What is chiral susceptibility probing?

OSAKA UNIVERSITY
Live Locally, Grow Globally

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for JLQCD collaboration

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(and preliminary results in Nf=2+1simulations)

Acknowledgments

Resources:

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- Oakforest-PACS [JCAHPC]

  HPCI projects : hp170061, hp180061, hp190090, hp200086, hp210104,

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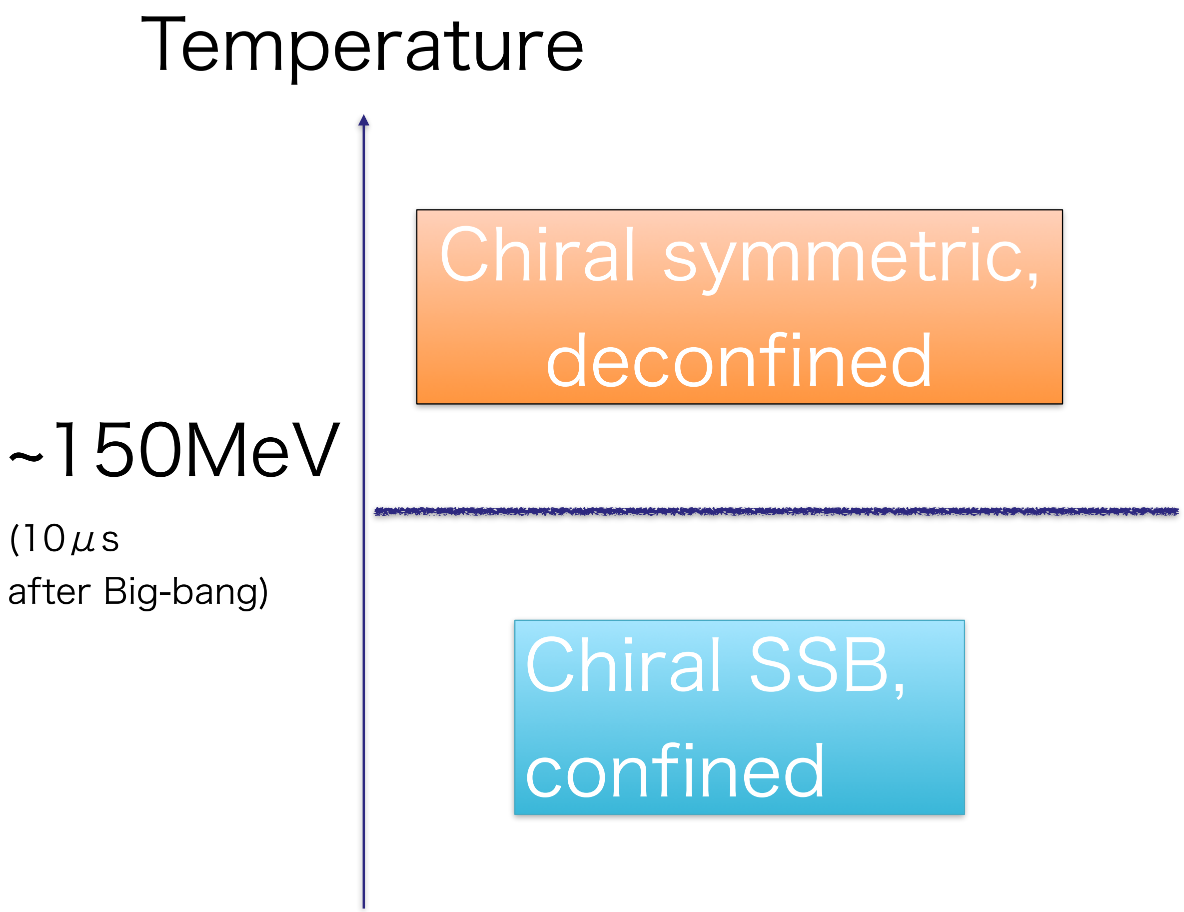
- Polarie/Grand Chariot (hp200130)
- Flow
- SQUID
- Program for Promoting Researches on the Supercomputer Fugaku, Simulation for basic science: from fundamental laws of particles to creation of nuclei Joint
- Institute for Computational Fundamental Science (JICFuS)

QCD phase transition



Chiral condensate (at m=0) probes $SU(2)_L x SU(2)_R$ symmetry breaking/ restoration :

For T>Tc, $\langle \bar{q}q \rangle = 0$

For T<Tc, $\langle \bar{q}q \rangle \neq 0$

Chiral susceptibility

QCD partition function

A : gluon fields

$$Z(m) = \int [dA] \det(D(A) + m)^{N_f} e^{-S_G(A)}$$

chiral condensate

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m)$$

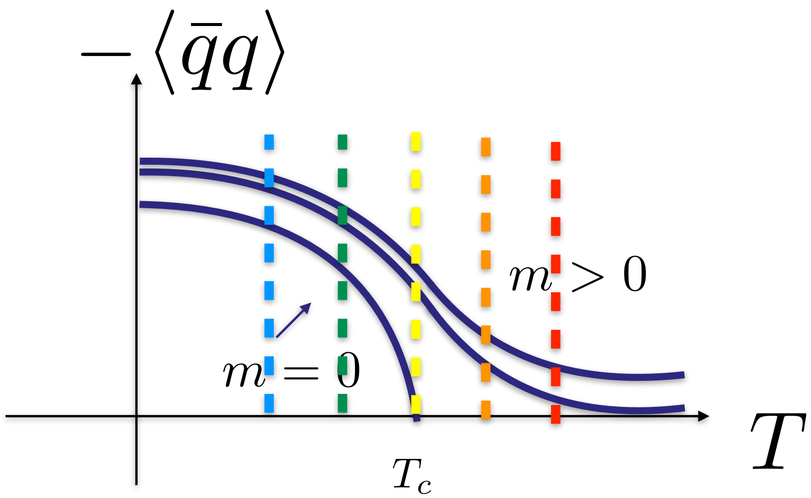
chiral susceptibility

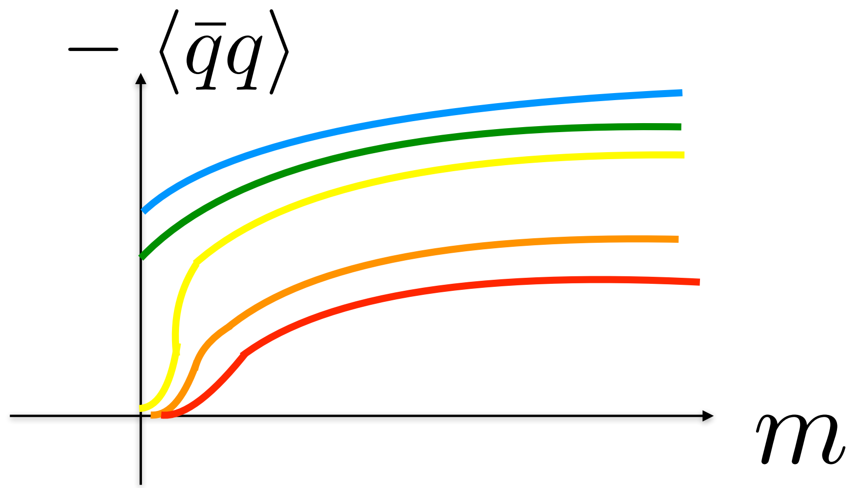
$$\chi(m) = -\frac{\partial}{\partial m} \langle \bar{q}q \rangle (m)$$

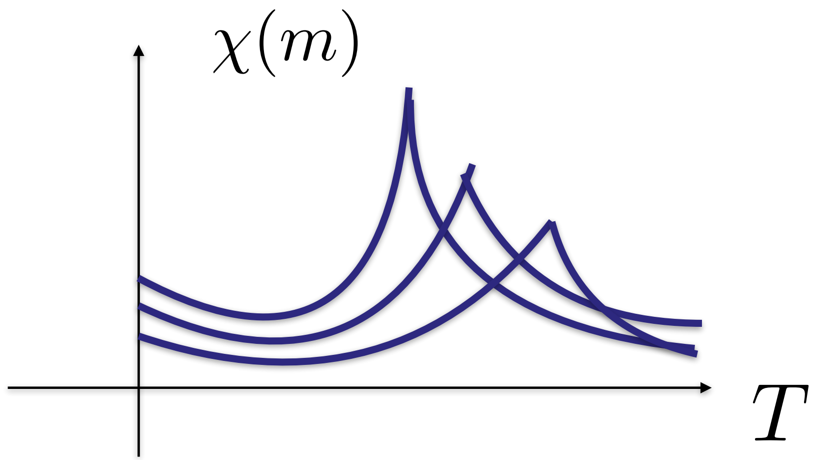
In this talk, $N_f = 2 \quad (m_u = m_d = m)$

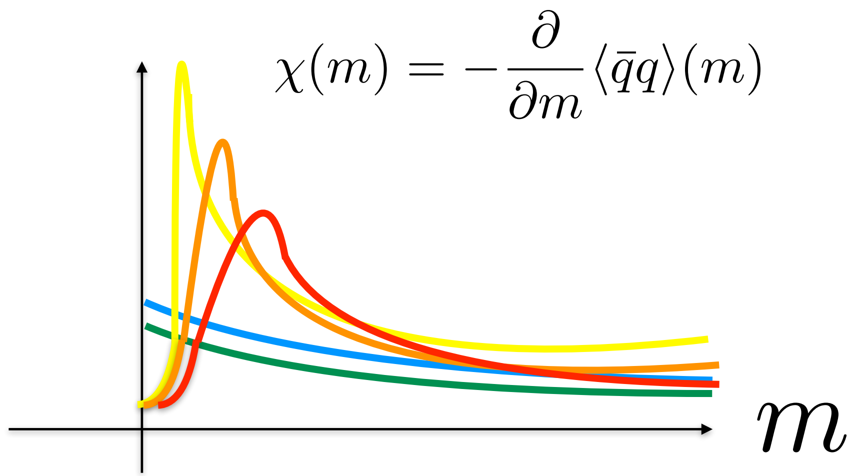
* strange quark is just a spectator.

Temperature(T) and mass(m) dependence



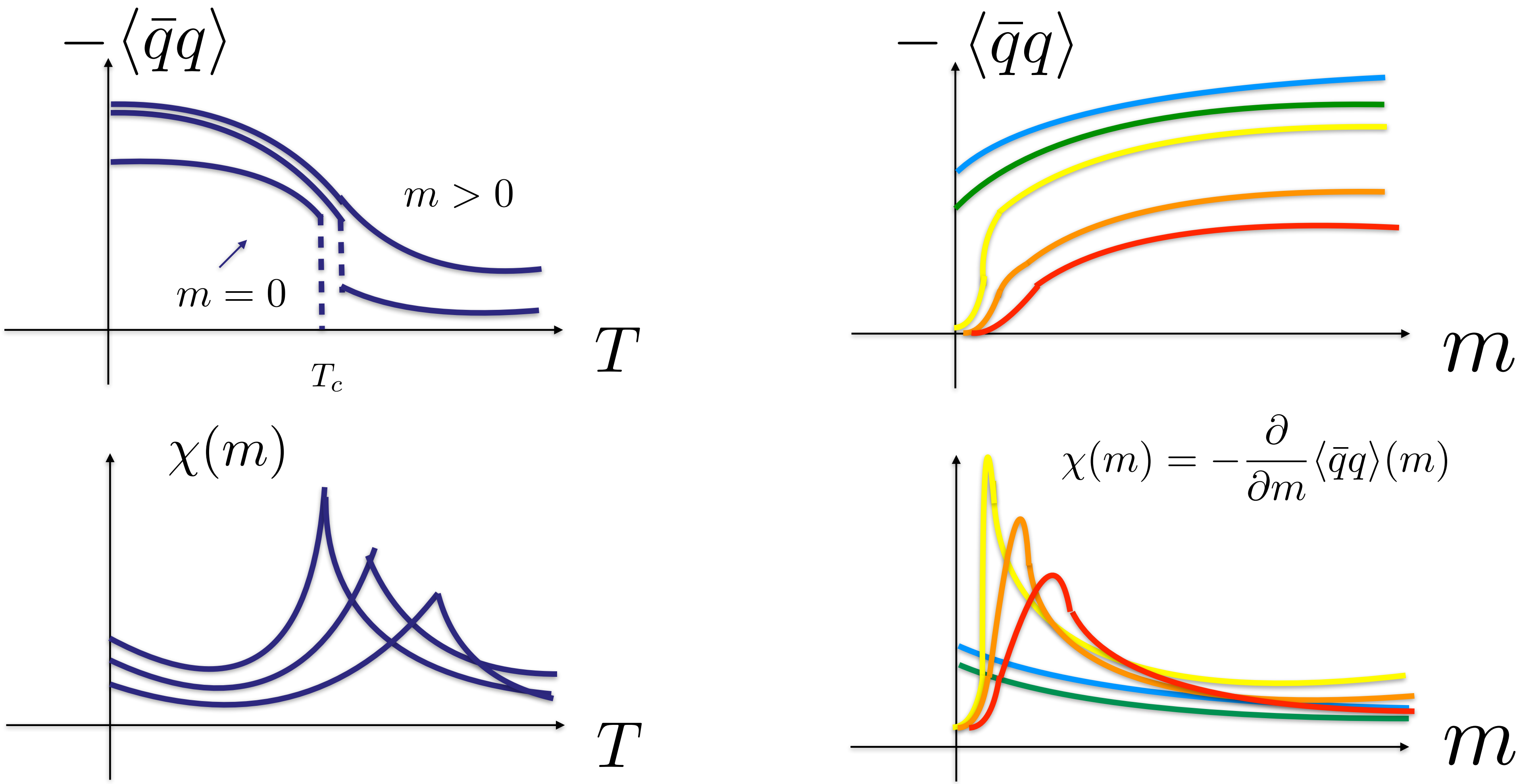






When the transition is 1st order

* But finite V effect makes the transition not sharp.



Chiral phase transition

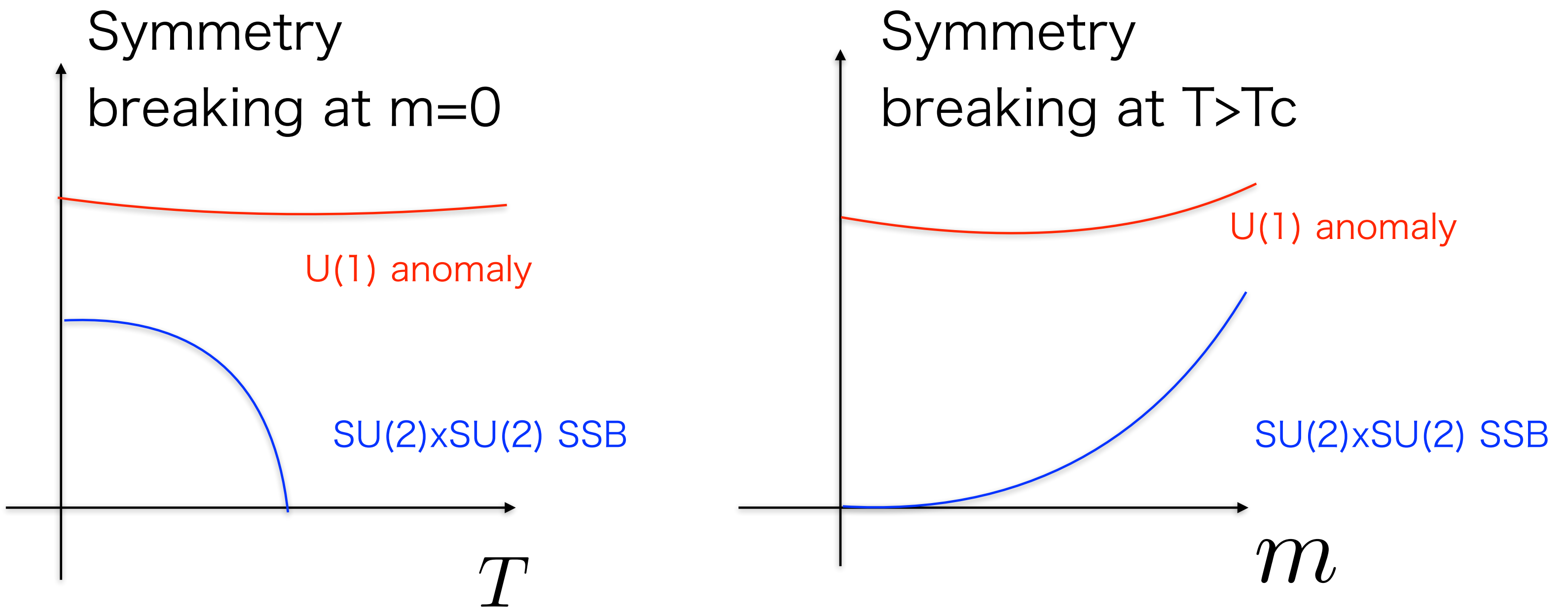
Chiral condensate probes
$SU(2)_L \times SU(2)_R$ symmetry breaking/restoration :

For T<Tc, $\langle \bar{q}q \rangle \neq 0$ For T>Tc, $\langle \bar{q}q \rangle = 0$

But $\langle \bar{q}q \rangle$ also breaks $U(1)_A$ symmetry.

Question:
How much does $U(1)_A$ (anomaly) contribute to the transition?

Naive expectation: U(1) anomaly exists at any energy scale (does not change much)



You may think that T and m dependences of chiral condensate should reflect $SU(2)_L x SU(2)_R$ breaking rather than U(1) anomaly.

But in early days of QCD

QCD founders in 70's and 80's thought

instanton → axial U(1) anomaly → SU(2)xSU(2) breaking.

Callan, Dashen & Gross 1978:

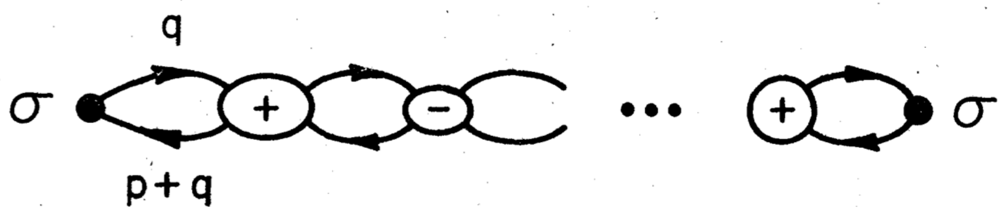


FIG. 9. The structure of the diagrams that produce a tachyon in the σ channel. The + (−) blobs refer to the effective determinantal four-fermion interaction induced by instantons (anti-instantons).

If this inverse is true, we should have

instanton disappears → anomaly disappears → SU(2)xSU(2) restored.

It has been difficult issue.

Analytic method:

Semi-classical QCD instantons are not enough to describe the low-energy dynamics of QCD.

Lattice simulations :

Staggered fermions explicitly breaks

$SU(2)_L \times SU(2)_R \times U(1)_A \rightarrow U(1)_{A'}$

Wilson fermion explicitly breaks

$SU(2)_L \times SU(2)_R \times U(1)_A \rightarrow SU(2)_V$

Moreover, we found that lattice artifacts are enhanced at high temperature (even for domain-wall fermions) [JLQCD 2015, 2016]

Our work

In this work we study chiral condensate and its susceptibility in 2- and 2+1-flavor QCD with exactly chiral symmetric Dirac operator.
We separate the axial U(1) breaking (in particular topological) effect from others in a clean way.

Our result shows that
signal of chiral susceptibility is dominated by axial U(1) breaking effect (at T>=Tc),
rather than $SU(2)_L \times SU(2)_R$.

Contents

✔ 1. Introduction

We simulate the chiral phase transition of Nf=2 QCD with chiral fermions to investigate the role of axial U(1) anomaly.

2. $U(1)_A$ contribution to chiral susceptibility

3. Numerical results

4. Summary

Dirac eigenmode decomposition

$$Z(m) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)} = \int [dA] \prod_\lambda (i\lambda(A)+m)^{N_f} e^{-S_G(A)}$$

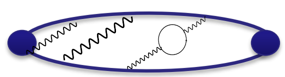
O(100) eigenvalues can be computed on the lattice.

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m) = \frac{1}{V} \left\langle \sum_\lambda \frac{1}{i\lambda(A)+m} \right\rangle,$$

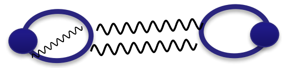
chiral susceptibility

$$\chi(m) = \frac{1}{N_f V} \frac{\partial^2}{\partial m^2} \ln Z(m) = \chi^{con.}(m) + \chi^{dis.}(m),$$

$$\chi^{con.}(m) = -\frac{\partial}{\partial m_{valence}} \langle \bar{q}q \rangle \Big|_{m_{valence}=m}$$



$$\chi^{dis.}(m) = -\frac{\partial}{\partial m_{sea}} \langle \bar{q}q \rangle \Big|_{m_{sea}=m}$$



Chiral rotations (with angle π)

$$S^0(x) = \bar{q}q(x)$$

$SU(2)_L \times SU(2)_R$

$\exp(i\pi\gamma_5\tau_a/2)$

$$P^a(x) = \bar{q}i\gamma_5\tau^a q(x)$$

$U(1)_A$

$\exp(i\pi\gamma_5/2)$

$$P^0(x) = \bar{q}i\gamma_5 q(x)$$

$$S^a(x) = \bar{q}\tau^a q(x)$$

isospin
singlet

isospin
triplet

Relation to scalar susceptibility

$$L_{\mathrm{QCD}} = \left[\frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{q}\left(\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) + m\right)q\right]$$

$$\chi(m) = \frac{1}{N_f V}\frac{\partial^2}{\partial m^2}\ln Z(m, \theta = 0)$$

$$= -\sum_x \langle S^0(x) S^0(0)\rangle - V\langle S^0\rangle^2 \qquad S^0(x) = \bar{q}q(x)$$

Relation to pseudoscalar susceptibility

$$Z(m,\theta) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)+i\theta Q(A)}$$

$$= \int [dA] \det(D(A)+me^{i\gamma_5\theta/N_f})^{N_f} e^{-S_G(A)}$$ ← U(1)A rotation

$$\chi_{\mathrm{top.}}(m) = -\frac{1}{N_f V}\frac{\partial^2}{\partial\theta^2}\ln Z(m,\theta)|_{\theta=0} = m\left[\frac{\partial}{\partial\theta}\langle\bar{q}i\gamma_5 e^{i\gamma_5\theta/N_f}q\rangle\right]|_{\theta=0}$$

⇨ $$\frac{N_f}{m^2}\chi_{\mathrm{top.}}(m) = -\sum_x \langle P^0(x)P^0(0)\rangle - \frac{\langle\bar{q}q\rangle(m)}{m}.$$

$$P^0(x) = \bar{q}i\gamma_5 q(x)$$

$*N_f = 2$

Symmetry structure of scalar/pseudoscalar susceptibilities

$\chi(m)$

$SU(2)_L \times SU(2)_R$

$$\sum_x \langle S^0(x) S^0(0) \rangle \quad \longleftrightarrow \quad \sum_x \langle P^a(x) P^a(0) \rangle$$

iso-singlet $\quad U(1)_A \quad\quad U(1)_A \quad$ iso-triplet

$$\sum_x \langle P^0(x) P^0(0) \rangle \quad \longleftrightarrow \quad \sum_x \langle S^a(x) S^a(0) \rangle$$

$SU(2)_L \times SU(2)_R$

$$-\frac{N_f}{m^2}\chi_{\text{top.}}(m) - \frac{-\langle \bar{q}q \rangle (m)}{m}$$

See also LLNL/RBC Collaboration 2014, Nicola & Elvira 2018, Nicola 2020.

Connected/disconnected pseudoscalar susceptibilities

From a Ward-Takahashi identity $0 = \langle \delta^a_{SU(2)} P^a(0) \rangle - \langle \delta^a_{SU(2)} S P^a(0) \rangle,$

we have

$$m \sum_x \langle P^a(x) P^a(0) \rangle + \langle S^0 \rangle = 0.$$

Therefore,

$$\frac{N_f}{m^2} \chi_{\mathrm{top.}}(m) = -\sum_x \langle P^0(x) P^0(0) \rangle - \frac{\langle S(0) \rangle}{m}$$

$$= \sum_x \langle P^a(x) P^a(0) \rangle - \sum_x \langle P^0(x) P^0(0) \rangle$$

Connected/disconnected scalar susceptibilities

$$\chi(m) = \chi^{\text{con.}}(m) + \chi^{\text{dis.}}(m)$$

$$\chi^{\text{con.}}(m) = \sum_x \langle S^a(x) S^a(0) \rangle = \sum_x \langle S^a(x) S^a(0) - P^a(x) P^a(0) \rangle + \sum_x \langle P^a(x) P^a(0) \rangle$$

$$= -\Delta_{U(1)}(m) + \frac{-\langle \bar{q} q \rangle (m)}{m}$$

$$\chi^{\text{dis.}}(m) = \sum_x \langle S^0(x) S^0(0) - S^a(x) S^a(0) \rangle - V \langle S^0(0) \rangle^2$$

$$= \sum_x \langle \left[S^0(x) S^0(0) - \langle S^0(0) \rangle^2 - P^a(x) P^a(0) \right] + \left[P^a(x) P^a(0) - S^a(x) S^a(0) \right] \rangle$$

$$= \Delta^{(1)}_{SU(2)}(m) + \left[P^a(x) P^a(0) - P^0(x) P^0(0) \right] + \left[P^0(x) P^0(0) - S^a(x) S^a(0) \right] \rangle$$

$$= \Delta^{(1)}_{SU(2)}(m) + \frac{N_f \chi_{\text{top.}}(m)}{m^2} - \Delta^{(2)}_{SU(2)}(m)$$

Separating U(1)A breaking part

$$\chi(m) = \chi^{\mathrm{con.}}(m) + \chi^{\mathrm{dis.}}(m)$$

$$\chi^{\mathrm{con.}}(m) = \underbrace{-\Delta_{U(1)}(m) + \frac{\langle |Q(A)| \rangle}{m^2 V}}_{\text{U(1)A breaking contribution}} \underbrace{- \frac{-\langle \bar{q}q \rangle_{\mathrm{sub.}}(m)}{m}}_{\text{mixed}}$$

* quadratic divergence is subtracted using the data at reference quark mass mref=0.005.

$$\chi^{\mathrm{dis.}}(m) = \overbrace{\frac{N_f}{m^2}\chi_{\mathrm{top.}}(m)}^{\text{U(1)A breaking contribution}} \underbrace{+ \Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)}_{\text{SU(2)xSU(2) breaking}}$$

where $\Delta_{U(1)}(m) \equiv \sum_x \langle P^a(x)P^a(0) - S^a(x)S^a(0) \rangle$ axial U(1) susceptibility

$$\Delta^{(1)}_{SU(2)}(m) \equiv \sum_x \langle S^0(x)S^0(0) - P^a(x)P^a(0) \rangle \qquad \Delta^{(2)}_{SU(2)}(m) \equiv \sum_x \langle S^a(x)S^a(0) - P^0(x)P^0(0) \rangle$$

Lattice formulas

Using $\lambda_m = \text{eigenvalues of } H_m = \gamma_5[(1-m)D_{ov} + m]$

$$\Delta_{U(1)}(m) = \frac{1}{V(1-m^2)^2}\left\langle \sum_{\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

$$-\langle \bar{q}q \rangle = \frac{1}{V(1-m^2)}\left\langle \sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right\rangle.$$

$$\chi^{\text{dis.}}(m) = \frac{N_f}{V}\left[\frac{1}{(1-m^2)^2}\left\langle \left(\sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2}\right)^2 \right\rangle - |\langle \bar{q}q \rangle^{lat}|^2 V^2\right].$$

Remark.1 eigen functions do not matter.

Remark.2 chiral symmetry is essential for this decomposition.

Contents

Simulation setup (Nf=2)

Nf=2 flavor QCD

1/a = 2.6 GeV (0.075fm)

Symanzik gauge action

L=24,32,40,48 [1.8-3.6fm] (at T=220MeV)

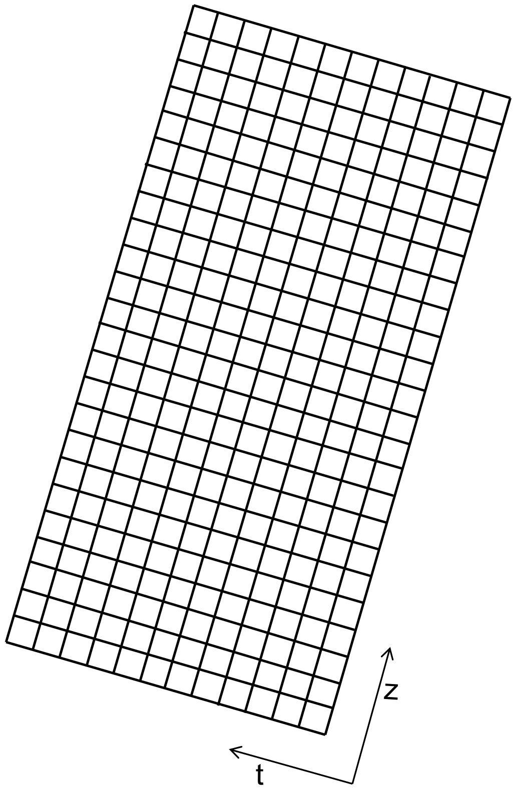
Mobius domain-wall fermions with m_{res}<1MeV

(and reweighted overlap fermion)

Quark mass from 3MeV (< phys. pt. ~4MeV) to 30MeV.

T=165 (~Tc), 195, 220, 260, 330 MeV (Lt=8,10,12,14,16)

Tc is estimated to be around 175MeV (from Polyakov loop)



Simulation codes : Irolro++ (https://github.com/coppolachan/Irolro)
Grid (https://github.com/paboyle/Grid)

Simulation setup (Nf=2+1) [preliminary]

Nf=2+1 flavor QCD

1/a = 2.453GeV

L=32 (2.58,fm)

Mobius domain-wall fermion with m_{res}<1MeV

(and reweighted overlap fermion)

up-down quark mass from phys. pt. ~4MeV to 30MeV.

strange quark mass at phys.pt.

T=153(~Tc), 175, 220 MeV

Low-mode approximation

In the eigenvalue summations,

$$\Delta_{U(1)}(m) = \frac{1}{V(1-m^2)^2}\left\langle \sum_{\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

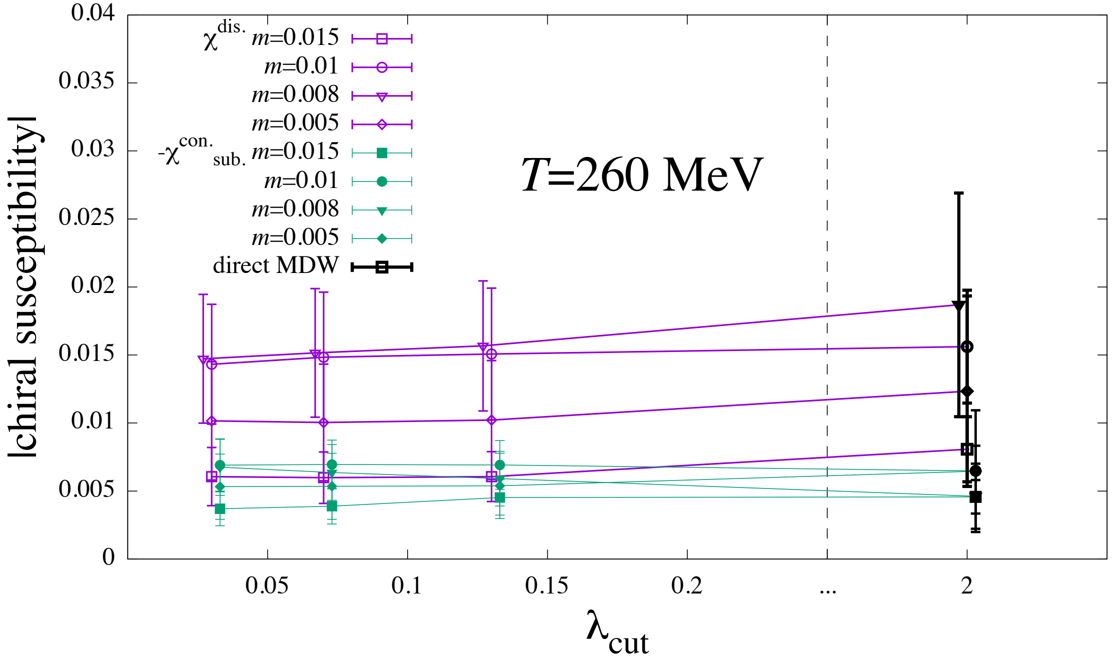
$$-\langle \bar{q}q \rangle = \frac{1}{V(1-m^2)}\left\langle \sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right\rangle.$$

$$\chi^{\text{dis.}}(m) = \frac{N_f}{V}\left[\frac{1}{(1-m^2)^2}\left\langle \left(\sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right)^2 \right\rangle - |\langle \bar{q}q \rangle^{lat}|^2 V^2\right].$$

where $\lambda_m = \text{eigenvalues of } H_m = \gamma_5[(1-m)D_{ov} + m]$

we truncate at 30-40th lowest mode ($\lambda_{\text{threshold}} \sim 150\text{–}300 \text{ MeV}$).

Low mode approximation



For T<= 260MeV, we find a good saturation and consistency with direct inversion of Mobius domain-wall Dirac operator (direct MDW) but T=330 MeV, it is not good; we use direct MDW.

AS index = eta invariant [cf. Yamaguchi's talk]

$$\mathrm{Ind} D_{\mathrm{ov}} = \eta(\gamma_5(D_{ov} + m))$$

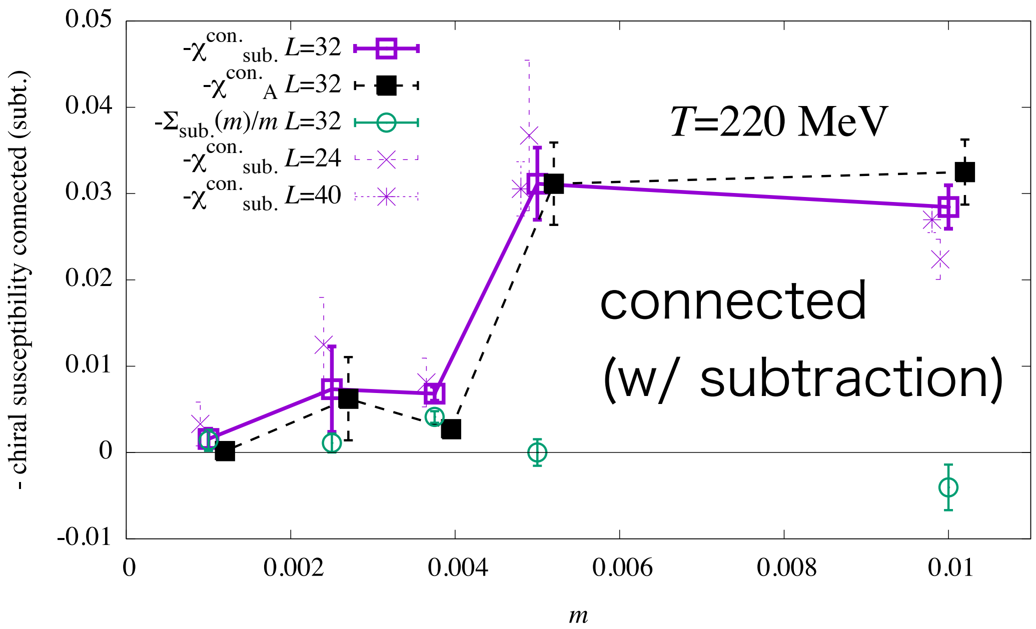
data_eigen — -zsh — 80×24

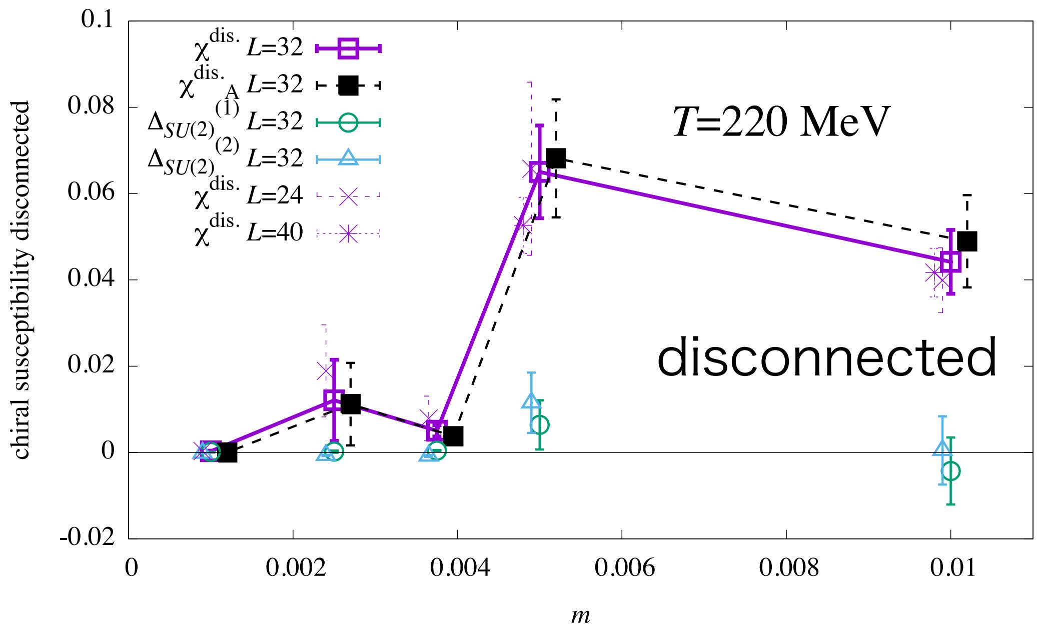
```
600 -0.005000025807555366 0.302609233991046 1.6064951657562172e-05
600 -0.0050002962669336 0.302609233991046 5.443187281609240 6e-05
600 0.005702514532165167 0.302609233991046 0.0027420539675677316
600 -0.005702652525898806 0.302609233991046 0.002742340941966567 5
600 0.008534378975613608 0.302609233991046 0.006916416732666278
600 -0.008534900200446262 0.302609233991046 0.0069170598938072675
600 0.011433627722244 2 0.302609233991046 0.010282533061119956
600 -0.01143389118468538 0.302609233991046 0.010282826023812 81
600 0.01482478910676644 0.302609233991046 0.013956333845655 7
600 -0.01482490275384444 0.302609233991046 0.01395645410647794 5
600 0.01927599474870225 0.302609233991046 0.018616461475477 65
600 -0.01927607127753628 0.302609233991046 0.018616540717500 83
```

data at beta=4.30, m=0.005, conf=600

Index=-2.

Nf=2 Result at T=220MeV





Open squares : data for chiral susceptibility

filled squares : axial U(1) anomaly part

crosses and stars : data on different Vs

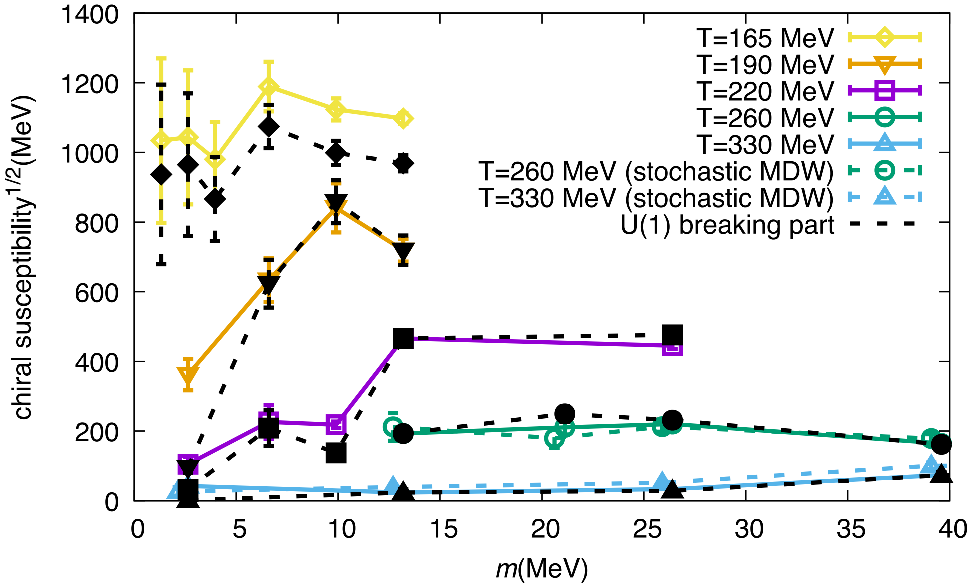
Axial U(1) anomaly dominates the signal:

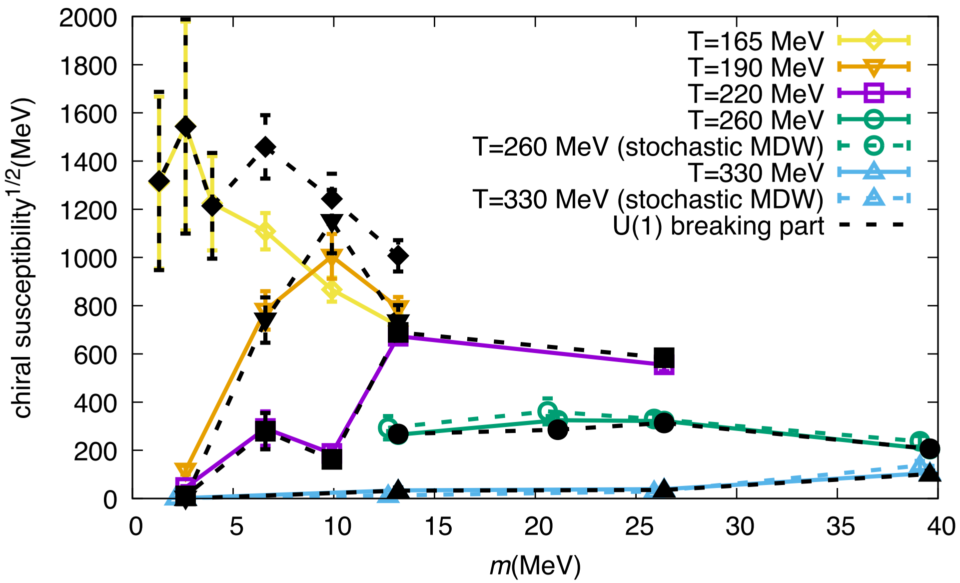
connected part ~ U(1) susceptibility

disconnected ~ topological susceptibilityx2/m^2.

Finite V effects look under control.

Nf=2 at different temperatures





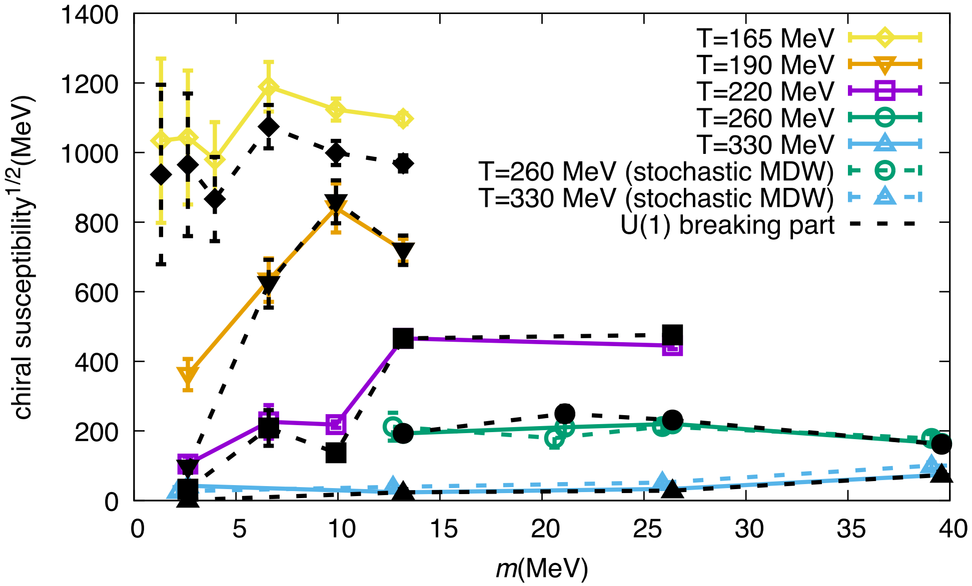
The dominance by axial U(1) anomaly is seen at 5 different Ts.

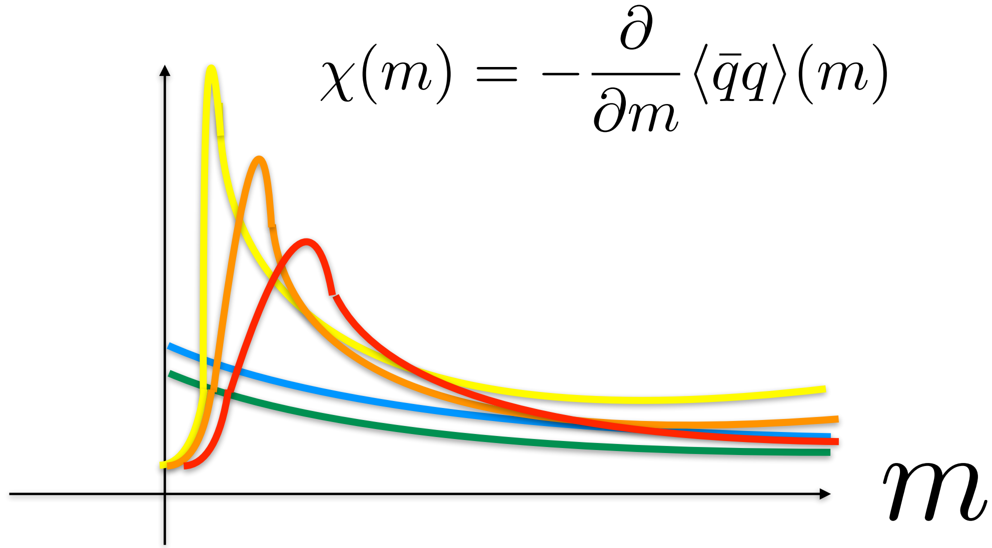
In fact, ~90% of the signal is from axial U(1) anomaly.

Also note that the chiral limit of anomaly part looks consistent with zero.

T=165 results are new.

Nf=2 at different temperatures





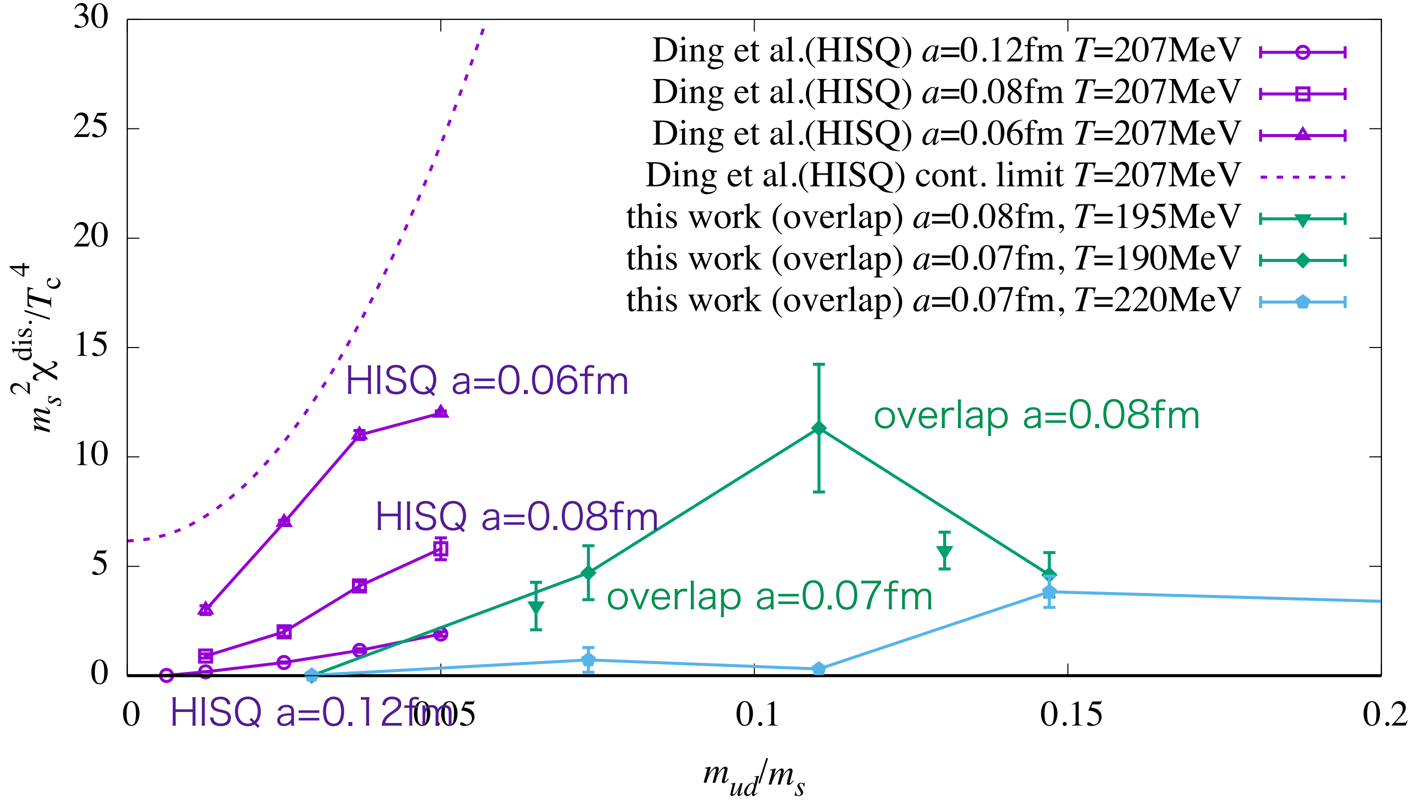
The dominance by axial U(1) anomaly is seen at 5 different Ts.

In fact, ~90% of the signal is from axial U(1) anomaly.

Also note that the chiral limit of anomaly part looks consistent with zero.

T=165MeV results are obtained from OFP/SQUID.

At different lattice spacings

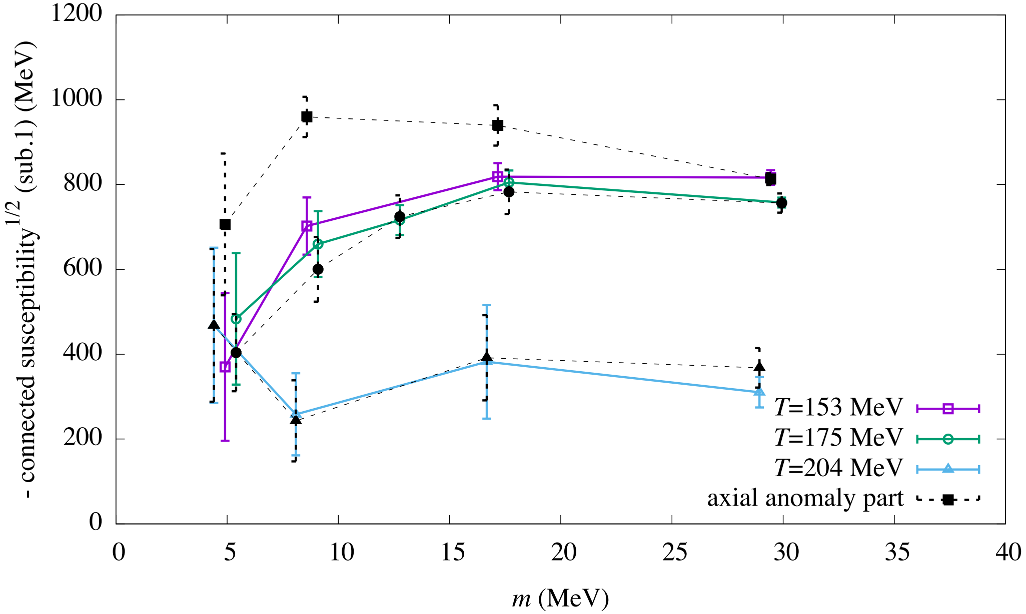


Our data at a=0.07fm and 0.08fm are consistent,

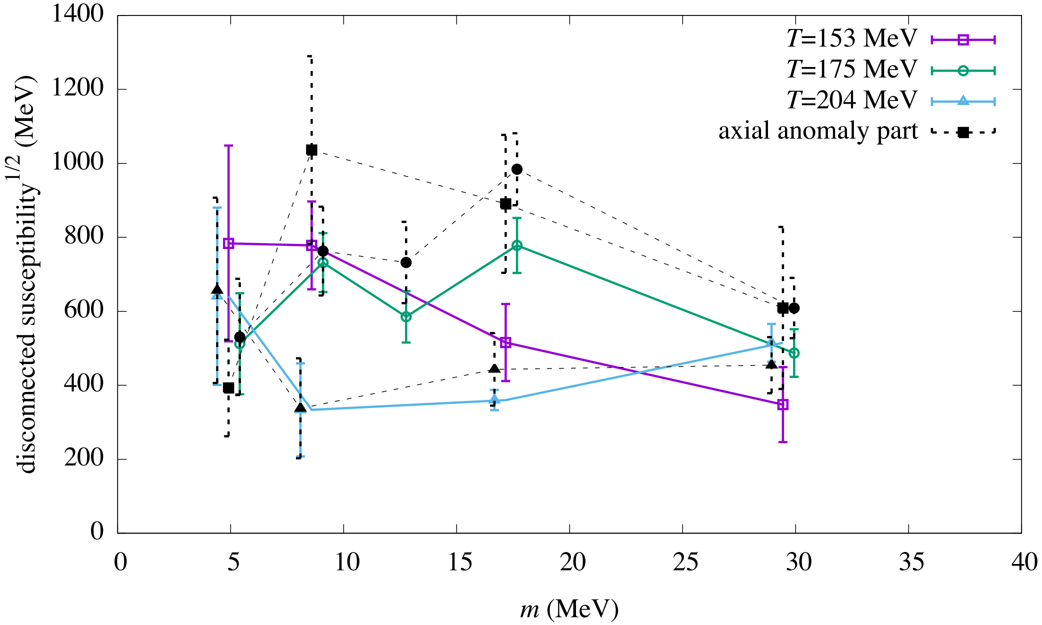
in contrast to HISQ results [Ding et al. 2020]

Nf=2+1 preliminary results

connected



disconnected



The dominance by axial U(1) anomaly is seen at 3 different Ts.

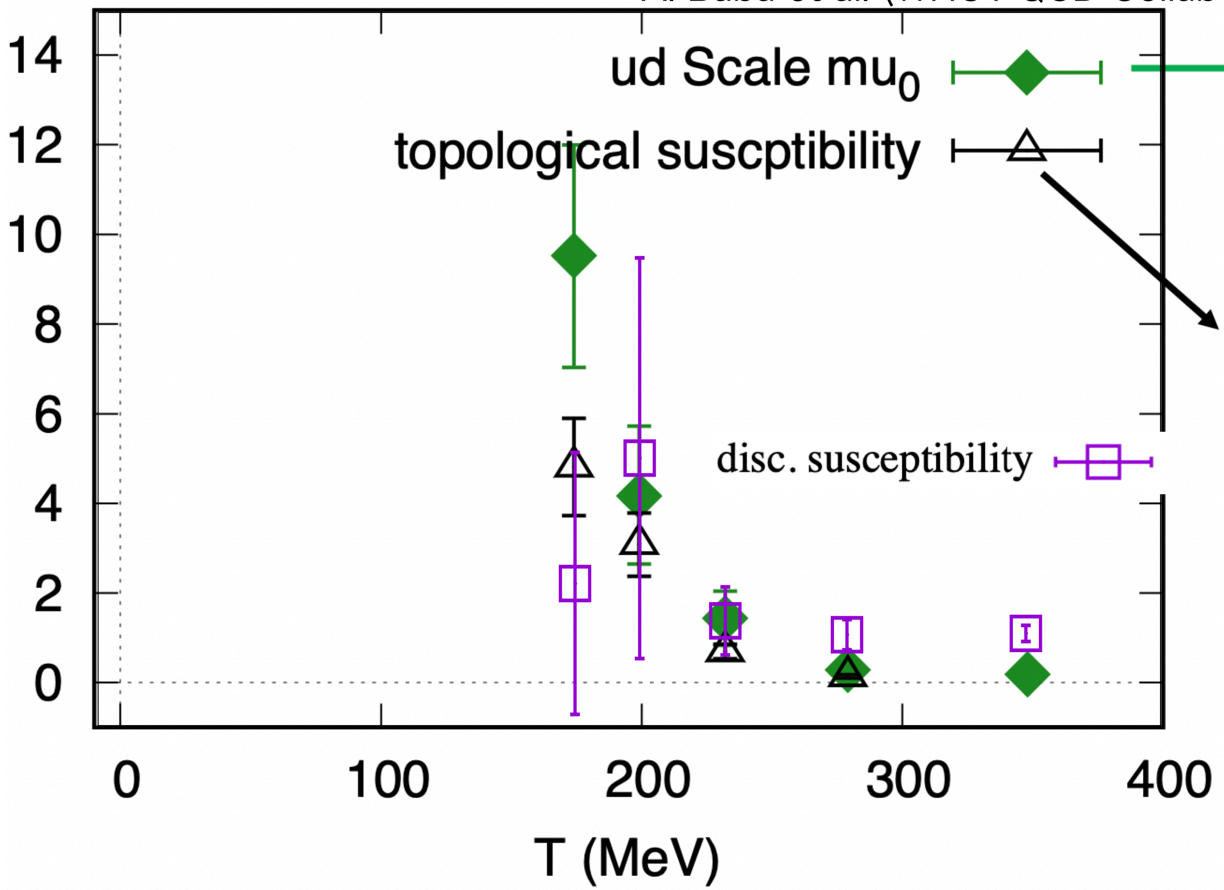
But the peaks look not significant.

The same in WHOT-QCD collaboration (?)

We thank A. Baba, S. Ejiri and K.Kanaya for providing us the data.

WHOT-QCD Collaboration, Phys. Rev. D 95, no.5, 054502 (2017)

A. Baba et al. (WHOT-QCD Collaboration), talk at 76th JPS annual meeting 2021



$\chi^{\pi-\delta}/T^2$

$\chi_t/(T^2 m_{ud}^2)$

$\chi^{\mathrm{dis.}}$ & $\chi_{\mathrm{top.}}/m^2$ are consistent.

Subtlety in the total contribution

$$\chi(m) = \chi^{\mathrm{con.}}(m) + \chi^{\mathrm{dis.}}(m)$$

$$\chi^{\mathrm{con.}}(m) = \underbrace{-\Delta_{U(1)}(m) + \frac{\langle |Q(A)| \rangle}{m^2 V}}_{} \underbrace{- \frac{-\langle \bar{q}q \rangle_{\mathrm{sub.}}(m)}{m}}_{\text{mixed}}$$

U(1)A breaking contribution

$$\chi^{\mathrm{dis.}}(m) = \overbrace{\frac{N_f}{m^2}\chi_{\mathrm{top.}}(m)}^{} \underbrace{+\Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)}_{}$$

SU(2)xSU(2) breaking

a large cancellation

O(1/V^1/2) effect

It is difficult to see what survives in the total contribution.

Contents

Summary

1. We simulate Nf=2 and 2+1 lattice QCD.
2. Chiral condensate/susceptibility are related to both SU(2)xSU(2) and $U(1)_A$.
3. In the spectral decomposition of the Dirac operator with exact chiral symmetry, we can separate the purely U(1) anomaly effect.
4. Connected/disconnected susceptibilities are dominated by U(1) breaking at T>=Tc.

    Connected part ~ axial U(1) susceptibility.

    Disconnected part ~ top. susceptibility x $2/m^2$

Axial U(1) anomaly may play more important role in the QCD phase diagram than expected.

Take-home message

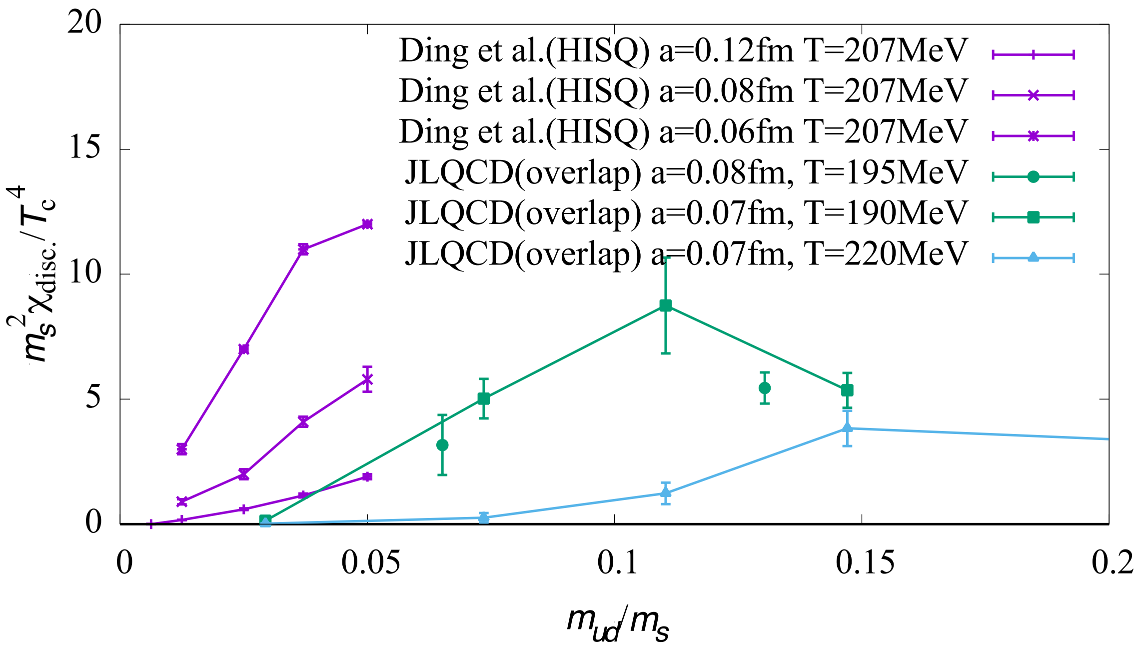
$$\frac{\partial}{\partial m}\langle\bar{\psi}\psi\rangle$$

is probing not only $SU(2)_L xSU(2)_R$ but also $U(1)_A$ breaking/restoration.

At T>=165MeV in Nf=2 QCD (>=153MeV in Nf=2+1),

U(1)A anomaly dominates the signal of connected/disconnected susceptibilities.

Comparison between Ding et al. 2010.14836 and JLQCD



1. Sharp decrease towards the chiral limit is similar.

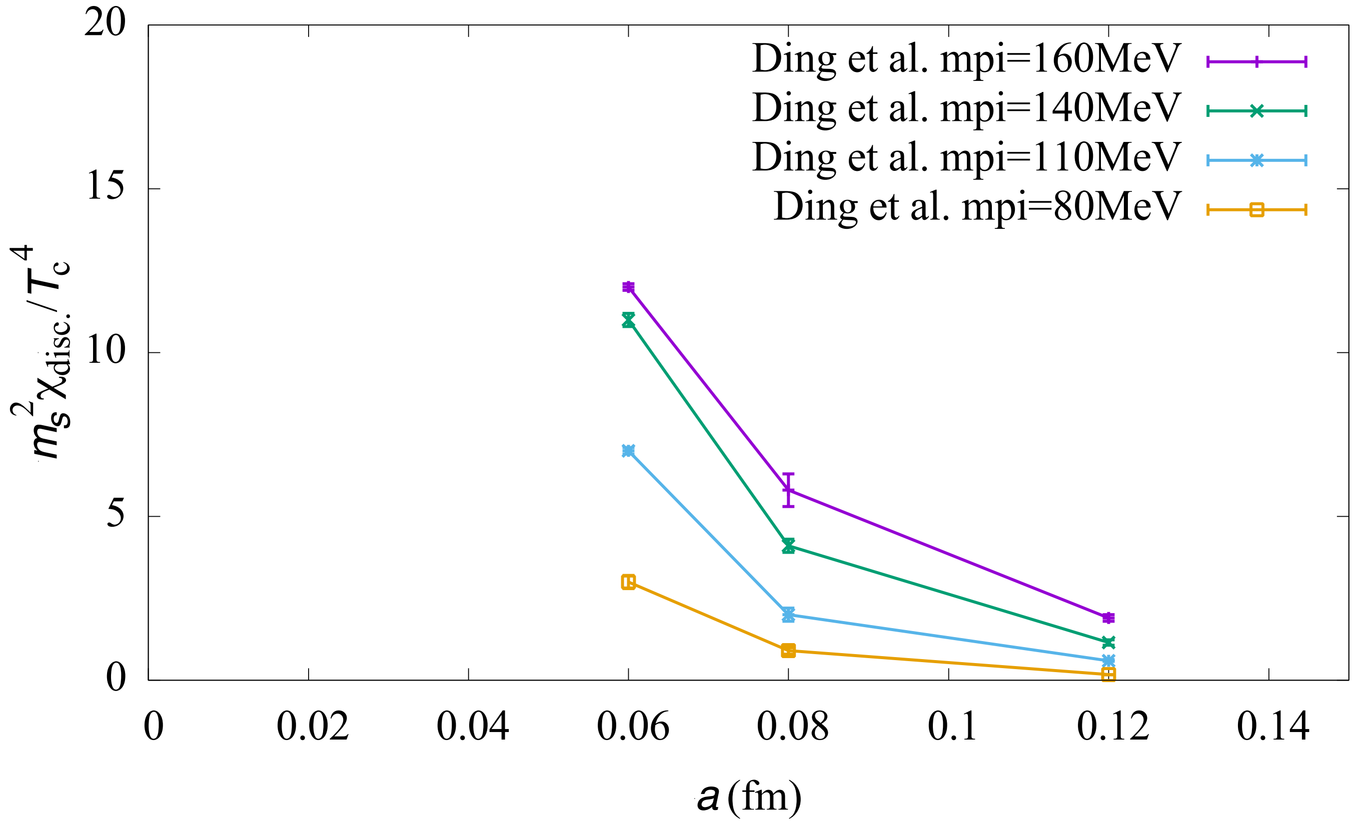
2. Ding et al. shows a sizable cut-off effect, while JLQCD's a=0.07fm (T=190MeV) and 0.08fm(T=195MeV) data are consistent.

Cutoff dependence of Ding et al. 2010.14836

We see a power-like increase.

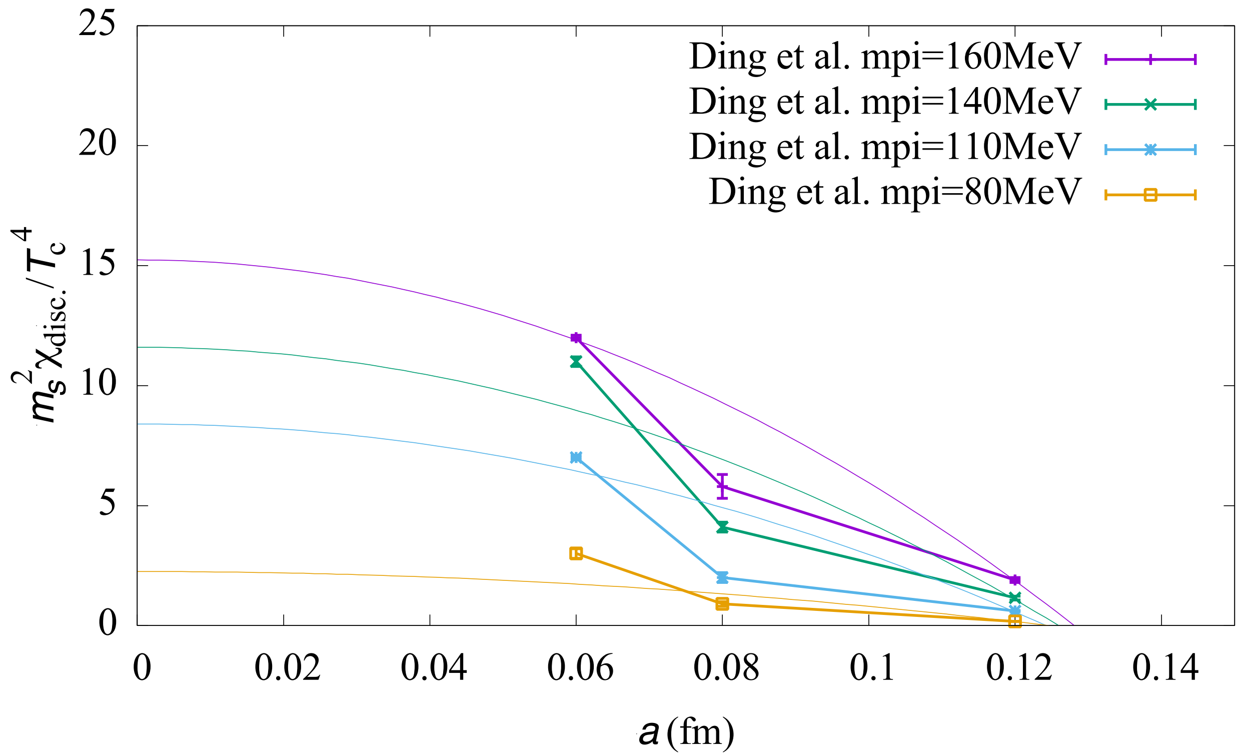
But they managed to obtain "continuum limits".

How?



2-parameter fit clearly fails.

χ2/d.o.f. > 100.



Global fit "magic".

More free parameters

(from 2 to 6.)

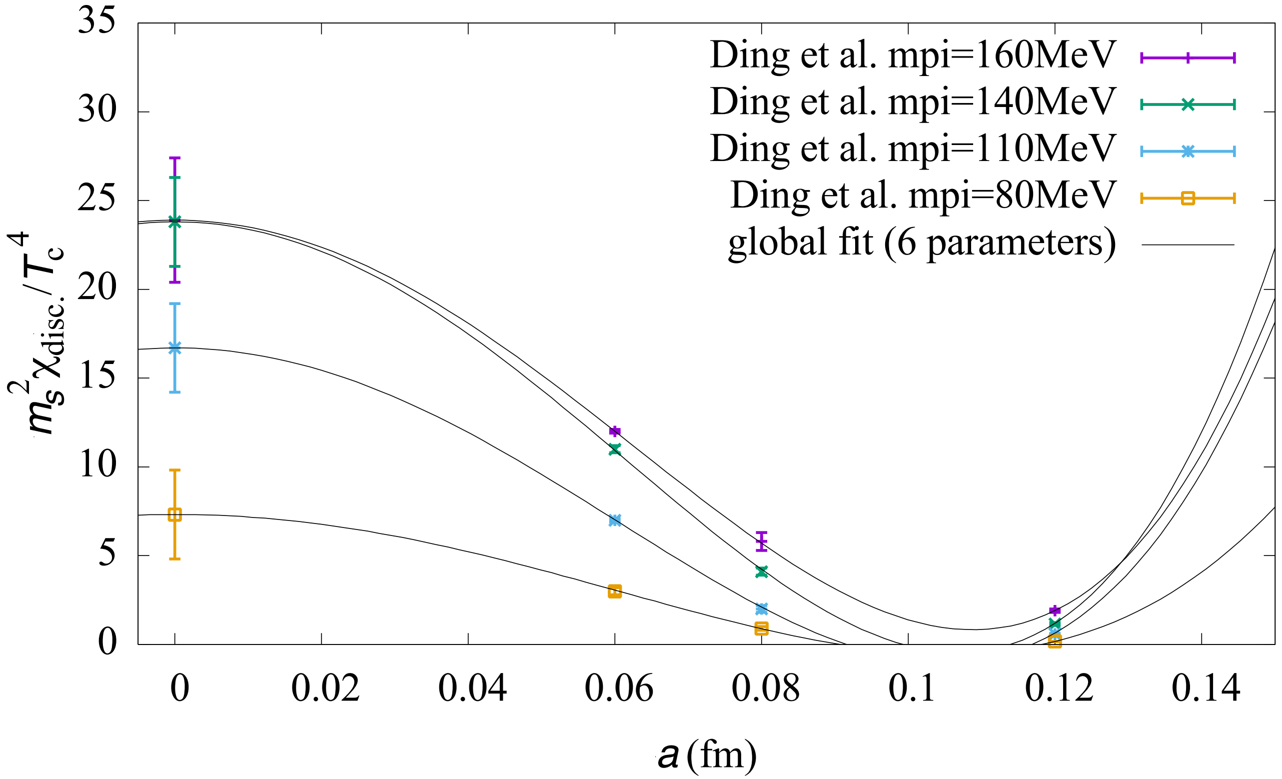
But the fit is unstable :

sensitive to the

$O(a^4)$ terms.

$|$negative $O(a^2)$ term$|$

~ $|$positive $O(a^4)$ term$|$
already at a~0.07fm.



Summary of my comments

1. Sharp decrease towards the chiral limit is similar to JLQCD's.
2. But data of Ding et al. 2010.14836 show a power-like increase towards the continuum limit (due to some lattice artifact?).
3. By a global fit "magic", Ding et al. obtained finite values (which are much bigger than raw values).
4. But the 6-parameter fit is unstable : sensitive to $O(a^4)$ terms.

John von Neumann said

"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk."