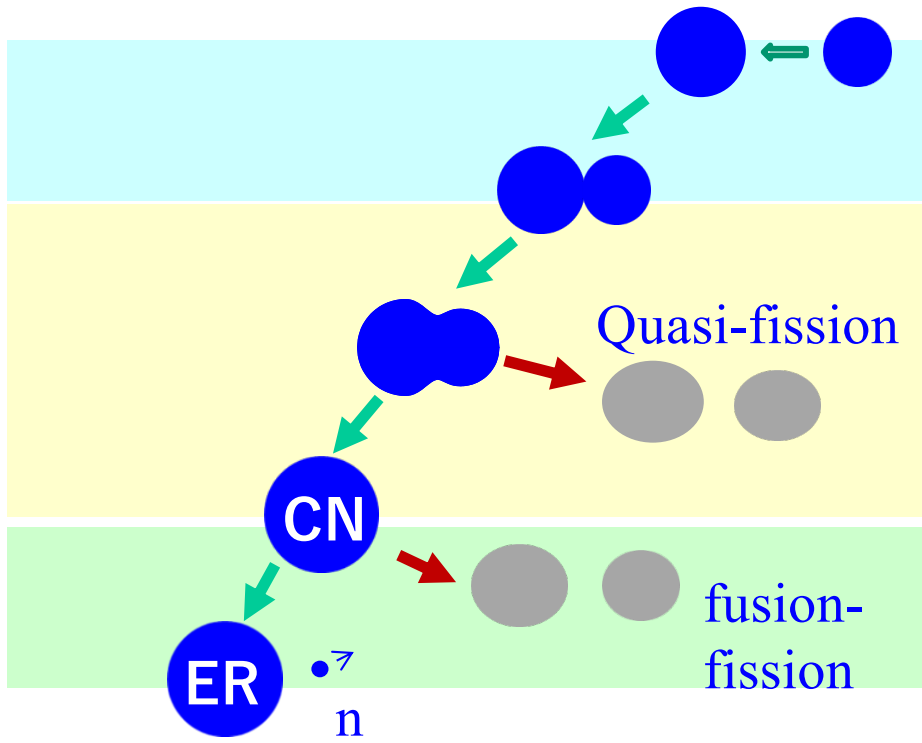
G. Scamps and C. Simenel,
Nature 564 (2018) 382

- discovered about 80 years ago (in 1938) by Hahn and Strassmann
- a primary decay mode of heavy nuclei

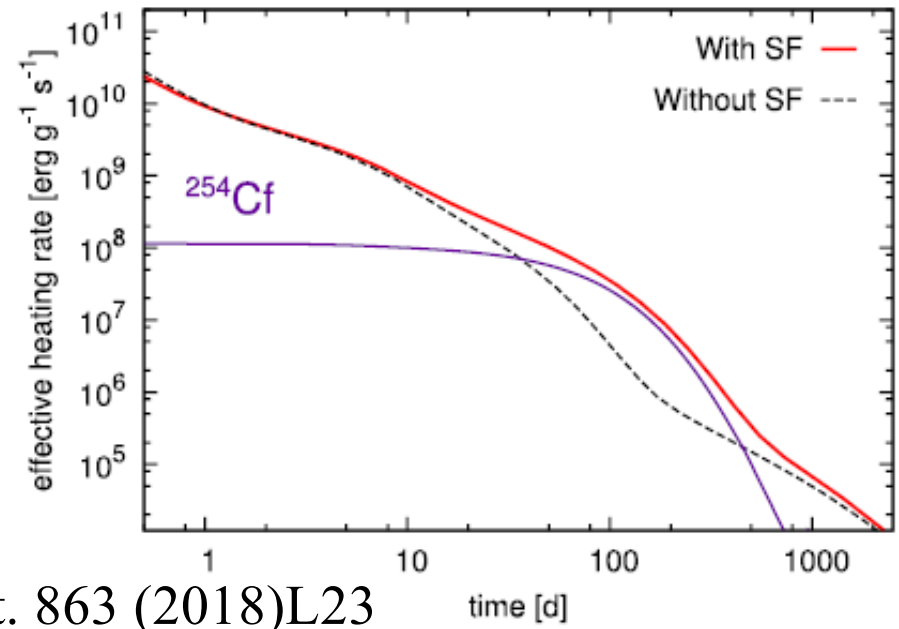
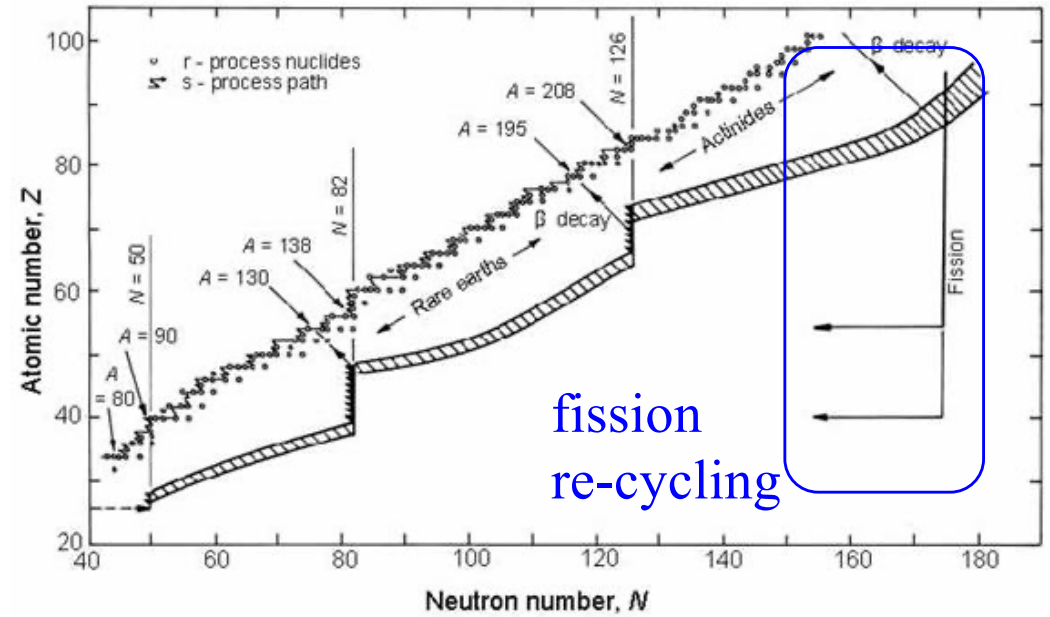


- **important role in:**
 - energy production
 - superheavy elements
 - r-process nucleosynthesis
 - production of neutron-rich nuclei
- cf. the previous talk by Suzuki-san

Superheavy elements



fission in r-process nucleosynthesis

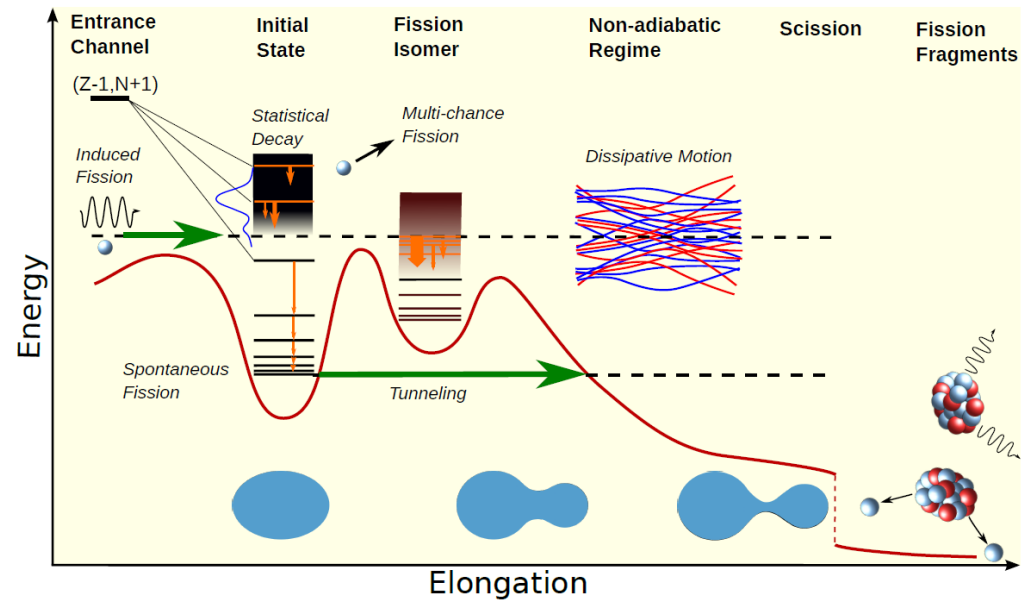
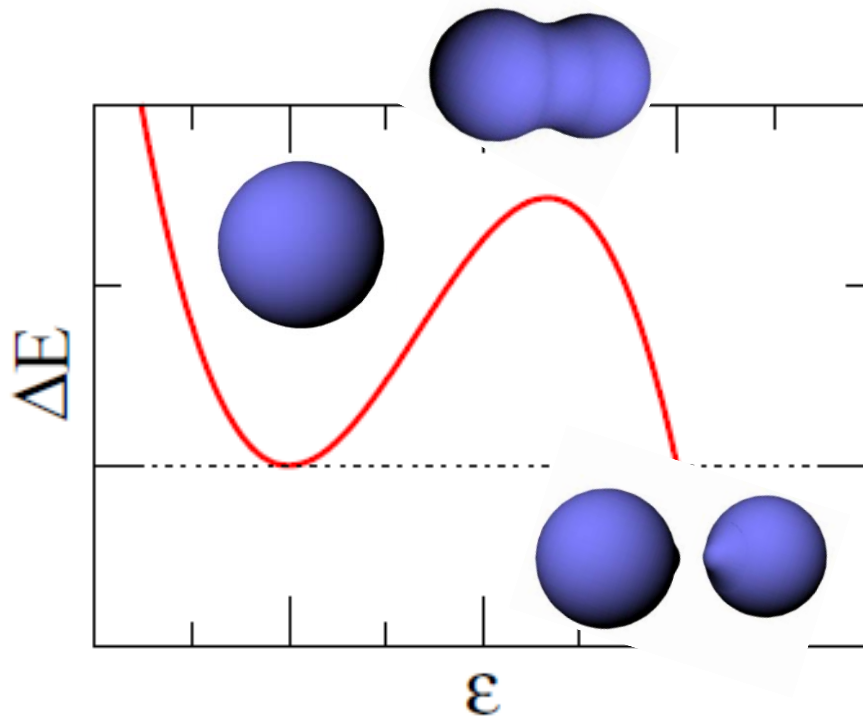


Y. Zhu et al.,
 Astrophys. J. Lett. 863 (2018)L23

➤ macroscopic understanding:

competition between the surface and the Coulomb energies

→ fission barrier



“Future of fission theory”

M. Bender et al., J. of Phys. G47, 113002 (2020)

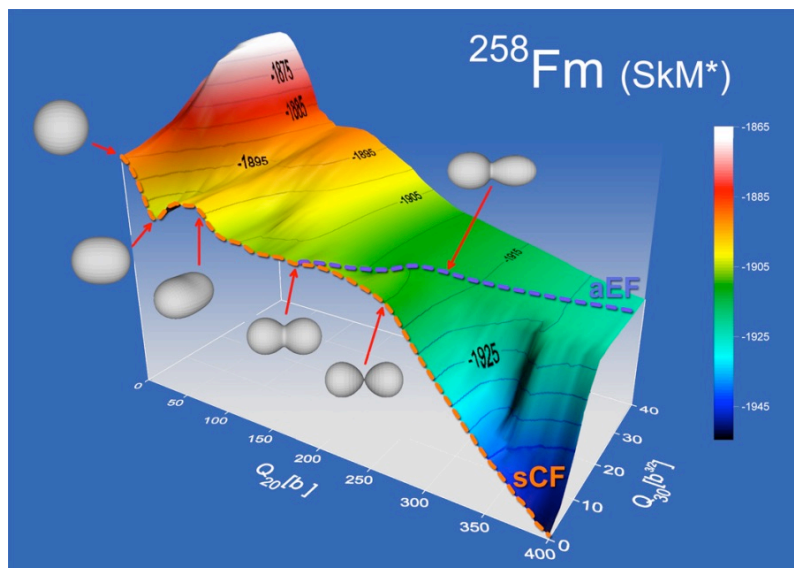
➤ a microscopic understanding:

large change of nuclear shape

→ microscopic description : far from complete

an ultimate goal of nuclear physics

➤ spontaneous fission



➤ induced fission

almost nothing has been developed for a microscopic theory

A. Staszczak, A. Baran, J. Dobaczewski, and W. Nazarewicz, PRC80 ('09) 014309

constrained Hartree-Fock (+B) method:

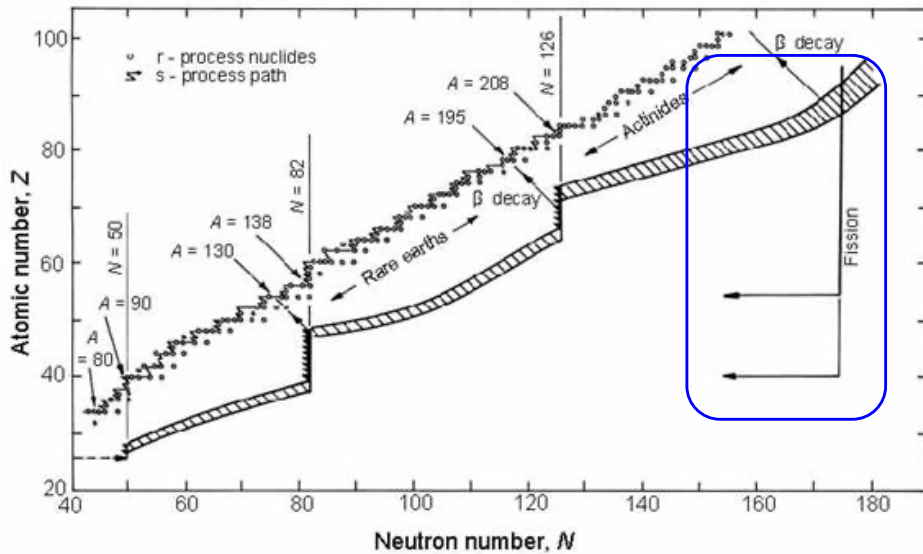
$$\delta \langle \Phi | H - \lambda Q_{20} | \Phi \rangle = 0$$

$$\rightarrow \Phi(Q_{20}), E(Q_{20})$$

$$\rightarrow P = \exp \left[-2 \int dq \sqrt{\frac{2B(q)}{\hbar^2} (V(q) - E)} \right]$$

Importance of a microscopic approach

➤ r-process nucleosynthesis

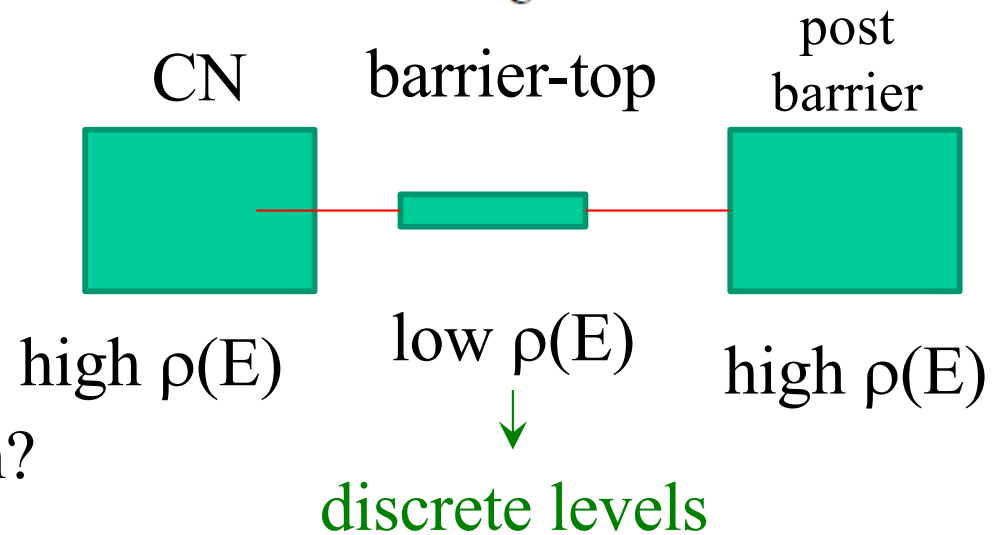
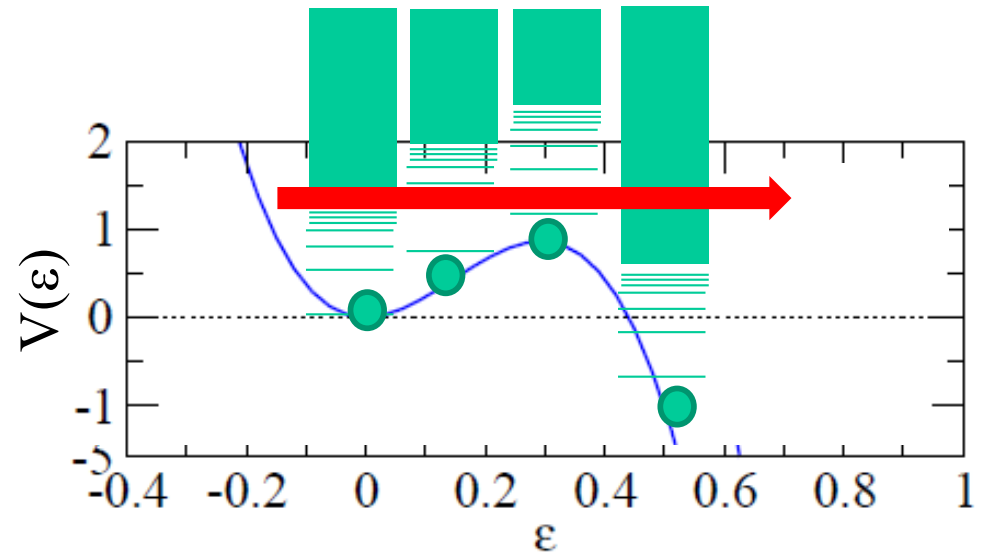


(neutron induced) fission of neutron-rich nuclei

→ low E^* and low $\rho(E^*)$

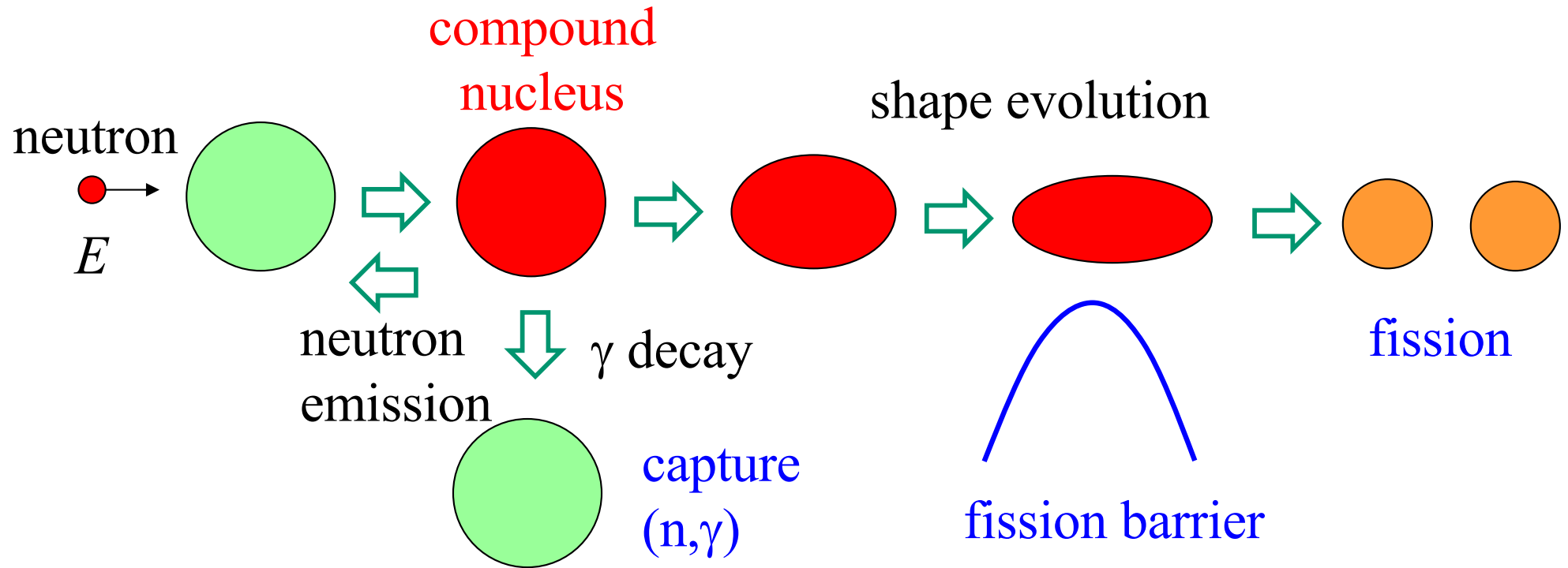
- ✓ Validity of statistical models?
- ✓ Validity of the Langevin approach?

➤ barrier-top fission

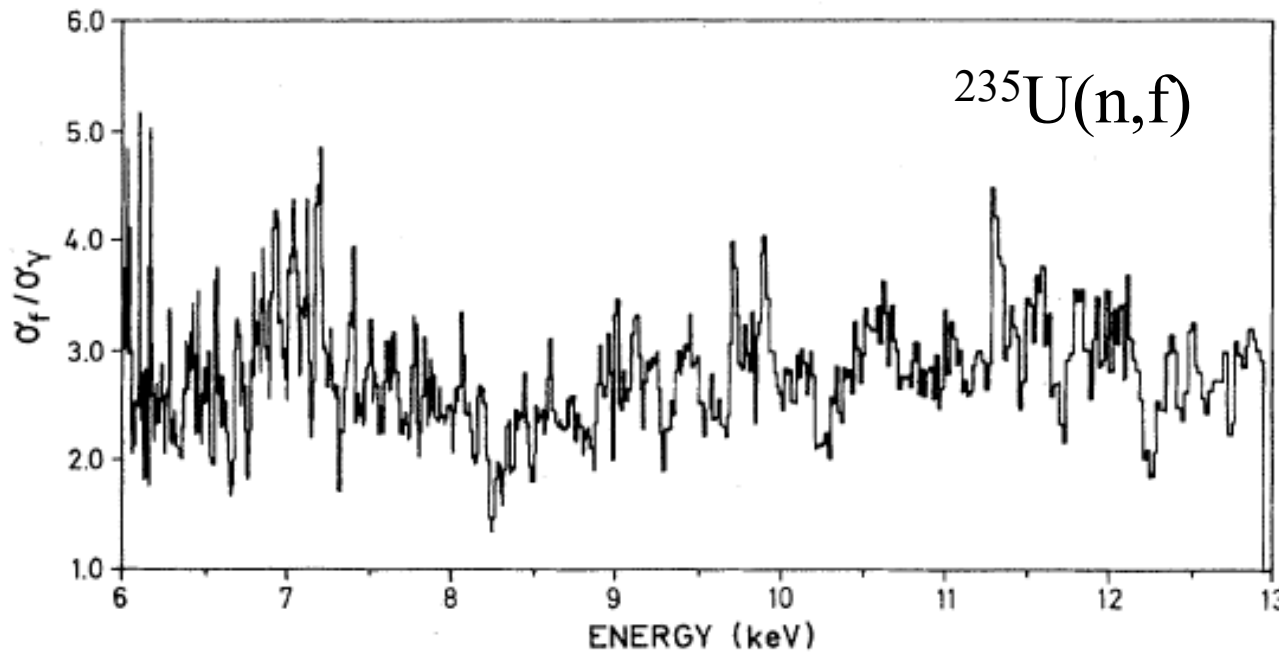
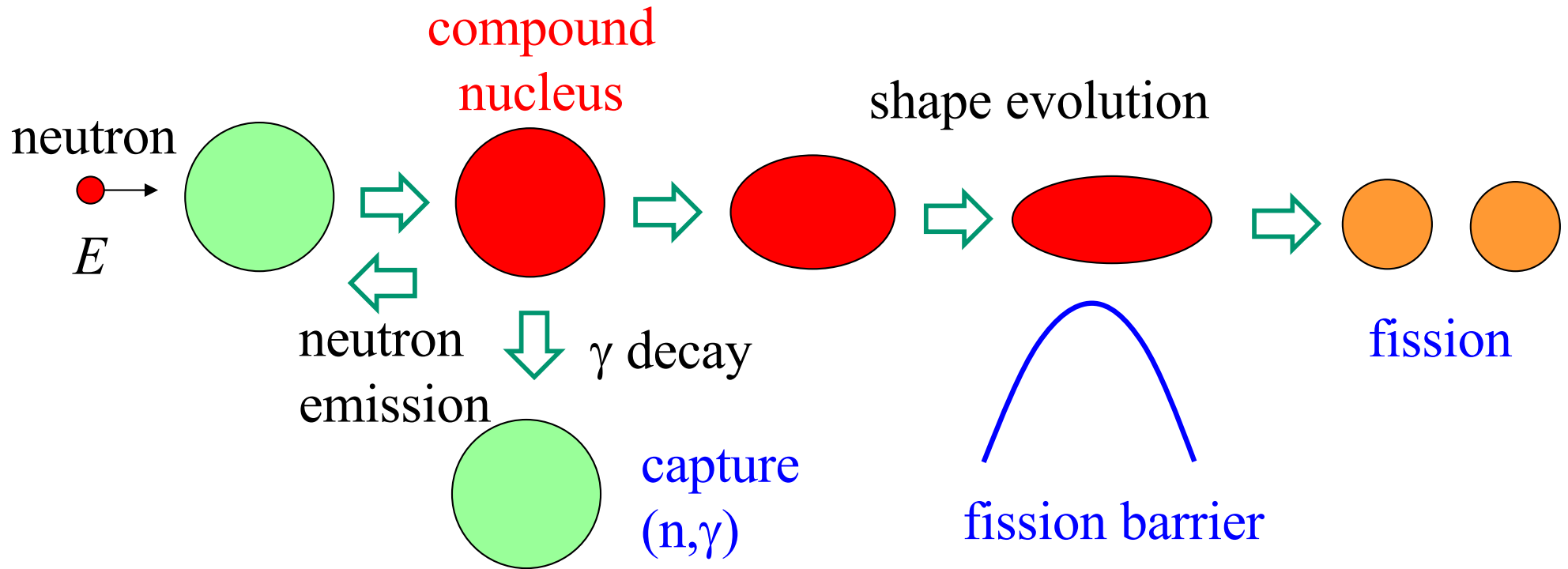


How to connect to a many-body Hamiltonian?

a process which we would like to discuss



a process which we would like to discuss

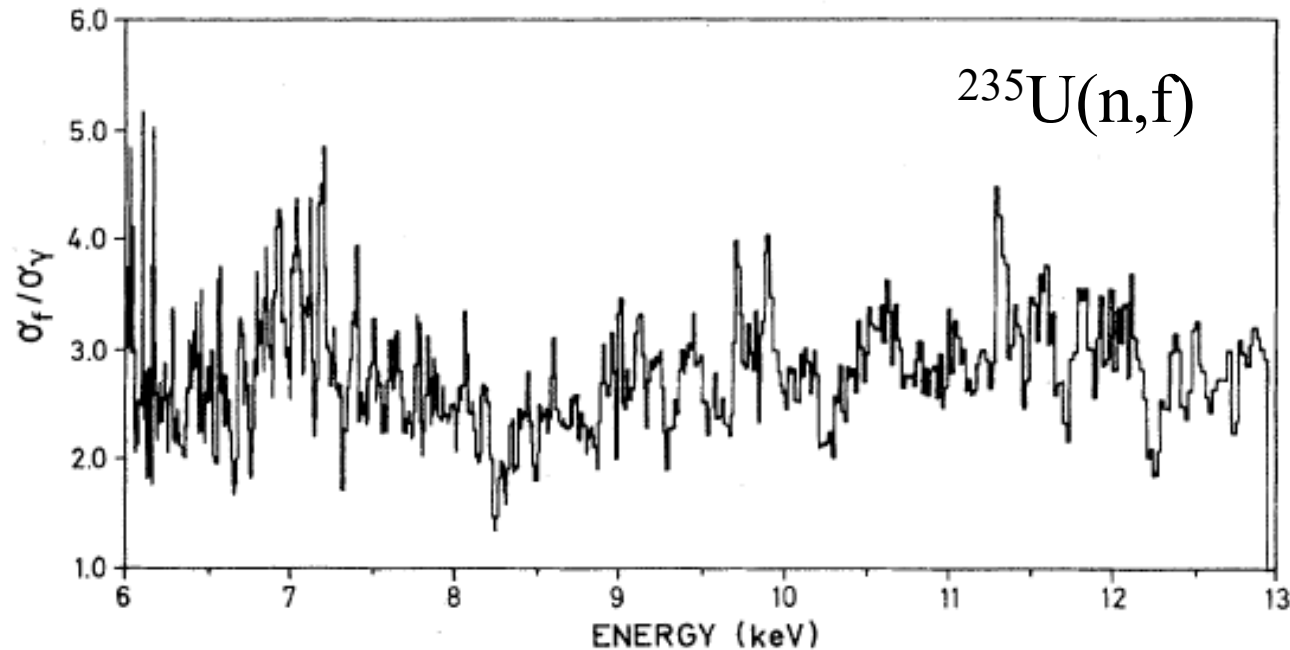


branching ratio

$$\alpha^{-1} = \frac{\sigma_f}{\sigma_\gamma}$$

sensitive to intermediate structure

M.S. Moore et al.,
PRC30 ('84) 214



branching ratio

$$\alpha^{-1} = \frac{\sigma_f}{\sigma_\gamma}$$

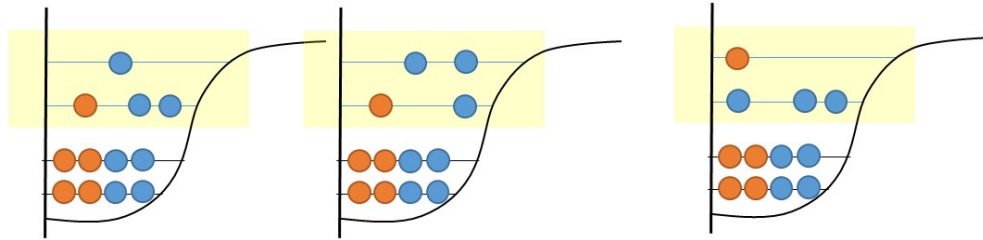
Important questions for r-process nucleosynthesis

- How will a fission barrier be modified for neutron-rich nuclei?
cf. Sasano-san' experiment
- What is an influence of pairing for (n,f) reactions?
- How does the branching ratio evolve towards n-rich nuclei?
(n,f) versus (n,γ)
- How does fission compete with alpha/cluster decays in neutron-rich heavy nuclei?

a microscopic approach may be crucial to address these questions

Shell model approach?

Shell model



$$|\Psi\rangle = v_1|m_1\rangle + v_2|m_2\rangle + v_3|m_3\rangle + \dots$$

Figure: Noritaka Shimizu (Tsukuba)

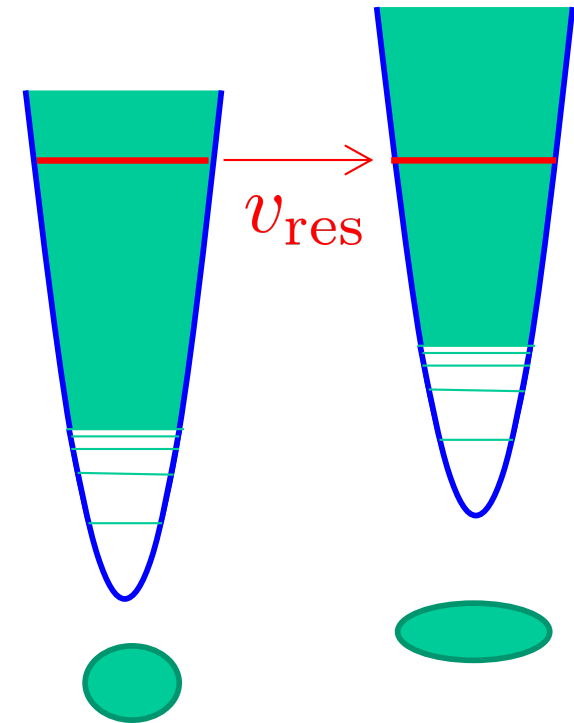
many-particle many-hole configurations
in a mean-field potential

→ mixing by residual interactions

$$|\Psi\rangle = \int dQ \sum_i f_i(Q) |\Phi_Q(i)\rangle$$

GCM with excited states

A similar approach
for nuclear fission?



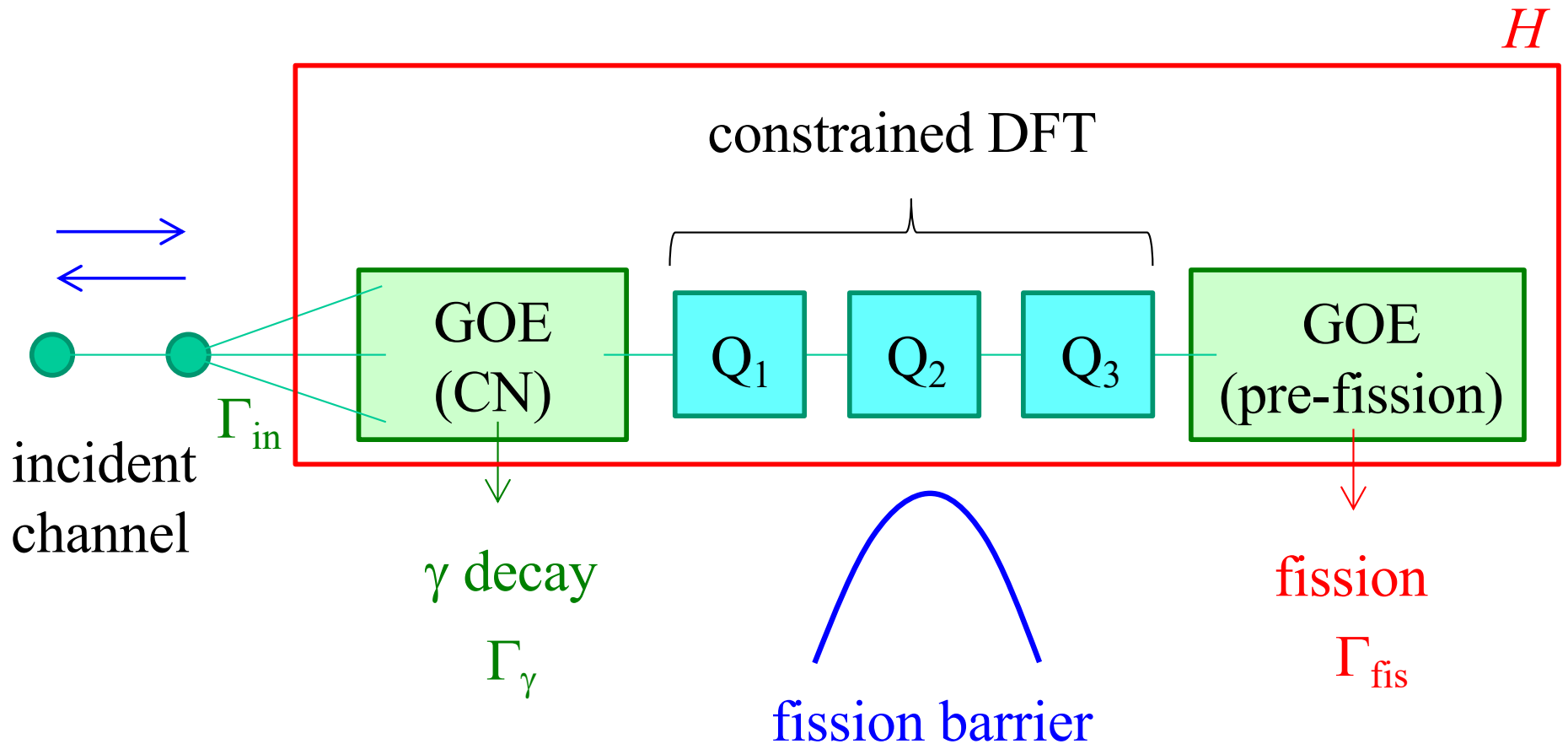
➤ Many-body configurations
in a MF pot. for each shape

➤ hopping due to res. int.

→ **shape evolution**

a good connection to
nuclear reaction theory

a process which we would like to discuss



Reaction theory (absorption probability):

$$T_{fis} = Tr[\Gamma_{in} G(E) \Gamma_{fis} G^{\dagger}(E)]$$

$$T_{cap} = Tr[\Gamma_{in} G(E) \Gamma_{\gamma} G^{\dagger}(E)]$$

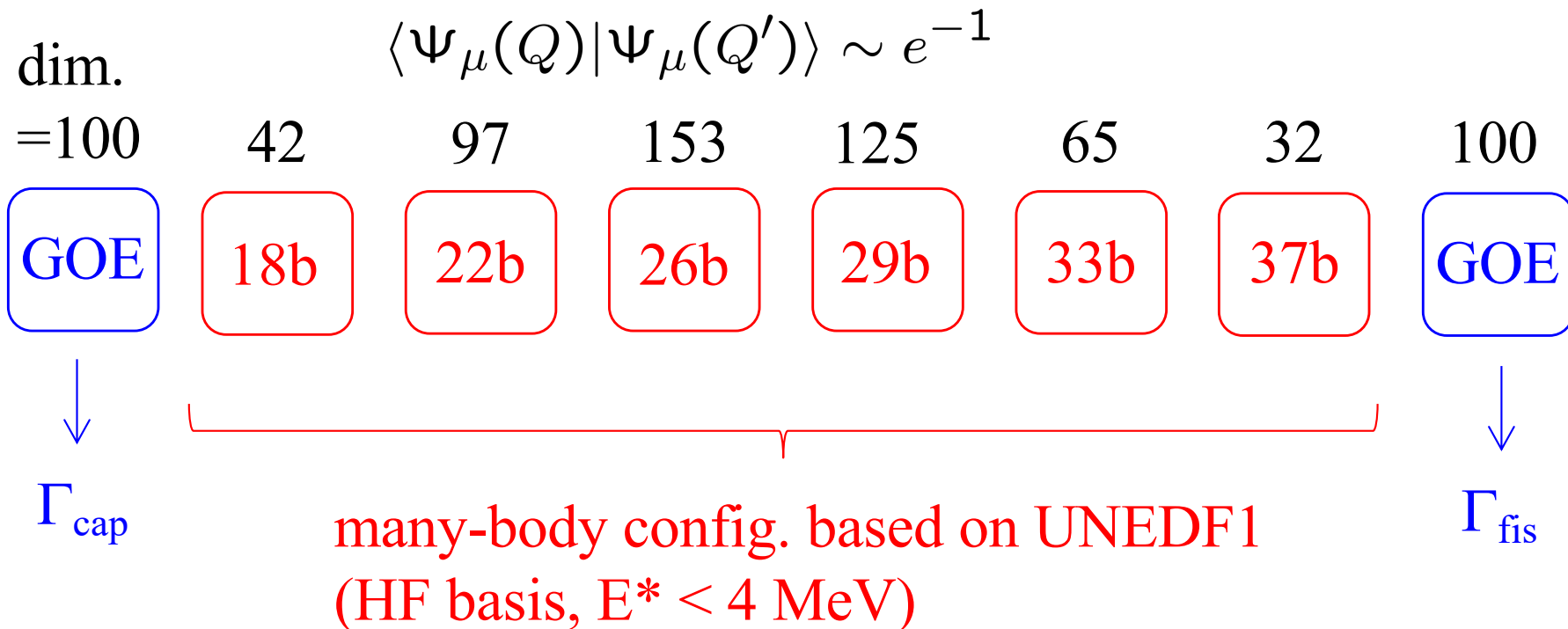
“Datta formula”

$$G(E) = [H - i\Gamma/2 - EO]^{-1}$$

Calculations for $^{235}\text{U}(n,f)$ based on Skyrme HF method

G.F. Bertsch and K.H., Phys. Rev. C107, 044615 (2023).

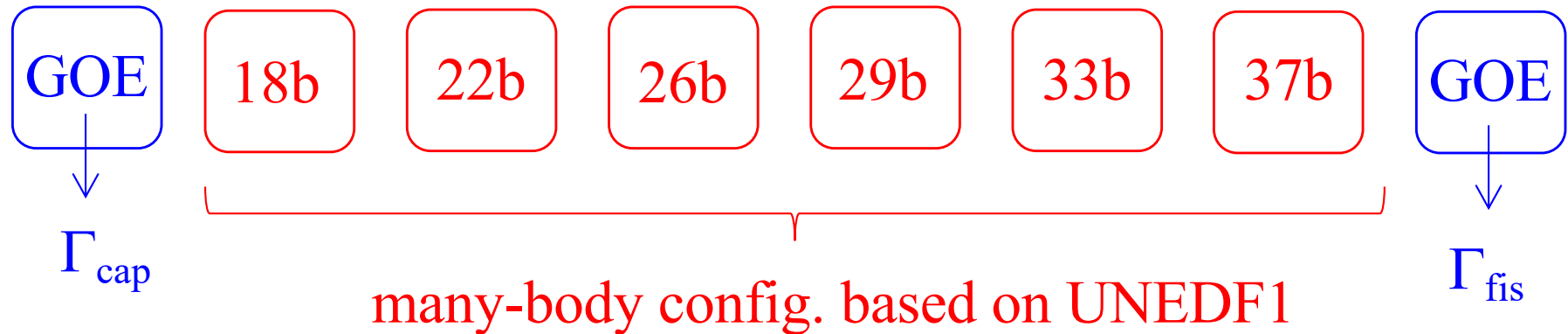
- Simplifications:
- ✓ ^{236}U : only neutron configurations, up to 4 MeV
 - ✓ Dynamics of the first barrier: axial symmetry
 - ✓ seniority-zero config. only: occupation of $(K, -K)$
 - ✓ a scaled fission barrier with $B_f = 4$ MeV



714x714 Hamiltonian matrix

Calculations based on Skyrme Hartree-Fock method

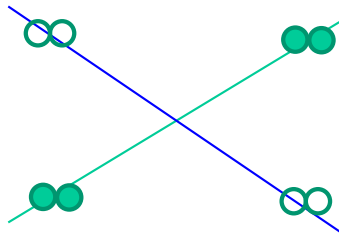
G.F. Bertsch and K.H., Phys. Rev. C107, 044615 (2023).



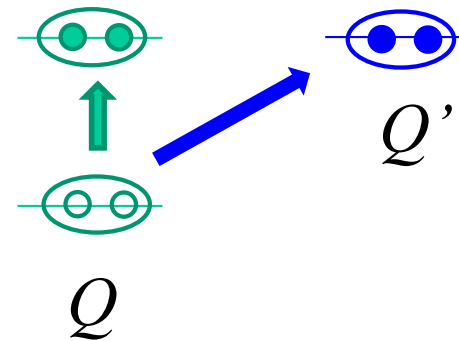
✓ overlap: $\langle \Psi_{\mu}(Q) | \Psi_{\mu}(Q') \rangle \sim e^{-1}$

✓ pairing: $v_{\text{pair}} = -GP^{\dagger}P$

✓ diabatic:



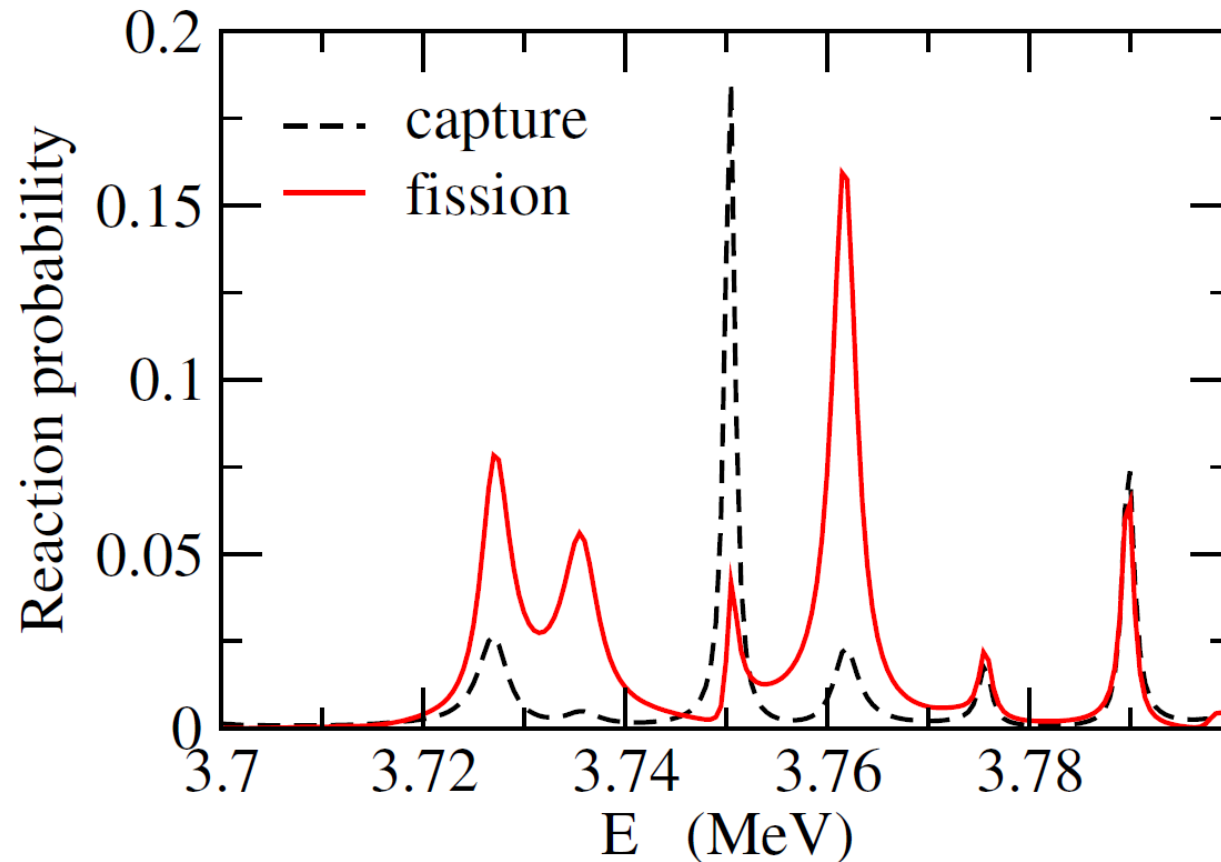
$$\frac{\langle \Psi_{\mu}(Q) | H | \Psi_{\mu}(Q') \rangle}{\langle \Psi_{\mu}(Q) | \Psi_{\mu}(Q') \rangle} \sim E_{\mu}(\bar{Q}) - h_2(\Delta Q)^2$$



✓ Γ_{cap} : exp. data (scaled according to N_{GOE}), Γ_{fis} : insensitivity

$$T_{\text{fis}}(E) = \text{Tr}[\Gamma_{\text{in}}G(E)\Gamma_{\text{fis}}G^\dagger(E)]$$

$$T_{\text{cap}}(E) = \text{Tr}[\Gamma_{\text{in}}G(E)\Gamma_{\gamma}G^\dagger(E)]$$



$$\Gamma_{\text{in}} = 0.01 \text{ MeV}$$

$$\Gamma_{\text{cap}} = 0.00125 \text{ MeV}$$

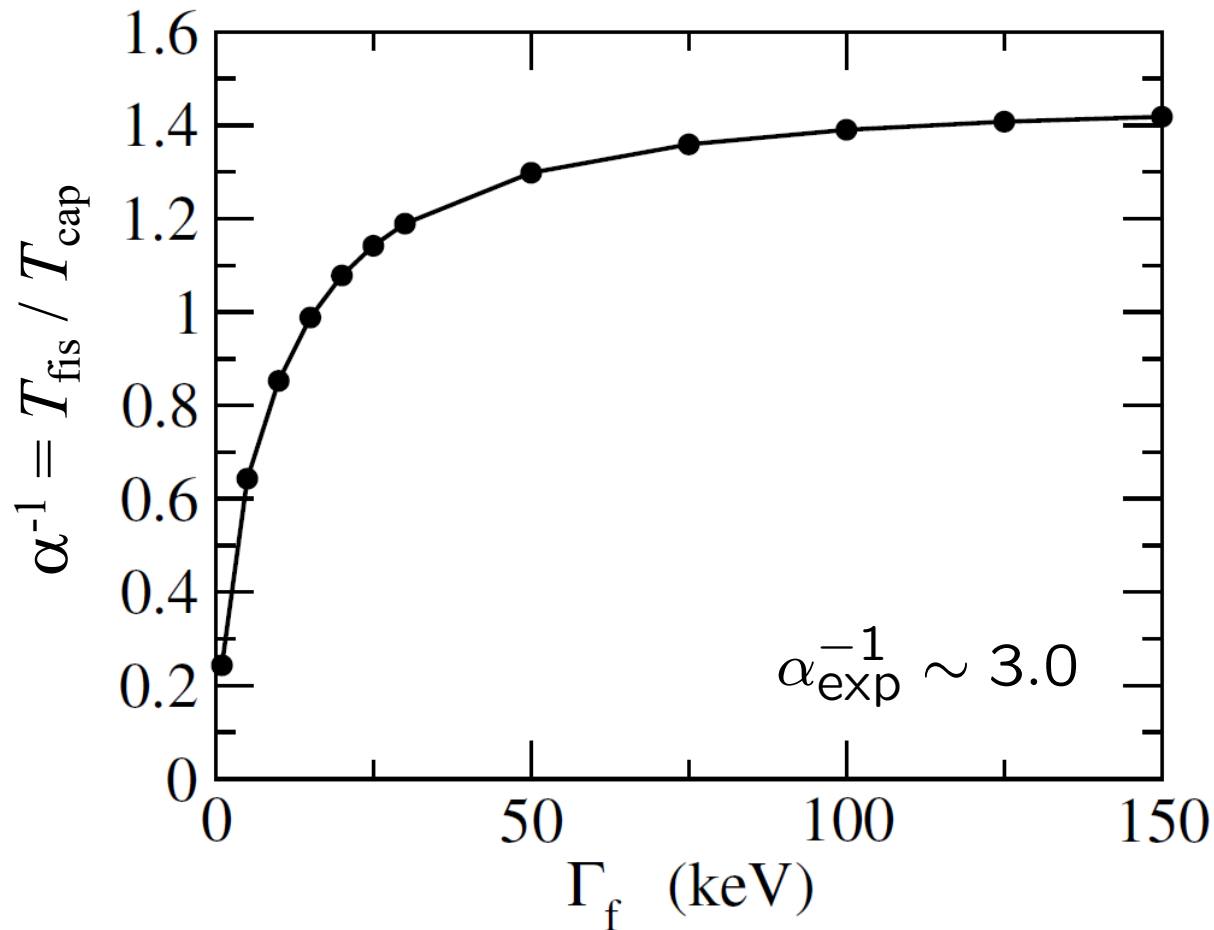
$$\Gamma_{\text{fis}} = 0.015 \text{ MeV}$$

energy average

$$\alpha^{-1} = \frac{\int_{\Delta E} T_{\text{fis}}(E')dE'}{\int_{\Delta E} T_{\text{cap}}(E')dE'}$$

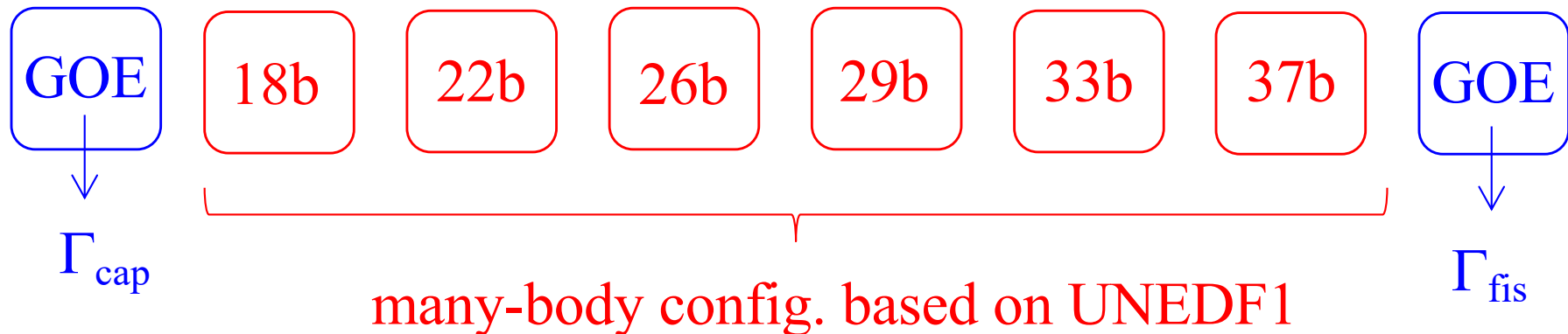
$$\Delta E = 0.5 \text{ MeV}$$

insensitivity property

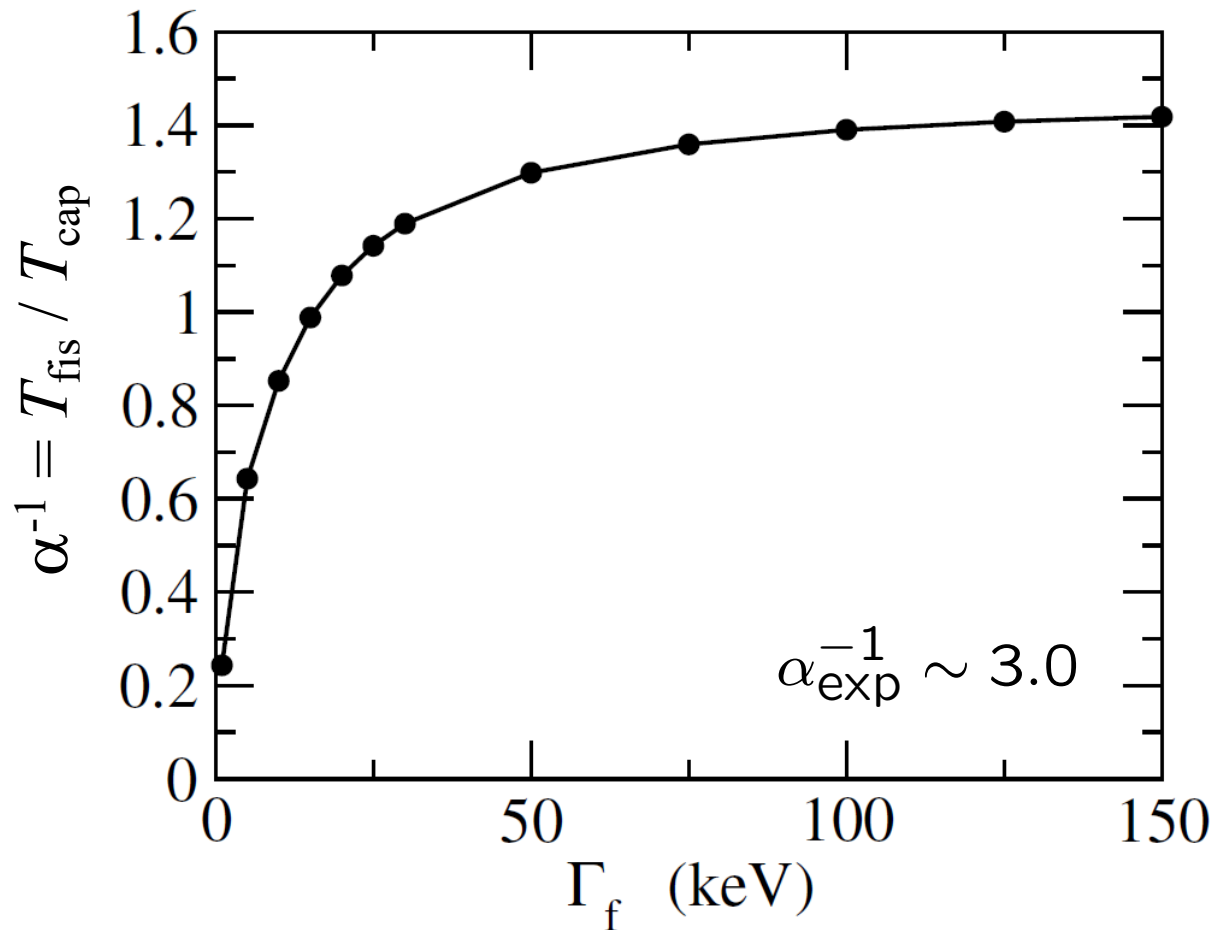


insensitive to Γ_f
(post-barrier dynamics)
→ the main assumption
of TST

cf. Analytic discussion
with a 2GOE+1Q model
K.H. and G.F. Bertsch,
arXiv: 2310.09537 (2023)



insensitivity property

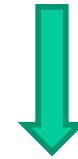


base set

$$G_{\text{pair}} = 0.2 \text{ MeV}$$

$$h_2 = 0.15 \text{ MeV}$$

$$\rightarrow \alpha^{-1} = 0.95$$



$$G_{\text{pair}} \rightarrow G_{\text{pair}}/2$$

$$G_{\text{pair}} = 0.1 \text{ MeV}$$

$$h_2 = 0.15 \text{ MeV}$$

$$\rightarrow \alpha^{-1} = 0.37$$

sensitive to the pairing, though less than in spontaneous fission

Further developments:

- seniority non-zero configurations with a schematic model
K. Uzawa and K. Hagino, PRC108 ('23) 024319
- an analytical derivation of transition-state-theory
K. Hagino and G.F. Bertsch, arXiv: 2310.09537
- validity of the discrete-basis formalism for barrier penetration
K. Hagino, arXiv: 2311.00925
G.F. Bertsch and K. Hagino, arXiv: 2401.10533
- fluctuation of fission width
K. Uzawa and K. Hagino, in preparation

Summary

r-process nucleosynthesis: fission of neutron-rich nuclei

requires a microscopic approach applicable to low E^* and $\rho(E^*)$

also for barrier-top fission

➔ a new approach: shell model + GCM

an application to induced fission of ^{236}U
based on Skyrme EDF

- ✓ neutron configurations only
- ✓ pairing and diabatic interactions
- ✓ truncation at 4 MeV

→ an importance of the pairing interaction

Future perspectives: seniority non-zero config. → pn res. interaction

K. Uzawa and K. Hagino, PRC108 ('23) 024319

a large scale calculation ($\sim 10^6$ dim.)

