

# Nuclear beta decay and electron capture in the origin of heavy elements

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# Outline

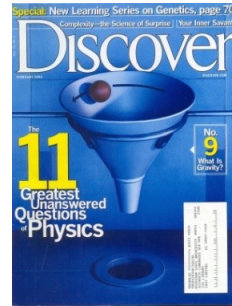
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- **How were heavy elements from iron to uranium made?**
- **Accurate Nuclear Physics Inputs for r-process**
  - **$\beta$ -decay half-lives**
- **Where does r-process happen?**
  - **Electron capture rates in core-collapse supernova**
- **Summary and Perspectives**

# How Were the Heavy Elements Made?

How were the heavy elements from iron to uranium made?

The 11 greatest unanswered questions of physics



## ● r-process

### ➤ Where does r-process happen?

**Supernova** Neutron star merger (NSM)



GW170817 NSM :

One of the main r-process sites

Nature **551**,64; 67; 75; 80 (2017)

Science **358**, 1559 (2017)

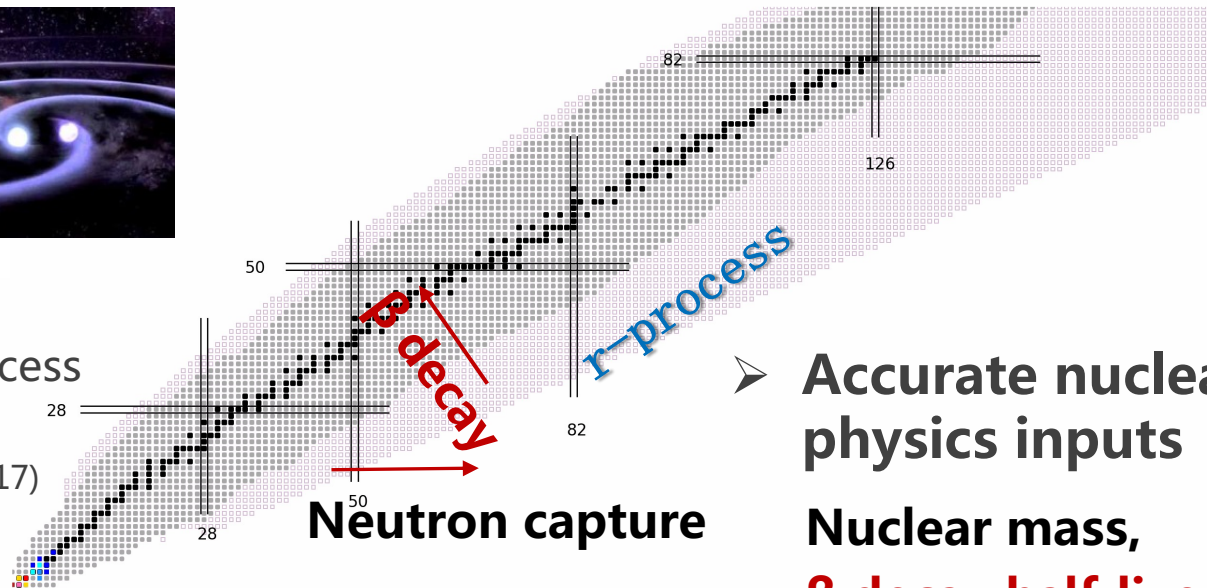
ApJL **848**, L17; L19 (2017)

NSM:

minimal contribution?

Nature **574**, 497; **569**,241 (2019)

arXiv:2102.05891v1



➤ Accurate nuclear physics inputs

Nuclear mass,  
 **$\beta$  decay half-lives,**  
Neutron-capture cross section,

...

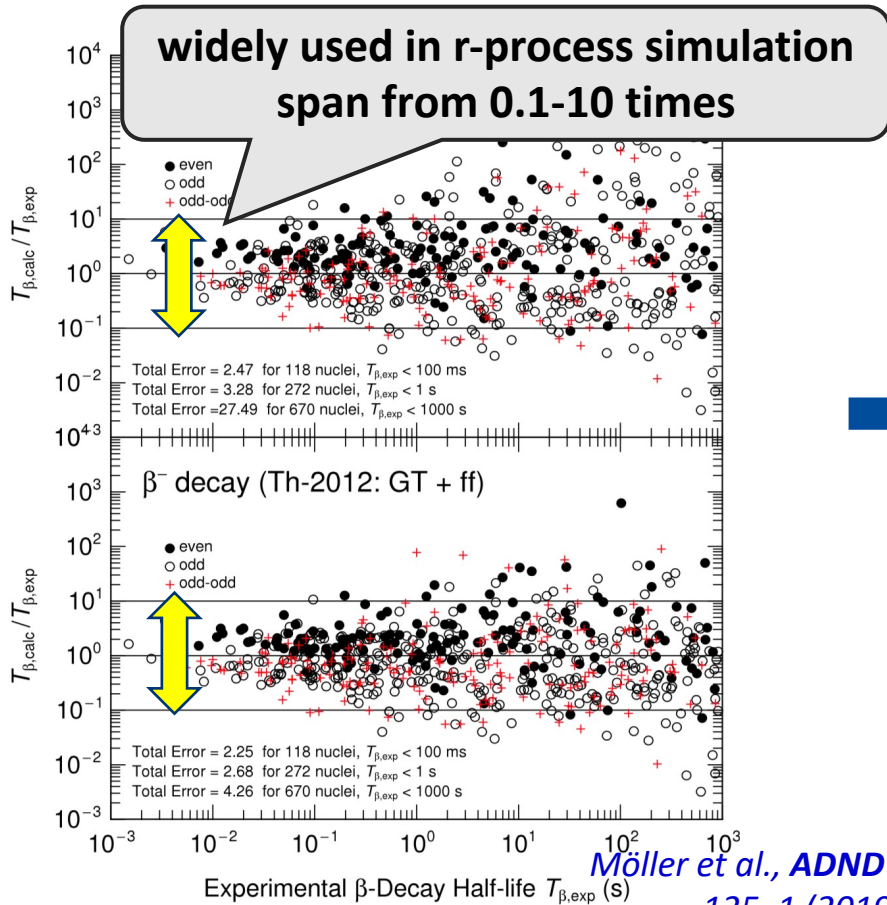
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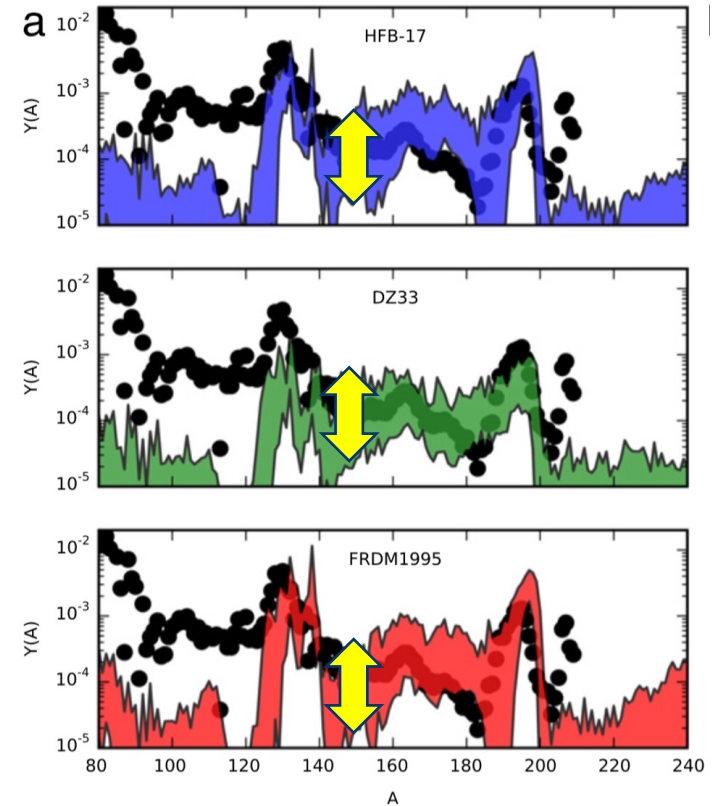
- How were heavy elements from iron to uranium made?
- **Accurate Nuclear Physics Inputs for r-process**
  - $\beta$ -decay half-lives
- Where does r-process happen?
  - Electron capture rates in core-collapse supernova
- Summary and Perspectives

# Consequences of uncertainties in $\beta$ -decay half-lives

- $\beta$ -decay half-lives by finite-range droplet model (FRDM) + quasiparticle random phase approximation (QRPA)
- Uncertainties caused by varying half-lives between  $10^{-1}$  - 10 times randomly



R-process abundance

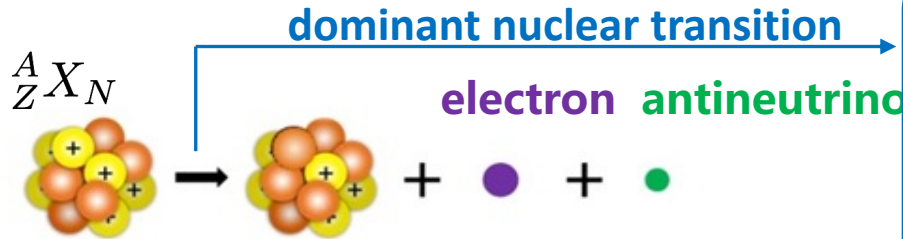


Mumpower et al., *PPNP* 86, 86 (2016)

Accurate description of nuclear  $\beta$ -decay half-lives is important for r-process study

# $\beta$ -decay Half-life Is a Hard Problem

- $\beta$ -decay



## Gamow-Teller transition

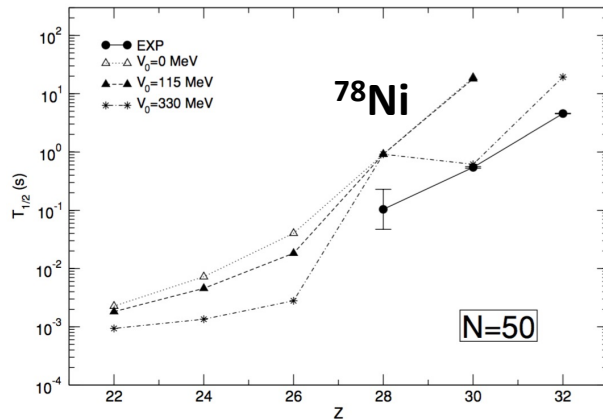
$T^-$   $\Delta S=1 \Delta L=0 \Delta T=1$

$\hat{O}_{GT^-} = \sum_{i=1}^A \vec{\sigma}(i) \cdot \tau_-(i)$

$Z+1, N-1$   $Z, N$

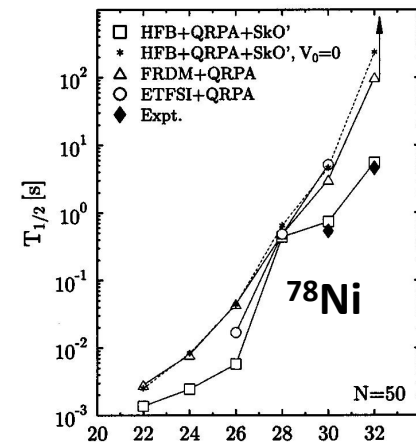
- RHB+QRPA

(quasiparticle random phase approximation)



Niksic, et al., PRC 71, 014308 (2005)

- Skyrme HFB+QRPA



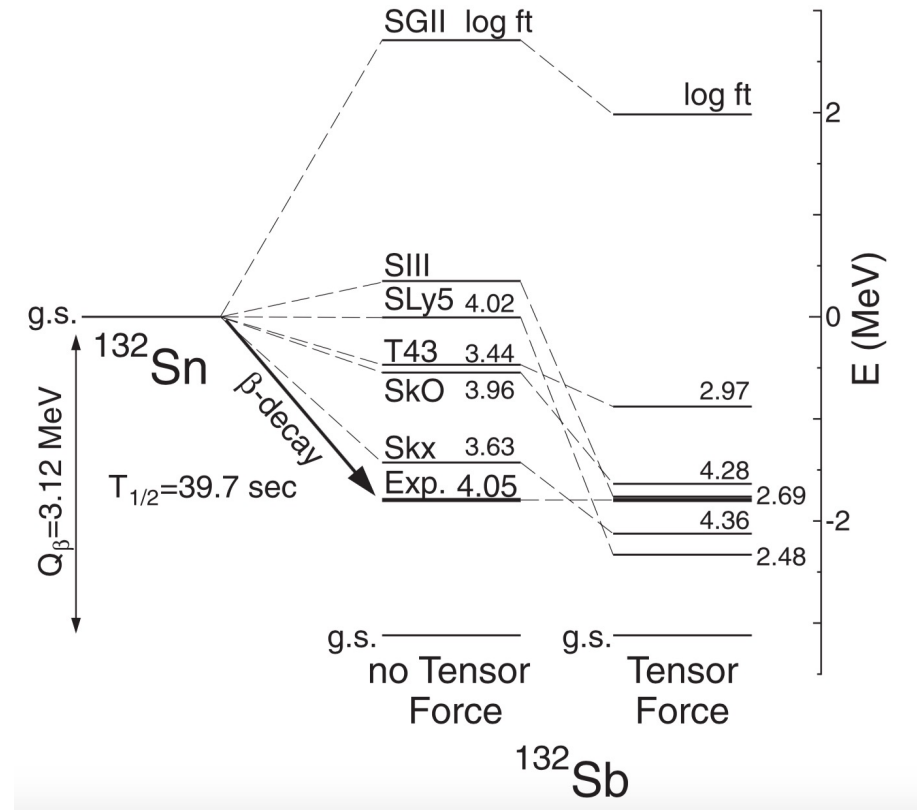
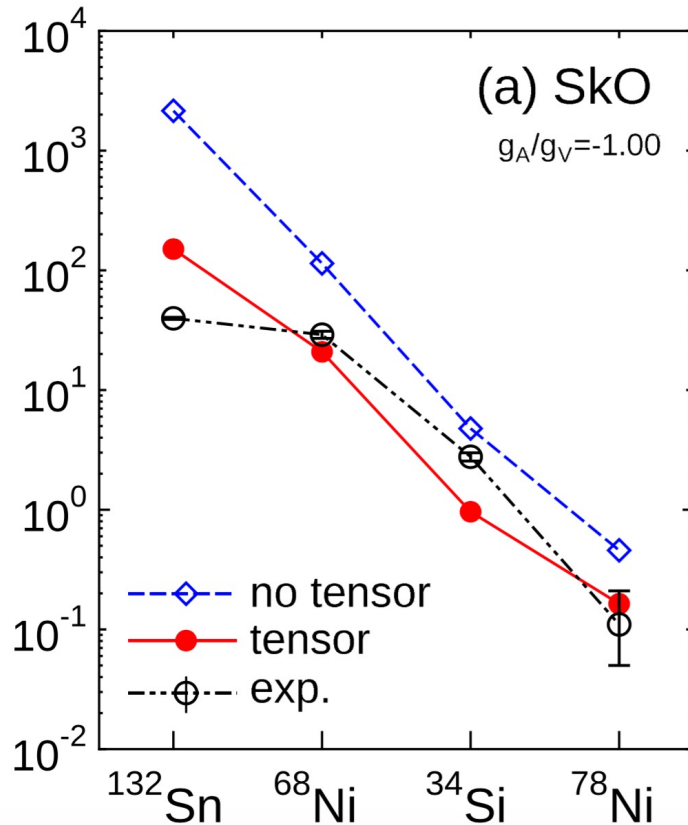
Engel, et al., PRC 60, 014302 (1999)

- Half-lives are **overestimated**
- Due to the nuclear structure part – Gamow-Teller transition

# What are missing in the calculation?

- Nuclear Force: tensor force**      **skyrme HF + RPA + tensor**

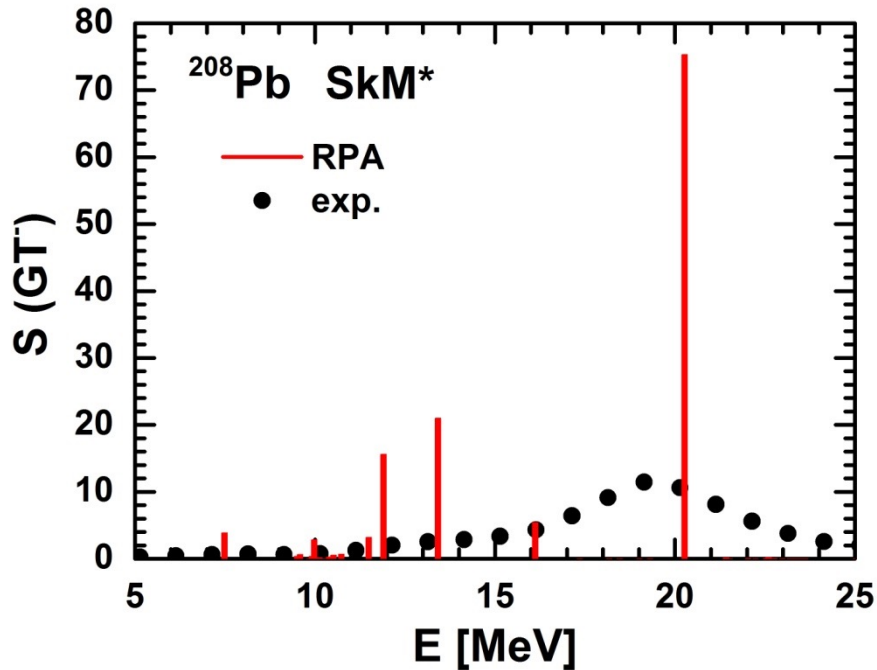
Minato and Bai, PRL 116, 089902 (2016)



- Nuclear Model: more correlations beyond RPA model**

# Limits of (Q)RPA Description

- (Q)RPA cannot describe the spreading width



- Spreading Width (Damping Width)  
energy and angular momentum of coherent vibrations  
→ more complicated states of 2p-2h, 3p-3h, ... character

- Correlations beyond RPA

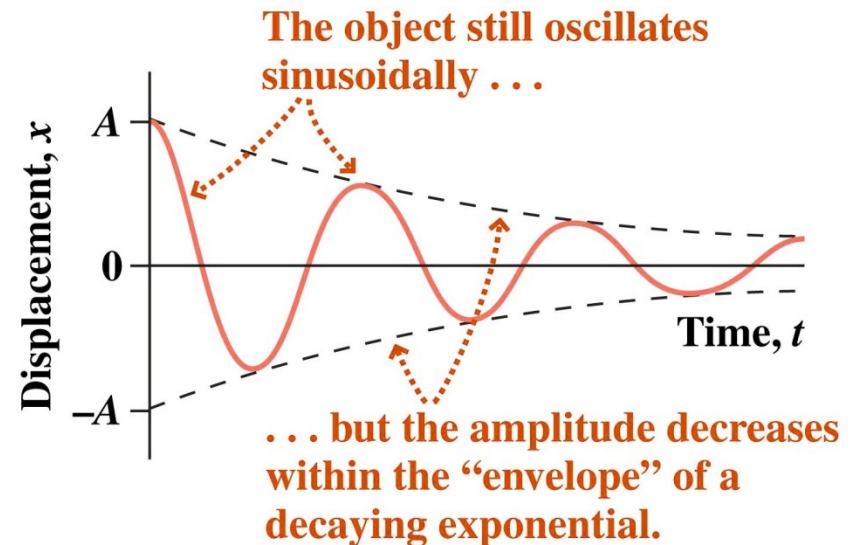
HF

RPA


1p1h

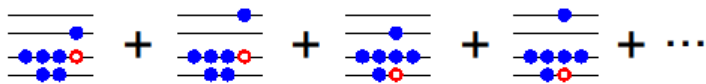
2p2h

...

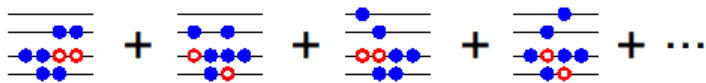


# Solution: RPA + PVC

HF 

1p1h 

**RPA**

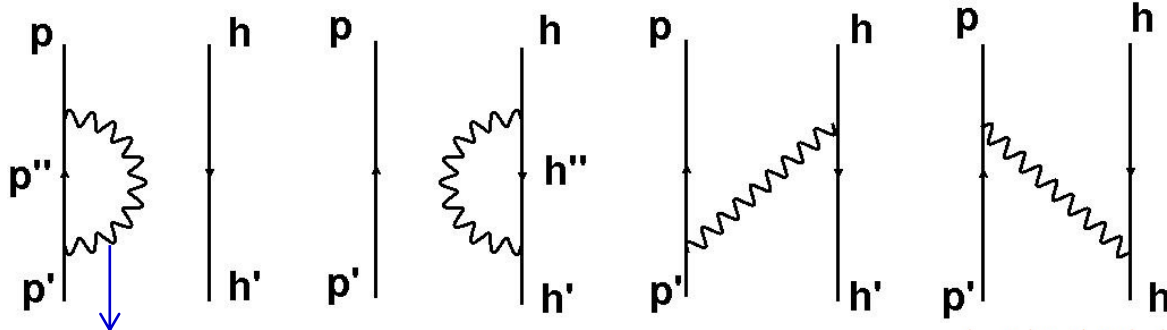
2p2h 

• **Second RPA** drozd et al., PR 197, 1 (1990)

Gambacurta et al., PRL 125, 212501 (2020)

• **RPA + PVC (particle vibration coupling)**

⋮



Low-lying vibration  
phonons  $|N\rangle$

$$W_{ph,p'h'}^{\downarrow}(\omega) = \sum_N \frac{\langle ph|V|N\rangle \langle N|V|p'h'\rangle}{\omega - \omega_N}$$

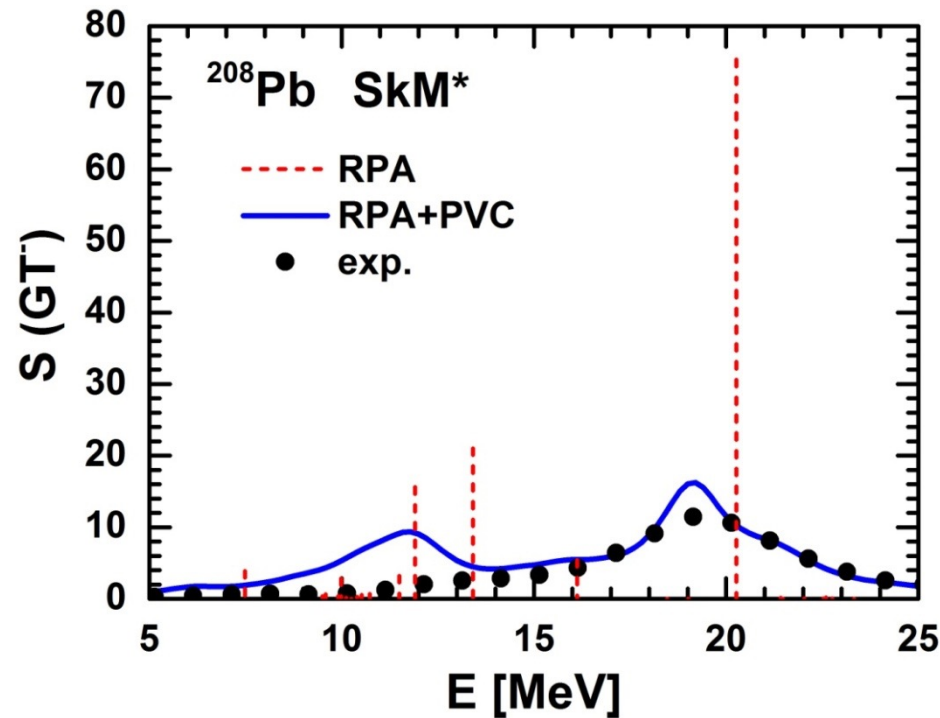
- RPA+PVC model based on Skyrme DFT

Colo et al., PRC 50, 1496 (1994); Niu et al., PRC 85, 034314 (2012)

- RPA+PVC model based on relativistic DFT Litvinova et al., PRC 75,064308 (2007)

# Gamow-Teller Resonance

- Improved description of GT resonance in  $^{208}\text{Pb}$

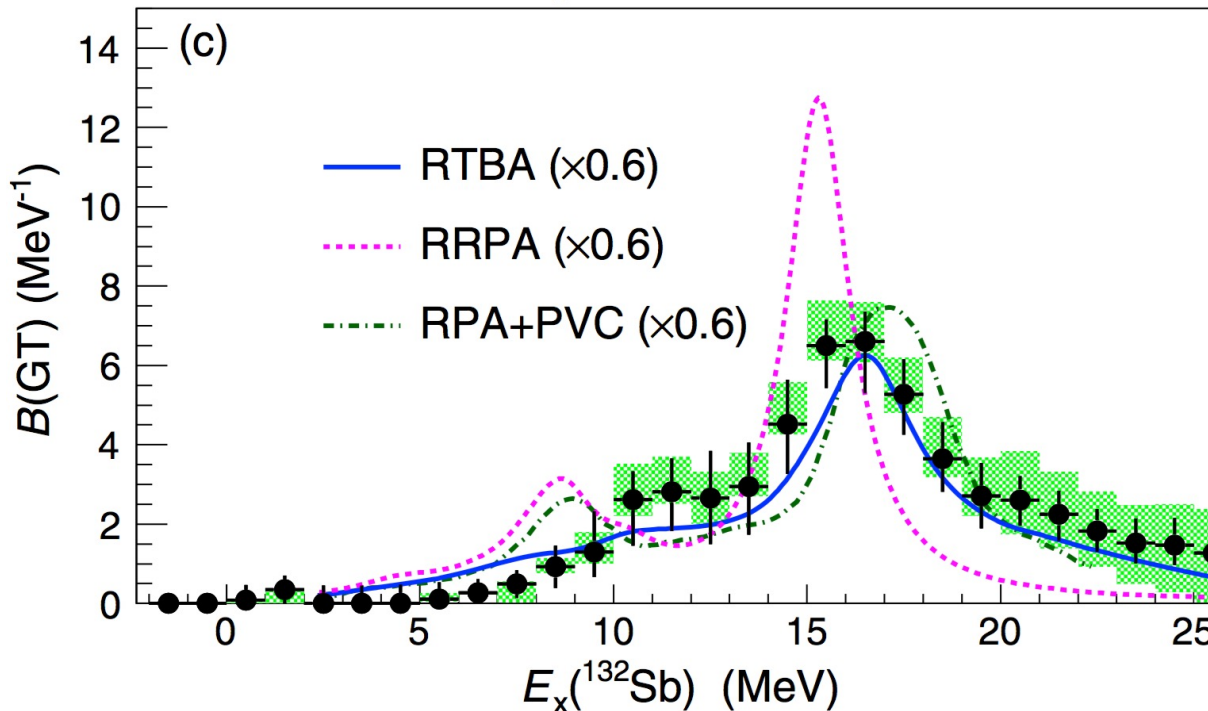


- ✓ Develop a spreading width
- ✓ Reproduce resonance lineshape

Y. F. Niu, G. Colo, and E. Vigezzi, *PRC* 90, 054328 (2014)

# Gamow-Teller Resonance

- Improved description of GT resonance in unstable nucleus  $^{132}\text{Sn}$



Yasuda, Sasano, et al., PRL 121, 132501 (2018)

**Exp:** (p,n) reaction @ RIBF, RIKEN

Yasuda, Sasano, et al., PRL 121, 132501 (2018)

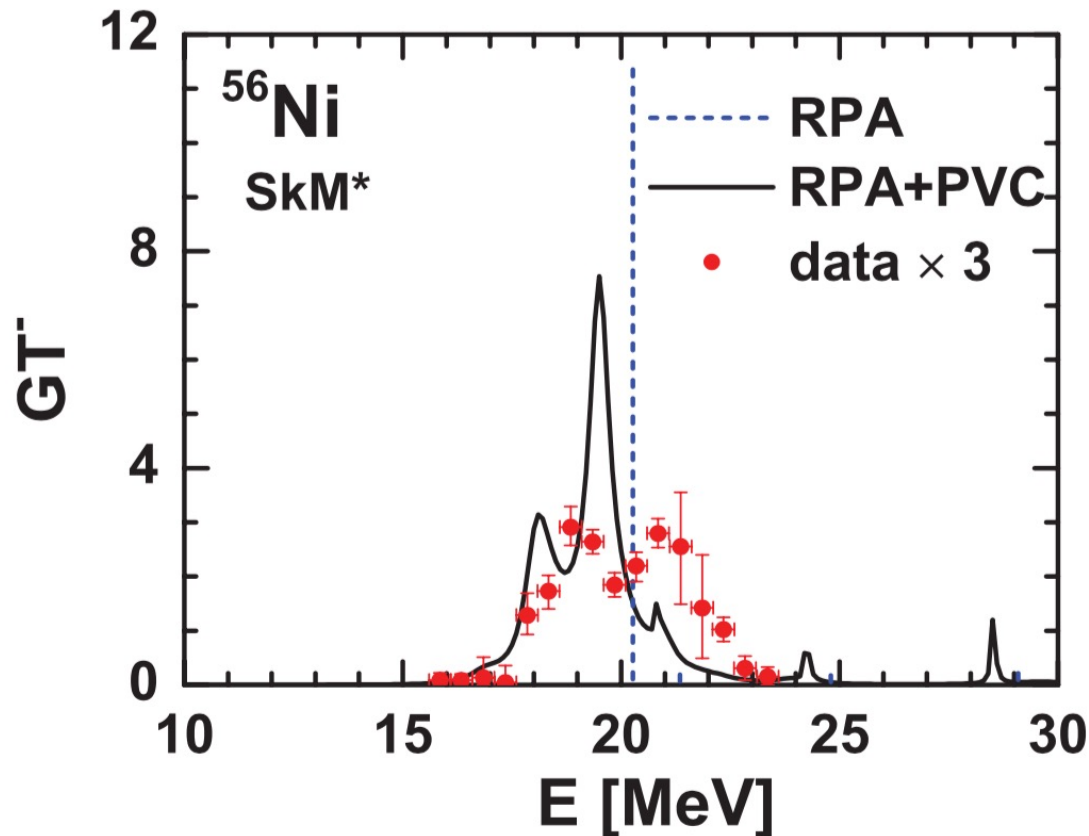
**RPA+PVC:** Y. F. Niu, G. Colo and E. Vigezzi, PRC 90, 054328 (2014)

**RTBA:** E. Litvinova et al., PLB 730, 307 (2014)

**RRPA:** H. Z. Liang, and Z. M. Niu private communication

# Gamow-Teller Resonance

- Reproduction of double-peak structure of GT resonance in  $^{56}\text{Ni}$

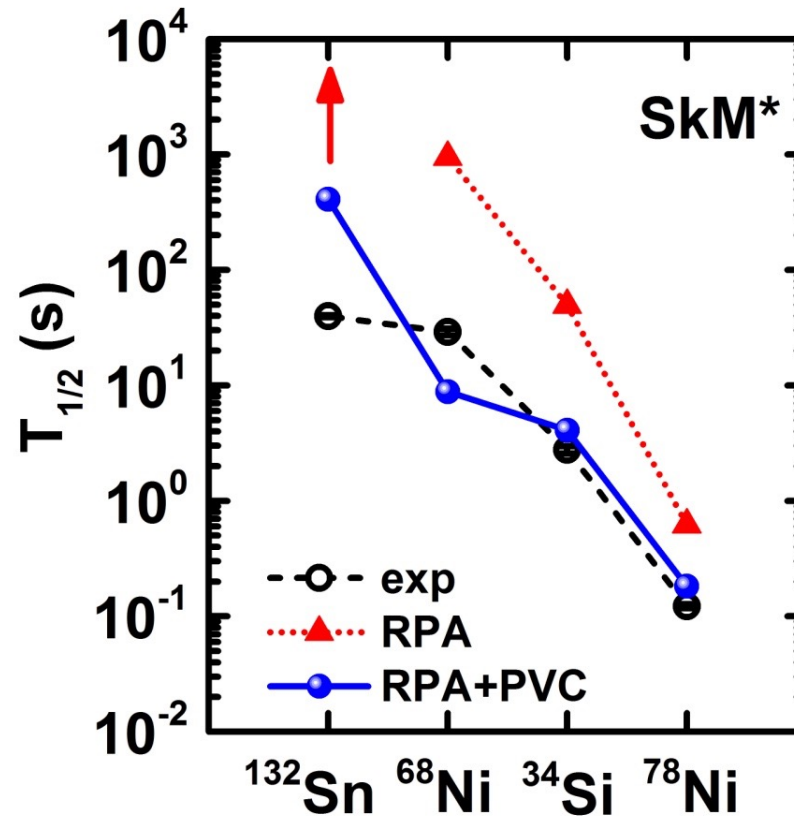


Y. F. Niu, G. Colo, M. Brenna, P.F. Bortignon, and J. Meng, **PRC** 85, 034314 (2012)

**Exp:** (p,n) reaction with  $T_p=296$  MeV @ NSCL, MSU  
Sasano et al., **PRL** 107, 202501 (2011)

# $\beta$ -Decay Half-Lives in Magic Nuclei

- Improved description of  $\beta$ -decay half-lives



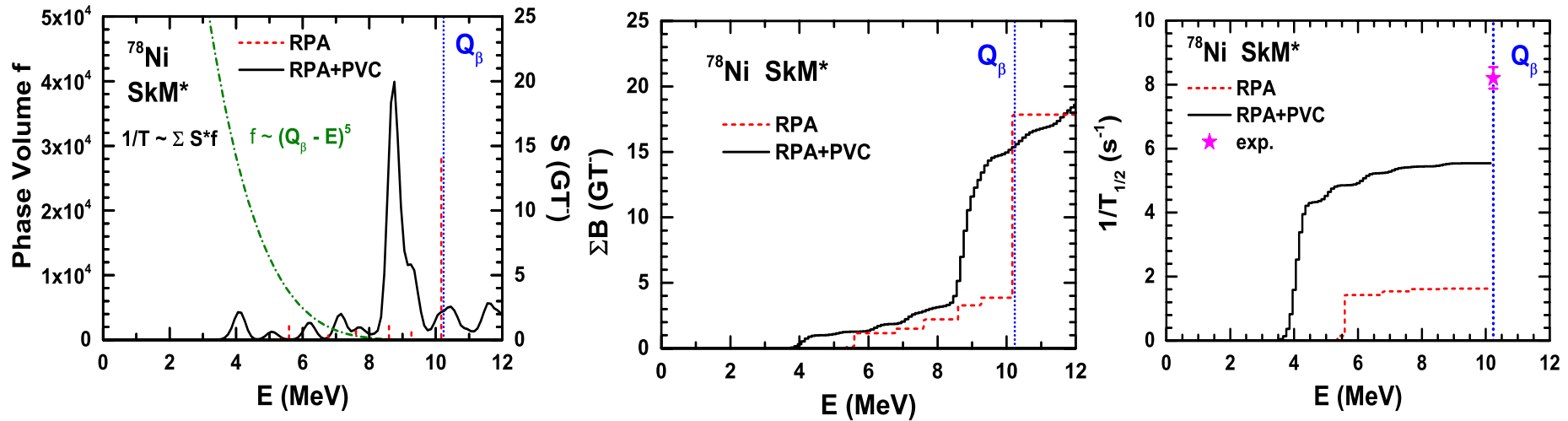
✓ Reduce half-lives systematically

✓ Reproduce  $\beta$ -decay half-lives

Y.F. Niu, Z. M. Niu, G. Colo, and E. Vigezzi, **PRL** 114, 142501 (2015)

**Exp:** G. Audi, F. G. Kondev, M. Wang, W. J. Huang, and S. Naimi, **CPC** 41, 030001 (2017)

# How PVC reduces half-lives?



Exp.: Xu, et al., PRL 113, 032505, 2014

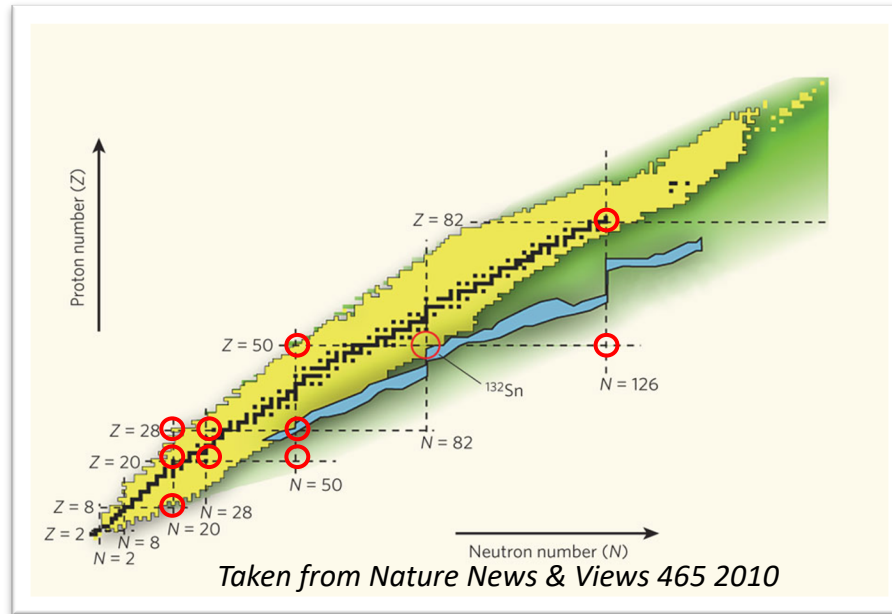
- Half-life

$$T_{1/2} = \frac{D}{g_A^2 \int^{Q_\beta} S(E) f(Z, \omega) dE},$$

- Phase Volume

$$f(Z, \omega_0) = \frac{1}{(m_e c^2)^5} \int_{m_e c^2}^{\omega_0} p_e E_e (\omega_0 - E_e)^2 F_0(Z + 1, E_e) dE_e.$$

# RPA+PVC: only for magic nuclei...

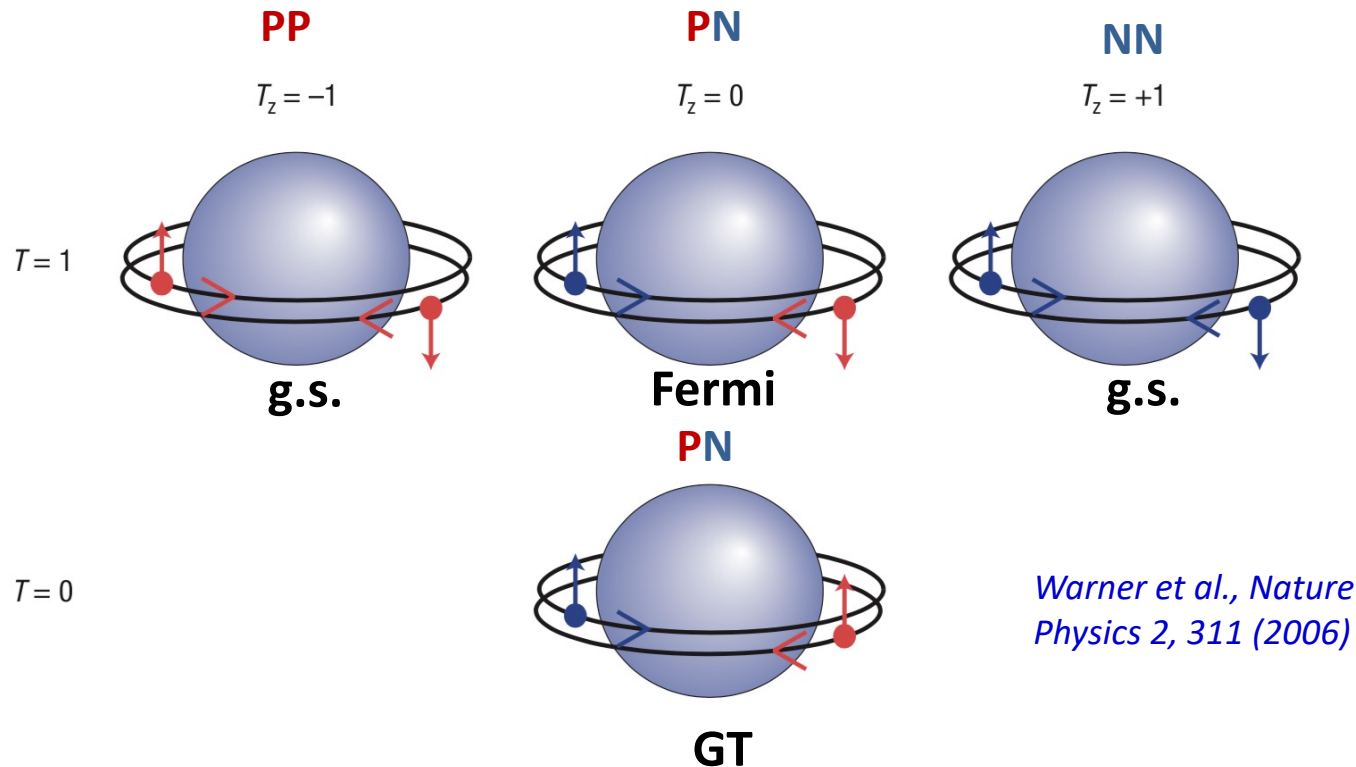


➤ To include pairing correlations for superfluid nuclei

Quasiparticle RPA + quasiparticle vibration coupling  
(**QRPA**) + (**QPVC**)

- ✓ for the study of Gamow-Teller resonance in superfluid nuclei
- ✓ for the study of  $\beta$ -decay half-lives in the whole isotopic chain

# Isvector and isoscalar pairing



- Isvector Pairing**

$$V_{T=1}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \frac{1 - P_\sigma}{2} \left( 1 - \frac{\rho(\mathbf{r})}{\rho_0} \right) \delta(\mathbf{r}_1 - \mathbf{r}_2),$$

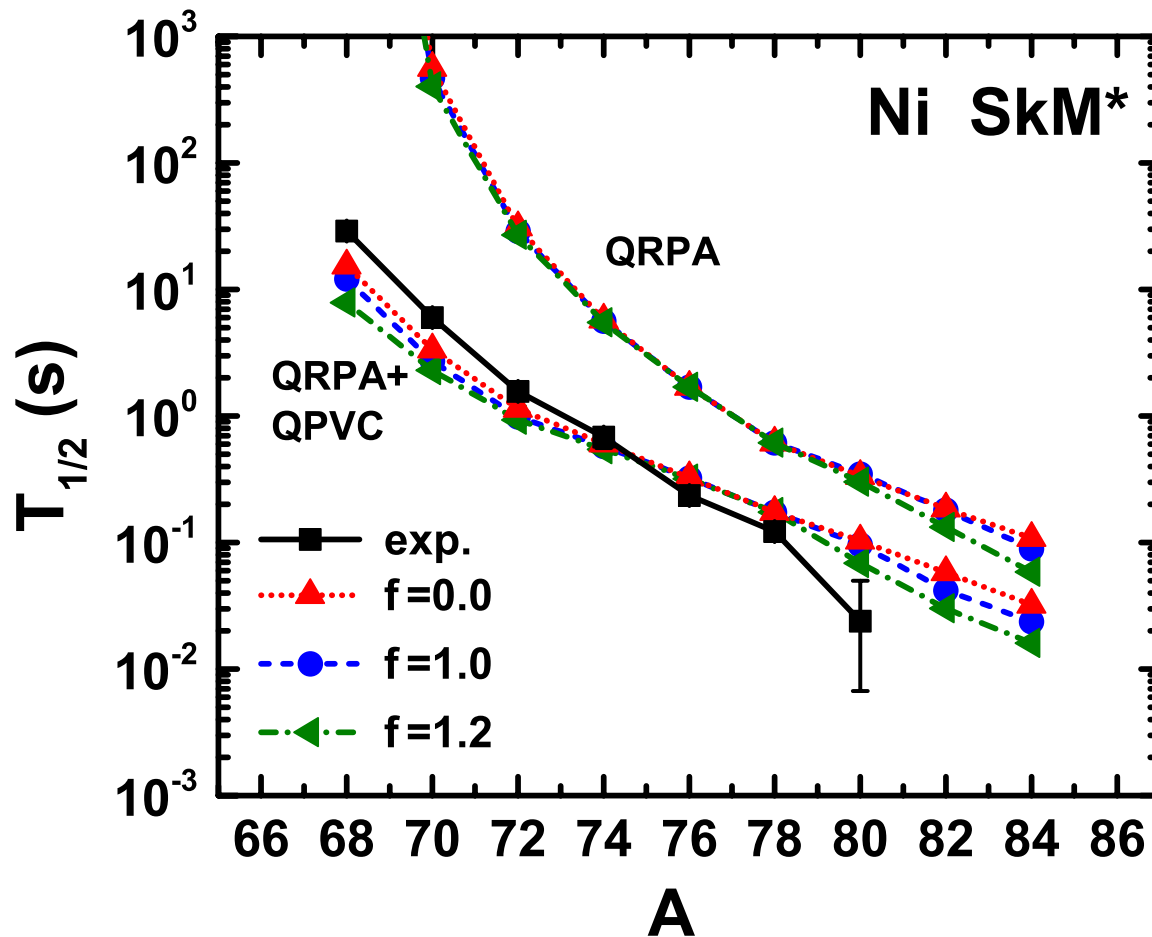
For ground state: pairing strength adjusted to reproduce empirical pairing gap

- Isoscalar Pairing**

$$V_{T=0}(\mathbf{r}_1, \mathbf{r}_2) = f V_0 \frac{1 + P_\sigma}{2} \left( 1 - \frac{\rho(\mathbf{r})}{\rho_0} \right) \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

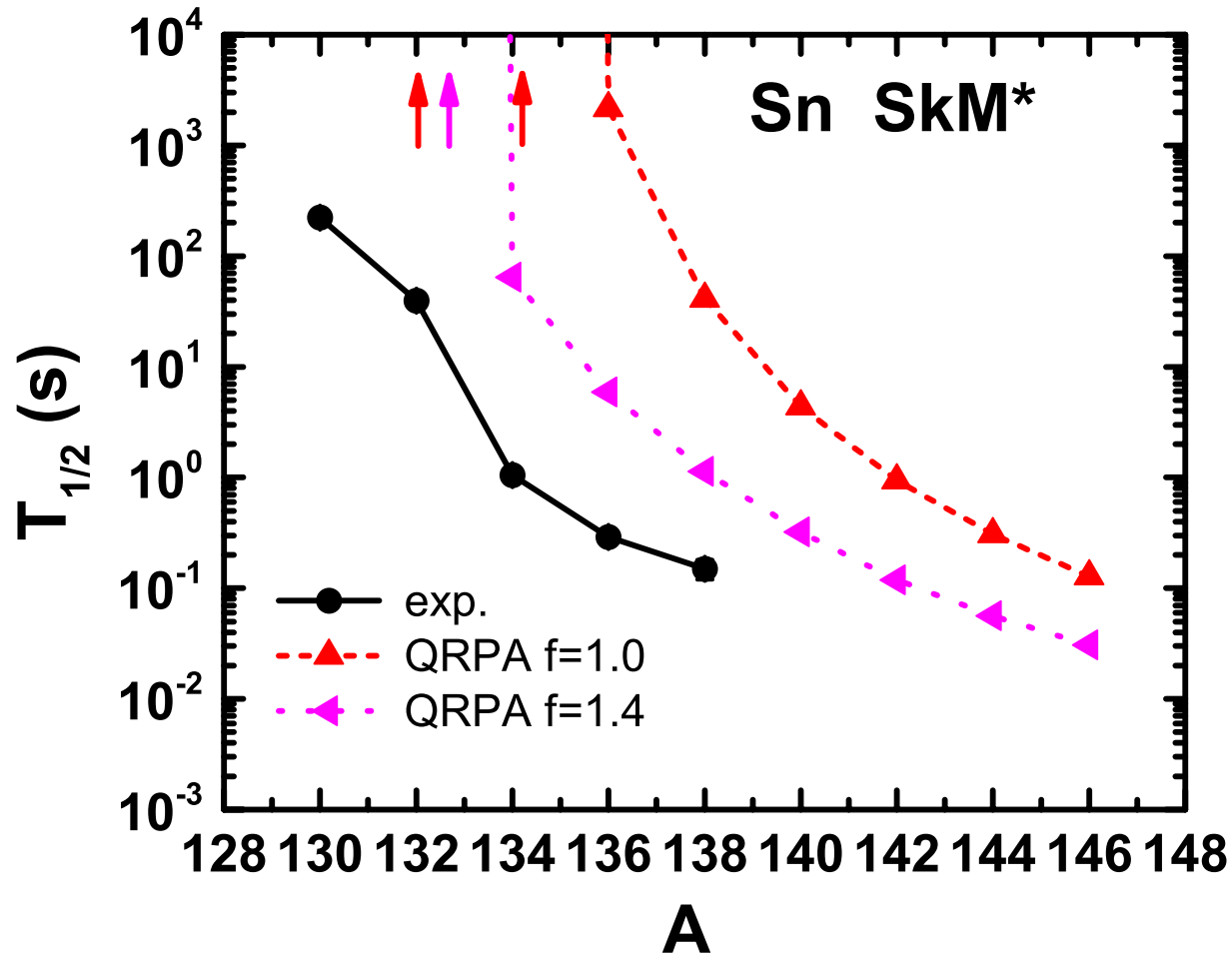
For GT: the same form as IV pairing, but with an adjustable pairing strength  $f$

# $\beta$ -Decay Half-Lives in Ni isotopes



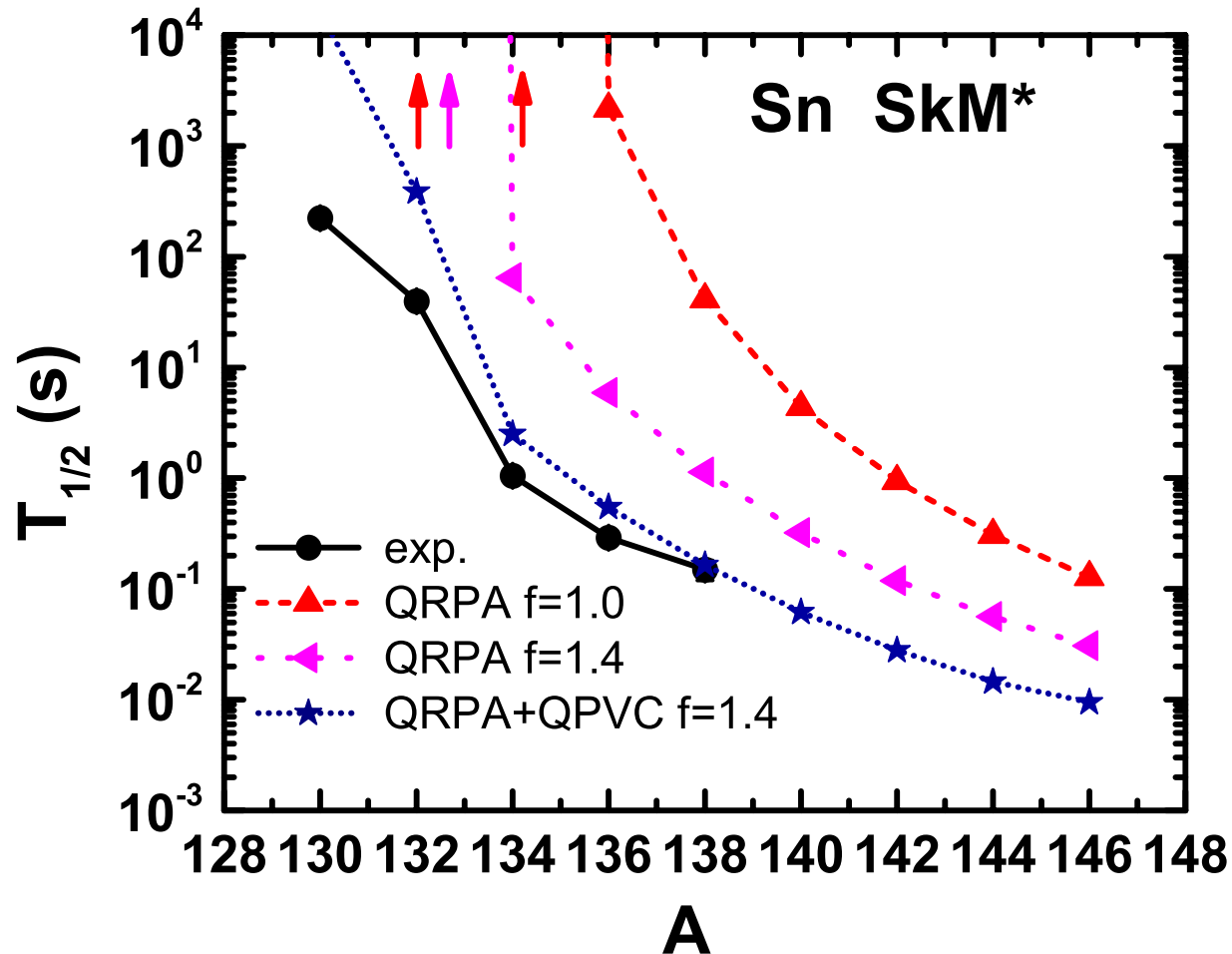
- Isoscalar Pairing:
  - similar at QRPA and QRPA+QPVC level
  - not so effective for Ni isotopes (nuclei before N=50 closed shell)
- QPVC:
  - reduce the half-lives

# $\beta$ -Decay Half-Lives in Sn isotopes



- Isoscalar Pairing:  
effective for Sn isotopes  
(nuclei above  $N=82$  closed shell)

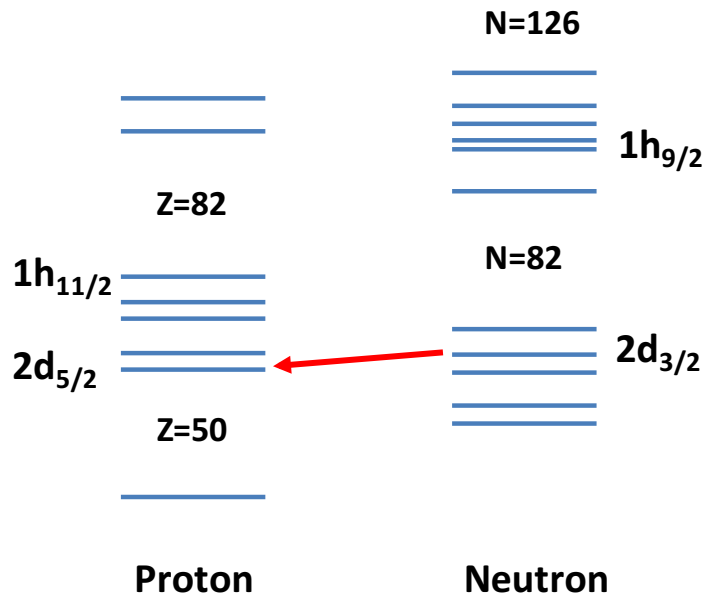
# $\beta$ -Decay Half-Lives in Sn isotopes



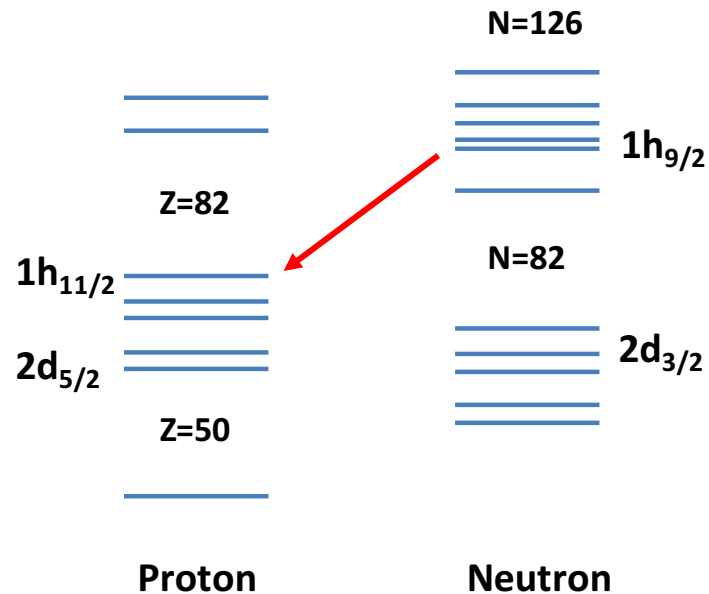
- Isoscalar Pairing:  
effective for Sn isotopes  
(nuclei above  $N=82$  closed shell)
- Isoscalar Pairing + QPVC:  
reduce the half-lives

# The role of IS pairing

- N < 82**



- N > 82**



$$V^{pp}(2d_{3/2} 2d_{5/2}, 2d_{3/2} 2d_{5/2})$$

$$-0.50 \text{ } (^{130}\text{Sn})$$

$$u^2(2d_{3/2})u^2(2d_{5/2}) + v^2(2d_{3/2})v^2(2d_{5/2})$$

$$0.069 \text{ } (^{130}\text{Sn})$$

ph type

$$V^{pp}(1h_{9/2} 1h_{11/2}, 1h_{9/2} 1h_{11/2})$$

$$-1.68 \text{ } (^{144}\text{Sn})$$

$$u^2(1h_{9/2})u^2(1h_{11/2}) + v^2(1h_{9/2})v^2(1h_{11/2})$$

$$0.89 \text{ } (^{144}\text{Sn})$$

pp type

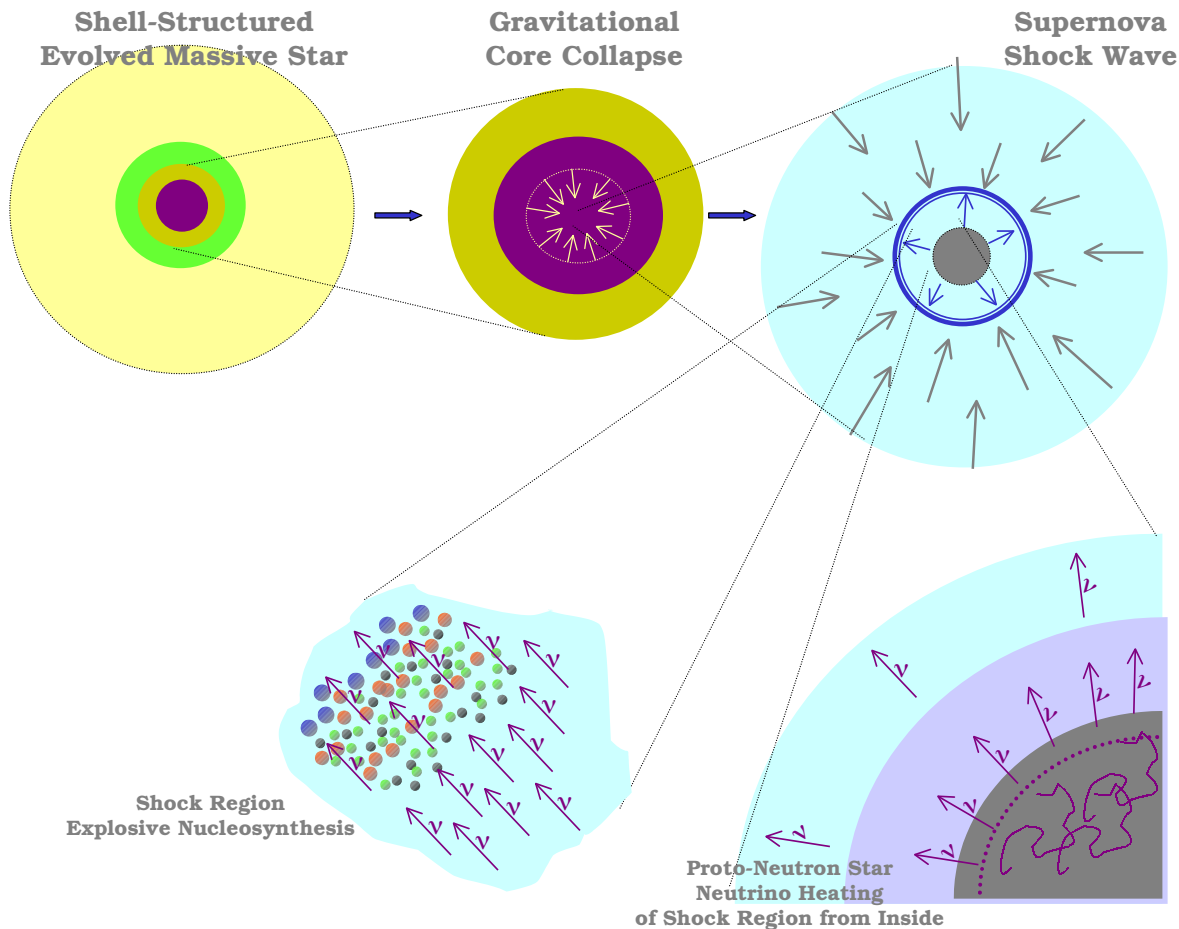
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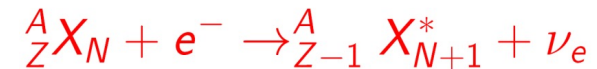
- How were heavy elements from iron to uranium made?
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- **Where does r-process happen?**
  - **Electron capture rates in core-collapse supernova**
- Summary and Perspectives

# Electron capture in core-collapse supernova

## Collapse of a massive star and a supernova explosion



## Electron capture (EC) on nucleus



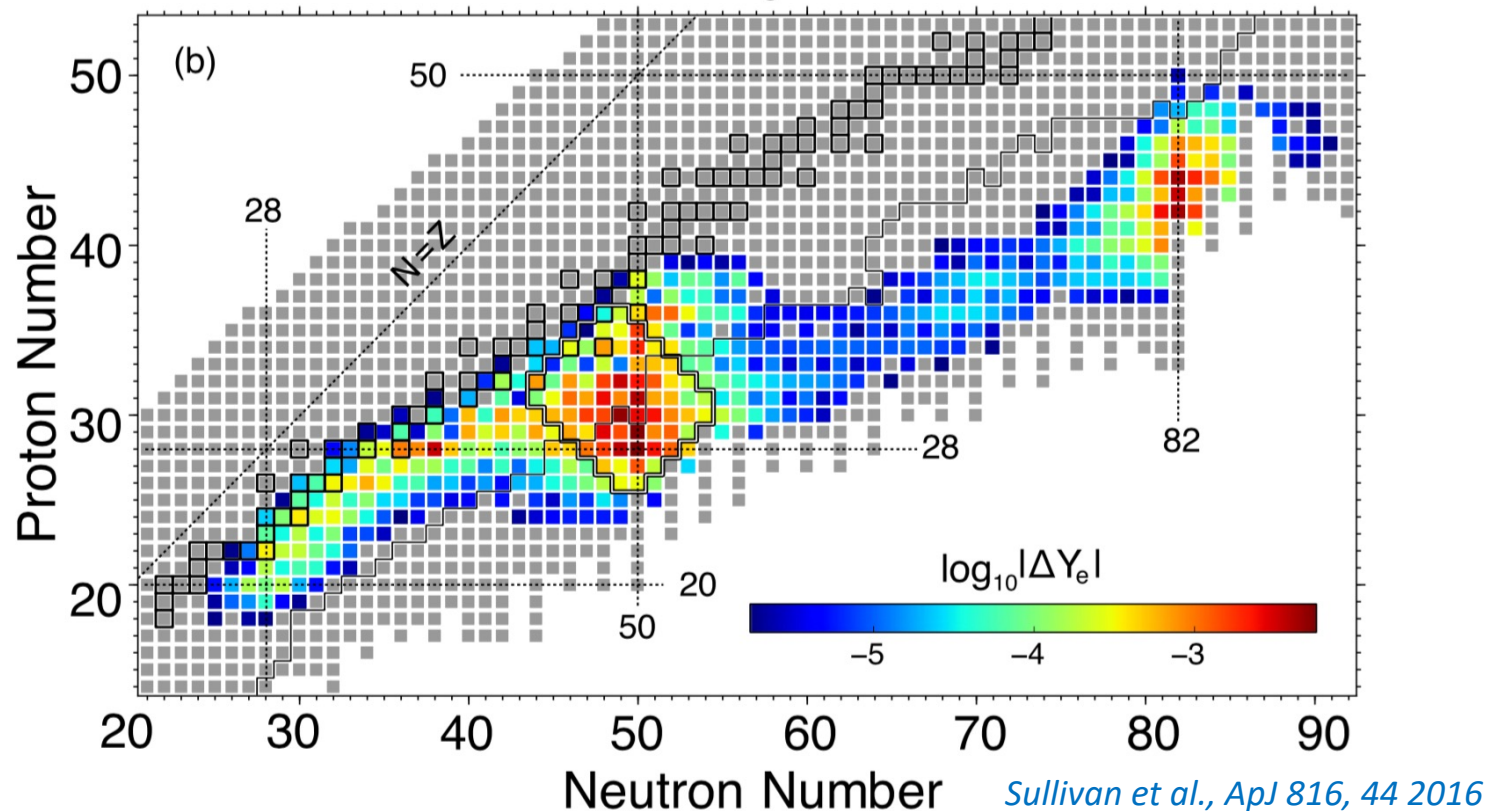
change  $Y_e$   
change entropy



affect the strength  
of bounce shock  
and supernova  
evolution

# Important electron-capturing nuclei

Top 500 electron-capturing nuclei with the largest absolute change to the electron fraction ( $Y_e$ ) up to neutrino trapping



- The integrated contribution to core deleptonization up to neutrino trapping

$$Y_e(t = t_{\text{trapping}}) \simeq Y_e(t = 0) - \sum_i \Delta Y_e^i$$

- Primary contributors: neutron rich nuclei near **N=50** and N=82 closed neutron shells

# Theoretical study of electron-capture rates

## Electron Capture Rate:

$$\lambda^{\text{ec}} = \frac{\ln 2}{6150 \text{ s}} \sum_J \sum_i \Phi_{Ji}^{(+)} F_i^{\text{ec}} = \sum_J \sum_i \lambda_{Ji}^{\text{ec}}$$

transition strength of **spin-isospin excitations** in  $T^+$  direction:  
Fermi, Gamow-Teller, Spin-Dipole transitions ...

phase space factor

- **Independent Particle Model (IPM)**

- ✓ **first tabulation of weak interaction rates  $21 \leq A \leq 60$**

*Fuller, Fowler, Newman ApJ 252, 715, 1982; ApJ 293, 1, 1985* **FFN**

- **Large Scale Shell Model**

- **sd shell nuclei  $17 \leq A \leq 39$**   $^{16}\text{O}$  core + effective interaction of Wildenthal

*Oda et al., ADNDT 56, 231, 1994* **ODA**

- **pf shell nuclei  $45 \leq A \leq 65$**  modified KB3 interaction

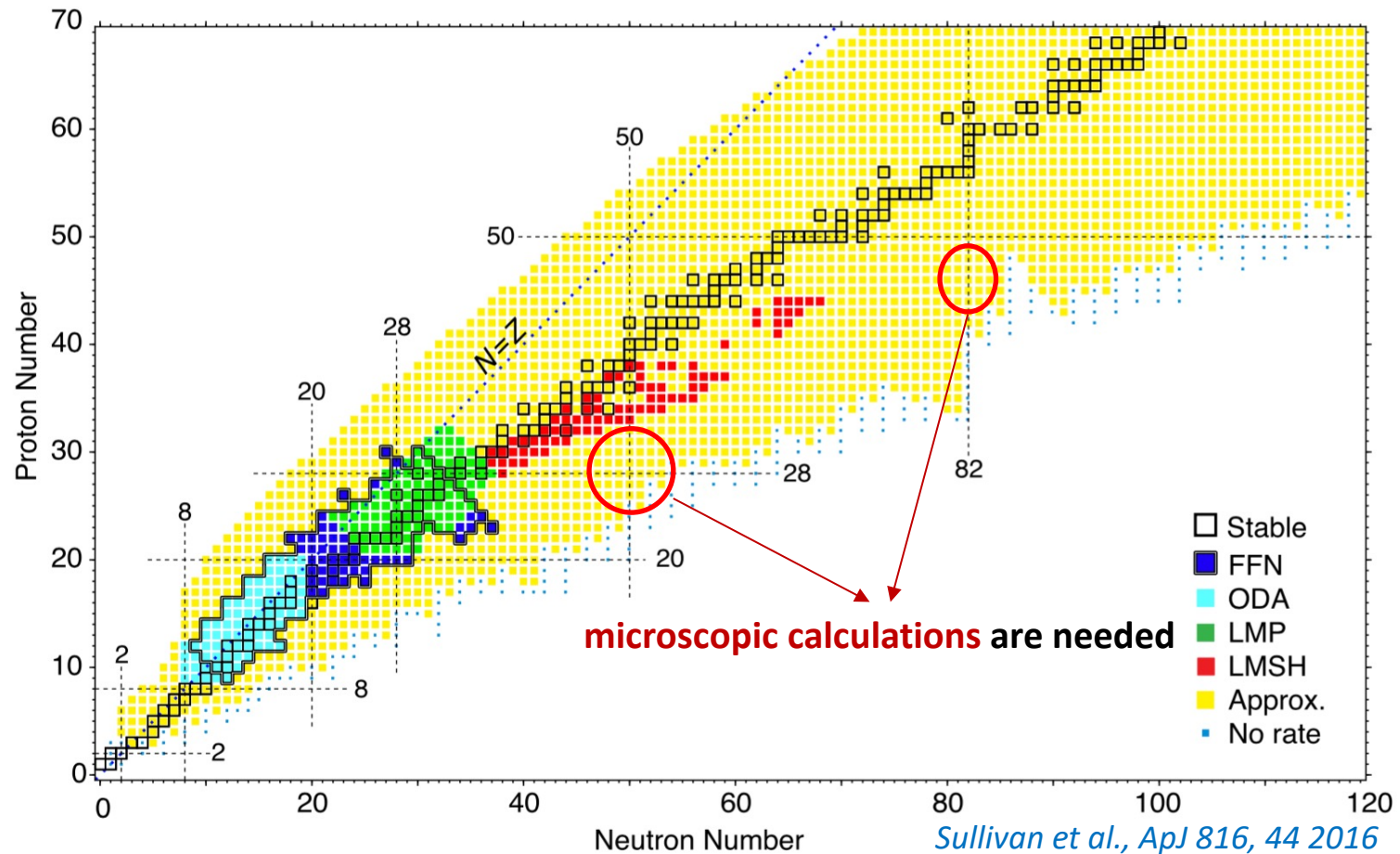
*Langanke and Martinez-Pinedo, NPA 673, 481 (2000); ADNDT 79, 1 (2001)* **LMP**

- **Hybrid Model**

- **Shell Model Monte Carlo (SMMC) + Random Phase Approximation(RPA)**  
**pfg/sdg shell nuclei  $65 \leq A \leq 112$**

*Langanke et al., PRL 90, 241102 (2003)* **LMSH**

# Theoretical study of electron-capture rates



- Approx. - Approximate Rates estimated by  $\lambda = \frac{(\ln 2)B}{K} \left( \frac{T}{m_e c^2} \right)^5 [F_4(\eta) - 2\chi F_3(\eta) + \chi^2 F_2(\eta)]$
- Fitted by shell model calculation for nuclei not far from stability line  
 $\Rightarrow$  For neutron rich nuclei, the formulas is not a good approximation

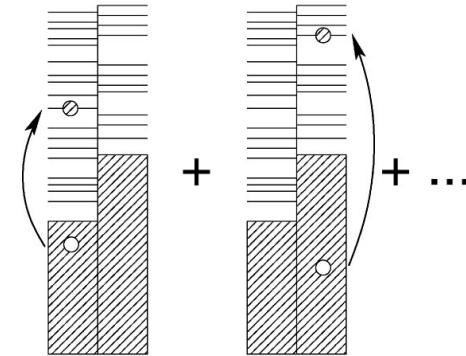
# Random Phase Approximation (RPA)

➤ **RPA:** widely used for the description of spin-isospin excitations

- The RPA excited state is generated by

$$Q_{\nu}^{\dagger} = \sum_{mi} X_{mi}^{\nu} a_m^{\dagger} a_i - \sum_{mi} Y_{mi} a_i^{\dagger} a_m$$

- Full 1p1h configuration space  $\Rightarrow$  almost whole nuclear chart



RPA

To study the electron capture in core-collapse supernova, inclusion of temperature effect is necessary! ( $T \sim 0 - 2$  MeV)

➤ **Finite Temperature RPA (FTRPA):** takes into account temperature self-consistently both in Hartree and RPA level

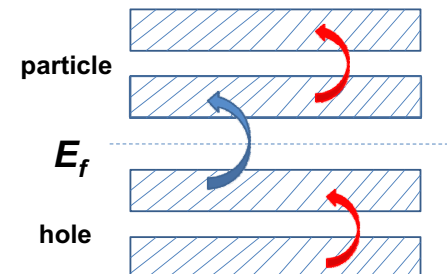
- Temperature is introduced by thermal occupation of each nucleon

$$f_{p(n)} = \frac{1}{1 + \exp\left(\frac{\epsilon_{p(n)} - \mu_{p(n)}}{kT}\right)}$$

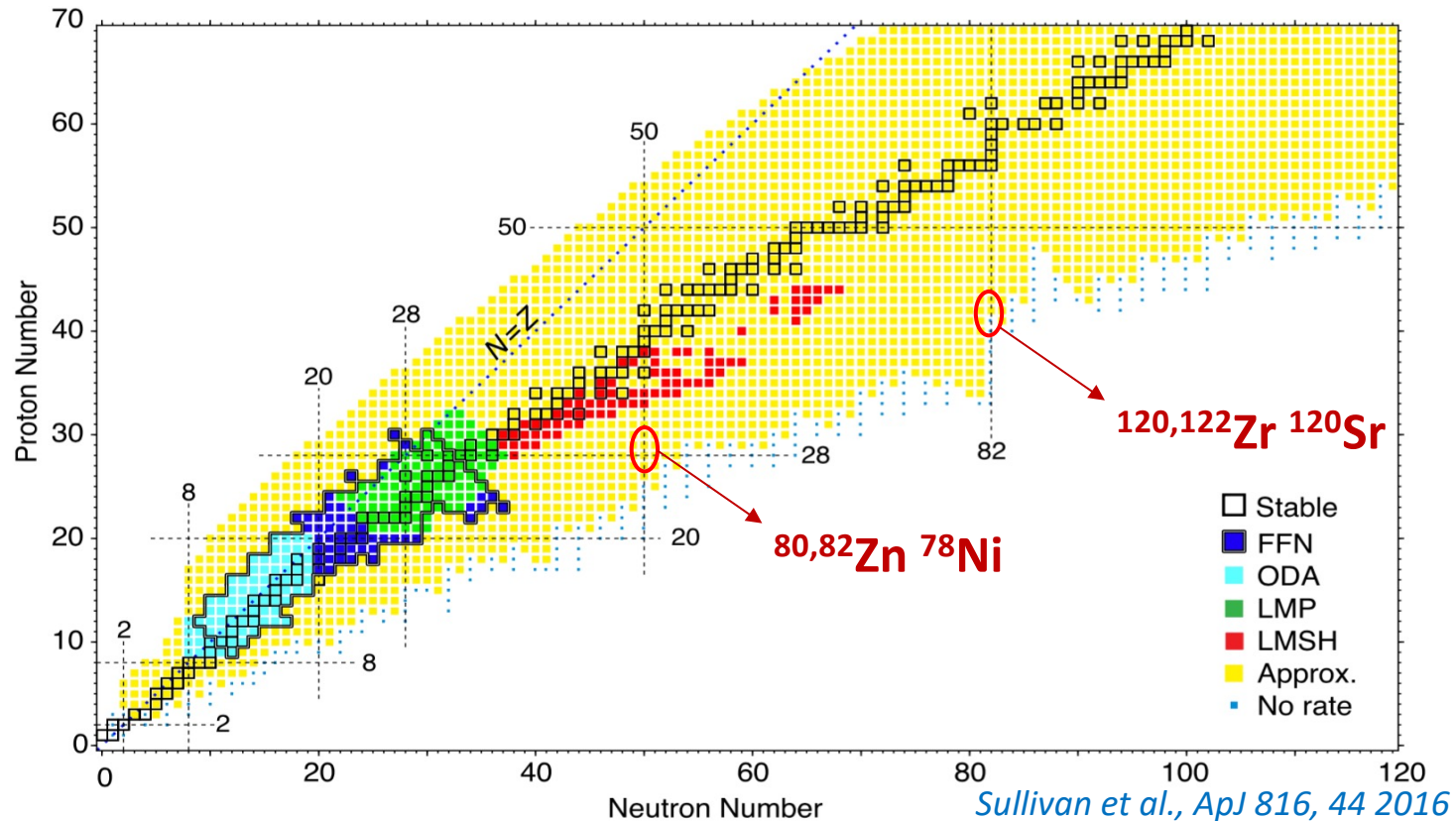
- Configuration space: p-h, p-p, and h-h pairs

*N. Paar et al., PRC 80, 055801 (2009)*

*Y. F. Niu et al., PLB 681, 315 (2009)*



# Electron capture study for important nuclei



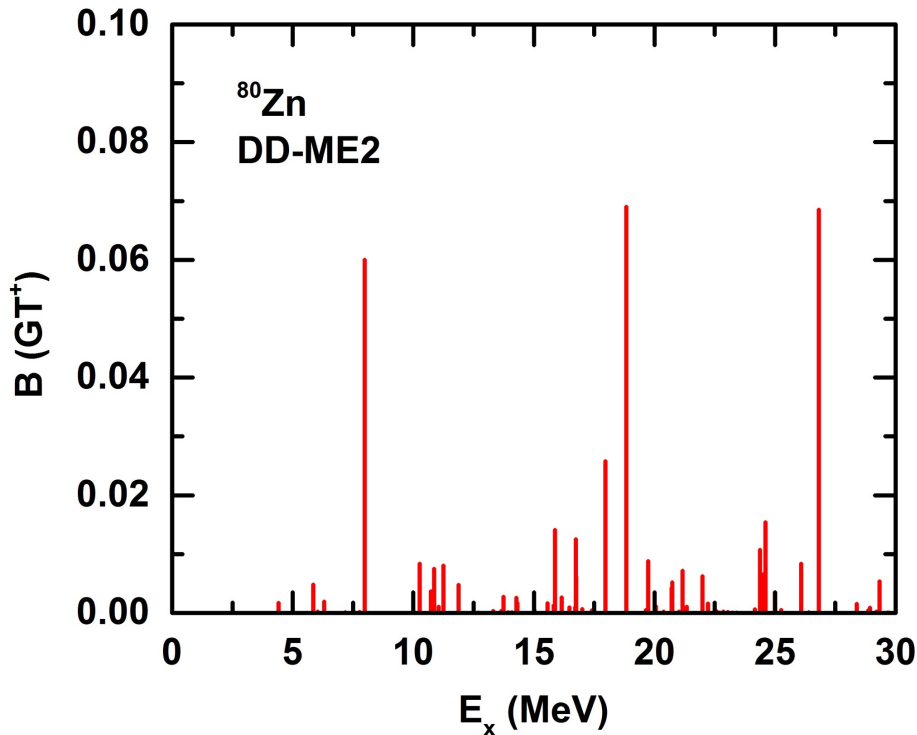
Finite temperature RPA (FTRPA) can provide a universal tool to study the electron capture for almost the whole nuclear chart, so the important nuclei for supernova explosion will be studied, including

$N \sim 50$ :  $^{78}\text{Ni}$   $^{80}\text{Zn}$   $^{82}\text{Zn}$

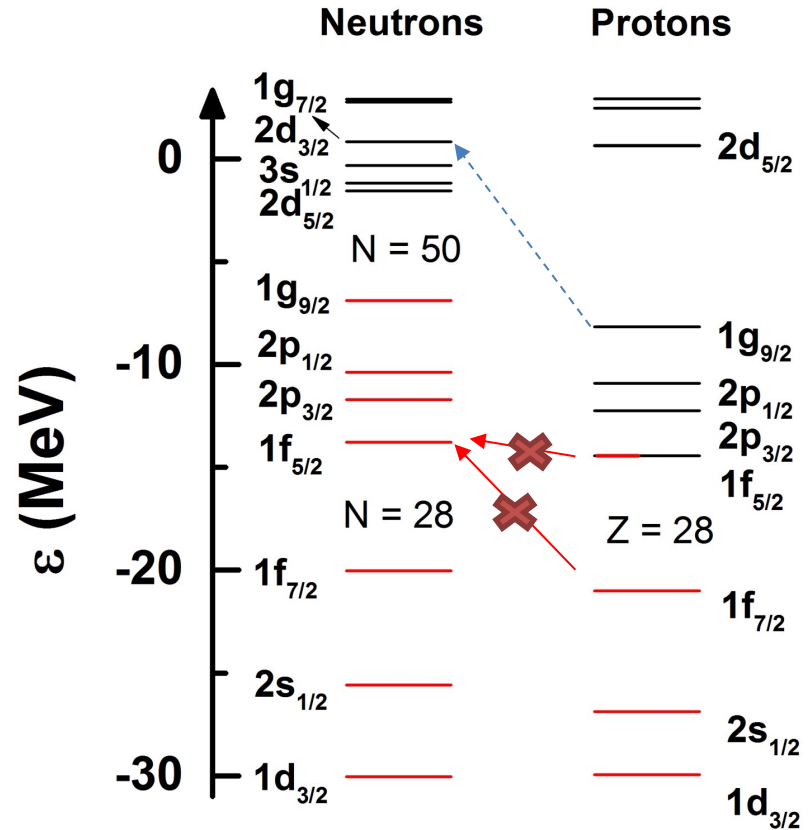
$N \sim 82$ :  $^{120}\text{Sr}$   $^{120}\text{Zr}$   $^{122}\text{Zr}$

# Gamow-Teller strength distribution ( $T^+$ )

- GT operator  $\hat{F}_{GT}^{\pm} = \sum_{i=1}^A \sigma(i) \tau_{\pm}(i)$   $J^{\pi} = 1^+$



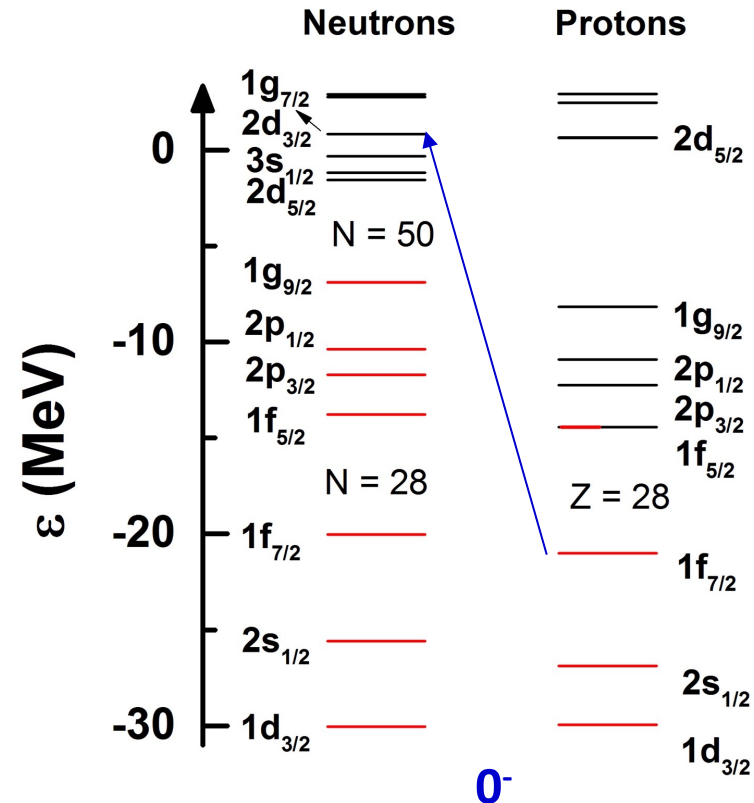
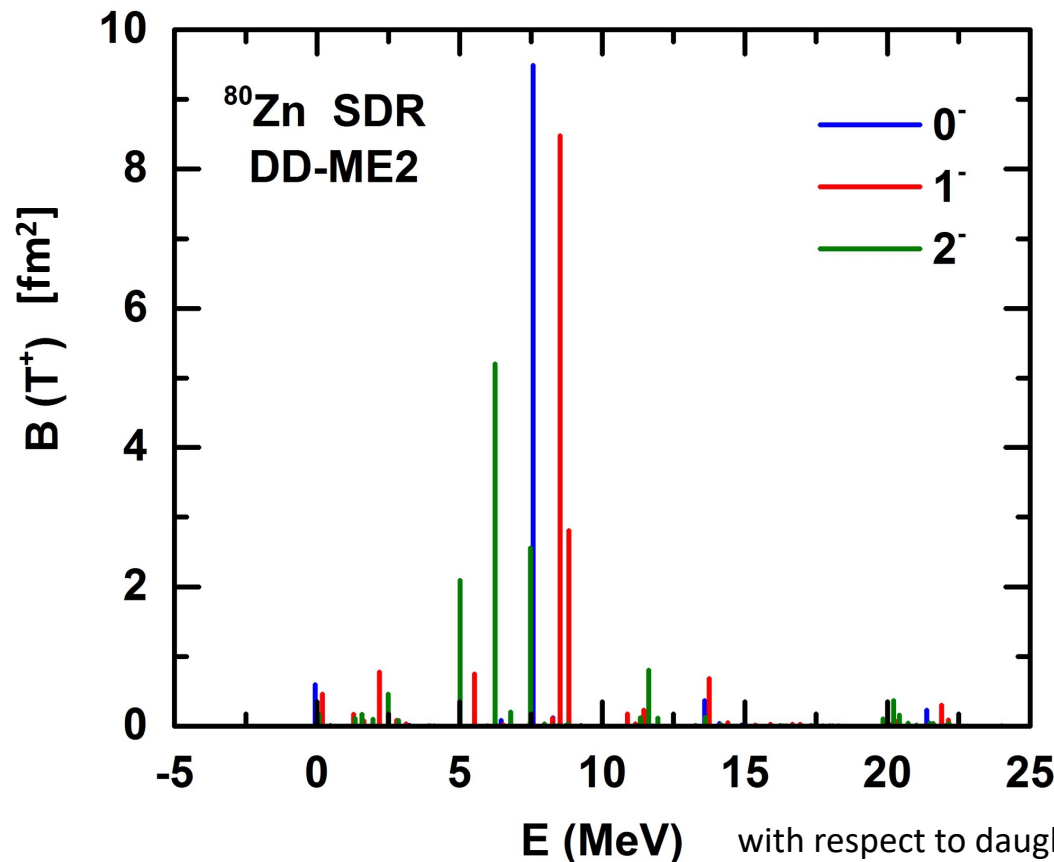
with respect to daughter nucleus  $^{80}\text{Cu}$



- ✓  $GT^+$  transitions are almost blocked (Ikeda sum rule = 60)
- ✓ Pairing correlations or transitions across major shells make little transition strength possible

# Spin-Dipole strength distribution ( $T^+$ )

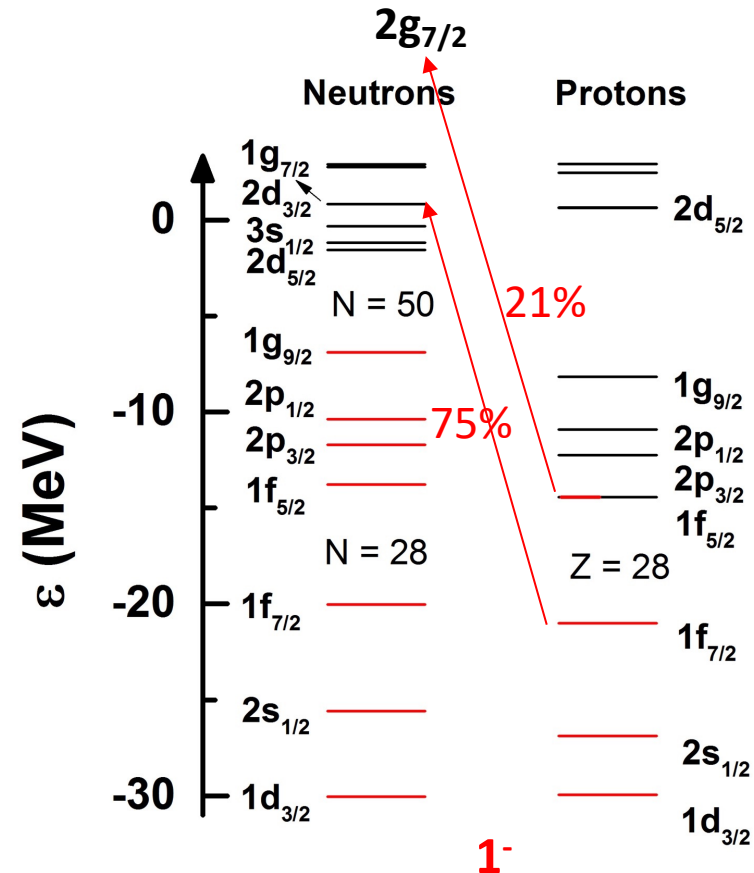
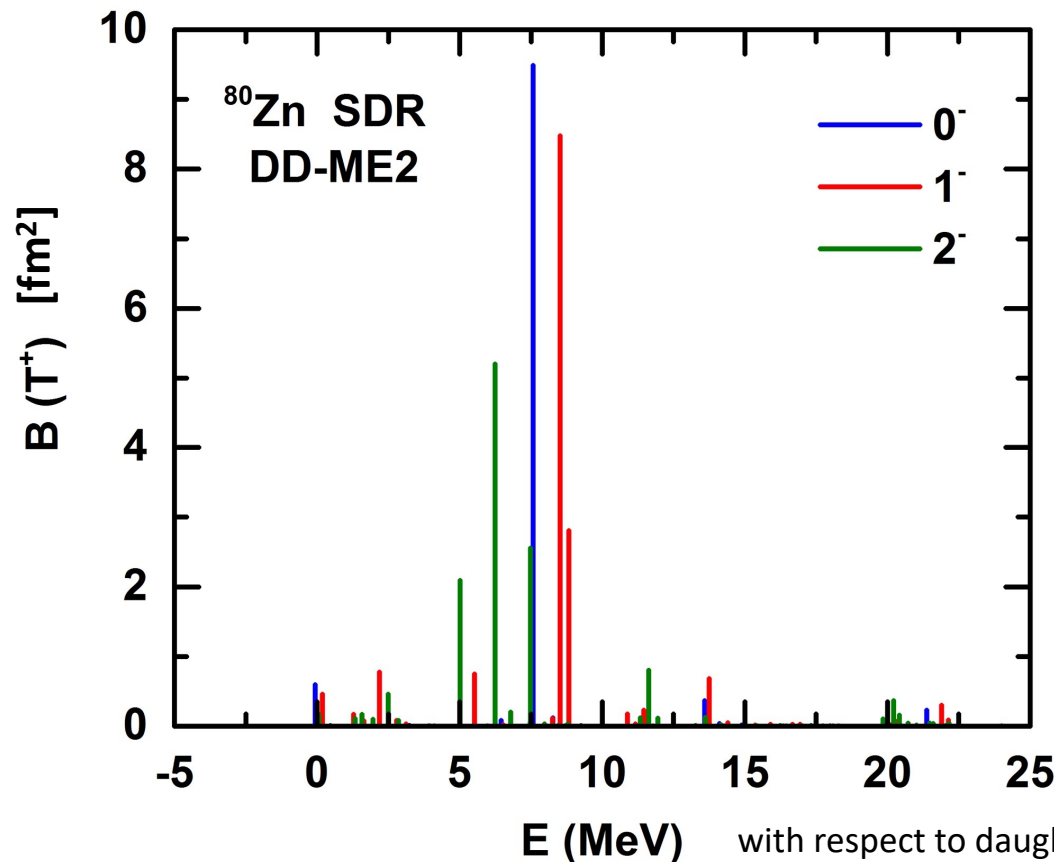
- SD operator  $\hat{F}_{\text{SDR}}^{\pm} = \sum_{i=1}^A [r_i Y_1(i) \otimes \sigma(i)]_{J=0,1,2} \tau_{\pm}(i) \quad J^{\pi} = 0^-, 1^-, 2^- \quad \Delta S=1, \Delta L=1$



- ✓ Spin-Dipole transitions have significant strength
- ✓ SD transitions will dominate EC cross section of  $^{80}\text{Zn}$

# Spin-Dipole strength distribution ( $T^+$ )

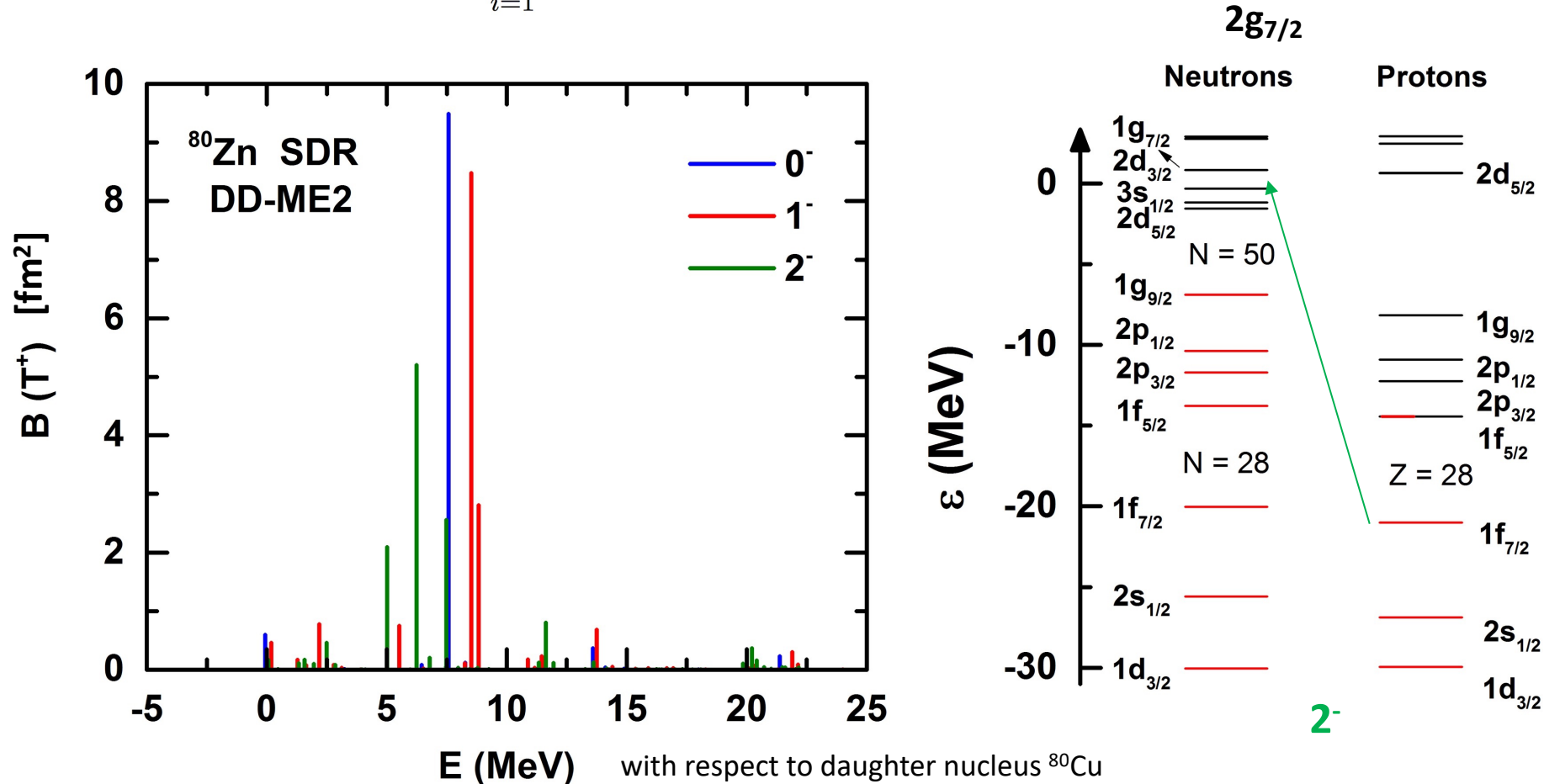
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# Spin-Dipole strength distribution ( $T^+$ )

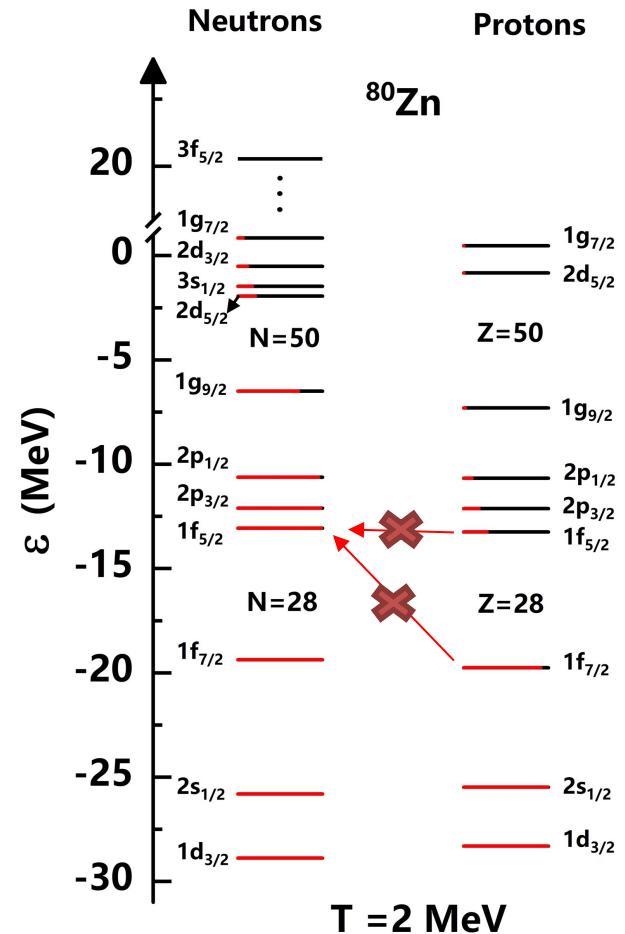
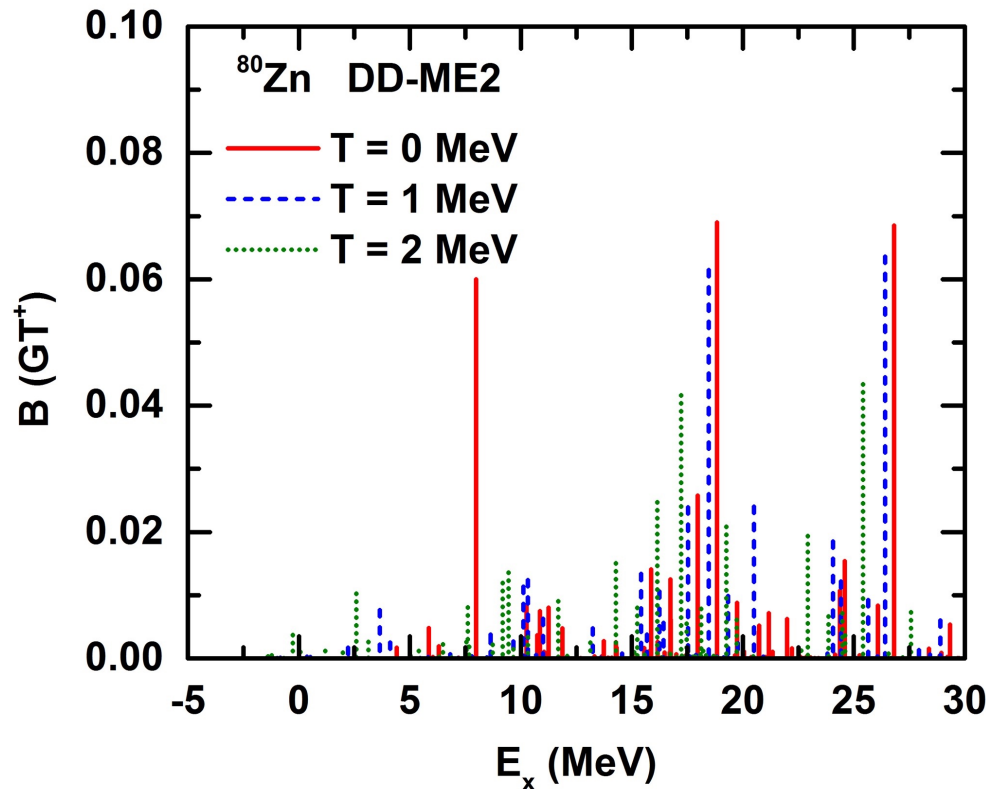
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# Temperature effects

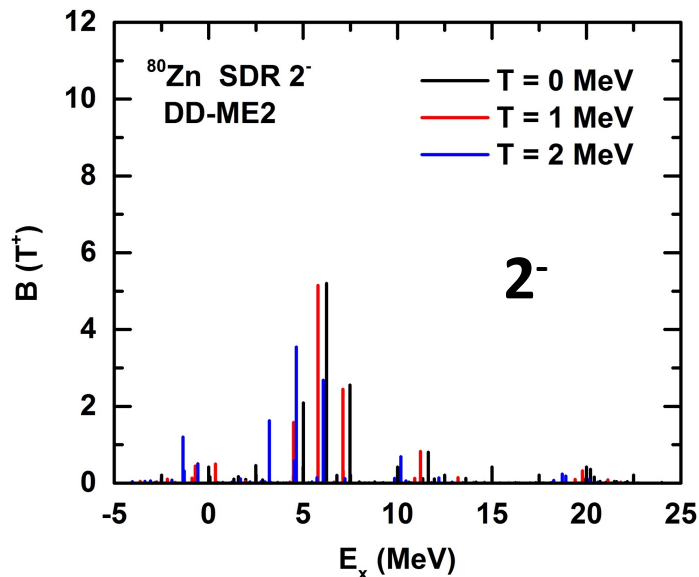
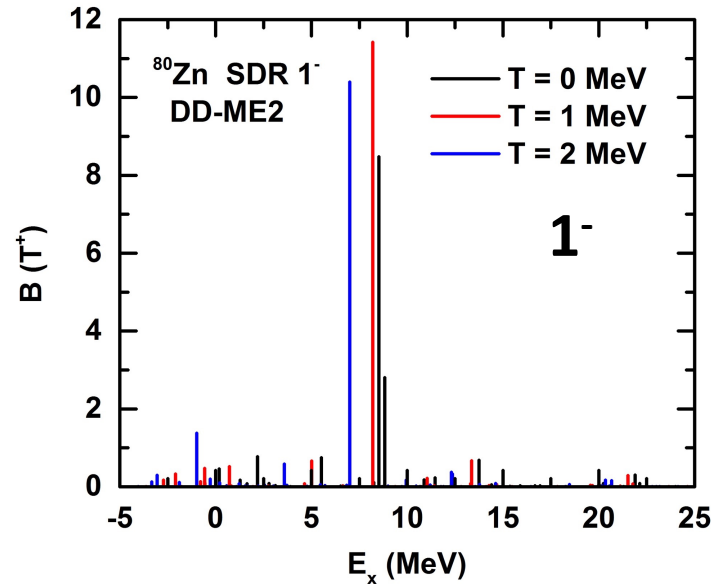
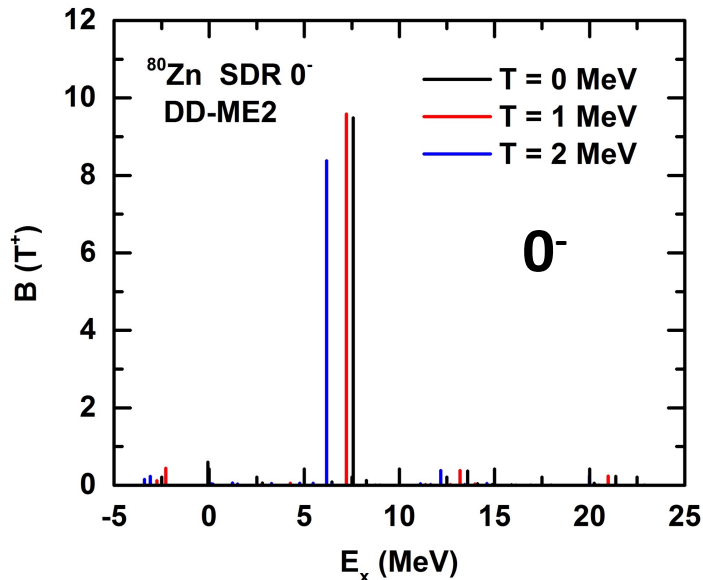
## GT transitions at finite temperature



- ✓ Even temperature cannot unblock the  $GT^+$  transition due to large neutron excess
- In stellar environment,  $GT^+$  still cannot contribute much to EC rates

# Temperature effects

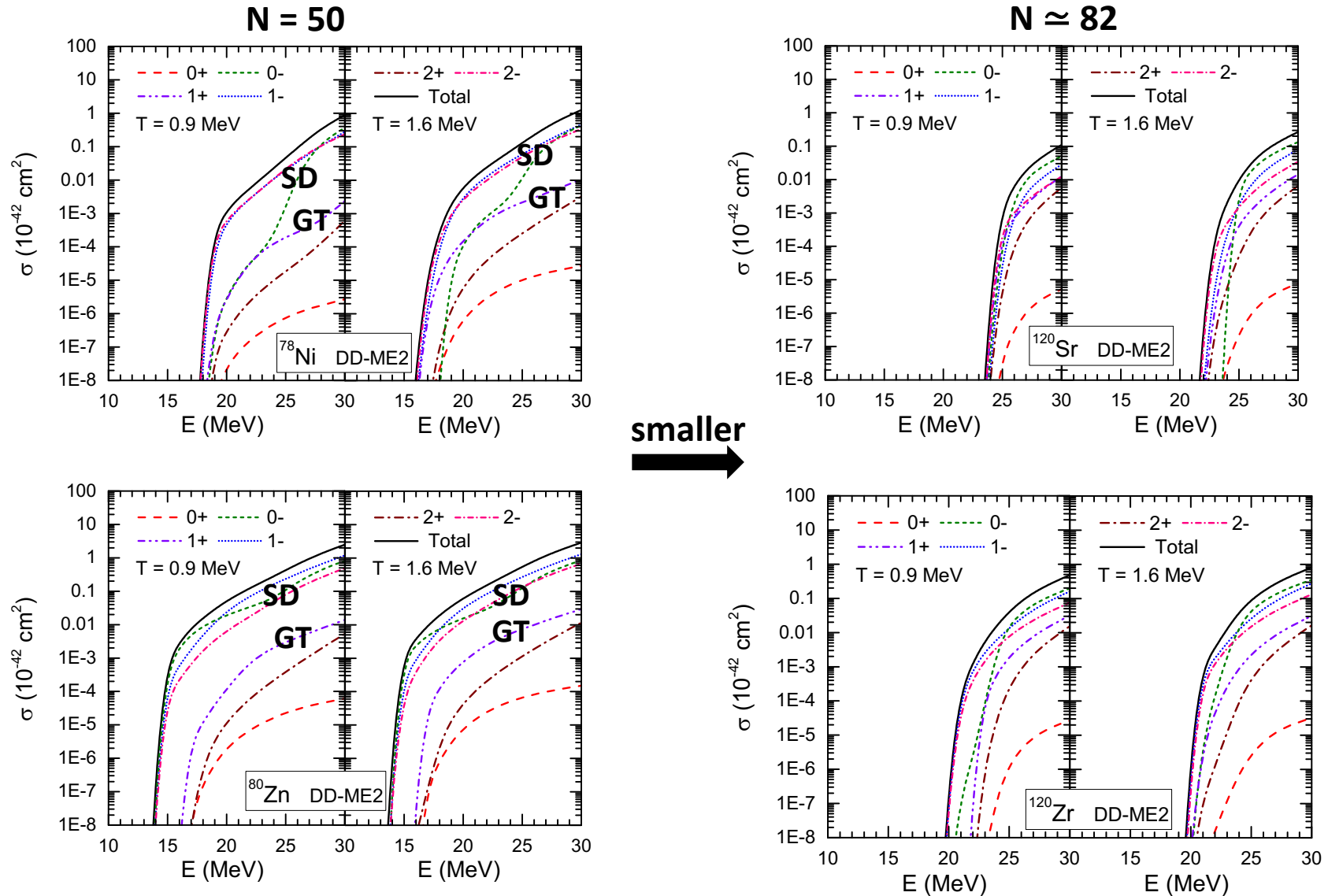
## Spin Dipole Transitions at finite temperature



- ✓ Temperature decreases energies, but changes are small.
- Spin-dipole transition data measured at Lab (zero temperature) can still be applied to EC study in supernova.

L. Guo, W. L. Lv, Y. F. Niu, D. L. Fang, B.S. Gao, K. A. Li, and X. D. Tang, in preparation

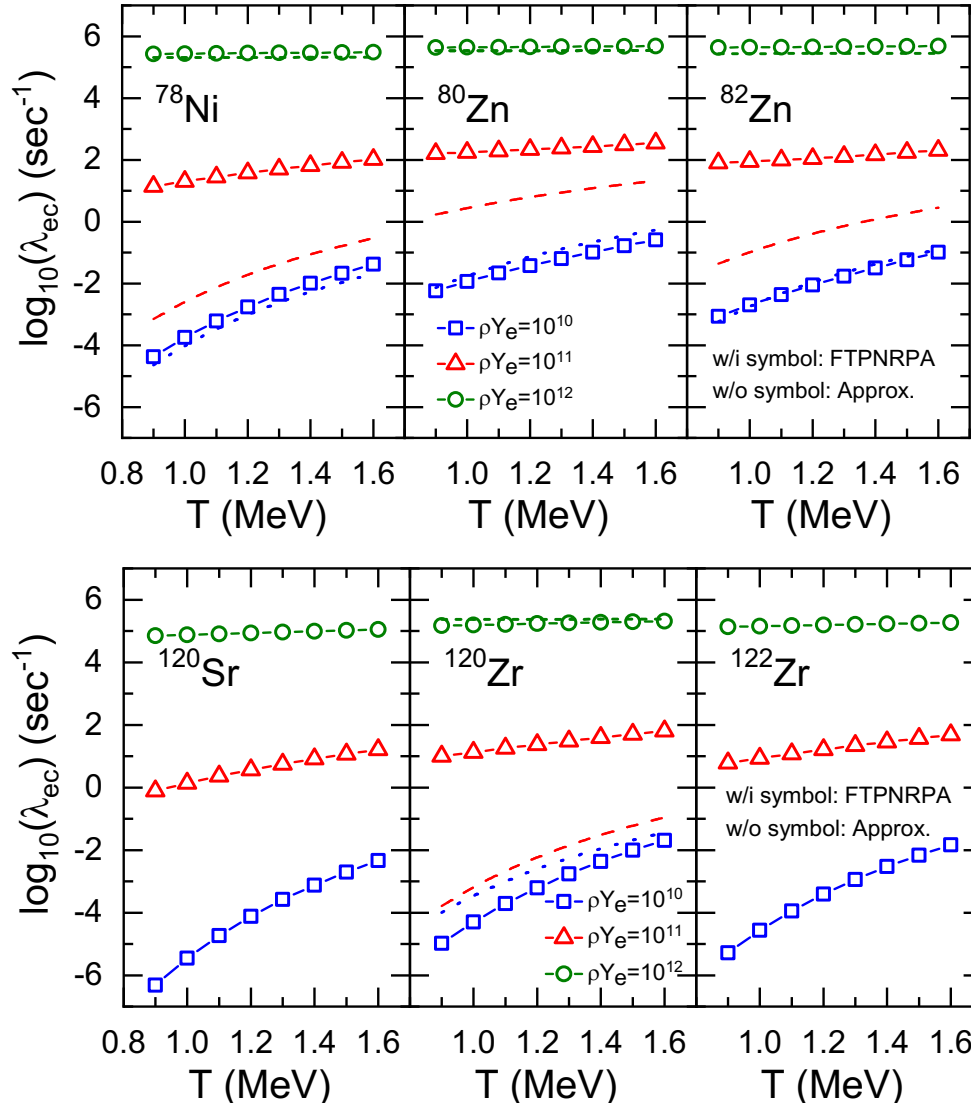
# Electron capture cross sections



- For these neutron rich nuclei, spin dipole transitions dominate the cross section
- Even at high temperatures, GT transitions are not considerably unblocked

# Electron capture rates

## Electron capture rates at different stellar environment



- With the increase of electron density, the EC rates are increased by several orders of magnitude.

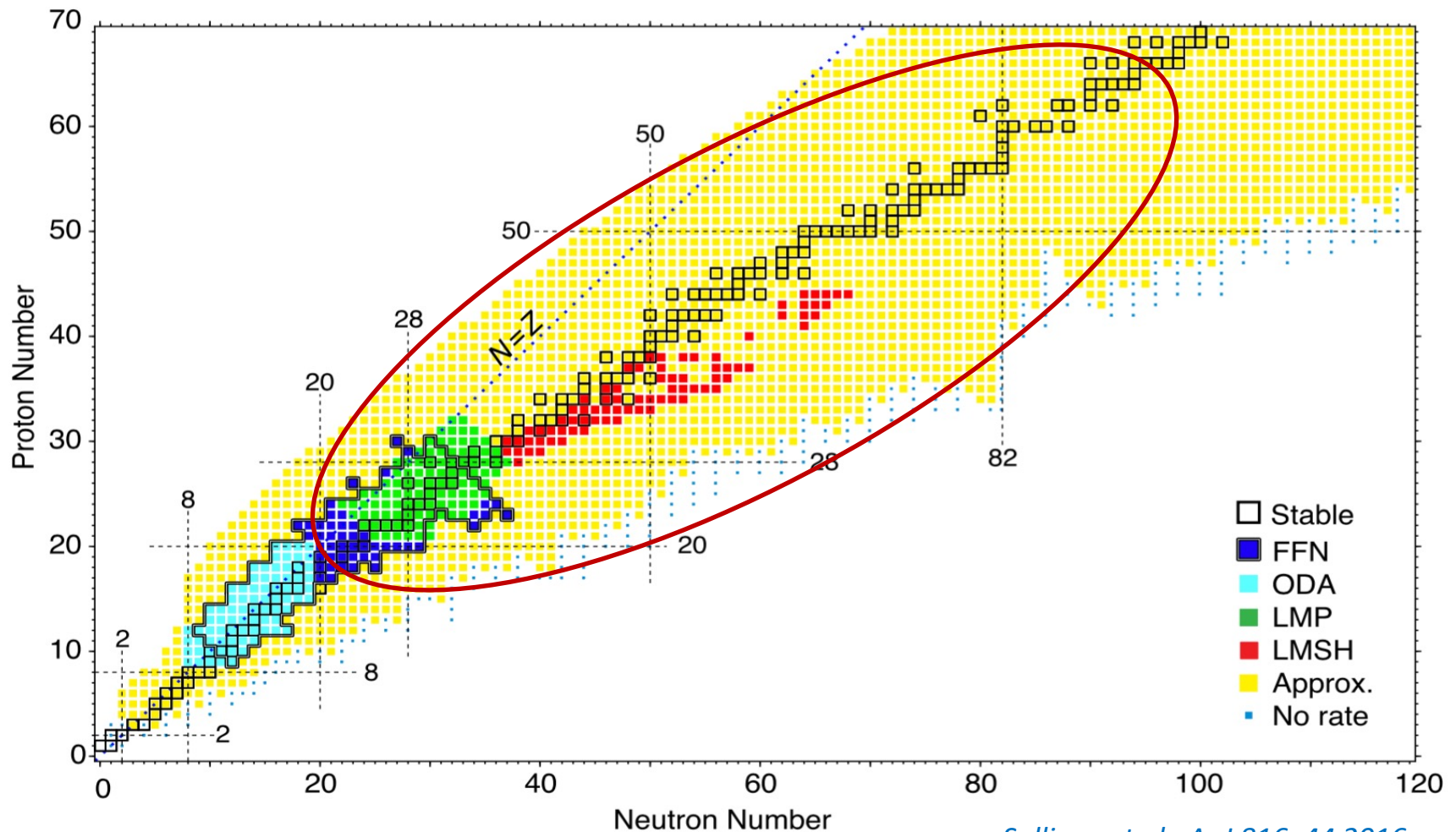
- At lower electron densities, the EC rates have big increase with temperature, but at high densities, the rate is not sensitive to temperature.

- Approx.

$$\lambda = \frac{(\ln 2)B}{K} \left( \frac{T}{m_e c^2} \right)^5 [F_4(\eta) - 2\chi F_3(\eta) + \chi^2 F_2(\eta)]$$

Rates from approximation formula at  $10^{11} \text{ g/cm}^3$  is much underestimated compared to our results

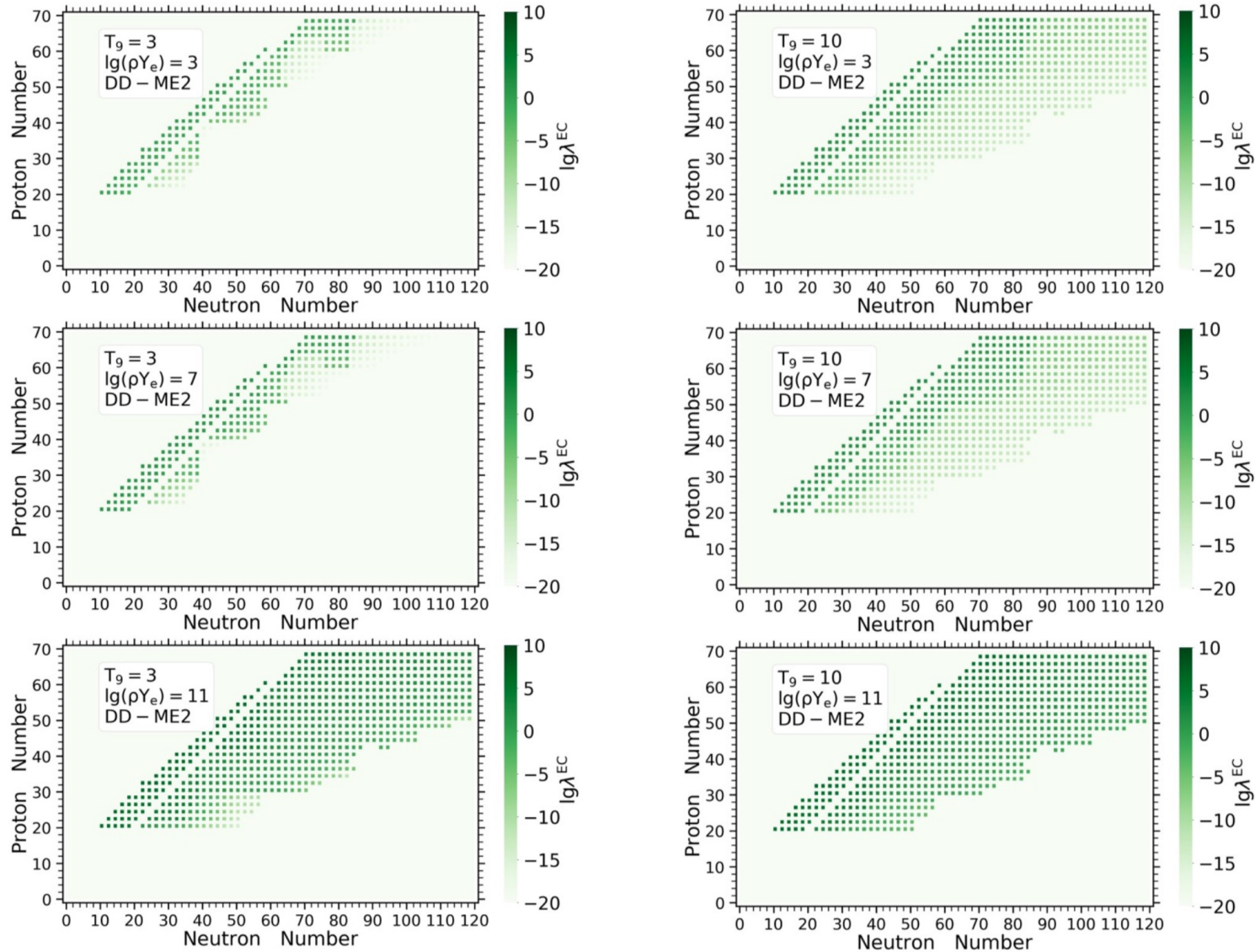
# Systematic calculations for $Z=20-68$ even-even nuclei



*Sullivan et al., ApJ 816, 44 2016*

- FTRRPA model is used for systematic calculation of EC rates for  $Z=20-68$  even-even nuclei
- Only GT transitions are considered for simplification.

# Systematic calculations for $Z=20-68$ even-even nuclei

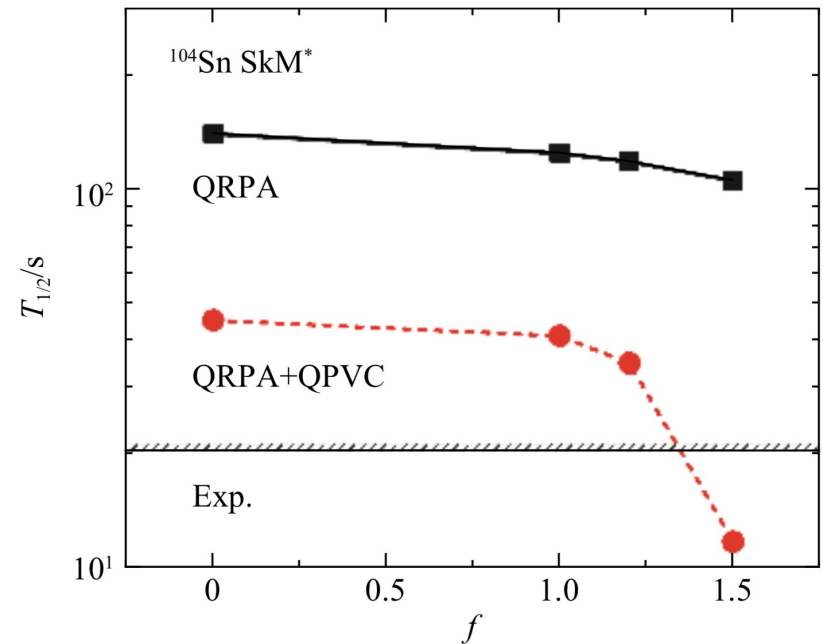
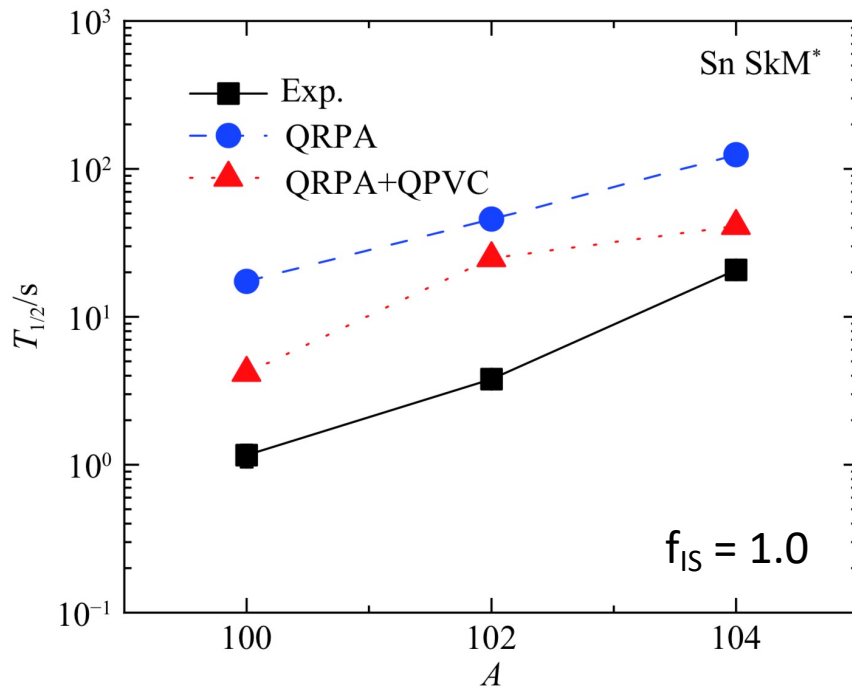


- At low temperature, FT-QRPA is needed. see talk by Ante Ravlic

# $\beta^+/\text{EC}$ Half-lives by QRPA+QPVC

➤  $\beta^+/\text{EC}$  Half-lives of neutron-deficient nuclei

➤ The effect of isoscalar pairing



- The  $\beta^+/\text{EC}$  half-lives are overestimated by one order of magnitude.
- QPVC reduces the half-lives of these nuclei.

- With the increase of isoscalar pairing strength,  $\beta^+/\text{EC}$  half-lives decrease.
- QPVC results decrease faster than QRPA.
- QRPA results cannot reproduce exp. even at large  $f_{\text{IS}}$ , while QPVC reproduces exp. at  $f_{\text{IS}} \sim 1.25$ .

# Summary and Perspectives

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Towards the understanding of origin of heavy elements

- Accurate Nuclear Physics Inputs:  $\beta$ -decay
  - ✓ Go beyond RPA/QRPA: we developed self-consistent RPA+PVC / QRPA+QPVC model
  - ✓ Successfully describe the GT resonance and  $\beta$ -decay half-lives in doubly magic nuclei and superfluid nuclei using the same Skyrme interaction
- Electron Capture Rates in core-collapse supernova
  - ✓ FTRPA provides a universal tool for the calculation of EC rates for core-collapse supernova

## Perspective:

- Extend QRPA+QPVC model to finite-temperature case, and apply it for EC study in core-collapse supernova

# Acknowledgement

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- Sichuan University: Bai Chunlin
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- University of Aizu & RIKEN: Hiroyuki Sagawa
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*Thank you!*