Gravitational turbulent instability of AdS

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energie atomique • energies alternatives

Based on: 1109.1825 [also: **1105.4167**]

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[Related work: Bizon, Rostworowski, 1104.3702 ; Holzegel, Smulevici 1110.6794]

Recent Advances in BH dynamics , YITP, Kyoto, Japan **2012**

Motivation :

- **The AdS / CFT correspondence relates a (d-1)-dim QFT with a d-dim theory of (quantum) gravity:**

  - Any gravitational phenomena should have an equivalent CFT description, and vice-versa.

  - General gravity is now a tool to study field theory open questions:
  holographic description of condensed matter systems;
  transport properties in strongly coupled field theories;
  hydrodynamic description of QFT; **quantum turbulence . . .**

  - Also works the other way around in its strong version:
  weak coupling CFT as a definition for non-perturbative String Theory.

- **Here, we want to study** ***far from equilibrium dynamics*** **in gravity, and try to understand its field theory interpretation.**
  Two options:
  1. **Full time evolution … hard!**
  2. **Poor's man approach:**

     break down of perturbation theory → onset of interesting dynamics.

Outline :

1. **Anti-de Sitter (AdS) properties. Standard lore & Heuristics**
2. **Outline of Perturbative construction**
3. **Linear Perturbations**
4. **General Structure of non-linear construction:**
   - **4a.** ***Geons***
   - **4b. Colliding Geons → AdS is non-linearly unstable**
5. **String Theory Embedding & Field theory implications**
6. ***Gravitational hairy black holes*** **with a** ***single*** $U(1)$**.**
7. **Conclusions & Open questions**

Anti-de Sitter spacetime :

Anti-de Sitter (AdS) space is a maximally symmetric solution of

$$S = \frac{1}{16\pi G}\int \mathrm{d}^d x\sqrt{-g}\left[R + \frac{(d-1)(d-2)}{L^2}\right]$$

which in *global* coordinates can be written as: ($\Lambda = -1/L^2$)

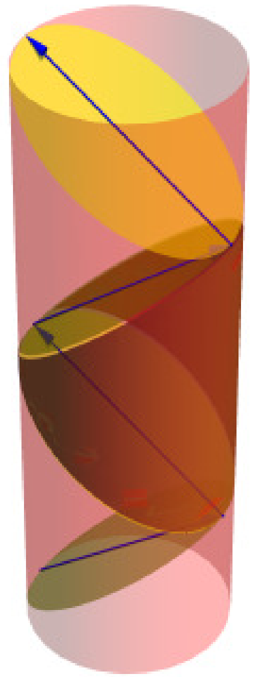
$$\mathrm{d}s^2 \equiv \bar{g}_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\left(\frac{r^2}{L^2}+1\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\frac{r^2}{L^2}+1} + r^2\mathrm{d}\Omega_{d-2}^2$$

Note that the Poincaré coordinates

$$\mathrm{d}s^2 = R^2(-\mathrm{d}\tau^2 + \mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x}) + \frac{L^2\mathrm{d}R^2}{R^2}$$

do <u>not</u> cover the entire spacetime. We will <u>not</u> use them.

Anti-de Sitter spacetime :

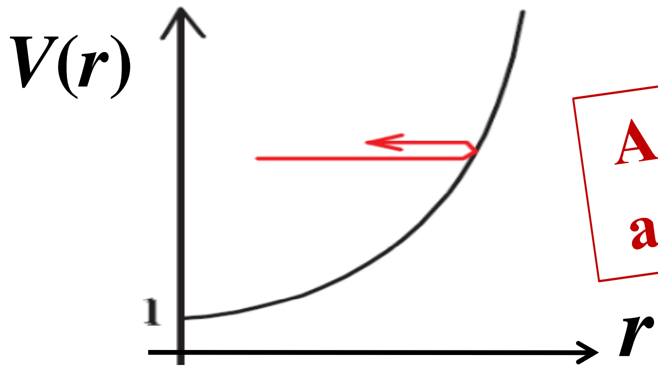


- **The turbulent instability will be described in <u>global</u> AdS**

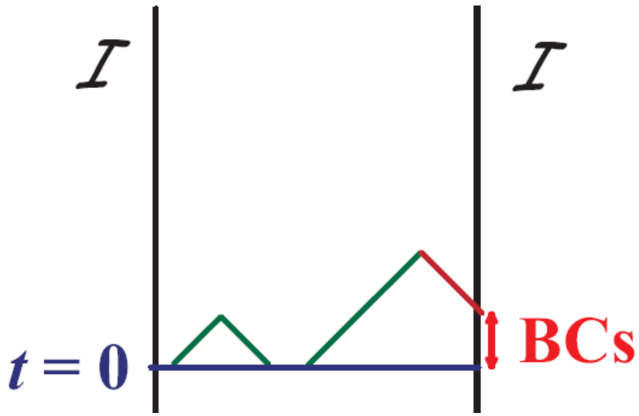
- **Conformally, global AdS is described by the interior of a cylinder**

  The dual field theory lives on $R_t \times S^{d\text{-}2}$.

- **With E, J preserving boundary conditions, waves bounce off at infinity and return in finite time.**



AdS behaves as a confining box



- **Poincaré coordinates** cover **only** the **brown-shaded region**;
  Poincaré horizon destroys confining box property;
  Therefore, the instability *should* <u>not</u> be present

A difference between Minkowski, dS & AdS :

- At the linear level,
  AdS spacetime is as stable as the Minkowski or de-Sitter (dS) spacetimes.

- For the Minkowski & dS spacetimes, it has been shown that
  small, but finite, perturbations remain small [Christodoulou-Klainerman `93]
  So, Minkowski & dS are also non-linearly stable

- Why has this not been shown for AdS ?
  well… because it is just NOT true !

- Claim: AdS is linearly stable but non-linearly unstable
  Generic small (but finite) perturbations of AdS become large
  and *eventually* form black holes.

- The energy cascades from low to high frequency modes
  in a manner reminiscent of the onset of turbulence.

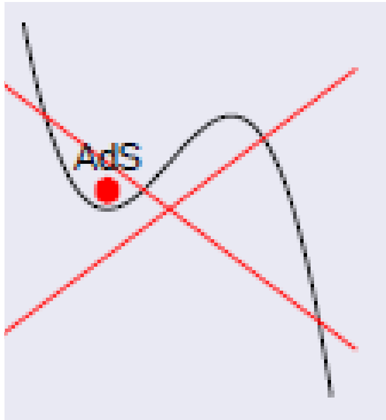
... *oops* :

- Doesn't this claim contradict the fact that AdS is supersymmetric ?

- Doesn't this contradict the fact that there is a
  positivity energy theorem for AdS ?

The short answer is NO :

- Positivity energy theorem: if matter satisfies the dominant energy condition, then $E \geq 0$ for all non-singular, asymptotically AdS initial data, being zero for AdS only.
  - This ensures that AdS cannot decay into state with lower E.
  - It does <u>not</u> ensure that a small amount of energy added to AdS will not generically form a small BH.
  - That is usually ruled out by arguing that waves disperse.
    … this does *not* happen in AdS because " it's a box ".



Example of NO relation between Positivity E theorem and non-linear stability

[Dafermos]

- **Consider the standard Einstein - scalar field action:**

$$S = \int \mathrm{d}^d x \sqrt{-g}\,\left[R - (\nabla\phi)^2\right]$$

There is a positivity energy theorem for all nonsingular asymptotically flat initial data, and **small finite perturbations remain small (non-linearly stable) [Christodoulou, Klainerman, 93]**

- **Consider now the same action, but with the wrong sign for the scalar field kinetic term:**

$$S = \int \mathrm{d}^d x \sqrt{-g}\,\left[R + (\nabla\phi)^2\right]$$

There is no positivity energy theorem,

… but Minkowski space is still non-linearly stable

This shows that there is NO relation between Pos. E. Th. & non-linear stability

AdS is a 2nd example where

solution is non-linearly unstable although there is Positivity E Theorem

Why is AdS unstable? (Heuristics)

- Dafermos & Holzegel: linearized perturbations of AdS do not decay
  … suggests that non-linear corrections will grow in time
- Anderson: AdS acts like a confining finite box.
  Any generic finite excitation added to this box might be expected to
  explore *all* configurations consistent with the conserved charges of AdS
  ... including small black holes.
- Special (fine tuned) solutions might *not* lead to formation of black holes:
  - We will see that for each linearized gravitational mode
    there will be an associated non-linear solution: a *geon* .
  - These solutions are special since they are exactly periodic in time
    and invariant under a single continuous symmetry (single KVF).
  - (AdS) Geons are analogous to gravitational plane waves (flat background)
- We then expect colliding geons to behave like colliding Grav. plane waves :
  …well, colliding exact plane waves produce singularities (BHs) [Penrose '71]

Perturbative construction of geons (1)

- **Expand the metric around global AdS as** $g = \bar{g} + \sum_i \epsilon^i h^{(i)}$
- **At each order i in perturbation theory, the Einstein equations yield:**

$$\tilde{\Delta}_L h^{(i)}_{ab} = T^{(i)}_{ab}$$

where $T^{(i)}$ depends on $\{h^{(j \leq i-1)}\}$ and their derivatives

$$2\tilde{\Delta}_L h^{(i)}_{ab} \equiv -\bar{\nabla}^2 h^{(i)}_{ab} - 2\bar{R}_a{}^c{}_b{}^d h^{(i)}_{cd} - \bar{\nabla}_a \bar{\nabla}_b h^{(i)} + 2\bar{\nabla}_{(a}\bar{\nabla}^c h^{(i)}_{b)c}$$

- **Any smooth symmetric two-tensor $\{h^{(i)}, T^{(i)}\}$ can be expressed as a sum of fundamental building blocks, $\mathcal{T}^{\ell m}_{ab}$, ie we can expand $\{h^{(i)}, T^{(i)}\}$ according to how they transform under coordinate transformations on the S^{d-2}**
- **For concreteness, set $d = 4$. There are three sectors of perturbations:**
  - ***Scalar-type perturbations*: perturbations are constructed from *scalar* (spherical) harmonics on S^2: $Y_{\ell m}$**
  - ***Vector-type perturbations*: perturbations are constructed from *vector* harmonics on S^2: $\star_{S^2} \nabla Y_{\ell m}$**
  - *Tensor-type perturbations*: irrelevant here; only exist in $d \geq 5$.

Perturbative construction of geons (2)

- We go beyond linear order: need real representation for $Y_{\ell m}$:

$$Y^c_{\ell m} = \cos\phi \, \mathrm{L}^m_\ell(\theta) \text{ and } Y^s_{\ell m} = \sin\phi \, \mathrm{L}^m_\ell(\theta)$$

- Technically, work with Kodama-Ishibashi '03 *gauge invariant* formalism.

  That is, work with gauge –invariant *scalars* that obey master equations.

  [see Kodama-Ishibashi '03 for linear order $i = 1$]

- At each order, we can reduce the metric perturbations to 4 gauge invariant functions satisfying

$$\Box_2 \Phi^{\alpha,(i)}_{\ell m}(t,r) + V^{(i)}_\ell(r)\Phi^{\alpha,(i)}_{\ell m}(t,r) = \tilde{T}^{\alpha,(i)}_{\ell m}(t,r)$$

  where $\alpha \in \{c,s\}$ and $\Box_2$ is the d'Alembertian in the (t, r) orbit space.

- Metric perturbation 2-tensor recovered through a linear differential map:

$$h_{ab} = h_{ab}(\Phi) \qquad \text{(in a given gauge)}$$

- Choice of initial data relates $\Phi^{c,(i)}_{\ell m}$ and $\Phi^{s,(i)}_{\ell m}$:

  Left with 2 PDEs to solve … well, for each $\{ \ell, m \}$ building block

Perturbative construction of geons (3)

Boundary conditions:

- Regularity at the origin ($r = 0$) requires (at least) the decay:

$$\Phi_{\ell m}^{\alpha,(i)} \sim \mathcal{O}(r^{\ell})$$

- Close to the AdS conformal boundary (as $r \to \infty$)

$$\Phi_{\ell m}^{\alpha,(i)}(t,r) \sim R_{\ell m}(t) + \frac{S_{\ell m}(t)}{r} + \ldots .$$

Surprisingly, if we want to keep the boundary metric fixed (ie, if we want the perturbations to preserve *global* AdS asymptotics), we need to choose:

$$S_{\ell m}(t) = 0$$

This is also the choice that gives finite energy perturbations for the standard definition of "gravitational energy"

Linear Perturbations ($i = 1$)

- **At the linear level, $T = 0$, we can decompose our perturbations in t as**

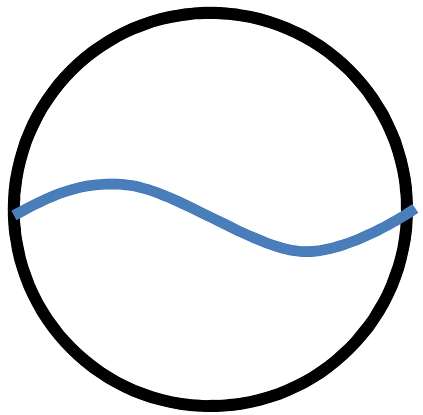
$$\Phi^{\alpha,(i)}_{\ell m}(t,r) = \Phi^{\alpha,(i),c}_{\ell m}(r)\cos(\omega_\ell t) + \Phi^{\alpha,(i),s}_{\ell m}(r)\sin(\omega_\ell t)$$

= 0 (initial data choice)

- **Because AdS acts like a confining box,**

only certain frequencies are allowed to propagate (p is radial overtone):

$$\omega_\ell L = 1 + \ell + 2p$$



These are the so-called normal modes of (global) AdS.

- **$\mathrm{Im}\,\omega = 0$ → AdS is linearly stable**

General Structure of higher order ($i > 1$)

1. **Start with a given perturbation** $\Phi^{\alpha,(i),\kappa}_{\ell m}(r)$ **, and determine the corresponding** $h^{(i)}_{\ell m}(t,r,\theta,\phi)$ **through the KI linear differential map**
   [Kodama-Ishibashi '03]

   $$h_{ab} = h_{ab}(\Phi) \qquad \text{(in a given gauge)}$$

2. **Compute** $T^{(i+1)}_{ab}$ **, in RHS of Einstein eq** $\tilde{\Delta}_L h^{(i+1)}_{ab} = T^{(i+1)}_{ab}$

   and decompose it as a sum of the fundamental building blocks $\mathcal{T}^{\ell m}_{ab}$

3. **Compute the source term** $\tilde{T}^{\alpha,(i+1)}_{\ell m}(t,r)$ **in the RHS of KI master eq**

   $$\Box_2 \Phi^{\alpha(i+1)}_{\ell m}(t,r) + V^{(i)}_\ell(r)\Phi^{\alpha(i+1)}_{\ell m}(t,r) = \tilde{T}^{(i+1)}_{\ell m}(t,r)$$

   and determine $\Phi^{\alpha,(i+1)}_{\ell m}(t,r)$

General Structure of higher order ($i > 1$)

3. Compute the source term $\tilde{T}_{\ell m}^{\alpha,(i+1)}(t,r)$ and determine $\Phi_{\ell m}^{\alpha,(i+1)}(t,r)$

4. If $\tilde{T}_{\ell m}^{\alpha,(i+1)}(t,r)$ has an harmonic time dependence $\cos(\omega t)$, then $\Phi_{\ell m}^{\alpha,(i+1)}(t,r)$ will exhibit the same dependence,

   EXCEPT when ω agrees with one of the normal frequencies of AdS:

$$\Phi_{\ell m}^{\alpha,(i+1)}(t,r) = \Phi_{\ell m}^{\alpha,(i+1),c}(r)\cos(\omega t) + \Phi_{\ell m}^{\alpha,(i+1),s}(r)\, t\, \sin(\omega t)$$

The latter mode is said to be *RESONANT*.

5. If for a given perturbation one can construct $\Phi_{\ell m}^{\alpha,(i)}$ to any order i, without ever introducing a term growing linearly in time, the solution is said to be stable; otherwise it is unstable.

Construction 1 : single Geon

[ℓ, m : quantum # $Y_{\ell m}(\theta,\phi)$]

1. Start with a single mode $\ell = m = 2$ $(\omega_2 L = 3)$ initial data [a normal mode] .
2. At 2^{nd} order there are no resonant modes: solution is regular everywhere
3. At 3^{rd} order, there is a resonant mode, but one can set the amplitude of of the growing mode to zero by changing the ω slightly:

$$\omega L = 3 - \frac{14703}{17920}\epsilon^2$$

- The structure of the equations indicate that there is only one resonant term at each odd order, and that the amplitude of the growing mode can be set to zero by correcting the frequency

- One can compute the asymptotic charges to fourth order, and they obey to the first law of thermodynamics:

$$E_g = \frac{3J_g}{2L}\left(1 - \frac{4901\,J_g}{7560\pi L^2}\right), \quad \omega_2 = \frac{3}{L}\left(1 - \frac{4901\,J_g}{3780\pi L^2}\right)$$

This also defines our expansion parameter ϵ :

$$J_g = \frac{27}{128}\pi L^2 \epsilon^2$$

Construction 1: single Geon (2)

- **We adjust our initial data such that the time dependence of our linear mode can always be recast as $\cos(\omega t - m\phi)$ which is invariant under:**

$$K = \frac{\partial}{\partial t} + \frac{\omega}{m}\frac{\partial}{\partial \phi}$$

Single **Killing vector field (KVF) of geon !**

∂_t , ∂_ϕ of original AdS are not KVFs !

- **At non-linear level, we have the same type of symmetry … but ω changes .**

  So, it's stationary (periodic) but not axisymmetric neither time-independent !

- **Resonances occur because normal modes of AdS take integer values.**

  Geons are likely to be "more" stable than AdS because the normal modes of the Geons correspond to continuous deformations of the AdS normal modes.

Construction 2: linear combination of Geons

1. Start with linear combination of $\ell = m = 2$ $(\omega_2 L = 3)$ <u>*and*</u> $\ell = m = 4$ $(\omega_4 L = 5)$

2. Like in the single mode initial data, at second order there are no resonant modes and the solution can be rendered regular everywhere

3. At third order, there are four resonant modes:
   - The amplitude of the growing modes in two of the resonant modes can be removed by adjusting the frequency of the initial data ($\omega_2 L = 3 + \ldots$ and $\omega_4 L = 5 + \ldots$) like we did for single mode initial data
   - The amplitude of the growing mode of smallest frequency $(\omega_1 L = 1)$ is automatically zero
   - The amplitude of the growing mode with the largest frequency can<u>not</u> be set to zero ($\omega L = 7$, $\ell = m = 6$) !

AdS is non-linearly unstable !

Construction 2: linear combination of Geons (2)

- The frequency $\omega L = 7$ of the growing mode is higher than any of the frequencies we started with: $\omega_2 L = 3$ and $\omega_4 L = 5$ **!**

- The energy (amplitude) is thus transferred to modes of higher frequency

- Expect this to continue: When the $\omega L = 7,\ \ell = m = 6$ mode grows, it will source even higher frequency modes with growing amplitude

Conjecture:

The endpoint of this gravitational turbulent instability is a rotating AdS black hole

- Timescale for BH formation given by breakdown of perturbation theory:

$$\epsilon^3 t \sim \epsilon \quad \rightarrow \quad t_{BH} \sim 1/\epsilon^2$$

Further support for the conjecture:
time evolution of similar spherical scalar field instability in AdS

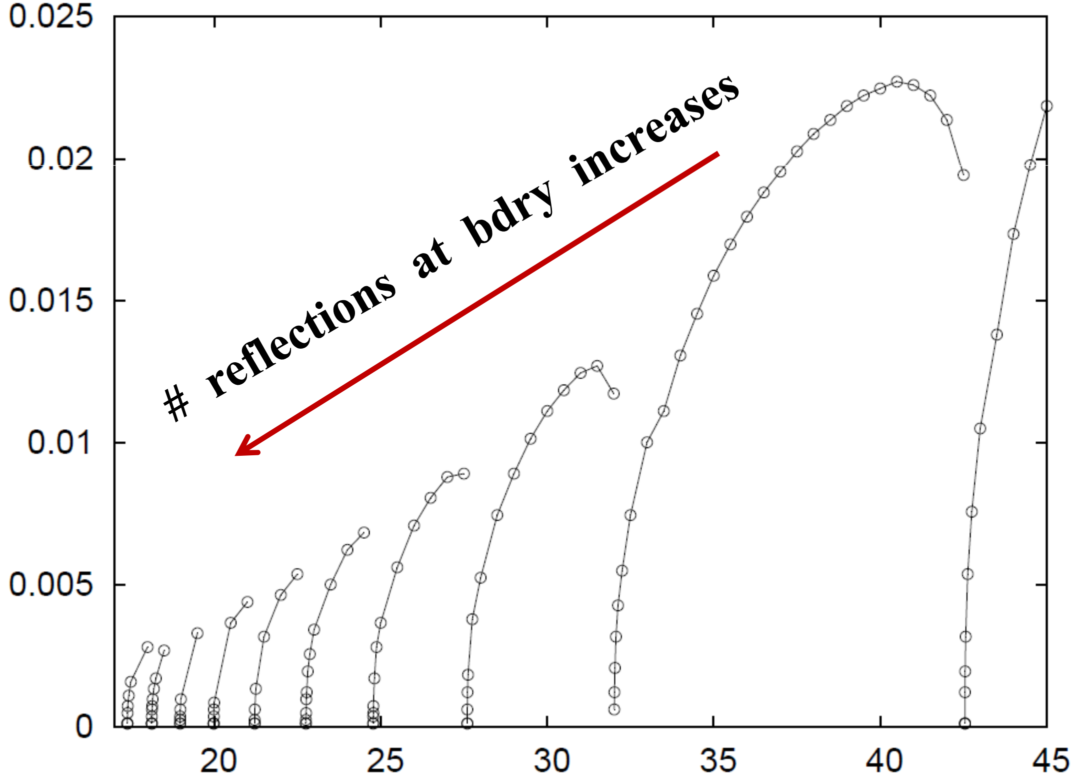
- **Time evolution of Spherical scalar field collapse in AdS**

  [Bizon-Rostworowski '11, Garfinkle '11]

- **No matter how small the initial amplitude is ,**

  the curvature at the origin grows and a small black hole forms.

Horizon radius



Amplitude α of initial perturbation

- At $r_H \sim 0$ a naked singularity forms (but very fine-tuned initial data).

- Same critical behavior as Choptuik (BHs so small that don't see AdS radius)

- In the flat case Choptuik told us:

  Initial scalar field profile $\Phi \sim \alpha$ f(r).

  .Small α → waves scatter and die-off at ∞

  .Critical α^* → naked singularity forms.

  Near it: $M_{BH} \sim (\alpha-\alpha^*)\gamma$ with $\gamma \sim 0.37$

  .Large α → large BH forms

Description within String theory

- **Consider II B string theory on $AdS_5 \times S^5$, with AdS length scale L**
- **There are two energy scales:**
  the Planck energy E_P and the string energy E_S, with $E_S < E_P$ ($E_P = N^2/L$)
- **Possibilities:**
  - **If the initial energy is larger than $E > E_P$, one forms a 5D AdS BH**
  - **If the initial energy is $E_{corr} < E < E_P$, one forms a 10D black hole**

    Here, E_{corr} is the energy of a BH of the string scale size

    [Susskind, Horowitz-Polchinski]
  - **If the initial energy is $E_s < E < E_{corr}$, one forms an excited string**
  - **If $E < E_S$, cascade stops at freq. $\omega = E$: gets a gas of particles in AdS**

- **Thus, at the quantum level there is no continuous cascade or instability!**
  The instability is probably *not* present at finite N
  $\Longrightarrow$ no source of a problem for the dual field theory
- **But, what is the dual description of the instability at large N ?**

Field theory implications

- **Fact that one evolves to state of max entropy (BH forms, 2nd law → S↗) can be viewed as thermalization (evolution towards equilibrium) ; not in the canonical ensemble (*T* is not fixed!), but in the microcanonical ensemble since *E*, *J* is fixed by our BCs**

- **All field theories with a gravity dual will show this cascade of energy like the onset of turbulence**

- **Interesting observation:**
  - **In 2+1 dimensions, classical turbulence has an inverse energy cascade due to an extra conserved quantity - the enstrophy.** This is responsible for hurricanes and other weather phenomena
  - **Our gravitational system is dual to a strongly coupled quantum theory**
  - **Our results indicate that in 2+1 strongly coupled QFT there is a standard energy cascade.**

Caveat: This intuition comes from solving the Navier Stokes equations in 2+1 dimensions. Because our regime is non-hydro, we don't know how to define enstrophy.

Field theory implications (2)

- **More intriguing, from the CFT perspective, is the existence of Geons**
- **At the linear level, these are spin-2 excitations**
- **A nonlinear geon is like a bose condensate of these excitations**

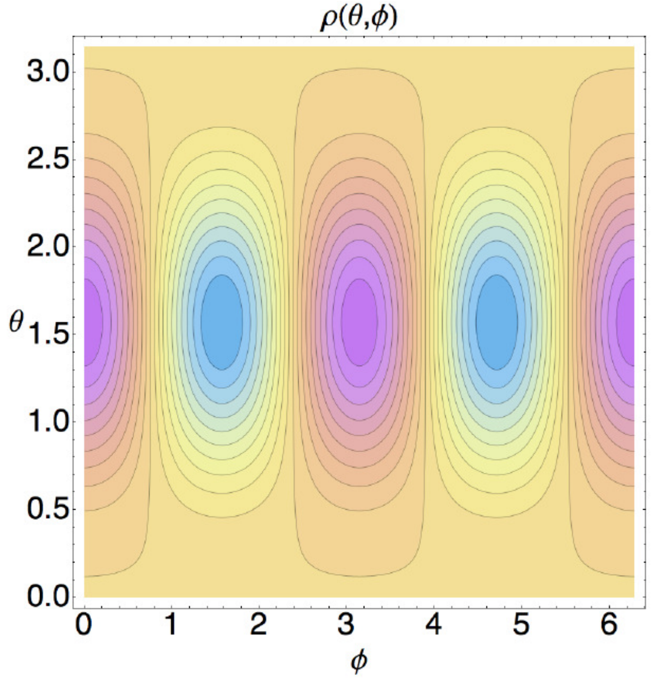
These high energy states do NOT thermalize !! … no BH forms, no decay in *t* …

- **The boundary stress-tensor contains regions of negative and positive energy density around the equator.**

It is invariant under :

$$K = \frac{\partial}{\partial t} + \frac{\omega}{m}\frac{\partial}{\partial \phi}$$

which is timelike near the poles but spacelike near the equator



Gravitational hairy BHs with a single $U(1)$

- **One can add a small black hole inside a geon: only constraint is that the Killing field of the Geon must be null on the horizon:**

$$\Omega_H = \frac{\omega}{m}$$

- **There are many Geons (*m's*); thus whole new class of BHs w/ single $U(1)$: they are stationary but not axisymmetric neither time independent !**

  - **This seems to contradict rigidity theorems**

    [Hawking,'72; Hollands, Ishibashi, Wald, '06; Isenberg, Moncrief, '06]

    which show that stationary black holes must be axisymmetric…

    (RT assumes $\exists$ stationary KV ∂_t that is not normal to H^+ … $\Longrightarrow \exists\ \partial_\phi$)

  - **Well, these theorems are not applicable to these BHs, since our (stationary) single KVF generates the horizon, ie it is normal to horizon**

- **Aside note: *Scalar hairy* BHs with single $U(1)$ *explicitly* constructed in**

  [OD, Horowitz, Santos 1105.4167]

The Kerr-AdS BH is *NOT* the unique stationary black hole in AdS

Gravitational hairy BHs with a single $U(1)$

- These black holes can be seen as metastable configurations in a time evolution towards the endpoint of superradiance

- **Superradiance:**
  - If a wave $e^{-i\omega t + i m \phi}$ scatters off a rotating black hole with $\omega < m\,\Omega_H$, it can return with a larger amplitude → superradiance
  - In AdS, the outgoing wave reflects at infinity, and the process repeats itself → superradiance instability

- **What is the endpoint ?**
  1. *Single* unstable mode:
     the final state will be the rotating BH with a single $U(1)$;
     one has numerical evidence from simpler systems (scalar hairy BH)
     [OD, Horowitz, Santos 1105.4167]
  2. *Superposition* of modes:
     – superradiance cause low ω modes to grow; high ω are absorbed
     – turbulent instability will cause higher ω modes to be created

Gravitational hairy BHs with a single $U(1)$

- **Superradiance : What is the endpoint ?**

  1. *Single* unstable mode:
     the final state will be the rotating BH with a single $U(1)$;
     one has numerical evidence from simpler systems (scalar hairy BH)
  2. *Superposition* of modes:
     superradiance cause them to grow ;
     turbulent instability will cause higher frequency modes to be created

- **Two possibilities for time evolution of superposition of modes :**

  1. **If the BH absorbs the higher frequency modes faster than they can be created, might stabilize with gravitational waves sloshing around outside the BH - unlikely for small BHs**
  2. **Otherwise , the BH exterior might continue to evolve toward higher and higher frequency**

Conclusions & Open questions

Conclusions:

- **AdS spacetime is non-linearly unstable:**
  generic small perturbations become large and (probably) form black holes
- **For each linearized gravity mode, there is an exact, nonsingular geon**
- **Dual field theory shows generic turbulent cascade to maximum entropy state but there are special states (geons) that do not thermalize**

Open questions:

- **Understand why the energy cascade in 2+1 quantum theory is different from the classical theory**
- **Prove a singularity theorem for AdS**
- **Understand the late time behavior of the superradiant instability**
- **Understand the space of CFT states that do not thermalize**

Thanks !

… Canonical vs microcanonical ensembles

1. Shouldn't thermal AdS always dominate the ensemble over small BH ?

Well this is indeed true (Hawking-Page) in canonical ensemble <u>but</u> our BCs fix E, J *not* T… in the microcanonical ensemble things are different !

2. So far, only classical solutions, …. What happens if include Hawking radiation?

$R_{BH} << L \rightarrow C < 0 \rightarrow$ shouldn't they evaporate completely *even* for $E > E_{corr}$?

- **NO, not always correct, since it could result in a decrease in entropy:**
  [OD, Horowitz, Santos 1105.4167]
  - **Small spherical BH (d=5) with radius R has $E \sim R^2/G$ and $S \sim R^3/G$**
  - **Gas with energy E in AdS_5 is $S_g \sim (E L)^{4/5}$.**
  - **If all the E in BH went into the gas (microcononical ensemble: E fix), S would increase only if $R/L < (L_P/L)^{3/7}$** L_P: Planck length
- **Our BHs can be much larger than this, so they will start to evaporate but then quickly come into equilibrium with their Hawking radiation.**

Thermodynamic model for BHs with single $U(1)$

- **Leading order thermodynamics can be determined modeling the single Killing vector field (KVF) BH by a *non-interacting mixture* of a Kerr BH and a geon.**

- **Absence of interaction means that the E, J of the final BH are simply the sum of the charges of its individual constituents:**
  $E = E_K + E_g, \quad J = J_K + J_g.$

- **Geon has only one KVF and we place a Kerr BH with a Killing horizon at its center; geon's KVF must coincide with the horizon generator of the final BH. That is, the angular velocity of the later must be**

$$\Omega_H = \frac{\omega}{m}$$

- **This thermodynamic equilibrium condition also follows from maximizing the entropy for a given total E and J.**

- **Combine these conditions with the leading order thermodynamics of the two components. It follows that, at leading order, the geon component carries all the rotation of the system and the Kerr component stores all the entropy.**

Region in phase space where the single KVF BH exists

- **Single KVF BH expected to bifurcate from the Kerr family at a curve that describes the onset of the superradiant instability. This occurs at an angular velocity that saturates the superradiant condition, $\omega \leq m\,\Omega$**

- **At bifurcation curve, Kerr & single KVF BH thermodynamics coincide. In a phase diagram {*E*, *J*}, this determines the upper bound curve of the region where single KVF BH exist:**

$$E|_{bif} \simeq \frac{r_+}{2} + \frac{r_+^3}{2L^2}\left(1 + \frac{\omega^2 L^2}{m^2}\right)$$

$$J|_{bif} \simeq \frac{1}{2} r_+^3 \frac{\omega}{m}$$

As we move down from this curve, the Kerr contribution weakens and the leading order thermodynamics of the system is increasingly dominated by geon's component. In the limit, the lower bound curve of single KVF BH is expected to be the geon curve.