Imaging quantum interferences in heavy-ion elastic scattering

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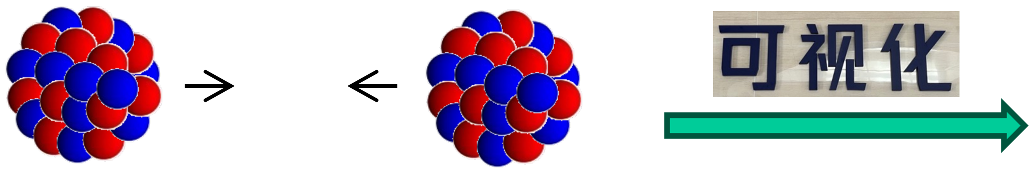
Kyoto University（京都大学）, Kyoto, Japan



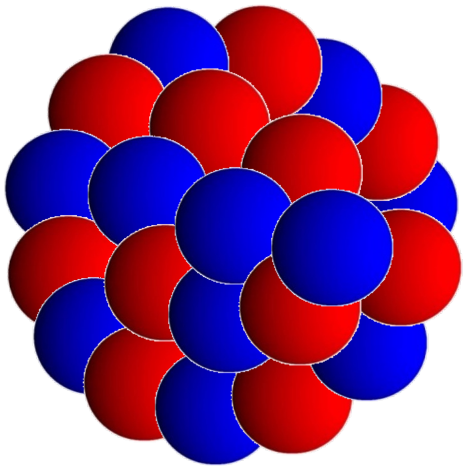
Collaborator: Takuya Yoda (particle theory, Kyoto University)

1. Introduction: interferences in nuclear reactions
2. A new attempt: visualization of nuclear reactions
3. Summary

K. Hagino and T. Yoda, PLB848, 138326 (2024).

A seminar at Sun Yat-sen University（中山大学）, March 13, 2024

Low energy nuclear reactions



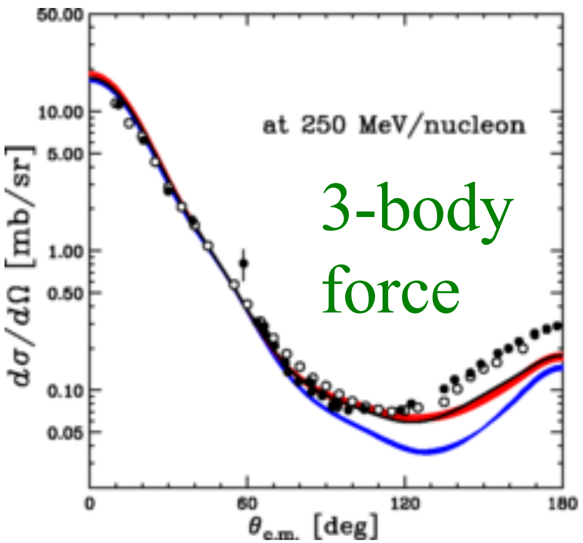
- Nuclei as quantum many-body systems

  $\longleftarrow$ in terms of nucleon d.o.f.

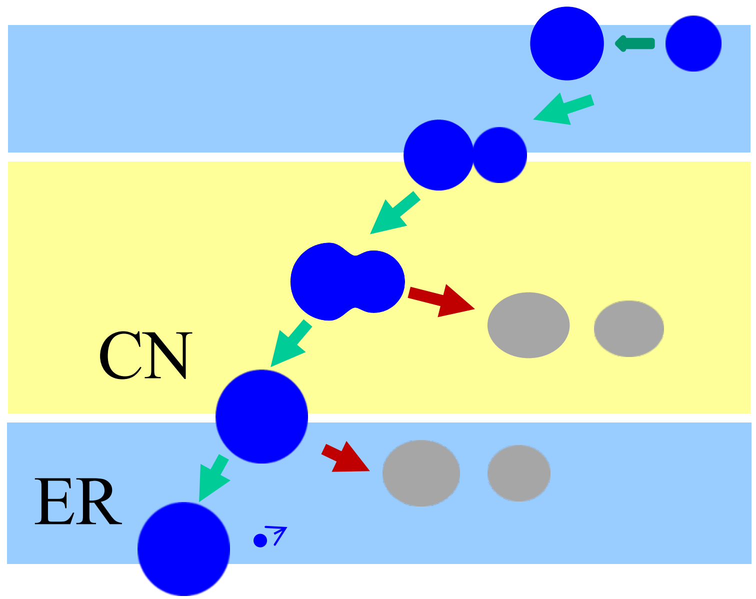
  - static properties: nuclear structure $E < 0$
  - dynamics: nuclear reactions $E > 0$

✓ Nuclear Reactions as a tool to investigate nuclear structure

knock-out reactions



K. Sekiguchi et al., PRC89('14)064007

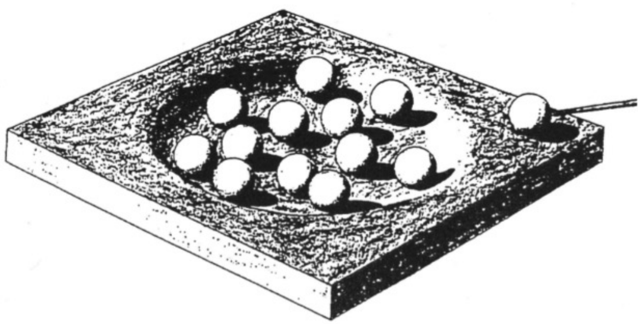


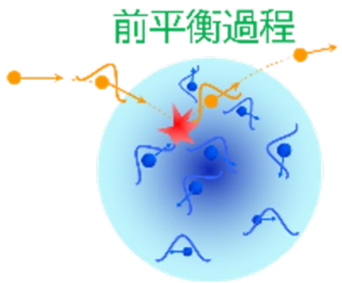
a synthesis of SHE

Two aspects of nuclear reactions

✓ a tool for nuclear structure ← this is often emphasized….

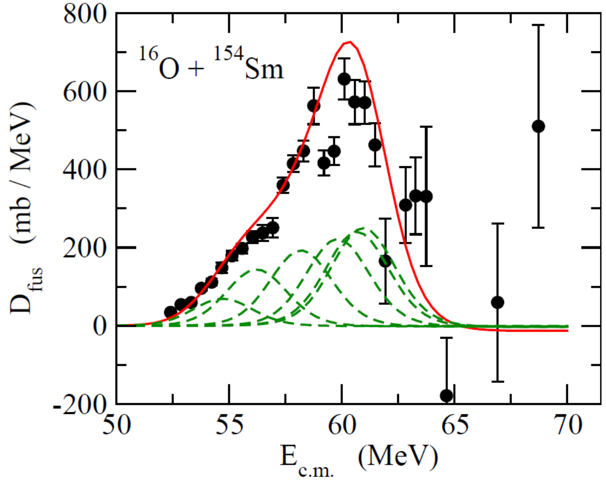
✓ reaction dynamics ⬅ this talk





pre-equilibrium reactions

fusion barrier distribution



nucleus: a composite system

✓ a rich reaction processes

✓ a rich interplay between nuclear structure and reaction

✓ elastic scattering

✓ inelastic scattering

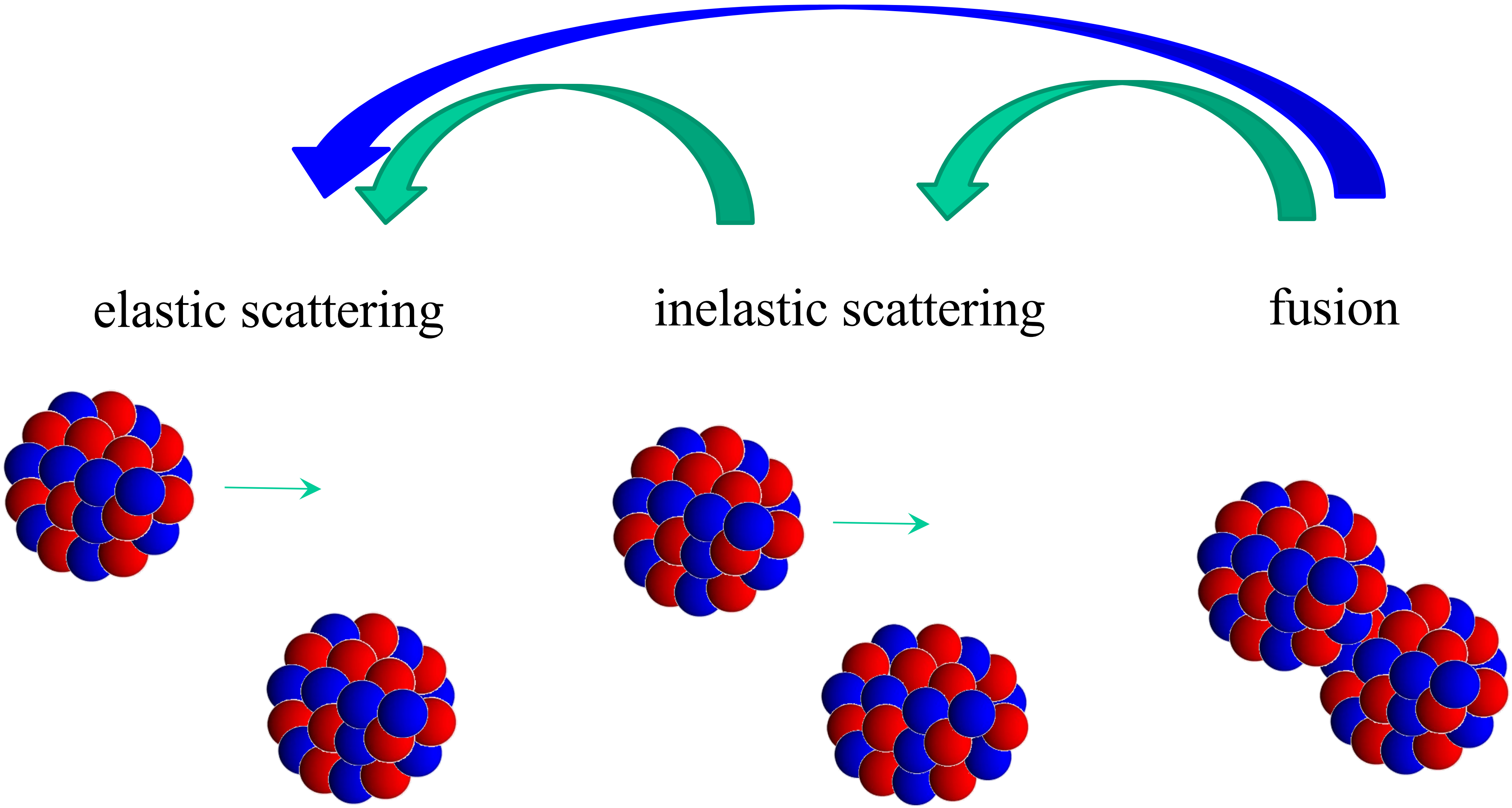
✓ transfer reactions

✓ fusion reactions

✓ g.s. properties (mass, size, shape….)

✓ excitations

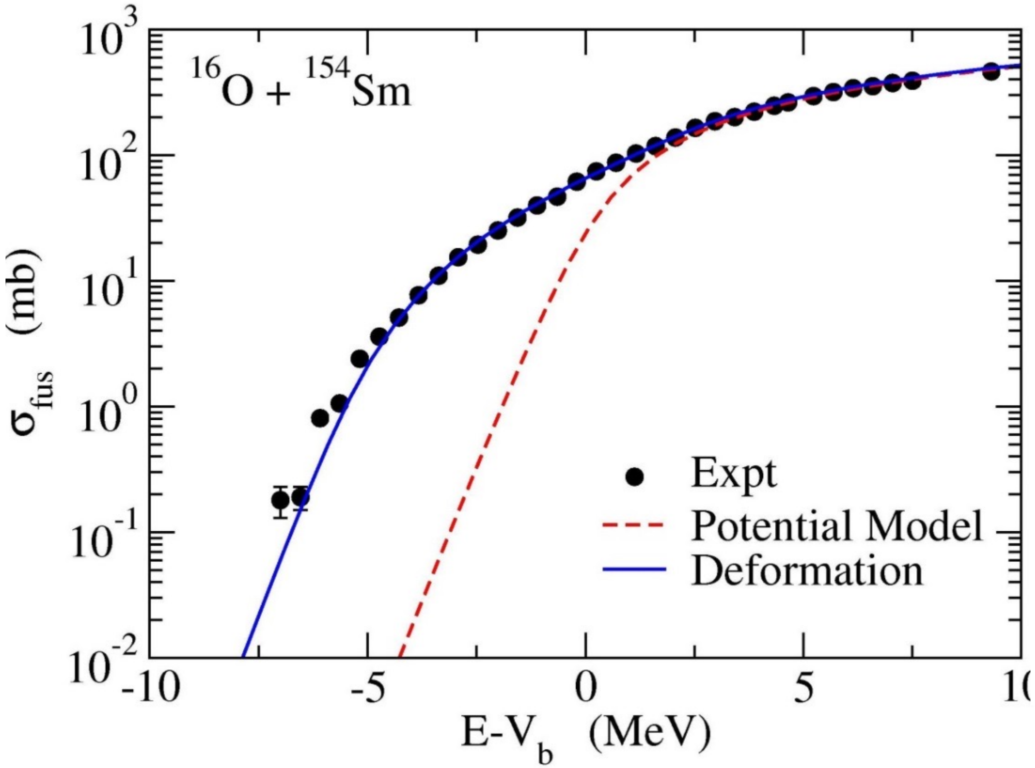
quantum many-body dynamics (nuclear reactions)

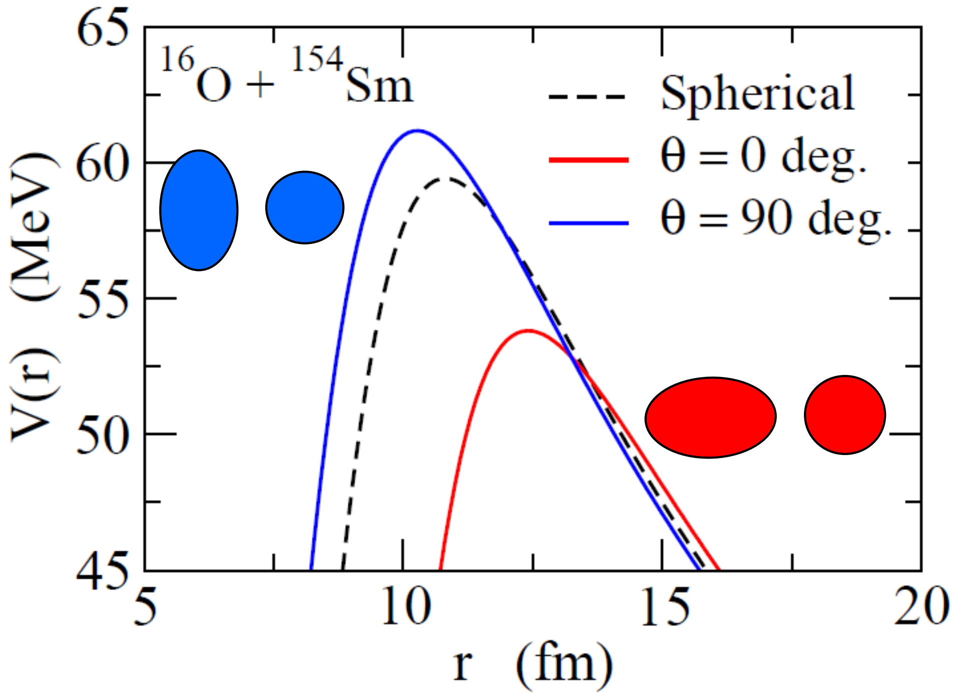


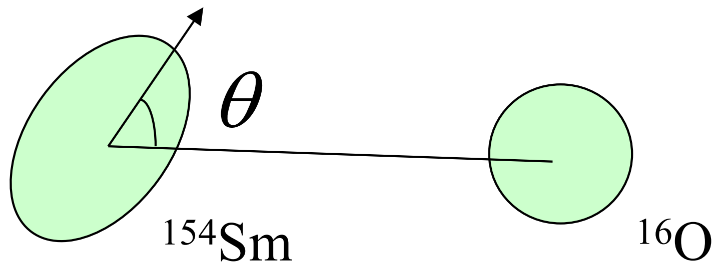
physics of nuclear reactions:
a unified description of these processes

Subbarrier enhancement of fusion cross sections

A typical example of the interplay between structure and reaction

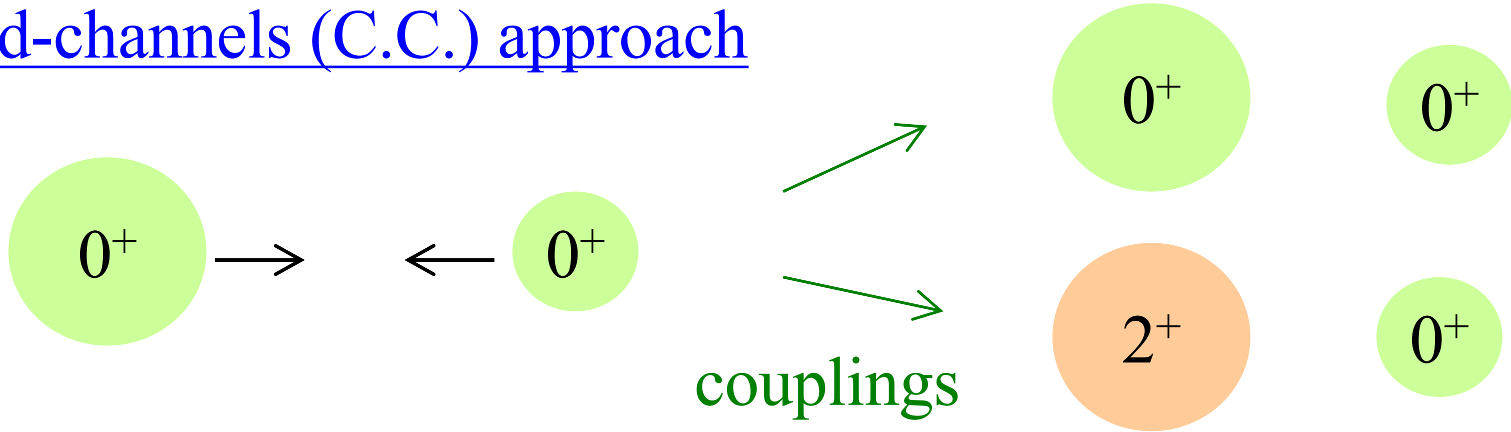






K.H., N. Takigawa, PTP128 (2012) 1061

Coupled-channels (C.C.) approach



a recent review of C.C. approach (Hagino, Ogata, and Moro)

Prog. Part. Nucl. Phys. 125 (2022) 103951

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ELSEVIER

Review

Coupled-channels calculations for nuclear reactions: From exotic nuclei to superheavy elements

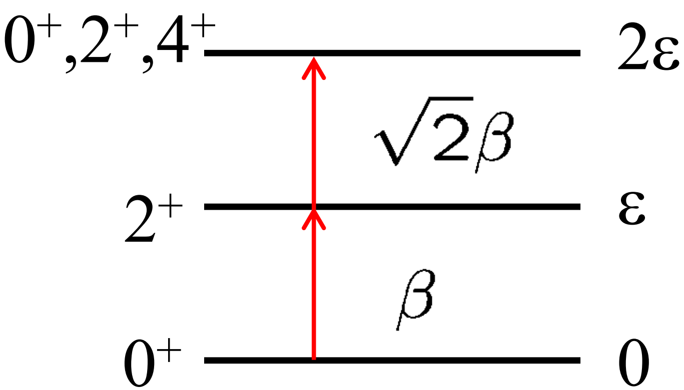
Check for updates

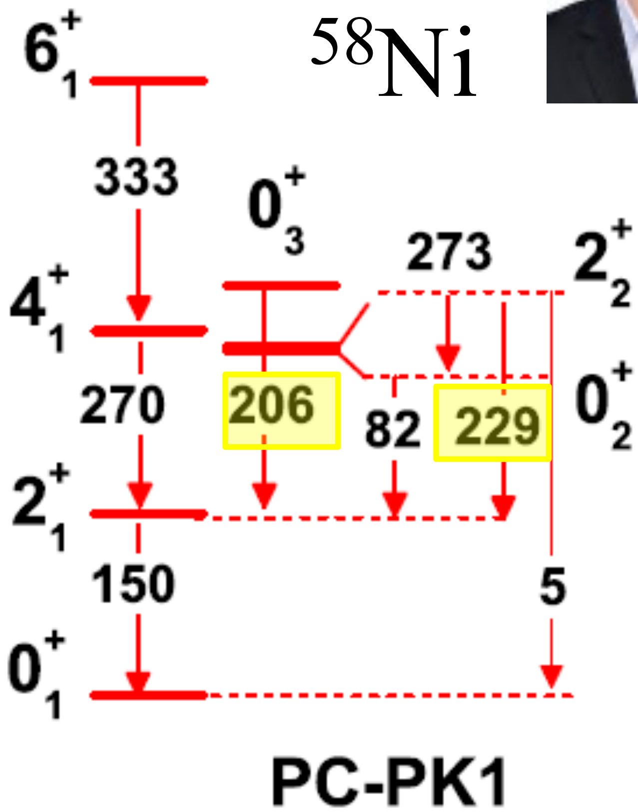
K. Hagino [a,*], K. Ogata [b,c,d], A.M. Moro [e,f]

Semi-microscopic modelling of subbarrier fusion reactions

K.H. and J.M. Yao, PRC91('15) 064606

simple harmonic oscillator

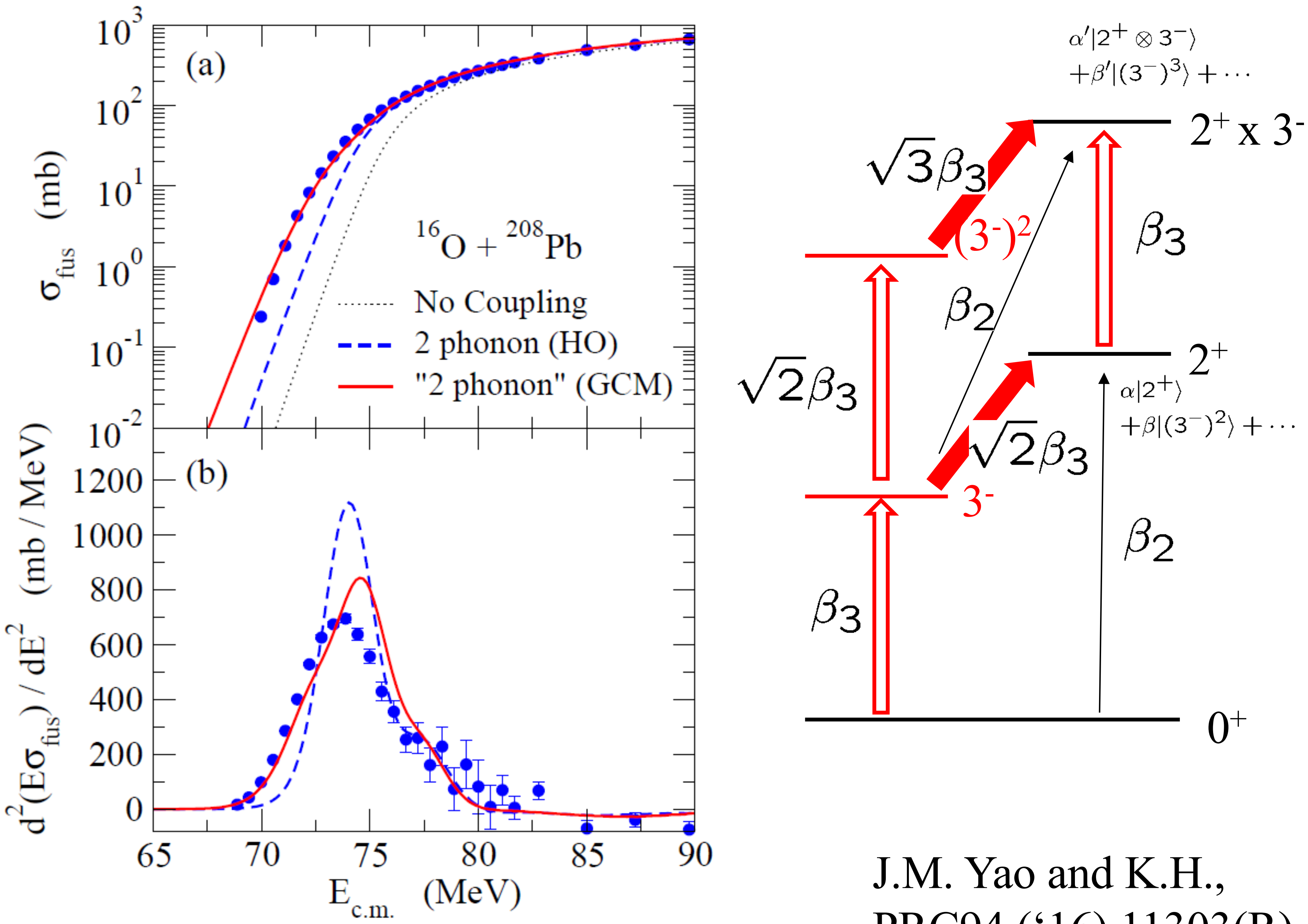




Beyond-mean-field method

anharmonicity of phonon spectra

→ C.C. calculations with
a phenomenological potential



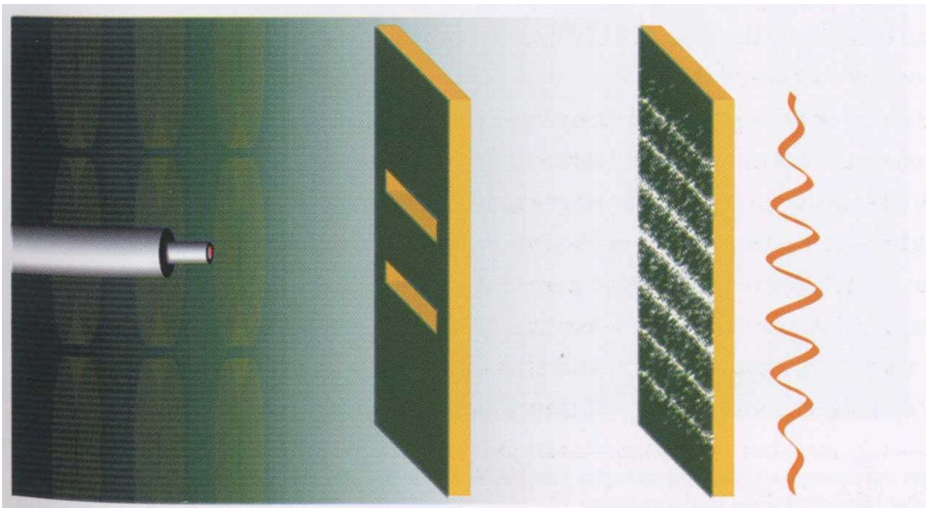
J.M. Yao and K.H., PRC94 ('16) 11303(R)

Nuclear Reactions

nucleus: a composite system
- ✓ a rich reaction processes
- ✓ a rich interplay between nuclear structure and reaction

- ✓ elastic scattering
- ✓ inelastic scattering
- ✓ transfer reactions
- ✓ fusion reactions
- ✓ …….

Another aspect of nuclear reactions
: a variety of quantum mechanical natures



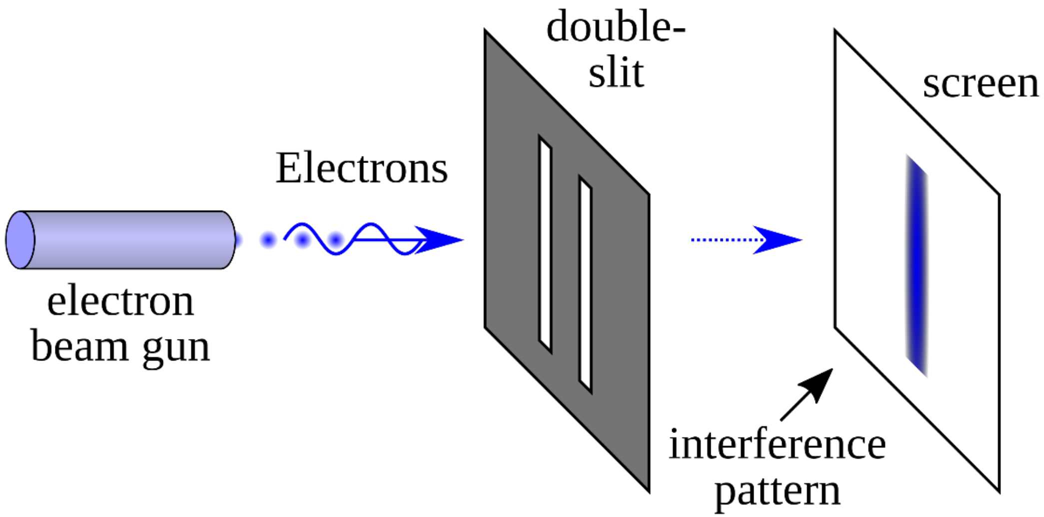
a figure from “Quantum Theory” by Jim Al-Khalili

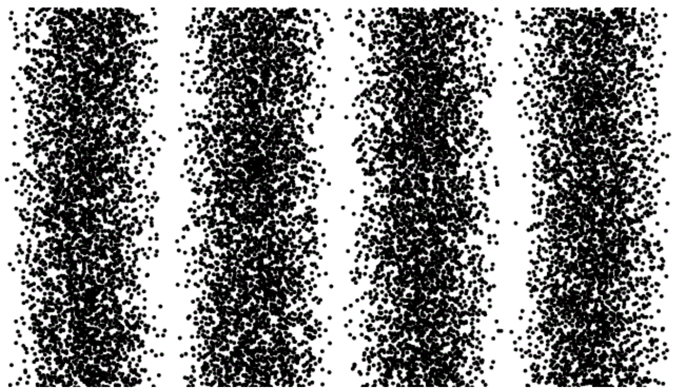
Manifestation of Quantum Nature in Nuclear Reactions

a superposition principle $\psi = \alpha\psi_1 + \beta\psi_2$

$$\to |\psi|^2 = |\alpha\psi_1|^2 + |\beta\psi_2|^2 + \underbrace{(\alpha\psi_1)^*(\beta\psi_2) + (\alpha\psi_1)(\beta\psi_2)^*}_{\text{interference}}$$

when two processes are in principle indistinguishable
→ take square after adding two amplitudes

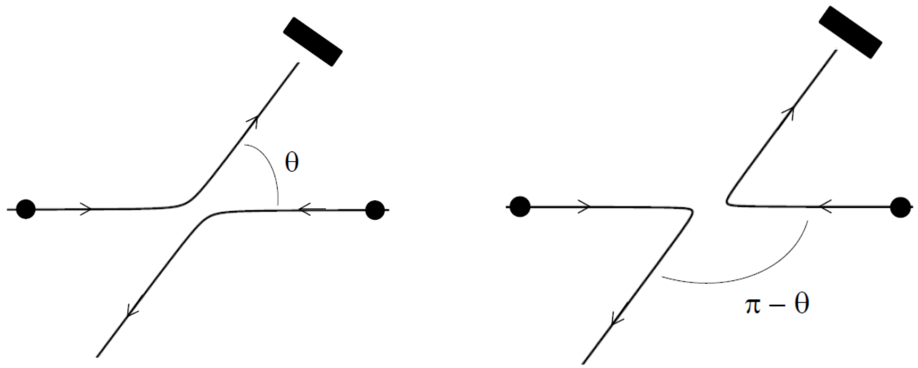




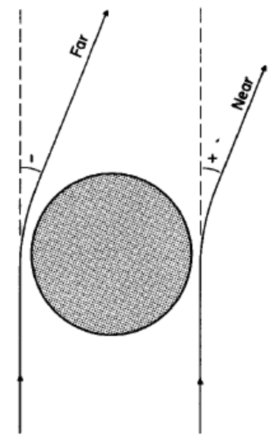
Wikipedia

Interference phenomena in Nuclear Reactions

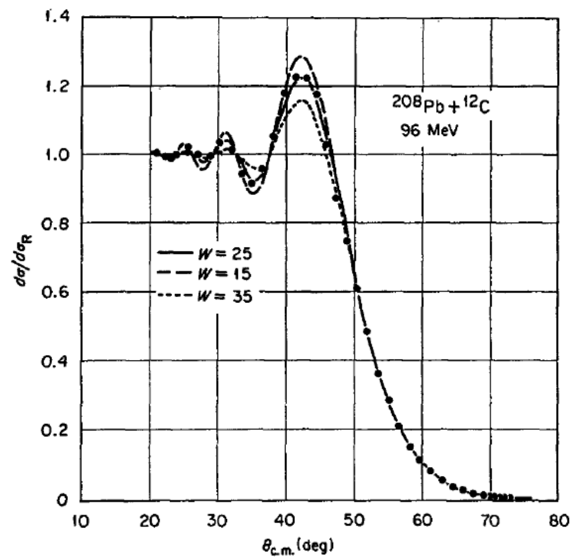
(i) Mott Scattering



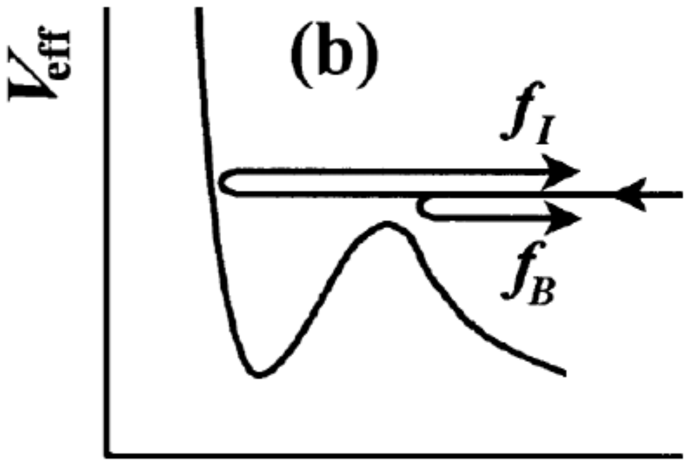
(iii) Near-far interference



(ii) Nuclear-Coulomb interference



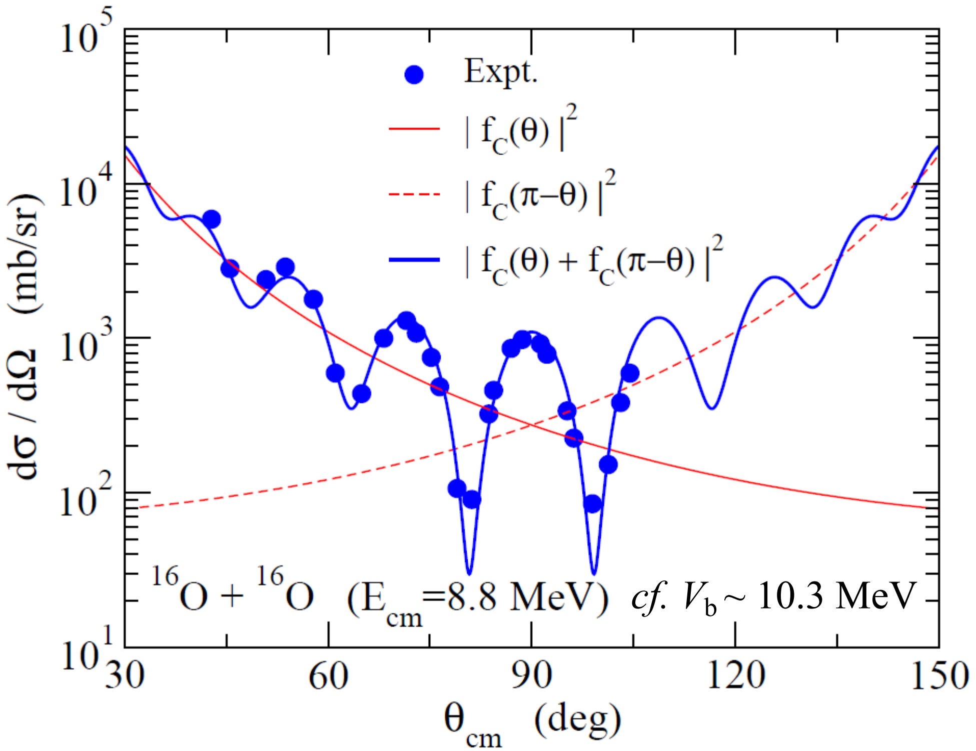
(iv) barrier-wave internal-wave interference

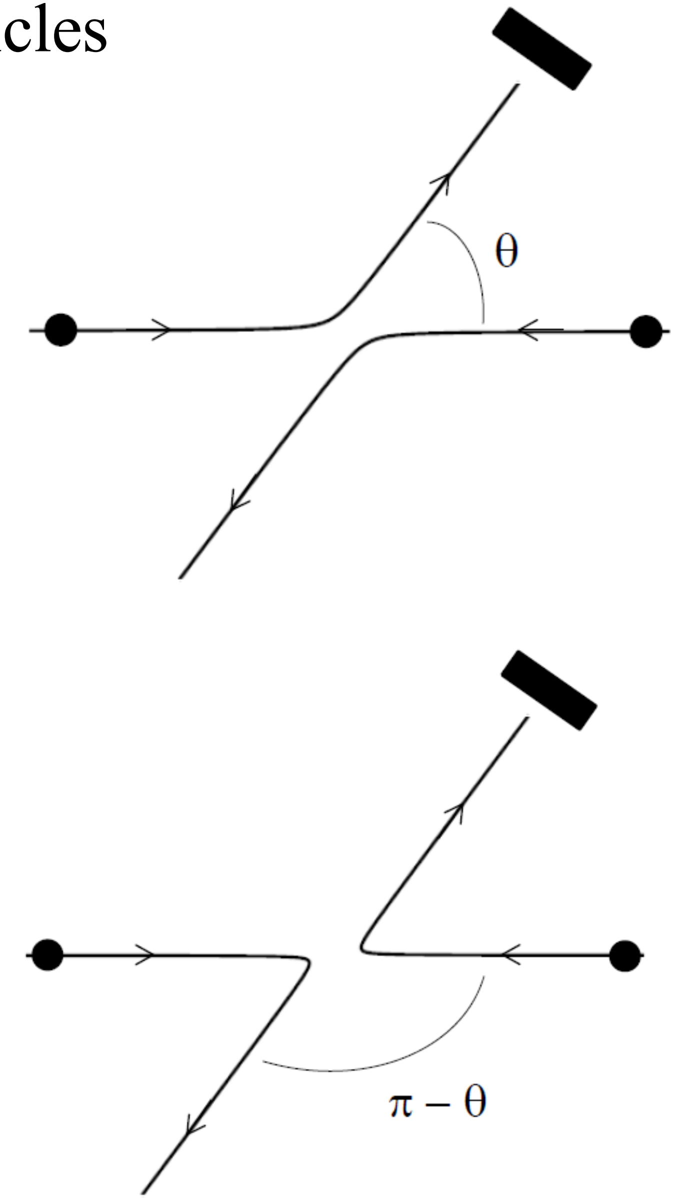


Manifestation of Quantum Nature in Nuclear Reactions

Mott Scattering: scattering of identical particles

$$\frac{d\sigma}{d\Omega} = |f(\theta) \pm f(\pi - \theta)|^2$$





expt: D.A. Bromley et al., Phys. Rev. 123 ('61)878

Interference phenomena in Nuclear Reactions

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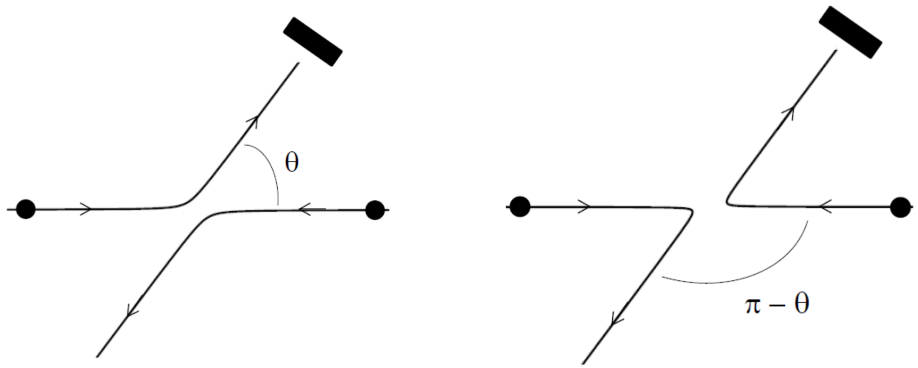

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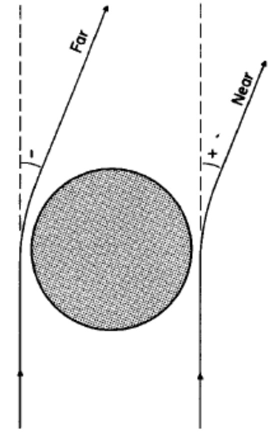

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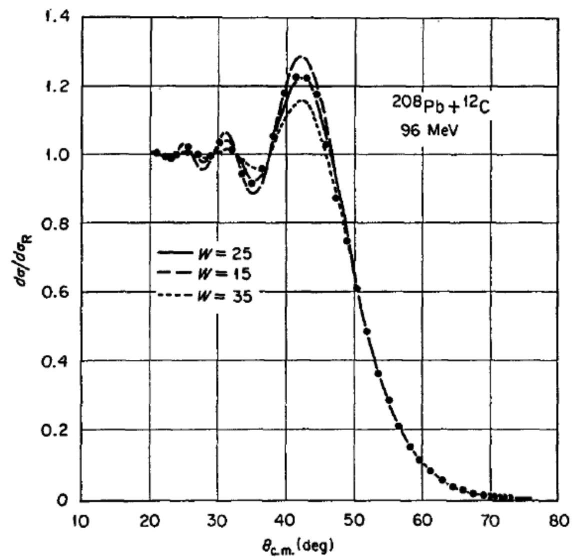

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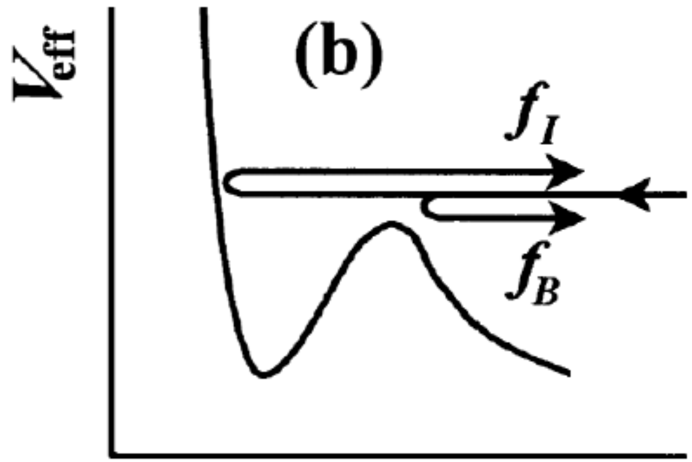
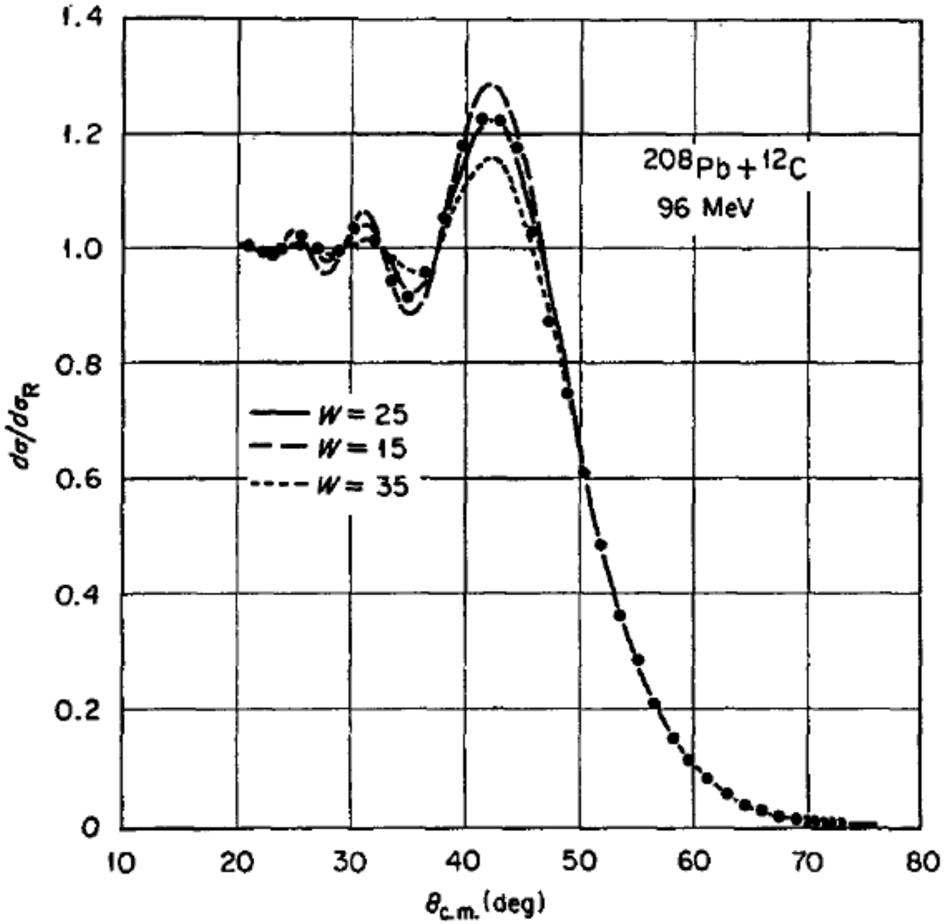


- Coulomb-Nuclear interference

$$f(\theta) = f_C(\theta) + f_N(\theta) \;\rightarrow\; \frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

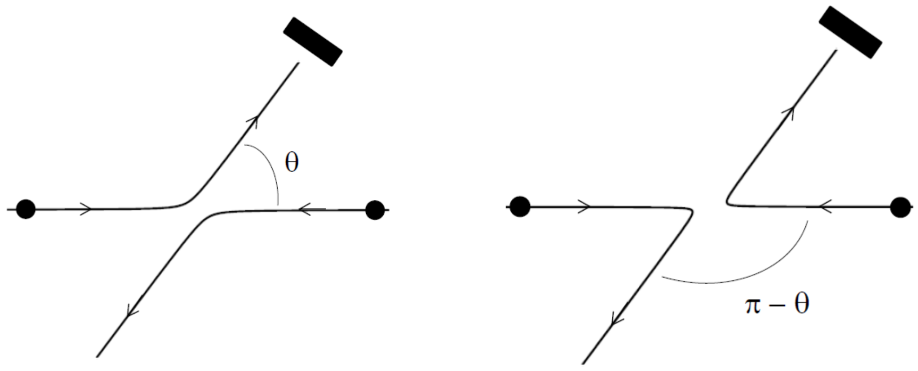


J.B. Ball et al., NPA252 ('75) 208

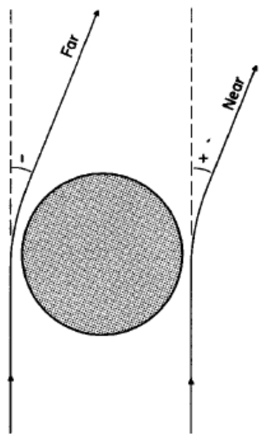
a special case: Fresnel oscillations ($S_l = 0$ ($l < l_g$); $S_l = e^{2i\sigma l}$ ($l > l_g$))

Interference phenomena in Nuclear Reactions

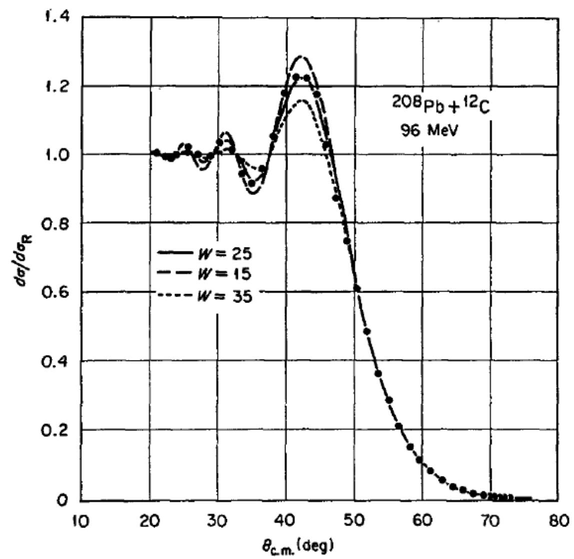
(i) Mott Scattering



(iii) Near-far interference



(ii) Nuclear-Coulomb interference



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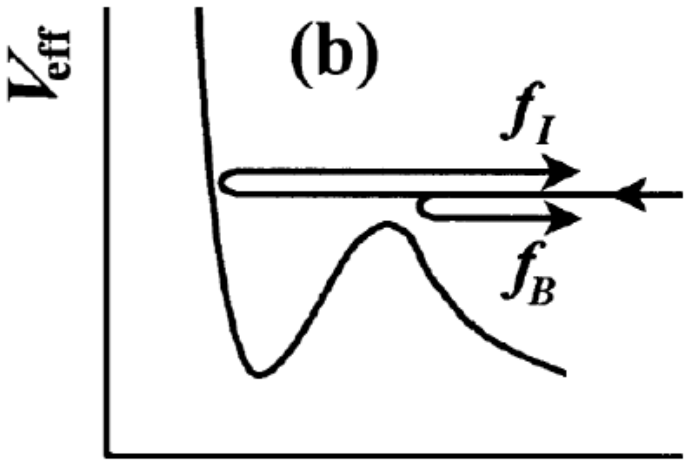
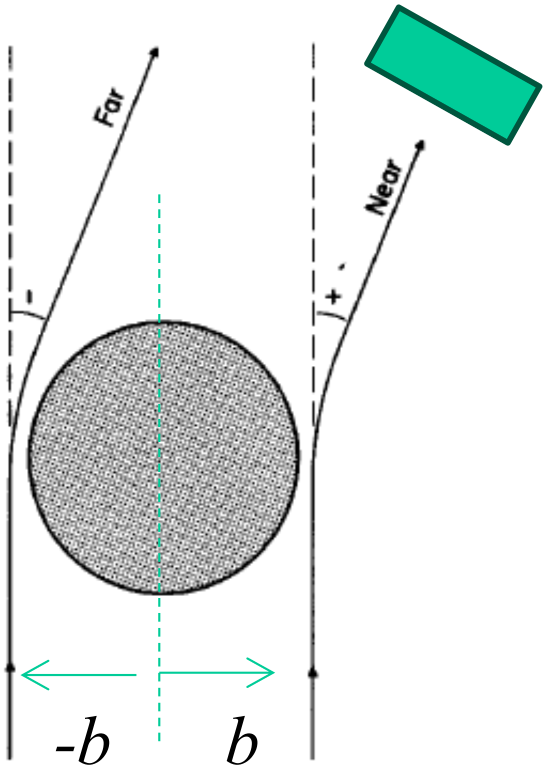
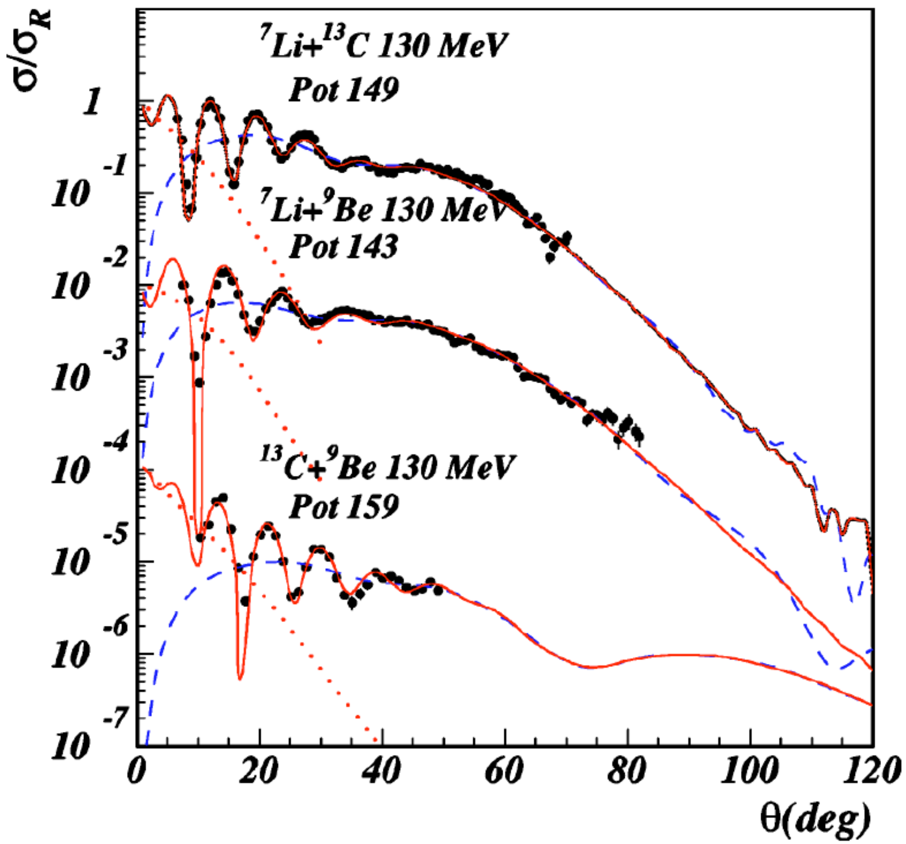


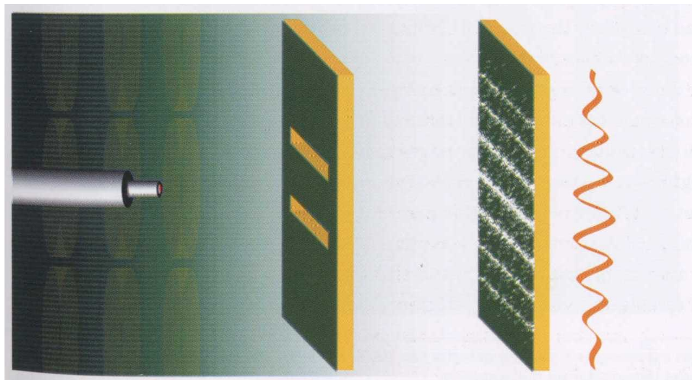
➢ near side - far side interference





R.C. Fuller, PRC12(‘75)1561
N. Rowley and C. Marty, NPA266(‘76)494
M.S. Hussein and K.W. McVoy, Prog. in Part. and Nucl. Phys. 12 (‘84)103

F. Carstoiu et al., PRC70 (‘04) 054610



Interference phenomena in Nuclear Reactions

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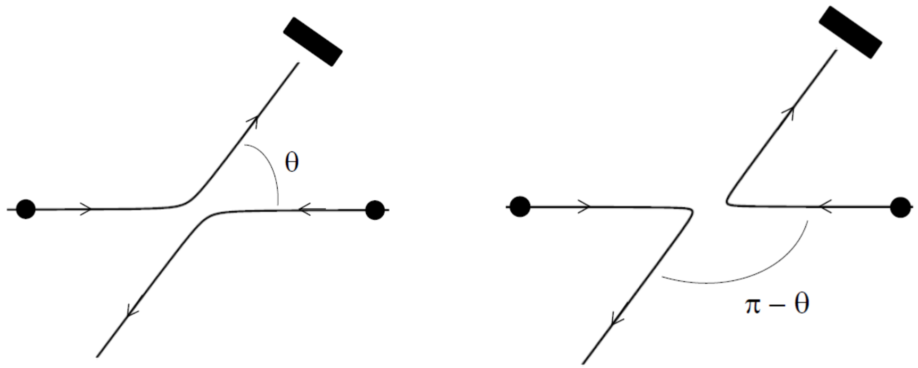
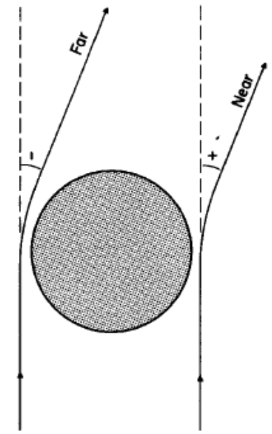


(iii) Near-far interference



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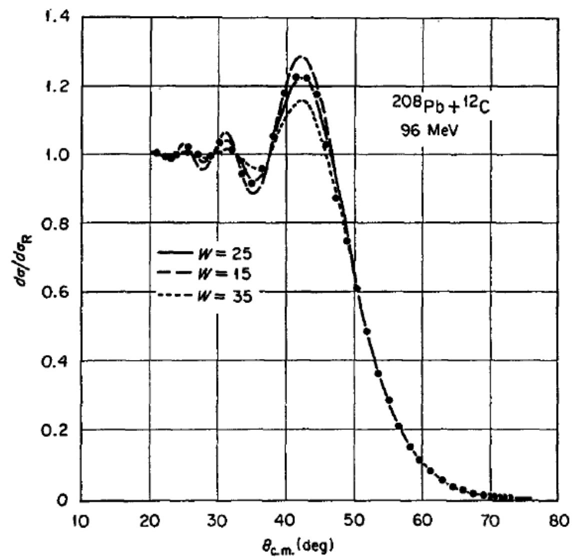

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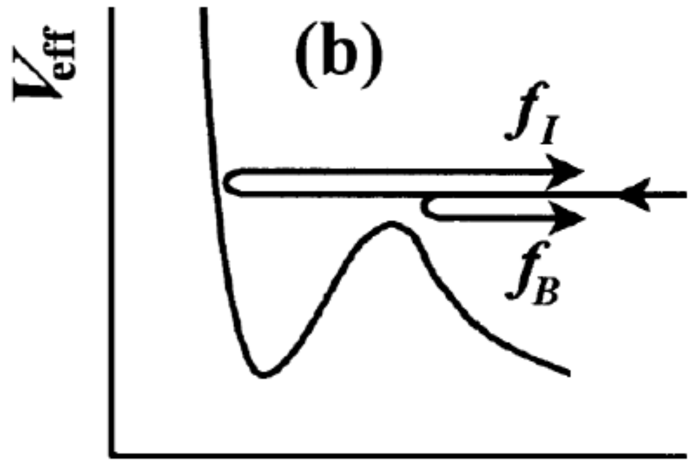
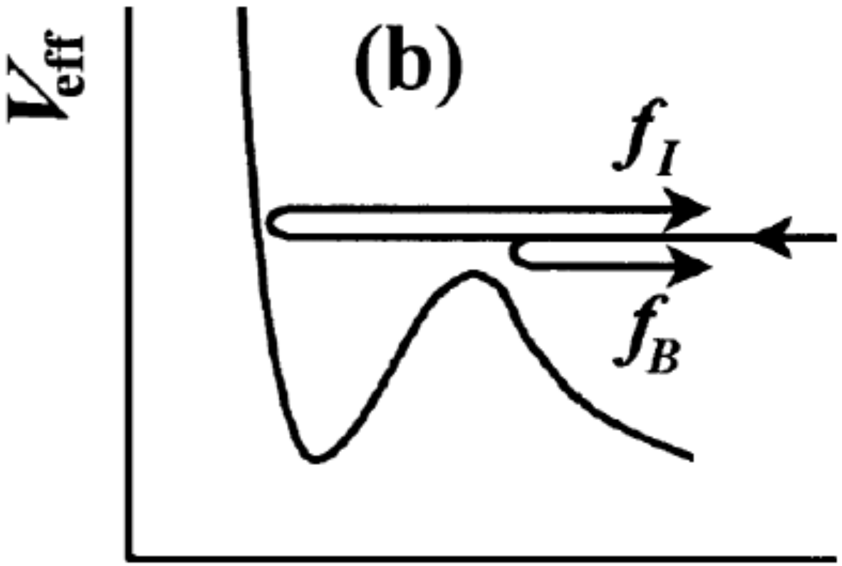
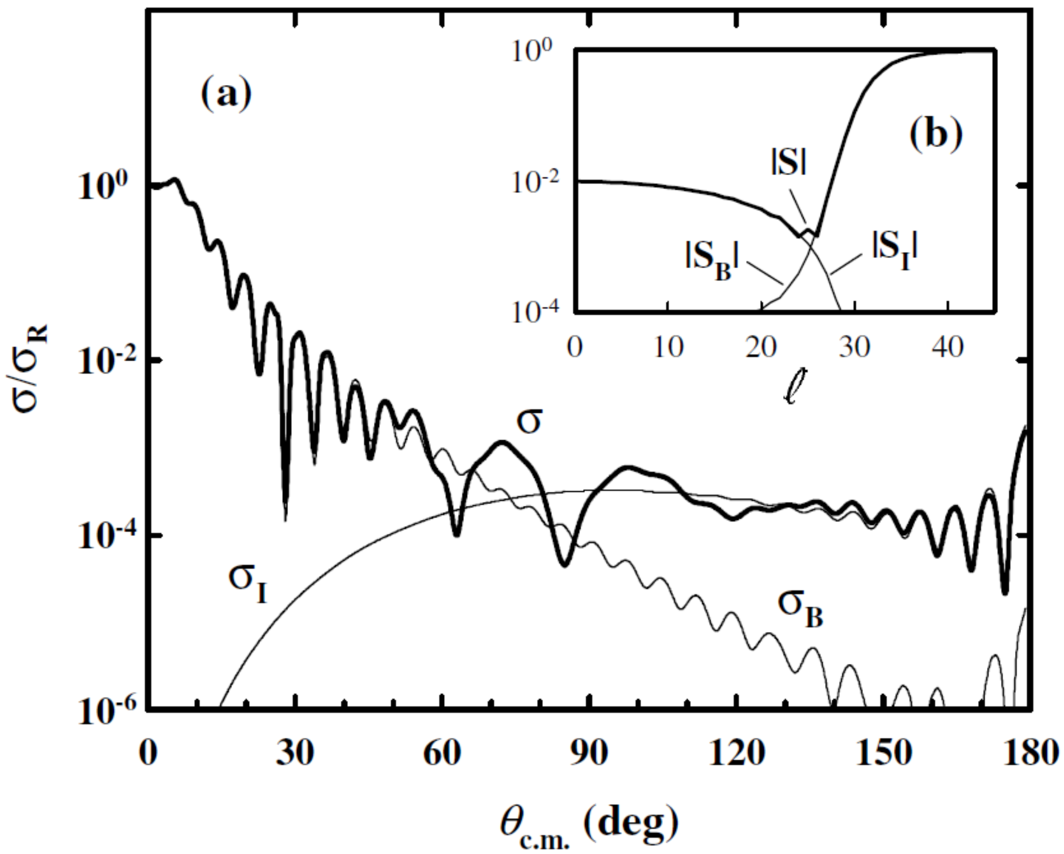


➢ barrier wave – internal wave interference

cf. D.M. Brink and N. Takigawa, NPA279 ('77) 159



$^{16}O+^{16}O$ at 124 MeV



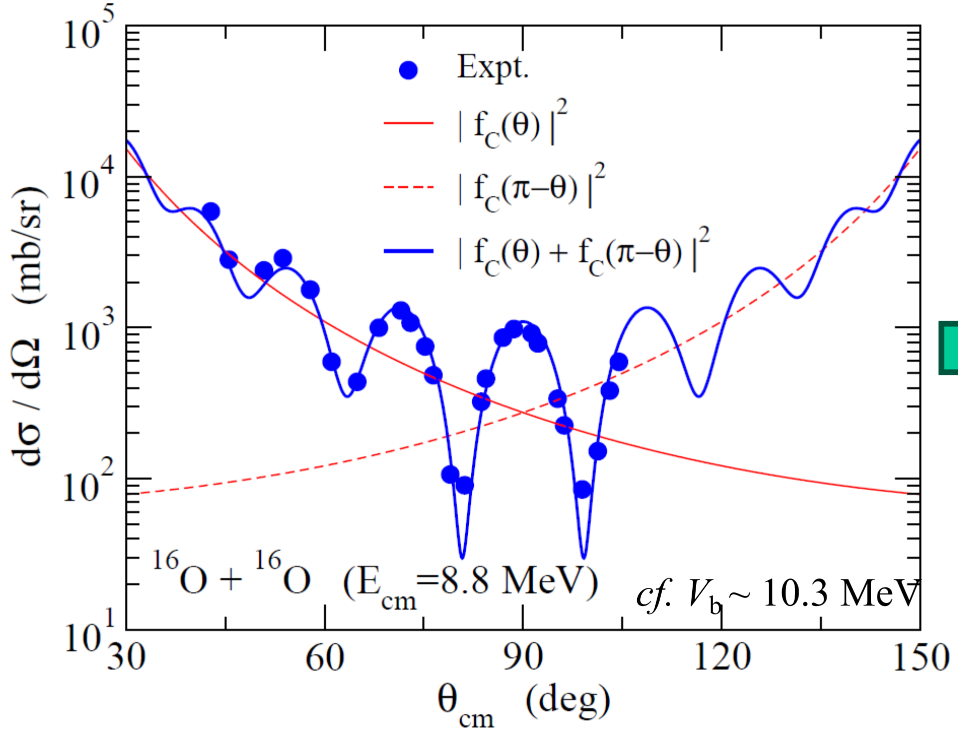
F. Michel et al., PRL85 ('00) 1823

anomalous large angle scattering

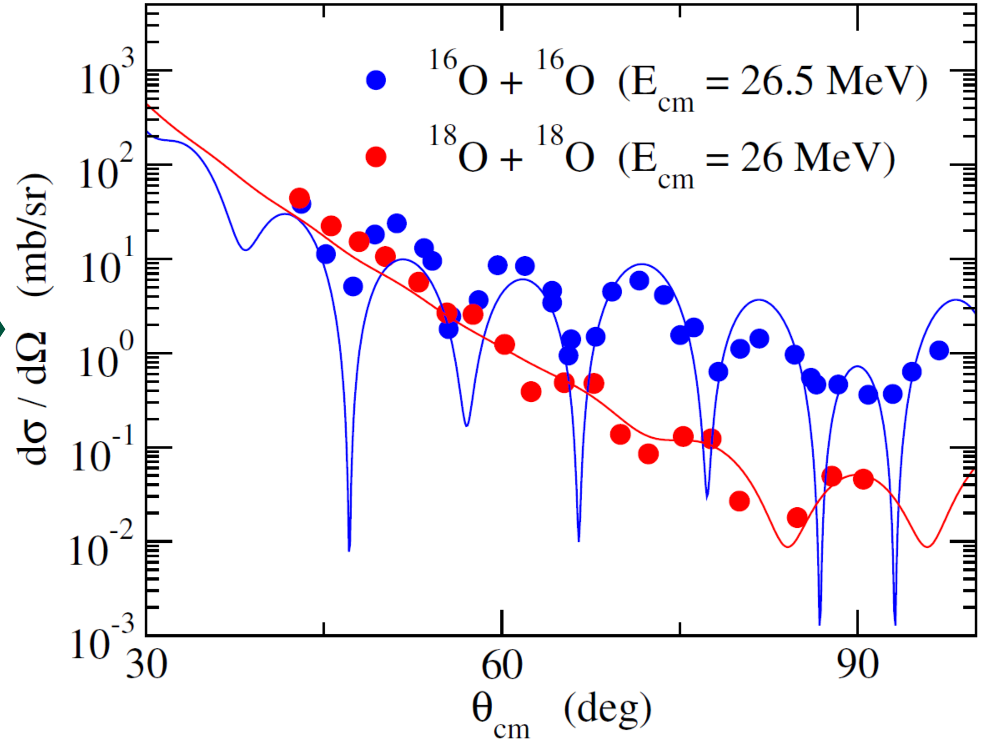
cf. α + ^{40}Ca scattering

$^{16}O+^{16}O$ system

$E < V_b$

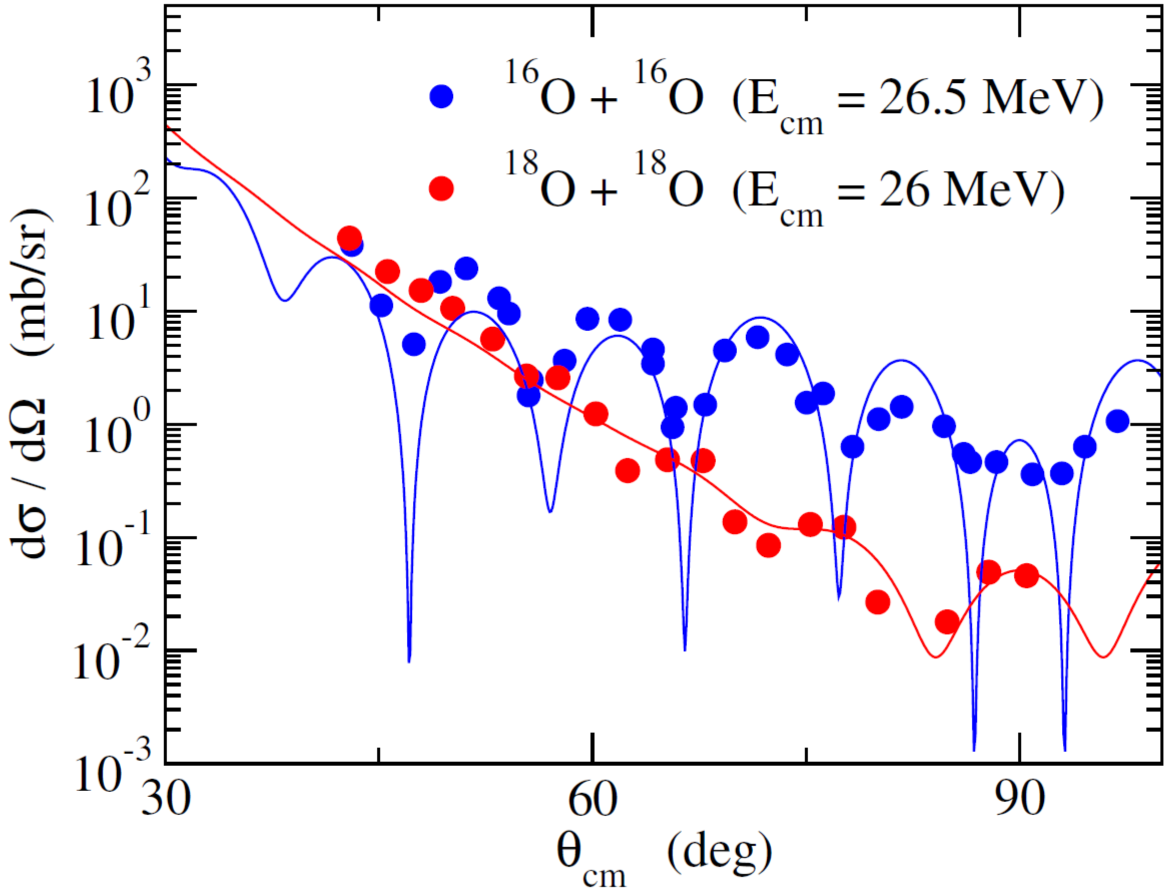


$E > V_b$



expt: D.A. Bromley et al., Phys. Rev. 123 ('61)878

Comparison between $^{16}O+^{16}O$ and $^{18}O+^{18}O$



^{16}O, ^{18}O: I^{π} (g.s.) = 0^{+}
(both are bosons)

$V_b \sim 10.3$ MeV

$\longrightarrow \quad E_{cm} \sim 2.5\ V_b$

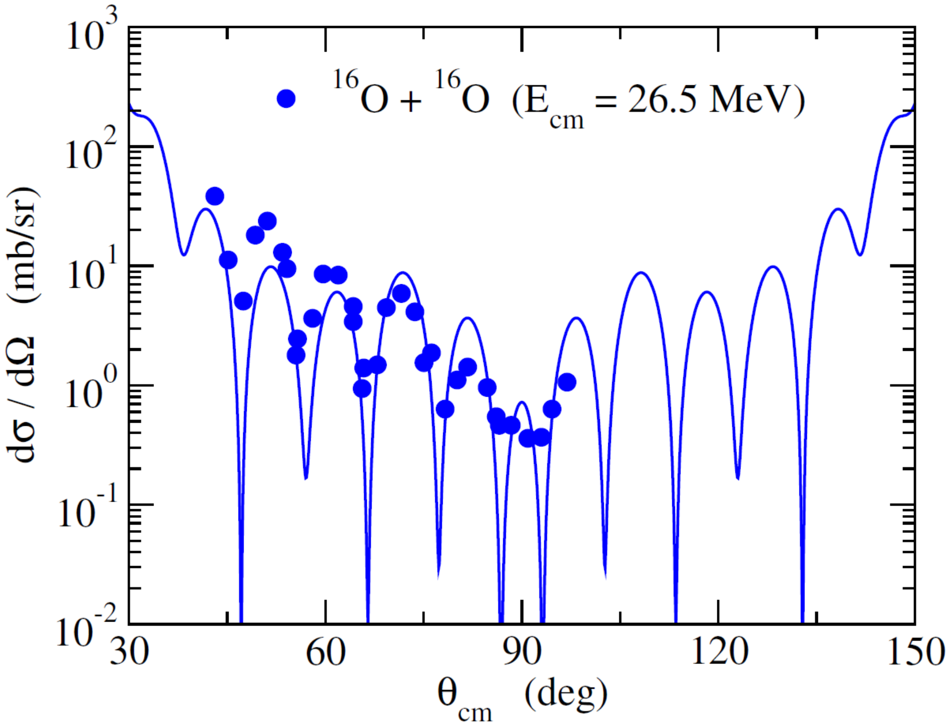
$^{18}O+^{18}O$: much less pronounced interference pattern

$^{18}O = ^{16}O$ (double closed shell) + 2n

→ stronger coupling to environment

➡ manifestation of (environmental) decoherence

Optical potential model calculation



Optical potential model calculations

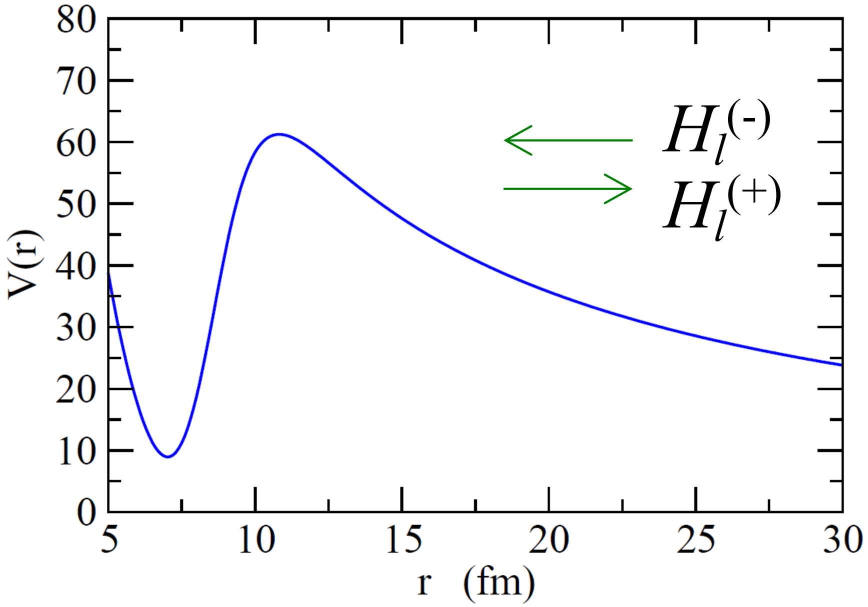
$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{l(l+1)\hbar^2}{2\mu r^2}+V(r)-\underline{iW(r)}-E\right]u_l(r)=0$$

an imaginary part → absorption

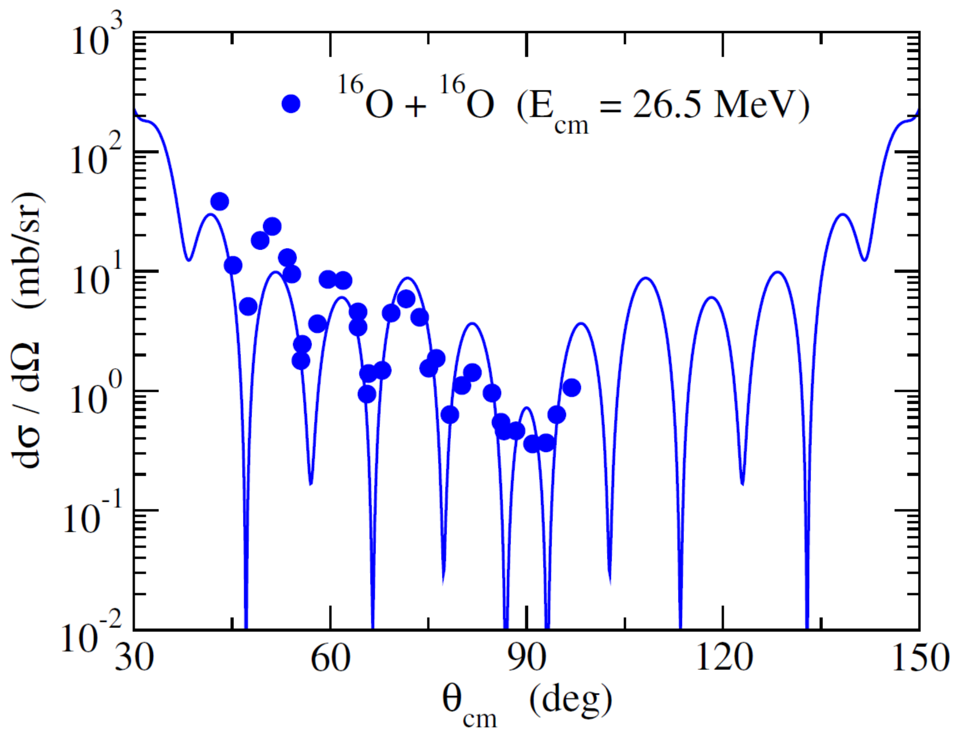
$$u_l(r)\to H_l^{(-)}(kr)-S_lH_l^{(+)}(kr),\quad r\to\infty$$

$$f(\theta)=f_C(\theta)+\sum_l e^{2i\sigma_l}(2l+1)\frac{S_l-1}{2ik}P_l(\cos\theta)$$

$$\frac{d\sigma}{d\Omega}=|f(\theta)|^2$$

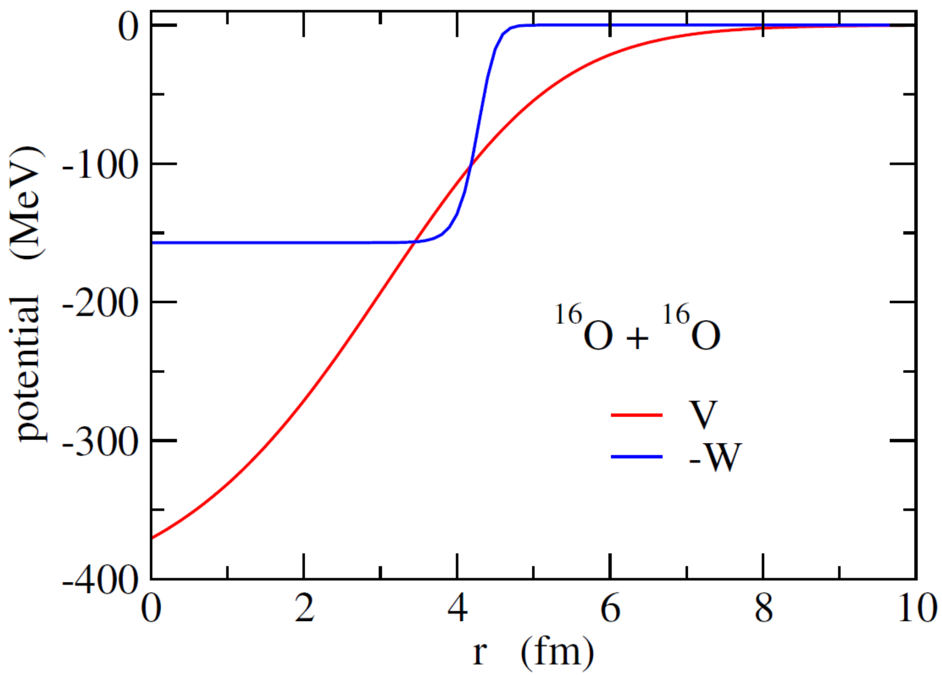


Optical potential model calculation



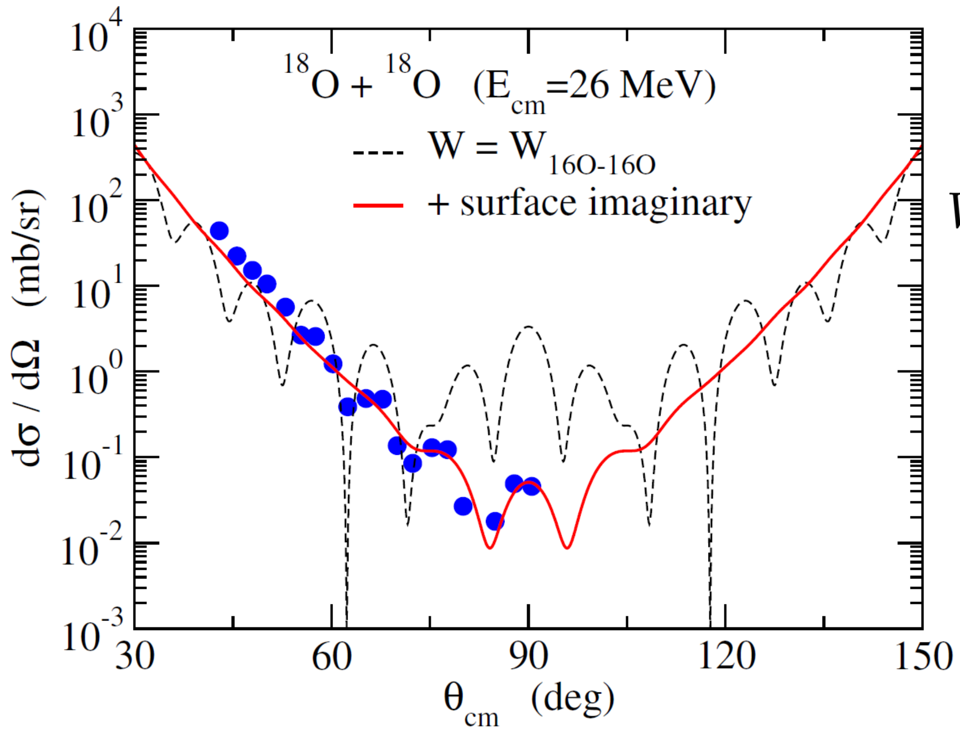
$$V(r) = -\frac{V_0}{(1+\exp[(r-R)/a])^2}$$

$$W(r) = \frac{W_0}{(1+\exp[(r-R_W)/a_W])^2}$$



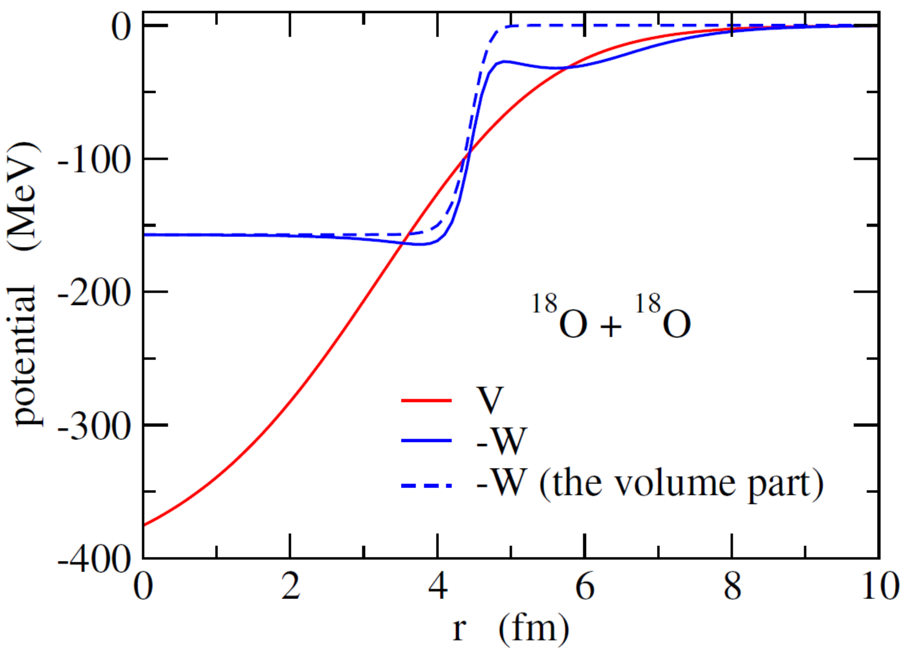
an opt. pot. model calculation with a deep WS^2 potential.

Optical potential model calculation



$$V(r) = -\frac{V_0}{(1+\exp[(r-R)/a])^2}$$

$$W(r) = \frac{W_0}{(1+\exp[(r-R_W)/a_W])^2} + \frac{d}{dr}\frac{W_{0S}}{1+\exp[(r-R_{WS})/a_{WS}}$$

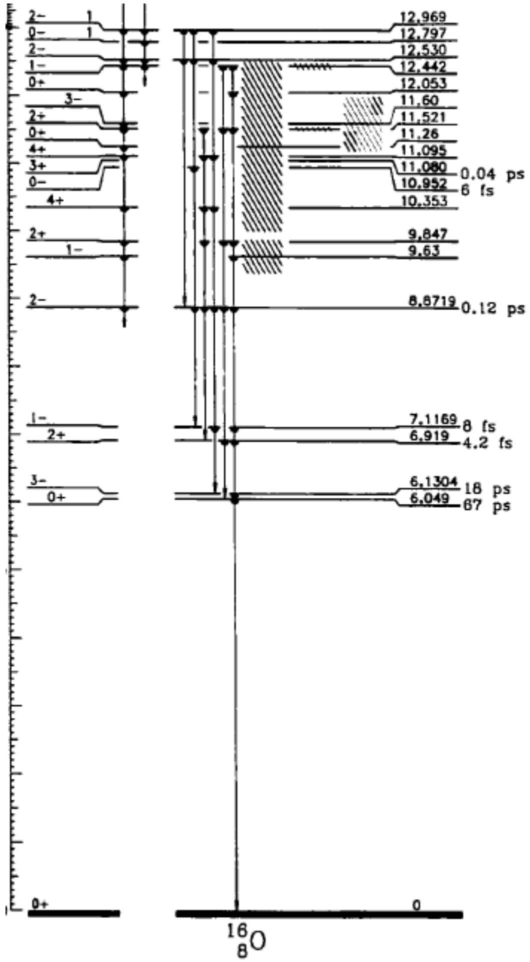


However, the same opt. pot. does not fit ^{18}O+^{18}O

↓

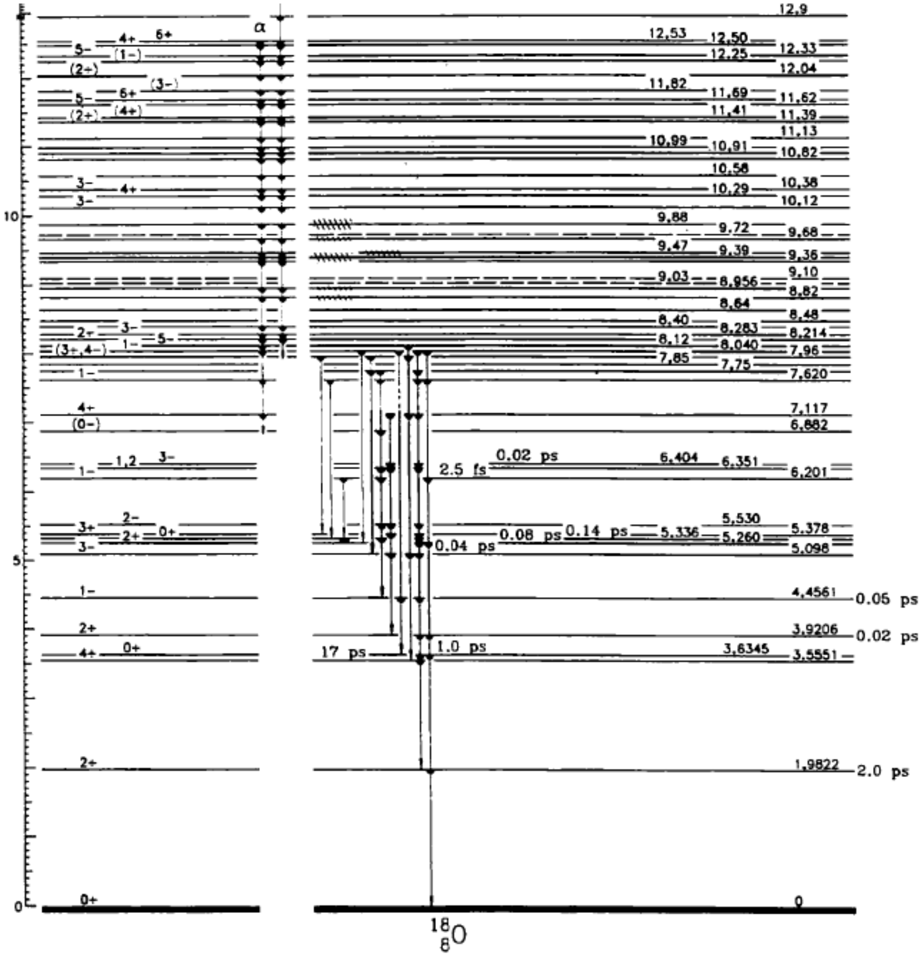
need to increase *W*
(with a surface imaginary pot.)

Spectra up to E* = 13 MeV



^{16}O

20 levels

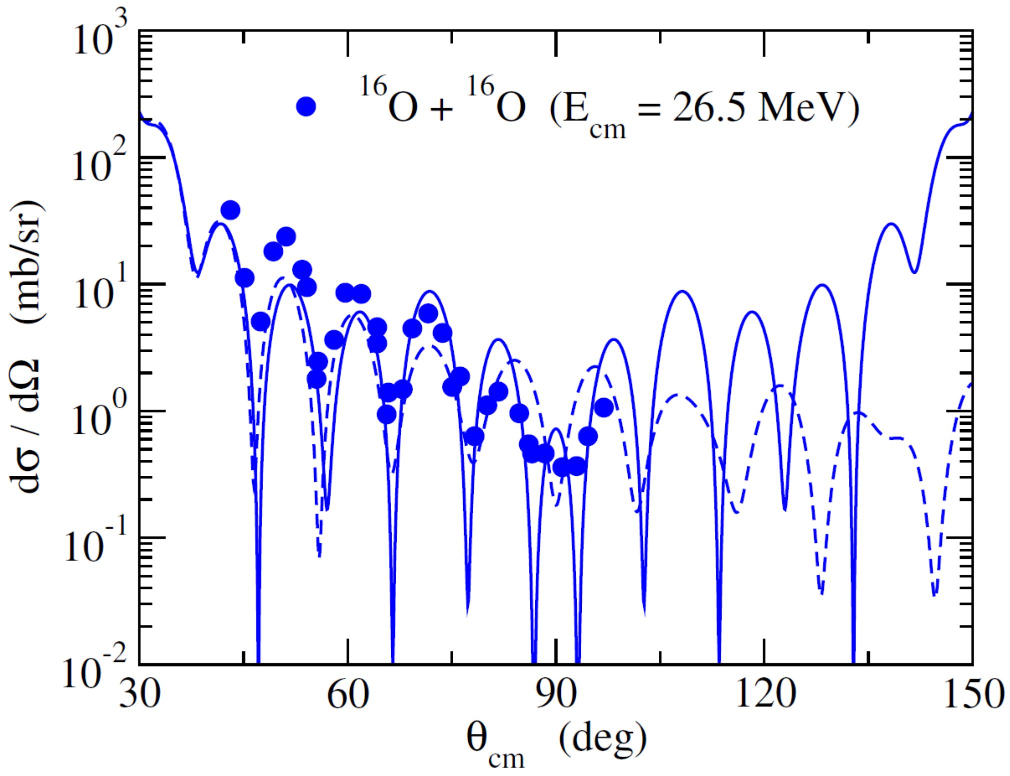


^{18}O

56 levels

cf. the number of oepn channels, F. Haas and Y. Abe, PRL46('81)1667

Origins of oscillations

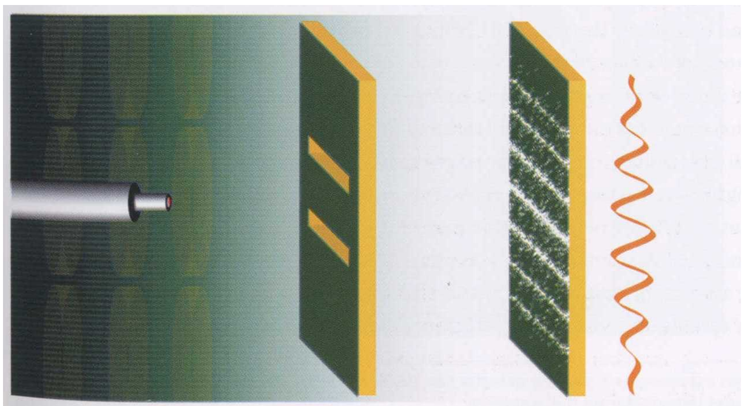


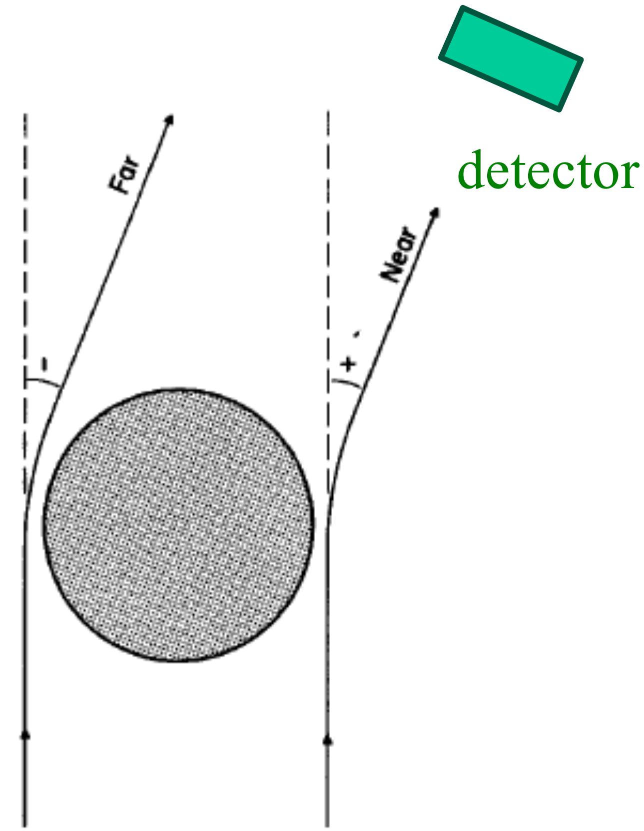
strong oscillations even in unsymmetrized cross sections

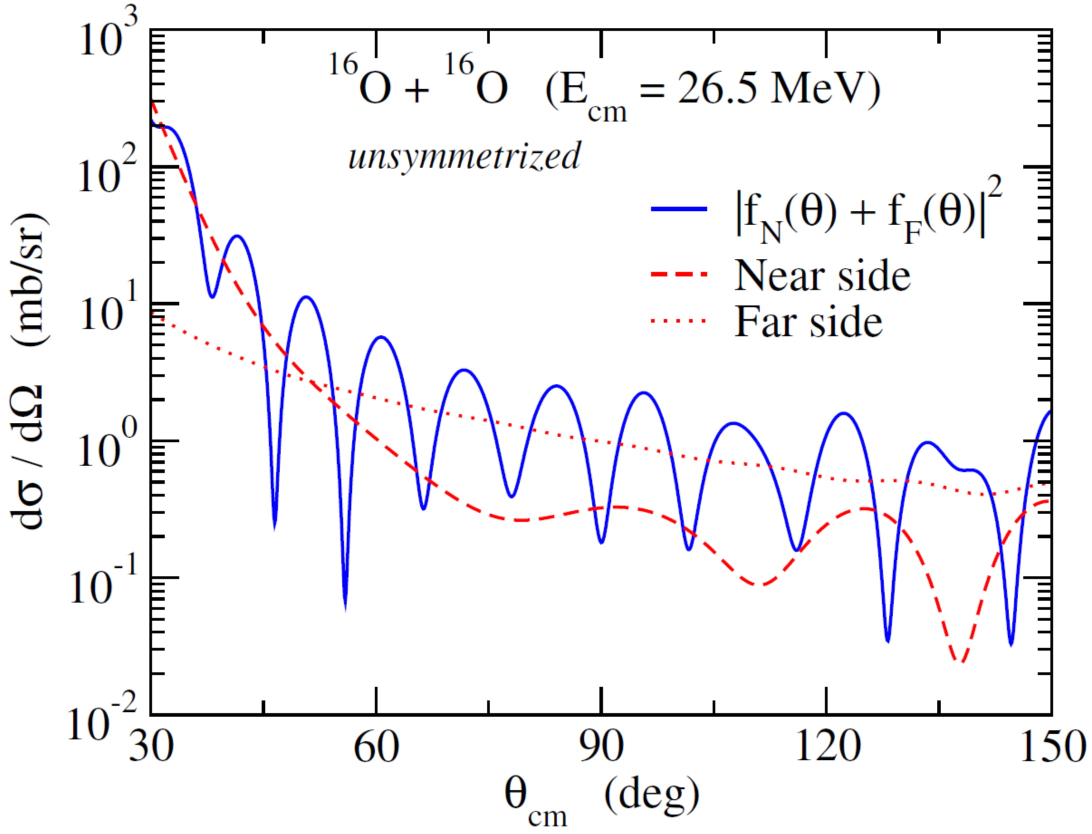
↓

✓ symmetrization: minor

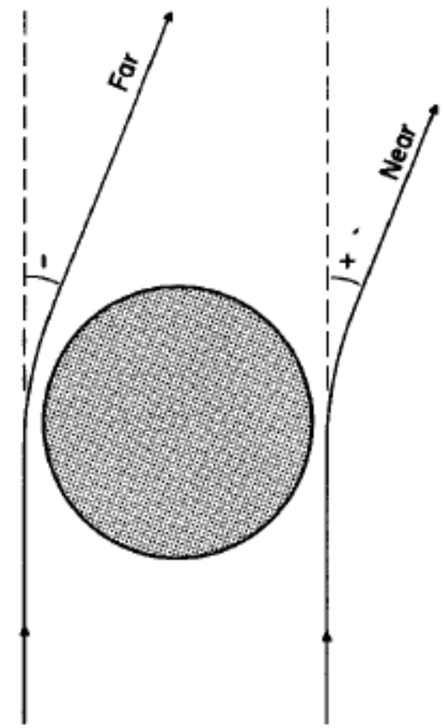
✓ the main origin:
near-side-far-side interference







near side-far side interference



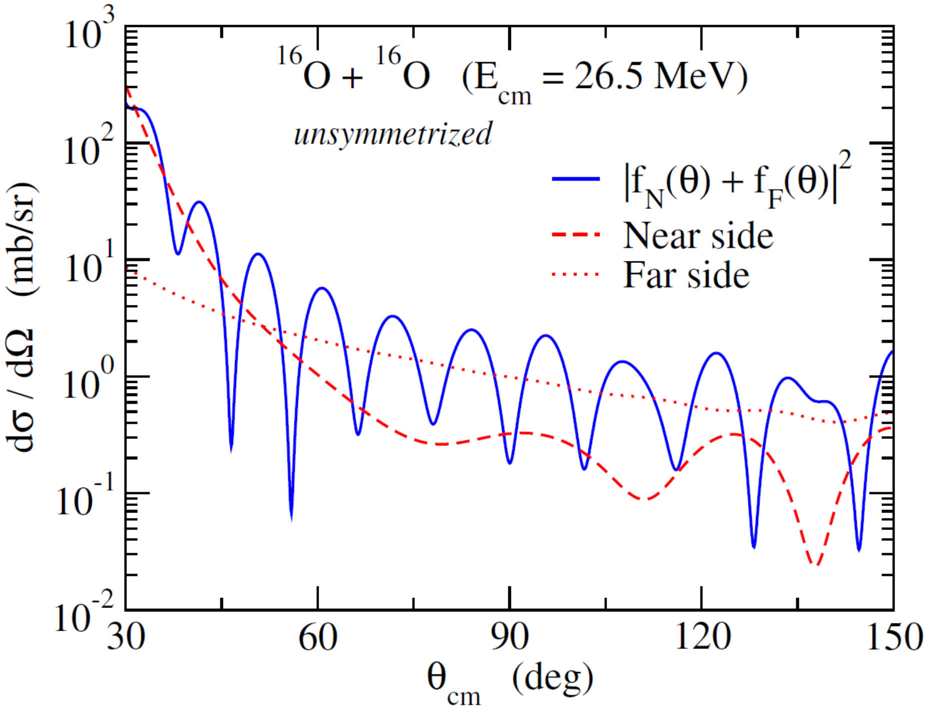
$$P_l(\cos\theta) \to \frac{1}{2}\left[P_l(\cos\theta) \mp i\frac{2}{\pi}Q_l(\cos\theta)\right]$$

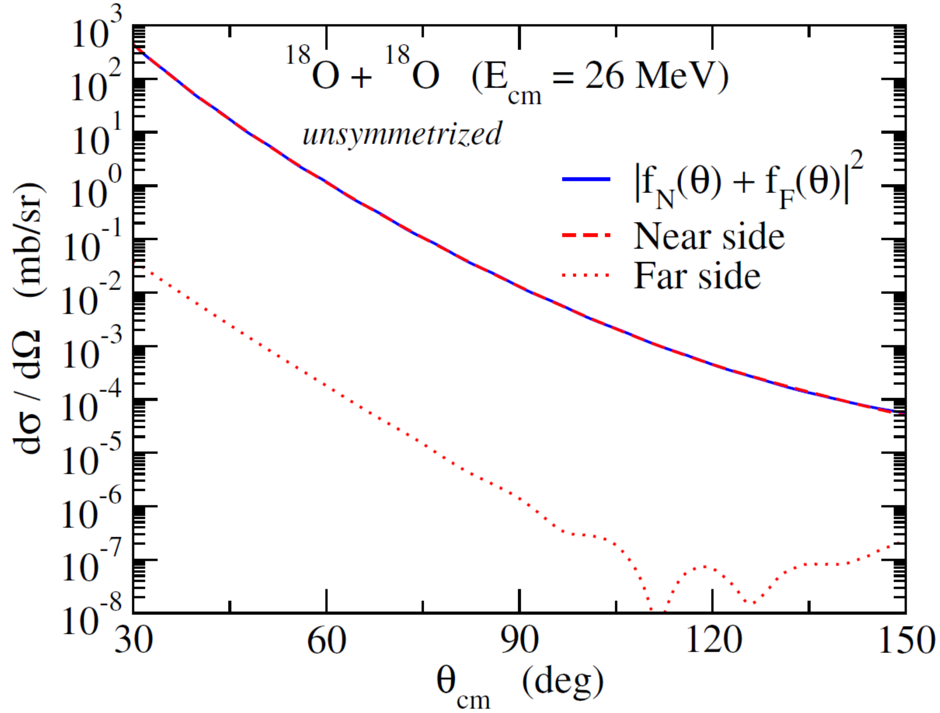
N

F

Q_l: Legendre function of the second kind

R.C. Fuller, PRC12, 1561 (1975)

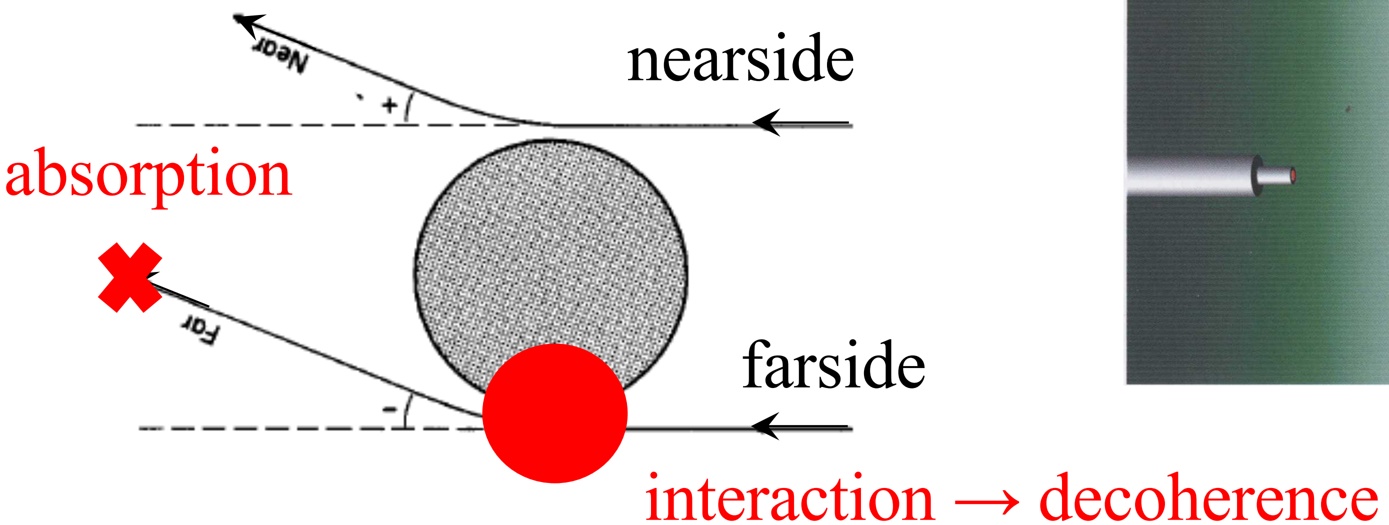


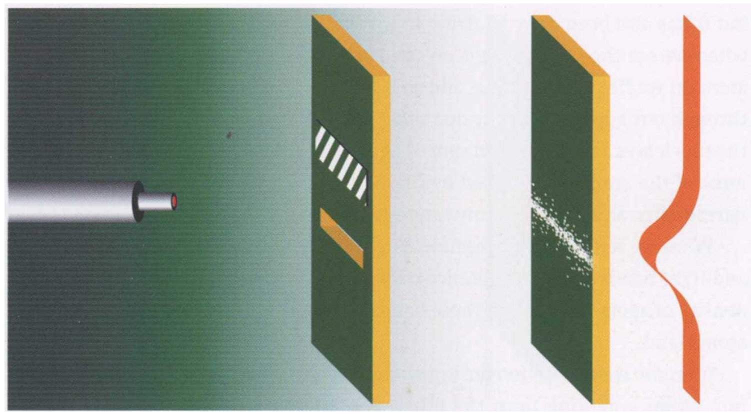


the far-side component is largely damped in $^{18}O + ^{18}O$ due to absorption

→almost no interference oscillations

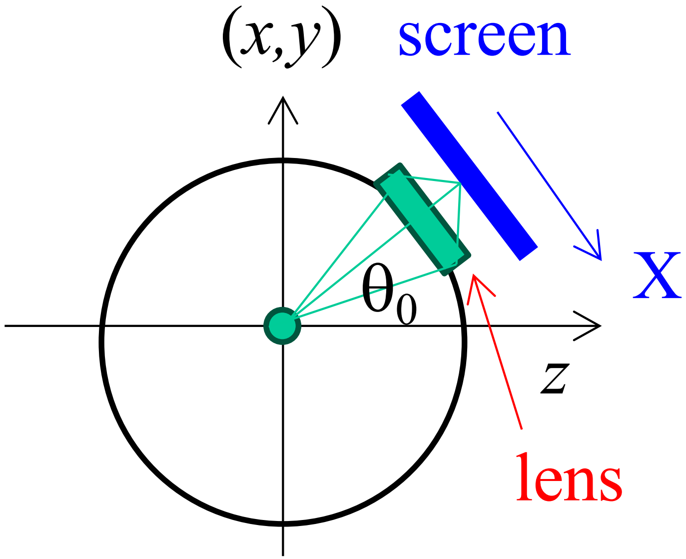
cf. a single slit



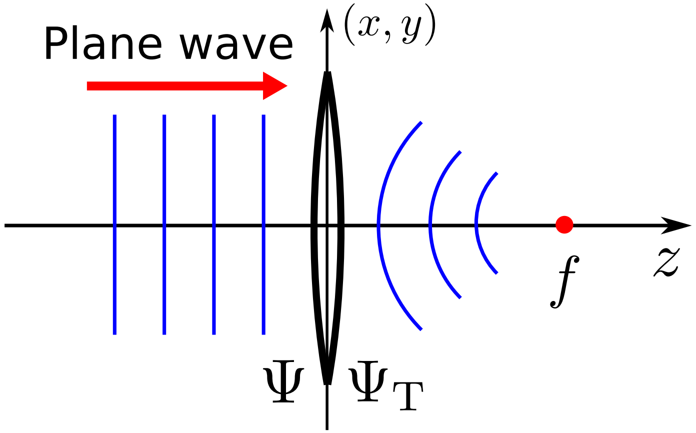


Imaging of nuclear reactions

K. Hagino and T. Yoda,
PLB848, 138326 (2024).



"condensing" scattering waves with a lens



K. Hashimoto et al., PRD101, 066018 (2020)

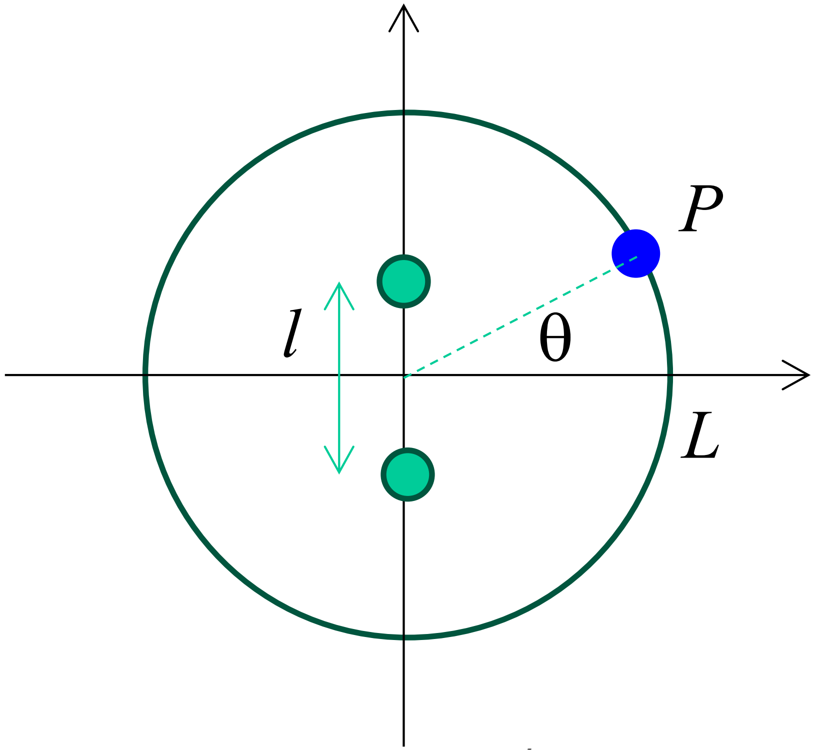
Fourier transform of scattering amplitude

$$\Phi(X,Y) \propto \int_{\theta_0-\Delta\theta}^{\theta_0+\Delta\theta} d\theta \int_{\varphi_0-\Delta\varphi}^{\varphi_0+\Delta\varphi} d\varphi\, e^{ik((\theta-\theta_0)X+(\varphi-\varphi_0)Y)} f(\theta,\varphi)$$

$$I(X,Y) = |\Phi(X,Y)|^2$$

Application to a double slit problem

K. Hashimoto, Y. Matsuo, and T. Yoda, PTEP2023, 043B04 (2023)

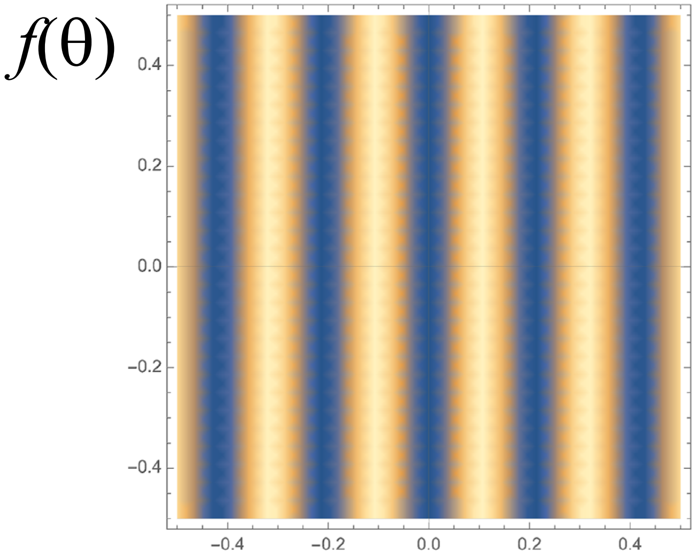


the amplitude at P

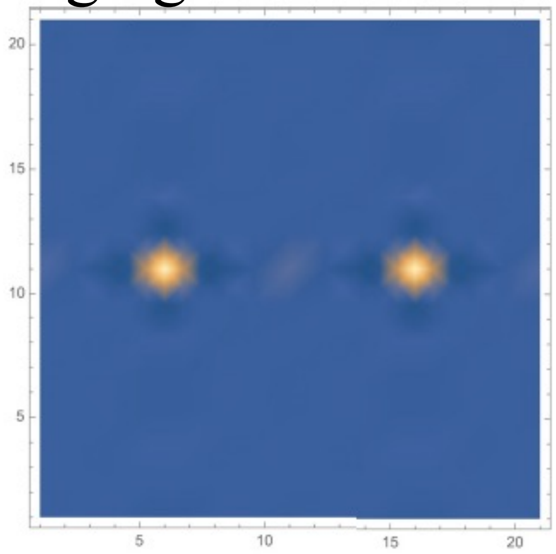
$$f(\theta) = f_1 + f_2$$

$$f_i = A \sin\left(\frac{2\pi}{\lambda} l_i - \omega t\right)$$

$$l_i \sim L\left(1 \pm \frac{l}{2L}\sin\theta\right)$$



imaging

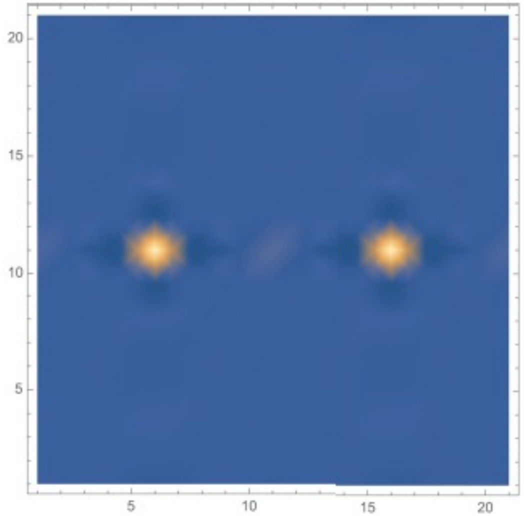


peaks at

$$\pm\frac{l}{2}\sin\theta_0$$

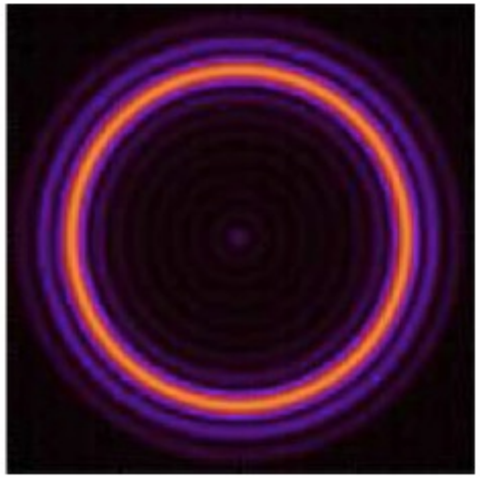
applications in particle physics

Takuya Yoda（世田拓也）



a double slit problem

scattering of string

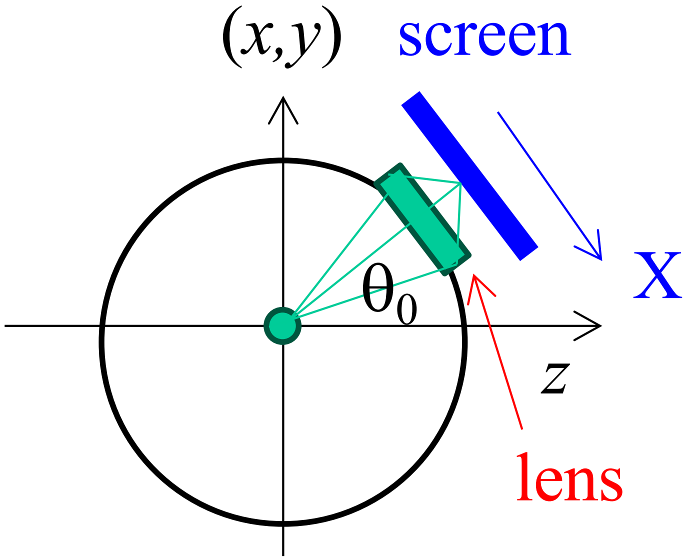


imaging black holes through AdS/CFT

K. Hashimoto, Y. Matsuo, and T. Yoda, PTEP2023, 043B04 (2023)
K. Hashimoto, S. Kinoshita, and K. Murata, PRL123, 031602 (2019)
PRD101, 066018 (2020)

Imaging of nuclear reactions

K. Hagino and T. Yoda,
PLB848, 138326 (2024).

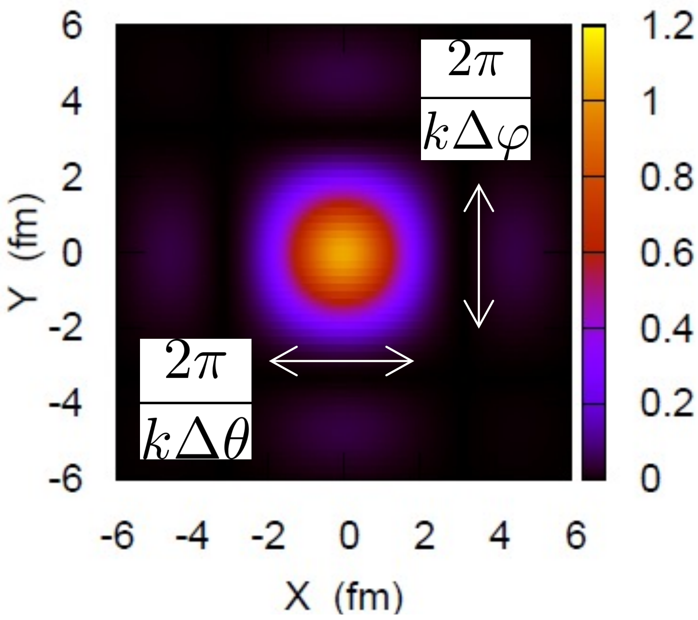


Fourier transform of scattering amplitude

$$\Phi(X,Y) \propto \int_{\theta_0-\Delta\theta}^{\theta_0+\Delta\theta} d\theta \int_{\varphi_0-\Delta\varphi}^{\varphi_0+\Delta\varphi} d\varphi \times e^{ik((\theta-\theta_0)X+(\varphi-\varphi_0)Y)} f(\theta,\varphi)$$

$$I(X,Y) = |\Phi(X,Y)|^2$$

for a flat distribution, $f(\theta,\phi)$ = const.,



$$\int_{\varphi_0-\Delta\varphi}^{\varphi_0+\Delta\varphi} d\varphi e^{ik(\varphi-\varphi_0)Y} = 2\Delta\varphi \frac{\sin(kY\Delta\varphi)}{kY\Delta\varphi}$$

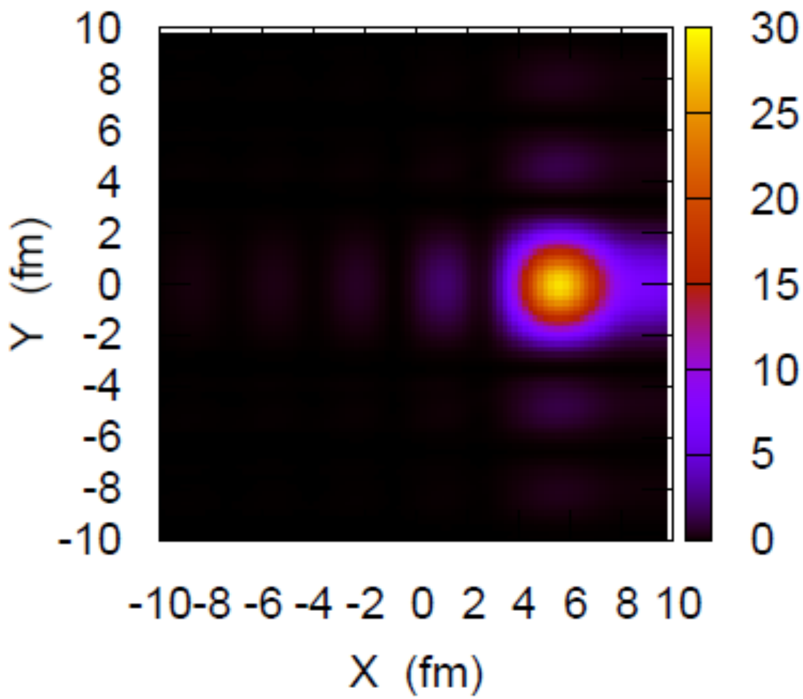
Imaging of nuclear reactions

Fourier transform of scattering amplitude

$$\Phi(X,Y) \;\propto\; \int_{\theta_0-\Delta\theta}^{\theta_0+\Delta\theta} d\theta \int_{\varphi_0-\Delta\varphi}^{\varphi_0+\Delta\varphi} d\varphi\, e^{ik((\theta-\theta_0)X+(\varphi-\varphi_0)Y)} f(\theta,\varphi)$$

$$I(X,Y) = |\Phi(X,Y)|^2$$

for the Rutherford scattering, $f(\theta,\phi) = f_C(\theta,\phi)$,



$^{16}O+^{16}O$ at E_{cm} = 8.8 MeV

θ_0= 90 deg.

$\Delta\theta = \Delta\phi$ = 30 deg.

↓

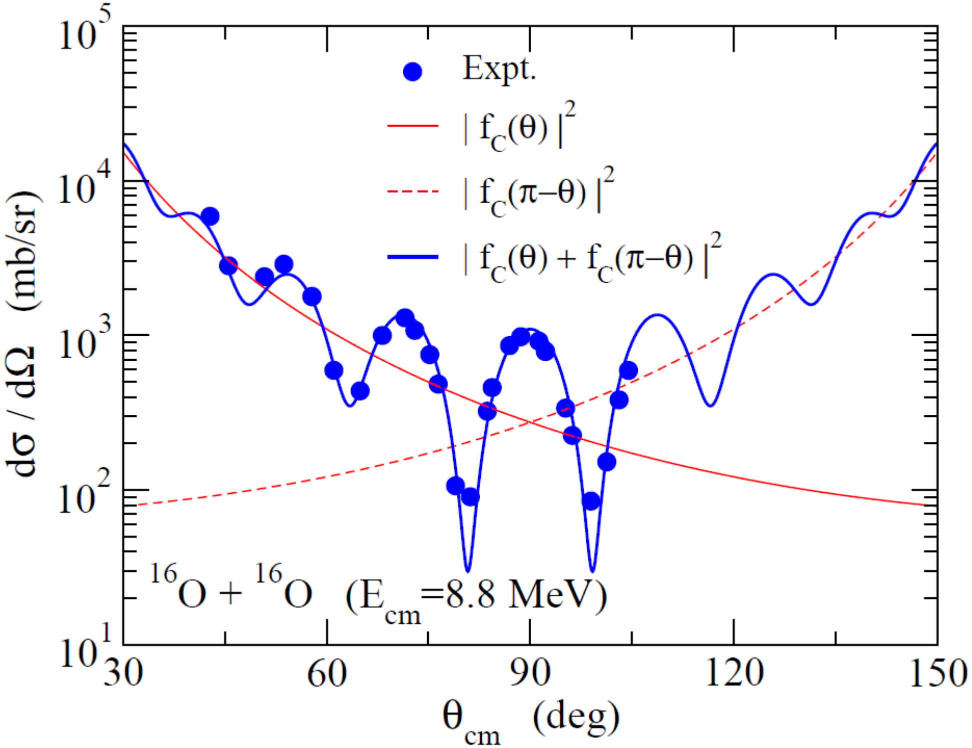
b_{cl} = 5.24 fm ~ X_{peak}

Imaging of nuclear reactions

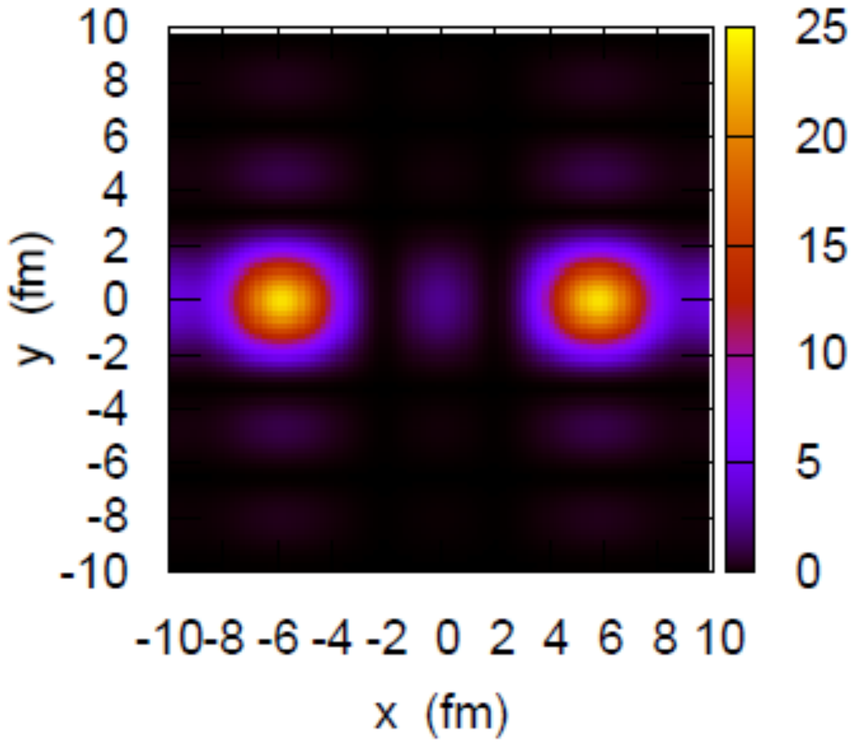
$$\Phi(X,Y) \propto \int_{\theta_0-\Delta\theta}^{\theta_0+\Delta\theta} d\theta \int_{\varphi_0-\Delta\varphi}^{\varphi_0+\Delta\varphi} d\varphi\, e^{ik((\theta-\theta_0)X+(\varphi-\varphi_0)Y)} f(\theta,\varphi)$$

Imaging of Mott scattering

$$I(X,Y) = |\Phi(X,Y)|^2$$



θ_0=90 deg., $\Delta\theta$=$\Delta\phi$=30 deg.

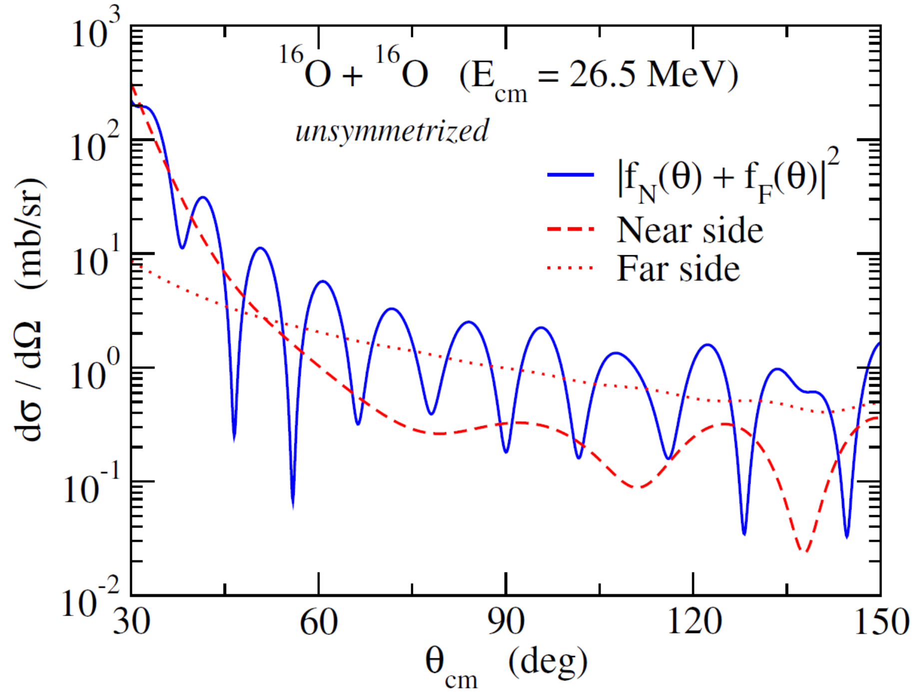


(note) for θ_0=90 deg.,

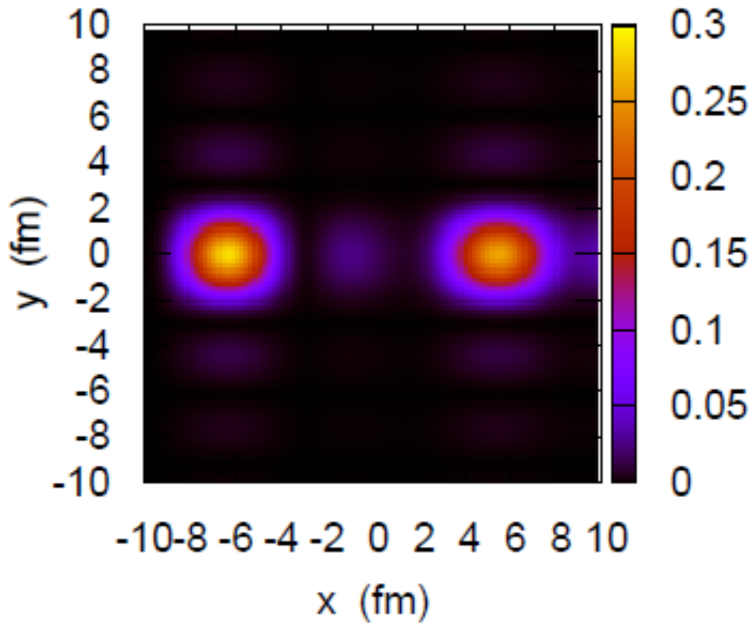
$$\Phi_\theta(X,Y) = \Phi_{\pi-\theta}(-X,Y)$$

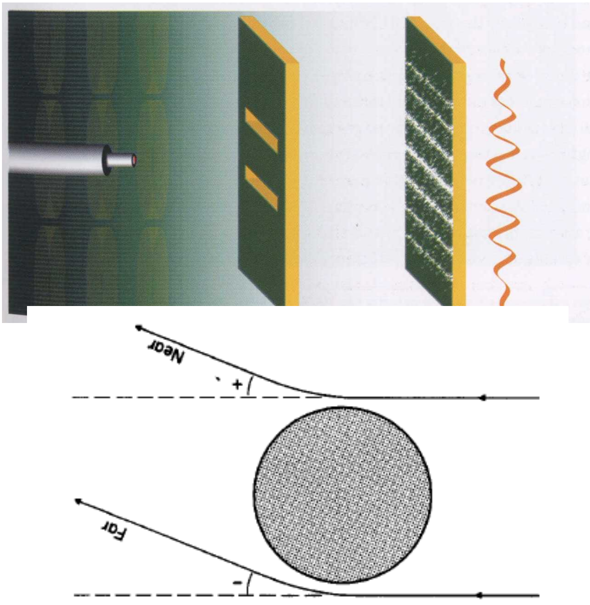
Imaging of nuclear reactions

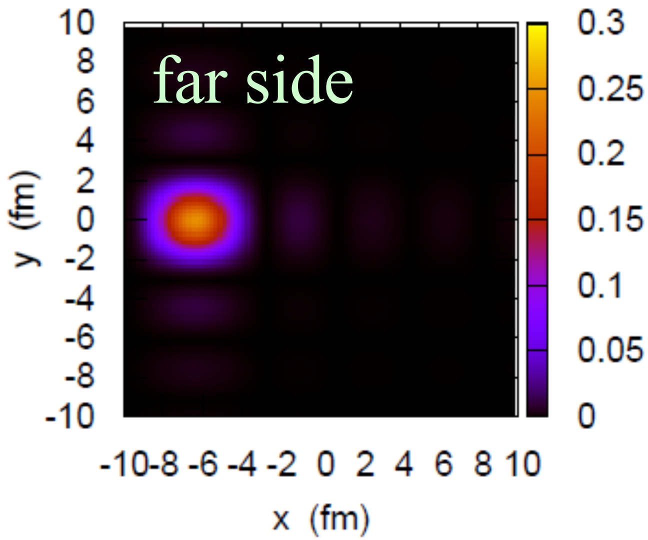
K. Hagino and T. Yoda,
PLB848, 138326 (2024).

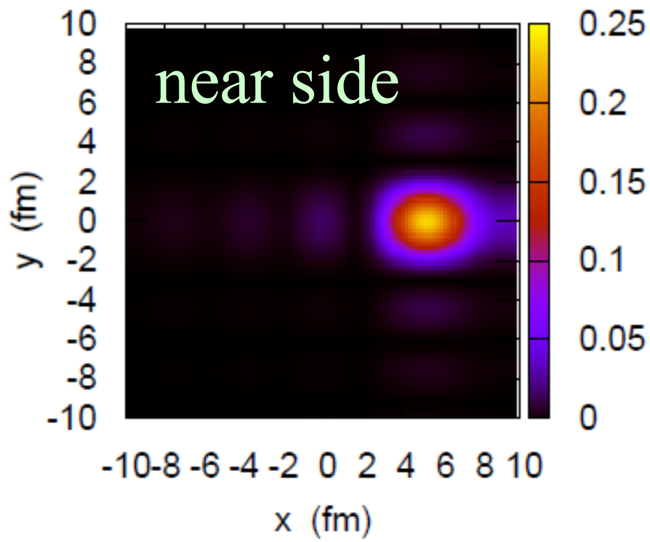


θ_0=55 deg., $\Delta\theta$=15 deg.



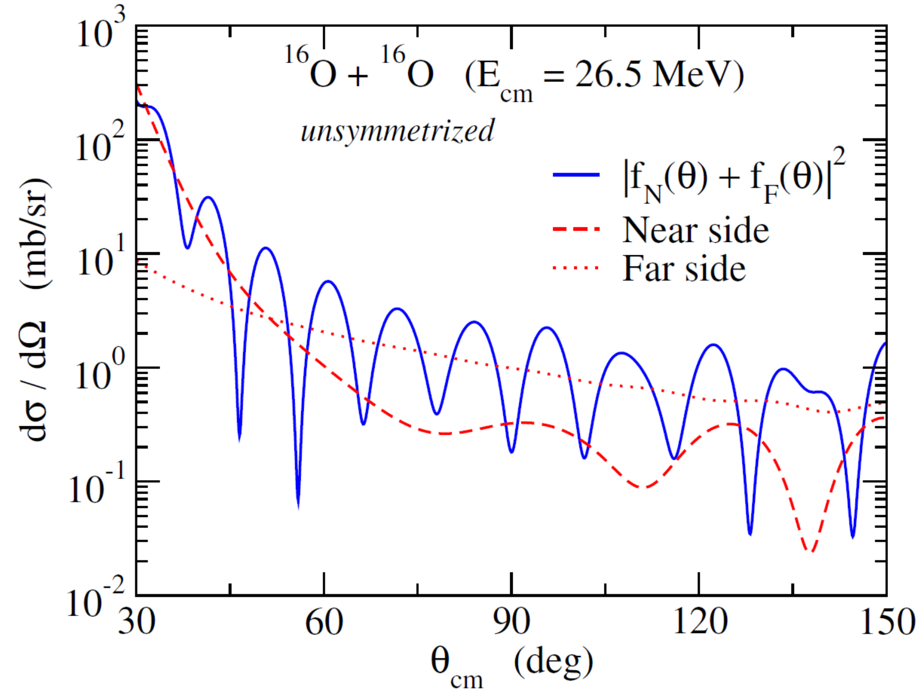


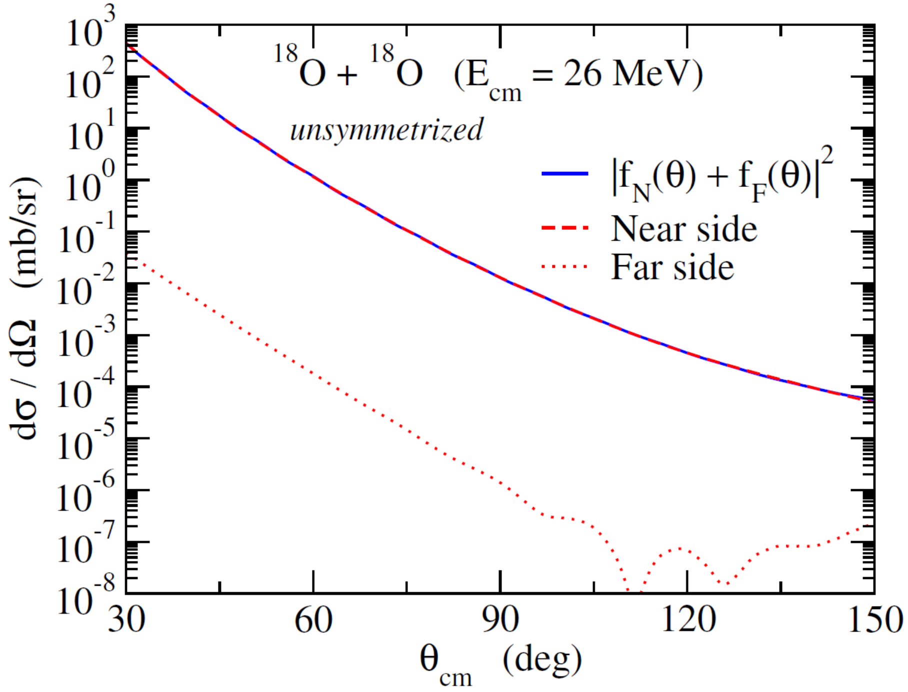


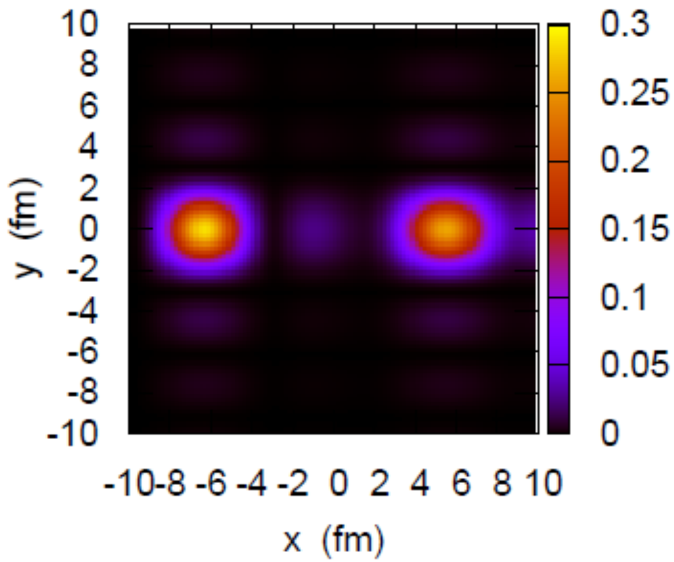


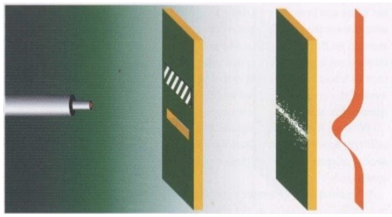
Imaging of nuclear reactions

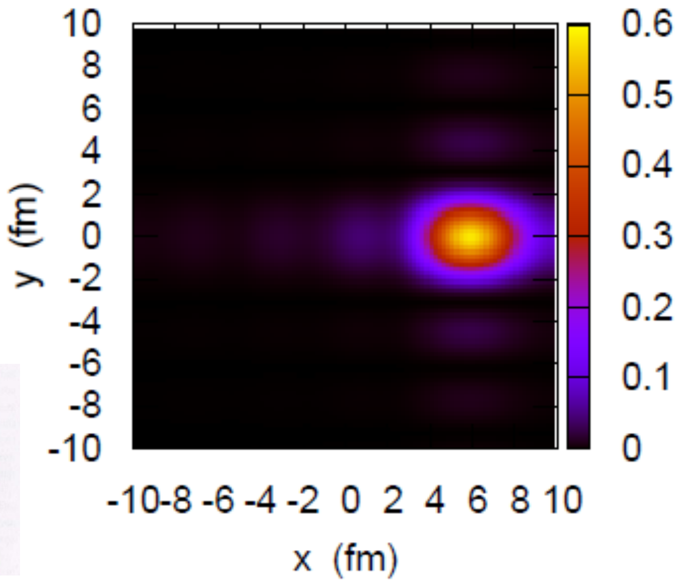
K. Hagino and T. Yoda, PLB848, 138326 (2024).







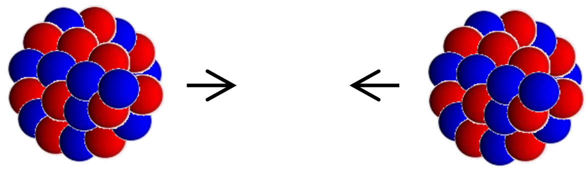




Summary

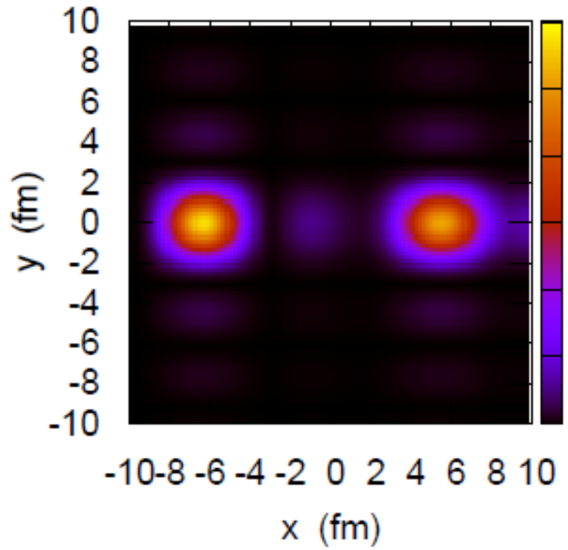
Nuclear Reactions as quantum many-body phenomena

- ✓strong interplay with nuclear structure
- ✓several nuclear intrinsic motions
- ✓Coupled-channels approach
- ✓a variety of interference phenomena
  - scattering of identical nuclei
  - Coulomb-nuclear interference
  - farside-nearside interference
  - barrier-wave-internal-wave interference



✓Imaging: a new approach

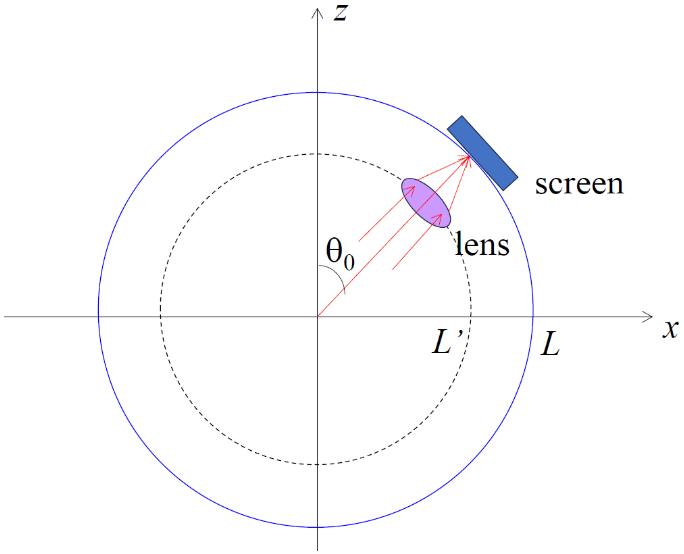
- a Fourier transform of scatt. amplitudes
- an intuitive way to understand physics of interferences



Ongoing work: inelastic scattering?
with Kyoungsu Heo (Soongsil University)


謝謝 ！

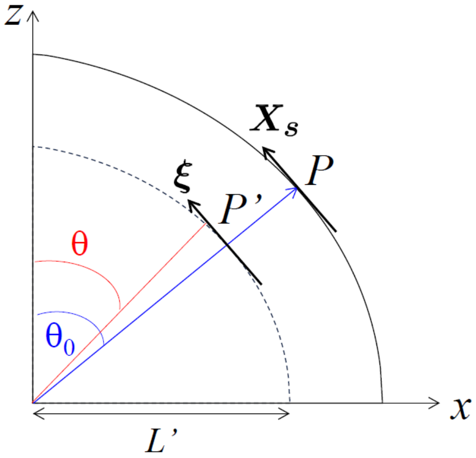
$$\Phi(X,Y) \propto \int_{\theta_0-\Delta\theta}^{\theta_0+\Delta\theta} d\theta \int_{\varphi_0-\Delta\varphi}^{\varphi_0+\Delta\varphi} d\varphi\, e^{ik((\theta-\theta_0)X+(\varphi-\varphi_0)Y)} f(\theta,\varphi)$$



$$\Psi_s(X_s,Y_s) = \int_{-d_\xi}^{d_\xi} d\xi \int_{-d_\eta}^{d_\eta} d\eta\, A(\xi,\eta)\, e^{-ikr}$$

$$\begin{aligned} r &= \sqrt{(X_s-\xi)^2+(Y_s-\eta)^2+(L-L')^2} \\ &\sim L-L'+\frac{(X_s-\xi)^2+(Y_s-\eta)^2}{2(L-L')} \\ &\sim L-L'+\frac{X_s^2+Y_2^s}{2(L-L')}+\frac{\xi X_s+\eta Y_s}{L-L'} \end{aligned}$$

(the size of the lens: much smaller than *L-L'*)



$$\xi \sim -L'(\theta-\theta_0)$$

$$\begin{aligned} X &\equiv -L'X_s/(L-L') \\ Y &\equiv L'\sin\theta_0 Y_s/(L-L') \end{aligned}$$