

The hydrodynamics of stratified ultrarelativistic outflows and the origin of GRB X-ray plateaus

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ABSTRACT

The origin of the X-ray plateau phase observed in a large fraction of gamma-ray burst afterglows remains debated. We present a novel analytic framework for the hydrodynamics of ultrarelativistic, radially stratified outflows interacting with an external medium. By explicitly accounting for a continuous distribution of Lorentz factors within the ejecta, we derive analytic expressions describing the evolution of a long-lived, mildly relativistic reverse shock and determine its crossing time. Then, we compute the resulting synchrotron emission from both the forward and reverse shocks. The forward shock naturally produces a shallow, long-lasting X-ray decay consistent with the observed properties of X-ray plateaus, including the Dainotti relation, without requiring prolonged central-engine activity or an additional high-energy emission component. We further show that reproducing the observed plateau durations requires a broad distribution of ejecta Lorentz factors, extending down to $\gamma_{\min} \sim 70\text{--}100$, consistent with the ultrarelativistic outflow that powers the prompt γ -ray emission. The reverse shock generates a long-lived millimetre emission component that outshines the forward shock emission at these wavelengths. Both the plateau and reverse shock emission terminate smoothly once the slowest ejecta are processed, marking a transition to the standard Blandford–McKee self-similar evolution. Such stratified outflows are expected on physical grounds, as the ultrarelativistic ejecta responsible for the prompt γ -ray emission are unlikely to be launched with a single Lorentz factor. This model provides a unified picture in which the same outflow powers the prompt emission, the X-ray plateau, and the subsequent afterglow evolution.

Key words: radiation mechanisms: non-thermal – relativistic processes – gamma-ray burst: general.

1 INTRODUCTION

Gamma-ray bursts (GRBs) are the most luminous transient events in the Universe, powered by ultrarelativistic (UR) outflows launched during the collapse of massive stars or the merger of compact objects (see T. Piran 2005; P. Mészáros 2006; E. Waxman 2006; E. Nakar 2007; E. Berger 2014; P. Kumar & B. Zhang 2015, for reviews). While the prompt γ -ray emission encodes the physics of internal dissipation within the outflow, the long-lasting afterglow is produced when the relativistic blast wave interacts with the external medium and accelerates non-thermal electrons that radiate via synchrotron emission (P. Mészáros & M. J. Rees 1997; E. Waxman 1997a; R. Sari, T. Piran & R. Narayan 1998; J. Granot & R. Sari 2002). The pre-*Swift* (N. Gehrels et al. 2004; D. N. Burrows et al. 2005) standard model predicted a relatively smooth, declining afterglow light curve shaped by the deceleration of a relatively uniform relativistic blast wave. The early time X-ray observations provided by the *Swift* X-Ray Telescope (XRT) revealed a far richer phenomenology than expected (J. A. Nousek et al. 2006; E.-W. Liang, B.-B. Zhang & B. Zhang 2007; E. Troja et al. 2007). A large fraction of GRB afterglows exhibit an extended phase of unusually shallow flux decay, often lasting $10^3\text{--}10^5$ s, before transitioning to the familiar steeper power-law decline. This feature, commonly referred to as the plateau phase, typically exhibits temporal slopes, α , in the range $0.3 \lesssim \alpha \lesssim 0.6$, with little to no spectral evolution across the break (E.-W. Liang et al. 2007; S. Ronchini et al. 2023; X.-Y. Li et al. 2026b).

The physical origin of the plateau remains uncertain. Several mechanisms have been proposed. A long-lived energy injection into the external shock is one of the most common explanations. This can be powered either by fallback accretion, by spin-down of a newly formed magnetar, or by the slower portion of an outflow with a broad, and relatively low ($\sim$ tens), initial Lorentz factor (LF) distribution, capable of supplying a sustained luminosity that gradually declines over time (B. Zhang & P. Mészáros 2002; J. Granot & P. Kumar 2006; J. A. Nousek et al. 2006; Y. W. Yu & Z. G. Dai 2007; S. Dall’Osso et al. 2011; B. D. Metzger et al. 2011; T. Matsumoto et al. 2020). Such an injection alters the dynamics of the forward shock (FS), producing a flatter temporal decay without modifying the underlying synchrotron spectrum. However, this standard refreshed-shock interpretation encounters significant difficulties. The shallow decay

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and long duration of the plateau imply that the energy injected at late times, either by continued central-engine activity or by slower ejecta, substantially exceeds the initial kinetic energy of the fast component responsible for the prompt γ -ray emission. As a result, evaluating the energy budget at the end of the plateau should naturally lead to an inferred radiative efficiency well below the canonical $\eta \sim 10$ per cent, in tension with observational constraints (N. Wygoda et al. 2016; P. Beniamini & E. Nakar 2019), whereas comparing the prompt γ -ray output to the kinetic energy available immediately after the prompt phase, instead, requires an extremely high prompt efficiency (K. Ioka et al. 2006) far above the efficiencies typically expected from internal-shock models (S. Kobayashi & R. Sari 2001).

Several explanations attribute the X-ray plateau to geometric effects rather than to the dynamics of the external shock. One possibility is that the plateau represents high-latitude emission (HLE) from the prompt phase, arising from angular structure in the prompt-emitting region and producing an extended tail of the prompt radiation (G. Oganessyan et al. 2020). Another class of geometric models invokes off-axis viewing of structured jets, in which the early afterglow is flattened as progressively more energetic regions of the jet become visible to the observer (D. Eichler & J. Granot 2006; P. Beniamini et al. 2020a). Both interpretations generically predict weaker and softer prompt emission for plateau GRBs; observationally, however, GRBs with X-ray plateaus show prompt energetics comparable to those of GRBs without plateaus. HLE is intrinsically tied to the prompt phase and therefore does not naturally explain the subsequent transition to an FS-dominated afterglow, whereas most observed plateaus evolve smoothly into a standard external-shock decay with little or no spectral evolution across the break. Off-axis structured-jet models, on the other hand, can reproduce plateau-like light curves only for specific combinations of jet structure and viewing angle, thereby placing stringent constraints on the parameter space. These considerations suggest that geometric effects alone are unlikely to provide a generic explanation for the majority of X-ray plateaus.

Another proposed explanation associates the plateau phase with the coasting stage of the relativistic outflow, prior to significant blast wave deceleration. In this scenario, a shallow decay can arise naturally if the jet propagates in a wind-like environment and has a relatively low initial LF, ~ 40 , which extends the coasting phase and produces a light curve with a temporal slope comparable to that observed during the plateau (H.-D. Lei et al. 2011; R. Shen & C. D. Matzner 2012; P. C. Duffell & A. I. MacFadyen 2015; H. Dereli-Bégué et al. 2022). This mechanism does not operate in a constant-density interstellar medium (ISM) environment, where the emission during the coasting phase rises in time. Moreover, reproducing plateaus in this framework requires an extremely extended wind medium $\sim 10^{18}$ cm, and that the entire outflow has such low LFs, which is in tension with prompt-emission constraints for typical long GRBs (Y. Lithwick & R. Sari 2001). More complex scenarios include evolving shock microphysics (J. Granot et al. 2006; K. Ioka et al. 2006; A. Panaitescu et al. 2006), reverse shock (RS)-dominated emission (F. Genet, F. Daigne & R. Mochkovitch 2007; Z. L. Uhm & A. M. Beloborodov 2007), prior emission (R. Yamazaki 2009), a thick layer with relativistic RS (K. Leventis, R. A. M. J. Wijers & A. J. Horst 2014; Y. Kusafuka & K. Asano 2025), or a thin layer with a Newtonian RS (S. Kobayashi & B. Zhang 2007).

The interaction of relativistic GRB ejecta with the external medium is commonly described within the FS–RS framework. In this picture, a UR outflow drives an FS into the ambient medium while an RS propagates back into the ejecta, with the dynamics governed by the shell deceleration scale and by whether the RS is relativistic or Newtonian (R. Sari & T. Piran 1995). The subsequent evolution of the FS is well described by the self-similar Blandford–McKee (BM) solution (R. D. Blandford & C. F. McKee 1976), which, together with the FS–RS system, forms the basis of the standard theoretical description of GRB afterglow dynamics. Despite its success, the canonical picture relies on idealized assumptions about the structure of the ejecta, most notably the treatment of the outflow as a shell characterized by a single LF and a sharp radial profile. It is physically expected that relativistic ejecta produced in GRBs are not characterized by a single LF, but instead span a finite distribution of velocities (M. J. Rees & P. Mészáros 1998; R. Sari & P. Mészáros 2000; J. Granot & P. Kumar 2006; T. Laskar et al. 2015). Even if the central engine launches material with a complicated time history, differential expansion quickly leads to an effective radial stratification: faster portions of the outflow propagate to larger radii, while slower portions lag behind. Such a structure may arise from intrinsic variability of the engine and/or internal dissipation within the flow, including internal shocks that redistribute energy and momentum. As the ejecta drive an FS into the external medium and the shocked region decelerates, progressively slower material can catch up and transfer its energy to the blast wave. The effective energy of the external shock increases over time, leading to a more gradual deceleration and a correspondingly shallower afterglow decay. Such stratification can significantly modify the dynamics of the RS, making it long-lived and mildly relativistic. Once the remaining energy in the slowest material becomes small compared to the accumulated blast wave energy, the evolution should approach the standard BM solution (R. D. Blandford & C. F. McKee 1976). While a small number of plateaus terminate in extremely steep declines that are inconsistent with typical external-shock decline, $1 \lesssim \alpha \lesssim 1.5$ (B. Zhang et al. 2006; E.-W. Liang et al. 2007; E. Troja et al. 2007; A. K. Ror et al. 2025; V. Swain et al. 2026), such cases are not common. The majority of observed X-ray plateaus appear broadly compatible with interpretations based on the dynamics and structure of the relativistic outflow (S. Ronchini et al. 2023).

Several empirical observations further motivate considering GRB outflows with a broad LF distribution. Some short GRBs display extended emission (J. P. Norris & J. T. Bonnell 2006), i.e. a softer and less variable component lasting ~ 10 –100 s, and a subset of these events exhibit X-ray plateaus (B. P. Gompertz, P. T. O’Brien & G. A. Wynn 2014; S. Kisaka & K. Ioka 2015; T. Matsumoto et al. 2020). In addition, *Einstein Probe* soft X-ray monitoring has revealed prompt activity extending beyond the γ -ray T_{90} (N. Fraija et al. 2026; A. Li et al. 2026a), followed by an X-ray plateau, demonstrating that prompt emission can persist at softer energies and later times than inferred from γ -ray data alone. While these observations do not uniquely determine the duration of central-engine activity, they motivate modelling the UR ejecta as radially stratified, rather than as a single-LF shell.

In this work, we revisit the hydrodynamics of GRB outflows interacting with an external medium, extending the standard single-LF shell description to radially stratified ejecta (G. Sadeh & E. Waxman 2025). Adopting a quasi-static analytic approach, we derive compact expressions for the shock dynamics, recover the canonical results in the appropriate limits, and develop an analytic description of the transition between relativistic and Newtonian RSs. We then apply this framework to provide a physical interpretation of the GRB X-ray plateau, in which the UR outflow responsible for the prompt emission possesses a broad distribution of LFs, extending down to $\gamma_{\min} \sim 70$ –100. In this picture, a mildly relativistic long-lived RS propagates through the ejecta, gradually transferring energy to the

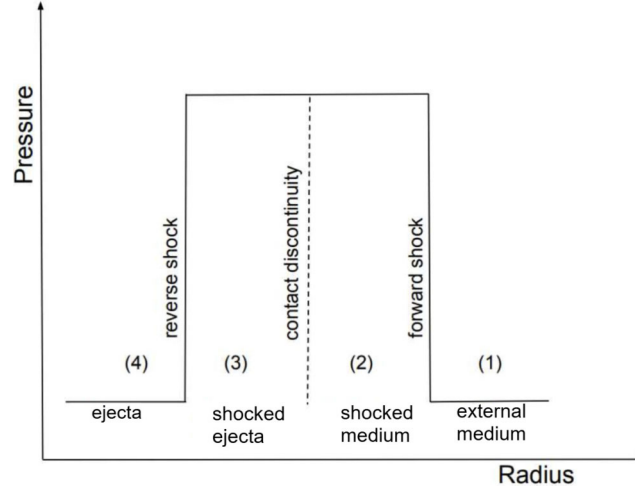


Figure 1. A schematic illustration of the FS-RS structure, showing the four dynamical regions: (1) unshocked external medium, (2) shocked external medium, (3) shocked ejecta, and (4) unshocked ejecta. The pressure in the unshocked regions is negligible compared to that in the shocked regions, $p_1, p_4 \ll p_2, p_3$. In our analytic treatment, the pressure and velocity are approximated as uniform throughout the shocked region, while the density is taken to be uniform within each of the regions (2) and (3), separated by the contact discontinuity.

shocked region over a time-scale set by the mass and minimum LF of the stratified outflow. The plateau ends once the RS completes its passage through the ejecta, after which the blast wave transitions to the standard BM self-similar evolution smoothly. This model directly links the plateau duration to physical properties of the outflow and provides a unified interpretation of the prompt emission, the X-ray plateau, and the subsequent afterglow evolution, without invoking prolonged central-engine activity, finely tuned angular jet structures, or uniformly low LFs of ~ 30 .

The paper is organized as follows. In Section 2, we develop an analytic framework for the hydrodynamics of radially stratified UR outflows interacting with an external medium and derive the resulting FS and RS evolution and duration. In Section 3, we compute the synchrotron break frequencies and emission from both the FS and RS under standard microphysical assumptions. In Section 4, we apply the stratified-outflow model to the observed properties of X-ray plateaus, deriving constraints on the ejecta stratification and the allowed parameter space, and discuss the implications of these constraints for the associated broad-band emission. Finally, our conclusions are summarized in Section 5.

2 FORWARD-REVERSE SHOCK DYNAMICS

2.1 Analytic expressions for the shocked plasma LFs

We first summarize the solution of the problem in which a shell of UR cold dense matter with an LF $\gamma_4 \gg 1$ collides with a stationary cold external medium. Two shocks form: an FS that propagates into the external medium, and an RS that propagates into the shell (see Fig. 1 for a schematic illustration). The deceleration imposed by the RS reduces kinetic energy in the ejecta, which is subsequently transmitted to the heated ionized plasma within the shocked external medium. While the FS is UR (assuming the density of the shell is larger than the external medium density), the RS is not necessarily so. Therefore, careful analysis is required to fully characterize its hydrodynamic evolution.

We approximate the shocked layers behind the shocks with uniform flow profiles (e , ρ , and γ), given by the shock jump conditions (R. D. Blandford & C. F. McKee 1976),

$$\rho' = \frac{\hat{\gamma} + 1}{\hat{\gamma} - 1} \rho, \quad e = (\gamma - 1) \rho' c^2, \quad (1)$$

where ρ and ρ' are the mass densities of the unshocked and shocked material, respectively, both measured in their local rest frames, and γ is the LF of the shocked fluid measured in the unshocked frame. Such an approximation was successfully tested against fully relativistic hydrodynamics numerical calculations of mildly relativistic (G. Sadeh, O. Guttman & E. Waxman 2023; G. Sadeh 2024), highly relativistic (G. Sadeh & E. Waxman 2025), and UR (S. Kobayashi, T. Piran & R. Sari 1999; Y. Kusafuka & K. Asano 2025) ejecta. The adiabatic index $\hat{\gamma}$ relates the internal energy density, e , and pressure, p , via $p = (\hat{\gamma} - 1)e$. Because the RS may be mildly relativistic or Newtonian and the heating is inefficient, we adopt an effective adiabatic index of the form

$$\hat{\gamma} = \frac{4 + \left(1 + \frac{e}{\rho' c^2}\right)^{-1}}{3}, \quad (2)$$

which smoothly interpolates between $\hat{\gamma} = 4/3$ in the UR limit and $\hat{\gamma} = 5/3$ in the Newtonian limit, providing a good approximation to the Synge equation of state (EoS) for an electron-proton plasma (J. L. Synge 1957; P. Kumar & J. Granot 2003; A. Mignone & J. C.

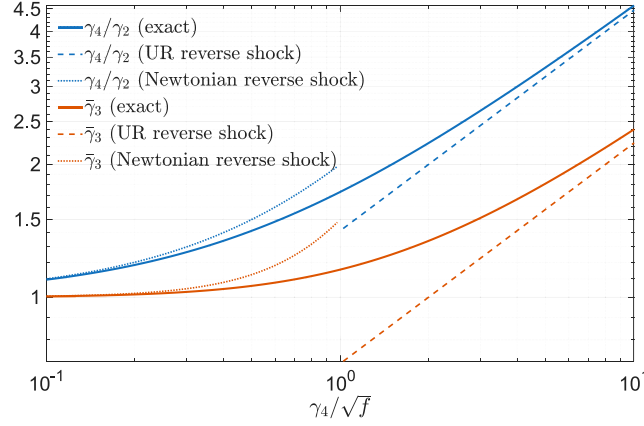


Figure 2. Exact analytic solutions for the LF ratio, γ_4/γ_2 , and for the shocked-ejecta LF, $\tilde{\gamma}_3$ (measured in the unshocked ejecta frame), shown as functions of the single dimensionless parameter $\gamma_4/\sqrt{f}$ (with $f \equiv \rho_4/\rho_1$). Also shown are the asymptotic limits: the UR RS regime ($\gamma_4^2 \gg f$, equation 7) and the Newtonian RS regime ($\gamma_4^2 \ll f$, equation 8).

McKinney 2007; Z. L. Uhm 2011; G. Sadeh 2024). For this EoS, the shocked density simplifies to

$$\rho' = \frac{4\gamma/3 + 4/3}{1/3\gamma + 1/3} \rho = 4\gamma\rho, \quad (3)$$

which is valid in both the UR and Newtonian regimes. Applying these relations to the forward- and reverse-shocked regions yields

$$\begin{aligned} \rho_2 &= 4\gamma_2\rho_1, \\ \rho_3 &= 4\tilde{\gamma}_3\rho_4, \\ e_2 &= (\gamma_2 - 1)\rho_2 c^2 \approx 4\gamma_2^2 \rho_1 c^2, \\ e_3 &= (\tilde{\gamma}_3 - 1)\rho_3 c^2 = 4(\tilde{\gamma}_3^2 - \tilde{\gamma}_3)\rho_4 c^2, \\ p_2 &= \frac{e_2}{3} \approx \frac{4}{3}\gamma_2^2 \rho_1 c^2, \\ p_3 &= \left(\frac{1 + \frac{1}{\tilde{\gamma}_3}}{3}\right) e_3 = \frac{4}{3}(\tilde{\gamma}_3^2 - 1)\rho_4 c^2, \end{aligned} \quad (4)$$

where the numerical subscripts denote the regions shown in Fig. 1. γ_2 and γ_4 are measured in the external medium frame, and $\tilde{\gamma}_3 = \gamma_2\gamma_4 \left(1 - \sqrt{1 - (1/\gamma_2^2) - (1/\gamma_4^2) + (1/\gamma_2^2\gamma_4^2)}\right) \approx 1/2((\gamma_4/\gamma_2) + (\gamma_2/\gamma_4))$ is the LF of the shocked ejecta measured in the unshocked ejecta frame, assuming equal velocities across the contact discontinuity. Imposing pressure balance across the contact discontinuity, $p_2 \simeq p_3$, yields a closed relation between γ_2 , γ_4 , and the density ratio $f \equiv \rho_4/\rho_1$, following the notation of R. Sari & T. Piran (1995),

$$\begin{aligned} \gamma_2^2 &= f[\tilde{\gamma}_3^2 - 1] = f\left[\frac{1}{4}\left(\frac{\gamma_2^2}{\gamma_4^2} + \frac{\gamma_4^2}{\gamma_2^2}\right) - \frac{1}{2}\right], \\ (4\gamma_4^2 - f)\gamma_2^4 + 2f\gamma_4^2\gamma_2^2 - f\gamma_4^4 &= 0. \end{aligned} \quad (5)$$

Remarkably, this equation admits a compact analytic solution (see also B. Zhang 2018; Z.-L. Zhang et al. 2022),

$$\frac{\gamma_4}{\gamma_2} = \sqrt{1 + 2\frac{\gamma_4}{\sqrt{f}}}, \quad \tilde{\gamma}_3 = \frac{1}{2} \left(\frac{1}{\sqrt{1 + 2\frac{\gamma_4}{\sqrt{f}}}} + \sqrt{1 + 2\frac{\gamma_4}{\sqrt{f}}} \right). \quad (6)$$

The applicability of this solution requires $\gamma_2 \gg 1$, which implies $\gamma_4/\sqrt{1 + 2\gamma_4/\sqrt{f}} \gg 1$. Two limiting regimes are of particular interest. In the UR RS limit, $\gamma_4^2 \gg f$, we recover the standard results of R. Sari & T. Piran (1995),

$$\gamma_2 \approx \frac{f^{1/4}\gamma_4^{1/2}}{\sqrt{2}}, \quad \tilde{\gamma}_3 \approx \frac{\gamma_4^{1/2}}{\sqrt{2}f^{1/4}}. \quad (7)$$

While in the Newtonian RS limit, $\gamma_4^2 \ll f$, the use of a variable EoS leads to

$$\gamma_2 \approx \gamma_4 \left(1 - \frac{\gamma_4}{\sqrt{f}}\right), \quad \tilde{\gamma}_3 \approx 1 + \frac{\gamma_4^2}{2f}, \quad (8)$$

slightly different from the result in R. Sari & T. Piran (1995). In Fig. 2, we show the exact solutions for γ_4/γ_2 and $\tilde{\gamma}_3$ as functions of $\gamma_4/\sqrt{f}$, together with their asymptotic UR and Newtonian limits. These relations form the basis for the quasi-static treatment of the RS evolution developed in the following subsection.

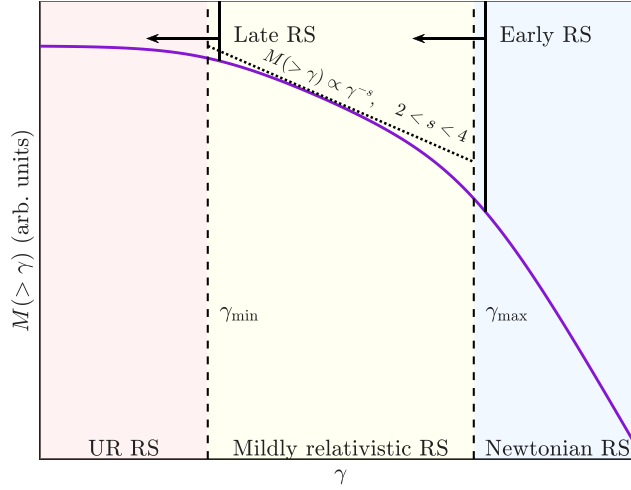


Figure 3. Schematic illustration of the ejecta LF structure. The cumulative mass profile $M(> \gamma)$ is shown as a function of LF, featuring a steep high- γ tail above a characteristic LF $\gamma_{\max}$ and a shallow, approximately constant profile at lower $\gamma_{\min}$. The RS initially propagates through the steep part of the ejecta and subsequently enters the shallow region, as indicated by the arrows. The figure is qualitative and not drawn to scale.

2.2 Stratified structure

Throughout this work, we use isotropic-equivalent energies and masses. This approximation is justified by the UR nature of the outflow and the typical opening angles inferred for GRB jets ($\theta_j \sim 0.1 \gg 1/\Gamma$), which ensure that the early-time dynamics and emission are effectively insensitive to lateral expansion and jet-edge effects. As a result, the hydrodynamic evolution during the phases of interest can be accurately described within a spherically symmetric framework.

In general, any relativistic outflow is expected to exhibit a distribution of LFs. Consequently, the cumulative mass above a given cut-off, $M_{\text{iso}}(> \gamma)$, should decline steeply above some maximum LF, $\gamma_{\max}$, while approaching a constant value below a minimum LF, $\gamma_{\min}$. A schematic illustration of this structure is shown in Fig. 3. The RS initially propagates through the steep high- γ tail of the ejecta, before reaching the shallower, low- γ region. Below, we present a quasi-static analysis of relativistic ejecta propagating into a cold external medium, parametrized by a cumulative mass profile $M_{\text{iso}}(> \gamma) \propto \gamma^{-s}$, valid for $s > 1$, so the cumulative energy above any arbitrary low LF reaches a constant (see also R. Sari & P. Mészáros 2000). The unshocked ejecta is modelled as a sequence of radially stratified shells, each coasting at an LF γ_4 . We assume that an FS-RS shock structure is established and maintained throughout the evolution. This parametrization provides an adequate description of the ejecta structure across the different stages of the RS propagation, while reproducing the correct temporal scaling of the shocked-plasma LF in the limits corresponding to steep ($s \rightarrow \infty$) and shallow ($s \rightarrow 1$) ejecta profiles. For moderate values of $2 \lesssim s \lesssim 4$, the characteristic width of the ejecta is given by $\sim (R)/(2\gamma_{\min}^2)$, where R is the FS radius.

We consider relativistic ejecta characterized by a cumulative mass profile:

$$M_{\text{iso}}(> \gamma_4) = M_{\text{iso}} \left(\frac{\gamma_4}{\gamma_{\min}} \right)^{-s}, \quad s > 1. \quad (9)$$

A shell with a single LF is recovered in the limit $s \rightarrow \infty$, for which $\gamma_4 \rightarrow \gamma_{\min}$. The isotropic equivalent energy of the ejecta is set by

$$\begin{aligned} E_{\text{iso}}(> \gamma_4) &= \int \frac{dM_{\text{iso}}(> \gamma_4)}{d\gamma_4} (\gamma_4 - 1) c^2 d\gamma_4 \\ &\approx \frac{s}{s-1} \gamma_4 M_{\text{iso}}(> \gamma_4) c^2 \equiv E_{\text{iso}} \left(\frac{\gamma_4}{\gamma_{\min}} \right)^{1-s}, \end{aligned} \quad (10)$$

where we assumed $\gamma_4 \gg 1$ and E_{iso} is the total isotropic equivalent energy. The total energy conservation equation follows from conservation of the energy-momentum tensor, $\partial_\mu T^{\mu 0} = 0$, such that the conserved quantity can be written as

$$\int T^{00} dV = \int ((\varepsilon + p)\gamma^2 - p) dV = \int ((\varepsilon + p)\gamma^2 - p) \frac{dM}{\rho\gamma}, \quad (11)$$

where ε includes the rest mass energy. The kinetic energy carried by ejecta with LFs exceeding a given γ_4 is transferred to the combined energy of the shocked external medium and the shocked ejecta,

$$E_{\text{iso}}(> \gamma_4) = E_2 + E_3, \quad (12)$$

where E_i denotes the total energy in region i (see Fig. 1). The total energies in regions (2) and (3) are given by (G. Sadeh 2024)

$$\begin{aligned} E_2 &= \frac{4}{3} \gamma_2^2 M_{\text{ext,iso}} c^2, \\ E_3 &= \gamma_2 \left(\frac{4\tilde{\gamma}_3 - \frac{1}{\tilde{\gamma}_3}}{3} \right) M_{\text{iso}}(> \gamma_4) c^2. \end{aligned} \quad (13)$$

Energy conservation then implies¹

$$\frac{s}{s-1} \gamma_4 M_{\text{iso}}(> \gamma_4) = \frac{4}{3} \gamma_2^2 M_{\text{ext,iso}} + \gamma_2 \left(\frac{4\tilde{\gamma}_3 - \frac{1}{\tilde{\gamma}_3}}{3} \right) M_{\text{iso}}(> \gamma_4). \quad (14)$$

The swept-up external medium mass is therefore

$$M_{\text{ext,iso}} = \frac{3}{4\gamma_4} M_{\text{iso}}(> \gamma_4) \left[\frac{s}{s-1} x^2 - \frac{2}{3} \left(x^2 + \frac{1}{x^2+1} \right) \right], \quad (15)$$

where we have defined $x \equiv \gamma_4/\gamma_2$.

In the UR/Newtonian RS limits, the swept-up external medium mass reduces to

$$M_{\text{ext,iso}} = \begin{cases} \frac{3}{2\sqrt{f}} M_{\text{iso}}(> \gamma_4) \left[\frac{s}{s-1} - \frac{2}{3} \right], & \text{UR RS limit,} \\ \frac{3}{4\gamma_4} M_{\text{iso}}(> \gamma_4) \left[\frac{s}{s-1} - 1 \right], & \text{Newtonian RS limit.} \end{cases} \quad (16)$$

The shocked external medium mass, $M_{\text{ext,iso}}$, and the shocked ejecta mass, $M_{\text{iso}}(> \gamma_4)$, can be related to their corresponding mass densities as

$$\rho_1 = \frac{(3-k)M_{\text{ext,iso}}}{4\pi R^3}, \quad \rho_4 = \frac{\gamma_4 s M_{\text{iso}}(> \gamma_4)}{4\pi (R_{\text{cd}} - \Delta_{\text{ej}})^3}, \quad (17)$$

where we consider an external density profile $\rho_1 = Ar^{-k}$, R_{cd} is the contact discontinuity radius, and Δ_{ej} the thickness of the shocked ejecta layer. To estimate the thickness of the shocked external medium layer, Δ_{ext} , we impose mass conservation across the FS,

$$\begin{aligned} \frac{4\pi}{3-k} \rho_1 R^3 &= \frac{4\pi}{3} 4\gamma_2^2 \rho_1 (R^3 - (R - \Delta_{\text{ext}})^3), \\ \Delta_{\text{ext}} &= R \left(1 - \frac{\left(\frac{4(3-k)}{3} \gamma_2^2 - 1 \right)^{(1/3)}}{\left(\frac{4(3-k)}{3} \gamma_2^2 \right)^{(1/3)}} \right) \approx \frac{R}{4(3-k)\gamma_2^2}, \end{aligned} \quad (18)$$

where the final approximation assumes $\gamma_2 \gg 1$. The thickness of the shocked ejecta layer, Δ_{ej} , follows from mass conservation across the RS,

$$\begin{aligned} M_{\text{iso}}(> \gamma_4) &= \frac{4\pi}{3} 4\gamma_2 \tilde{\gamma}_3 \rho_4 (R_{\text{cd}}^3 - (R_{\text{cd}} - \Delta_{\text{ej}})^3), \\ \Delta_{\text{ej}} &= R_{\text{cd}} \left(1 - \frac{\left(\frac{\gamma_2 \gamma_4 s}{3} \times 4\tilde{\gamma}_3 \right)^{(1/3)}}{\left(1 + \frac{\gamma_2 \gamma_4 s}{3} \times 4\tilde{\gamma}_3 \right)^{(1/3)}} \right) \approx \frac{R_{\text{cd}}}{2s(\gamma_4^2 + \gamma_2^2)}, \end{aligned} \quad (19)$$

where the final expression applies in the relativistic limit. Since the contact discontinuity satisfies $R - \Delta_{\text{ext}} = R_{\text{cd}}$, we obtain

$$(R_{\text{cd}} - \Delta_{\text{ej}})^3 \approx R^3 \left(1 - \frac{3}{4(3-k)\gamma_2^2} - \frac{3}{2s(\gamma_4^2 + \gamma_2^2)} \right). \quad (20)$$

Finally, the density ratio between the unshocked ejecta and the ambient medium is given by

$$\begin{aligned} f = \frac{\rho_4}{\rho_1} &= \frac{s\gamma_4}{3} \frac{M_{\text{iso}}(> \gamma_4)}{M_{\text{ext,iso}}} \frac{R^3}{(R_{\text{cd}} - \Delta_{\text{ej}})^3} \\ &\approx \frac{4\gamma_4^2 s}{9 \left[\frac{s}{s-1} x^2 - \frac{2}{3} \left(x^2 + \frac{1}{x^2+1} \right) \right]}. \end{aligned} \quad (21)$$

Using equation (6), the following implicit expression is derived:

$$s(x^2 - 1)^2 = 9 \left[\frac{s}{s-1} x^2 - \frac{2}{3} \left(x^2 + \frac{1}{x^2+1} \right) \right]. \quad (22)$$

¹Note that we avoid the energy conservation problem introduced in Z. L. Uhm (2011), which also exists in K. Nakamura & T. Shigeyama (2006), as we impose it to derive the blast wave evolution. This method was verified against full numerical relativistic hydrodynamical computations in G. Sadeh et al. (2023).

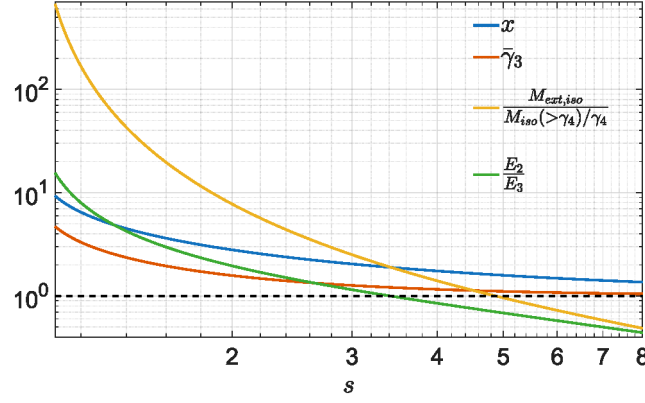


Figure 4. In blue: Numerical solution of equation (22) showing the dependence of the LF ratio $x \equiv \gamma_4/\gamma_2$ on the ejecta power-law index s . In red: The RS LF in the frame of the unshocked ejecta, $\bar{\gamma}_3 = (1/2)(x + (1/x))$. In yellow: The ratio of the swept-up external medium mass, M_{ext} , to the canonical deceleration mass, $M_{\text{iso}}(> \gamma_4)/\gamma_4$ (M. J. Rees & P. Meszaros 1992). In green: The ratio between the shocked external medium energy, E_2 , and the shocked ejecta energy, E_3 . All are shown as a function of the ejecta power-law index s . For $s \gg 1$, corresponding to a Newtonian RS, the outflow experiences negligible deceleration due to the RS; consequently, most of the energy remains in the ejecta (E_3).

Notably, the ratio $x \equiv \gamma_4/\gamma_2$ depends only on the power-law index s , and is independent of the absolute values of γ_4 and γ_2 . In Fig. 4, we solve equation (22) numerically to obtain $x(s)$ along with $\bar{\gamma}_3(s)$. For moderate values of $2 \lesssim s \lesssim 4$, as expected when the RS propagates into the shallower part of the ejecta, the RS is mildly relativistic, with $x(s)$ of order a few (E. Waxman & J. N. Bahcall 2000). We emphasize that, for any specific choice of γ_4 , the condition $\gamma_2 \gg 1$ must be satisfied for the present analysis to remain valid. Equation (15) can be recast in the compact form

$$M_{\text{ext,iso}} = \frac{M_{\text{iso}}(> \gamma_4)}{\gamma_4} \frac{s(x(s)^2 - 1)^2}{12}. \quad (23)$$

In Fig. 4, we also plot the ratio of the swept-up external medium mass, $M_{\text{ext,iso}}$, to the canonical estimate for the external medium mass required for deceleration (M. J. Rees & P. Meszaros 1992), $M(> \gamma_4)/\gamma_4$, along with the ratio between the energy in the shocked external medium, E_2 , to the energy in the shocked ejecta, E_3 , both as a function of s . For $s \gg 1$ ($x \rightarrow 1$), corresponding to a Newtonian RS, the outflow is not significantly decelerated by the RS.

We now turn to the temporal evolution of the shocked-plasma LF, $\gamma_2(t)$. As long as the FS remains UR, the swept-up external medium mass for an external density profile $\rho_1 = Ar^{-k}$, with $k < 3$ and A being constant, is

$$M_{\text{ext,iso}} = \frac{4\pi}{3-k} AR^{3-k} \approx \frac{4\pi}{3-k} A \left(\frac{2\gamma_2^2 ct}{1+z} \right)^{3-k}, \quad (24)$$

where t is the observer time, related to the shock radius by $t = (1+z)R/(\zeta c\gamma_2^2)$, with z as the cosmological redshift. For self-similar BM evolution $\zeta \approx 4$, while for full coasting $\zeta = 1$, assuming emission arrives mainly from an angle $\sim(1/\gamma)$ relative to the line of sight (E. Waxman 1997b; A. Panaitescu & P. Mészáros 1998; R. Sari 1998). We adopt here $\zeta = 2$ as the deceleration is intermediate.

Combining equations (10), (23), and (24) yields (same temporal scaling as R. Sari & P. Mészáros 2000)

$$\gamma_2 = t^{(3-k)/(2k-7-s)} \left[\frac{48\pi 2^{3-k} c^{5-k} x^{1+s} A \gamma_{\text{min}}^{1-s}}{(3-k)(1+z)^{3-k} (x^2 - 1)^2 (s-1) E_{\text{iso}}} \right]^{1/(2k-7-s)}. \quad (25)$$

This expression correctly reproduces the relevant asymptotic limits. In the limit $s \rightarrow \infty$, corresponding to a shell with a single LF, the RS has a negligible dynamical effect and $\gamma_2 \simeq \text{const}$. Conversely, for $s \rightarrow 1$, the RS has already crossed most of the energetic ejecta, the energy from the slower parts adds up only logarithmically, and the flow asymptotically approaches the instantaneous energy injection case, given by the self-similar BM solution (R. D. Blandford & C. F. McKee 1976), for which $\gamma_2 \propto t^{(3-k)/(2k-8)}$. Thus, while the RS propagates through the steep part of the ejecta profile, one has $\gamma_2 \simeq \gamma_4 \simeq \text{const}$. As the shock encounters progressively shallower ejecta layers, the temporal evolution of γ_2 transitions smoothly towards the standard BM deceleration law.

The RS crossing time is set by

$$t_{\text{cross}} = \frac{(1+z)x^2}{2c\gamma_{\text{min}}^2} \left[\frac{(3-k)E_{\text{iso}}}{48\pi A c^2 \gamma_{\text{min}}^2} \times (s-1)(x(s)^2 - 1)^2 \right]^{1/(3-k)}. \quad (26)$$

For the cases of a uniform ISM ($k = 0$) and a wind profile produced by a constant mass-loss rate ($k = 2$), this expression reduces to

$$t_{\text{cross}} = \begin{cases} t_\gamma \times \left(\frac{x^6}{12} \times (s-1)(x(s)^2 - 1)^2 \right)^{1/3} \equiv t_\gamma \times h_\gamma(s), & \text{for } k = 0, \\ t_a \times \left(\frac{x^2}{27} \times (s-1)(x(s)^2 - 1)^2 \right) \equiv t_a \times h_a(s), & \text{for } k = 2, \end{cases} \quad (27)$$

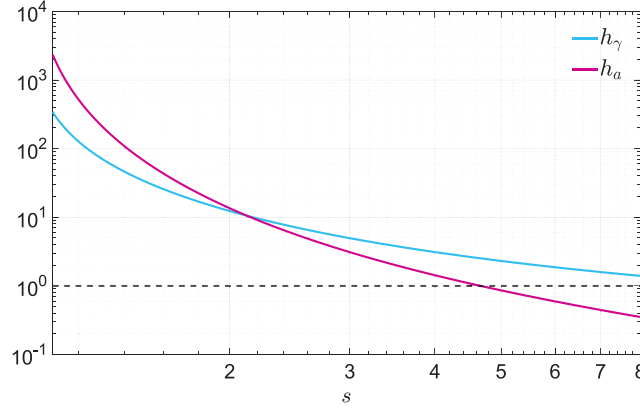


Figure 5. The corresponding correction factors for the RS crossing time, h_γ (uniform ISM; $k = 0$) and h_a (wind environment; $k = 2$), are shown. For moderate values $2 \lesssim s \lesssim 4$, relevant when the RS propagates through the shallow part of the ejecta, the RS is mildly relativistic and the crossing time is significantly extended relative to the standard thin-shell estimates.

with

$$t_\gamma \equiv 9 \times 10^2 (1+z) E_{\text{iso},53}^{(1/3)} n_{-2}^{-(1/3)} \gamma_{\text{min},100}^{-(8/3)} \text{ s}, \quad (28)$$

$$t_a \equiv 3.3 \times 10^2 (1+z) E_{\text{iso},53} A_{10}^{-1} \gamma_{\text{min},100}^{-4} \text{ s},$$

where n is the constant number density of the external medium in case $k = 0$, $\gamma_{\text{min},100} = \gamma_{\text{min}}/100$, and for the other parameters we adopt the following notation $q_x = q/10^x$. t_γ is the thin-shell deceleration time for a uniform ISM (R. Sari & T. Piran 1995; R. Sari 1997), and t_a is the corresponding deceleration time for a wind environment (R. Shen & C. D. Matzner 2012; H. Dereli-Bégué et al. 2022). In Fig. 5, we plot the correction factors h_γ and h_a as functions of the ejecta power-law index s . For moderate values $s \simeq 2$, as expected when the RS propagates through the shallow part of the ejecta, the crossing time is increased by approximately an order of magnitude relative to the standard estimates t_γ and t_a . In practice, this implies that when the ejecta has a non-negligible radial width, as expected for an outflow with a distribution of LFs, the transition from the coasting phase to the BM phase is not abrupt. Instead, an intermediate stage emerges in which the deceleration proceeds gradually.

3 SYNCHROTRON EMISSION FROM THE SHOCKED REGIONS

Following the hydrodynamic evolution of the shocked regions, we calculate the synchrotron emission produced by both the FS and RS during the phase in which the RS propagates through the stratified ejecta. We map this dynamical evolution to observable synchrotron emission within the standard afterglow framework, explicitly accounting for the modified dynamics induced by the ejecta stratification. The resulting temporal scalings are consistent with those first derived by R. Sari & P. Mészáros (2000). For brevity, in the main text, we present normalized expressions only for the two benchmark cases $k = 0$ and $k = 2$. The corresponding temporal scalings for arbitrary k are summarized in Appendix B. The observed X-ray emission is dominated by synchrotron radiation from the forward-shocked external medium. The RS primarily contributes at lower frequencies, most notably in the millimetre (mm) band, while its role at high energies is negligible. Its main influence on the X-ray emission is therefore indirect, through its regulation of the energy transfer from the stratified ejecta to the FS.

We adopt the standard assumptions commonly used in GRB afterglow modelling (R. Sari 1997; E. Waxman 1997a, b; R. Sari et al. 1998): fixed fractions ε_e and ε_B of the post-shock internal energy density are transferred to relativistic electrons and magnetic fields, respectively. We further assume those fractions are the same for the shocked external medium and the shocked ejecta, $\varepsilon_{e/B}^f = \varepsilon_{e/B}^r$, where f/r notation corresponds to the FS/RS. The electrons are accelerated into a power-law distribution, $N(\gamma_e) d\gamma_e \propto \gamma_e^{-p} d\gamma_e$, $\gamma_e \geq \gamma_m$, with $p \geq 2$, and

$$\gamma_m^f = \varepsilon_e \gamma^f \frac{l_p}{p-1} \frac{m_p}{m_e} \equiv \varepsilon_e \gamma^f f_p \frac{m_p}{m_e}, \quad (29)$$

$$\gamma_m^r = \varepsilon_e (\gamma^r - 1) f_p \frac{m_p}{m_e} = \left(\frac{(x-1)^2}{2x\gamma^f} \right) \gamma_m^f, \quad (30)$$

where we simply denote γ_2 as γ^f , γ_3 as γ^r . l_p is given by (G. Sadeh et al. 2023)

$$l_p = \frac{p-2}{1 - (\gamma_M/\gamma_m)^{2-p}}, \quad l_p \xrightarrow{p \rightarrow 2} \frac{1}{\ln(\gamma_M/\gamma_m)}, \quad (31)$$

where $\gamma_M \approx \sqrt{3e/(\sigma_T B)}$ is the maximum electron LF in the power law, assuming that the acceleration time equals the synchrotron cooling time (Z. G. Dai & K. S. Cheng 2001). We further assume that radiative losses are negligible, so that the blast wave evolution remains adiabatic. This phenomenological description, capturing the post-shock relativistic plasma conditions, finds support across a diverse spectrum of observations and plasma calculations (see R. Blandford & D. Eichler 1987; U. Keshet & E. Waxman 2005; U.

Keshet 2006; E. Waxman 2006; A. Spitkovsky 2008; L. Sironi & A. Spitkovsky 2009; A. M. Bykov & R. A. Treumann 2011; L. Sironi, A. Spitkovsky & J. Arons 2013; M. Pohl, M. Hoshino & J. Niemiec 2020). Following equation (4) for the internal energy density behind the FS, the comoving magnetic field there is

$$B^f = (32\pi\epsilon_B(\gamma^f)^2 \rho c^2)^{1/2}, \quad (32)$$

where we denoted ρ_1 as ρ . Pressure equality between the two shocked regions yields the following expression for the comoving magnetic field in the shocked ejecta (using equation 4):

$$B^r = \frac{B^f}{\sqrt{1 + \frac{1}{\gamma^r}}} = \frac{\sqrt{x^2 + 1}}{x + 1} B^f. \quad (33)$$

The accelerated electrons emit synchrotron radiation. Electrons with an LF above γ_c lose a significant part of their energy during the dynamical time. It is given by

$$\gamma_c = \frac{3\pi m_e c(1+z)}{\sigma_T B^2 t \gamma^f}. \quad (34)$$

The characteristic synchrotron frequency associated with electrons of LF γ_e , averaged over an isotropic pitch-angle distribution and over the specific power contribution,² P_v^{syn} , is (G. B. Rybicki & A. P. Lightman 1979)

$$\nu_{\text{syn}}(\gamma_e) = \frac{\gamma^f \gamma_e^2 eB}{4m_e c(1+z)}. \quad (35)$$

Substituting the expressions above and using equations (6) and (25), the characteristic frequencies corresponding to γ_m ($\nu_m \equiv \nu_{\text{syn}}(\gamma_m)$) and γ_c ($\nu_c \equiv \nu_{\text{syn}}(\gamma_c)$) are

$$\nu_m^f = \begin{cases} 5.8 \times 10^{14} \epsilon_{e,-1}^2 \epsilon_{B,-2}^{1/2} n_{-2}^{(s-1)/(2(s+7))} E_{\text{iso},53}^{4/(s+7)} \gamma_{\text{min},100}^{(4(s-1))/(s+7)} \\ \times (f_{s0,2})^4 f_{p,-1}^2 (1+z)^{(5-s)/(s+7)} t_h^{-(12)/(s+7)} \text{ Hz}, & \text{for } k=0, \\ 2.1 \times 10^{14} \epsilon_{e,-1}^2 \epsilon_{B,-2}^{1/2} A_{10}^{(s-1)/(2(s+3))} E_{\text{iso},53}^{2/(s+3)} \gamma_{\text{min},100}^{(2(s-1))/(s+3)} \\ \times (f_{s2,2})^2 f_{p,-1}^2 (1+z)^{2/(s+3)} t_h^{-(s+5)/(s+3)} \text{ Hz}, & \text{for } k=2. \end{cases} \quad (36)$$

$$\nu_c^f = \begin{cases} 8.6 \times 10^{15} \epsilon_{B,-2}^{-(3/2)} n_{-2}^{-(3s+13)/(2(s+7))} E_{\text{iso},53}^{-(4)/(s+7)} \gamma_{\text{min},100}^{(4(1-s))/(s+7)} \\ \times (f_{s0,2})^{-4} (1+z)^{(s-5)/(s+7)} t_h^{-(2(s+1))/(s+7)} \text{ Hz}, & \text{for } k=0, \\ 1.9 \times 10^{17} \epsilon_{B,-2}^{-(3/2)} A_{10}^{-(3s+13)/(2(s+3))} E_{\text{iso},53}^{2/(s+3)} \gamma_{\text{min},100}^{(2(s-1))/(s+3)} \\ \times (f_{s2,2})^2 (1+z)^{-(2s+4)/(s+3)} t_h^{(s+1)/(s+3)} \text{ Hz}, & \text{for } k=2. \end{cases} \quad (37)$$

$$\nu_m^r = \frac{(x-1)^4 \sqrt{x^2+1}}{4x^2(x+1)(\gamma^f)^2} \nu_m^f \equiv \frac{f_x^v}{(\gamma^f)^2} \nu_m^f \quad (38)$$

$$= \begin{cases} 5.8 \times 10^9 \epsilon_{e,-1}^2 \epsilon_{B,-2}^{1/2} n_{-2}^{(s+3)/(2(s+7))} E_{\text{iso},53}^{2/(s+7)} \gamma_{\text{min},100}^{(2(s-1))/(s+7)} \\ \times f_{x,-1}^v (f_{s0,2})^2 f_{p,-1}^2 (1+z)^{-(s+1)/(s+7)} t_h^{-(6)/(s+7)} \text{ Hz}, & \text{for } k=0, \\ 2.1 \times 10^9 \epsilon_{e,-1}^2 \epsilon_{B,-2}^{1/2} A_{10}^{1/2} f_{x,-1}^v f_{p,-1}^2 t_h^{-1} \text{ Hz}, & \text{for } k=2. \end{cases} \quad (39)$$

$$\nu_c^r = \frac{(x+1)^3}{(x^2+1)^{3/2}} \nu_c^f \approx 2.4 \nu_c^f, \quad (40)$$

where t_h is the time measured in hours, $f_x^v \approx 0.05$, and

$$f_{s0} = (1 \text{ h})^{-(3)/(7+s)} \left[\frac{1.6 \times 10^{-24} x^{1+s} 100^{1-s}}{(x^2-1)^2(s-1)} \right]^{-1/(7+s)} \approx 52, \quad (41)$$

$$f_{s2} = (1 \text{ h})^{-(1)/(3+s)} \left[\frac{8.1 \times 10^{-10} x^{1+s} 100^{1-s}}{(x^2-1)^2(s-1)} \right]^{-1/(3+s)} \approx 40. \quad (42)$$

In Appendix A, we derive the self-absorption frequencies for both the FS and RS (see also E. Nakar & T. Piran 2004), resulting in

$$\nu_a^f = \begin{cases} 1.6 \times 10^9 \epsilon_{e,-1}^{-1/5} \epsilon_{B,-2}^{(4s+20)/(5(s+7))} E_{\text{iso},53}^{8/(5(s+7))} \gamma_{\text{min},100}^{(8(s-1))/(5(s+7))} \\ \times f_{a,1}^f f_{s0,2}^{8/5} (1+z)^{-(5(s+11))/(5(s+7))} t_h^{(3(s-1))/(5(s+7))} \text{ Hz}, & \text{for } k=0, \\ 6 \times 10^8 \epsilon_{e,-1}^{-1/5} A_{10}^{(4(s+20))/(5(s+3))} E_{\text{iso},53}^{-(8)/(5(s+3))} \gamma_{\text{min},100}^{(8(1-s))/(5(s+3))} \\ \times f_{a,1}^f f_{s2,2}^{-(8/5)} (1+z)^{(3(s+1))/(5(s+3))} t_h^{-(5(s+7))/(5(s+3))} \text{ Hz}, & \text{for } k=2. \end{cases} \quad (43)$$

$$\nu_a^r = \begin{cases} 10^{11} \epsilon_{e,-1}^{(2(p-1))/(p+4)} \epsilon_{B,-2}^{(p+2)/(2(p+4))} n_{-2}^{((p+6)(s+3))/(2(p+4)(s+7))} \\ \times E_{\text{iso},53}^{(2(p+6))/(p+4)} \gamma_{\text{min},100}^{(2(p+6)(s-1))/(p+4)} f_{a0,11}^r f_{s0,2}^{(2(p+6))/(p+4)} \\ \times (1+z)^{(6(p+6))/(p+4)(s+7)-1} t_h^{(2(s+7)-6(p+6))/(p+4)(s+7)} \text{ Hz}, & \text{for } k=0, \\ 10^{11} \epsilon_{e,-1}^{(2(p-1))/(p+4)} \epsilon_{B,-2}^{(p+2)/(2(p+4))} A_{10}^{(p+6)/(2(p+4))} \\ \times f_{a2,11}^r (1+z)^{4/(2(p+4))} t_h^{-1} \text{ Hz}, & \text{for } k=2, \end{cases} \quad (44)$$

²A simple average over the specific power contribution, $\bar{\nu} = (\int \nu P_v^{\text{syn}} d\nu) / (\int P_v^{\text{syn}} d\nu)$.

where

$$f_a^f = \frac{(p+2)(p-1)f_p^{-(5/3)}}{3p+2} \stackrel{p=2.2}{\approx} 9.7, \quad (45)$$

$$f_{a0}^r \stackrel{p=2.2, s=3}{\approx} 5.8 \times 10^{10}, \quad (46)$$

$$f_{a2}^r \stackrel{p=2.2, s=3}{\approx} 8.5 \times 10^{10}. \quad (47)$$

For full expressions, see Appendix A. The typical synchrotron specific power for a single electron is (G. B. Rybicki & A. P. Lightman 1979)

$$P_{\nu, \max} = P_{\nu}^{\text{syn}}(\nu_{\text{syn}}) = 0.7 \frac{e^3 B \gamma^f}{m_e c^2}, \quad (48)$$

independent of the electron energy. The total number of radiating electrons in the swept-up external medium is (see equation 24)

$$N_e^f \approx \frac{M_{\text{ext, iso}}}{m_p} = \frac{4\pi r_A}{(3-k)m_p} \left(\frac{2(\gamma^f)^2 c t}{1+z} \right)^{3-k}, \quad (49)$$

while for the shocked ejecta, it is given by (following equation 23)

$$N_e^r \approx \frac{M_{\text{iso}}(>\gamma_4)}{m_p} = \frac{12\chi\gamma^f}{s(\chi^2-1)^2} N_e^f. \quad (50)$$

The peak observed flux is obtained by

$$F_{\nu, \max} = \frac{(1+z)N_e P_{\nu, \max}}{4\pi D_L^2}, \quad (51)$$

where D_L is the luminosity distance. For the FS, we have

$$F_{\nu, \max}^f = \begin{cases} 1.2 \times 10^{-1} \varepsilon_{B,-2}^{1/2} n_{-2}^{(3s+5)/(2(s+7))} E_{\text{iso}, 53}^{8/(s+7)} \gamma_{\text{min}, 100}^{(8(s-1))(s+7)} D_{L, 28}^{-2} \\ \times f_{s0, 2}^8 (1+z)^{(10-2s)/(s+7)} t_h^{(3(s-3))/(s+7)} \text{Jy}, & \text{for } k=0, \\ 1.7 \times 10^{-2} \varepsilon_{B,-2}^{1/2} A_{10}^{(3(s+5))/(2(s+3))} E_{\text{iso}, 53}^{2/(s+3)} \gamma_{\text{min}, 100}^{(2(s-1))/(s+3)} D_{L, 28}^{-2} \\ \times f_{s2, 2}^2 (1+z)^{(s+5)/(s+3)} t_h^{-(2)/(s+3)} \text{Jy}, & \text{for } k=2, \end{cases} \quad (52)$$

$$F_{\nu, \max}^f = \begin{cases} 1.2 \times 10^{-1} \varepsilon_{B,-2}^{1/2} n_{-2}^{(3s+5)/(2(s+7))} E_{\text{iso}, 53}^{8/(s+7)} \gamma_{\text{min}, 100}^{(8(s-1))(s+7)} D_{L, 28}^{-2} \\ \times f_{s0, 2}^8 (1+z)^{(10-2s)/(s+7)} t_h^{(3(s-3))/(s+7)} \text{Jy}, & \text{for } k=0, \\ 1.7 \times 10^{-2} \varepsilon_{B,-2}^{1/2} A_{10}^{(3(s+5))/(2(s+3))} E_{\text{iso}, 53}^{2/(s+3)} \gamma_{\text{min}, 100}^{(2(s-1))/(s+3)} D_{L, 28}^{-2} \\ \times f_{s2, 2}^2 (1+z)^{(s+5)/(s+3)} t_h^{-(2)/(s+3)} \text{Jy}, & \text{for } k=2, \end{cases} \quad (52)$$

while for the RS

$$F_{\nu, \max}^r = \frac{12\chi\sqrt{\chi^2+1}\gamma^f}{s(\chi^2-1)(\chi+1)} F_{\nu, \max}^f \equiv f_x^F \gamma^f F_{\nu, \max}^f, \\ = \begin{cases} 12\varepsilon_{B,-2}^{1/2} n_{-2}^{(3(s+3))/(2(s+7))} E_{\text{iso}, 53}^{9/(s+7)} \gamma_{\text{min}, 100}^{(9(s-1))(s+7)} D_{L, 28}^{-2} \\ \times f_x^F f_{s0, 2}^9 (1+z)^{(13-2s)/(s+7)} t_h^{(3(s-6))/(s+7)} \text{Jy}, & \text{for } k=0, \\ 1.7\varepsilon_{B,-2}^{1/2} A_{10}^{(3(s+3))/(2(s+3))} E_{\text{iso}, 53}^{3/(s+3)} \gamma_{\text{min}, 100}^{(3(s-1))/(s+3)} D_{L, 28}^{-2} \\ \times f_x^F f_{s2, 2}^3 (1+z)^{(s+6)/(s+3)} t_h^{-(3)/(s+3)} \text{Jy}, & \text{for } k=2, \end{cases} \quad (53)$$

where $f_x^F \stackrel{s=3}{\approx} 0.5$. Using the expressions derived above, the observed synchrotron flux can be computed following the standard prescription (R. Sari et al. 1998). For the FS: $\nu_a^f < \nu_m^f < \nu_c^f$:

$$F_{\nu}^f = \begin{cases} \left(\frac{\nu}{\nu_m^f} \right)^{1/3} F_{\nu, \max}^f, & \text{for } \nu < \nu_m^f, \\ \left(\frac{\nu}{\nu_m^f} \right)^{(1-p)/2} F_{\nu, \max}^f, & \text{for } \nu_m^f < \nu < \nu_c^f, \\ \left(\frac{\nu_c^f}{\nu_m^f} \right)^{(1-p)/2} \left(\frac{\nu}{\nu_c^f} \right)^{-(p/2)} F_{\nu, \max}^f, & \text{for } \nu_c^f < \nu, \end{cases} \quad (54)$$

while for the RS: $\nu_m^r < \nu_a^r < \nu_c^r$:

$$F_{\nu}^r = \begin{cases} \left(\frac{\nu_a^r}{\nu_m^r} \right)^{(1-p)/2} \left(\frac{\nu}{\nu_a^r} \right)^{5/2} F_{\nu, \max}^r, & \text{for } \nu < \nu_a^r, \\ \left(\frac{\nu}{\nu_m^r} \right)^{(1-p)/2} F_{\nu, \max}^r, & \text{for } \nu_a^r < \nu < \nu_c^r, \\ \left(\frac{\nu_c^r}{\nu_m^r} \right)^{(1-p)/2} \left(\frac{\nu}{\nu_c^r} \right)^{-(p/2)} F_{\nu, \max}^r, & \text{for } \nu_c^r < \nu. \end{cases} \quad (55)$$

The temporal and spectral evolution of the observed flux depends on both the external density profile, parametrized by k , and the relevant spectral regime. Adopting the standard parametrization $F_{\nu} \propto t^{-\alpha} \nu^{-\beta}$, we summarize the corresponding closure relations between the

Table 1. Temporal and spectral indices of the synchrotron flux, $F_\nu \propto t^{-\alpha} \nu^{-\beta}$, expected from both the FS and RS emission resulting from the hydrodynamic evolution derived in Section 2. $s \rightarrow \infty$ corresponds to the coasting phase ($\gamma \sim \text{const}$), while $s \rightarrow 1$ corresponds to the adiabatic BM phase, where most of the energy is in the shocked external medium.

Emission region	Spectral regime	β	α ($k = 0$)	α ($k = 2$)	$\alpha(k)$
Forward shock	$\nu < \nu_m$	$\frac{1}{3}$	$\frac{3s+1}{s+7}$	$\frac{1-s}{3(s+3)}$	$\frac{4ks+2k-9s-3}{3(s+7-2k)}$
	$\nu_m < \nu < \nu_c$	$\frac{p-1}{2}$	$\frac{6p-3s-3}{s+7}$	$\frac{5p+sp-s-1}{2(s+3)}$	$\frac{kps-7kp+5ks+5k+24p-12s-12}{4(s+7-2k)}$
	$\nu > \nu_c$	$\frac{p}{2}$	$\frac{6p-2s-2}{s+7}$	$\frac{5p+sp-2s-2}{2(s+3)}$	$\frac{kps-7kp+2ks+2k+24p-8s-8}{4(s+7-2k)}$
	$\nu < \nu_a$	$\frac{5}{2}$	$\frac{2s+8}{s+7}$	$\frac{2(s+3)}{5s+9}$	$\frac{-ks+7k-8s-32}{4(s+7-2k)}$
Reverse shock	$\nu_a < \nu < \nu_c$	$\frac{p-1}{2}$	$\frac{3p-3s+3}{s+7}$	$\frac{3p+sp-s+3}{2(s+3)}$	$\frac{kps-3kp+5ks-3k+12p-12s+12}{4(s+7-2k)}$
	$\nu > \nu_c$	$\frac{p}{2}$	$\frac{3p-2s+4}{s+7}$	$\frac{3p+sp-2s+2}{2(s+3)}$	$\frac{kps-3kp+2ks-6k+12p-8s+16}{4(s+7-2k)}$

temporal decay index α and the spectral index β in the cases of a uniform ISM ($k = 0$), a wind profile produced by a constant mass-loss rate ($k = 2$), and for an arbitrary k , for both the FS and RS in Table 1 (see also R. Sari & P. Mészáros 2000).

After the crossing time, the FS emission is expected to follow the standard decelerating blast wave temporal properties, which are given in Table 1, by taking $s = 1$ (R. Sari et al. 1998; B. Zhang & P. Mészáros 2004; R. Shen & C. D. Matzner 2012).

4 CONSTRAINTS FROM THE OBSERVED PROPERTIES OF X-RAY PLATEAUS

In this section, we demonstrate that the stratified-outflow model developed in Sections 2 and 3 reproduces the typical observed properties of X-ray plateaus, while providing a prediction for the emergent emission of the long-lived RS. Rather than modelling individual bursts, we translate the phenomenology of plateau samples into generic observational constraints and examine whether these are satisfied for typical model parameters.

X-ray plateaus observed by *Swift* exhibit several robust and well-established properties (e.g. J. A. Nousek et al. 2006; B. Zhang et al. 2006; R. Shen & C. D. Matzner 2012; S. Ronchini et al. 2023). During the plateau phase, the X-ray flux decays slowly, $F_X \propto t^{-\alpha_{X,1}}$, $\alpha_{X,1} \sim 0.3$ – 0.6 , with typical observed flux of $F_\nu(1 \text{ keV}) \sim 0.1$ – $10 \mu\text{Jy}$. The plateau ends at a break time $t_b \sim 10^3$ – 10^5 s , and is followed by a smooth transition to a steeper decay with a temporal index $\alpha_{X,2} \sim 1.0$ – 1.5 , consistent with standard external-shock evolution. Spectrally, the X-ray emission during the plateau is well described by a single power law, $F_\nu \propto \nu^{-\beta_{X,1}}$, with a typical spectral index $\beta_{X,1} \sim 1.0$ – 1.2 , corresponding to an electron power-law index of $p \sim 2.0$ – 2.4 . Importantly, the spectral index shows little or no evolution across the plateau break, $\beta_{X,1} \simeq \beta_{X,2}$, in the majority of events, indicating that the break is dynamical rather than spectral in origin. Any explanation must therefore account for the above-mentioned properties simultaneously.

4.1 Plateau slope

In the stratified-outflow model, the ejecta mass above LF γ follows a power-law distribution $M(> \gamma) \propto \gamma^{-s}$. As slower ejecta catches up with the decelerating blast wave, the total energy of the shocked region increases with time, modifying the synchrotron temporal decay. Within the model framework, the X-ray emission is dominated by the FS emission. For a given external density profile and spectral regime, the temporal index α_X can be written explicitly as a function of s and the electron power-law index p (see Table 1). We focus on the case in which the X-ray band lies above both the peak and cooling frequencies, $\nu_X > \max(\nu_m, \nu_c)$, as motivated by the observed X-ray spectral slopes $\beta_X \simeq 1.0$ – 1.2 . For representative values $p \simeq 2$ – 2.4 and ISM-like external medium ($k = 0$), stratification indices of

$$s \simeq 2.5\text{--}4 \quad (56)$$

naturally yield $\alpha_X \simeq 0.3$ – 0.6 , consistent with the observed plateau distribution. For a wind-like external medium ($k = 2$), reproducing the same observed plateau slopes requires larger inferred values, $s > 4$. We stress, however, that the ejecta stratification index, s , is an intrinsic property of the outflow and does not itself depend on the external medium. Rather, because the closure relations depend on both s and k (Table 1), the value of s inferred from a given observed plateau slope depends on the assumed external density profile. In particular, for the same ejecta stratification and electron index, p , a wind-like external medium produces a steeper decline than an ISM-like medium. Accordingly, the wind and ISM cases should not be interpreted as implying different intrinsic ejecta populations, but rather as different mappings between the observed plateau properties and the underlying stratification index.

4.2 Plateau duration

The plateau duration corresponds to the time over which slower ejecta continues to contribute energy to the shock. Once this reservoir is exhausted, the system approaches the standard self-similar deceleration regime, producing a smooth transition to the post-plateau decay without an abrupt flux drop. The plateau duration is therefore set by the RS crossing time t_{cross} (equation 27).

(i) ISM ($k = 0$) case:

For a given stratification index $s \simeq 2.5\text{--}4.0$, inferred from the plateau slopes, we find $h_\gamma(s) \simeq 3\text{--}10$. The observed plateau duration $t_b \sim 10^3\text{--}10^5$ s leads to

$$(1+z)E_{\text{iso},53}^{(1/3)} n_{-2}^{-(1/3)} \gamma_{\text{min},100}^{-(8/3)} \simeq \frac{t_b}{9 \times 10^2 h_\gamma(s)} \lesssim 1. \quad (57)$$

For typical parameters of long GRBs, $z \simeq 1.5\text{--}2.5$, $E_{\text{iso},53} \approx 1$, $n_{-2} \simeq 10\text{--}100$, we find

$$\gamma_{\text{min}} \sim 70\text{--}100. \quad (58)$$

The LF of the forward-shocked plasma at the end of the plateau is smaller than the minimum ejecta LF by a factor $x(s)$, namely $\gamma^f(t_b) = \gamma_{\text{min}}/x(s)$, and is therefore typically only a few tens, consistent with the standard BM estimate at t_b .

(ii) Wind ($k = 2$) case:

For $s > 4$, inferred from the plateau slopes, we find $h_a(s) \lesssim 1$. The observed plateau duration $t_b \sim 10^3\text{--}10^5$ s leads to

$$(1+z)E_{\text{iso},53} A_{10}^{-1} \gamma_{\text{min},100}^{-4} \simeq \frac{t_b}{3.3 \times 10^2 h_a(s)} \lesssim 60. \quad (59)$$

For typical parameters of $z \simeq 1.5\text{--}2.5$, $E_{\text{iso},53} \approx 1$, $A_{10} \simeq 1$, a lower value of $\gamma_{\text{min}} = x(s)\gamma^f(t_b) \sim 50$ is required to match the observed plateau duration, consistent with the coasting-in-wind model (R. Shen & C. D. Matzner 2012). The combination of $s > 4$ and $\gamma_{\text{min}} \sim 50$ implies that most of the ejecta energy resides in the slowest material, while the fraction carried by ejecta with LF > 100 is strongly suppressed.

In the remainder of this work, we focus on the ISM case, which is independently supported by the plateau flux and spectral constraints discussed below.

4.3 Spectral regime

We consider the regime in which the observed X-ray band lies above both the peak and cooling frequencies, $\nu_X > \max(\nu_m, \nu_c)$, as motivated by the observed X-ray spectral slopes $\beta_X \simeq 1.0\text{--}1.2$. Using the expressions for ν_m^f and ν_c^f derived in Section 3, we now examine the conditions under which this ordering is satisfied during the plateau phase.

For stratification indices $s \simeq 2.5\text{--}4.0$ and observation time of $t \sim 10^3$ s, the expression for ν_m^f shows that even for maximal choice of parameters (e.g. $\varepsilon_e = 0.3$, $\varepsilon_B = 10^{-2}$), ν_m^f lies well below the X-ray band already at the beginning of the plateau. As a result, the condition $\nu_m^f < \nu_X$ is naturally satisfied over the relevant parameter space and does not impose an additional constraint.

The cooling frequency provides a more restrictive bound. Using the expression for ν_c , the requirement that the X-ray band remains above the cooling break during the plateau imposes a lower bound on the magnetic energy fraction ε_B . Using the inferred values of $s \simeq 2.5\text{--}4.0$, $\gamma_{\text{min}} \simeq 100$, and typical parameters of $z \simeq 1.5\text{--}2.5$, $E_{\text{iso},53} \approx 1$, $n_{-2} \simeq 10\text{--}100$, this condition implies

$$\varepsilon_B \gtrsim 10^{-3}. \quad (60)$$

Both characteristic frequencies decline monotonically with time during the plateau phase. Consequently, if the condition $\nu_X > \max(\nu_m, \nu_c)$ is satisfied at $t \sim 10^3$ s, it remains satisfied throughout the plateau.

4.4 Plateau flux

In the regime $\nu_X > \max(\nu_m, \nu_c)$, the FS synchrotron emission is

$$F_X = F_{\nu,\text{max}}^f (\nu_c^f)^{1/2} (\nu_m^f)^{(p-1)/2} \nu_X^{-(p/2)}, \quad (61)$$

which explicitly states that the X-ray flux is determined by the combination of $F_{\nu,\text{max}}^f$, ν_m^f , and ν_c^f . Substituting the expressions for $F_{\nu,\text{max}}^f$, ν_m , and ν_c derived in Section 3, and fixing $\gamma_{\text{min}} = 100$, $s \simeq 2.5\text{--}4.0$ shows that for $p \simeq 2.0\text{--}2.4$ the X-ray flux depends primarily on the electron energy fraction ε_e and the isotropic-equivalent energy E_{iso} . The dependence on ε_B and the external density normalization is weaker due to compensating scalings between $F_{\nu,\text{max}}^f$, ν_m^f , and ν_c^f , while the dominant redshift dependence enters through the luminosity distance $D_L^{-2}(z)$. Observed X-ray plateaus have fluxes $F_{\nu,X}(1 \text{ keV}) \sim 0.1\text{--}10 \mu\text{Jy}$ at observer times $t \sim 10^3\text{--}10^5$ s. Matching this range for $z \simeq 1.5\text{--}2.5$ yields

$$\varepsilon_e \sim 0.05\text{--}0.5 \quad (62)$$

for typical GRB energetics, with $E_{\text{iso}} \sim 10^{52.5}\text{--}10^{53.5}$ erg. Lower values of ε_e underproduce the observed X-ray flux, while significantly larger values require unusually low energies. Additionally, joint X-ray and optical analyses show that, for the majority of events, the temporal and spectral properties are consistent with a common synchrotron origin in a refreshed FS (X.-Y. Li et al. 2026b).

The stratified-outflow model presented here is also consistent with the Dainotti relation (M. G. Dainotti, V. F. Cardone & S. Capozziello 2008; M. G. Dainotti et al. 2010) between the X-ray plateau luminosity and its end time. In our framework, the plateau duration (identified

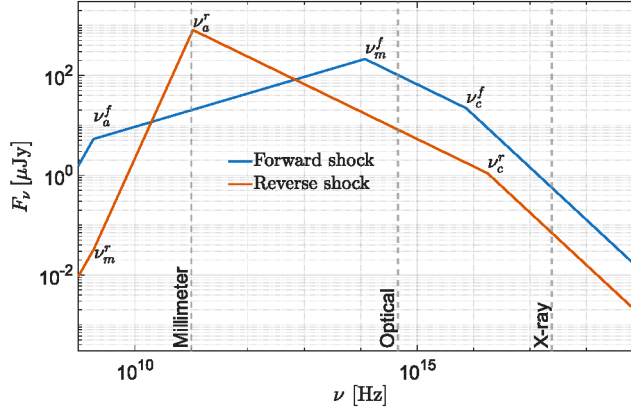


Figure 6. Representative broad-band synchrotron spectra of the FS (blue) and RS (red) at $t \simeq 1$ h for fiducial parameters $p = 2.1$, $\varepsilon_e = 0.1$, $\varepsilon_B = 10^{-2}$, $n = 10^{-1} \text{ cm}^{-3}$, $E = 10^{53} \text{ erg}$, $\gamma_{\min} = 100$, $s = 3$, and $z = 2$. Vertical dashed lines indicate mm, optical, and X-ray bands. The characteristic frequencies of the FS and RS are also provided. The FS dominates the optical and X-ray emission, whereas the RS peaks at mm wavelengths and outshines the FS at low frequencies, producing a bright, long-lived mm component during the X-ray plateau phase.

with the RS crossing time) is most sensitive to the minimum LF of the ejecta, $\gamma_{\min}$. For example, combining equations (27) and (61) for typical values of $s = 3$, $p = 2$ yields

$$F_X \propto \varepsilon_e E_{\text{iso}} (1+z) t_{\text{cross}}^{-1} D_L^{-2}. \quad (63)$$

This anticorrelation between the X-ray luminosity at the end of the plateau and its duration is consistent with the observations. Given the additional variations in electron power index and the detailed ejecta stratification, the relation is naturally expected to exhibit a significant scatter.

4.5 Post-plateau decline

In the majority of GRBs, the decay following the plateau phase is broadly consistent with the standard adiabatic evolution of an external FS, with post-plateau temporal slopes falling within the range expected for normal afterglow decay. This behaviour is observed in both X-ray and optical bands when the emission is consistent with a common external-shock origin, reinforcing the interpretation that many plateaus are associated with refreshed shocks rather than a fundamentally distinct emission mechanism.

A smaller subset of bursts, however, exhibits a post-plateau decline, observed by *Swift*/XRT, that is significantly steeper than predicted by simple FS models (e.g. GRB 060614A, GRB 050730A, GRB 120404A, GRB 130408A, GRB 130609B, GRB 170317A, GRB 180224A, and GRB 180620A; A. K. Ror et al. 2025; V. Swain et al. 2026). In these cases, the steepening may be consistent with a jet break near the end of the plateau phase, so that the light-curve transitions directly from the plateau into the jet-spreading regime. This interpretation is particularly plausible for plateaus associated with lower effective LFs of the forward-shocked plasma at the end of the plateau, $\gamma^f = \gamma_{\min}/\chi(s)$, increasing the probability of an early jet break. Such a scenario naturally links the plateau properties to the jet geometry and does not require invoking an internal origin for the emission.

4.6 Reverse shock emission

The parameter ranges inferred from the X-ray plateau analysis have direct and robust implications for the RS emission. While the FS naturally dominates the X-ray band, the RS generically produces a much brighter component at low frequencies, with its peak emission falling in the mm band during the plateau phase. This behaviour follows from two general properties of the RS in stratified outflows. First, at a given observer time, the RS processes a substantially larger number of electrons than the FS, since it sweeps through the ejecta rather than only the external material (equation 50). Second, the typical electron LF in the shocked ejecta is significantly lower than in the FS (equation 6), which shifts the characteristic synchrotron frequencies of the RS to much lower values (equation 38). As a result, the RS spectrum peaks at frequencies well below those of the FS, while maintaining a comparable or larger peak flux (R. Sari & P. Mészáros 2000).

Fig. 6 illustrates a representative broad-band spectrum of the FS and RS at $t \simeq 1$ h, while Fig. 7 illustrates representative light curves in the X-ray, optical, and mm bands, during (and shortly after) the plateau phase. Both are shown for fiducial parameters consistent with our inferred bounds: $p = 2.1$, $\varepsilon_e = 0.1$, $\varepsilon_B = 10^{-2}$, $n = 10^{-1} \text{ cm}^{-3}$, $E = 10^{53} \text{ erg}$, $\gamma_{\min} = 100$, $s = 3$, and $z = 2$. In these examples, the FS emission peaks in the optical band and dominates the X-ray emission, while the RS peak lies in the mm range, where it outshines the FS by more than an order of magnitude. At higher frequencies, the RS contribution declines rapidly and becomes negligible in the X-ray band. The prominence of the RS in the mm band is therefore a robust consequence of the ejecta stratification required to reproduce the X-ray plateau slopes. During the plateau phase, the RS mm emission is expected to persist as long as the RS crosses progressively slower ejecta, and to fade once this process ceases. This behaviour implies a clear, testable prediction: GRBs exhibiting X-ray plateaus should be accompanied by a bright, long-lived mm component that is absent or significantly weaker once the plateau ends. Thus, early ($\sim$ h) and sustained mm observations, particularly with facilities such as AtLAST (Atacama Large Aperture Submillimeter Telescope; T.

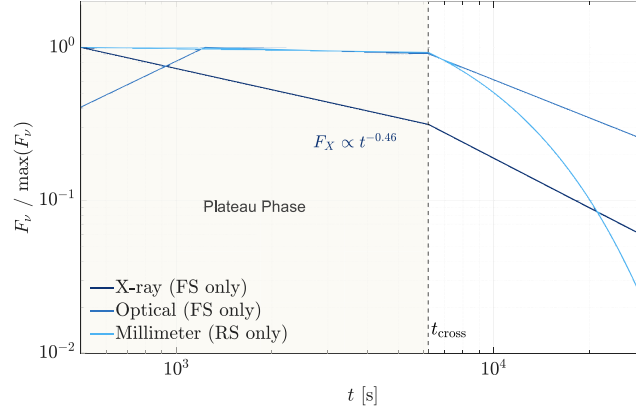


Figure 7. Representative synchrotron X-ray, optical, and mm light curves for fiducial parameters $p = 2.1$, $\varepsilon_e = 0.1$, $\varepsilon_B = 10^{-2}$, $n = 10^{-1} \text{ cm}^{-3}$, $E = 10^{53} \text{ erg}$, $\gamma_{\min} = 100$, $s = 3$, and $z = 2$. As shown in Fig. 6, the FS dominates the optical and X-ray emission, whereas the RS peaks at mm wavelengths and outshines the FS at low frequencies, producing a bright, long-lived mm component during the X-ray plateau phase. After the RS crossing, the X-ray and optical light curves follow the typical decline from FS with standard adiabatic evolution, whereas the RS emission should decline exponentially.

Mroczkowski et al. 2025) or ALMA (Atacama Large Millimeter/submillimeter Array; T. Laskar et al. 2018, 2019), can provide a sensitive probe of the stratified-ejecta interpretation of X-ray plateaus, complementary to constraints obtained from the X-ray band alone.

5 CONCLUSIONS

In this work, we developed a novel analytic framework for treating the hydrodynamics of UR, radially stratified outflows interacting with an external medium. By explicitly accounting for a continuous distribution of LFs within the ejecta, we derived closed-form expressions describing the dynamics of a long-lived, mildly relativistic RS propagating through the outflow (Section 2). Within this framework, we inferred the RS crossing time (equation 27), which is $\sim$ an order of magnitude larger than previous estimates based on the thin-shell approximation. Then, in Section 3, we computed the resulting FS and RS synchrotron emission, focusing in particular on a constant-density ISM ($k = 0$) and a wind-like ($k = 2$) external medium.

We find that the interaction of stratified relativistic ejecta with the external medium naturally produces FS emission with a long-lasting, shallow X-ray decay, whose duration and temporal slope are consistent with the observed properties of X-ray plateaus, without invoking an additional energy reservoir beyond the initial ejecta, a wind-like external medium, or an additional high-energy emission component (Section 4). The appearance of a plateau is associated with an intermediate range of the stratification index s (similar to J. Granot & P. Kumar 2006). In practice, the cumulative ejecta distribution $M(> \gamma)$ (or equivalently $E(> \gamma)$) is not expected to follow a single power law over the entire LF range: by construction, it must be steep at the largest LFs and gradually flatten towards the smallest LFs. In our model, an effective slope in the range $2.5 \lesssim s \lesssim 4$ during the relevant phase leads to a prolonged RS crossing time and, consequently, a shallow X-ray decay. For steeper stratification ($s \gg 4$), the crossing time approaches the thin-shell limit, producing a more impulsive evolution with a rapid transition to the standard adiabatic blast wave decay. The model is also consistent with the observed anticorrelation between X-ray plateau luminosity and duration (M. G. Dainotti et al. 2008, 2010), without requiring additional assumptions about prolonged central-engine activity. The RS associated with the same stratified ejecta is predicted to generate a bright, long-lived emission component at lower frequencies. For the parameter ranges inferred from the plateau slopes and durations, the RS spectrum peaks in the mm band and outshines the FS emission at these wavelengths during the plateau phase, while remaining negligible in the X-ray band. As a result, mm observations during the plateau phase can provide a particularly stringent test of the stratified-ejecta scenario, since the FS alone is expected to be faint at these wavelengths (e.g. T. Laskar et al. 2018). Conversely, the absence of an mm excess during the plateau would place strong constraints on the allowed microphysical parameters or on the role of ejecta stratification in shaping the observed X-ray plateaus.

The transition from the plateau phase to the standard adiabatic blast wave evolution occurs smoothly once the RS has crossed the slowest ejecta, and the resulting post-plateau temporal and spectral behaviour is consistent with that inferred from broad-band afterglow observations. However, a small fraction of events show a significantly steeper post-plateau decline, which in some cases may be explained by a jet break occurring close to the end of the plateau (A. K. Ror et al. 2025). Taken together, the plateau flux level shows that once the ejecta stratification index s and the minimum LF $\gamma_{\min}$ are fixed by the plateau slope and duration, the resulting microphysical parameters and energetics fall within standard ranges (Section 4). Additionally, the same region of parameter space that reproduces the plateau morphology also satisfies the required spectral ordering throughout the plateau phase, without the need for fine-tuning.

A key aspect of this model is that the stratified structure required to power the plateau is expected on physical grounds. UR outflows (with $\text{LF} \gtrsim 100$) responsible for the prompt γ -ray emission are unlikely to be launched with a single LF. Instead, variability in the central engine and internal dissipation naturally lead to ejecta with a broad LF distribution. In addition, modestly off-axis viewing angles could further flatten and extend the observed light curve, enhancing the plateau without qualitatively altering the underlying dynamics (P. Beniamini, J. Granot & R. Gill 2020b). Taken together, this model provides a unified explanation in which the same outflow that produces the prompt emission also gives rise to the X-ray plateau and its subsequent transition to the BM phase. The plateau thus reflects the hydrodynamic evolution of a stratified relativistic outflow rather than prolonged central-engine activity or fine-tuned geometric effects,

offering a physically economical interpretation of one of the most persistent features of GRB afterglows. An additional strength of this framework is that it provides a consistent energy budget for the plateau phase without introducing tensions in the inferred radiative efficiency. Because the energy that powers the plateau is already carried by the same UR outflow responsible for the prompt emission, there is no need to invoke a large reservoir of previously hidden kinetic energy or sustained late-time energy injection. The gradual release of energy from the stratified ejecta naturally produces the plateau while maintaining prompt radiative efficiencies within the range inferred from observations.

Note added during review. After submission of this manuscript, G. K. Keating et al. (2026) reported rapid-response 1.3 mm observations of GRB 260127A, detecting a bright mm counterpart within ~ 13 min of the burst that declined rapidly on day time-scales. This behaviour is broadly consistent with the early mm component predicted in this work.

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DATA AVAILABILITY

The data underlying this article will be shared following a reasonable request to the corresponding author.

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APPENDIX A: SELF-ABSORPTION FREQUENCIES

We estimate the self-absorption frequency at time t as the frequency for which the optical depth is $\tau_\nu = \alpha_\nu \Delta_\tau = 1$, where α_ν and Δ_τ are the typical absorption coefficient and the typical path-length traversed by photons through the shocked plasma, dominating the emission of radiation observed at time t .

To derive the self-absorption frequency in the observer frame, we first approximate the absorption coefficient (G. B. Rybicki & A. P. Lightman 1979). For the FS $\nu_a \ll \nu_m$,

$$\alpha_\nu^f = g_a^f(p) \rho (\gamma_m^f)^{-(5/3)} (1+z)^{-(2/3)} (\gamma^f)^{(5/3)} \nu^{-(5/3)} (B^f)^{(2/3)}, \quad (\text{A1})$$

where

$$g_a^f(p) = \frac{(p+2)(p-1)}{(3p+2)} \frac{12e^3 \int_0^\pi \sin^{(5/3)}(y) dy}{2^{(7/3)} m_p m_e^2 c^2 \Gamma(\frac{1}{3})} \left[\frac{4\pi m_e c}{3e} \right]^{(1/3)}. \quad (\text{A2})$$

For the RS $v_m \ll v_a$,

$$\alpha_v^r = g_a^r(s, p) \rho_{ej} (\gamma_m^r)^{p-1} (1+z)^{-(p+2)/(2)} (\gamma^f)^{(p+2)/(2)} v^{-(p+4)/(2)} (B^r)^{(p+2)/(2)}, \quad (A3)$$

where

$$g_a^r(s, p) = 2 \left(x + \frac{1}{x} \right) \frac{(p-1)\sqrt{3}e^3}{16\sqrt{\pi}m_p m_e^2 c^2} \left[\frac{3e}{2\pi m_e c} \right]^{(p/2)} \\ \times \frac{\Gamma\left(\frac{p+6}{4}\right)}{\Gamma\left(\frac{p+8}{4}\right)} \Gamma\left(\frac{3p+22}{12}\right) \Gamma\left(\frac{3p+2}{12}\right). \quad (A4)$$

The photons are emitted in the shocked plasma while it expands relativistically with LF γ_f . The typical distance they go through in the shocked external medium is

$$\Delta_\tau^f \approx \Delta_{ext} \gamma_f^2, \quad (A5)$$

and in the shocked ejecta is

$$\Delta_\tau^r \approx \Delta_{ej} \gamma_f^2. \quad (A6)$$

The self-absorption frequency, ν_a , is simply defined by $\tau_\nu(\nu = \nu_a) = 1$. For the relevant parameter space, $\nu_a^r \gg \nu_a^f$. Thus, even though the photons emitted from the shocked ejecta go through the shocked external medium, they are not affected by its self-absorption.

APPENDIX B: GENERAL SYNCHROTRON SCALINGS

Here, we summarize the temporal scalings of the synchrotron observables for a general external-density profile $\rho_1 = Ar^{-k}$, with $k < 3$, during the phase in which the RS propagates through the stratified ejecta. The temporal scaling of the FS LF and radius are

$$\gamma_f \propto t^{-(3-k)/(s+7-2k)}, \quad R \propto t^{(s+1)/(s+7-2k)}. \quad (B1)$$

Using equations (29)–(35) and the same assumptions adopted in the main text, the characteristic synchrotron quantities scale with observer time as follows. For the FS,

$$\nu_m^f \propto t^{(7k-ks-24)/(2(s+7-2k))}, \quad (B2)$$

$$\nu_c^f \propto t^{((3k-4)(s+1))/(2(s+7-2k))}, \quad (B3)$$

$$F_{\nu, \max}^f \propto t^{(k-3ks+6s-6)/(2(s+7-2k))}. \quad (B4)$$

For the RS,

$$\nu_m^r \propto t^{(3k-ks-12)/(2(s+7-2k))}, \quad (B5)$$

$$\nu_c^r \propto t^{((3k-4)(s+1))/(2(s+7-2k))}, \quad (B6)$$

$$F_{\nu, \max}^r \propto t^{(3(k-ks+2s-4))/(2(s+7-2k))}. \quad (B7)$$

For the self-absorption frequencies in the spectral ordering relevant here, $\nu_a^f < \nu_m^f < \nu_c^f$ and $\nu_m^r < \nu_a^r < \nu_c^r$, we obtain

$$\nu_a^f \propto t^{(3s-4ks-2k-3)/(5(s+7-2k))}, \quad (B8)$$

$$\nu_a^r \propto t^{(3kp-kps-6ks+10k-12p+4s-44)/(2(p+4)(s+7-2k))}. \quad (B9)$$

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