Constraining the stellar weak-interaction rates within the relativistic energy density functional theory

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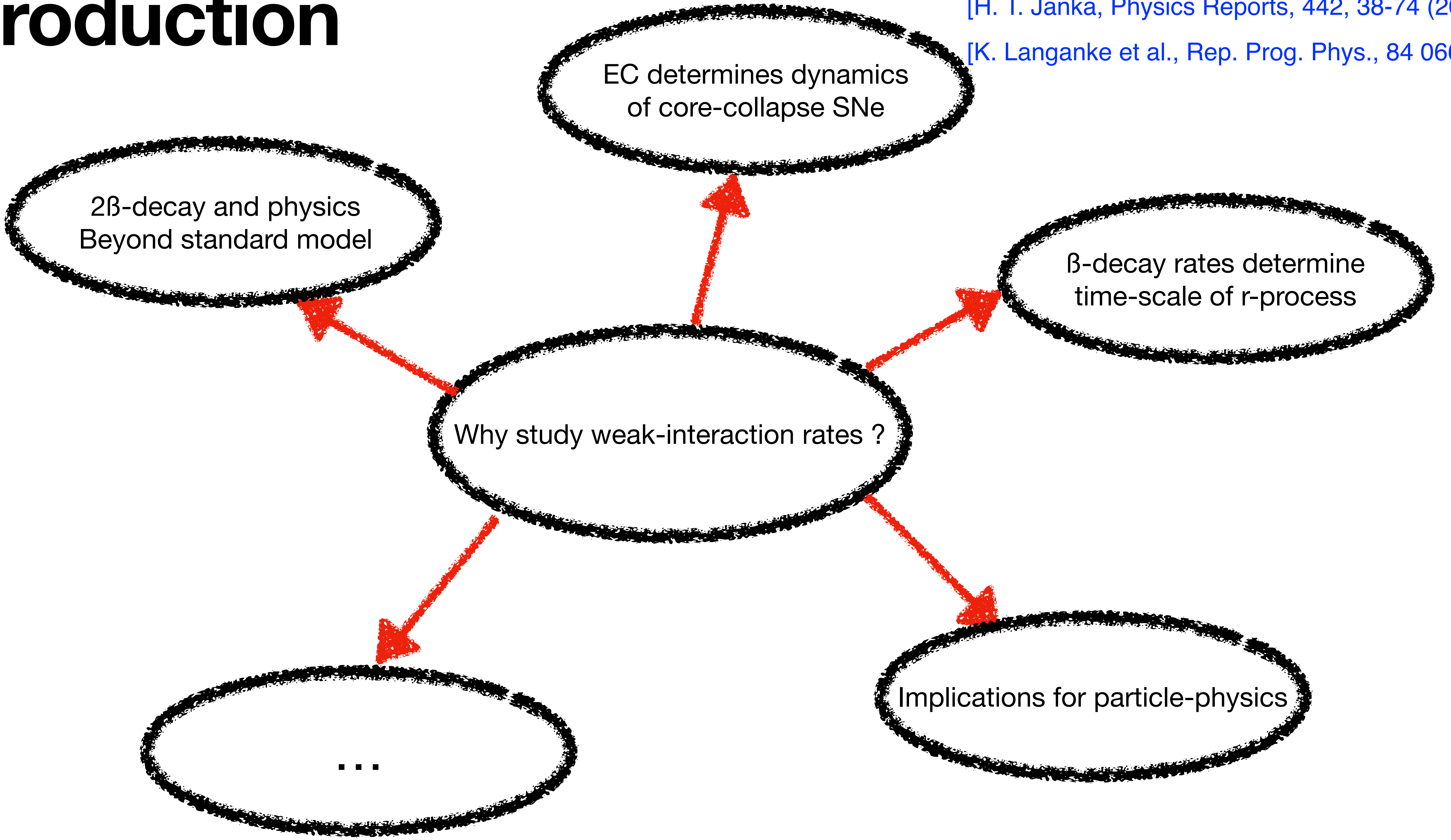
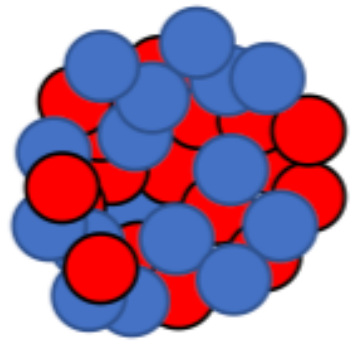
Introduction

[H. T. Janka, Physics Reports, 442, 38-74 (2007)]

[K. Langanke et al., Rep. Prog. Phys., 84 066301(2021)]



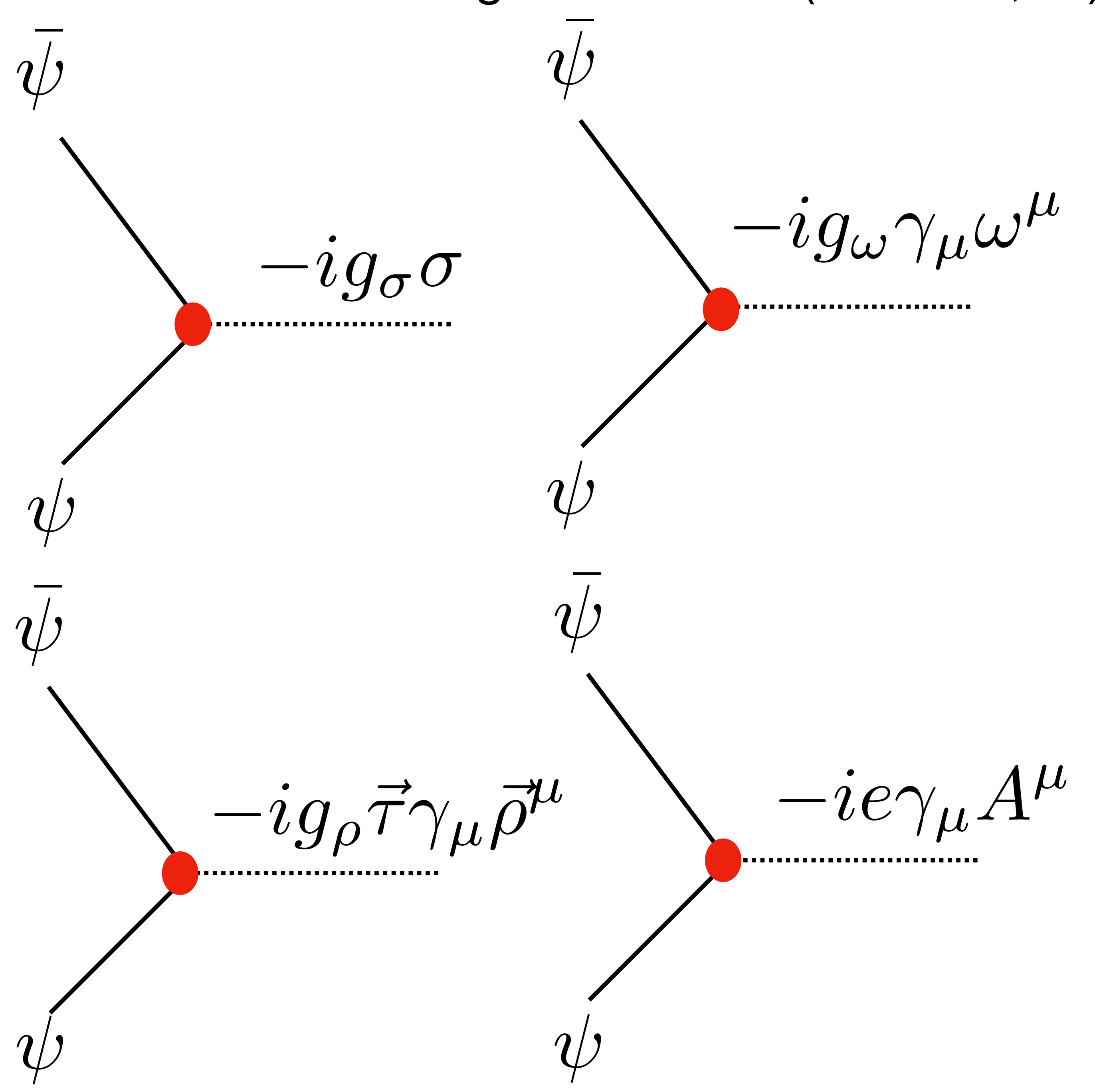
My model of choice



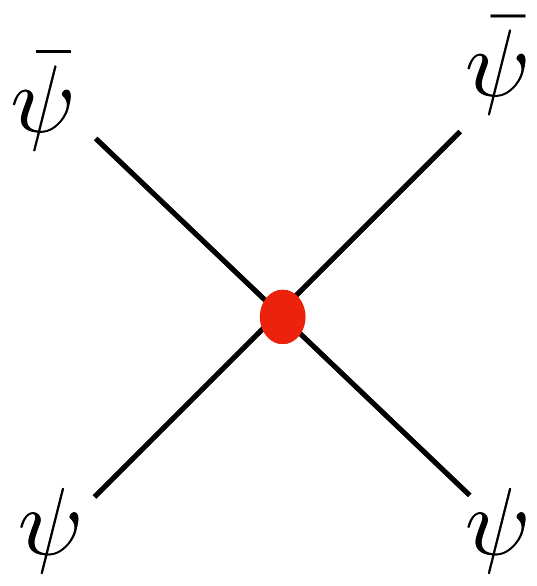
[P. Ring, Prog. Part. Nucl. Phys. 37, 193 (1996)]

Relativistic EDF

Meson-exchange interaction (DD-ME2, …)



Point-coupling interaction (DD-PC1, DD-PCX, …)



Derivative coupling (D3C*, D3C, …)

$$\gamma_\mu \to \Gamma_\mu = \gamma^\nu g_{\mu\nu} + \gamma^\nu Y_{\mu\nu} - g_{\mu\nu} Z^\nu$$

$$1 \to \Gamma = 1 + \gamma_\mu u_\nu Y^{\mu\nu} - u_\mu Z^\mu$$

$$Y^{\mu\nu} = \frac{\Gamma_V}{m^4} m_\omega^2 \omega^\mu \omega^\nu, \; Z^\mu = \frac{\Gamma_S}{m^2}\omega^\mu \sigma$$

Start from the model Lagrangian density:

$$\mathcal{L} = \mathcal{L}_{nuc} + \mathcal{L}_{mes} + \mathcal{L}_{int}$$

Write down the mean-field EDF:

$$\mathcal{H} = T^{00} = \frac{\partial \mathcal{L}}{\partial \dot{q}_j}\dot{q}_j - \mathcal{L} \longrightarrow E_{RMF}[\psi, \bar{\psi}, \sigma, \omega_\mu, \vec{\rho}_\mu, A_\mu] = \int d^3\boldsymbol{r}\mathcal{H}(\boldsymbol{r})$$

At finite-temperature perform minimisation of grand-canonical potential:

$$\Omega = E_{RMF} - TS - \sum_q \lambda_q N_q$$

Entropy (S)

Chemical potential (λ_q)

Derive mean-field equations:

Assuming even-even nuclei, time-reversal symmetry and charge conservation !

$$\frac{\delta\Omega}{\delta\rho} = 0 \rightarrow \{-i\boldsymbol{\alpha}\cdot\nabla + \beta M^*(\boldsymbol{r}) + V(\boldsymbol{r})\}\,\psi_i(\boldsymbol{r}) = \varepsilon_i\psi_i(\boldsymbol{r})$$

Scalar field — Vector field

+ equations for meson fields (if ME interaction)

Sigma:

$$\left[-\nabla^2 + m_\sigma^2\right]\sigma = -g_\sigma(\rho_v)\rho_s$$

Omega:

$$\left[-\nabla^2 + m_\omega^2\right]\omega^0 = g_\omega(\rho_v)\rho_v$$

Rho:

$$\left[-\nabla^2 + m_\rho^2\right]\vec{\rho}^{\,0} = g_\rho(\rho_v)\rho_{tv}$$

Coulomb:

$$\nabla^2 A^0 = -e\rho_p$$

Densities at finite-temperature:

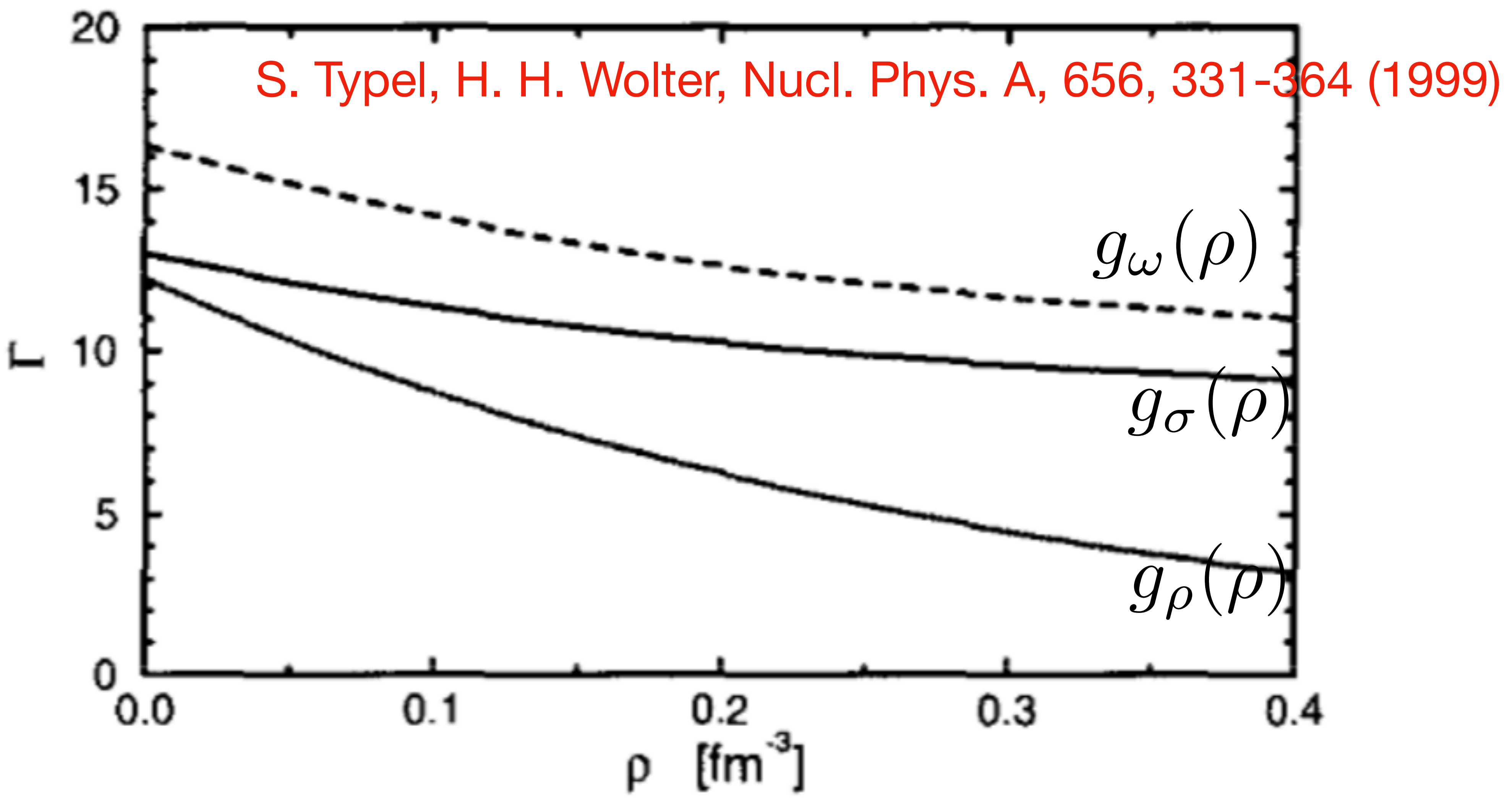
$$\rho_s(\boldsymbol{r}) = \sum_{i=1}^{A}\bar{\psi}_i(\boldsymbol{r})\psi_i(\boldsymbol{r})f_i$$

$$\rho_v(\boldsymbol{r}) = \sum_{i=1}^{A}\bar{\psi}_i(\boldsymbol{r})\gamma_0\psi_i(\boldsymbol{r})f_i$$

$$\rho_{tv}(\boldsymbol{r}) = \sum_{i=1}^{A}\bar{\psi}_i(\boldsymbol{r})\tau_3\gamma_0\psi_i(\boldsymbol{r})f_i$$

$$f_i = \left(1 + \exp\left(\frac{\varepsilon_i - \lambda_q}{k_B T}\right)\right)^{-1}$$

Couplings are density dependent: $g_i(\rho_v) = g_i(\rho_{sat})f_i(\rho_v/\rho_{sat})$



How to include pairing correlations?

$$E_{RHB}[\rho,\kappa] = E_{RMF}[\rho] + E_{pair}[\kappa]$$

$$\frac{1}{4}\sum_{n_1 n_1' n_1 n_2'} \kappa^*_{n_1 n_1'} \langle n_1 n_1' | V^{pp} | n_2 n_2' \rangle \kappa_{n_2 n_2'}$$

Pp interaction, e.g. Gogny D1, separable, delta …

Find a basis such that:

$$E_{RHB} \approx \sum_\nu E_\nu a^\dagger_\nu a_\nu$$

Bogoliubov-Valatin transformation:

$$\begin{pmatrix} a^\dagger \\ a \end{pmatrix} = \begin{pmatrix} U & V \\ V^* & U^* \end{pmatrix} \begin{pmatrix} c^\dagger \\ c \end{pmatrix}$$

Independent q.p. ensemble at FT:

$$\hat{\rho} = \prod_\nu \left[f_\nu a^\dagger_\nu a_\nu + (1 - f_\nu) a_\nu a^\dagger_\nu \right]$$

$$f_\nu = [1 + \exp(E_\nu / k_B T)]^{-1}$$

$$\rho_{nn'} = \langle \Phi | c^\dagger_{n'} c_n | \Phi \rangle \rightarrow \langle c^\dagger_{n'} c_n \rangle_T$$

$$\kappa_{nn'} = \langle \Phi | c_{n'} c_n | \Phi \rangle \rightarrow \langle c_{n'} c_n \rangle_T$$

$$\rho = U^T f U^* + V^\dagger (1 - f) V$$

$$\kappa = U^T f V^* + V^\dagger (1 - f) U$$

Bogoliubov-Valatin density:

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1 - \rho^* \end{pmatrix}$$

$$\frac{\partial \Omega}{\partial \mathcal{R}} \rightarrow 0$$

FT-RHB equation

$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

Dirac mean-field

Pairing field

Alternatively we can solve the FT-BCS equations

$$n_k = v_k^2(1 - f_k) + f_k u_k^2, \text{with } u_k^2 + v_k^2 = 1$$

There are effectively two approaches:

1) Diagonal approximation:

$$\Delta_{kk'} = \Delta_k \delta_{kk'}$$

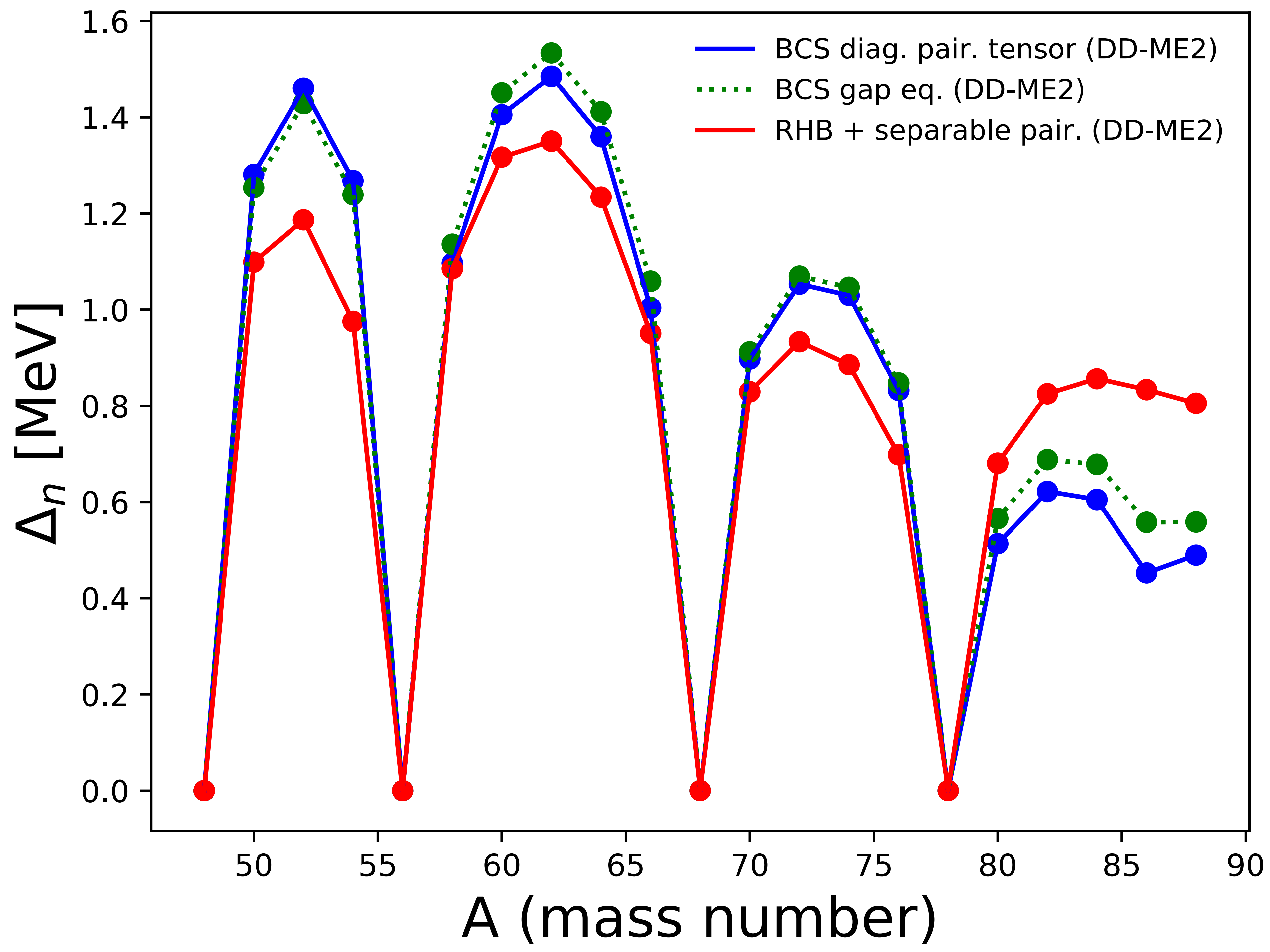
$$\Delta_k = \sum_{k'} \int d^3\boldsymbol{r} F_k(\boldsymbol{r})^* V^{pp}_{k\bar{k}k'\bar{k}'} F_{k'}(\boldsymbol{r})$$

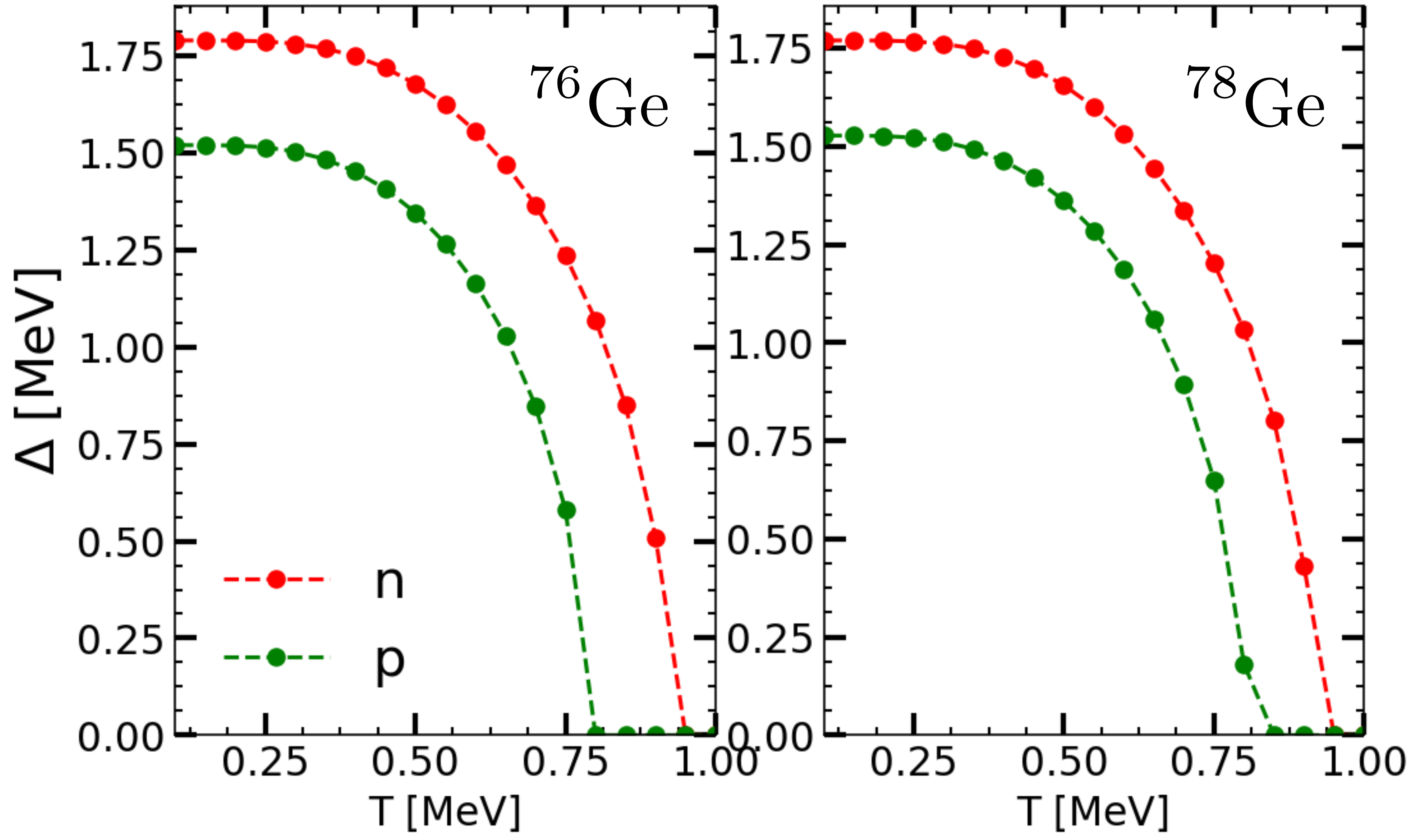
2) Solving the gap equation:

$$E_k = \sqrt{(\varepsilon_k - \lambda_q)^2 + \Delta_k^2}$$

$$\Delta_k = \frac{1}{2} \sum_{k'>0} V^{pp}_{k\bar{k}k'\bar{k}'} \frac{\Delta_{k'}}{E_{k'}}$$

Ni chain





Calculating excited states at finite-temperature

- Time-dependent HF:

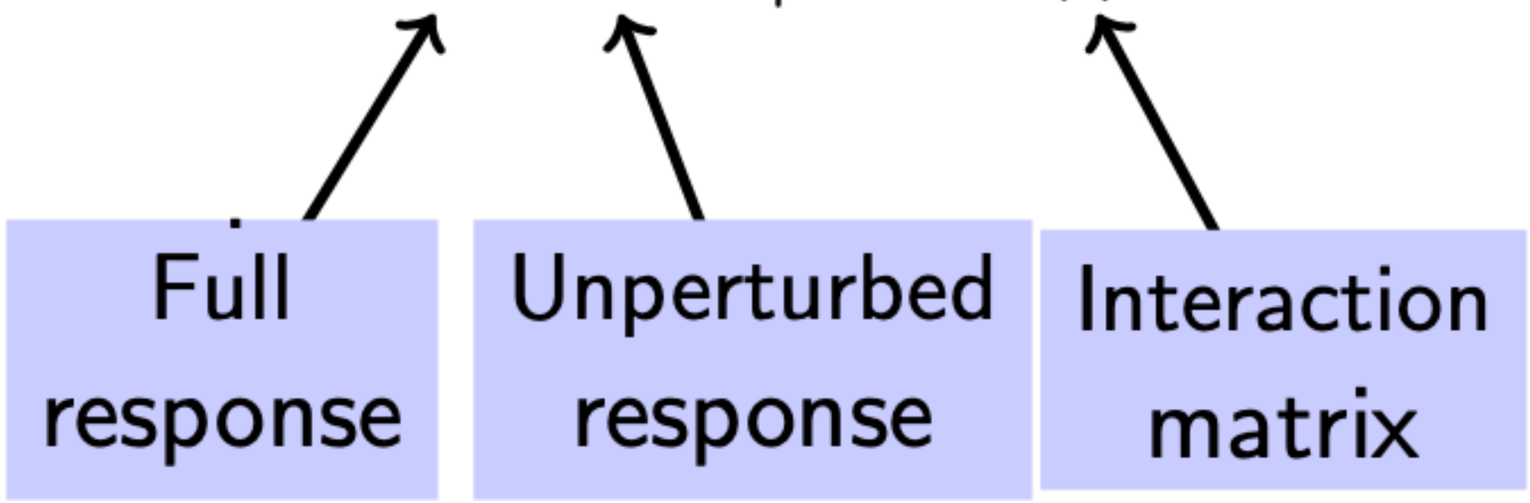
$$i\dot{\mathcal{R}}(t) = [\mathcal{H}(\mathcal{R}) + \mathcal{F}(t), \mathcal{R}(t)]$$

- Linearization of density-matrix

$$\mathcal{R}(t) = \mathcal{R}^0 + (\delta\mathcal{R}e^{-iEt} + \text{h.c.})$$

- Bethe-Salpeter equation

$$\mathbb{R} = \mathbb{R}^0 + \mathbb{R}^0 \cdot \mathbb{W} \cdot \mathbb{R}$$

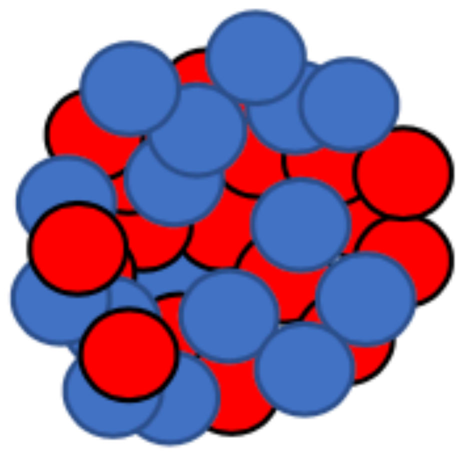


[P. Ring et al., Nucl. Phys. A, 419:261 294 (1984)]

[A. Ravlic, Y.F. Niu et al., PRC 104, 064302 (2021)]

$$\mathcal{F}(t) = \hat{F}e^{-i\omega t} + \text{ h.c.}$$

$$\hat{F} = \sum_{ij} F_{ij} c_i^\dagger c_j$$



$$(N, Z), \rho^0, \kappa^0$$

Alternative derivation leads to matrix FT-QRPA

- 2qp phonon operator:

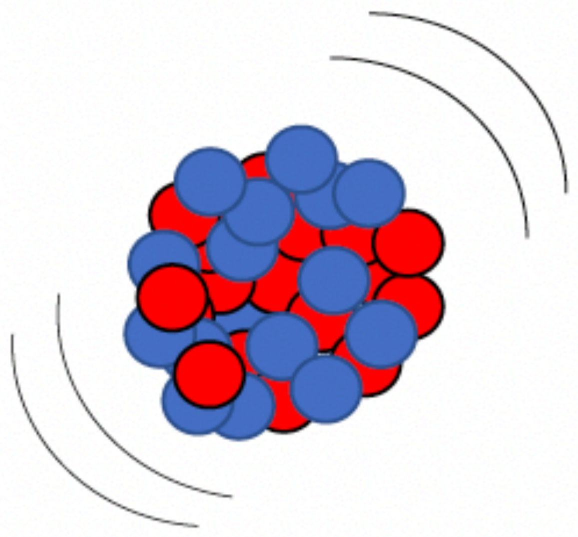
$$\Gamma_\nu^\dagger = \sum_{pn} \left[X^\nu_{pn} a^\dagger_p a^\dagger_n - Y^\nu_{pn} a_n a_p \underbrace{+ P^\nu_{pn} a^\dagger_p a_n - Q^\nu_{pn} a^\dagger_n a_p}_{\text{Only for } T > 0} \right]$$

Solve equation of motion (EoM)

$$\begin{pmatrix} \tilde{C} & \tilde{a} & \tilde{b} & \tilde{D} \\ \tilde{a}^+ & \tilde{A} & \tilde{B} & \tilde{b}^T \\ -\tilde{b}^+ & -\tilde{B}^* & -\tilde{A}^* & -\tilde{a}^T \\ -\tilde{D}^* & -\tilde{b}^* & -\tilde{a}^* & -\tilde{C}^* \end{pmatrix} \begin{pmatrix} \tilde{P} \\ \tilde{X} \\ \tilde{Y} \\ \tilde{Q} \end{pmatrix} = E_\nu \begin{pmatrix} \tilde{P} \\ \tilde{X} \\ \tilde{Y} \\ \tilde{Q} \end{pmatrix}$$

Terms is red contribute at T > 0

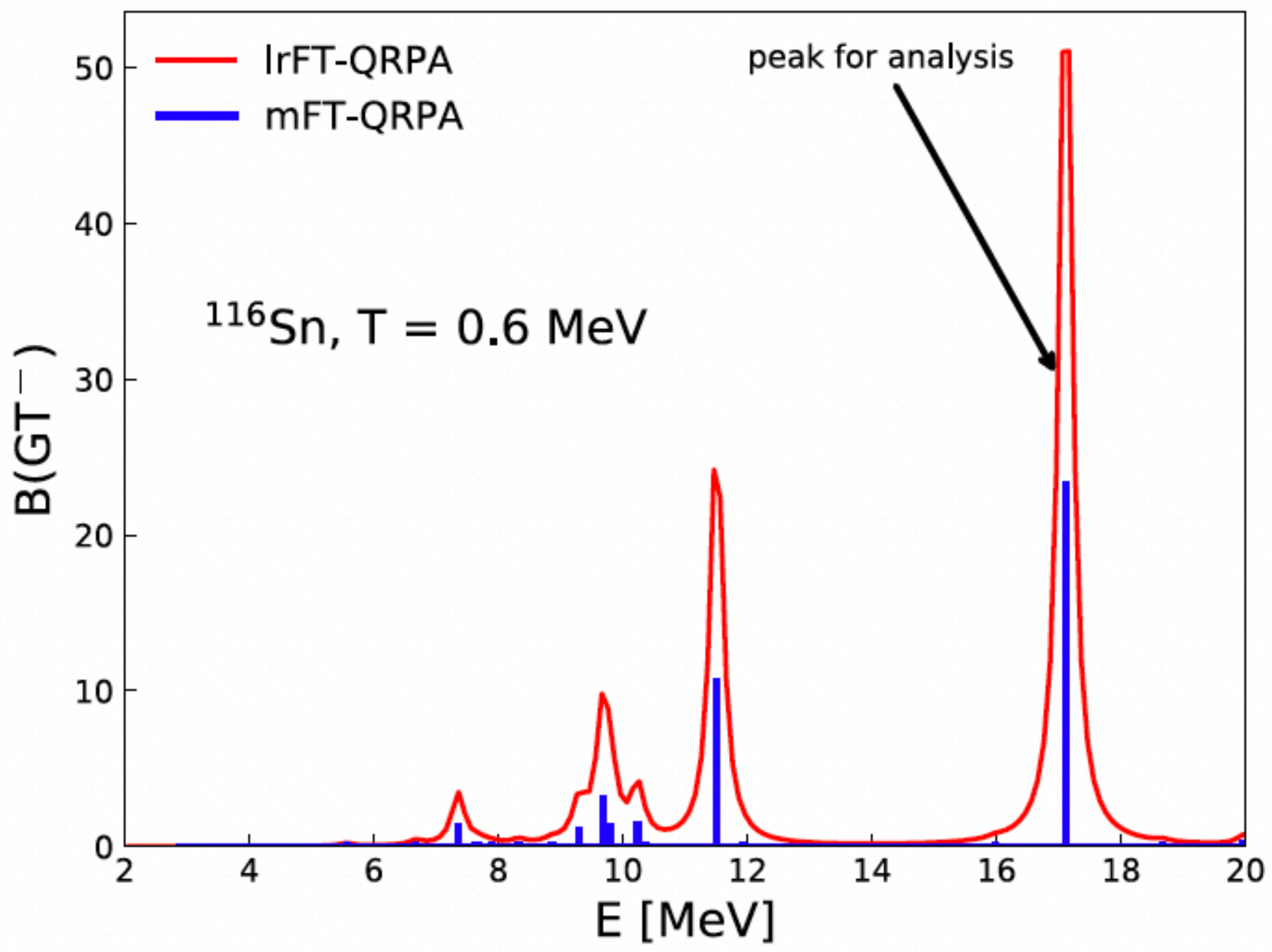
[H. Sommermann, Ann. of Phys. 151, 163 (1983)]



$$(N \pm 1, Z \mp 1), \rho(t) = \rho^0 + \delta\rho(t)$$

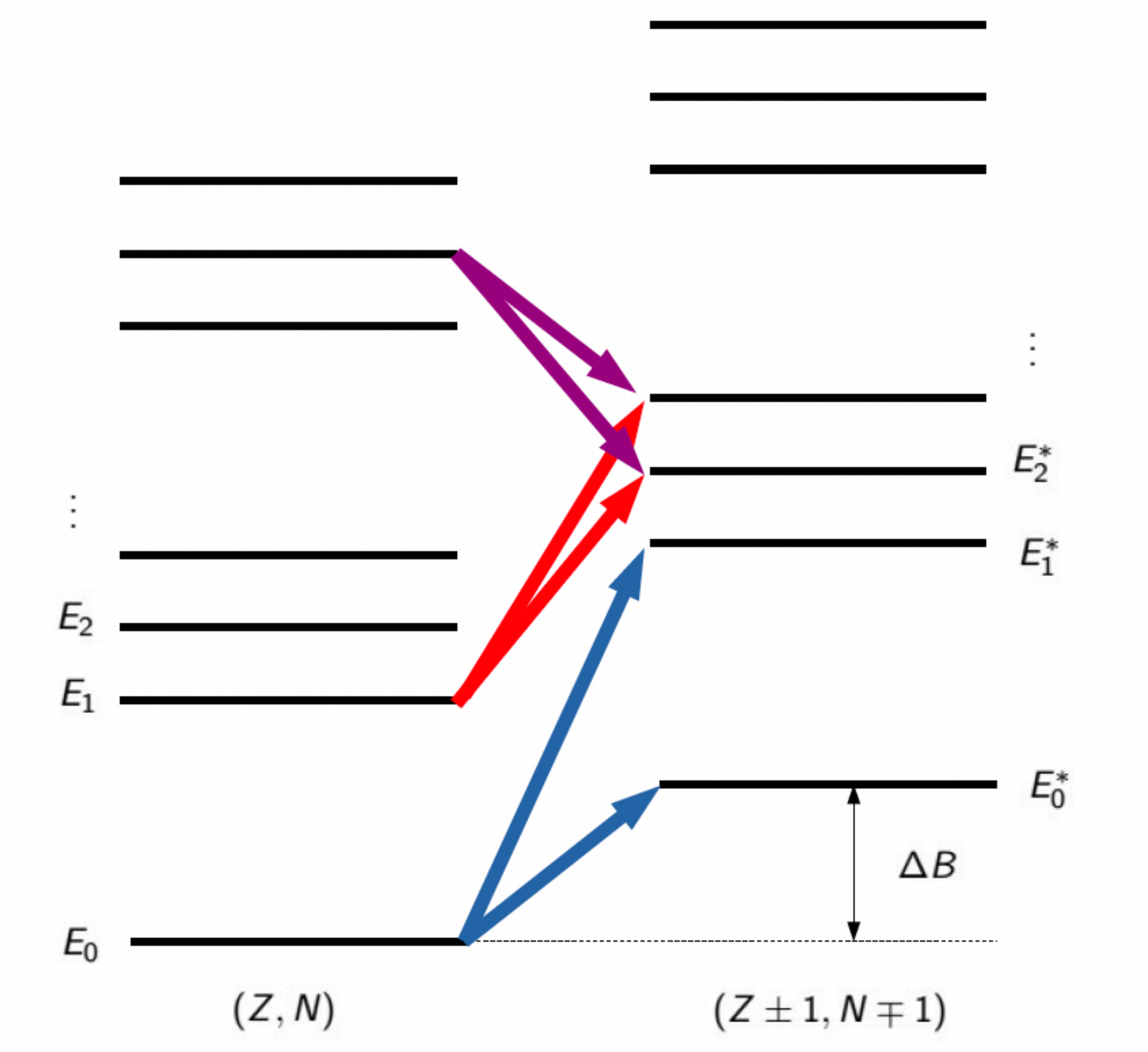
$$\kappa(t) = \kappa^0 + \delta\kappa(t)$$

$$B(\mathrm{GT}^-) = |\langle i || \boldsymbol{\sigma} \vec{\tau}_- || \tilde{0} \rangle|^2$$



Interpretation of strength function at finite-temperature

[E. M. Ney, A. Ravlic, J. Engel, N. Paar, in preparation]



Transitions at T = 0

Transitions allowed for T > 0

De-excitations (T > 0)

- Physical strength function:

Final and initial state energy

$$\tilde{S} = \sum_{if} p_i |\langle f|\hat{F}|i\rangle|^2 \delta(E - E_f + E_i)$$

Boltzmann factor

- FT-(PN)RQRPA strength function:

Ensemble averaged Matrix element

$$R^{QRPA}(E) = \sum_n \frac{|\langle[\Gamma^n, \hat{F}]\rangle|^2}{E - \Omega_n + i\eta} - \frac{|\langle[\Gamma^n, \hat{F}^\dagger]\rangle|^2}{E + \Omega_n + i\eta}$$

FT-PNRQRPA eigenvalue

- Physical strength function as approximated by FT-(PN)RQRPA:

Detailed balance factor

$$\tilde{S}(E) \approx -\frac{1}{\pi}\mathrm{Im}\left[\frac{R^{QRPA}(E)}{1 - e^{-\beta(E - \lambda_n + \lambda_p)}}\right]$$

Isoscalar (T=0) pairing strength

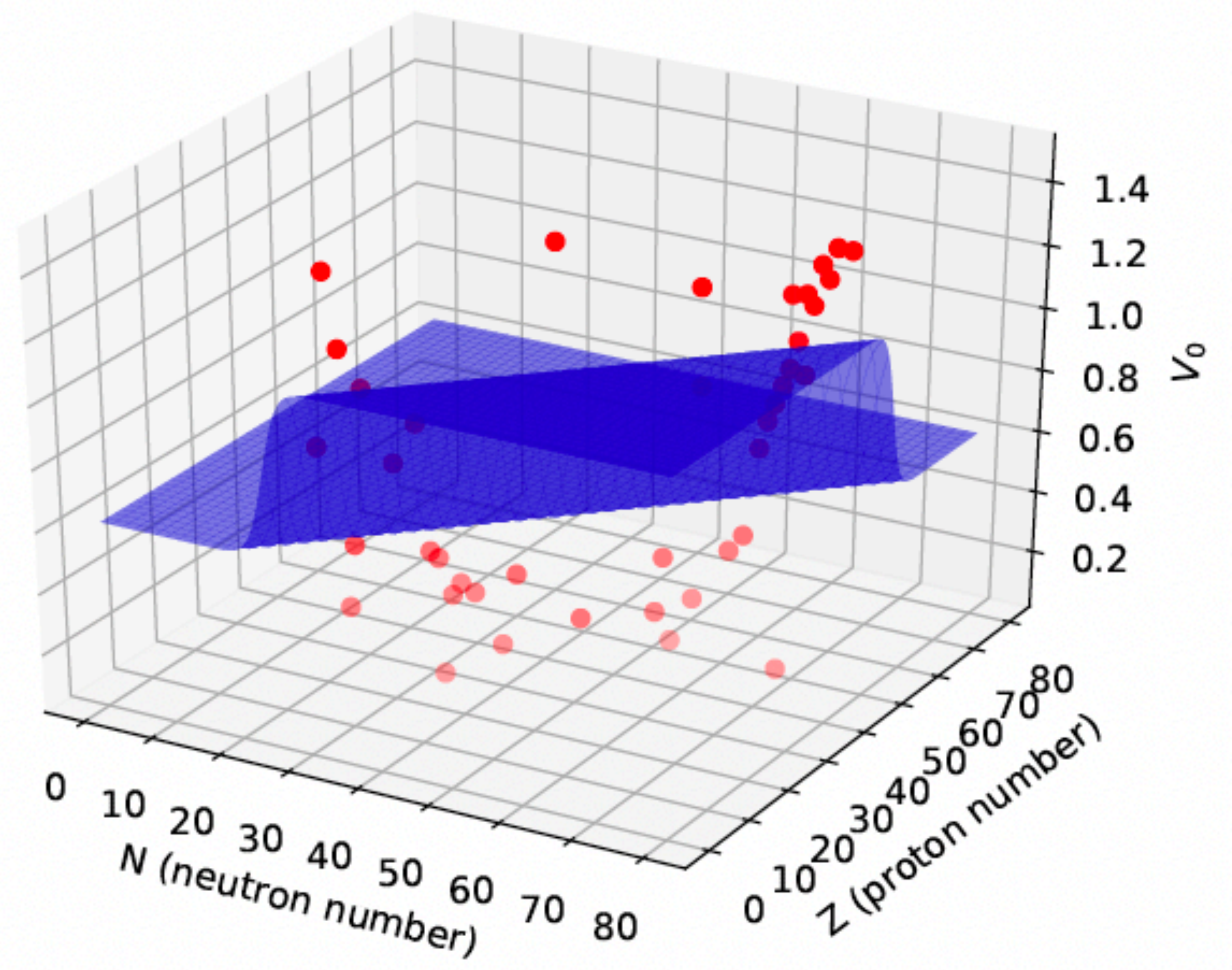
Check discussion in Ref. [N. Popara, A. Ravlic and N. Paar, arXiv:2107.08747, accepted for publication in PRC]

- Residual pairing interaction can be divided as:
  - **Isovector** (T=1,S=0), same as in FT-RHB(BCS)
  - **Isoscalar** (T=0,S=1), not constrained at the mean-field level
- We can take interaction strength V_0^{is} as a free parameter, and perform fit on the following functional form : [Z. M. Niu et al., PLB 723, 172 (2013).]

$$V_0^{is} = V_L + \frac{V_D}{1 + e^{a+b(N-Z)}}$$

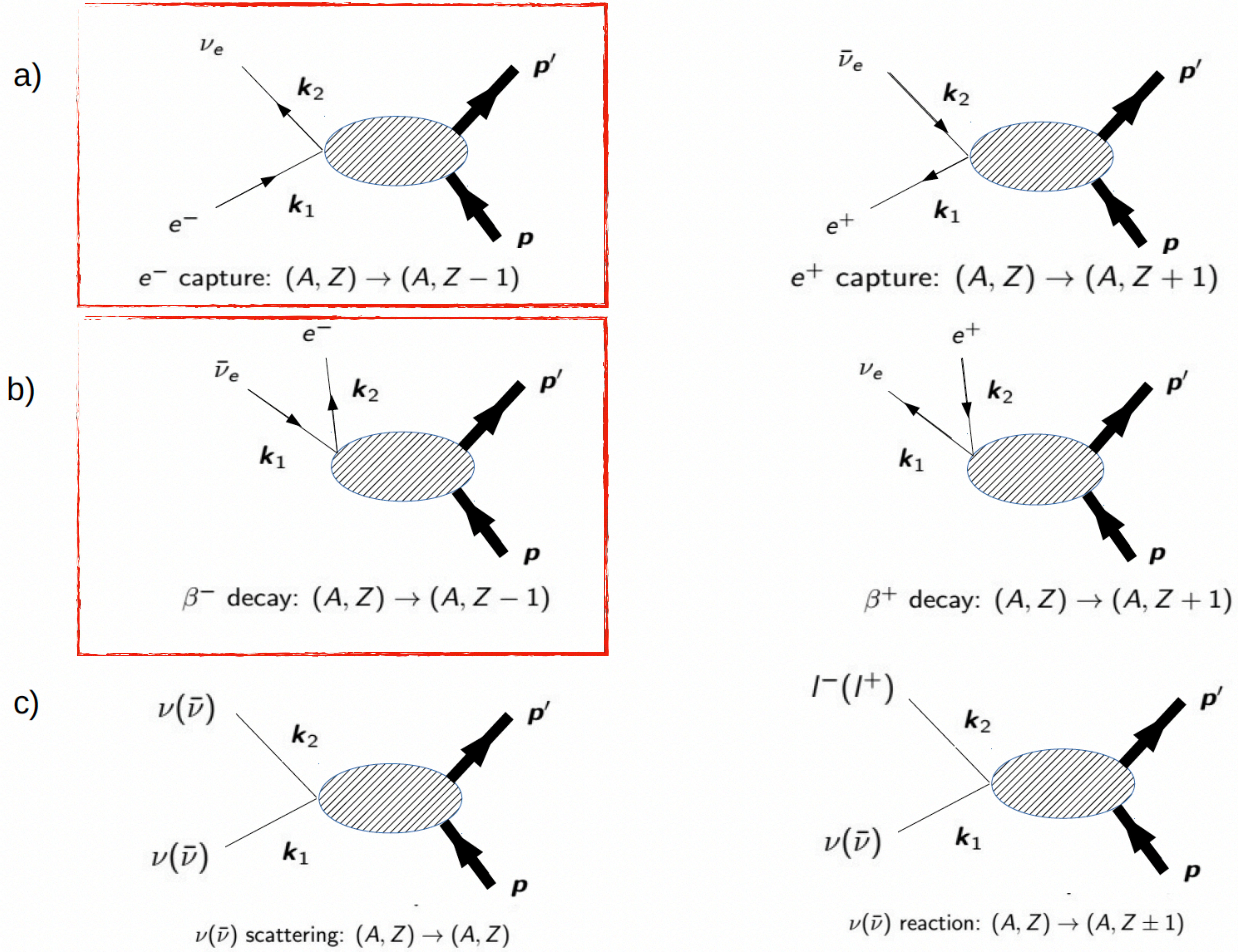
Parameters: a, b, V_L and V_D are fitted to exp. single beta-decay half-lives

DD-PCX functional:



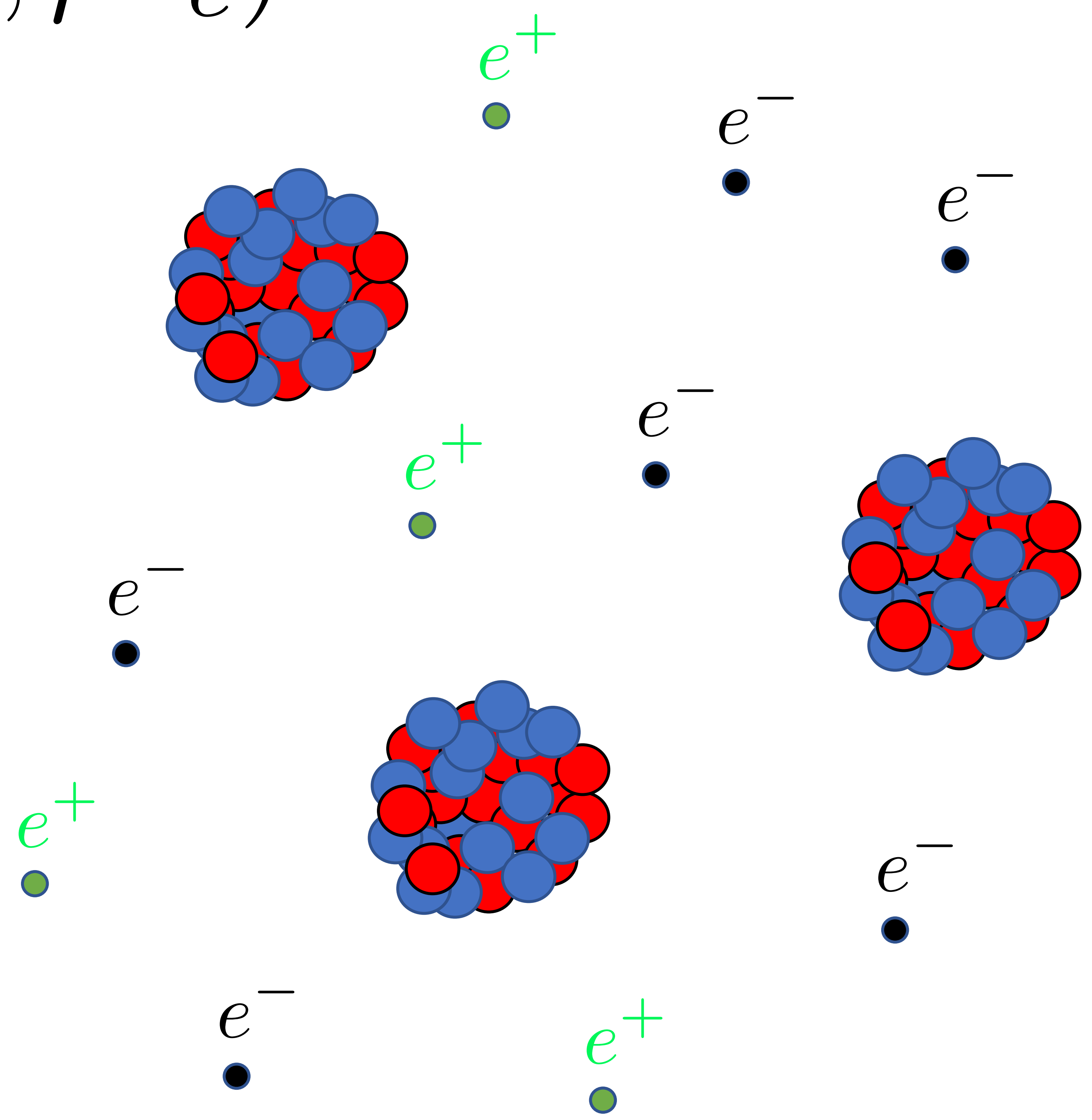
$$V_L = 0.6 \pm 0.1$$
$$V_D = 0.5 \pm 0.2$$
$$a = 22.5 \pm 29.5$$
$$b = -1.0 \pm 1.3$$

Weak-interaction rates

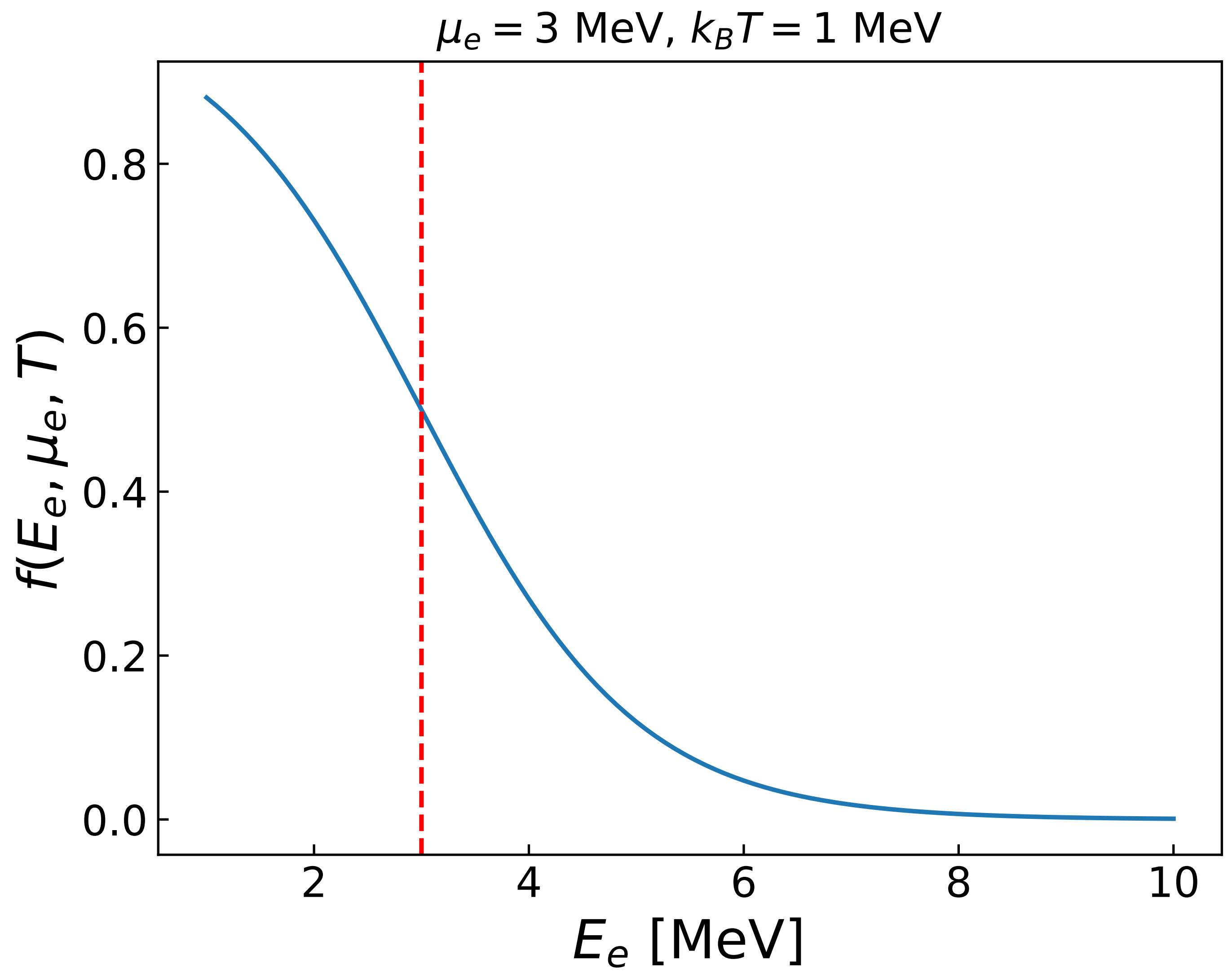


What are the assumptions of stellar environment ?

$$(T, \rho Y_e)$$



Fermi-Dirac distribution of electrons:



Electron chem. Potential determined from:

$$\rho Y_e = \frac{1}{\pi^2 N_A}\left(\frac{m_e c}{\hbar}\right)^3 \int_0^\infty (f_e - f_e^+) p^2 dp$$

[A. Ravlić, E. Yüksel, Y.F. Niu et al., PRC 102, 065804 (2020)]

[A. Ravlić, E. Yuksel, Y. F. Niu et al., PRC 102, 065804 (2020)]

Electron capture

- Reaction rates derived using the Walecka formalism

$$\underbrace{\hat{H}_W = -\frac{G}{\sqrt{2}} \int d^3\mathbf{r}\, j_\mu^{lept.}(\mathbf{r}) \hat{\mathcal{J}}_\mu(\mathbf{r}),}_{\text{current-current Hamiltonian}}$$

j_μ^{lept} - lepton current, $\hat{\mathcal{J}}_\mu(\mathbf{r})$ hadronic current

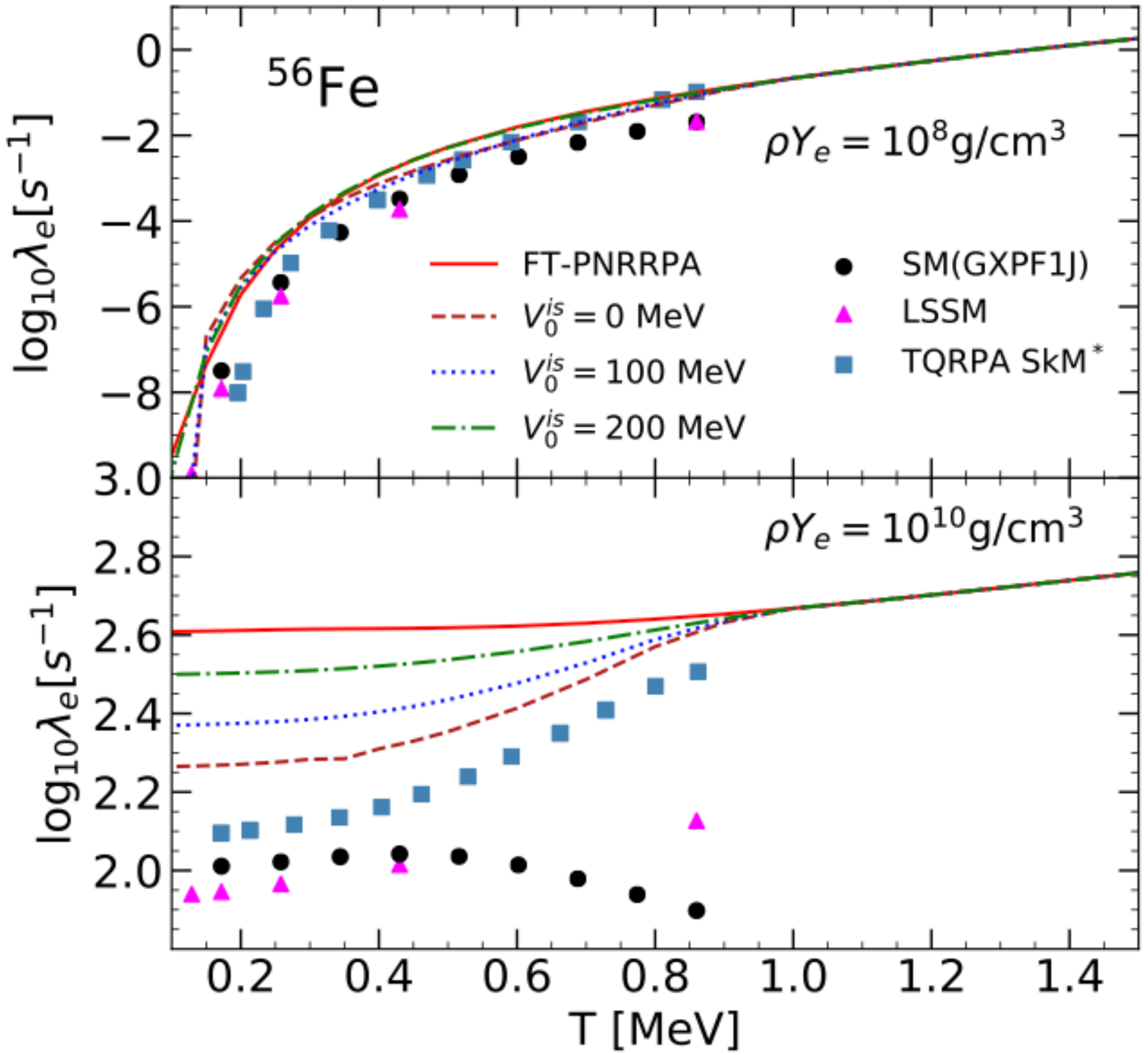
- Fermi Golden Rule:

$$\frac{d\sigma_{ec}}{d\Omega} = \frac{1}{(2\pi)^2}\Omega^2 E_\nu^2 \frac{1}{2} \sum_{lept.spin.} \underbrace{\frac{1}{2J_i+1} \sum_{M_i M_f} |\langle f|\hat{H}_W|i\rangle|^2}_{\text{matrix element of weak Hamiltonian}}$$

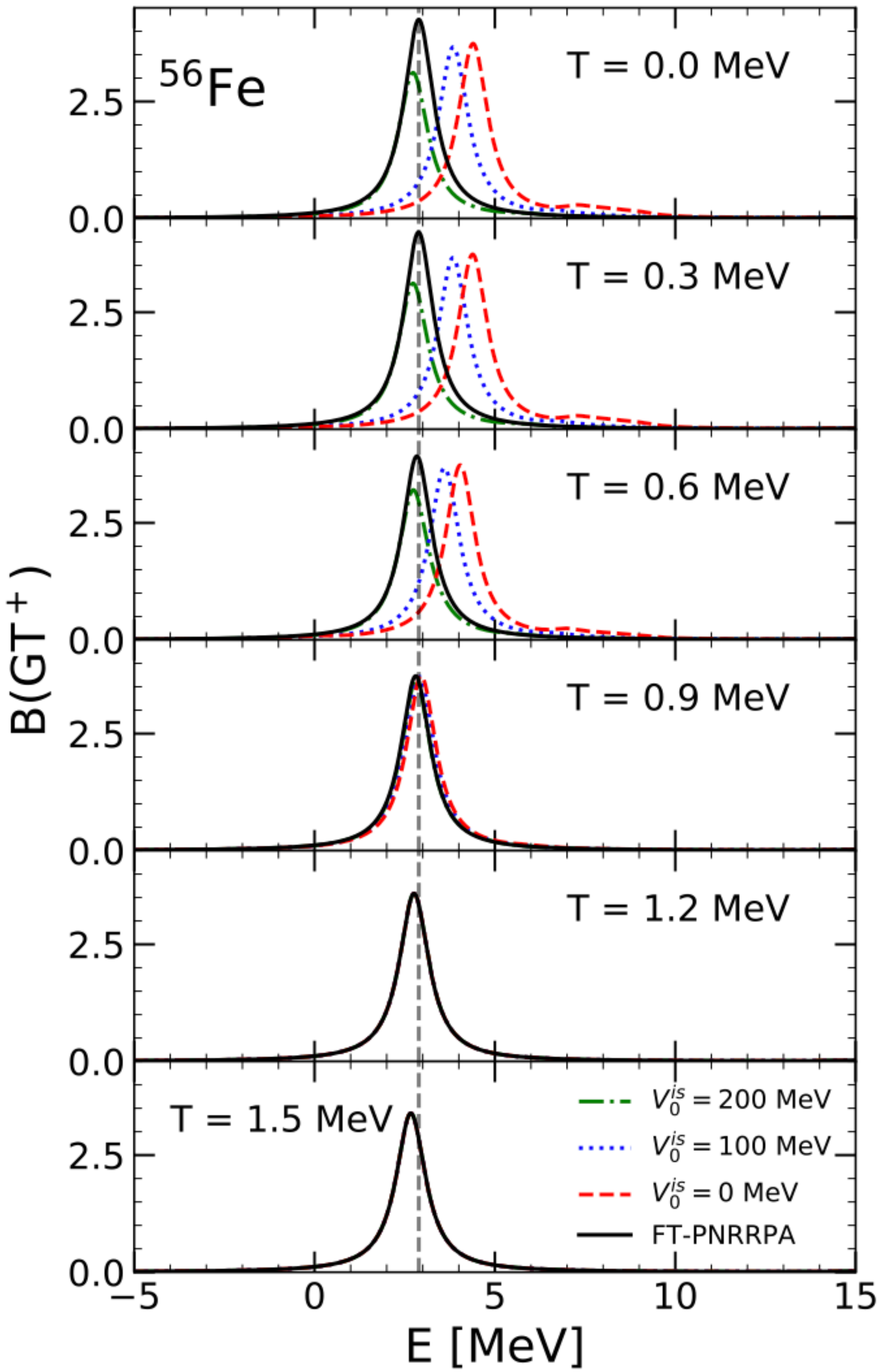
Matrix elements calculated within the FT-PNRQRPA

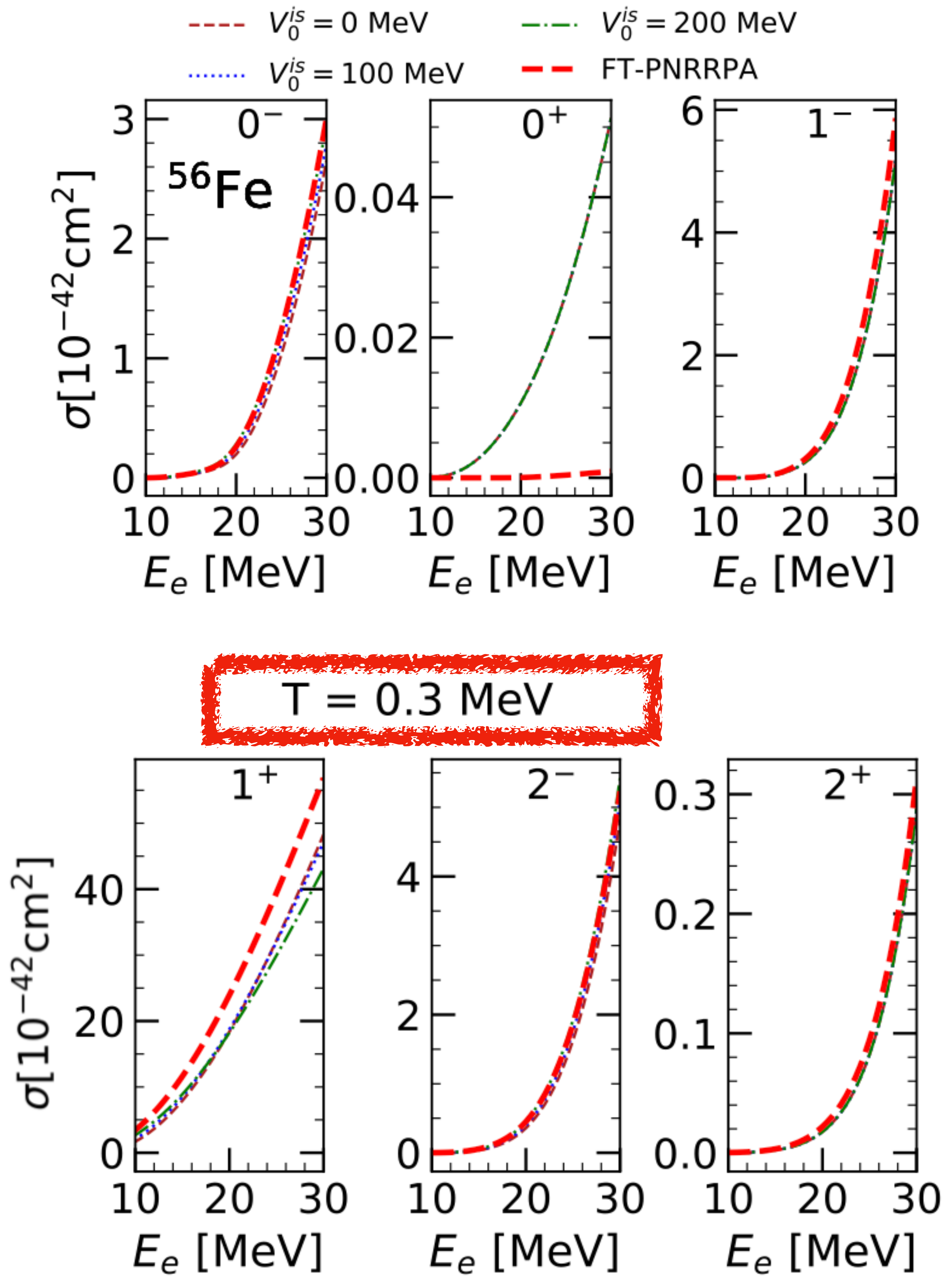
$$\lambda_{ec} = \frac{1}{\pi^2 \hbar^3} \int_{E_e^0}^{\infty} p_e E_e \sigma_{ec}(E_e) f(E_e, \mu_e, T) dE_e$$

Folding energy-weighted cross-section with Fermi-Dirac distribution of electrons



[A. Ravlić, E. Yuksel, Y. F. Niu et al., PRC 102, 065804 (2020)]





[A. Ravlić, E. Yuksel, Y. F. Niu and N. Paar, PRC 104, 054318 (2021)]

Beta-decay

Fermi function

FD factor

$$\lambda_\beta = \frac{\ln 2}{K} \int_0^{p_0} p_e^2 (W_0 - W)^2 F(Z,W) C(W) [1 - f(W)] dp_e$$

$$p_e = \sqrt{W^2 - 1}$$

$$W = E_e/(m_e c^2)$$

Shape factor:

$$C(W) = k + kaW + kb/W + kcW^2$$

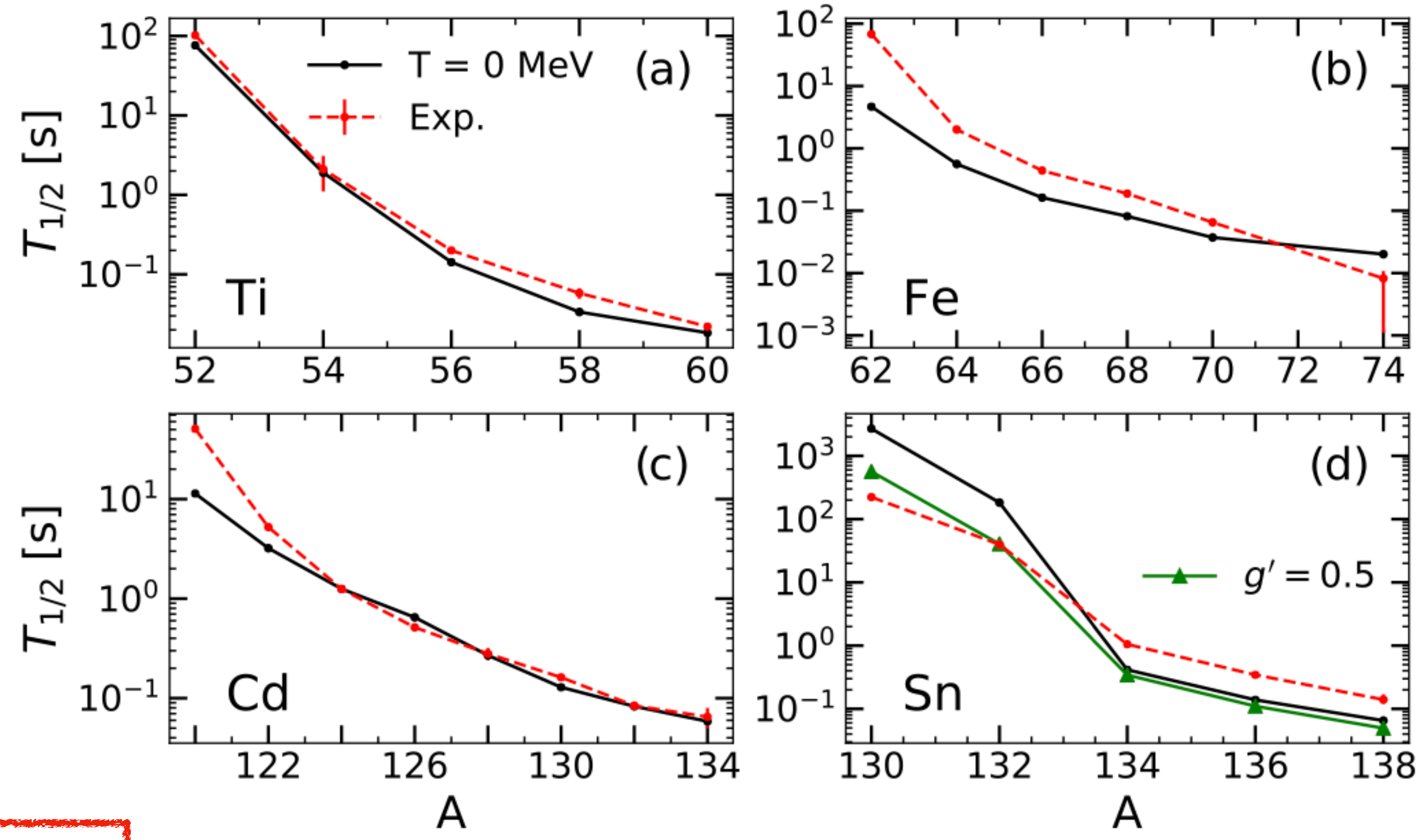
In case of allowed Gamow-Teller transition:

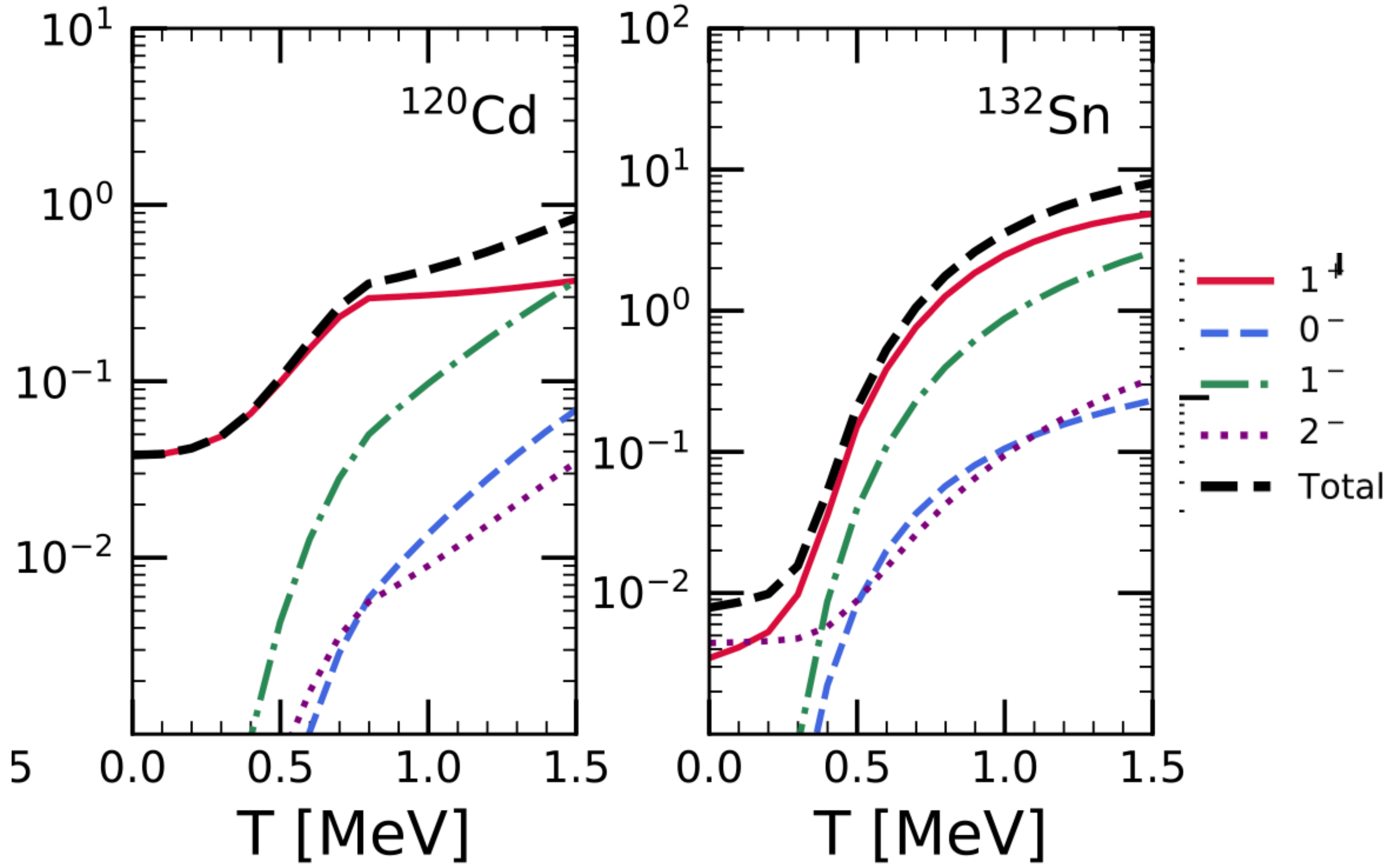
$$C(W) = B(\mathrm{GT}^-) = g_A^2 \frac{|\langle f || \boldsymbol{\sigma} \vec{\tau}_- | i \rangle}{2J_i + 1}$$

Matrix elements calculated within the FT-PNRQRPA

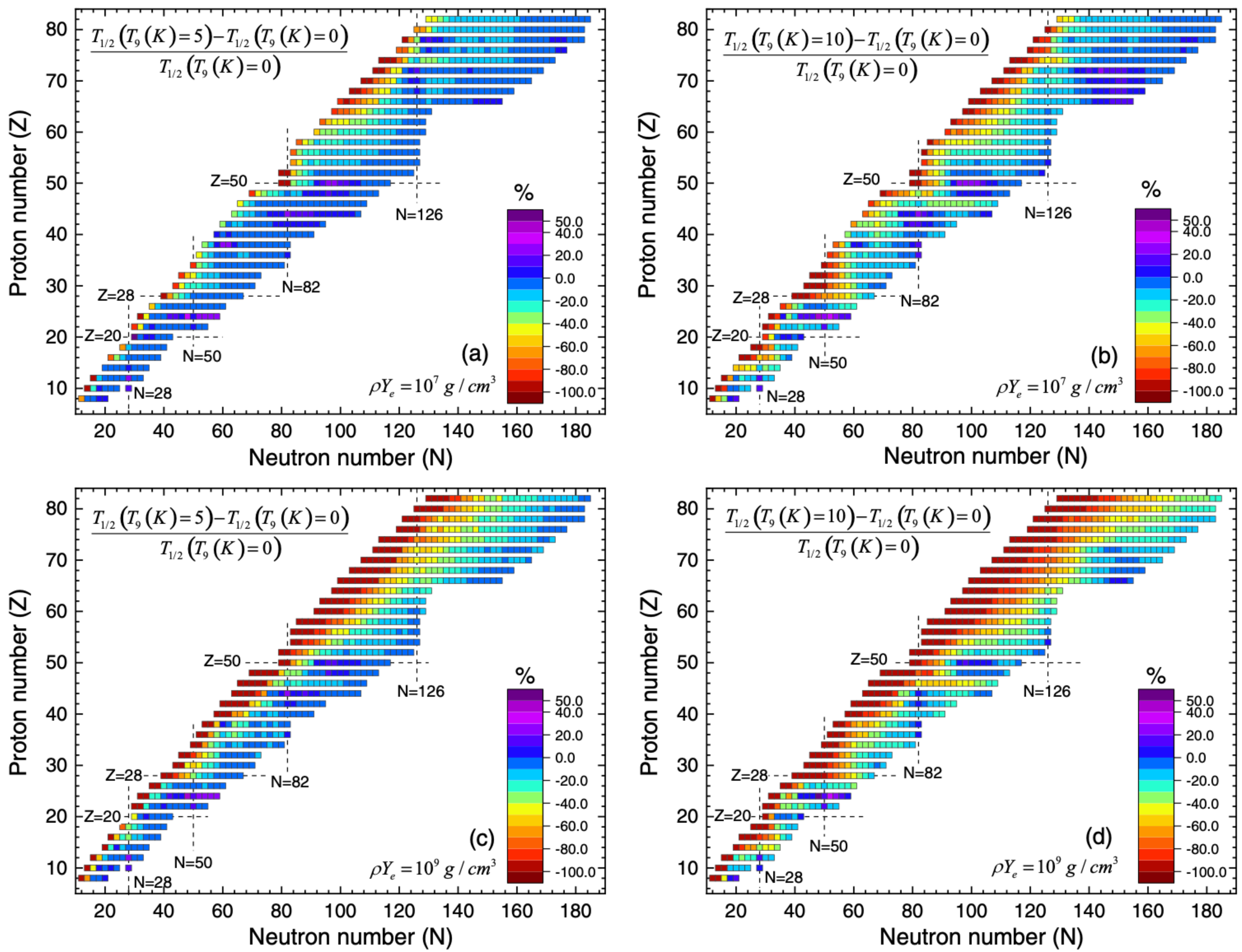
D3C* functional

[T. Marketin et al., PRC 93, 025805 (2016)]





[A. Ravlić, E. Yuksel, Y. F. Niu and N. Paar, PRC 104, 054318 (2021)]



- Nuclei with initially long half-lives at T = 0 are most impacted by the temperature change
- Nuclei closer to drip lines display more moderate changes compared to T = 0 case
- Almost all nuclei show decrease of half-life with increasing temperature
- Importance of de-excitations especially around pf-shell

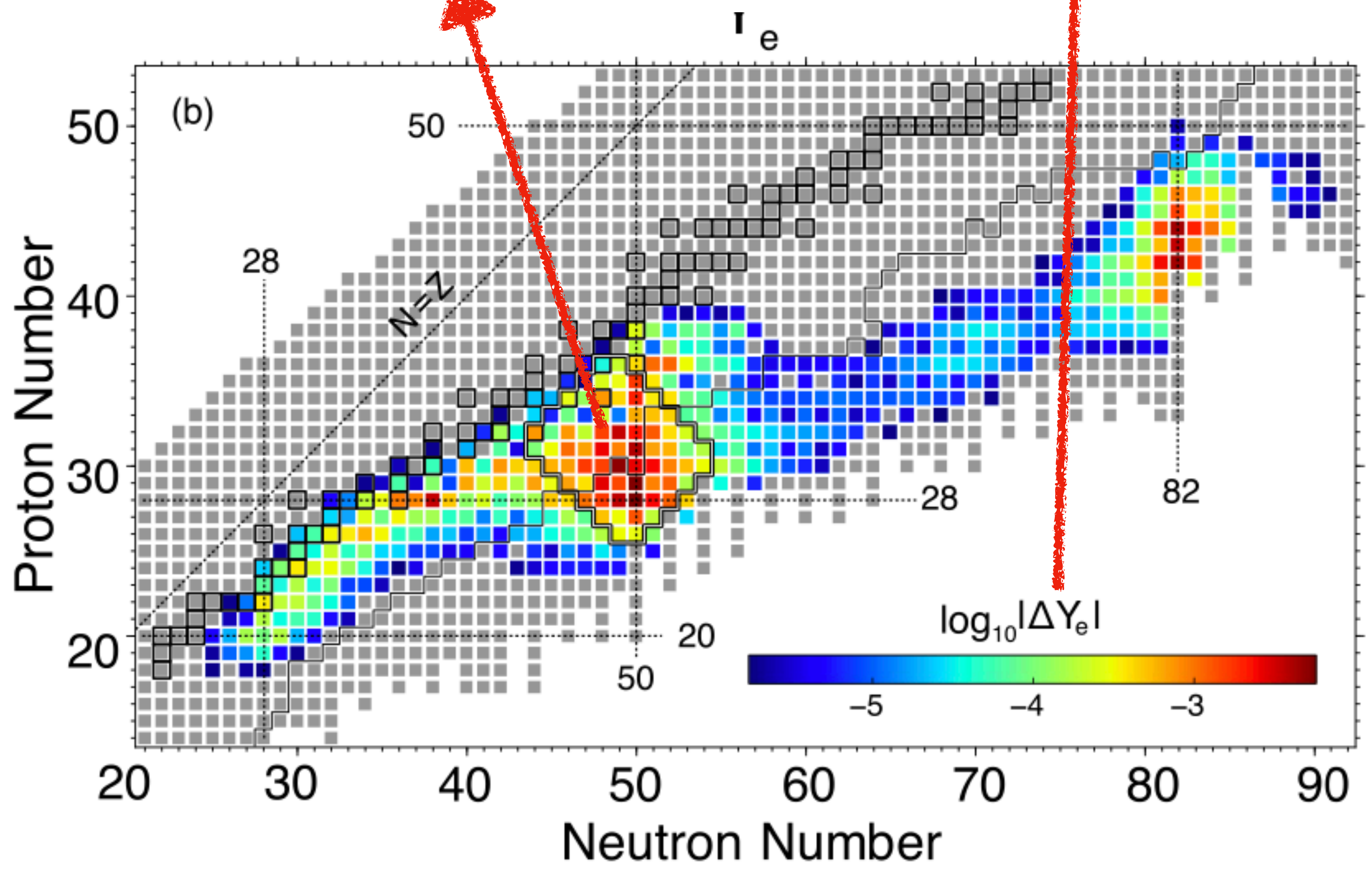
Implications for core-collapse SNe

[S. Giraud, E. M. Ney, A. Ravlic, R. G. T. Zegers, J. Engel, N. Paar et al., PRC 105, 055801 (2022), **Editors' Suggestion**]

Statistical re-sampling study:

Nuclei with the largest change of electron frac. up to neutrino trapping

"Diamond" region around N = 50



[C. Sullivan, E. O'Connor, R.G.T. Zegers et al., ApJ, 816:44 (14pp) (2016)]

Benchmark with EC rates On ^{86}Kr with shell-model calculations

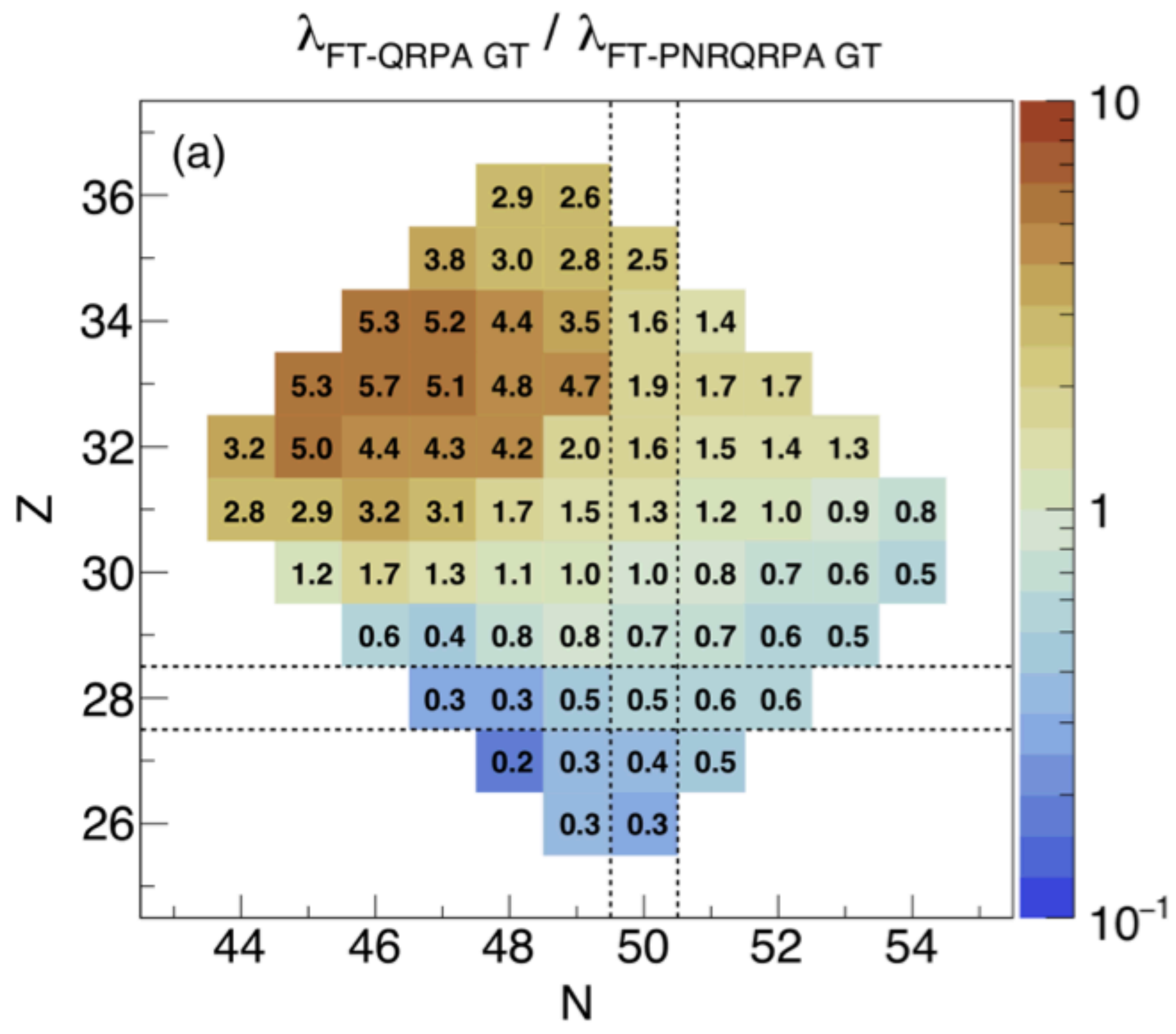
Employed theoretical models

FT-PNRQRPA
With relativistic D3C* interaction
(Spherical)

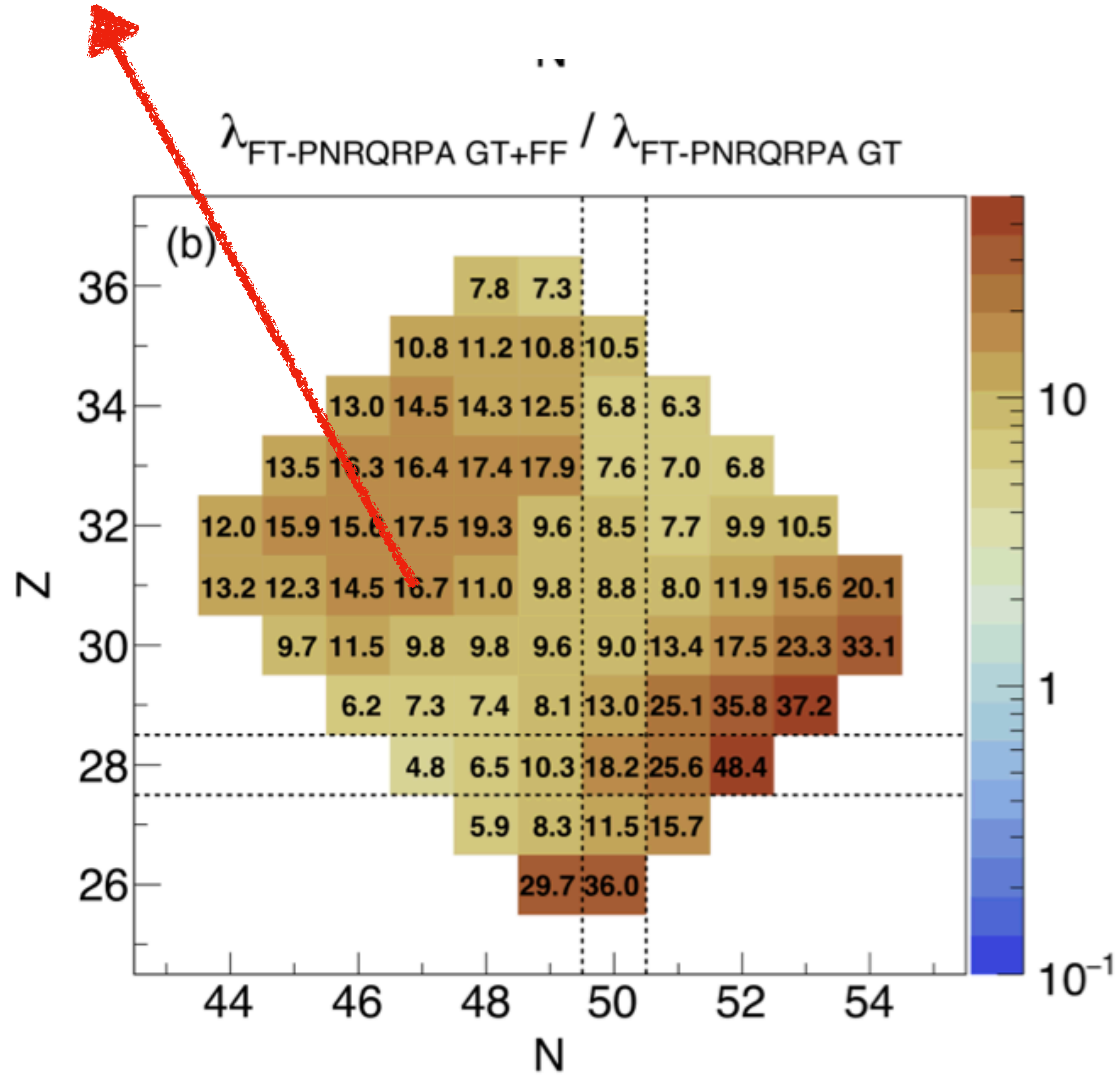
pnFAM FT-QRPA
With non-relativistic SkO' interaction
(ax-deformed)

$$T_9 = 10 \text{ GK}, \quad \rho Y_e = 10^{11} \text{ g/cm}^3$$

With increasing temperature ratio gets closer to one (Pauli blocking of GT)



Comparison of rates between relativistic and non-relativistic calculations



Importance of first-forbidden (FF) transitions (Only for relativistic calculation)

CCSNe simulations run with spherically-symmetric code GR1D (s15WW95 progenitor, NULIB data, SFHo EoS)

[C. Sullivan, E. O'Connor, R.G.T. Zegers et al., ApJ, 816:44 (14pp) (2016)]

EC rates for CCSNe are well constrained

Results for some important properties:

Evolution of electron-to-barion Fraction with barion density

Electron neutrino luminosity measured after bounce at R = 500 km

Central velocity as function of enclosed mass

Conclusion

- Developed FT-PNRQRPA (based on FT-BCS or FT-RHB) for studying highly-excited nuclei [A. Ravlić, E. Yuksel, Y. F. Niu and N. Paar, PRC 104, 054318 (2021), A. Ravlić, E. Yuksel, Y. F. Niu et al., PRC 102, 065804 (2020), A. Ravlic, Y.F. Niu et al., PRC 104, 064302 (2021)]
- Present theoretical models for weak-interaction rates applicable to CCSNe (high density and temperature, nuclei mostly spherical …)
  [S. Giraud, E. M. Ney, A. Ravlic, R. G. T. Zegers, J. Engel, N. Paar et al., PRC 105, 055801 (2022), **Editors' Suggestion**]
- However, there are astrophysical scenarios where transition-by-transition description is necessary —> nuclei in neutron star crust (low T, high density)
- To do: extend present model to axially-deformed nuclei

Collaborators:

THANK YOU FOR YOUR ATTENTION !

- **N. Paar** (Thesis supervisor, University of Zagreb, Croatia)
- **T. Oishi** (YITP, Kyoto University, Japan)
- **E. Yüksel** (Yildiz Technical University, Turkey)
- **Y. F. Niu** (Lanzhou University, China)
- **R. G. T. Zegers and S. Giraud** (MSU/NSCL/FRIB, USA)
- **E. M. Ney and J. Engel** (University of North Carolina, USA)
- **T. Nikšić** (University of Zagreb, Croatia)
- **G. Colò** (Università degli Studi di Milano and INFN Italy)
- **E. Khan** (Université Paris-Sud, IN2P3-CNRS UniversitéParis-Saclay France)