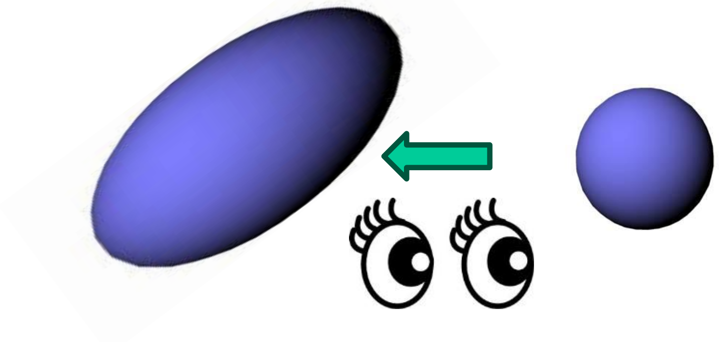
What does “taking a snapshot” actually mean?
: similarities between subbarrier fusion and relativistic HIC



Kouichi Hagino
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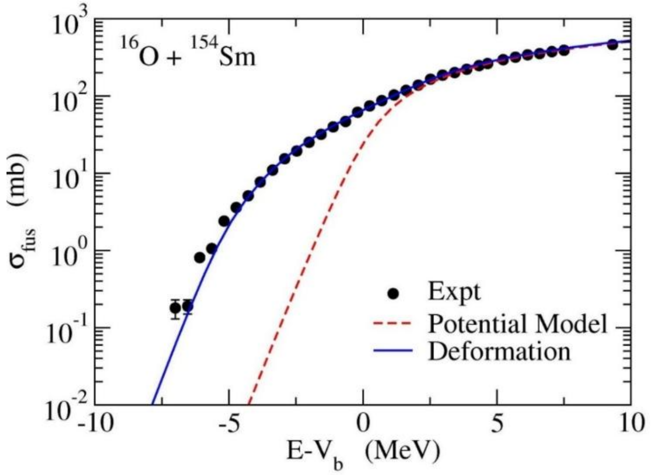
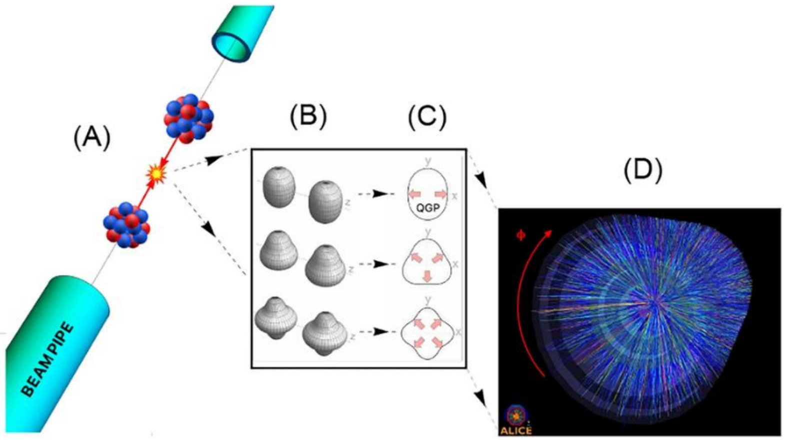




similarities between heavy-ion subbarrier fusion and relativistic HIC
in the context of “taking a snapshot”

YITP Molecular Workshop on ”Intersection of nuclear structure and high energy nuclear collisions 2026”, YITP, Kyoto University, 2026.4.13-24

Taking a Snapshot

taking snapshots of a “slow” motion with a high-speed camera

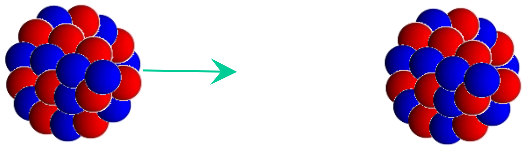
$$\tau_{\text{camera}} \ll \tau_{\text{motion}}$$

https://www.sony.jp/ichigan/products/ILCE-7M3/feature_3.html

(photos with a Sony camera α7III)

a *slow* mode
a *fast* mode

⇒ taking snapshots of a nucleus with a “fast” nuclear reaction

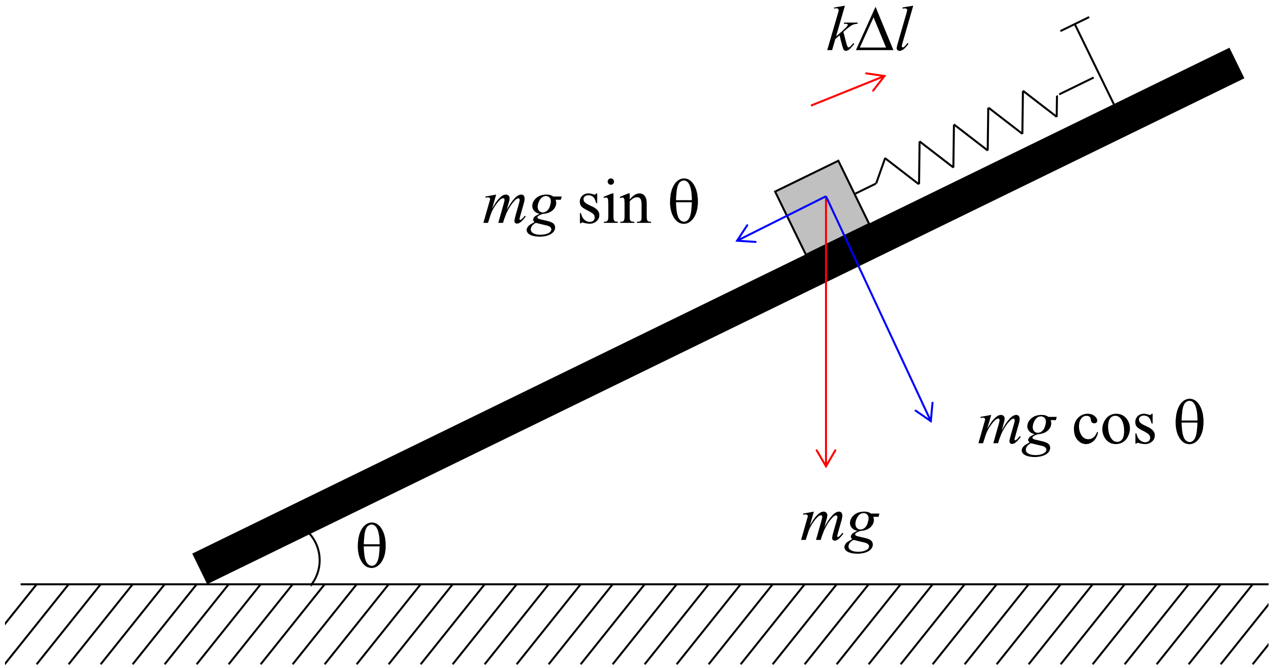


$$\tau_{\text{reaction}} \ll \tau_{\text{nucleus}}$$

Taking a Snapshot

What does “taking a snapshot” actually mean?

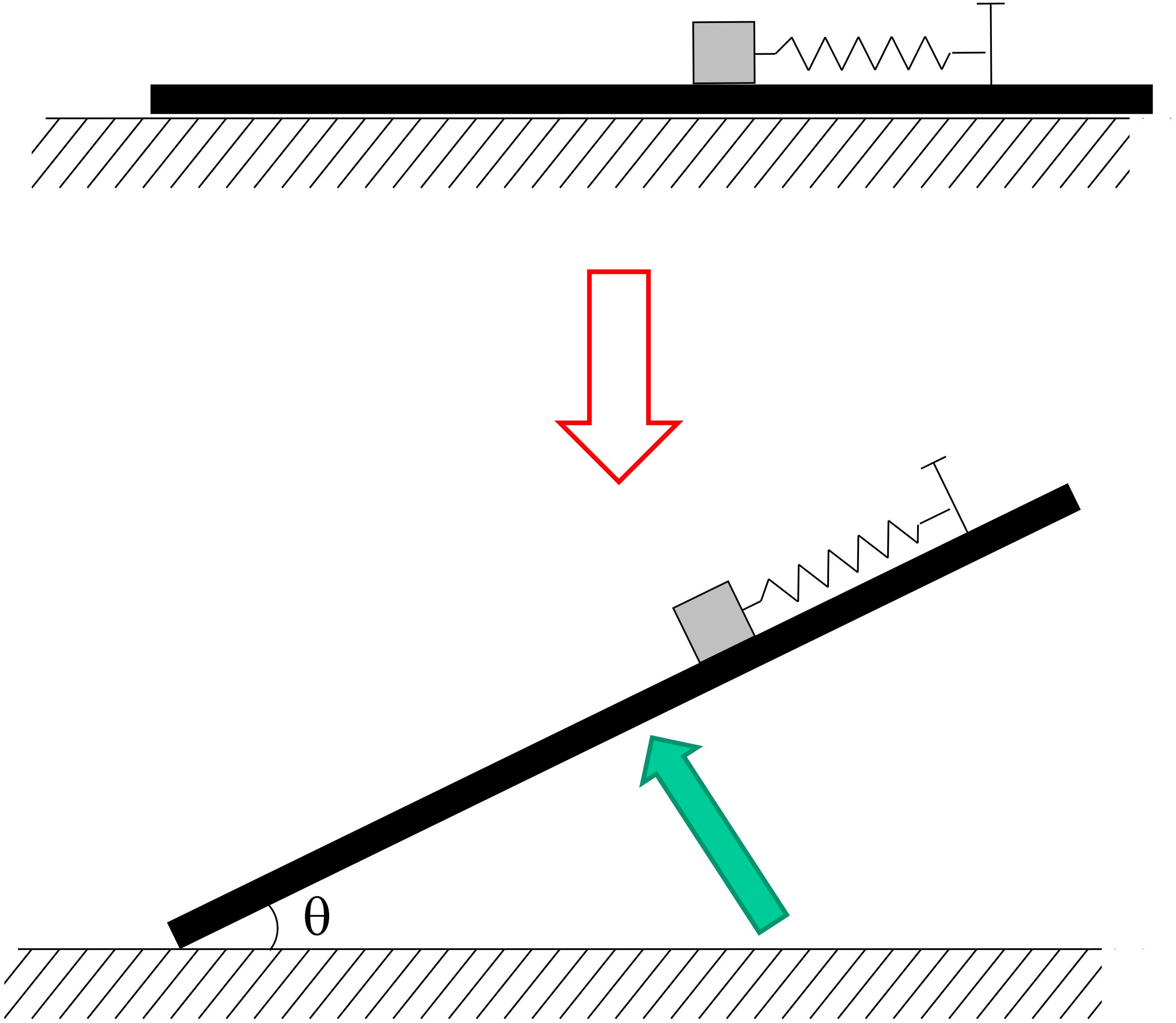
an illustrative example: a spring on a slope



the equilibrium position:

$mg \sin \theta = k\Delta l \;\; \rightarrow \Delta l = mg \sin \theta \,/\, k$

Taking a Snapshot



i) if the angle increases very slowly

→ the equilibrium point at every instance

$$\Delta l = mg \sin \theta / k$$

“adiabatic limit”

ii) if the angle increases very rapidly

→ the initial length ($\Delta l = 0$) is kept

“sudden limit”

Taking a Snapshot

Separation of degrees of freedom: Quantum Mechanics

a simple 1D model with 2 d.o.f.:

$$H = \underset{\text{rel. motion}}{\underline{H_f}} + \underset{\text{other d.o.f}}{\underline{H_s}} + \underset{\text{coupling}}{\underline{V(x_f, x_s)}}$$

$$H_s \phi_k(x_s) = \epsilon_k \phi_k(x_s)$$

S-matrix for the transition from ϕ_i to ϕ_f:

$$S_{fi} = \langle \phi_f | \hat{S} | \phi_i \rangle$$

for an <u>inclusive</u> process:

$$\begin{aligned} P_{\text{tot}} &= \sum_f |S_{fi}|^2 = \sum_f \langle \phi_i | \hat{S}^\dagger | \phi_f \rangle \langle \phi_f | \hat{S} | \phi_i \rangle \\ &= \langle \phi_i | \hat{S}^\dagger \hat{S} | \phi_i \rangle \end{aligned}$$

one can insert any completeness relation

$$1 = \sum_k |\psi_k\rangle\langle\psi_k| = \int dx_x |x_s\rangle\langle x_s|$$

$$\rightarrow \quad \begin{aligned} &\langle \phi_i | \hat{S}^\dagger \hat{S} | \phi_i \rangle \\ &= \sum_k \langle \phi_i | \hat{S}^\dagger | \psi_k \rangle \langle \psi_k | \hat{S} | \phi_i \rangle \\ &= \int dx_s \, \langle \phi_i | \hat{S}^\dagger | x_s \rangle \langle x_s | \hat{S} | \phi_i \rangle \end{aligned}$$

Taking a Snapshot

Separation of degrees of freedom: Quantum Mechanics

the adiabatic approximation:

$$H = \underbrace{H_f}_{\text{rel. motion}} + \underbrace{\cancelto{0}{H_s}}_{\text{other d.o.f}} + \underbrace{V(x_f, x_s)}_{\text{coupling}}$$

rel. motion ↓ fast

other d.o.f ↓ slow

coupling ↓ keep this as this depends on the fast variable

S is diagonal w.r.t. x_s

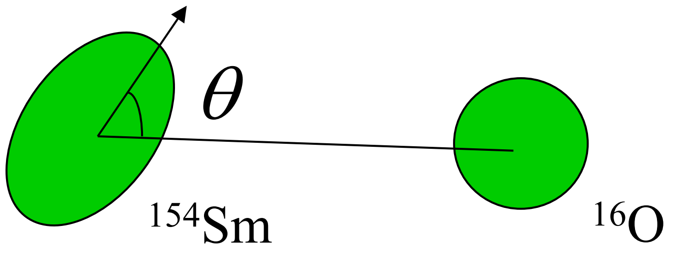
$$\langle x_s|\hat{S}|x'_s\rangle = S(x_s)\delta(x_s - x'_s)$$

(note) $\left\langle x_s \left| e^{-iHt/\hbar} \right| x'_s \right\rangle = e^{-i(H_f+V(x_f,x_s))t/\hbar}\delta(x_s - x'_s)$

$$\langle\phi_i|\hat{S}^\dagger\hat{S}|\phi_i\rangle = \int dx_s\,\langle\phi_i|\hat{S}^\dagger|x_s\rangle\langle x_s|\hat{S}|\phi_i\rangle$$

$$= \int dx_s \underbrace{|\phi_i(x_s)|^2}_{\text{the g.s. wf in the } x \text{ space}} \underbrace{|S(x_s)|^2}_{\text{the probability with a fixed } x_s}$$

$$\boxed{\sigma(E) = \int_0^1 d(\cos\theta)\sigma(E;\theta)}$$

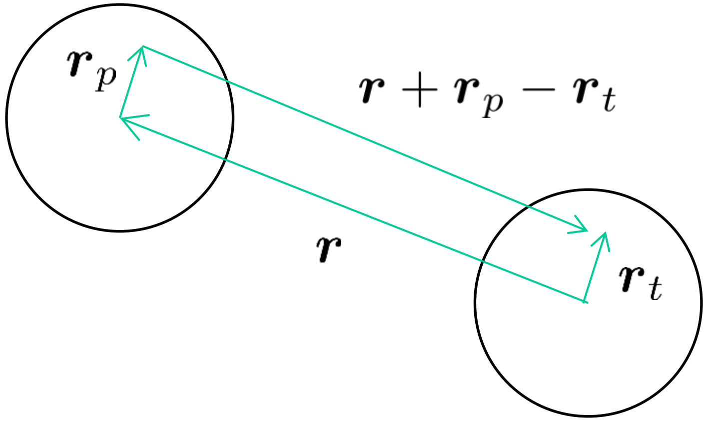


Taking a Snapshot

Separation of degrees of freedom: Quantum Mechanics

nuclear many-body Hamiltonian: $$H = T_{\rm rel} + \cancel{h}_p + \cancel{h}_t + \sum_{i\in p}\sum_{i\in t} v(\boldsymbol{r}+\boldsymbol{r}_i-\boldsymbol{r}_j)$$

adiabatic
approximation



$$\langle \Phi_0^{(p)}\Phi_0^{(t)}|\hat{S}^\dagger\hat{S}|\Phi_0^{(p)}\Phi_0^{(t)}\rangle = \int \prod_{i\in p} d\boldsymbol{r}_i \prod_{j\in t} d\boldsymbol{r}_j \left|\Phi_0^{(p)}(\{\boldsymbol{r}_i\})\right|^2 \left|\Phi_0^{(t)}(\{\boldsymbol{r}_j\})\right|^2 |S(\{\boldsymbol{r}_i\},\{\boldsymbol{r}_j\})|^2$$

reactions with fixed nucleon coordinates ← also in the Glauber model

Taking a Snapshot

Separation of degrees of freedom: Quantum Mechanics

nuclear many-body Hamiltonian: $$H = T_{\rm rel} + \cancel{h_p} + \cancel{h_t} + \sum_{i\in p}\sum_{i\in t} v(\boldsymbol{r}+\boldsymbol{r}_i-\boldsymbol{r}_j)$$

(note) In a similar way, one can construct the amplitude for elastic scattering:

$$\begin{aligned}\langle\Phi_0^{(p)}\Phi_0^{(t)}|e^{-iHt/\hbar}|\Phi_0^{(p)}\Phi_0^{(t)}\rangle \;&=\; \int\prod_{i\in p}d\boldsymbol{r}_i\prod_{j\in t}d\boldsymbol{r}_j\,\left|\Phi_0^{(p)}(\{\boldsymbol{r}_i\})\right|^2\left|\Phi_0^{(t)}(\{\boldsymbol{r}_j\})\right|^2 \\ &\quad\times\exp\left[-\frac{it}{\hbar}\left(T_{\rm rel}+\sum_{i\in p}\sum_{i\in t}v(\boldsymbol{r}+\boldsymbol{r}_i-\boldsymbol{r}_j)\right)\right]\end{aligned}$$

$$\sim 1-\frac{it}{\hbar}\left(T_{\rm rel}+\sum_{i\in p}\sum_{i\in t}v(\boldsymbol{r}+\boldsymbol{r}_i-\boldsymbol{r}_j)\right)$$

the double folding model
with one-body densities

$$\sim\quad 1-\frac{it}{\hbar}\left(T_{\rm rel}+\int d\boldsymbol{r}_p\boldsymbol{r}_t\,\rho_p(\boldsymbol{r}_p)\rho(\boldsymbol{r}_t)v(\boldsymbol{r}+\boldsymbol{r}_p-\boldsymbol{r}_t)\right)$$

$$\sim\quad \exp\left[-\frac{it}{\hbar}\left(T_{\rm rel}+\int d\boldsymbol{r}_p\boldsymbol{r}_t\,\rho_p(\boldsymbol{r}_p)\rho(\boldsymbol{r}_t)v(\boldsymbol{r}+\boldsymbol{r}_p-\boldsymbol{r}_t)\right)\right]$$

Taking a Snapshot

Separation of degrees of freedom: Quantum Mechanics

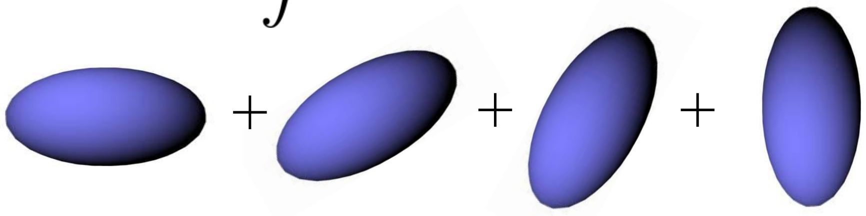
nuclear many-body Hamiltonian: $H = T_{\rm rel} + \cancel{h_p} + \cancel{h_t} + \sum_{i \in p} \sum_{i \in t} v(\boldsymbol{r} + \boldsymbol{r}_i - \boldsymbol{r}_j)$

$$\langle \Phi_0^{(p)} \Phi_0^{(t)} | \hat{S}^\dagger \hat{S} | \Phi_0^{(p)} \Phi_0^{(t)} \rangle = \int \prod_{i \in p} d\boldsymbol{r}_i \prod_{j \in t} d\boldsymbol{r}_j \left| \Phi_0^{(p)}(\{\boldsymbol{r}_i\}) \right|^2 \left| \Phi_0^{(t)}(\{\boldsymbol{r}_j\}) \right|^2 |S(\{\boldsymbol{r}_i\}, \{\boldsymbol{r}_j\})|^2$$

for a deformed nucleus

$$\Phi_0^{(t)}(\{\boldsymbol{r}_j\}) = \int d\Omega \, \hat{\mathcal{R}}(\Omega) \phi_0(\{\boldsymbol{r}'_j\}) = \int d\Omega \, \phi_\Omega(\{\boldsymbol{r}'_j\})$$

$$\prod_{j \in t} d\boldsymbol{r}_j = d\Omega \prod_{j \in t} d\boldsymbol{r}'_j$$



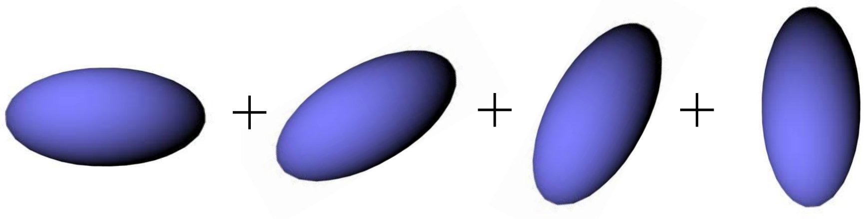
$$\langle \Phi_0^{(p)} \Phi_0^{(t)} | \hat{S}^\dagger \hat{S} | \Phi_0^{(p)} \Phi_0^{(t)} \rangle = \int d\Omega \int \prod_{i \in p} d\boldsymbol{r}_i \prod_{j \in t} d\boldsymbol{r}'_j \left| \Phi_0^{(p)}(\{\boldsymbol{r}_i\}) \right|^2 \left| \Phi_\Omega^{(t)}(\{\boldsymbol{r}'_j\}) \right|^2 \times \left| S(\{\boldsymbol{r}_i\}, \{\boldsymbol{r}'_j\}, \Omega) \right|^2$$

reactions with a fixed orientation angle

Taking a Snapshot

the ground state
of a deformed nucleus

$\Psi_{0+} =$



fast reactions

reactions with
a fixed angle

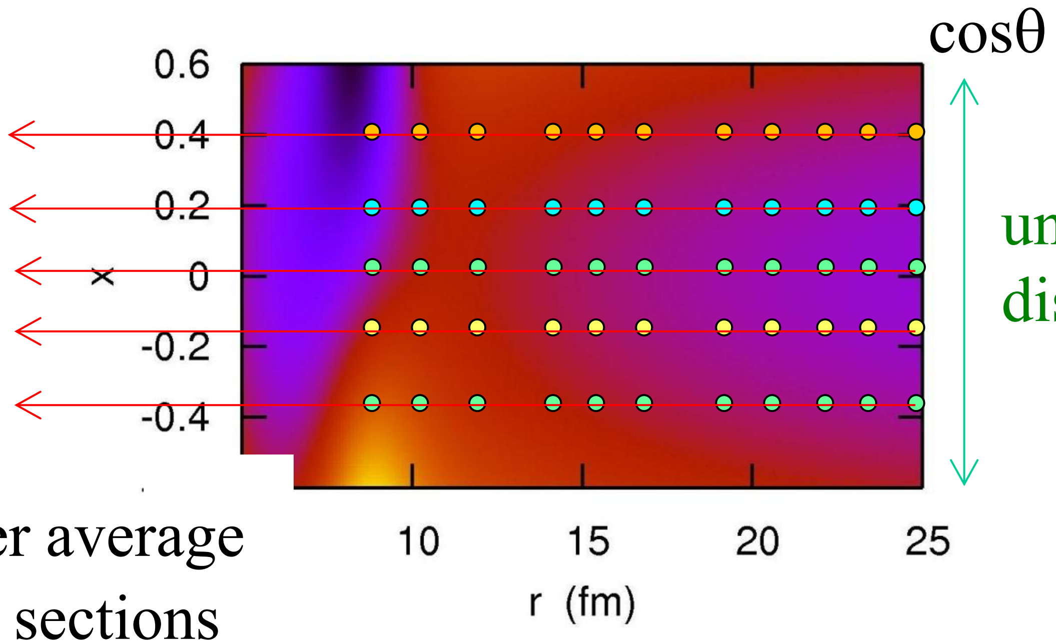
⇓

observables after average

- fusion cross sections
- flow pattern

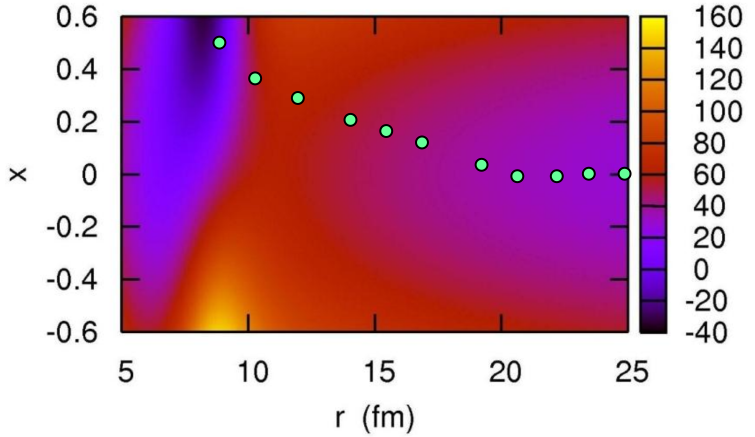
⇓

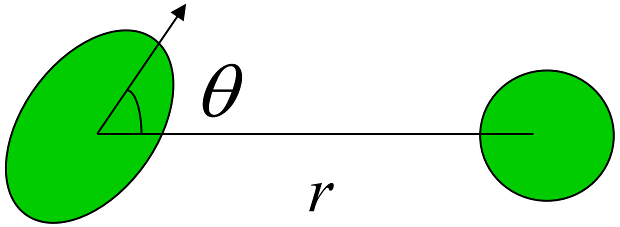
information on shapes
"snap shots"



uniformly
distributed

cf. slow reactions
→ the adiabatic path

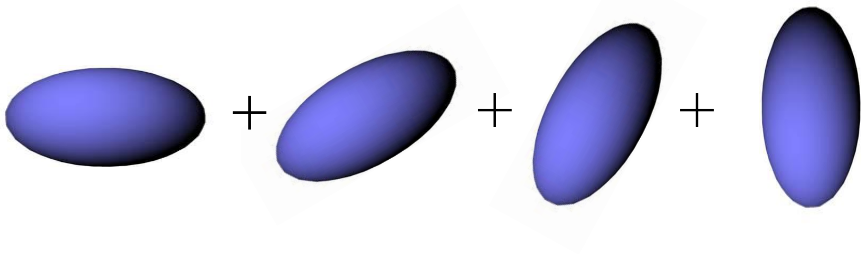




Taking a Snapshot

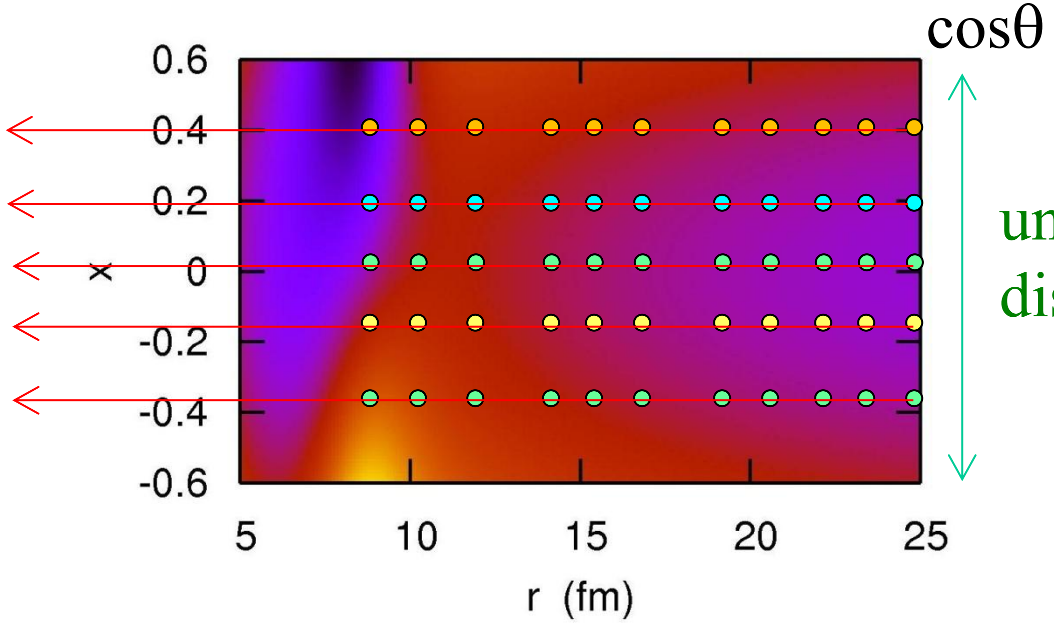
the ground state
of a deformed nucleus

$\Psi_{0+} =$

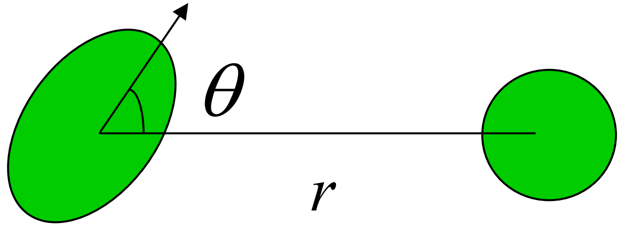


fast reactions

reactions with
a fixed angle



uniformly
distributed



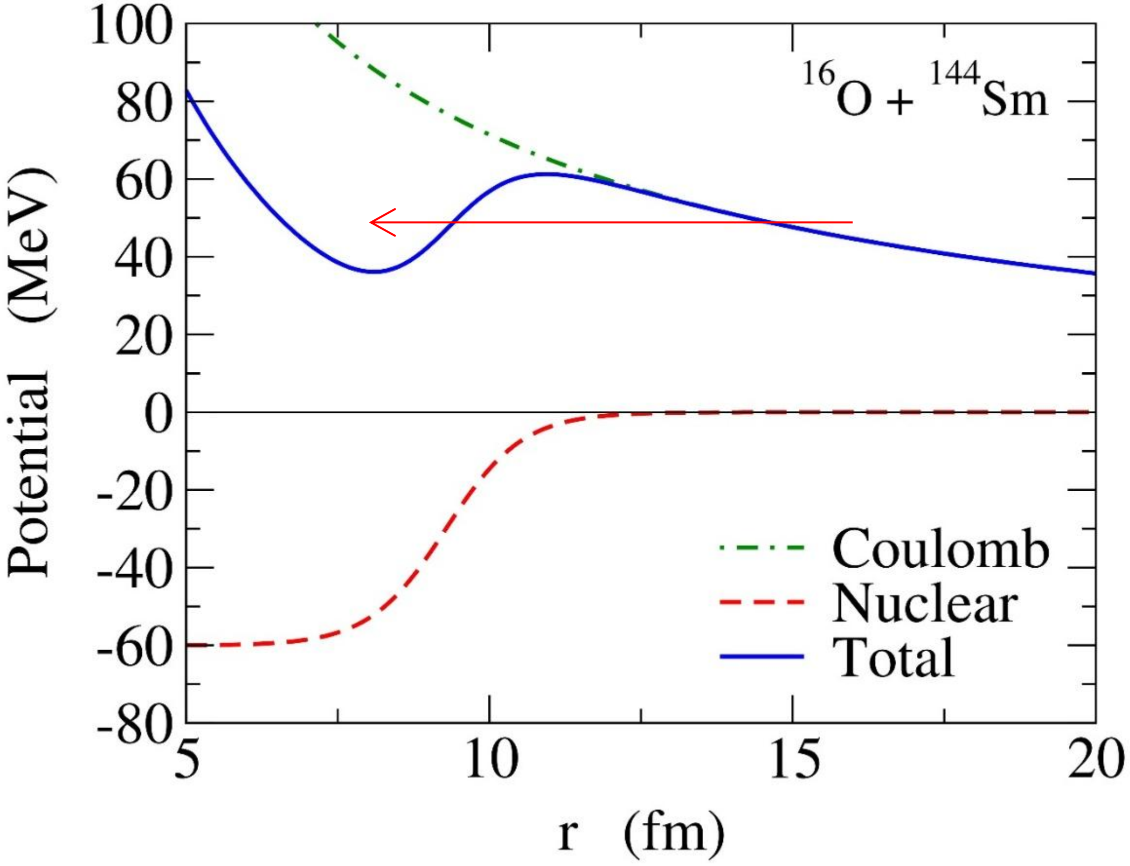
"Taking a snapshot"

- does not directly probe the nuclear shapes
- only through reaction observables
- thus does not literally mean "taking a snapshot"

reaction dynamics: important
(even though often ignored
in discussions)

Sub-barrier fusion reactions and quantum tunneling

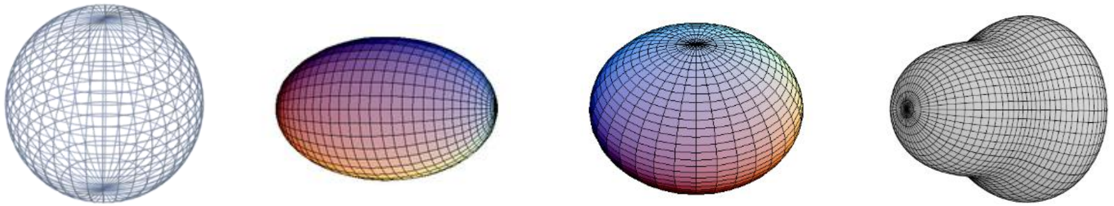
Coulomb barrier



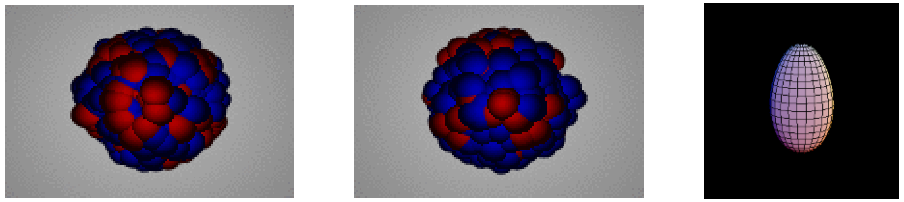
Fusion: takes place by overcoming the barrier
→ quantum tunneling when $E < V_b$

Fusion with quantum tunneling
with many degrees of freedom

- several nuclear shapes

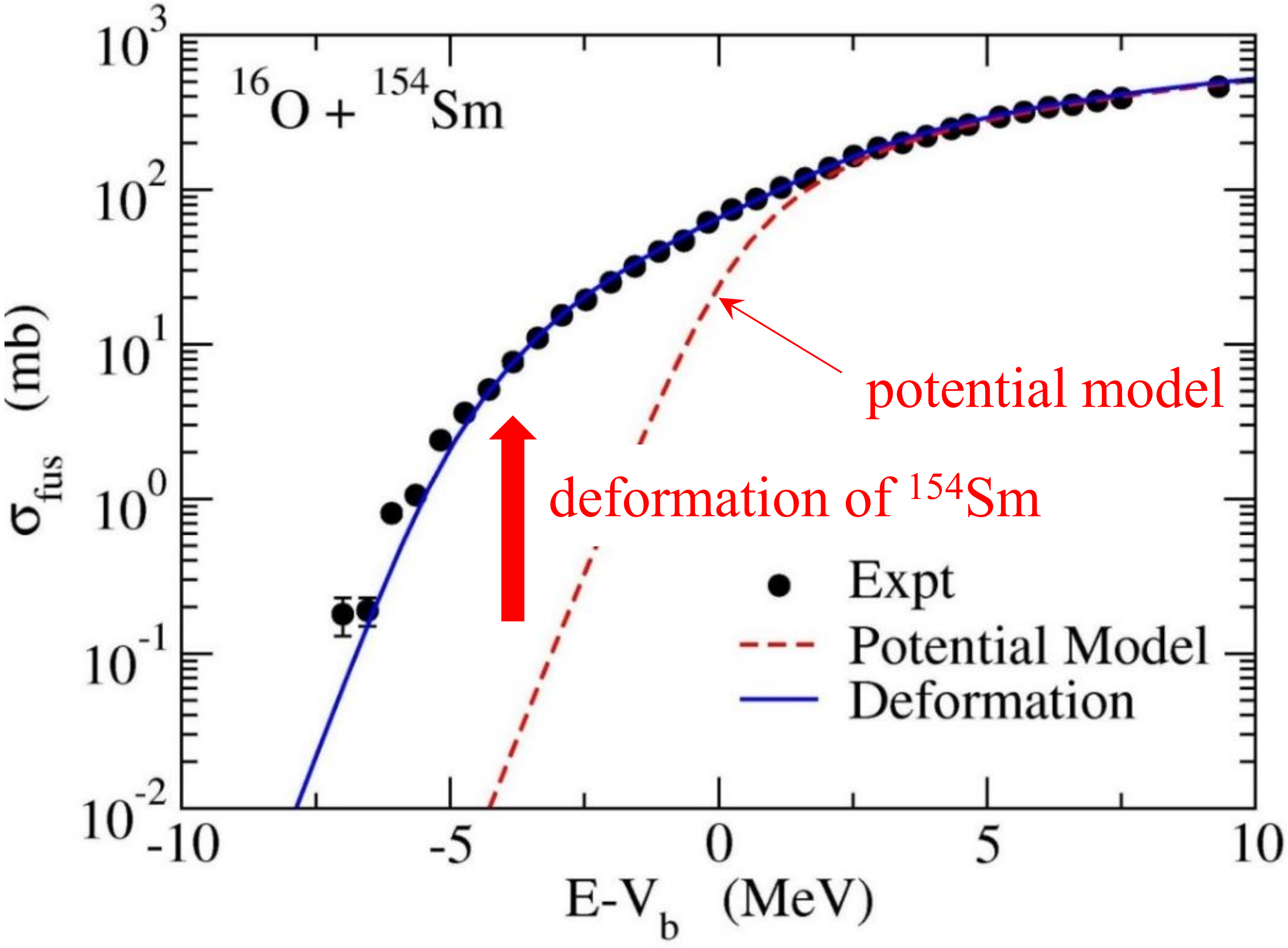


- several surface vibrations



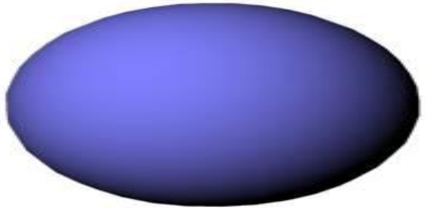
the strong E dep. of tunneling probabilities
→ nuclear structure effects are amplified

Sub-barrier fusion reactions and quantum tunneling

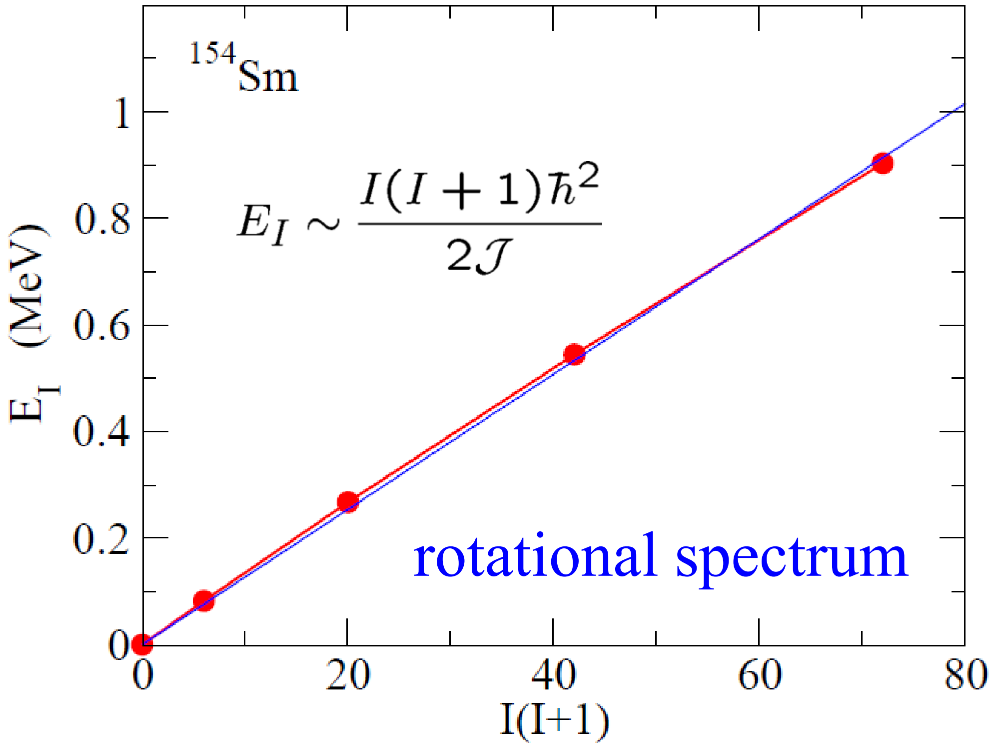


potential model

deformation of ^{154}Sm



^{154}Sm

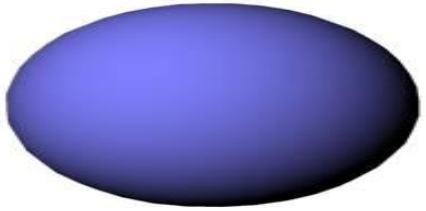


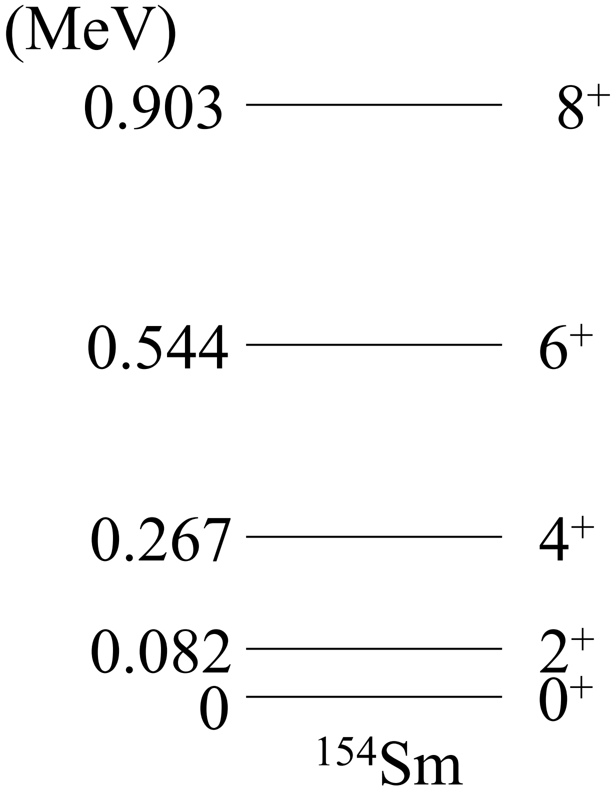
$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

rotational spectrum

K. Hagino and N. Takigawa, Prog. Theo. Phys.128 (2012)1061.

Effects of nuclear deformation on fusion





rotational spectrum

a small rotational energy

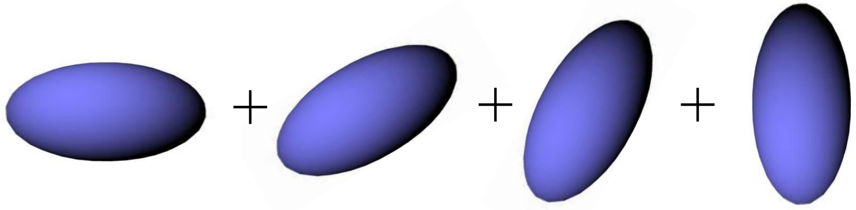
$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

→ a large moment of inertia *J*

→ rotation: a slow deg. of freedom

$$\begin{aligned} E_{\rm rot} &\sim E_{2^+} = 82 \text{ keV} \\ E_{\rm tunnel} &\sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV} \end{aligned}$$

$\Psi_{0^+} =$



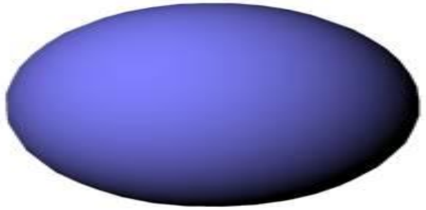
→ a spherical state in the lab. system

fix the orientation angle to calculate the fusion probability

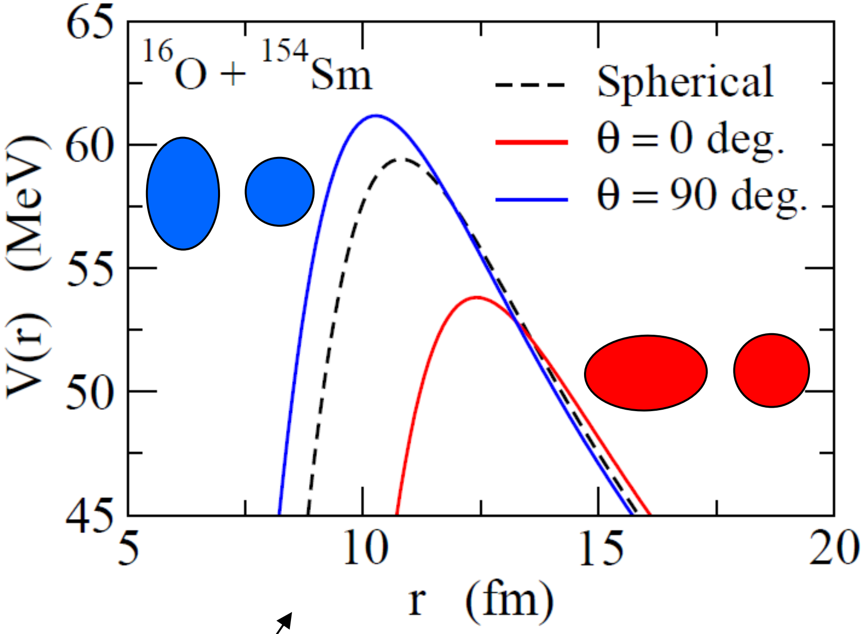
"a snapshot of a rotating nucleus"

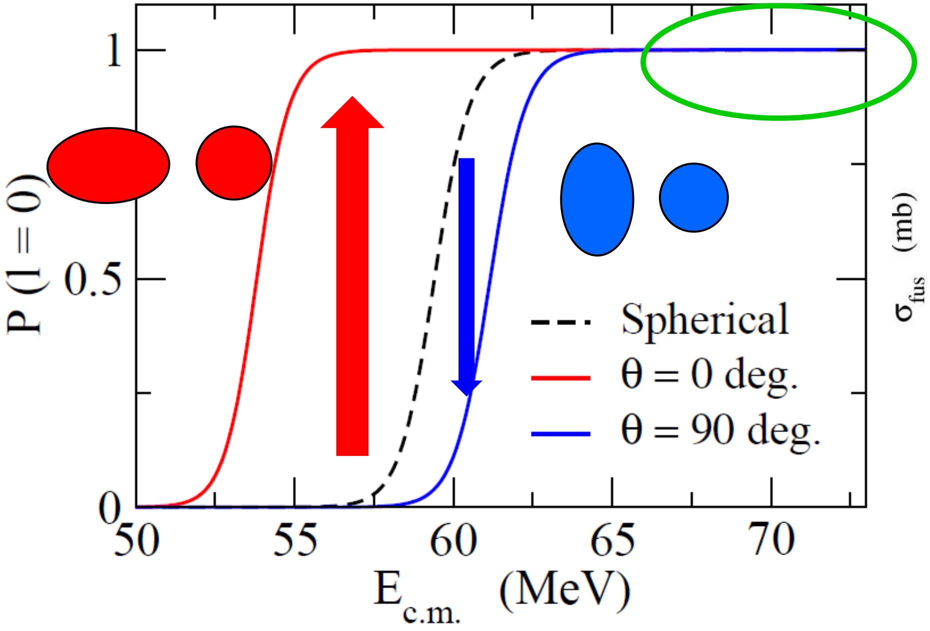
Effects of nuclear deformation on fusion

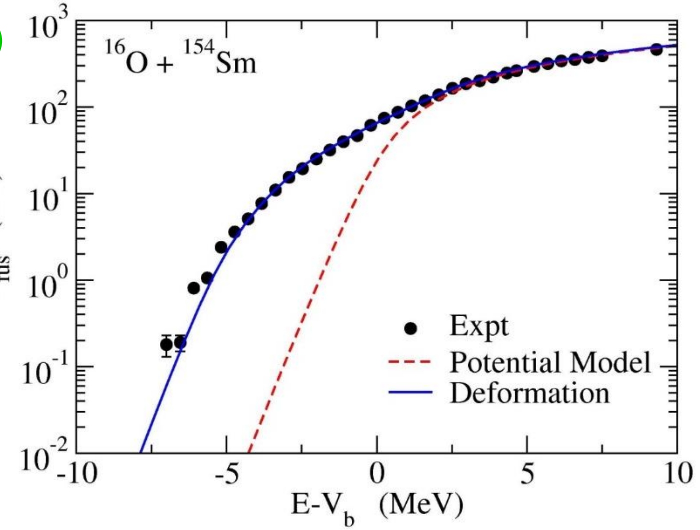
^{154}Sm : a typical deformed nucleus

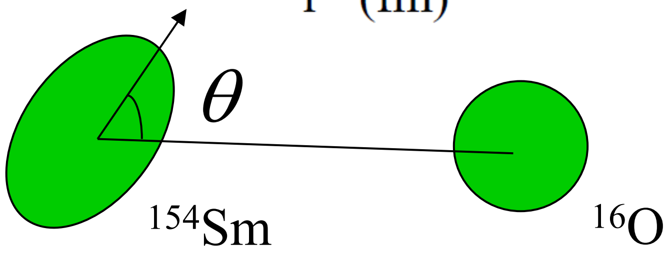


^{154}Sm









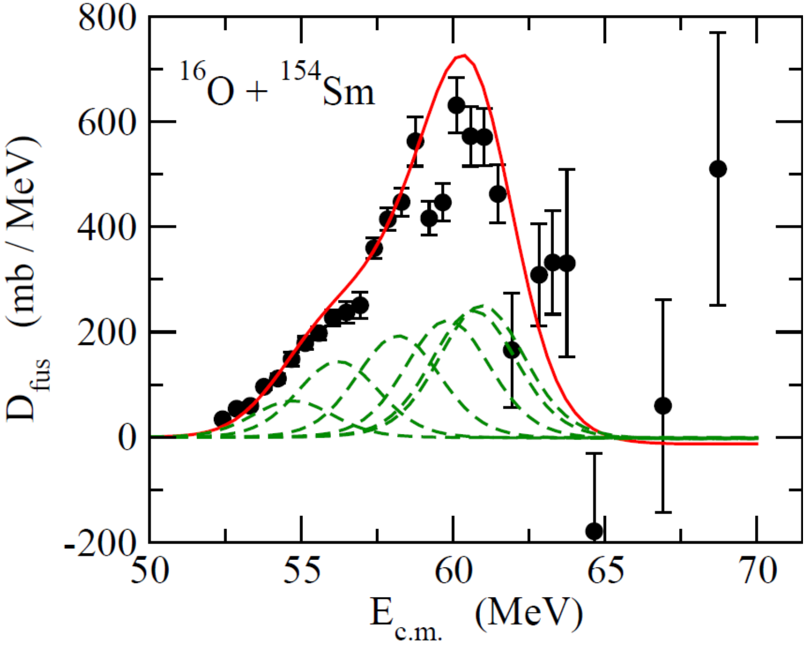
$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

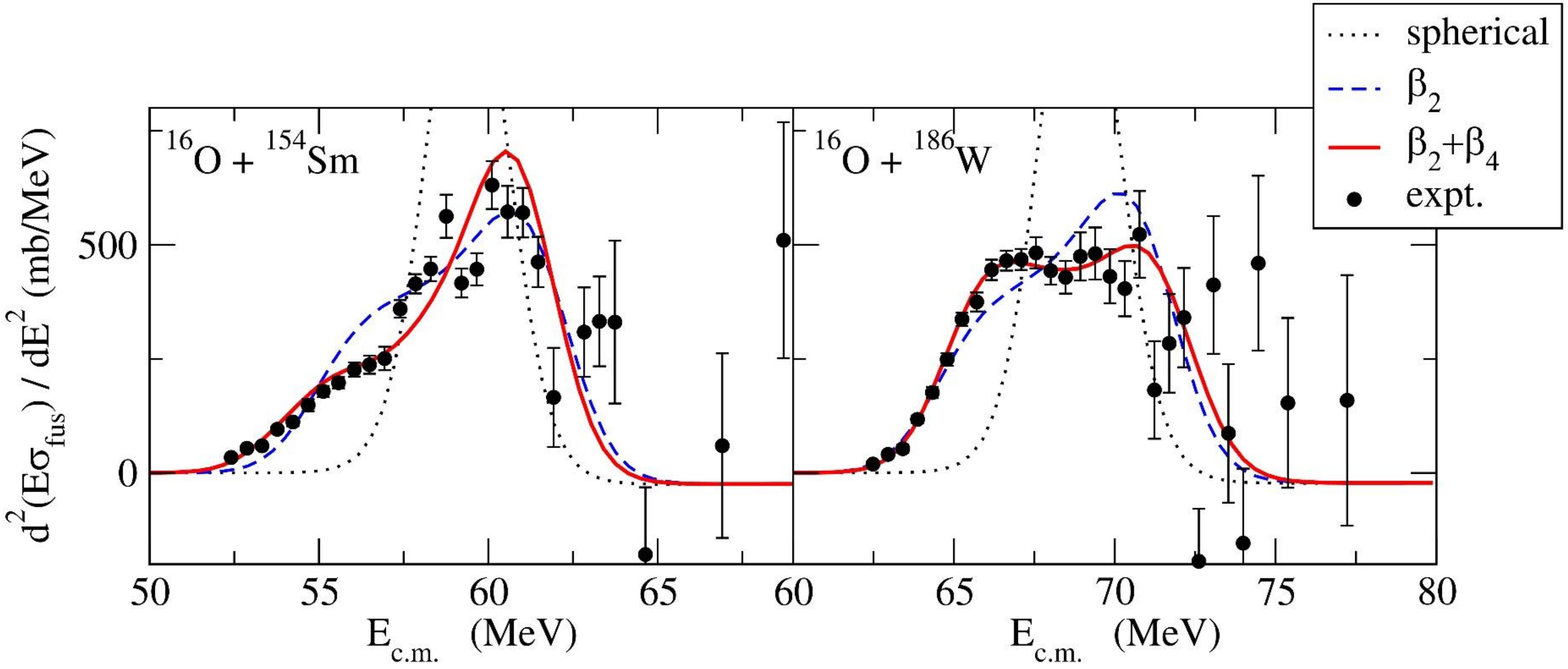
Fusion: strong interplay between nuclear structure and reaction

Fusion barrier distribution

$$D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25





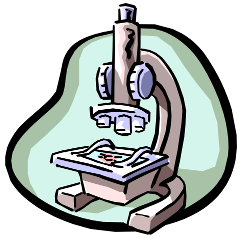
$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

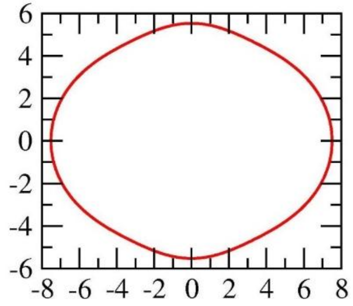
$\beta_2 = 0.33$

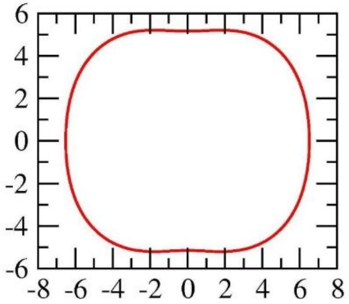
$\beta_4 = +0.05$

$\beta_2 = 0.29$

$\beta_4 = -0.03$



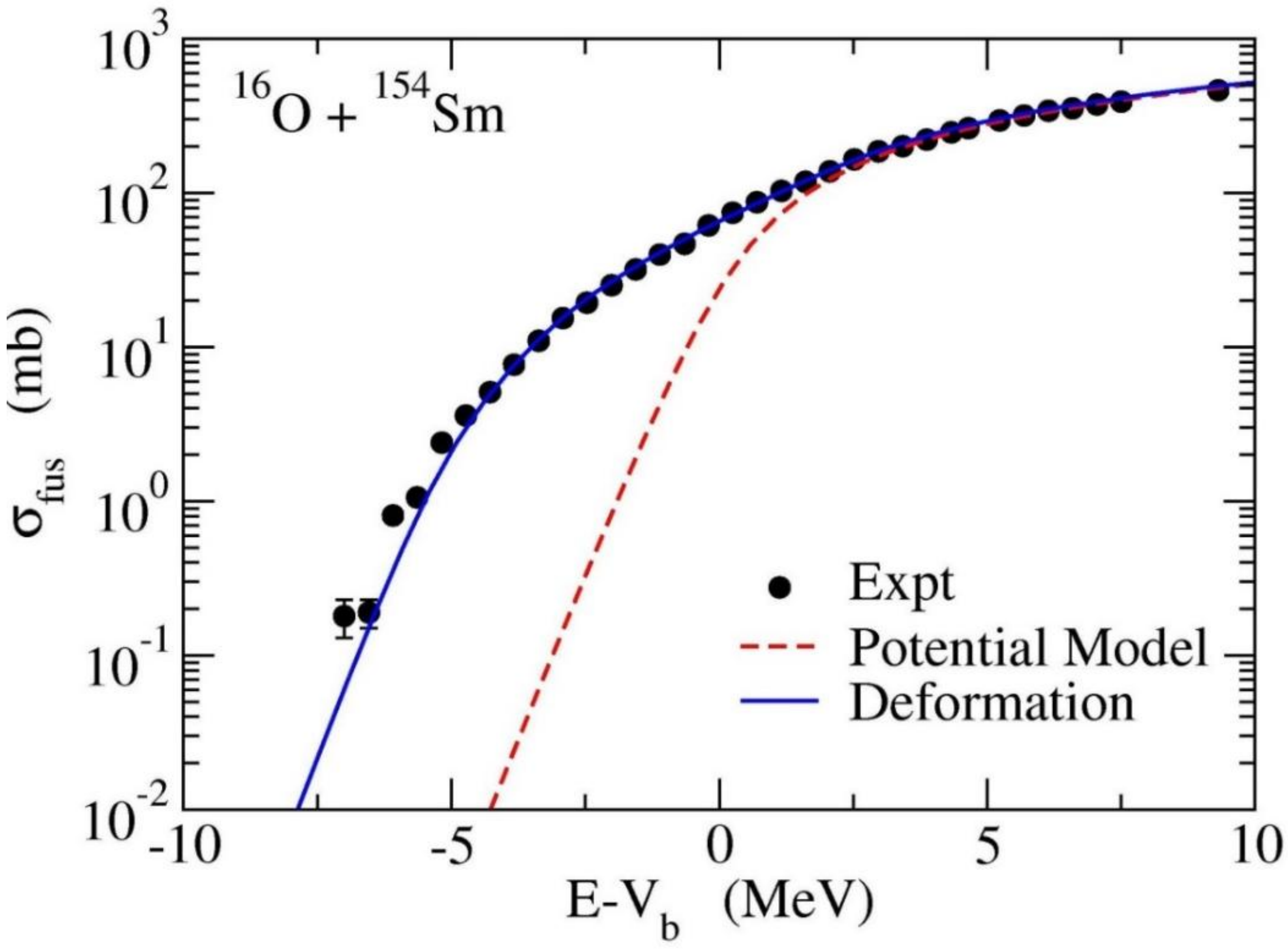


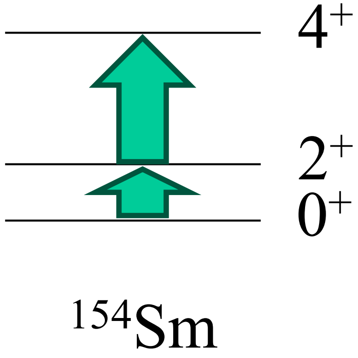


sensitive to the sign of β_4!

→ Fusion as a quantum tunneling microscope for nuclei

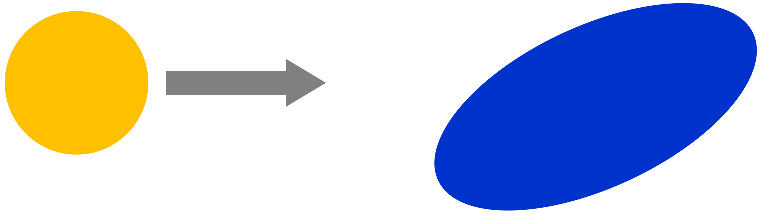
Coupled-channels calculations for fusion: more general



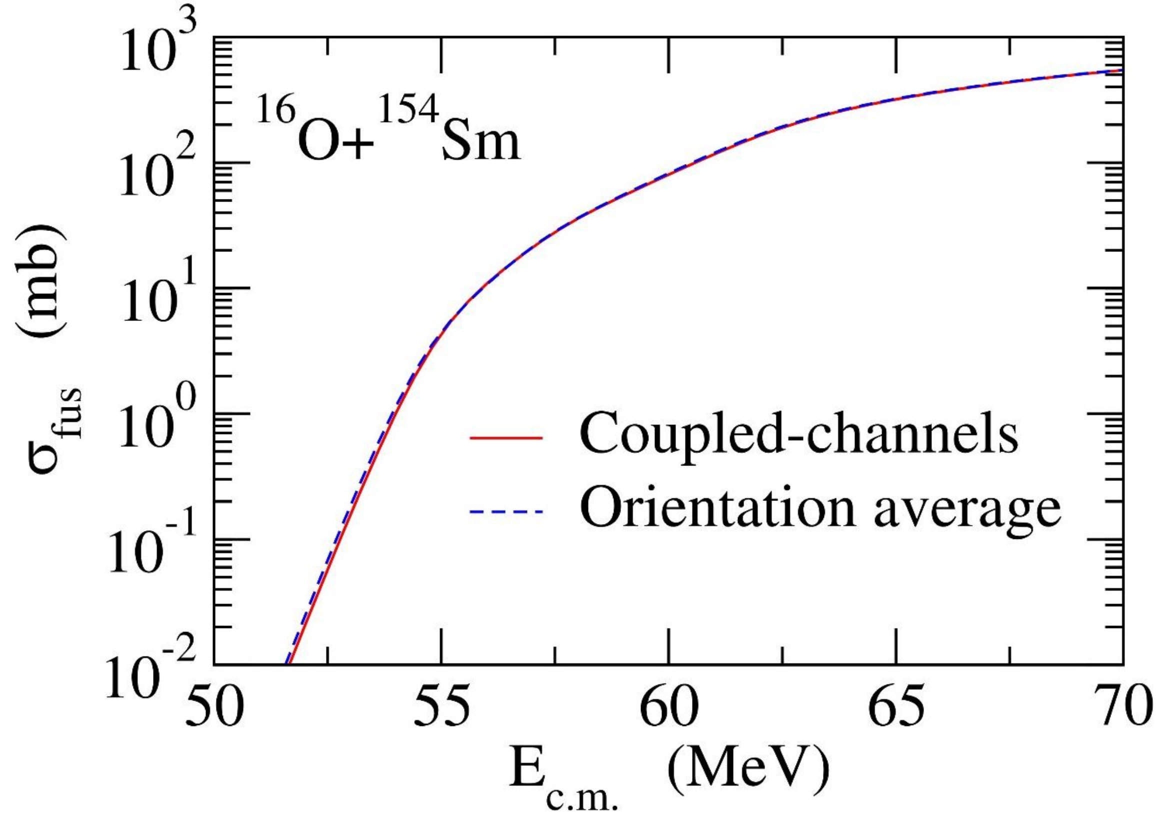


Inputs for the calculations

- potential parameters
- E_{2+}
- β_2, β_4,….

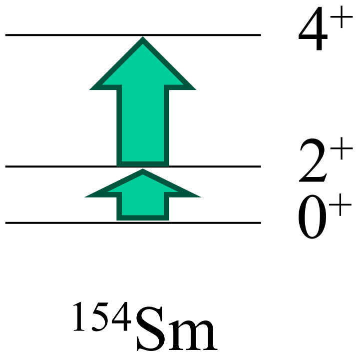


Equivalence between the Coupled-channels approach and the orientation average procedure



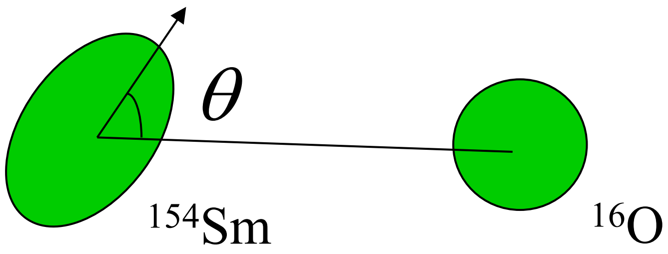
* a small deviation due to the finite *E** effect

coupled channels



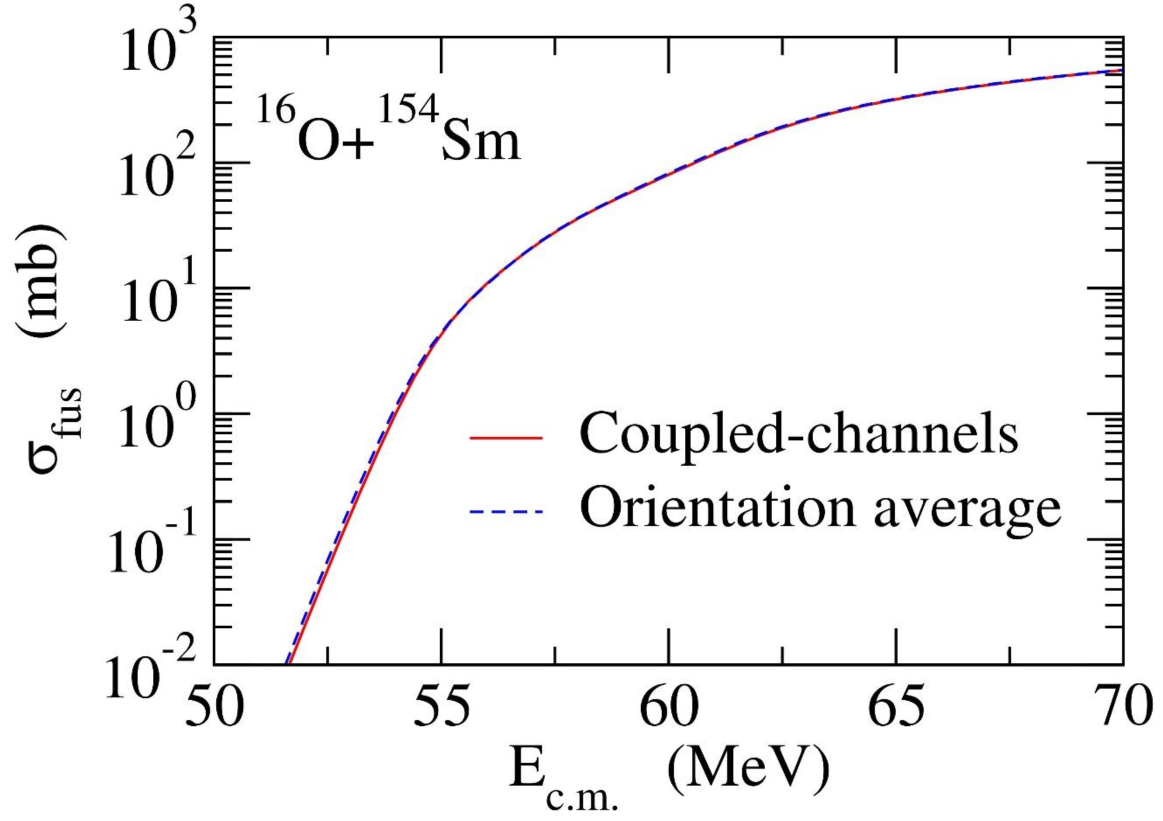
$$V_{II'}(r) = \langle I^+|V(r,\theta)|I'^+\rangle$$

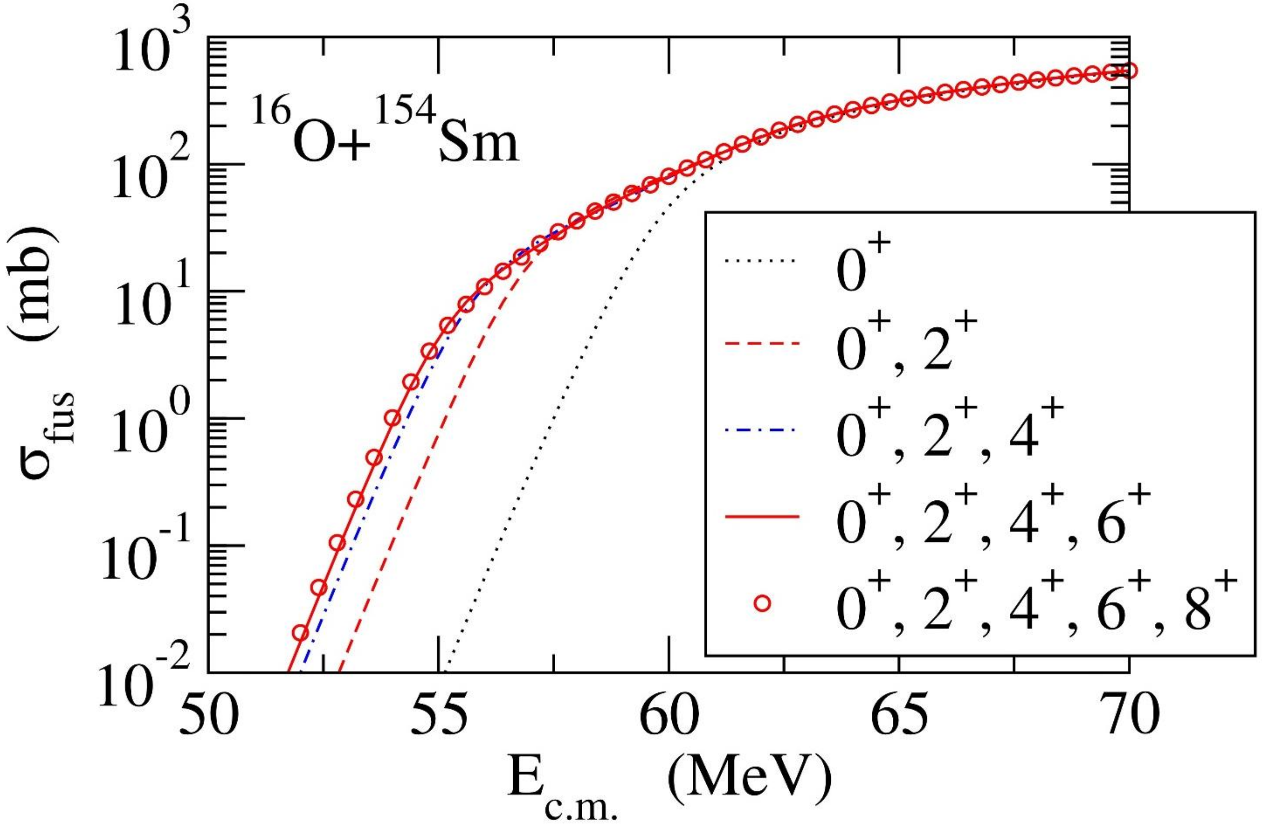
the orientation average formula

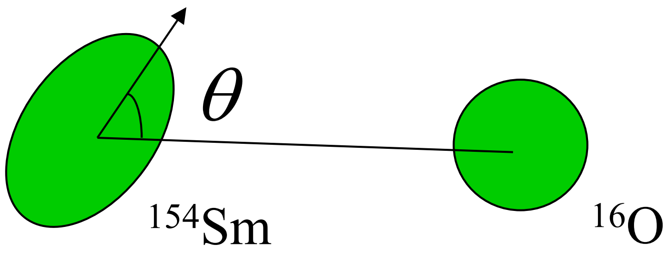


$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

Equivalence between the Coupled-channels approach and the orientation average procedure





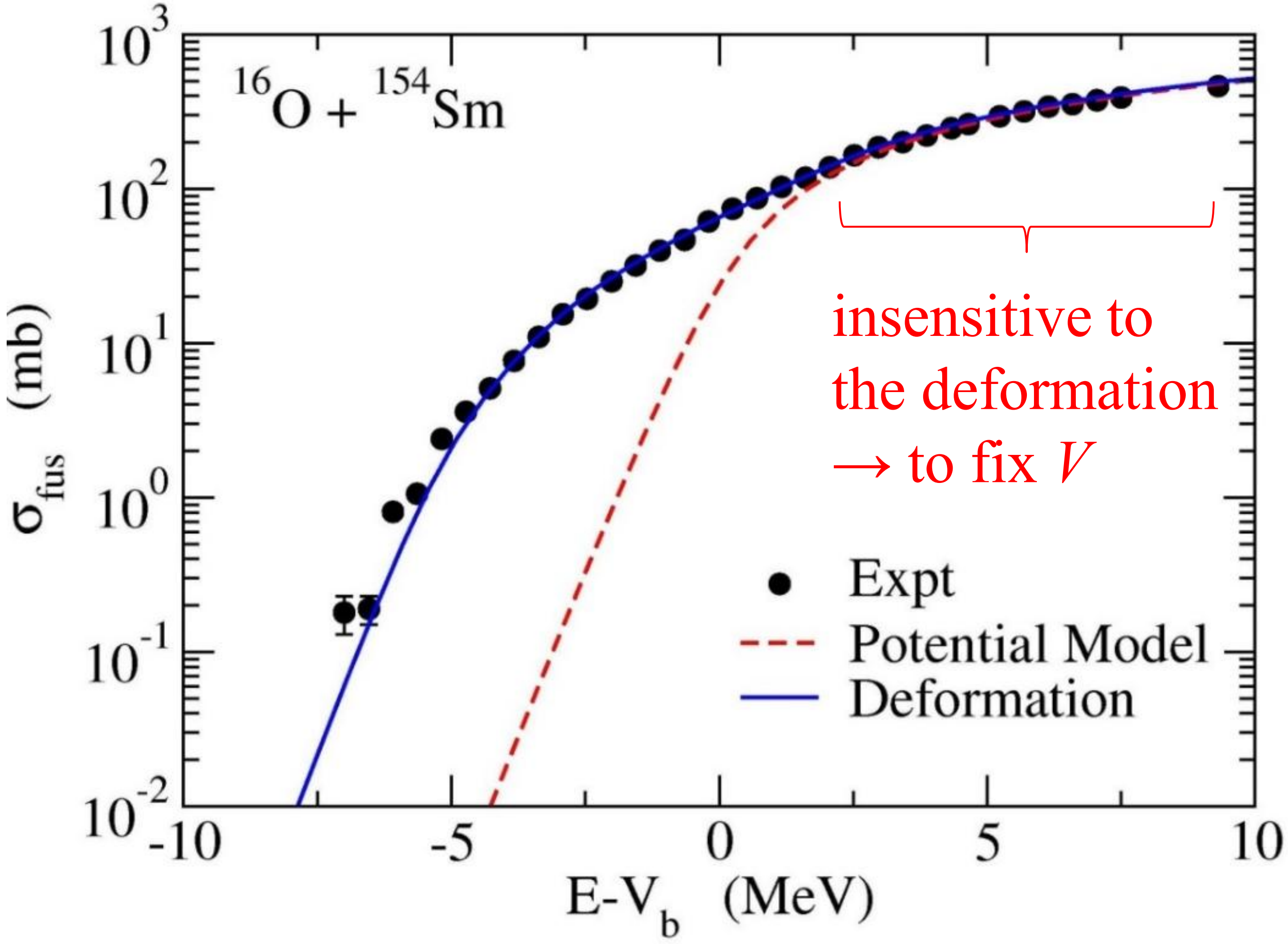


Notice: the weight is given by the g.s. wave function

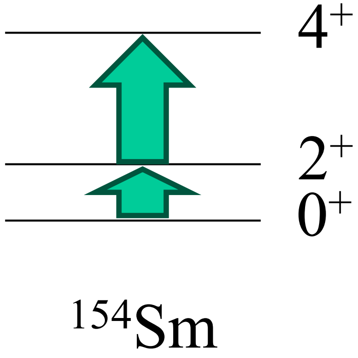
→ this does NEVER mean that a nucleus is not excited.

cf. $|\theta\rangle = \sum_I \langle Y_{I0}|\theta\rangle|Y_{I0}\rangle$

Coupled-channels calculations for fusion

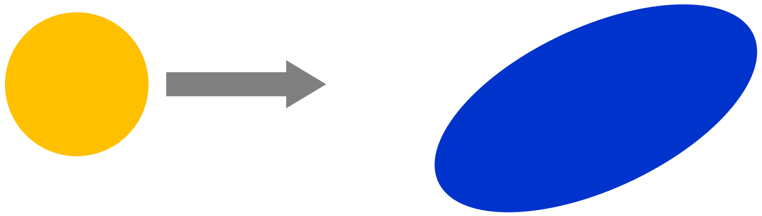


insensitive to the deformation $\rightarrow$ to fix V

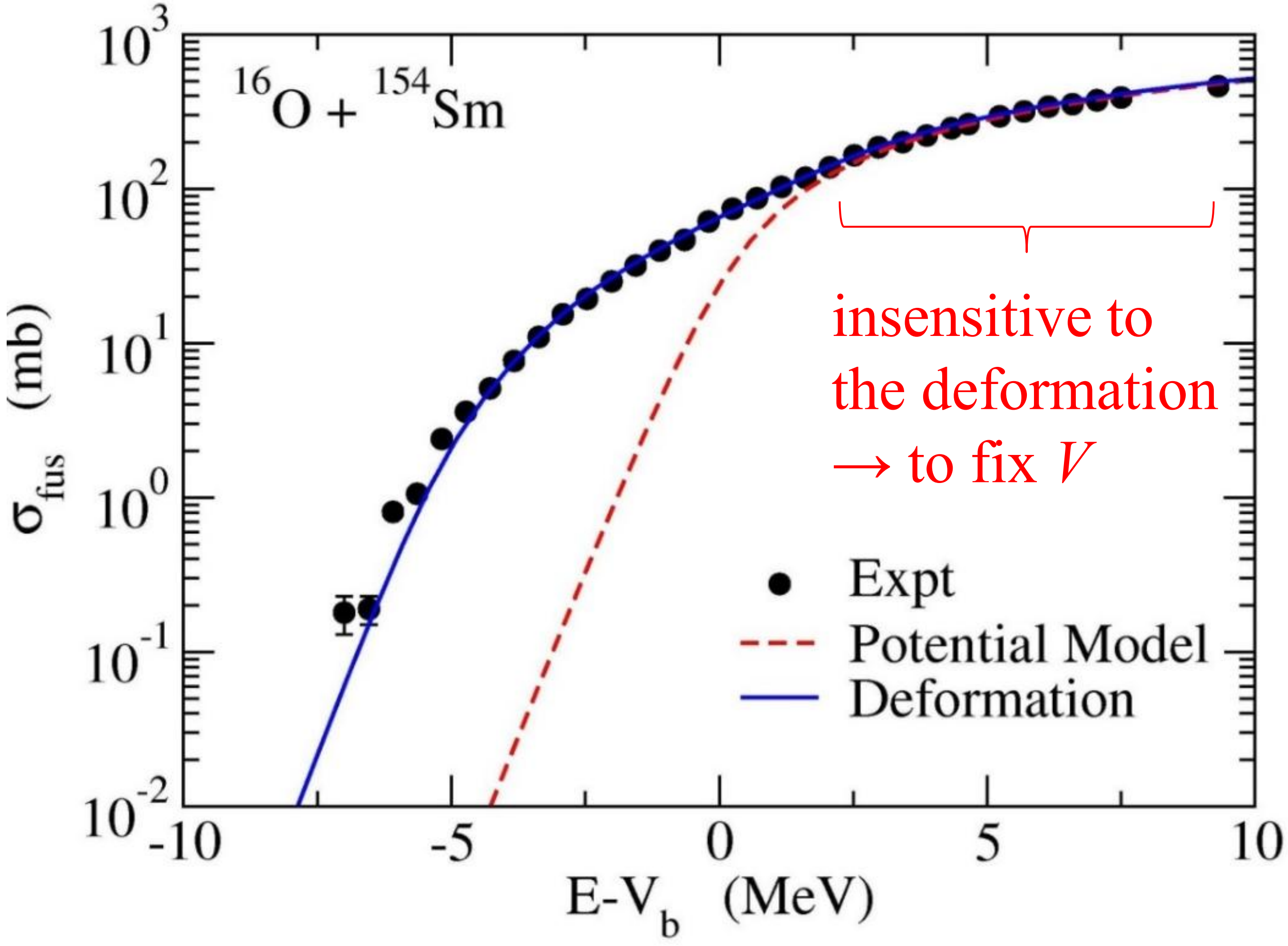


Inputs for the calculations

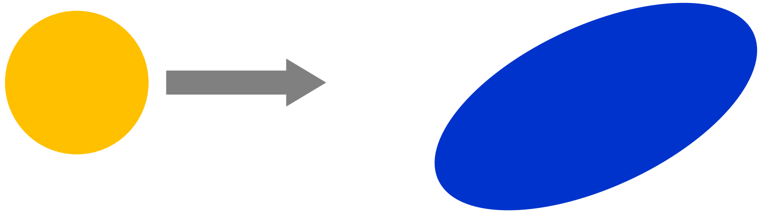
- potential parameters ✓
- E_{2+}
- $\beta_2, \beta_4, \ldots$

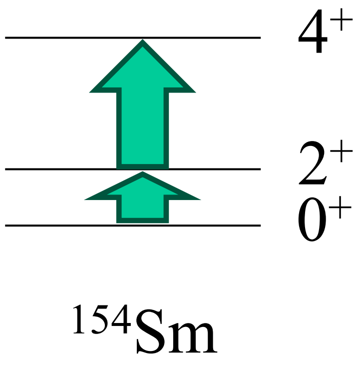


Coupled-channels calculations for fusion



insensitive to the deformation
→ to fix V





Inputs for the calculations

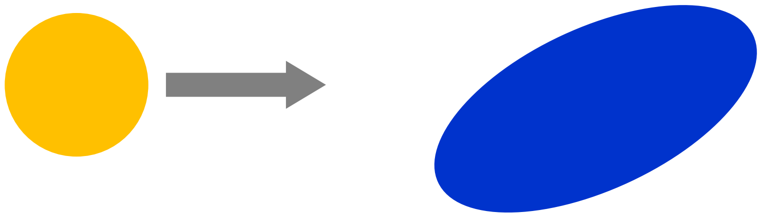
- potential parameters ✓
- E_{2+}
- β_2, β_4,….

✓ experimental data

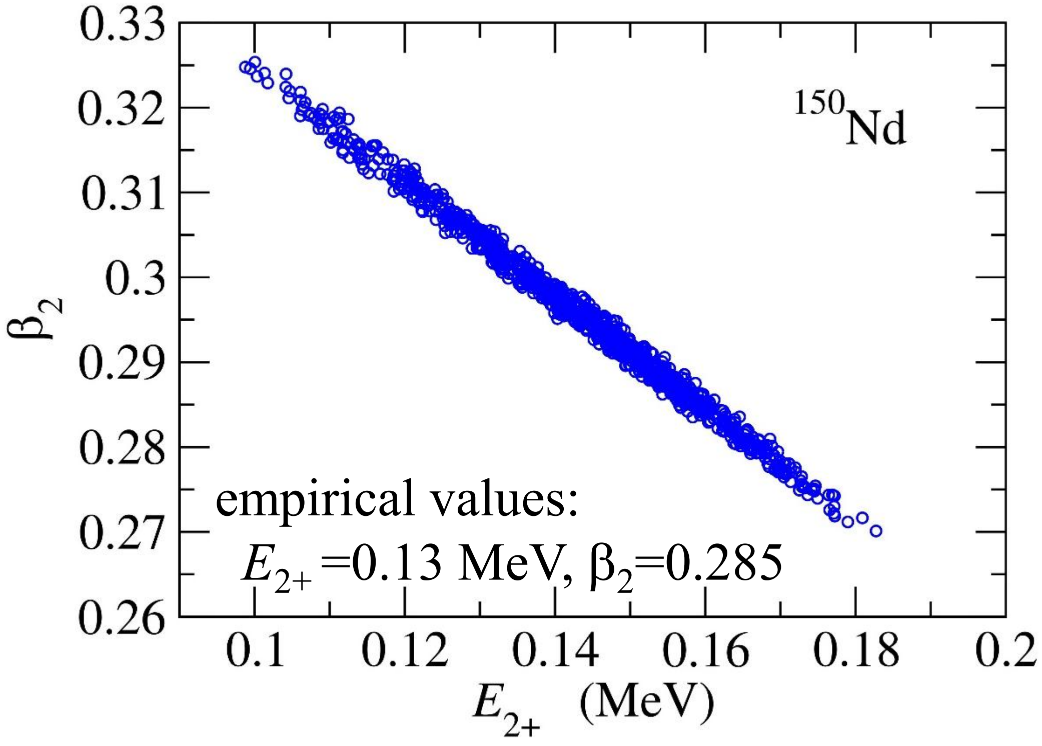
$$\beta_2 = \frac{4\pi}{3ZR^3}\sqrt{\frac{B(E2:0^+ \to 2^+)}{e^2}}$$

✓ beyond mean-field calculations

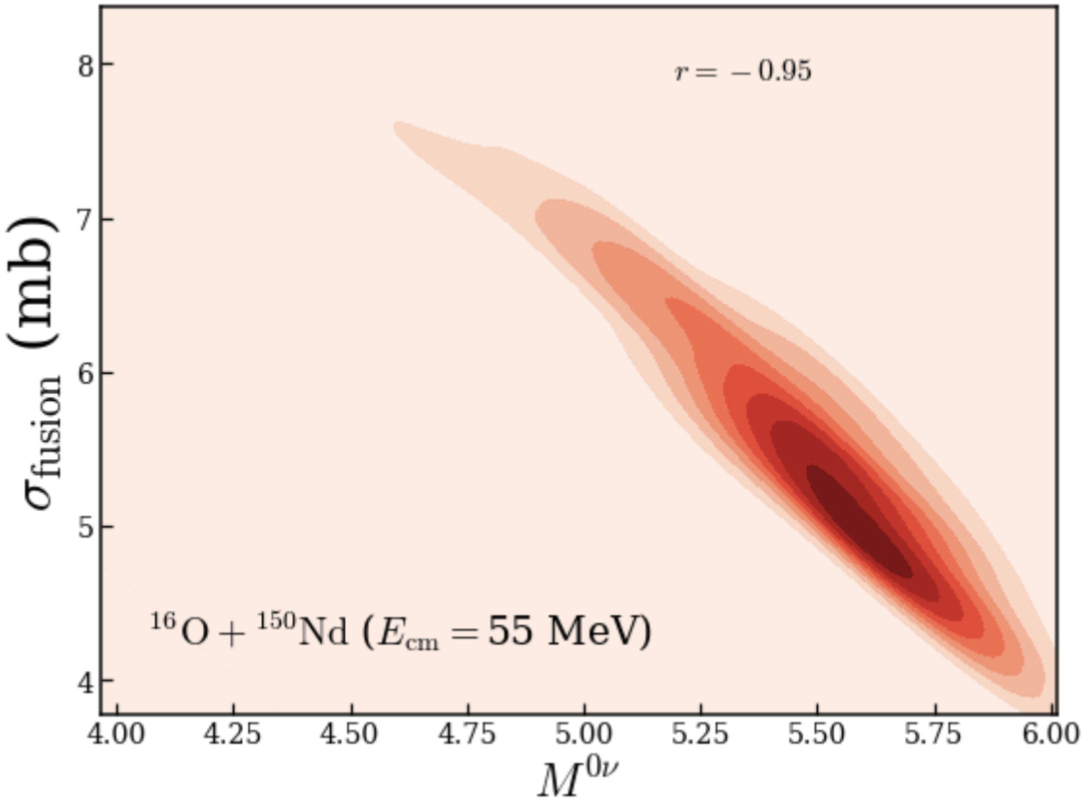
Coupled-channels calculations for fusion



beyond mean-field calculations with CDFT
with 1000 different parameter sets



for each parameter set of CDFT,
one can compute $\sigma_{\rm fus}$ and $M^{0\nu}$

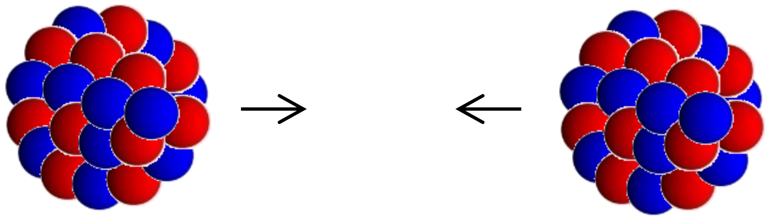


a strong correlation

cf. J.M. Yao's talk on Monday

J.M. Yao, X. Zhang, and K. Hagino,
in preparation (2026).

Summary



The meaning of “taking a snapshot”

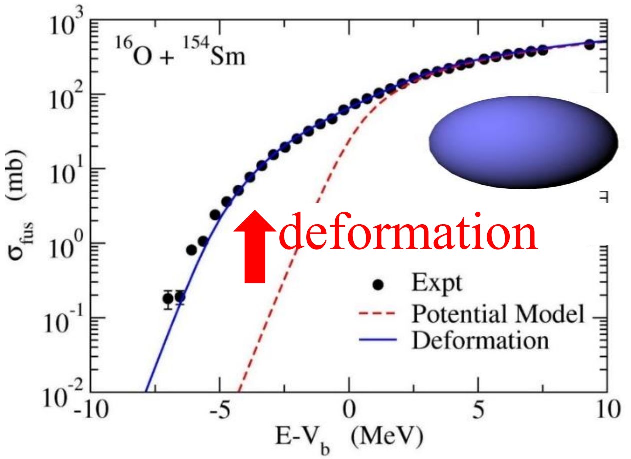
- ✓ fast collision → other d.o.f. are frozen in the coordinate space representation
- ✓ this does not mean that there is no excitations during the collision
- ✓ probing nuclear shape *through reaction observables*
- ✓ nuclear shapes are not directly probed

Heavy-ion fusion reactions around the Coulomb barrier

- ✓Strong interplay between nuclear structure and reaction
- ✓Quantum tunneling with various intrinsic degrees of freedom
- ✓Role of deformation in sub-barrier enhancement

→ a snapshot of the rotational motion

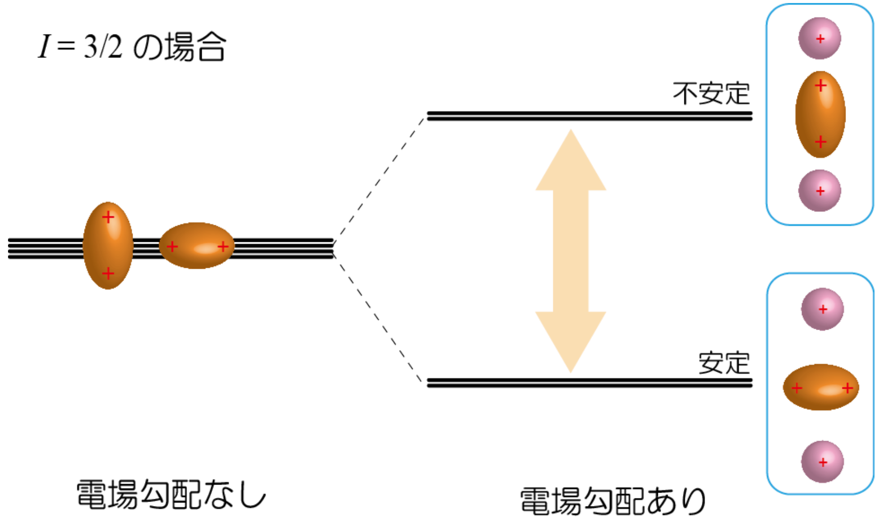
↓
amplified

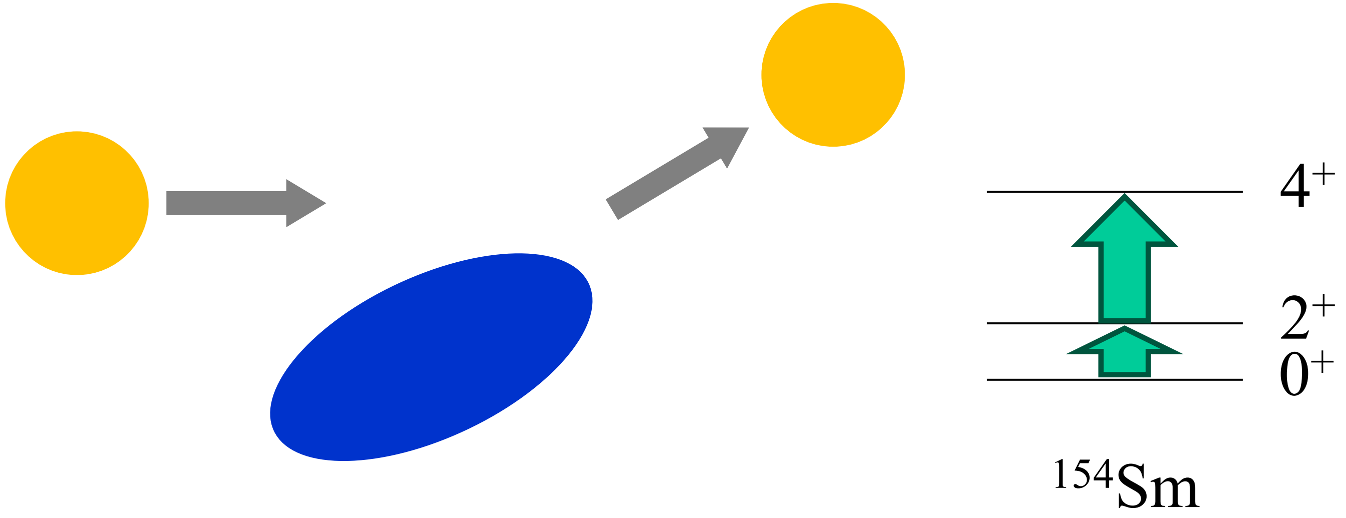


Discussions

One can measure nuclear shapes in various ways:

- ✓ quadrupole moment
- ✓ electromagnetic transition
- ✓ nuclear reactions (multiple Coulomb excitations and sub-barrier fusion reactions)

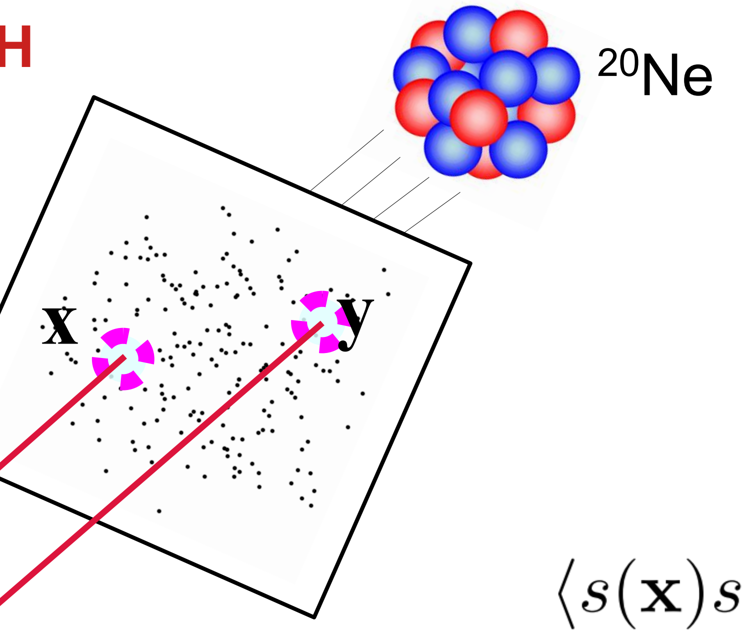


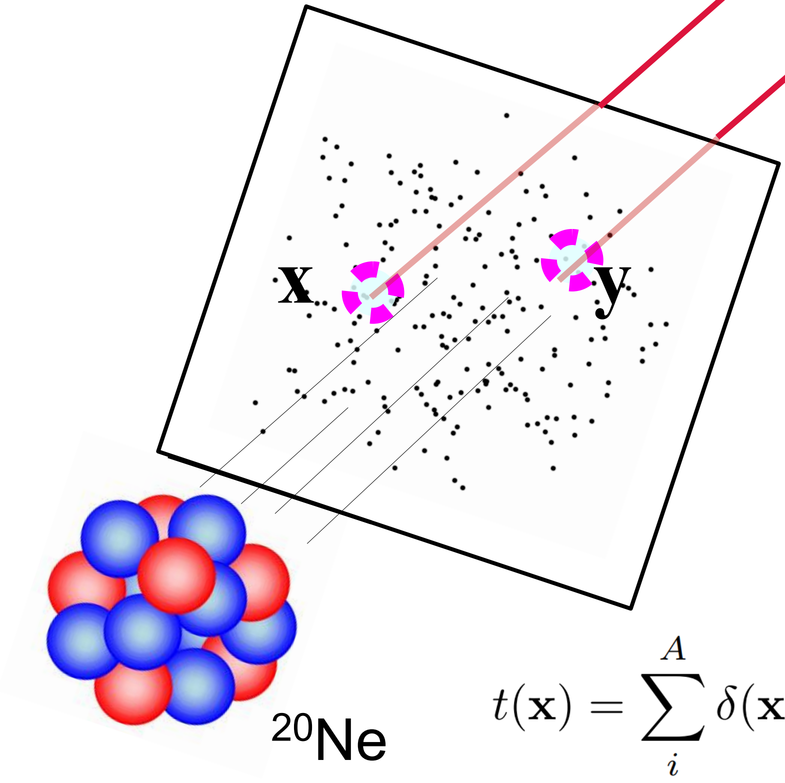


What is an advantage/a justification of using relativistic heavy-ion collisions to probe nuclear shapes? → What is the component beyond “just for fun”?

HIGH ENERGY APPROACH

$$|\Psi(\mathbf{r_1}, \mathbf{r_2}, \ldots, \mathbf{r_A})|^2$$





$$t(\mathbf{x}) = \sum_i^A \delta(\mathbf{x} - \mathbf{x}_i)$$

Ultra-central collisions (all nucleons involved)

Large-scale structures from incoming nuclei !

$$\langle s(\mathbf{x}) s(\mathbf{y}) \rangle \qquad |\mathbf{x} - \mathbf{y}| > 1/\Lambda_{\text{GeV}}$$