Microscopic modeling of a compound nucleus
: low-energy induced fission and astrophysical nuclear fusion reactions

Kouichi Hagino
Kyoto University

George F. Bertsch (Seattle)
Kotaro Uzawa (JAEA)
Kosei Nagao (Kyoto)

1. Introduction: macroscopic vs microscopic models
2. Microscopic calculations for low-energy induced fission
3. $^{12}C+^{12}C$ fusion reactions at astrophysical energies
4. Discussion: the validity of compound nucleus picture at low energies?
5. Summary

RIBF ULIC Mini-WS047 核分裂における非平衡性の起源の解明に向けて：微視的理論と実験的検証、3月18日～19日、理研

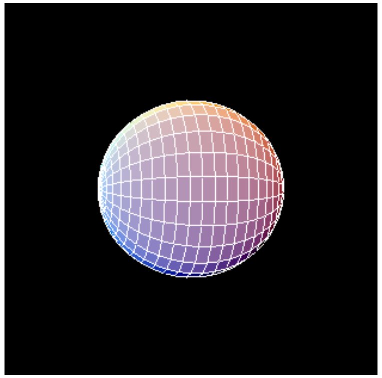
Introduction: macroscopic vs microscopic models

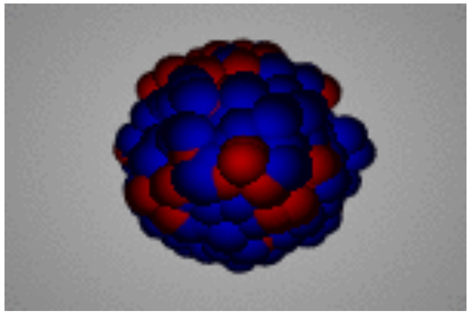
Macroscopic (phenomenological, empirical)

first extract important degrees of freedom ← phenomena

⇨ equations of motion (EOM) for such d.o.f.

simple, but extrapolations may be questionable





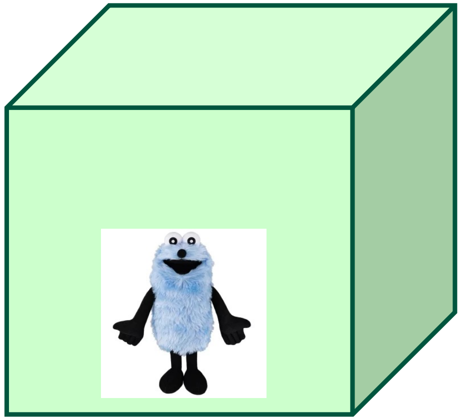
Semi-microscopic: EOM for macroscopic variables, but with parameters determined microscopically

Microscopic an approach with particle degrees of freedom

good for extrapolations, but often worse description

Introduction: macroscopic vs microscopic models

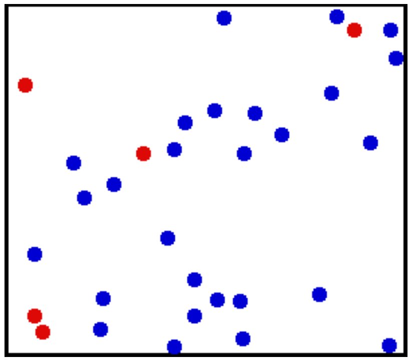
Macroscopic view



n-mols of gas
volume: V
temperature: T

pressure → $PV = nRT$

Microscopic view



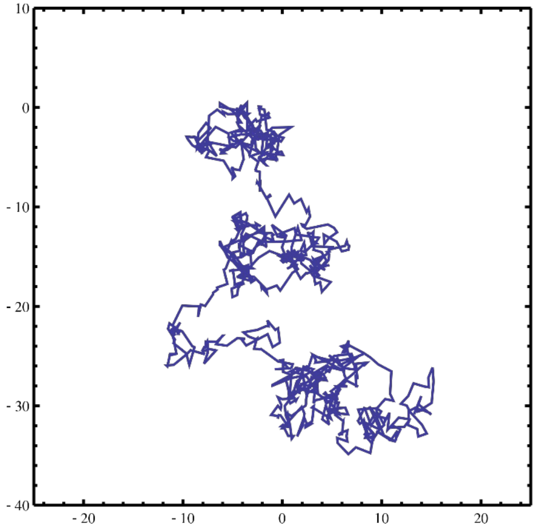
impulse（力積）$= 2mv_x$

the kinetic theory of gas
（気体の分子運動論）

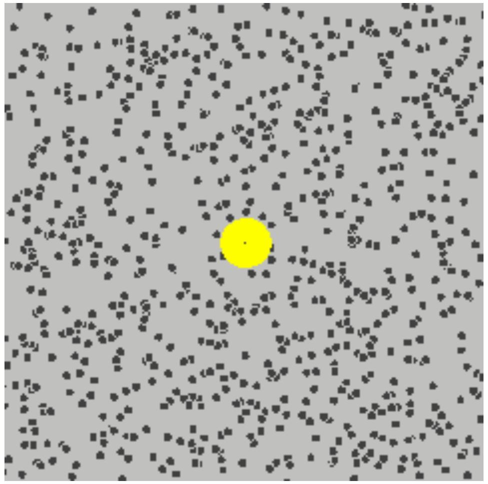
Wikipedia
https://wakariyasui.sakura.ne.jp/p/therm/kitai/bunnsi.html

Introduction: macroscopic vs microscopic models

Brownian motion



In reality:



In this talk:
Toward a microscopic description of

- low energy induced fission
- $^{12}C+^{12}C$ fusion

Langevin equation

$$ma = F - \gamma v + R(t)$$

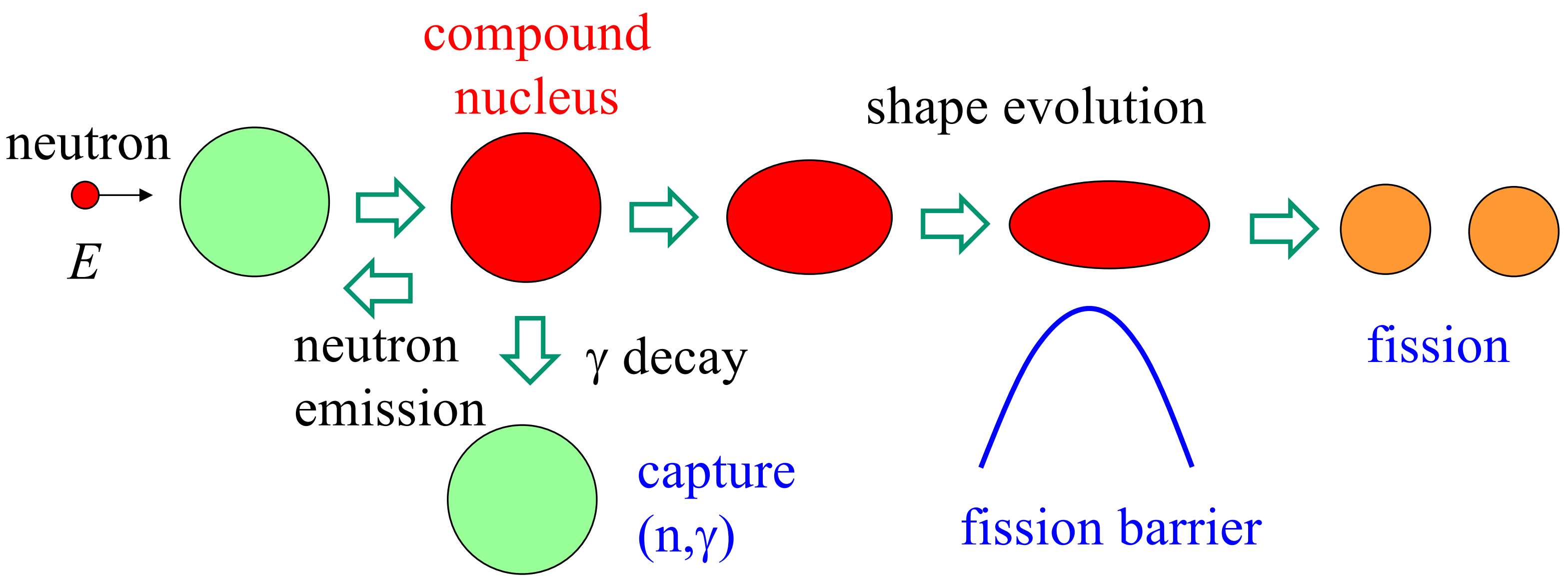
➡ A macroscopic approach

simultaneous treatment of a fine particle and atoms

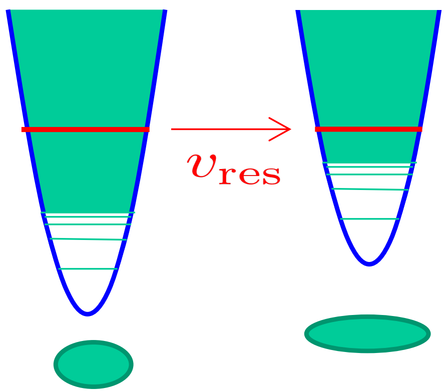
➡ a microscopic approach

A semi-microscopic approach: Langevin equation with a microscopically determined γ

Microscopic calculations for low energy induced fission



the basic idea:

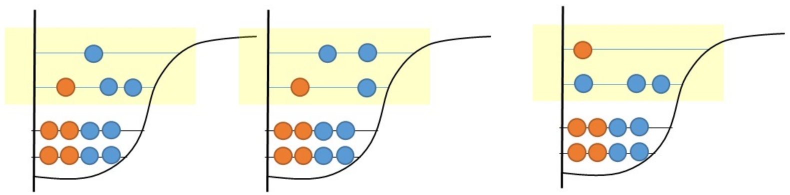


- Many-body configurations in a MF pot. at each shape
- hopping due to residual interactions

→ shape evolution

Microscopic calculations for low energy induced fission

Shell model approach for nuclear structure

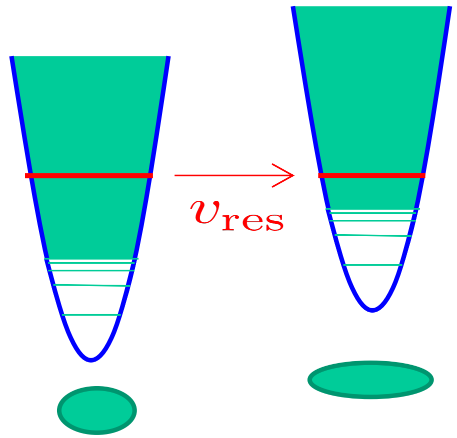


$$|\Psi\rangle = v_1|m_1\rangle + v_2|m_2\rangle + v_3|m_3\rangle + \cdots$$

Figure: Noritaka Shimizu (Tsukuba)

many-particle many-hole configurations
<u>in a given mean-field potential</u>
→mixing by residual interactions

for induced fission



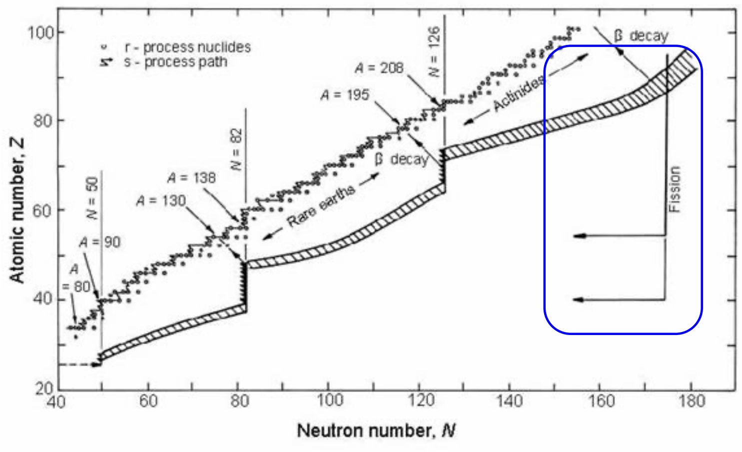
shape dependent mean-field potentials

G.F. Bertsch and K.Hagino, Phys. Rev. C107, 044615 (2023).
K. Uzawa and K. Hagino, Phys. Rev. C110, 014321 (2024).

Microscopic calculations for low energy induced fission

Why is a microscopic theory for fission important?

- r-process nucleosynthesis

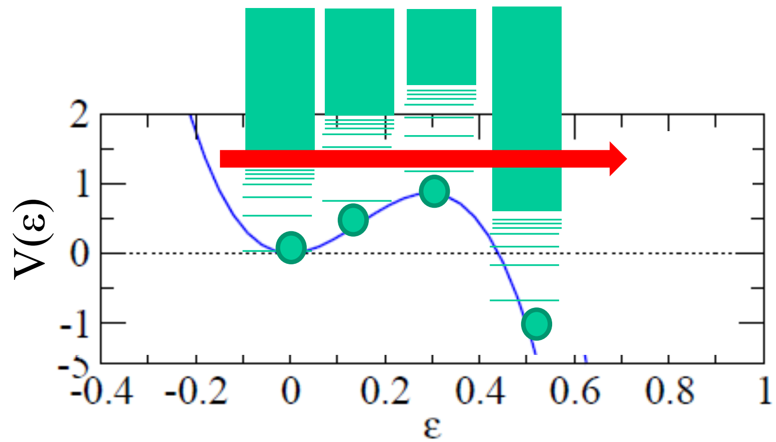


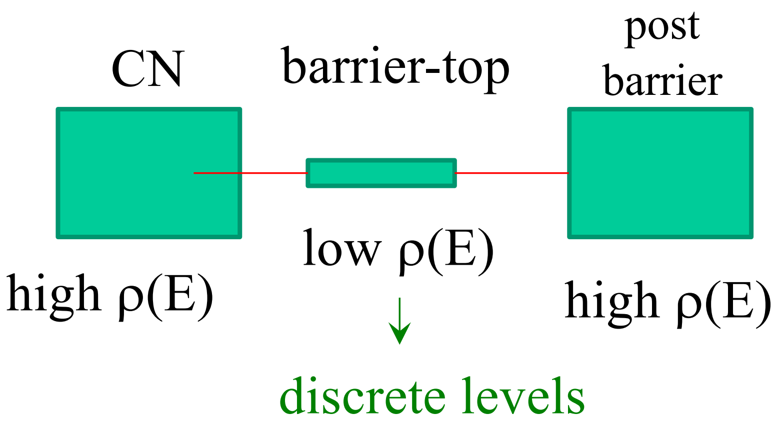
fission of neutron-rich nuclei

→ low E^* and low $\rho(E^*)$

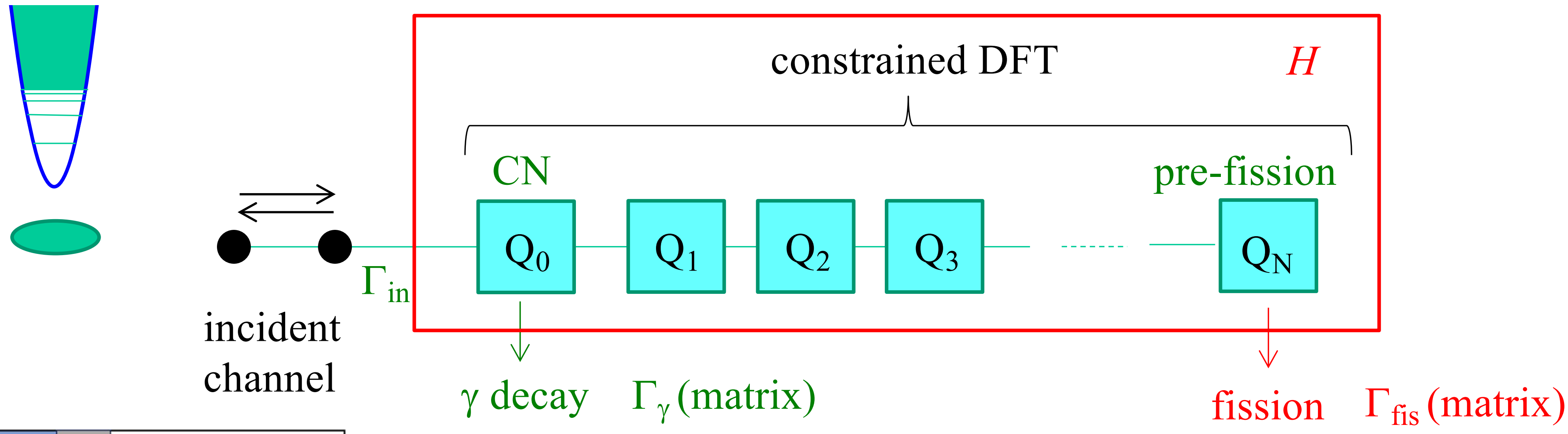
✓ Validity of statistical models?
✓ Validity of the Langevin approach?

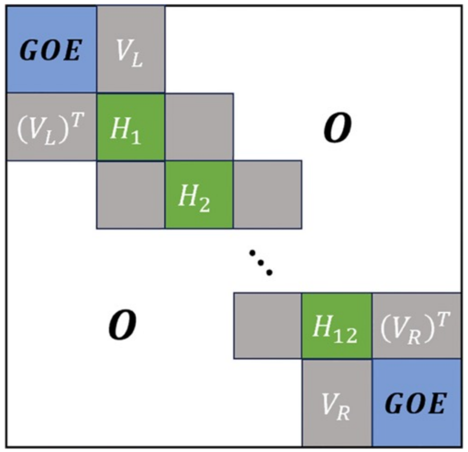
- barrier-top fission





Microscopic calculations for low energy induced fission





$$H = H_0 - GP^\dagger P + V_{\rm rand}$$

非平衡グリーン関数法

$$T_{CN\to{\rm fis}} = {\rm Tr}[\Gamma_n G \Gamma_{\rm fis} G^\dagger]; \quad G = (H - NE)^{-1}$$

some complications

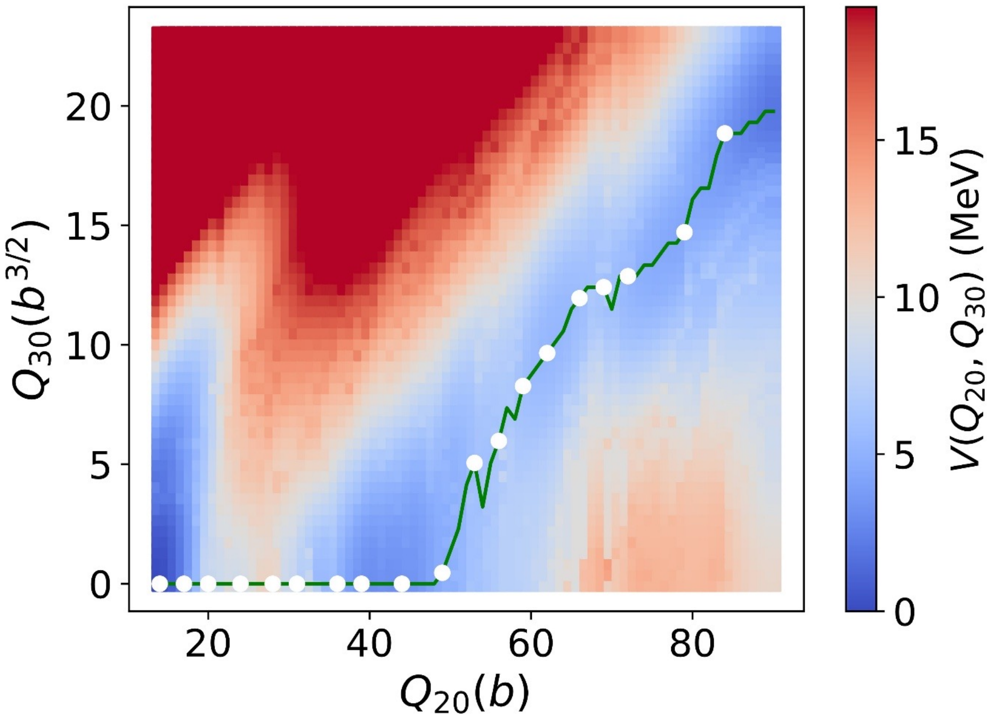
$$N_{\mu\mu'} = \langle \Psi_\mu(Q) | \Psi_{\mu'}(Q') \rangle$$

$$\frac{\langle \Psi_\mu(Q) | H_0 | \Psi_\mu(Q') \rangle}{\langle \Psi_\mu(Q) | \Psi_\mu(Q') \rangle} \equiv \langle \Psi_\mu(Q) | V_{\rm diabatic} | \Psi_\mu(Q') \rangle$$

Application to low-energy fission of ^{236}U

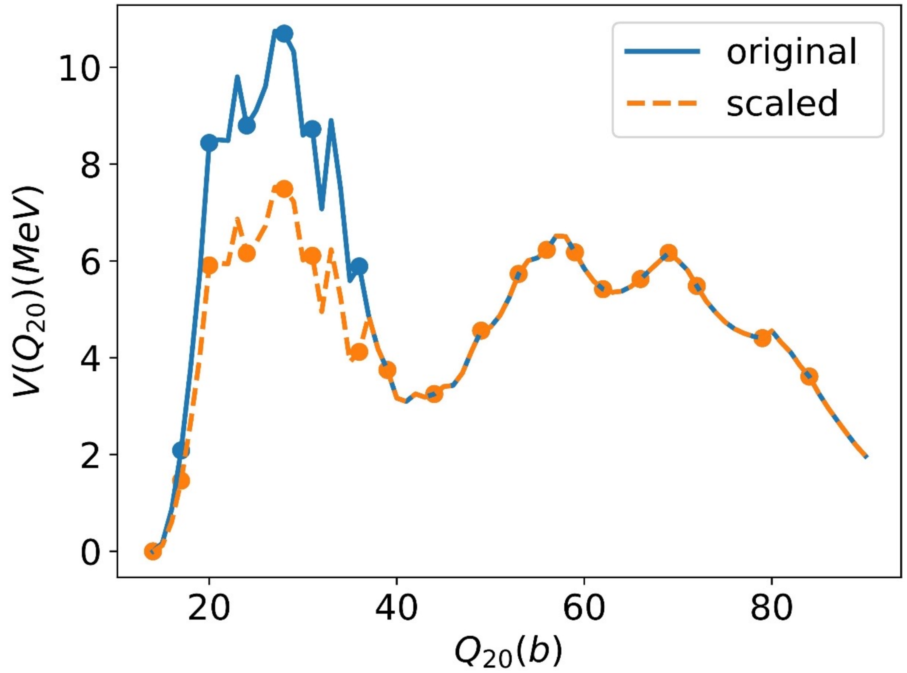
K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).

fission path based on Skyrme DFT with UNEDF1



$\langle \Psi_\mu(Q) | \Psi_\mu(Q') \rangle \sim 0.52$

fission barrier without pairing

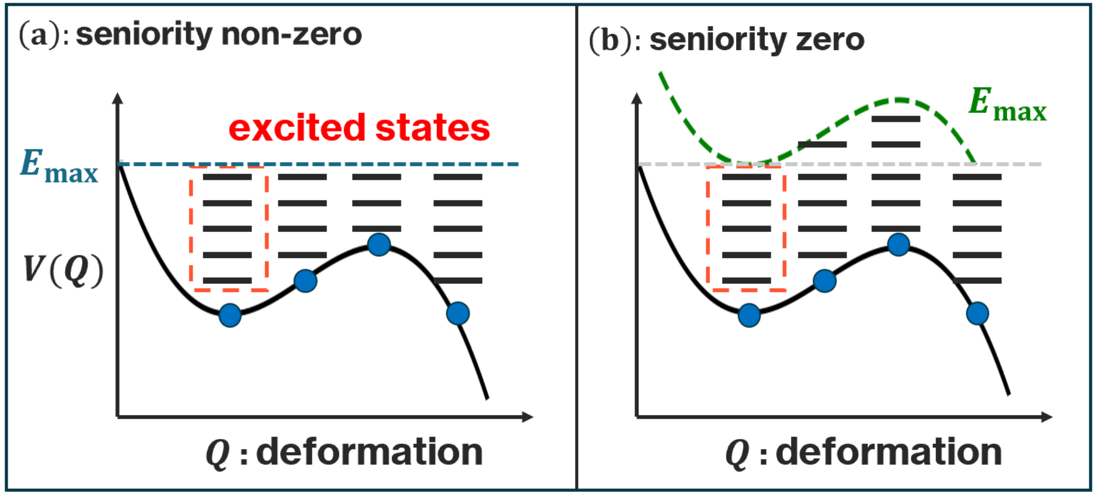


scale the first barrier
$\rightarrow B \sim B_{\text{empirical}}$ if with pairing

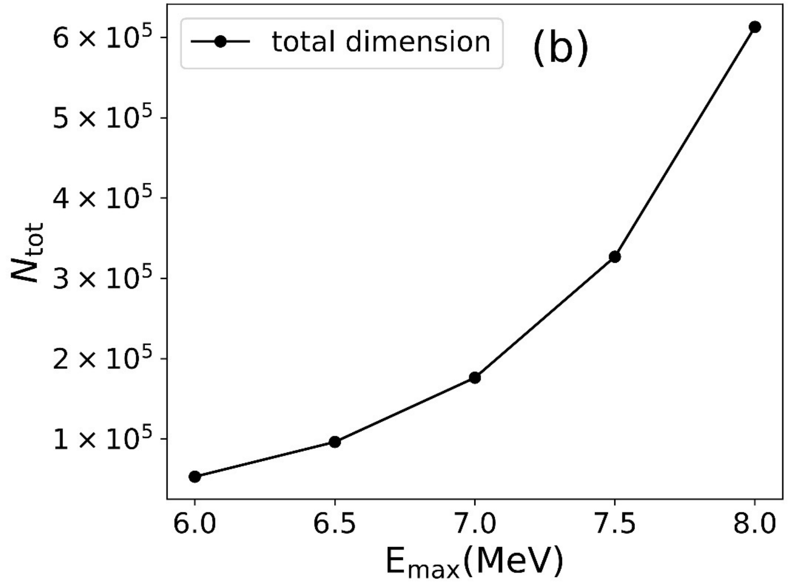
Application to low-energy fission of ^{236}U

K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).

np-nh configurations up to E_{max}



dimension of $H \sim O(10^5)$



$$H = H_0 - GP^\dagger P + V_{\rm rand}$$

- $G \leftarrow 0^+_2$ in ^{236}U for a given model space
- $V_{\rm rand} \leftarrow$ systematics
- $\Gamma_{\rm cap}$, $\Gamma_{\rm n} \leftarrow$ empirical
- $\Gamma_{\rm fis}$ for a pre-fission config. $\leftarrow$ insensitivity

inverse of a huge matrix

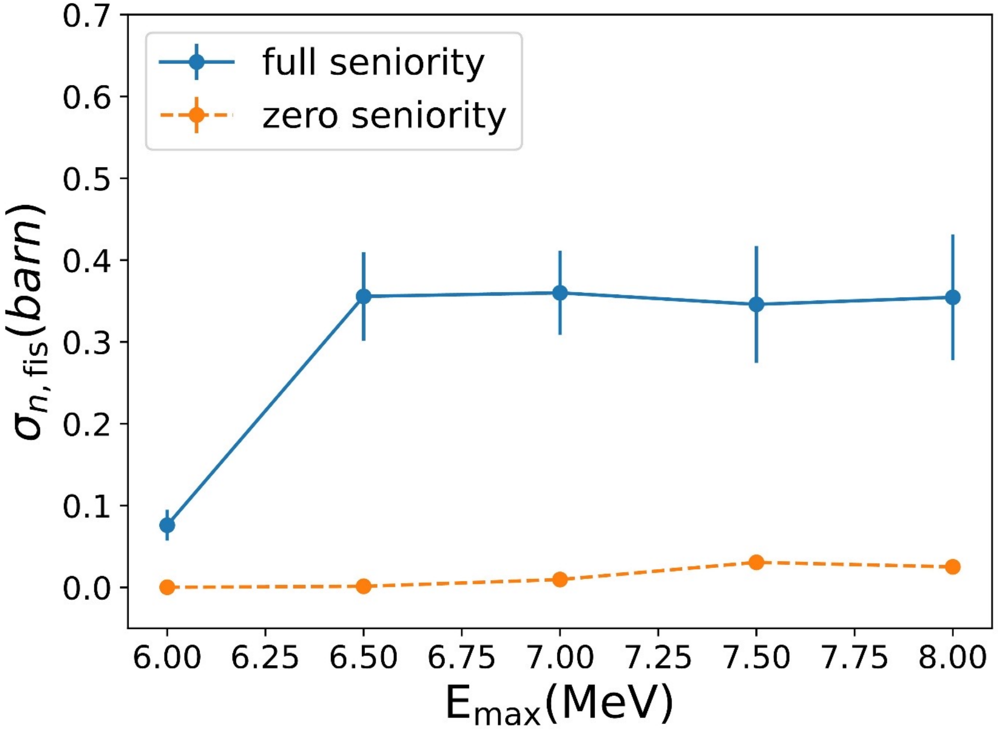
- LSMR method
- shift-invert Lanczos method

K. Uzawa and K. Hagino
PRE110, 055302 (2024).

Application to low-energy fission of ^{236}U

K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).

fission cross section (E = 10 keV)



convergence at $E_{max} \sim 6.5$ MeV

the empirical value: $\sigma_{n,fis} = 3.116$ b

→ reproduced within a factor of 10

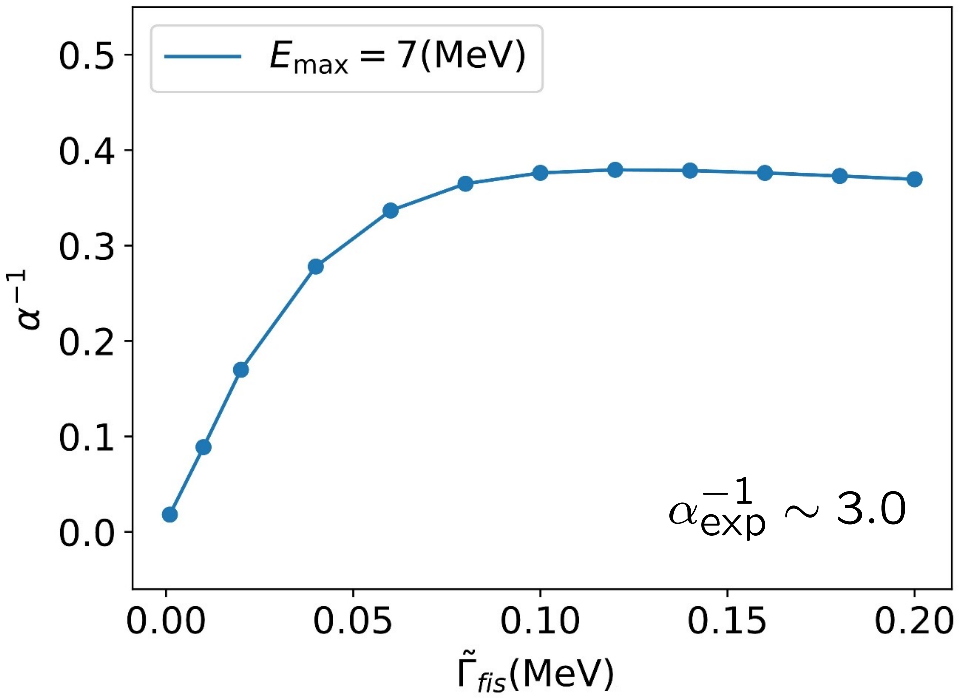
possible origins of the discrepancy:

- axial symmetry → triaxiality
- G = const. → $G(\beta)$
- Skyrme UNEDF1 → other functionals?
- collective coordinates: Q_2 and Q_3?
  e.g., the momentum dependence?

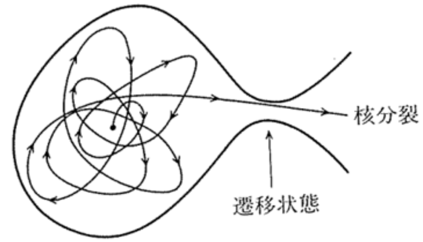
Application to low-energy fission of ^{236}U

K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).

fission-to-capture branching ratio



14 b → Γ_{cap} | 18b | 23b | 29b | .... | 84 b → Γ_{fis}



insensitivity property

insensitive to Γ_{fis} (post-barrier dynamics)

→the main assumption of the transition state theory (TST)

cf. K.Hagino and G.F. Bertsch, JPSJ 93, 064003 (2024).

cf. strong absorption in nuclear reactions

Application to low-energy fission of ^{236}U

K. Uzawa and K. Hagino,
Phys. Rev. C110, 014321 (2024).

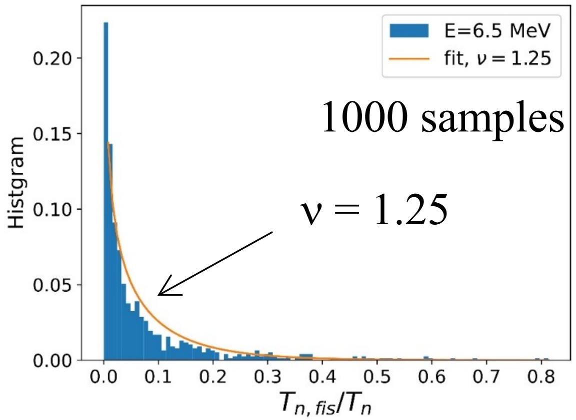
the effective number of freedom for fission

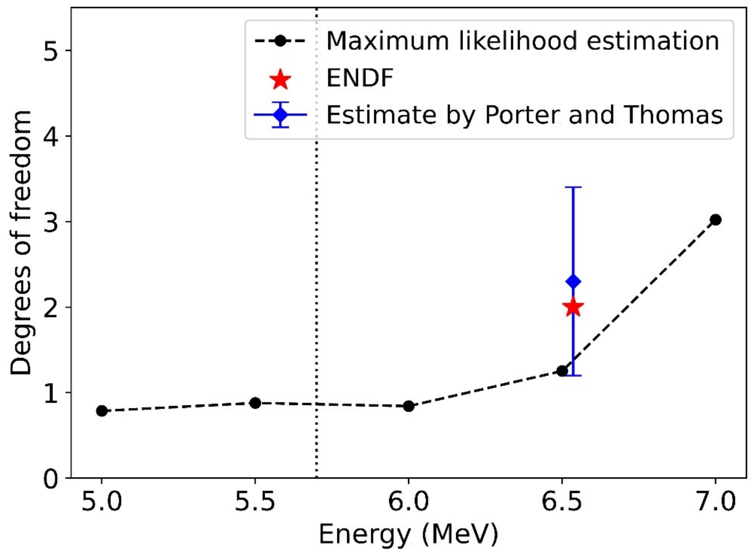
calc. with a smaller model space (seniority 0 only)

the distribution of $T_{\rm n,fis}$ → fit with the chi-square function

$$P_\nu(x) = \frac{\nu}{2\Gamma(\nu/2)}\left(\frac{\nu x}{2}\right)^{\nu/2-1} e^{-\nu x/2}$$

ν: # of d.o.f. (cf. ν=1: the PT distribution)





$$G(E_n) = \frac{1}{H - i\Gamma/2 - E_n} = \sum_\alpha \frac{|\phi_\alpha\rangle\langle\tilde{\phi}_\alpha|}{E_\alpha - E_n}$$

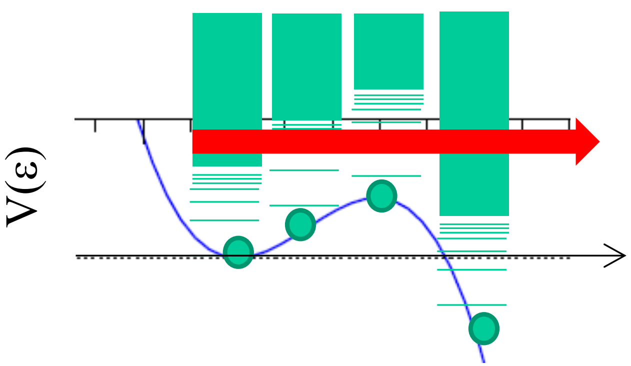
only a few eigenstates with Re(E_α) ~ E_n contribute

"transition states"

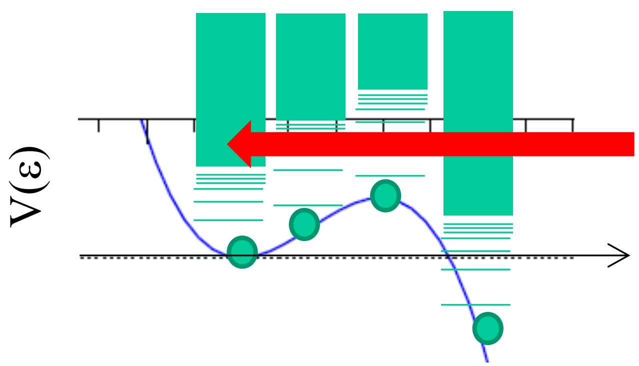
a small number of d.o.f. for induced fission ← transition state theory

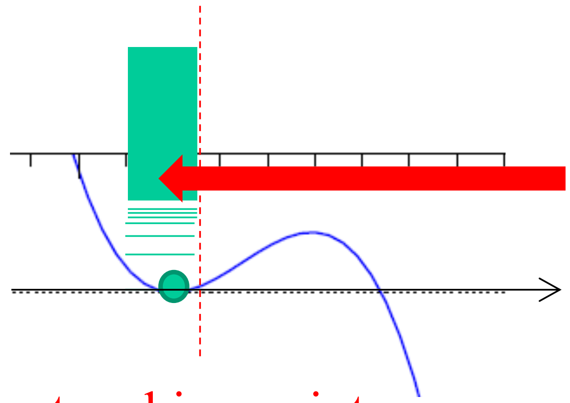
$^{12}C+^{12}C$ fusion reactions at astrophysical energies

fission



fusion





touching point

fusion for
superheavy elements

↓

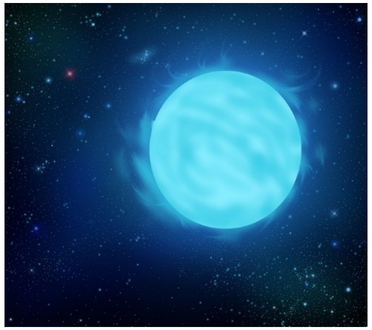
a microscopic description
for shape evolution

fusion of light nuclei
(nuclear astrophysical
reactions)

relative motion
+ compound nucleus

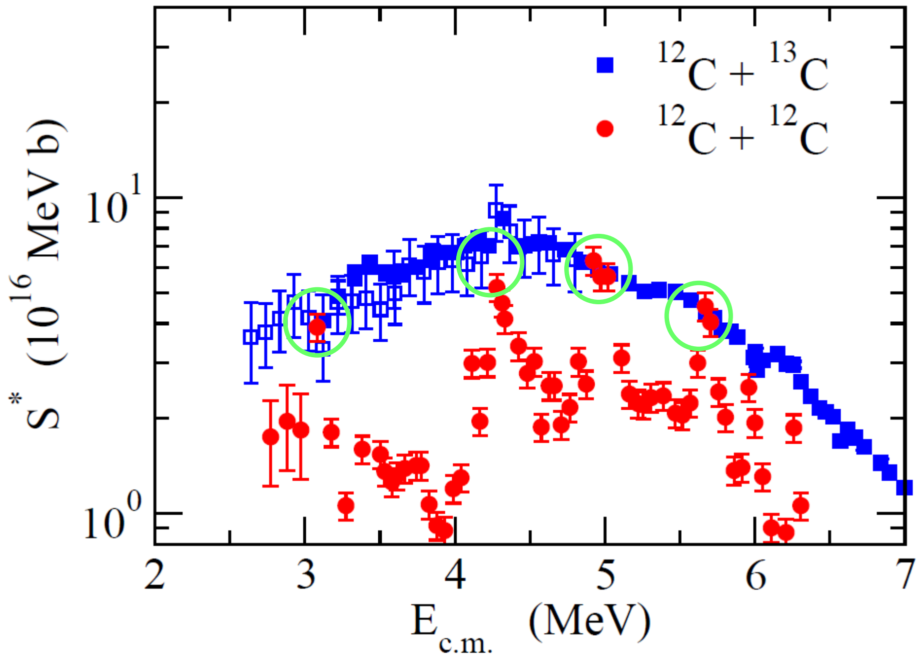
$^{12}C+^{12}C$ fusion reactions at astrophysical energies

$^{12}C+^{12}C$ fusion
: one of the key reactions in nuclear astrophysics



✓ Carbon burning in massive stars
✓ Type Ia supernovae
✓ X-ray superburst

$$S^*(E) = E\sigma(E)\, e^{2\pi(\eta-\eta_0)}$$



J.L. Jiang et al., PRL110, 072701 (2013).

difference between ^{24}Mg and ^{25}Mg

- ^{24}Mg: isolated CN resonances
- ^{25}Mg: overlapping CN resonances

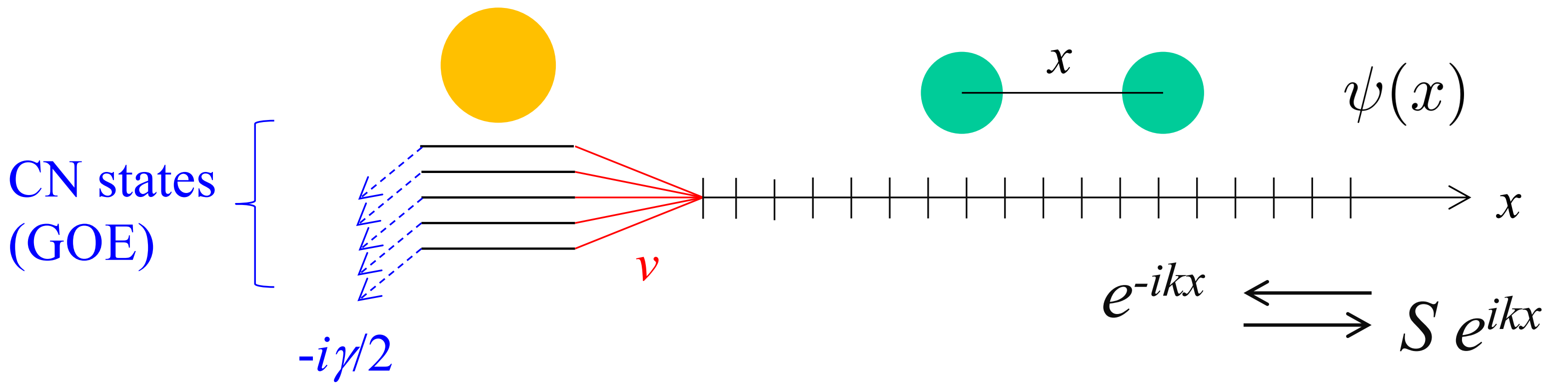
need a new reaction model

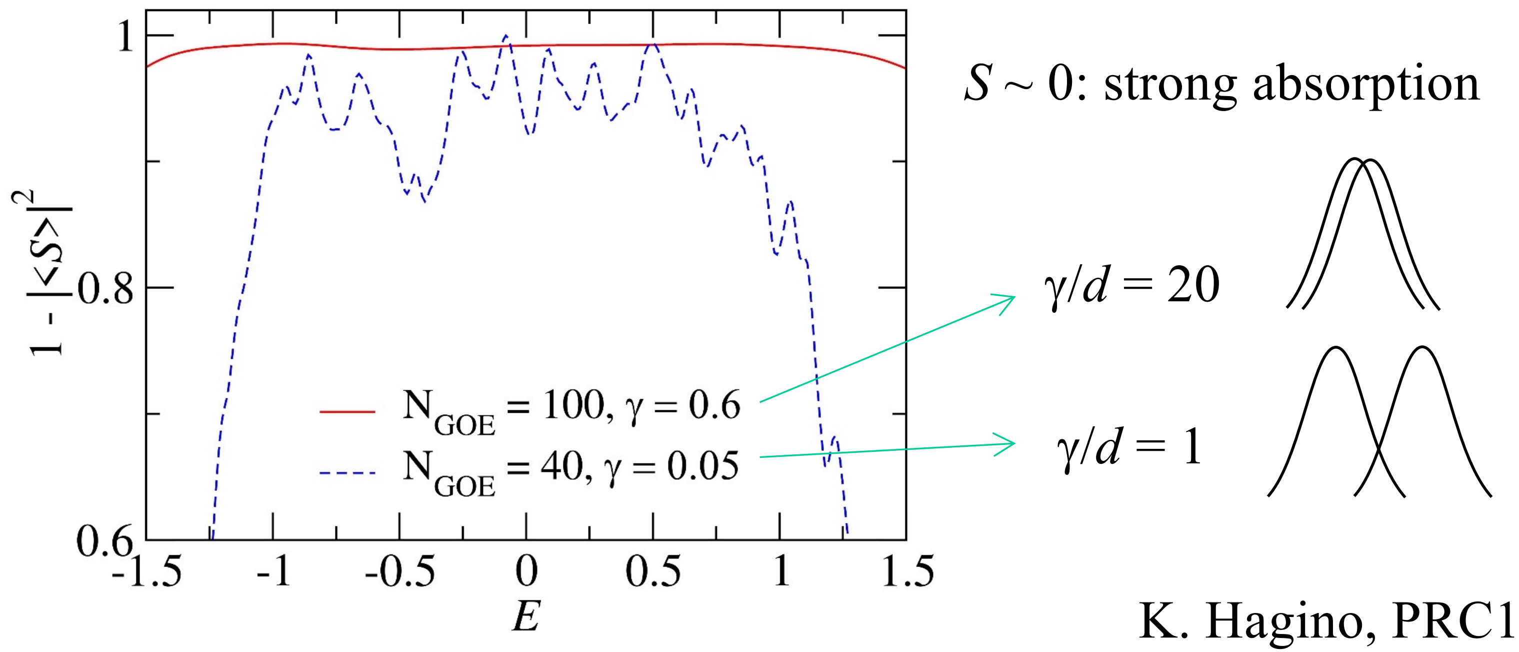
→

prominent resonance structure in $^{12}C+^{12}C$ fusion

✓ off-resonance: fusion inhibition
✓ on-resonance: match with $^{12}C+^{13}C$

a schematic model with a random matrix : a microscopic modeling of the imaginary part of $V_{\rm opt}$

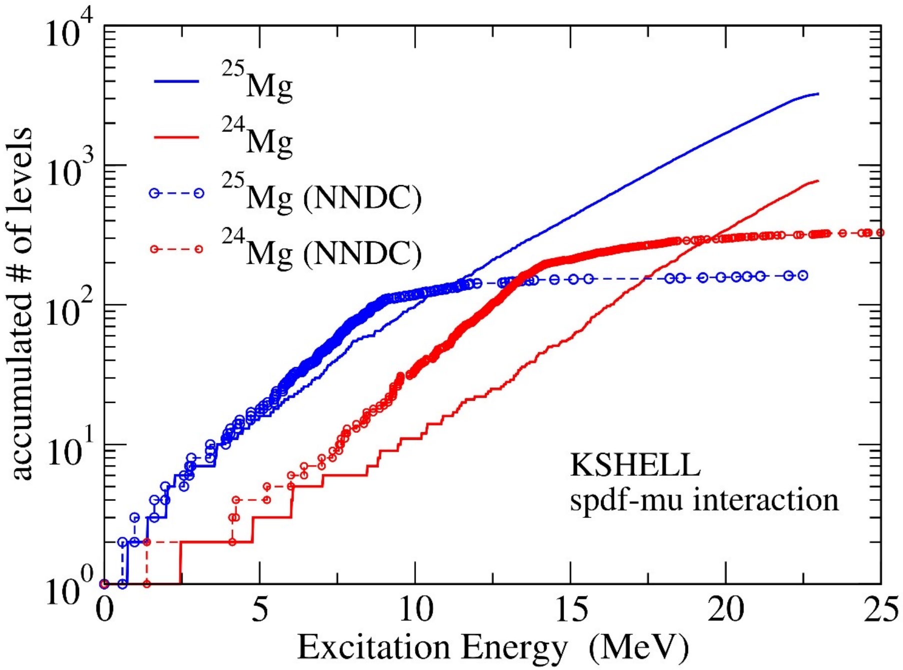


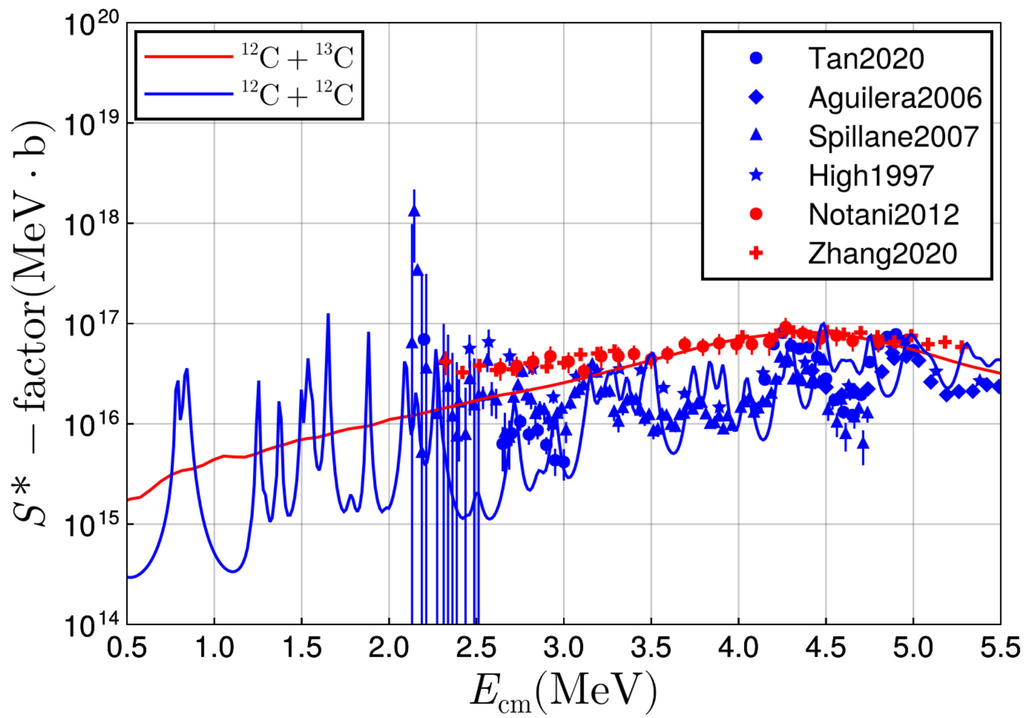


K. Hagino, PRC112, 034611 (2025).

a more realistic model with shell model   K. Nagao, K. Hagino, and K. Uzawa, in preparation.

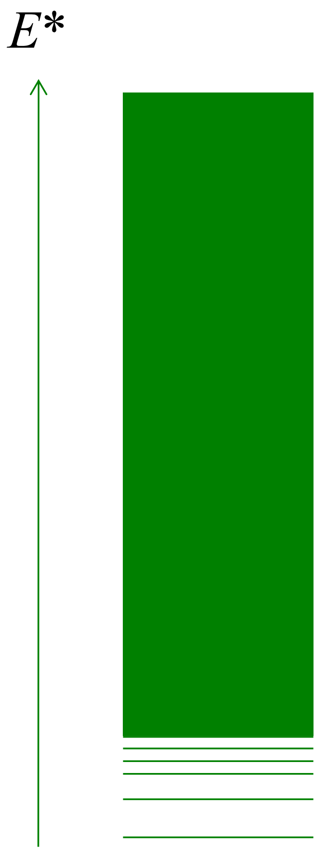
✓ the spectrum of CN states: RMT → shell model (KSHELL)
✓ decay widths → statistical model
✓ 3D calculations with the Coulomb barrier





Discussion: the validity of compound nucleus picture at low energies?

A question: at what E^* does the compound nucleus picture begin to hold?



quasi-continuum spectrum

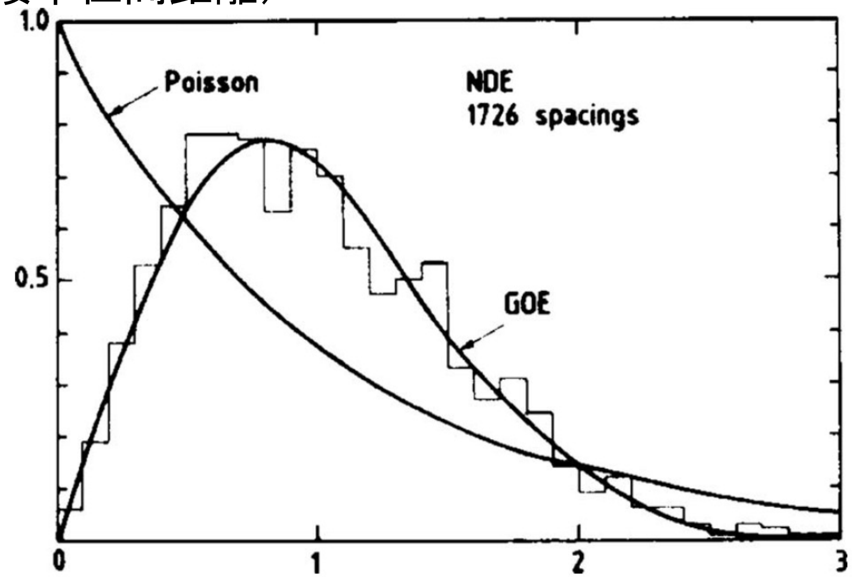
$\rightarrow$ statistical treatment

$$\rho(E) \sim e^{2\sqrt{aE^*}}$$

regular motion

random matrix: provides a good description of CN

the nearest neighboring level spacing distribution
（最近接準位間距離）

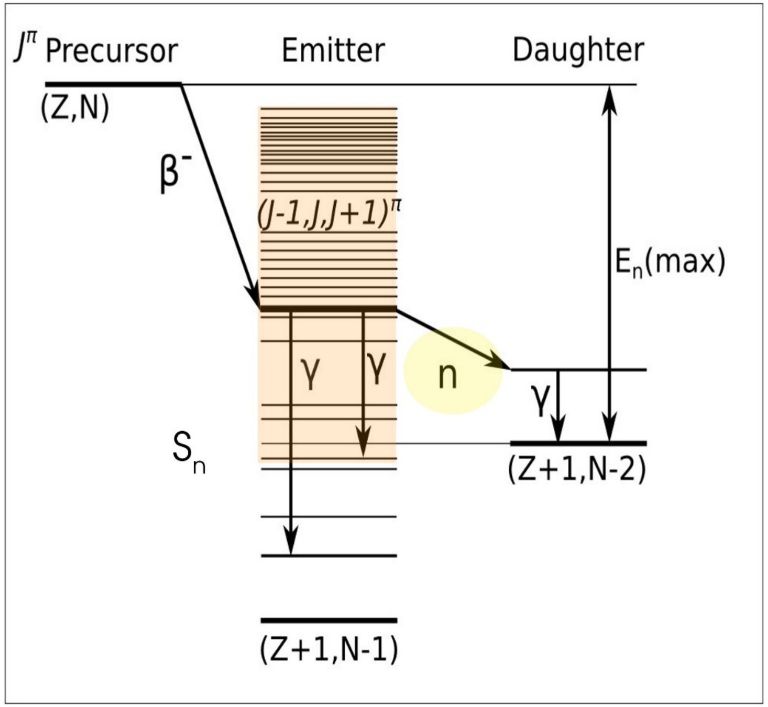


O. Bohigas et al., “Nuclear Data for Science and Technology”, p. 809.

Discussion: the validity of compound nucleus picture at low energies?

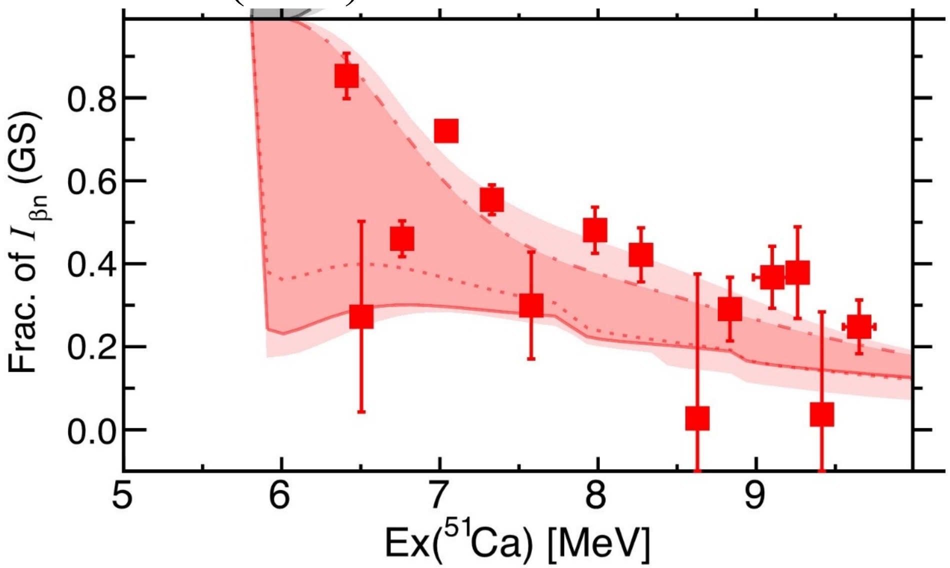
A question: at what *E** does the compound nucleus picture begin to hold?

beta-delayed neutron emission



a figure by R. Grzywacz (Tennessee)

^{51}K (3/2+) → ^{51}Ca** → ^{50}Ca* + n



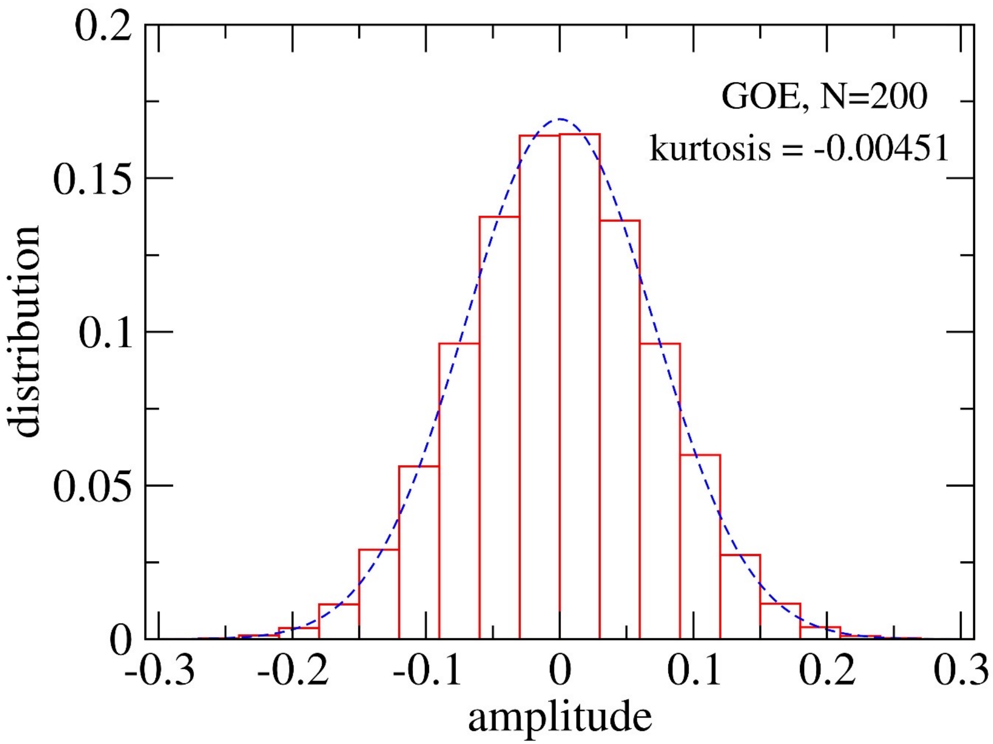
Z.Y. Xu, R. Grzywacz et al., PRL133, 042501 (2024)

Statistical model calculations seem to work well despite that the level density is low (10-20 states/MeV).

Discussion: the validity of compound nucleus picture at low energies?

A question: at what *E** does the compound nucleus picture begin to hold?

We address this question by investigating the kurtosis（尖度）of a distribution.



$$\psi_k = \sum_\mu C_\mu^{(k)} |\mu\rangle \quad \to \text{decay width} \sim |C|^2$$

Random Matrix (GOE):

$$P(x) = \sqrt{\frac{N}{2\pi}}\, e^{-Nx^2/2}; \quad \left(x = \{C_\mu^{(k)}\}\right)$$

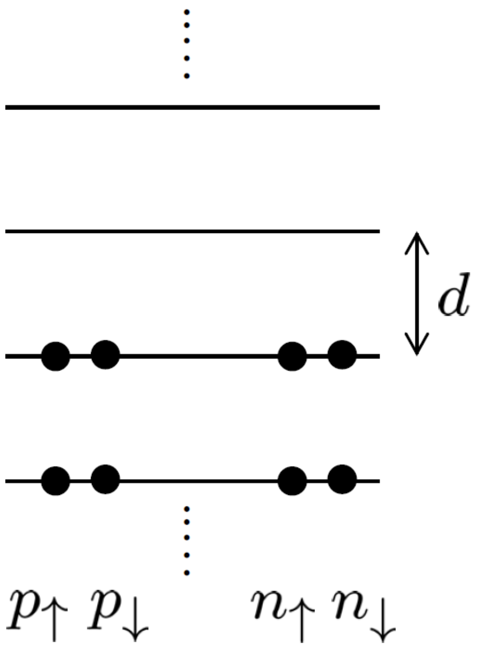
$$\text{kurtosis} \equiv \frac{\langle x^4 \rangle}{\langle x^2 \rangle^2} - 3$$

= 0 for a Gaussian distribution

Discussion: the validity of compound nucleus picture at low energies?

uniform spacing single-particle levels

K. Uzawa and K. Hagino,
Phys. Rev. C108, 024319 (2023).



$$H = H_0 - G \sum_{k,k'} a_k^\dagger a_{\bar{k}}^\dagger a_{\bar{k}'} a_{k'} - v \sum r a^\dagger a^\dagger a a$$

$$G = 0.3d, \quad v_{np} = 0.025d, \; v_{nn} = v_{pp} = 0.02d$$

(note) kurtosis = model space dependent, and basis dependent

$$|\psi_{N,k}\rangle = \sum_{\mu} C_{\mu}^{(k)} |\phi_{N,\mu}\rangle$$

$$[H_0 + H_{\text{pair}}]\,|\phi_{N,\mu}\rangle = E_{\mu} |\phi_{N,\mu}\rangle$$

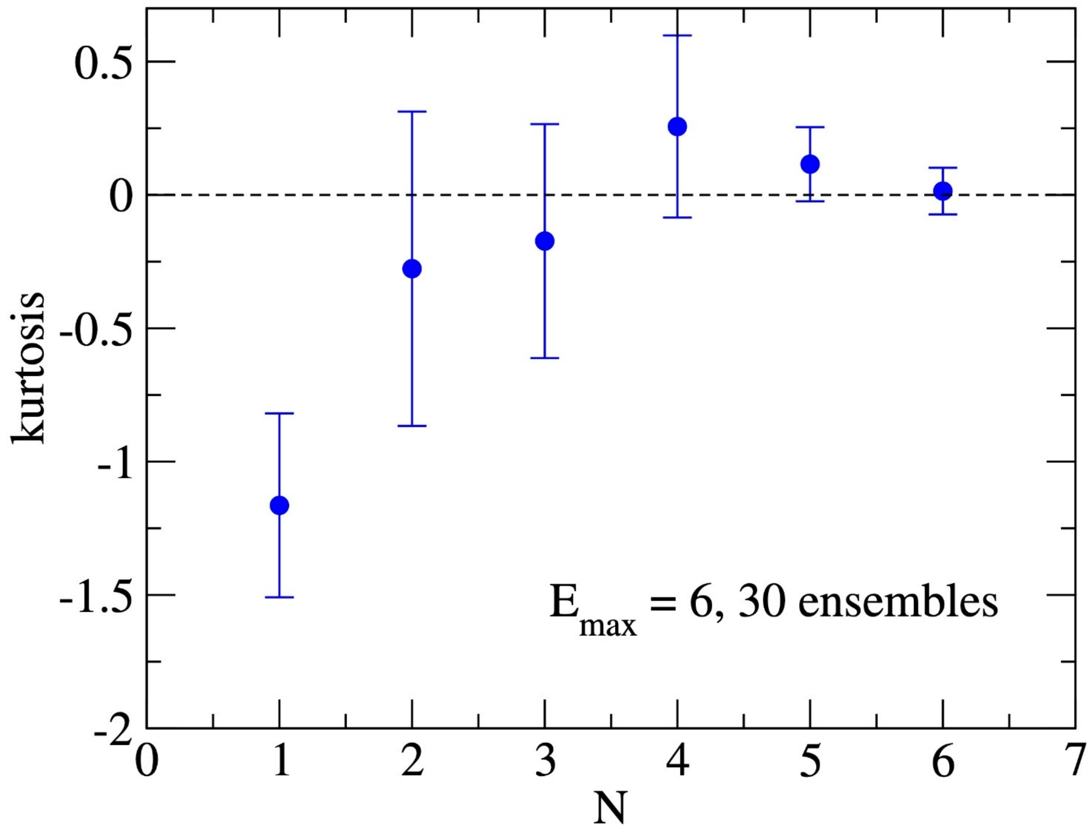
with $E_\mu \sim Nd$

mixing due to the random interaction → kurtosis

Discussion: the validity of compound nucleus picture at low energies?

uniform spacing model

K. Uzawa and K. Hagino, Phys. Rev. C108, 024319 (2023).



# of levels for K_{tot} = 0:

4 (N=1)

16 (N=2)

48 (N=3)

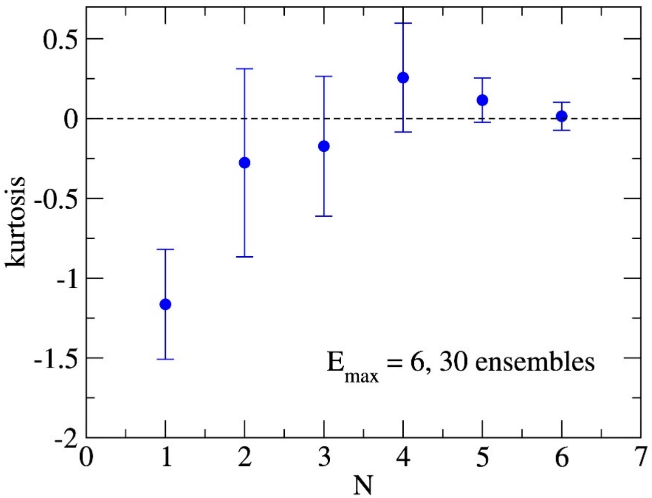
133 (N=4)

332 (N=5)

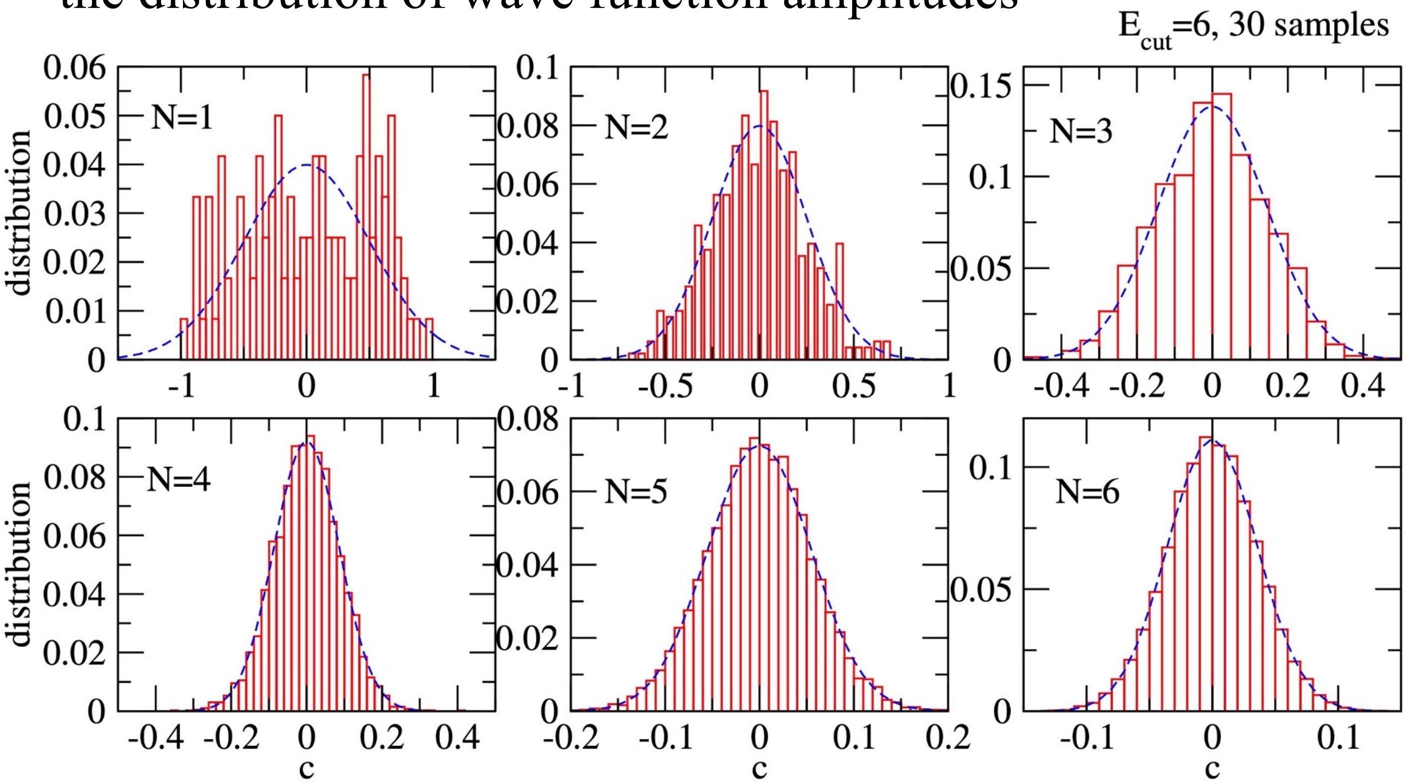
784 (N=6)

the kurtosis is consistent with 0 even for N = 2.

Discussion: the validity of compound nucleus picture at low energies?



the distribution of wave function amplitudes



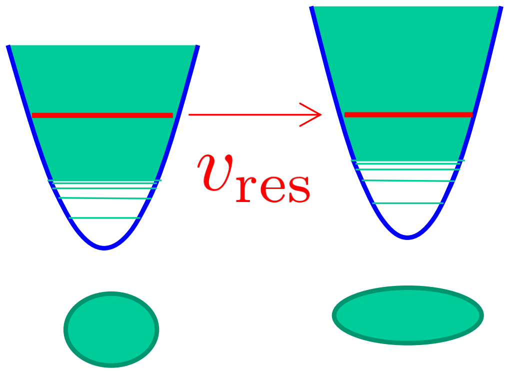
$$|\psi_{N,k}\rangle = \sum_{\mu} C_{\mu}^{(k)} |\phi_{N,\mu}\rangle$$

$$[H_0 + H_{\rm pair}] |\phi_{N,\mu}\rangle = E_{\mu} |\phi_{N,\mu}\rangle$$

$N = 2$: approximately Gaussian
$N = 6$: almost complete Gaussian

K. Hagino and K. Uzawa, in preparation.

Summary



- r-process nucleosynthesis: fission of neutron-rich nuclei

  requires a microscopic approach applicable to low E^* and $\rho(E^*)$

  ➡ a new approach: shell model + GCM

  an application to induced fission of ^{236}U based on Skyrme EDF

  → • the insensitive property
  • a small value of d.o.f.

  } ← the transition state theory

- energy dependence of fusion cross sections for ^{12}C+^{12,13}C

  ^{12}C+^{12}C : prominent resonance peaks
  ^{12}C+^{13}C : a much smoother energy dependence

  a new model with shell model calculations for the CN

  future direction: 1-channel → coupled-channels

- the validity of the compound nucleus picture at low excitation energies

  ✓ recent measurements of beta-delayed neutron emissions
  ✓ the kurtosis from an uniform spacing model → consistent with kurtosis ~ 0 even for N=2