1. Hairy black holes and solitons in global AdS_5

arXiv: 1112.4447

2. Scalar field condensation instability of AdS BHs

arXiv: 1007.3745

3. Black holes with only one Killing field

arXiv: 1105.4167

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energie atomique • energies alternatives

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Hairy black holes and solitons in global AdS_5

arXiv: 1112.4447

OD, Pau Figueras, Shiraz Minwalla,
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- AdS Abelian Higgs model: AdS Einstein Maxwell gravity interacting with a charged massless scalar field

$$S = \frac{1}{8\pi G_5}\int d^5x\sqrt{-g}\left[\frac{1}{2}\left(\mathcal{R}[g]+12\right) - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - |D_\mu\phi|^2\right]$$

$$D_\mu = \nabla_\mu - ieA_\mu$$

$$\ell \equiv 1$$

- Field content: gravity, Maxwell field and a charged complex scalar.

- Static and spherically symmetric solutions: expect a three parameter family of solutions parametrized by {M,Q,e}.

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{g(r)} + r^2\mathrm{d}\Omega_3^2, \qquad \mathcal{A} = \mathcal{A}_t(r)\mathrm{d}t, \qquad \phi = \phi(r).$$

- AdS Reissner-Nordstrom BH: $E,Q = E,Q\ (R,\mu)$

$$\phi(r) = 0,\ f = g$$

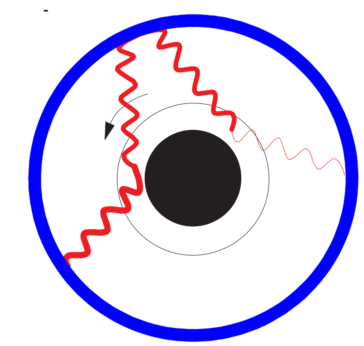
$$f(r) = \left(\frac{r^2}{\ell^2} - \frac{R^2}{\ell^2}\right)\left(1 + \frac{R^2+\ell^2}{r^2} - \frac{2}{3}\frac{R^2\ell^2\mu^2}{r^4}\right), \quad \text{and} \quad \mathcal{A}_t = \mu\left(1 - \frac{R^2}{r^2}\right)$$

Regular extremal limit, with near horizon geometry $AdS_2 \times S^3$, with $S_{ext} \neq 0$:

$0 \le \mu \le \mu_{ext}$ with $$\mu_{ext} = \sqrt{\frac{3}{2}}\sqrt{1+\frac{2R^2}{\ell^2}}$$

• AdS Reissner-Nordstrom BH has two instabilities:

1) Superradiant Instability:



If a wave $e^{-i\omega t}$ scatters off a charged black hole with $\mathbf{0 < \omega \leq e\,\mu}$,

it returns with a larger amplitude - superradiant scattering

In AdS, the outgoing wave reflects-off infinity, and the process repeats itself.

Multiple Superradiance / Reflection leads to instability.

Can we estimate the instability onset?

The scalar modes that can propagate in the RN-AdS background, in the limit of very small R,

are effectively the normal modes of global AdS: $\boldsymbol{\omega L_5 = 4 + 2p}$. Lowest mode has p = 0.

On the other hand, small extremal black holes require $\mu \leq \sqrt{\frac{3}{2}}$

Assuming that the instability first appears at extremality, we get superradiance condition:

$$4/L_5 < e\sqrt{\frac{3}{2}}$$

→ **Arbitrarily small extremal black holes suffer from the superradiant instability when** $e^2 > \frac{32}{3}$

2) Near-Horizon scalar condensation instability:

• Consider charged massive scalar field: $\square\phi - \mu^2\phi = 0$

Normalizable modes ⟶ scalar field must obey the Breitenlohner-Freedman (BF) bound:

$$\mu^2 \geq \mu^2\big|_{BF} \equiv -\frac{(d-1)^2}{4\ell^2}$$

• Take ***any*** extreme, asymptotically AdS_d BH whose near-horizon geometry contains an AdS_2 factor w/ radius L_{AdS_2}:

the BF bound associated to this AdS_2, $\mu^2\,|_{NH\,BF} = -1/4\,L^2_{AdS_2}$ is different from the BF of AdS_d.

In particular if: $$\mu^2\big|_{NH\,BF} > \mu^2 \geq \mu^2\big|_{BF}$$

then the asymptotic AdS_d space will be stable

but the near-horizon geometry is unstable.

⟶ This suggests that the full BH

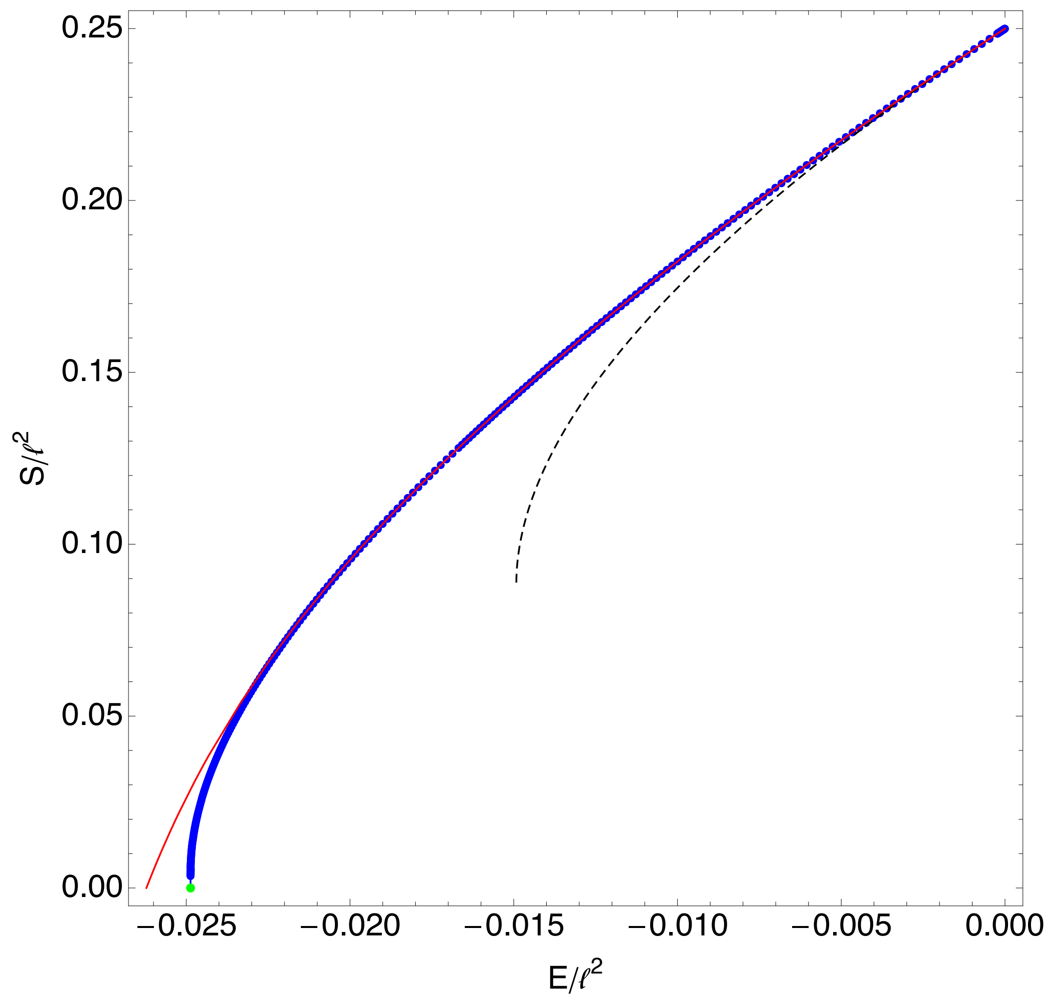
will be unstable against scalar condensation

⟶ Confirmed in

arXiv:1007.3745

A scalar field condensation instability of rotating AdS BHs

OD, Ricardo Monteiro, Harvey Reall, Jorge Santos



2) Near-Horizon scalar condensation instability:

- Return to the particular RN-AdS case where we start with massless scalar.
  linearized equation for charged ϕ on NH RN-AdS reduces to eq for a massive scalar with effective mass:

$$m_s^2 l_{AdS_2}^2 = -\frac{3e^2 R^2}{8} \frac{1+2R^2}{(1+3R^2)^2}$$

- AdS_2 is unstable whenever it violates the 2d BF bound: $$m_s^2 l_{AdS_2}^2 < -\frac{1}{4}$$

$\rightarrow$ extremal RN-AdS is unstable whenever $$e^2 \geq \frac{2(1+3R^2)^2}{3R^2(1+2R^2)}$$

- The RHS is a monotonically decreasing function of R. At large R, this reduces to

$$e^2 \geq \frac{2(1+3R^2)^2}{3R^2(1+2R^2)} \geq 3 + \mathcal{O}(1/R^2)$$

It follows that large extremal RN-AdS BHs are unstable when $e^2 > 3$.

The endpoint of the instability involves a condensate of the scalar field.

By the Hawking area increase theorem it also has a horizon.

Consequently, the endpoint of this instability is a hairy black hole.

RN-AdS BHs (apparently) stable for $e^2 < 3$

Very large extremal RN-AdS BHs are unstable when $e^2 > 3$.

Arbitrarily small extremal black holes suffer from the superradiant instability when $e^2 > 32/3$

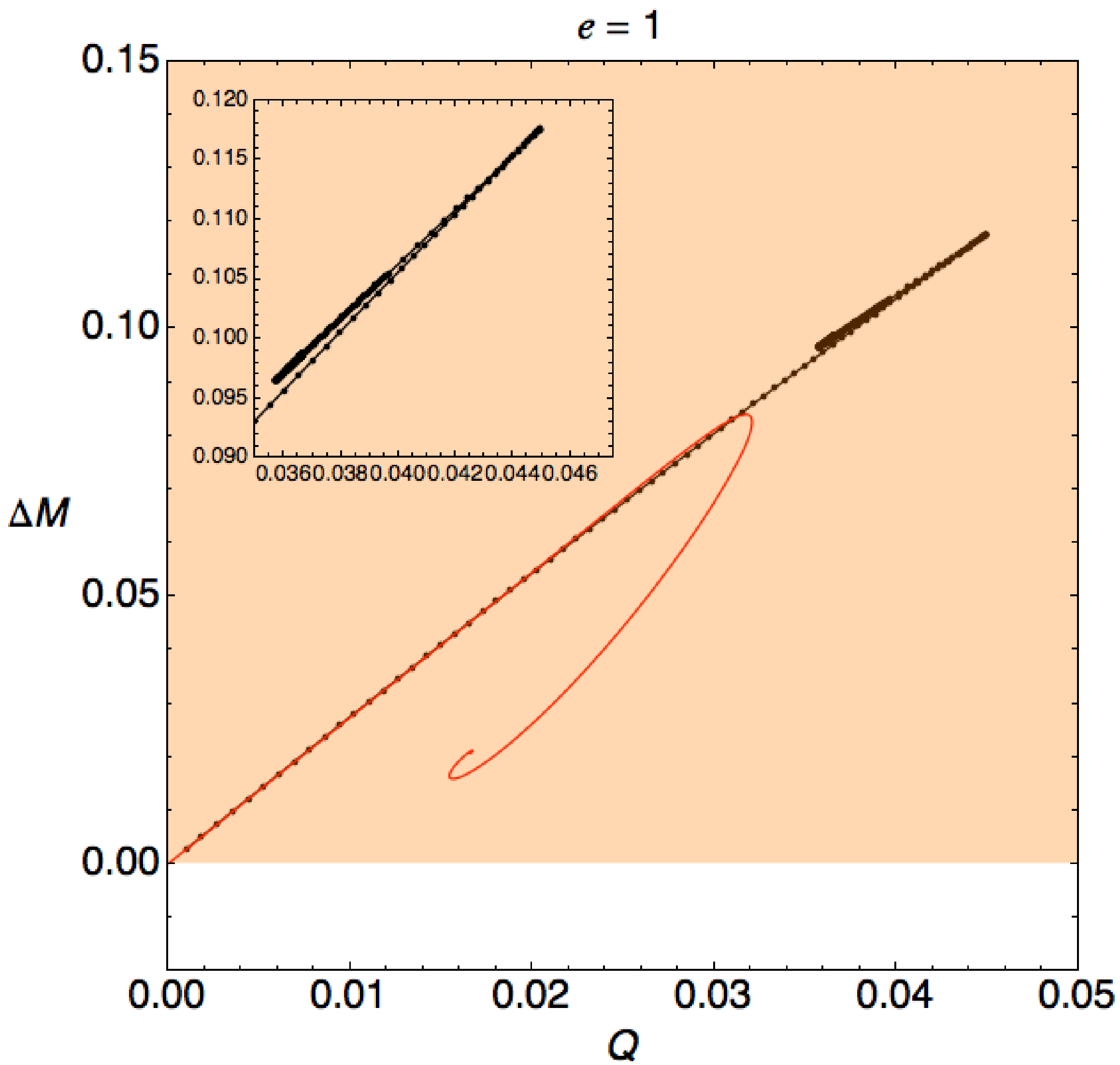
→ This suggests we should look into 3 regimes:

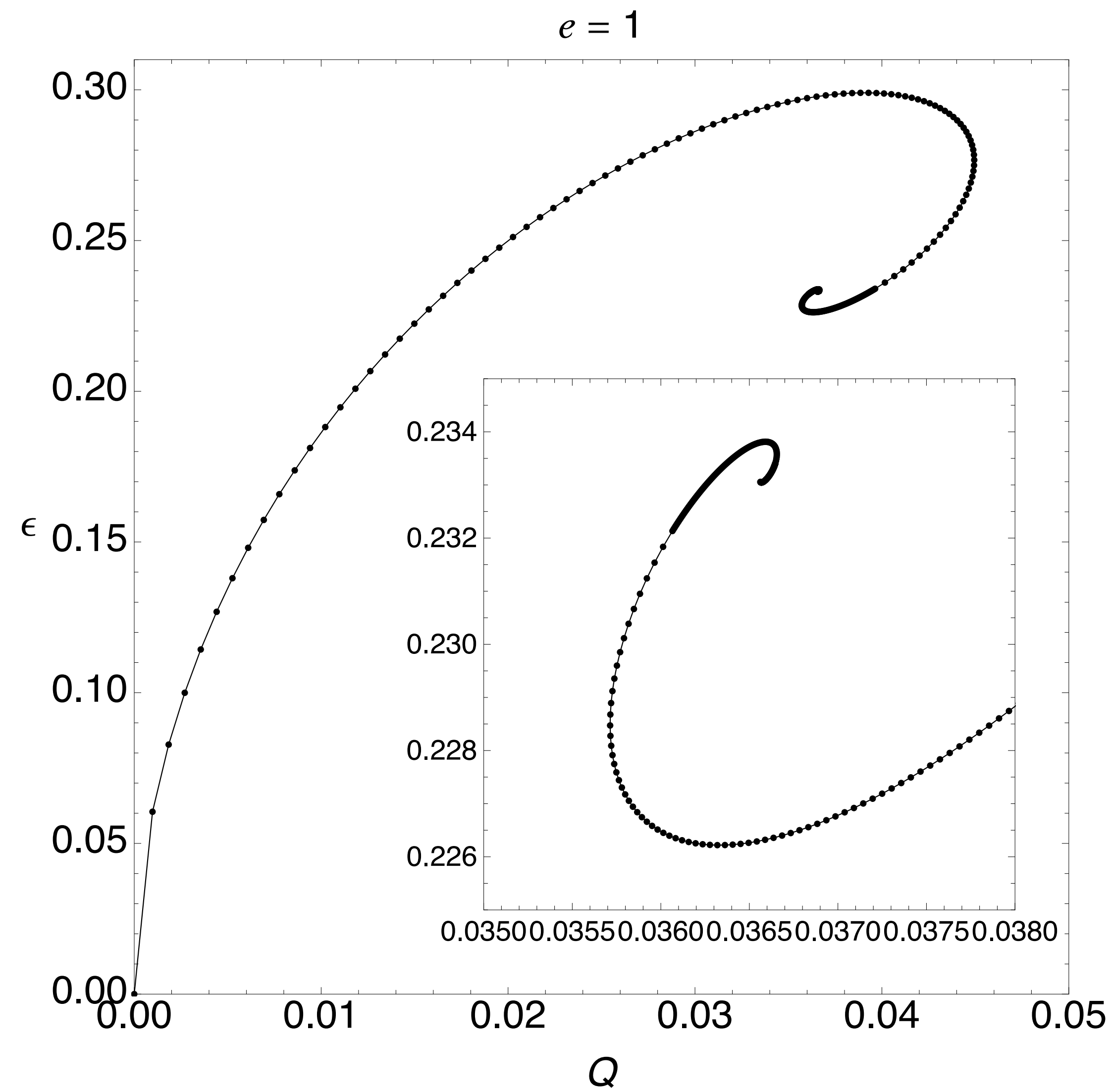
$$e^2 < 3$$

$$3 < e^2 < \frac{32}{3}$$

$$e^2 > \frac{32}{3}$$

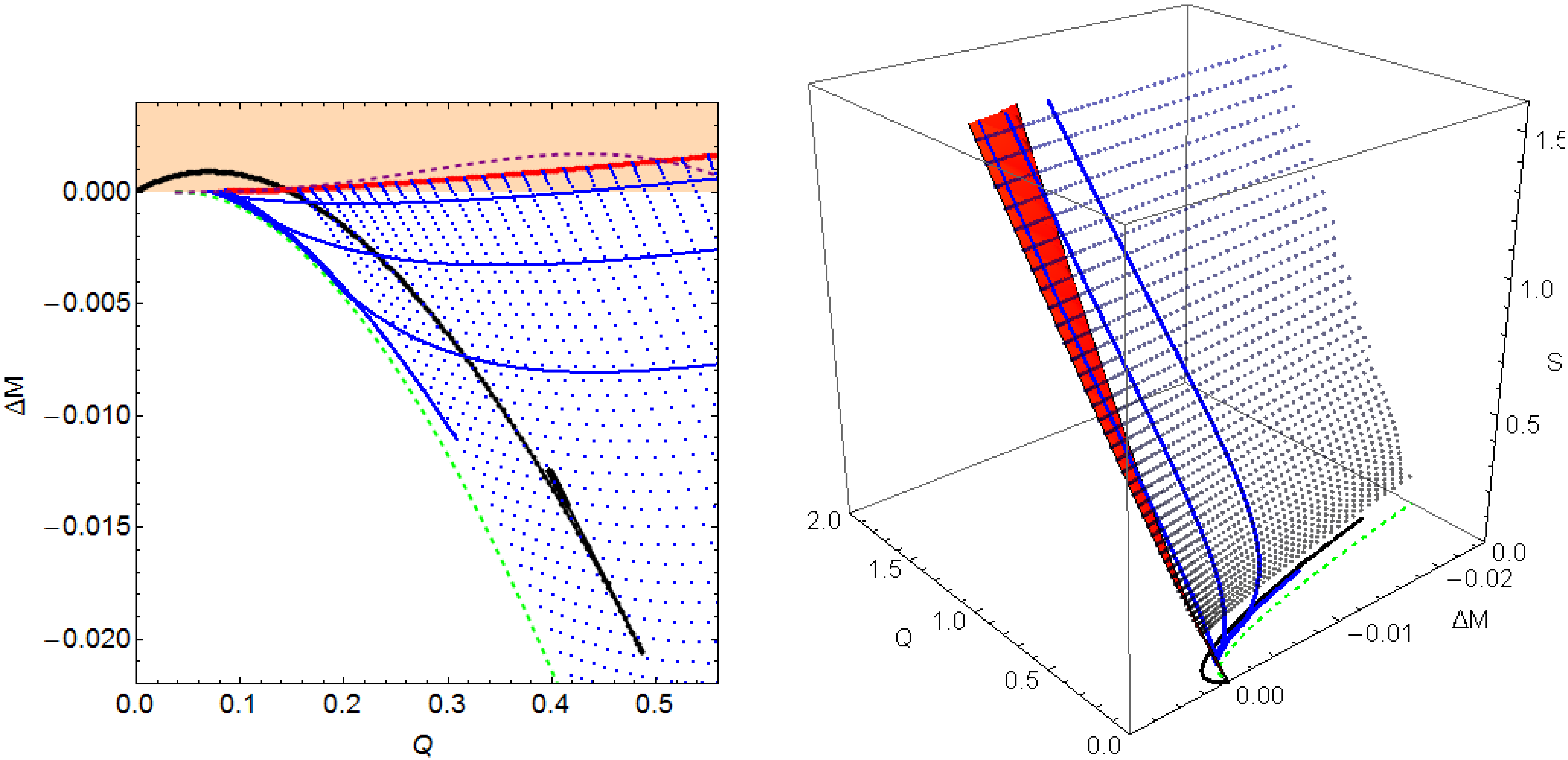
$e^2 < 3$





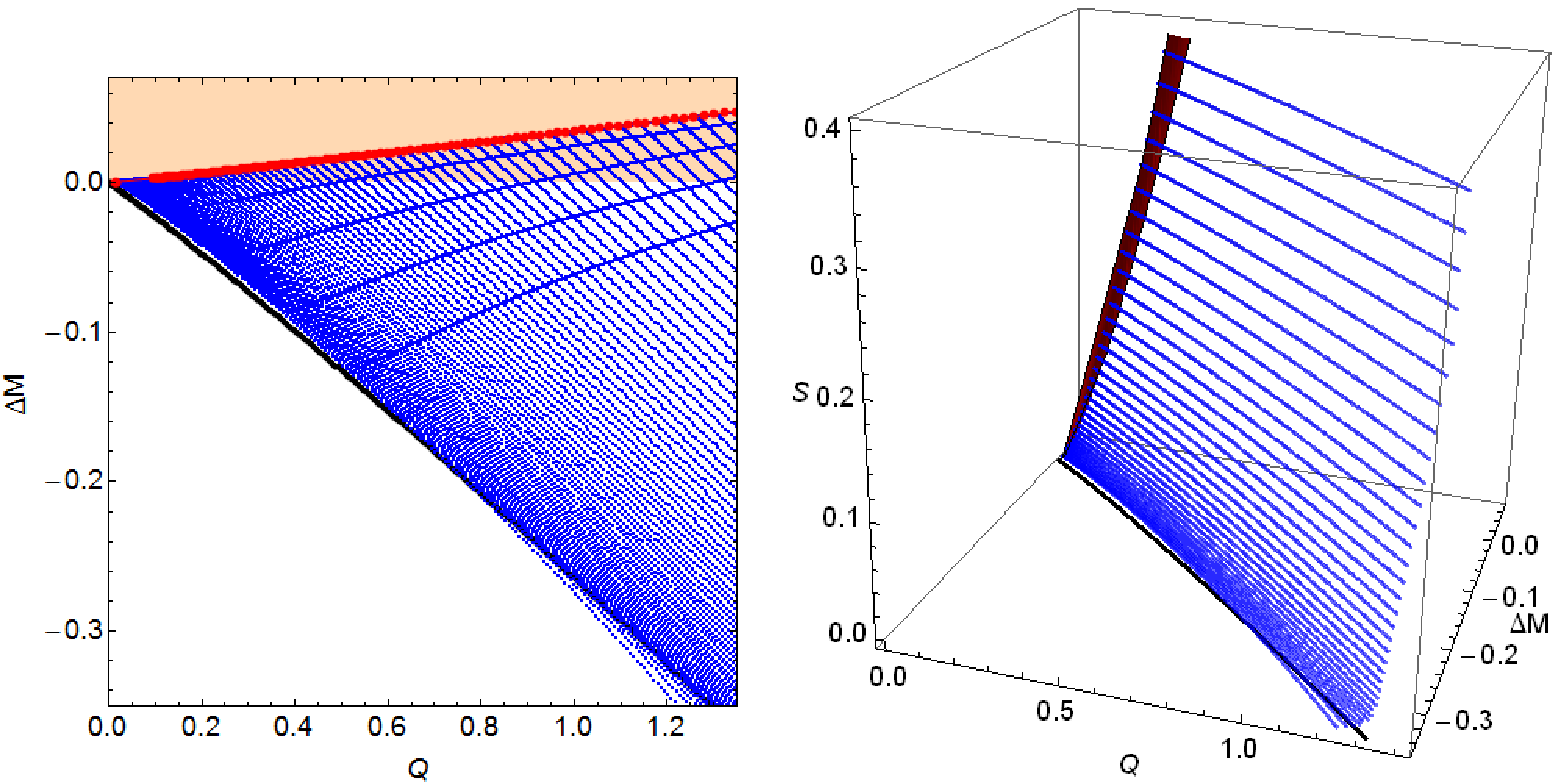
$\Delta M = M - M_{ext}$, where M_{ext} is the mass of the extremal RN AdS BH with the same charge Q

$$3 < e^2 < \frac{32}{3}$$



0.000
−0.005
−0.010
−0.015
−0.020
ΔM
0.0
0.1
0.2
0.3
0.4
0.5
Q
1.5
1.0
0.5
0.0
S
2.0
1.5
1.0
0.5
0.0
Q
0.00
−0.01
−0.02
ΔM

$$e^2 > \frac{32}{3}$$



ΔM
0.0
−0.1
−0.2
−0.3
Q
0.0
0.2
0.4
0.6
0.8
1.0
1.2
S
0.4
0.3
0.2
0.1
0.0
0.0
0.5
1.0
Q
0.0
−0.1
ΔM
−0.2
−0.3

CONCLUSION:

- First non-linear construction of Hairy BHs that bifurcate from original unstable BH family at the superradiant / NH scalar condensation merger curve
- Complex but interesting BH / soliton phase diagram structure ... Universal ?
- Study Time evolution of these instabilities to find their endpoint.

Black holes with only one Killing field

arXiv:1105.4167

OD, Gary Horowitz, Jorge Santos

• d=5 AdS Einstein gravity minimally coupled to 2 complex massless scalar fields Π :

$$S = \frac{1}{16\pi}\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left[R + \frac{12}{\ell^2} - 2\left|\nabla\vec{\Pi}\right|^2\right]$$

$$G_{ab} - 6\ell^{-2} g_{ab} = T_{ab}$$

$$\nabla^2\vec{\Pi} = 0.$$

• Look for boson star and BH solutions of this theory whose gravitational and scalar fields obey the ansatz:

$$\mathrm{d}s^2 = -f\,g\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f} + r^2\left[h\left(\mathrm{d}\psi + \frac{\cos\theta}{2}\mathrm{d}\phi - \Omega\mathrm{d}t\right)^2 + \frac{1}{4}\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)\right]$$

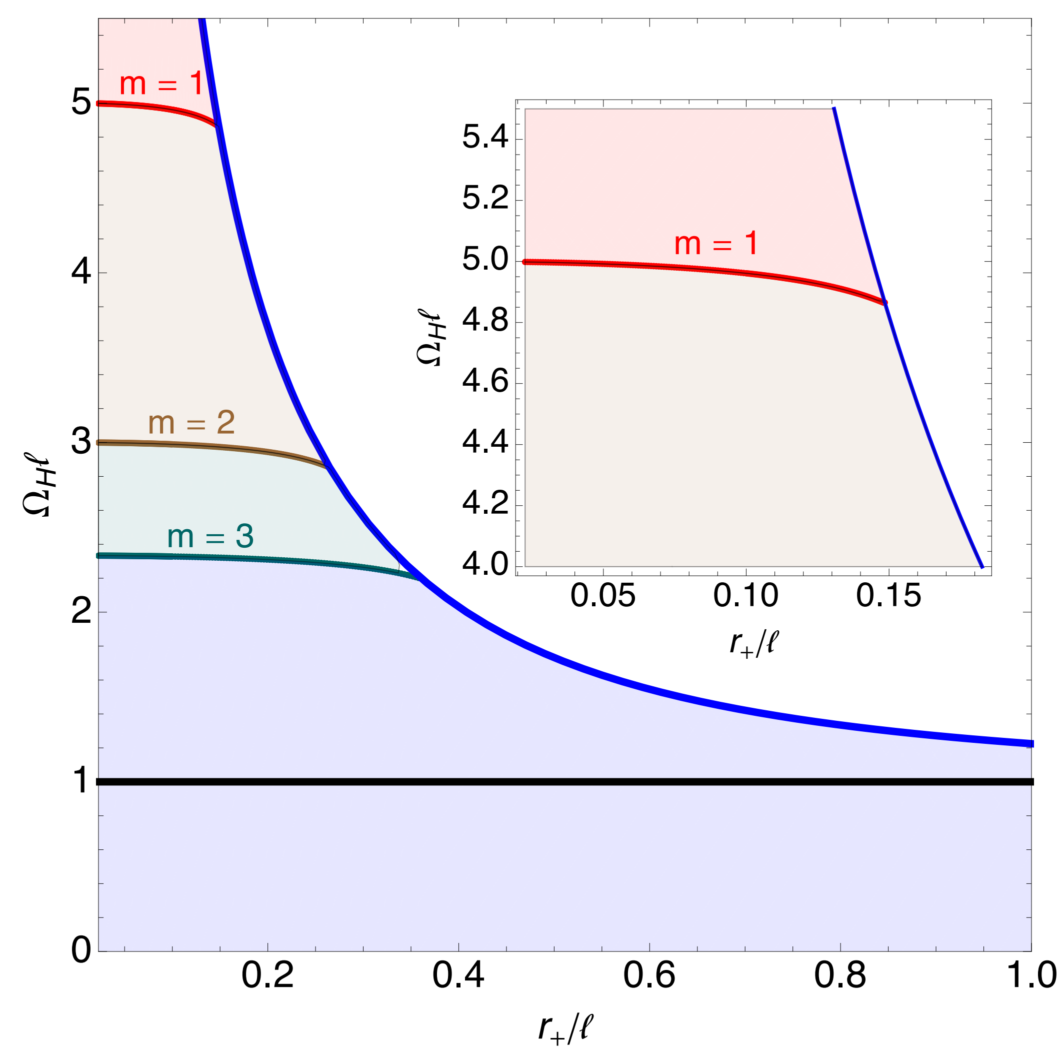
$$\vec{\Pi} = \Pi\, e^{-i\,\omega\,t + i\,\psi}\begin{bmatrix}\sin\left(\frac{\theta}{2}\right)\,e^{-i\frac{\phi}{2}}\\ \cos\left(\frac{\theta}{2}\right)\,e^{i\frac{\phi}{2}}\end{bmatrix}.$$

• **MP-AdS with equal J is case Π=0, $g = 1/h$.**

Unstable to m-superradiant modes above m-line.

Blue curve Extremal MP

• Boson stars are smooth horizonless solutions
(with harmonic time dependence)



• Symmetry of the solution must leave both the metric and matter fields invariant:

This metric has 5 linearly independent Killing vector fields, namely ∂_t, ∂_ψ and the three rotations of S^2.

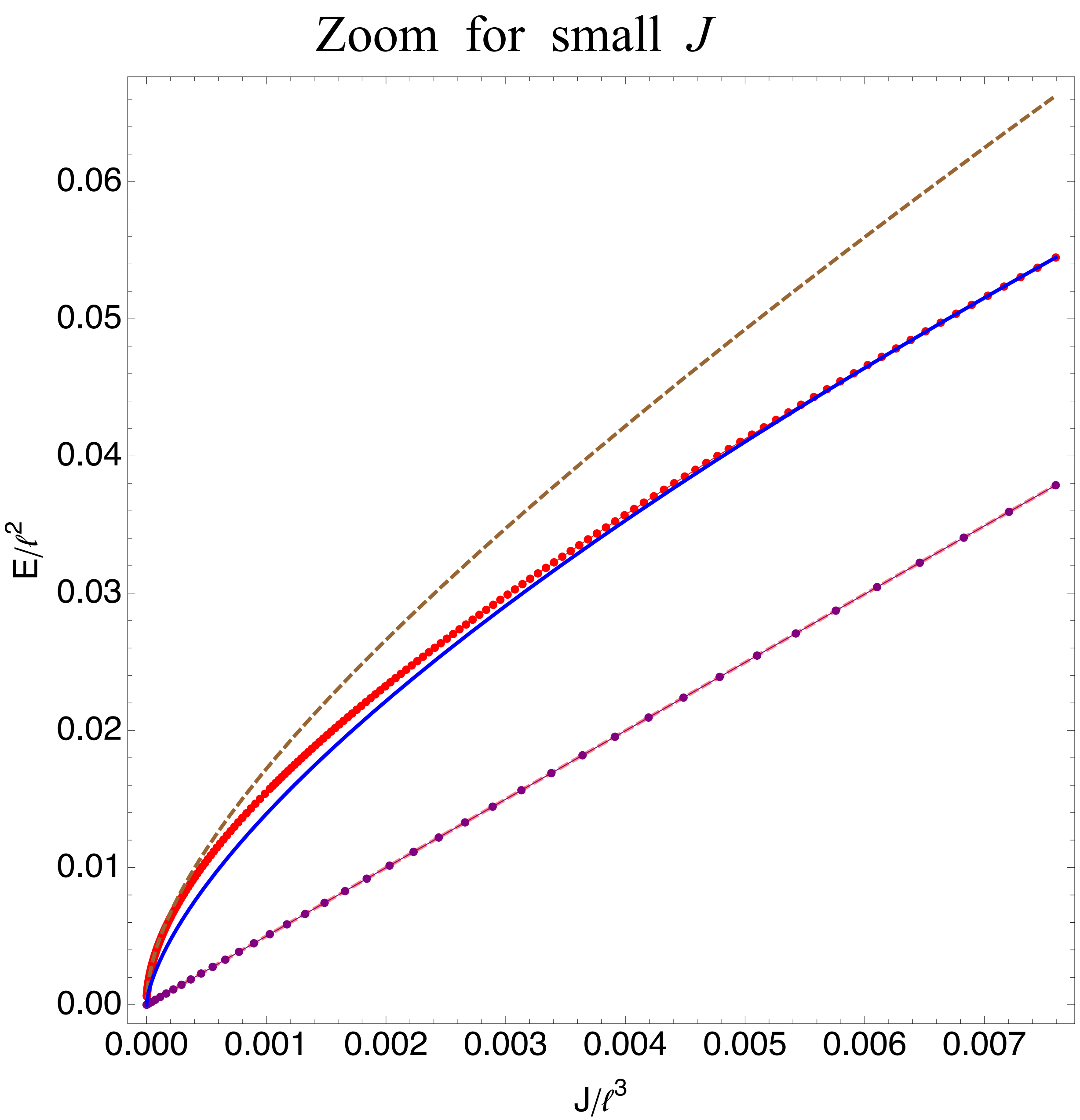
However, the only linear combination which leaves $\vec{\Pi}$ invariant is: $K = \partial_t + \omega \partial_\psi$

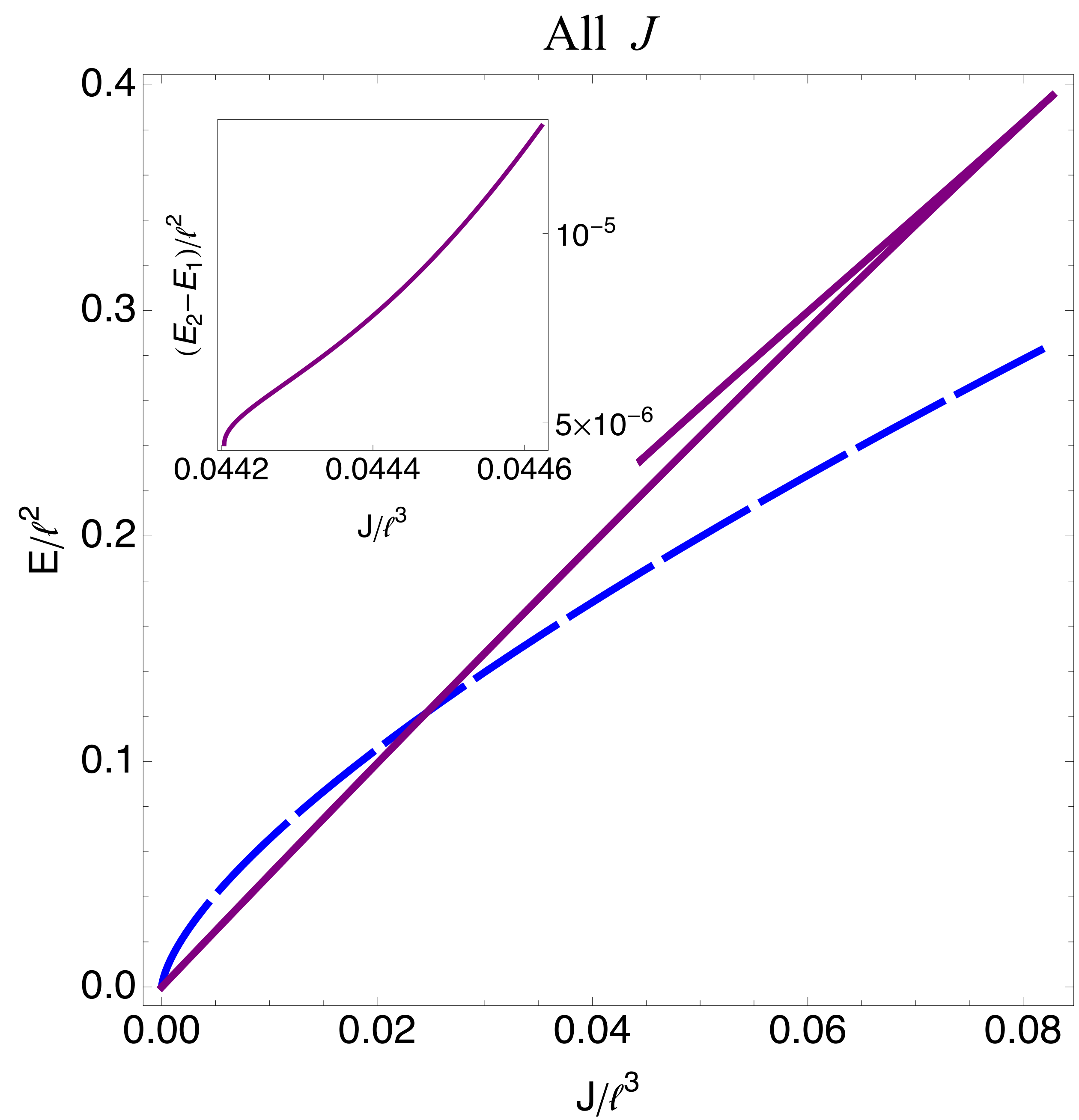
$$\mathcal{L}_K g = 0 \quad \text{and} \quad \mathcal{L}_K \Pi^\alpha = 0 \quad \text{for } \alpha = 1, 2$$

• Not usual to have solutions where matter fields have much less symmetry than the metric !

Doublet scalar field ansatz is special; it conspires in such a way that T_{ab} only depends on radial coord:

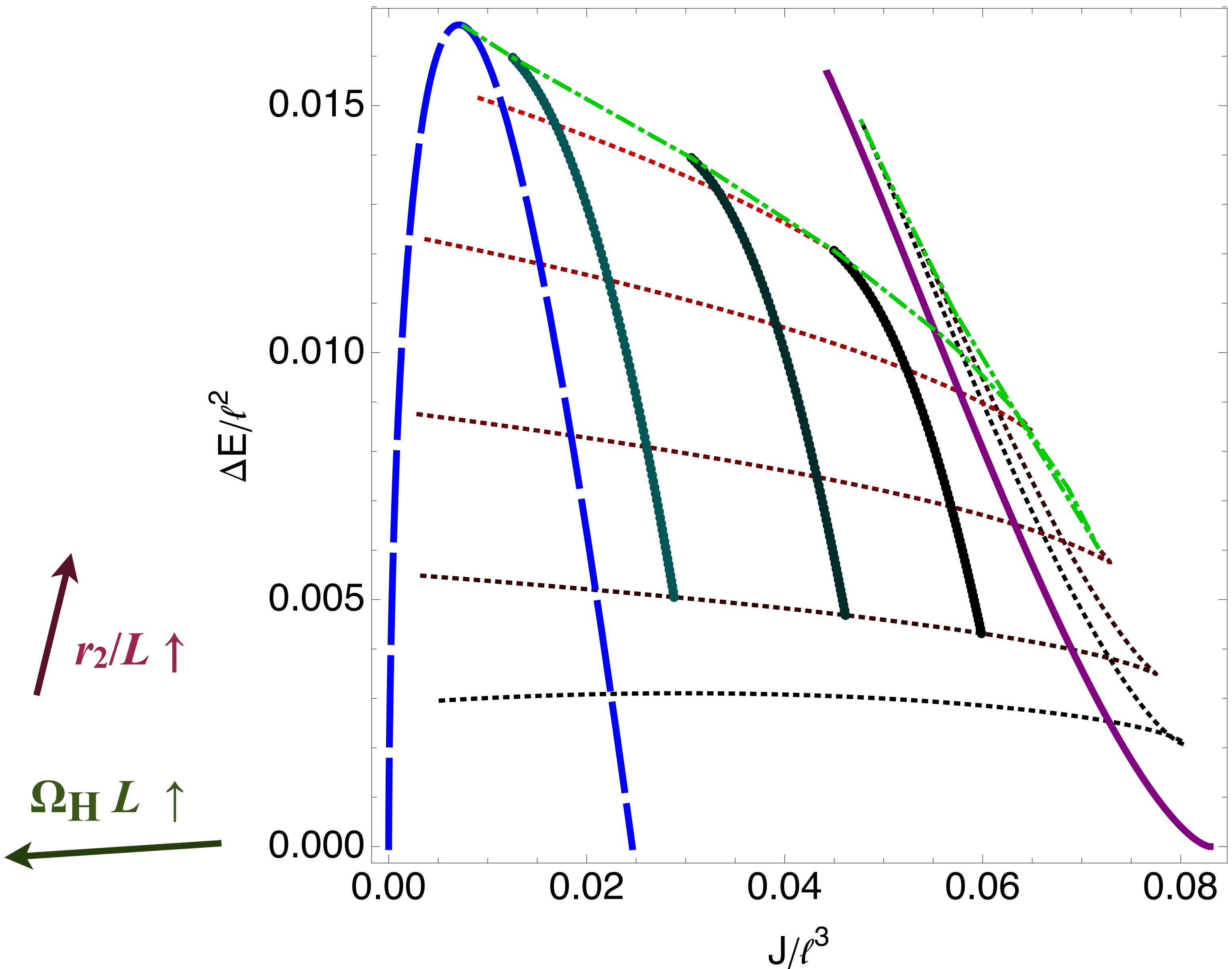
$$T_{ab} = \left(\partial_a \vec{\Pi}^* \partial_b \vec{\Pi} + \partial_a \vec{\Pi} \partial_b \vec{\Pi}^*\right) - g_{\rm ab} \left(\partial_c \vec{\Pi} \partial^c \vec{\Pi}^*\right)$$





- **Analytical estimate for the merger line,**
  Exact merger line,
- **Extremal line of the MP-AdS BHs,**
- **Analytical estimate for the bosons stars**
  Exact data for the bosons stars (Dots).

- **Energy of the boson star**
  as a function of its angular momentum
- **Extremal line of the MP-AdS BH**



- **Dashed blue curve: extremal MP-AdS**
  (regular MP-AdS only exist above it).
- **Solid purple curve: boson stars.**
- **Dotted red curves:**
  hairy BH lines of constant r_2/L
  ($r_1 \nearrow$ along const r_2 line)
- **Green "vertical" solid curves:**
  hairy BH lines of constant $\Omega_H L$.
- **Dotted-dashed green curve:**
  the best approximation to
  the **singular extremal hairy BHs.**

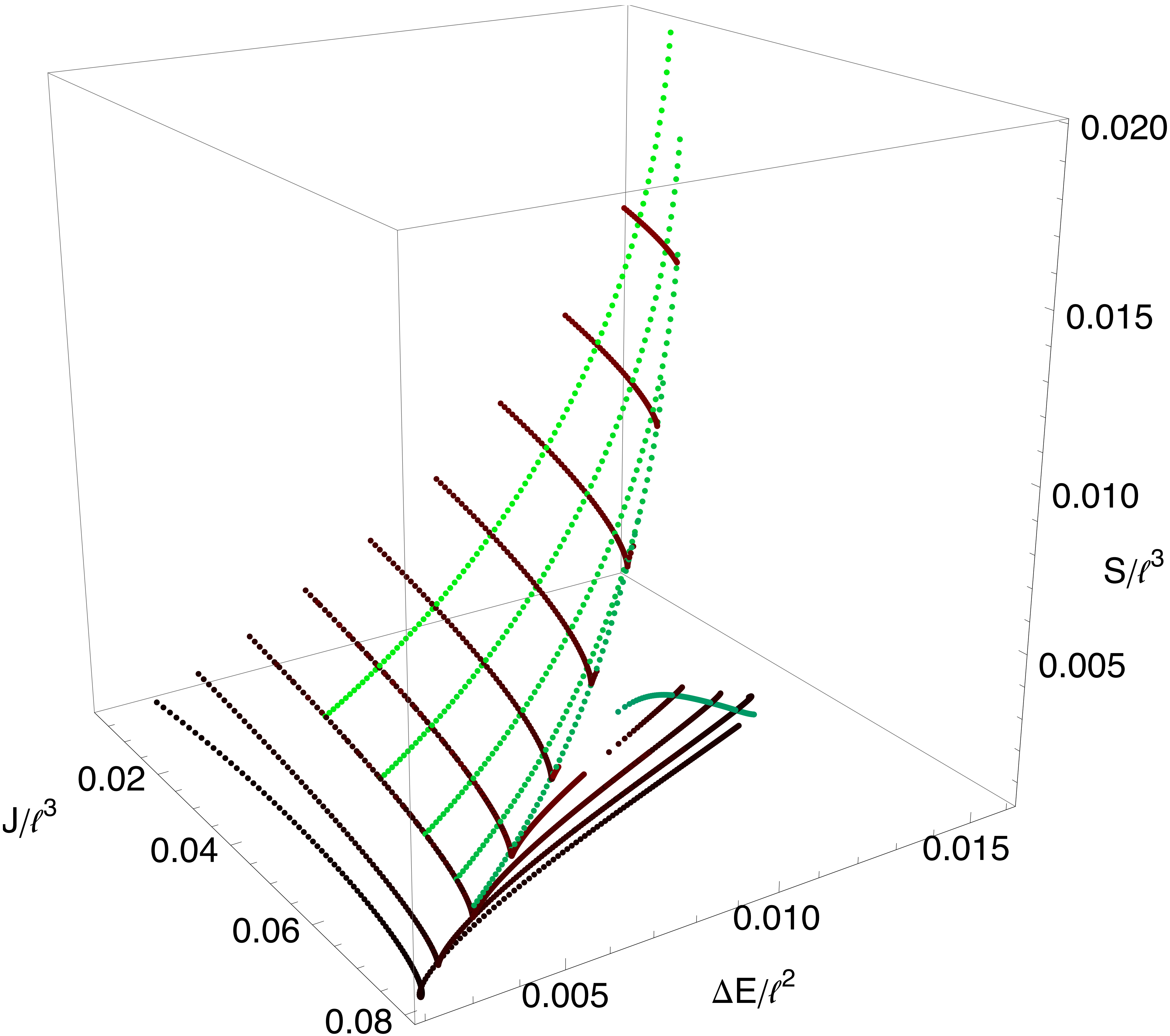
ΔE is the energy relative to the lower branch of boson stars

- Close to the merger, the $S_{MP} < S_{\text{hairy BH}}$. $S_{MP} = S_{\text{hairy BH}}$ at merger $\rightarrow$ 2nd order phase transition.
- However, for sufficiently large J, the MP-AdS coexist with hairy BHs, and $S_{MP} > S_{\text{hairy BH}}$.
  Moreover, the transition is now 1st order, because these solutions never merge for this range of *J*.
- In sum, in a 3d plot of $\{S/l^3, \Delta E/l^2, J/l^3\}$:

$J < J_{\text{crit}}$: the hairy BH family is a 2d surface bounded by the merger line and the boson star curve

$J > J_{\text{crit}}$: Surface continues but is now bounded by the boson star line & extremal hairy BH curve.

This 2d surface never intersects with itself and has a sequence of (regular) "cusp lines".



0.020
0.015
0.010
0.005
S/ℓ³
J/ℓ³
0.02
0.04
0.06
0.08
0.005
0.010
0.015
ΔE/ℓ²

CONCLUSION:

- BHs with a scalar field condensate & orbitating around horizon.
- ***First*** example of stationary **BH** with **single isometry**:

  it is stationary but not time symmetric nor axisymmetric

- This seems to contradict rigidity theorems

  [Hawking,'72; Hollands, Ishibashi, Wald, '06; Isenberg, Moncrief, '06]

  which show that stationary black holes must be axisymmetric...

  (RT assumes $\exists$ stationary KV ∂_t that is **not** normal to H ... $\Rightarrow$ $\exists\ \partial_\psi$)

  Well, these theorems are not applicable to these BHs, since our

  (stationary) single KVF generates the horizon, ie it is normal to horizon

- **What is the endpoint of the superradiant instability in this system ??????**