Beta-decay formula for the allowed and first forbidden transitions revisited

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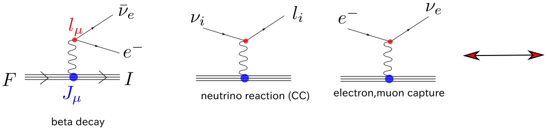
base on

Beta-decay formulas revisited (I): Prog. Theor. Exp. Phys. 2021,103D03
Beta-decay formulas revisited (II): (in preparation)

W. Horiuchi, TS, Y. Uesaka, K. Yoshida

Nuclear semi-leptonic process

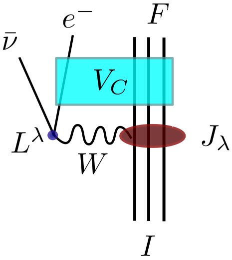


Nuclear structure
Nuclear Current

- beta decay : $E_0 \leq$ a few 10MeV
  - β decay in r-process (Coulomb effect of beta particle should be examined under extreme environment, high Q_β, large nuclear charge Z)
  - Low energy precision test of SM and physics BSM
  - spectrum of reactor neutrino (sterile neutrino)

- Neutrino reaction:
  LBL neutrino experiments:$E_\nu \sim GeV$ QE, RES, DIS, Supernova : $E_\nu \leq 50$MeV, Reactor neutrino, ..

- electron, muon capture
  Capture of muon from atomic orbit: energy transfer ~ 100MeV



Transition matrix element:

$$T_{fi} = \frac{G_F V_{ud}}{\sqrt{2}} \int d\boldsymbol{x} < F|(V_\mu(\boldsymbol{x}) - A_\mu(\boldsymbol{x}))|I > \bar{\psi}^{(-)}_{e^-}(\boldsymbol{x})\gamma^\mu(1-\gamma_5)\psi_\nu(\boldsymbol{x})$$

Nuclear current density   Lepton current density

- PLW lepton:J. D. Walecka (in Muon Physics II, V. W. Hughes, C.S.Wu ed. 1975)

$$T^J_{C,L,M,E}(q)$$

(νd reaction, S. X. Nakamura et al. PRC63 2001)

- Systematic expansion: H. Behrens, W. Bühring, NP A162(1971)111
H. Behrens and J. Jänecke, Landolt-Boernstein - Group I Elementary Particles, Nuclei and Atoms 1969

$$\sum_{\alpha,\beta,\gamma,\delta} c_{\alpha,\beta,\gamma,\delta}(\frac{r}{R})^{\alpha}(m_e R)^{\beta}(E_e R)^{\gamma}(\alpha Z)^{\delta}$$

- Transparent compact formula: K. Koshigiri et al. NPA340 (1980) , M. Morita, "beta decay and muon capture" (1973)

Objective:

Provide transparent, easy to use, practical formula/code on nuclear β decay

NLO improvement of 'conventional' LO
formula applicable to neutrino-reaction/electron-capture

- Formulation of nuclear beta decay
- Electron wave function LO and NLO

  Beta-decay formulas revisited (I): Prog. Theor. Exp. Phys. 2021,103D03

- First forbidden transition

  Explicit formula including velocity dependent current/weak magnetism within IA.

  - 1^- Current conservation and Siegert theorem
  - 0^- Interplay between A_0 and $\boldsymbol{A}$
  - 2^- Unique first forbidden transition and spectrum shape

  Example: beta decay of ^{160}Sn

  nuclear energy-density functional(EDF) (K. Yoshida, PTEP 2013(11)113D02)

  Role of first forbidden transition:M. T. Mustonen, J. Engel, PRC93,014304(2016)
  $A = 80, 160$ region: T. Shafer et al. PRC94,055802(2016)

Formulation ofnuclear beta decay

- Multipole operator Ξ_{JL} following K. Koshigiri et al. (Nucl. Phys. A340(1980)482)

$$< F||\Xi_{JL}||I >= \int_0^\infty drr^2 \rho_{JL}(r)[c_g G_{\kappa_e}(r) g_{\kappa_\nu}(r) + c_f F_{\kappa_e}(r) f_{\kappa_\nu}(r)]$$

  - Nuclear transition density - example of axial vector current –

$$\rho_{JL}(r) =< F|| \int d\Omega_r [Y_L(\hat{r}) \otimes \boldsymbol{A}(\boldsymbol{r})]_J ||I >$$

  - G_κ, F_κ:Electron, g_κ, f_κ: neutrino wave function

- decay rate:

$$\Gamma = \frac{(G_F V_{ud})^2}{\pi^2} \int_{m_e}^{E_0} dE_e p_e E_e (E_0 - E_e)^2 \sum_{J,\kappa_e,\kappa_\nu} \frac{1}{2J_i + 1} | \langle F \| \sum_L \Xi_{JL}(\kappa_e, \kappa_\nu) \| I \rangle |^2,$$

Electron Wave Function

Electron Dirac wave function $\psi_{-1} = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} G_{-1}(r) \\ -i\boldsymbol{\sigma}\cdot\hat{r}F_{-1}(r) \end{pmatrix} \chi_{s_e}$

$$
\begin{aligned}
G_{-1}(r) &= G_{-1}(0) + \int_0^r dr'(m_e + E_e - V_c(r'))F_{-1}(r') \\
F_{-1}(r) &= \frac{1}{r^2}\int_0^r dr'(m_e - (E_e - V_C(r')))G_{-1}(r')r'^2
\end{aligned}
$$

Leading order(LO)

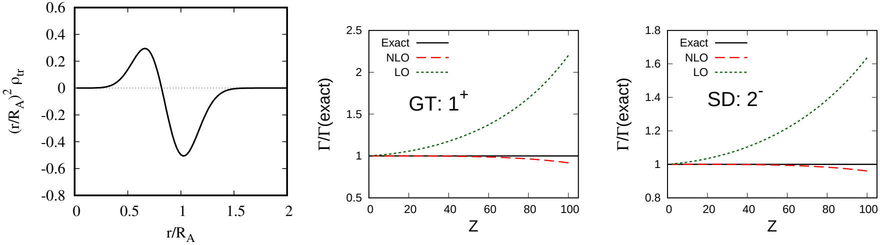
$$
G_{-1}^{\mathrm{LO}} = G_{-1}(0) = \alpha_{-1}
$$

Next to Leading order(NLO)

$$
G_{-1}^{\mathrm{NLO}} = \alpha_{-1} + \int_0^r dr'(m_e + E_e - V_c(r'))F_{-1}^{\mathrm{LO}}(r')
$$

- Recursion formula of the integral equation (M. E. Rose(1951), H. Behrens, W. Bühring(1971))

How good NLO for beta decay rate?



transition density

Decay rate for Gamow-Teller and Spin Dipole transition ($E_0 = 10$MeV)

- LO (LO approx. for electron, neutrino), NLO(NLO electron, full neutrino)
- NLO works well for a wide range of Z

  comment

J^π		Effective nuclear current
1^+	Allowed	$\boldsymbol{A} \quad + \quad (\ A_0\boldsymbol{r},\ \boldsymbol{V}\times\boldsymbol{r})$
0^-	First forbidden	$\boldsymbol{A}\cdot\boldsymbol{r},\ A_0$
1^-	First forbidden	$\boldsymbol{A}\times\boldsymbol{r},\ V_0\boldsymbol{r},\ \boldsymbol{V}$
2^-	Unique first forbidden	$[\boldsymbol{A}\otimes\boldsymbol{r}]_{(2)}$

Velocity dependent currents, weak magnetism in non-rela IA

$$\boldsymbol{V} \sim g_V\frac{\boldsymbol{p}}{M},\quad \frac{g_V+g_M}{2M}i\boldsymbol{\sigma}\times\boldsymbol{q}$$

$$A_0 \sim g_A\frac{\boldsymbol{p}'+\boldsymbol{p}}{2M}$$

(Induced pesudoscalar term $\propto q^\mu \sim m_l(e,\mu,\tau)$ is not included.)

- Microscopic model of transition density

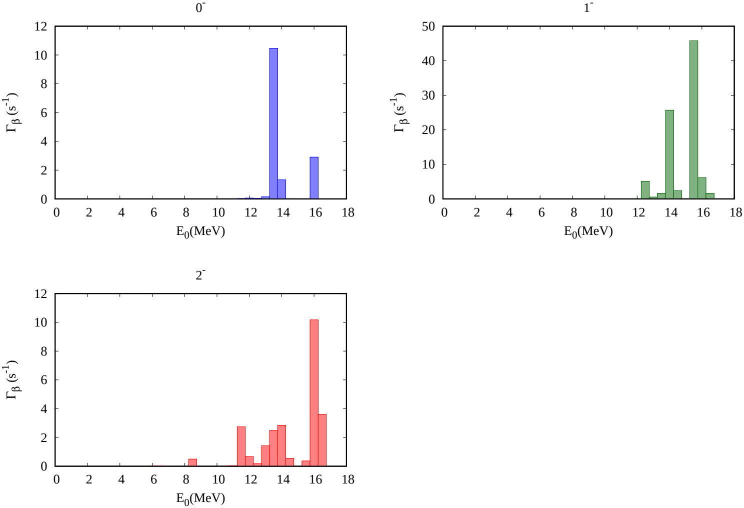
nuclear energy-density functional(EDF)(K. Yoshida, PTEP 2013(11)113D02)

example: ^{160}Sn($Z = 50$)

Ratio of decay rate using GT and SD operator

J^π		exact/LO	exact/NLO
1^+	allowed	0.869	1.00
0^-	first forbidden	0.874	1.00
1^-	first forbidden	0.895	1.00
2^-	unique first forbidden	0.857	1.00

Decay rate $0^-, 1^-, 2^-$



$\boldsymbol{V}$ for $\pi = (-1)^J$ transition

- E1 transition of Photo reaction : Use of Siegert theorem because of large MEC for $\boldsymbol{V}$ / small MEC for V_0.
- Electron scattering, neutirno reaction in PLW lepton

$$\boldsymbol{V} = \hat{q}(\boldsymbol{V}\cdot\hat{q}) + (\hat{q}\times\boldsymbol{V})\times\hat{q}$$

Eliminate longitudinal current and part of transverse electric current by using current conservation relation

$$\boldsymbol{q}\cdot\boldsymbol{V} + i[H, V_0] = 0$$

Note: weak charged current does not conserve when we include Coulomb, pn mass difference in nuclear Hamiltonian. $\rightarrow$ extended Siegert theorem.

- non trivial for distorted wave.

Procedure:

Ξ^{orig}_{LJ} + (FB trans) $\rightarrow T_{LJ}(q)$ and implement $T^L_J(q) = -\frac{\omega}{q}T^C_J(q)$ + ([FB trans]$^{-1}$) $\rightarrow \Xi^{New}_{LJ}$

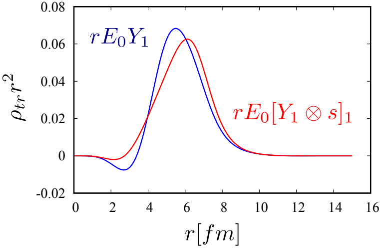
Original

$$\Xi \sim -Y_J V_0(\boldsymbol{r})\Phi(r) + i[Y_{J-1} \otimes \boldsymbol{V}(\boldsymbol{r})]_J \phi_{J-1}(r) + i[Y_{J+1} \otimes \boldsymbol{V}(\boldsymbol{r})]_J \phi_{J+1}(r)$$

New

$$\Xi \sim -Y_J V_0(\boldsymbol{r})(\Phi(r) - E_0\sqrt{\frac{2J+1}{J}}\frac{\int_0^r dr'(r')^{J+1}\phi_{J-1}(r')}{r^{J+1}}) + i[Y_{J+1} \otimes \boldsymbol{V}(\boldsymbol{r})]_J \tilde{\phi}_{J+1}(r)$$

Decay rate [s^{-1}]	Tot	(orig)	V	A
Full	89.0	139	30.0	40.0
LO	93.8	149	34.2	45.5

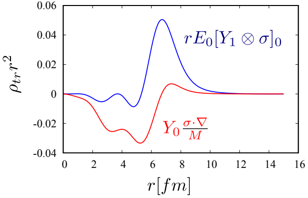


PLW approx.

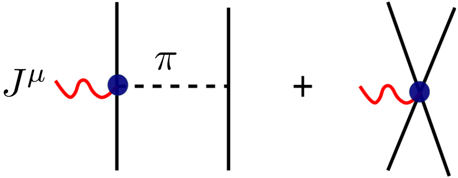
$$A_\mu L^\mu e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \sim A^0 L^0 - \boldsymbol{A}\cdot\boldsymbol{L}(-i\boldsymbol{q}\cdot\boldsymbol{r}) = L_0(A_0 + i\frac{E_0}{3}\boldsymbol{A}\cdot\boldsymbol{r})$$

Impulse approximation: $ig_A(-\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}}{M} + \frac{E_0}{3}\boldsymbol{\sigma}\cdot\boldsymbol{r})$

Decay rate [s^{-1}]	Tot	$g_A Y_0 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}}{M}$	$g_A[Y_1 \otimes \boldsymbol{\sigma}]_0$
Full	15.0	111	51.4
LO	14.1	123	60.3



- Cancelation between A_0 and $\boldsymbol{A}$ in IA
- Large pion exchange current is expected for A_0 (Kobodera-Delorme-Rho)



$$[Q_5, A^\mu] \sim \mathcal{O}(1) \text{ for } A_0$$

$A = 16$: about 40% enhancement, I. S. Towner, Ann. Rev. Nucl. Part. Sci. 36(1988)

$A = 205 \sim 212$: $\epsilon_{mec} \sim 2$, E. K. Warburton, PRC44 (1991)

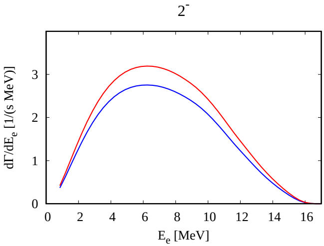
- MEC and quenching of GT: E. M. Ney et al. PRC105 (2022) unique first forbidden transition: dominant operator $r[Y_1 \otimes \boldsymbol{A}]_2$

$$C(E) \propto p_\nu^2 + p_e^2$$

Higher order operators $[Y_3 \otimes \boldsymbol{A}]_2, \quad [Y_2 \otimes \boldsymbol{V}]_2, \quad Y_2 A_0$

- Neutrino mass ^{187}Re R. Dvornicky et al. PRC83 (2011)
- Fierz interference term A. Glick-Magid et al. PLB767 (2017)

2^-

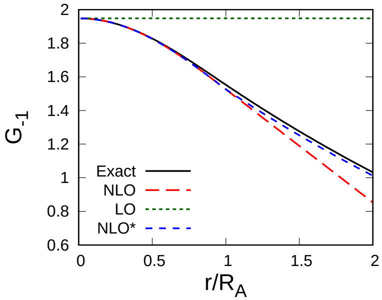


Full, LO

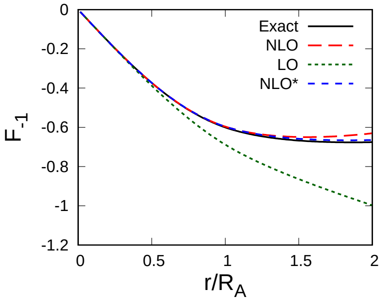
- Shape of beta ray spectrum is appreciably modified by including NLO.
- caution to use the unique first forbidden transition for BSM.

Summary

- Effect of Coulomb on beta decay rate is re-examined
- Formalism ready to apply to electron capture/neutrino reaction(electron,muon)
- Widely used formula(LO) overestimates the decay rate about 50-60%(compared with 'exact') for heavy nuclei ($Z \sim 80$)
- simple NLO formula (systematic iteration of integral equation) works well for schematic and microscopic transition densities.
- Formula for velocity dependent current (including induced weak magnetism term) is developed
  - 1^-: conservation of vector current is implemented.
  - 0^-: destructive interference between A_0 and $\boldsymbol{A}$ is found, suggesting possible role of MEC.
  - 2^-: shape of beta spectrum is very much modified for unique first forbidden transition by NLO.

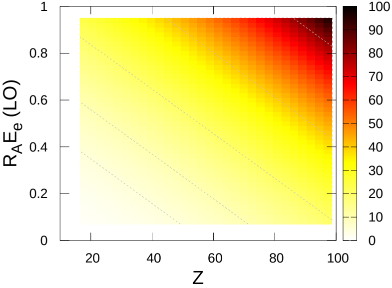


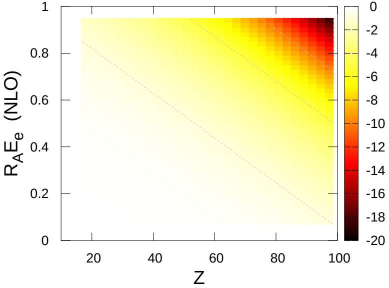
s-wave



p-wave

$Z = 82, A = 208$, $\kappa_e = -1$ and $E_e = 10$ MeV. (Uniform charge distribution)





Ratio $[G_{-1}(\text{approx.})/G_{-1}(\text{exact}) - 1] \times 100$. The LO wave function (left) and NLO wave function (right) is used for the approximate wave function.