On the intersection of the cluster, shell and collective models

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Tyger, Tyger burning bright,
In the forest of the night;
What immortal hand or eye,
Could frame thy fearful symmetry?

William Blake

I. Introduction
Structure models of atomic nuclei

Based on different physical pictures

Model	Degrees of freedom
shell	nucleon
collective	deformation param.
cluster	relative motion
(molecular)	(+ intern.struct.)

What is the relation of the fundamental structure models (in general)?

What is their common intersection (if any)?

I. Introduction

II. Single shell problem: the 1958 connection

III. Many shells problem: MUSY

IV. Features of MUSY

V. Applications

VI. Summary and conclusions

II. Connection-1958: SU(3)

Elliott: *Proc. Roy. Soc. A 245, 28,562 (1958)*

Spectra of light nuclei,
deformation + rotation from spherical shell model:
SU(3).

From shell model to cluster model:
Wildermuth-Kanellopoulos: *Nucl. Phys. 7, 150 (1958)*
Harm. osc. appr. $H_{SM} = H_{CM}$

Cluster-shell connection: SU(3).
Bayman-Bohr: *Nucl. Phys. 9, 596 (1958/59).*

A quadrupole collective or a cluster band is picked up from the spherical shell model basis by their special SU(3) symmetry.

For a single major-shell problem the connection between the shell, collective and cluster models is provided by an SU(3) dynamical symmetry:

$$U(3) \supset SU(3) \supset SO(3) .$$

III. Many major shells: MUSY
Multiconfigurational dynamical symmetry

Extension of the U(3) connection from 1958
(J. Cseh, Phys. Rev. C 103 (2021) 064322.)

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Intersection of the
-(No-core) Symplectic shell model
-Contracted symplectic collective model
-Microscopic and semimicroscopic cluster models

Symplectic shell model

G. Rosensteel, D. Rowe, PRL 38 (1977) 10

Extension of Elliott, microscopic coll. model

$$\begin{array}{ccccccc} \mathrm{Sp(6,R)} & \supset & \mathrm{U(3)} & \supset & \mathrm{SU(3)} & \supset & \mathrm{SO(3)} \\ |\,[n_1^s, n_2^s, n_3^s], [n_1^e, n_2^e, n_3^e], \rho, [n_1, n_2, n_3], (\lambda, \mu), & & K, & & L\rangle & & \end{array}$$

Multi-shell extension of the Elliott model, microscopic version of the collective model.

Symmetry-adapted no-core shell model

T. Dytrych et al. J. Phys. G 35 (2008) 123101

Symmetry-adapted (no-core) quartet model

J. Cseh, Phys. Lett. B **743**, 213 (2015).

Contracted symplectic model

(D.J. Rowe, G. Rosensteel, Phys. Rev. C 25 (1982) 3236(R);

O. Castanos, J. P. Draayer, Nucl. Phys. A 491 (1989) 349.)

$$U_s(3) \otimes U_e(6) \supset U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Simpler mathematical structure,
bosonized description,
algebraic collective model of the multi-shell problem.

Cluster model

Microscopic or semimicroscopic:
antisymmetrization.

Semimicroscopic algebraic cluster model

J. Cseh, G. Lévai, Ann. Phys. 230 (1994) 165.

$$U_{C_1}(3) \otimes U_{C_2}(3) \otimes U_R(4) \supset$$
$$U_c(3) \otimes U_R(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Microscopic model space, algebraic operators.
Internal cluster structure: Elliott model,
relative motion: modified vibron.

Spin-isospin sector:
in symplectic
contarcted symplectic
semimicroscopic algebraic cluster models

$$U^{ST}(4) \supset U^{S}(2) \otimes U^{T}(2)$$

Shell (quartet), cluster and collective states: representation labels of an algebra-chain

(J. Cseh, Phys. Rev. C 103 (2021) 064322.)

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

IV. Features of MUSY

A) Composite symmetry of a composite sytem:

1. U(3) dysy in each configuration,

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

2. symmetry transforming the configurations
   (in the pseudo space of particle indeces)
   identical spectra
   invariant or transformed H-operator

Particle scheme: n particles in HO states

P. Kramer, M. Moshinsky, in Group Theory and Its Applications (ed: E.M. Loebl, Academic Press, New York) p. 339 (1968).

$$C_{jk}^{st} = \eta_j^s, \xi_k^t$$

$$[C_{jk}^{st}, C_{j'k'}^{s't'}] = \delta^{ts'}\delta_{kj'}C_{jk'}^{st'} - \delta^{st'}\delta_{jk'}C_{j'k}^{s't}$$

Symmetry-group: U(3n).

Symmetry-group: U(3n).

Contractions with resp. s,t particle indices, and with resp. j,k space indices:

$$C_{jk} = \sum_{s=1}^{n} C_{jk}^{ss} \quad , \quad C^{st} = \sum_{j=1}^{3} C_{jj}^{st}$$

$$U(3n) \supset U(3) \otimes U(n)$$

$$\mathrm{U(n)} \supset \mathrm{U(n-1)} \supset \mathrm{O(n-1)} \supset \mathrm{S(n)},$$

$$U(3) \supset SU(3) \supset SO(3)$$

B) Dual symmetry breaking

1. U(3) and SU(3) symmetry is dynamically broken by e.g. LL.

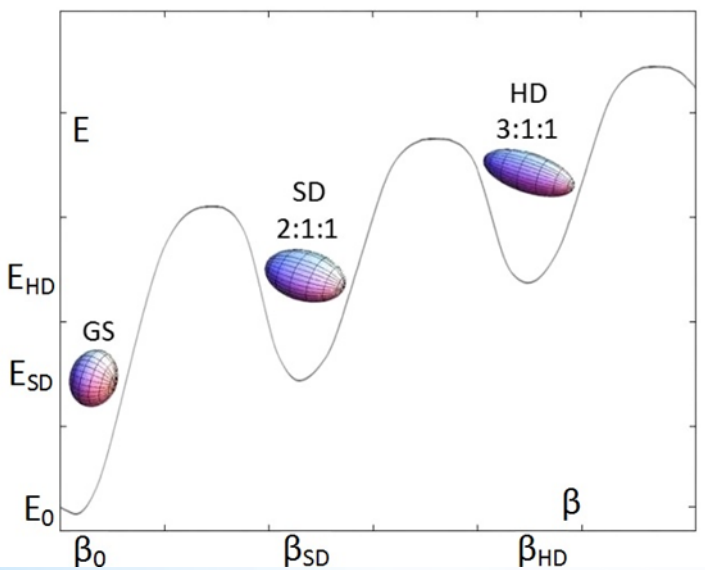
2. SO(3) is spontaneously broken in the eigenvalue problem of the intrinsic H. Deformed intrinsic state. J. Cseh, Phys. Lett. B. 793, 59 (2019)

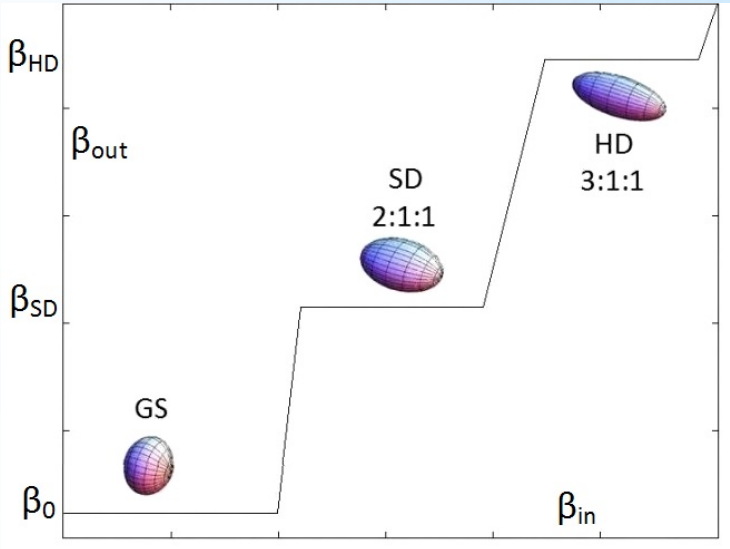
Dual symmetry breaking in many other models as well.

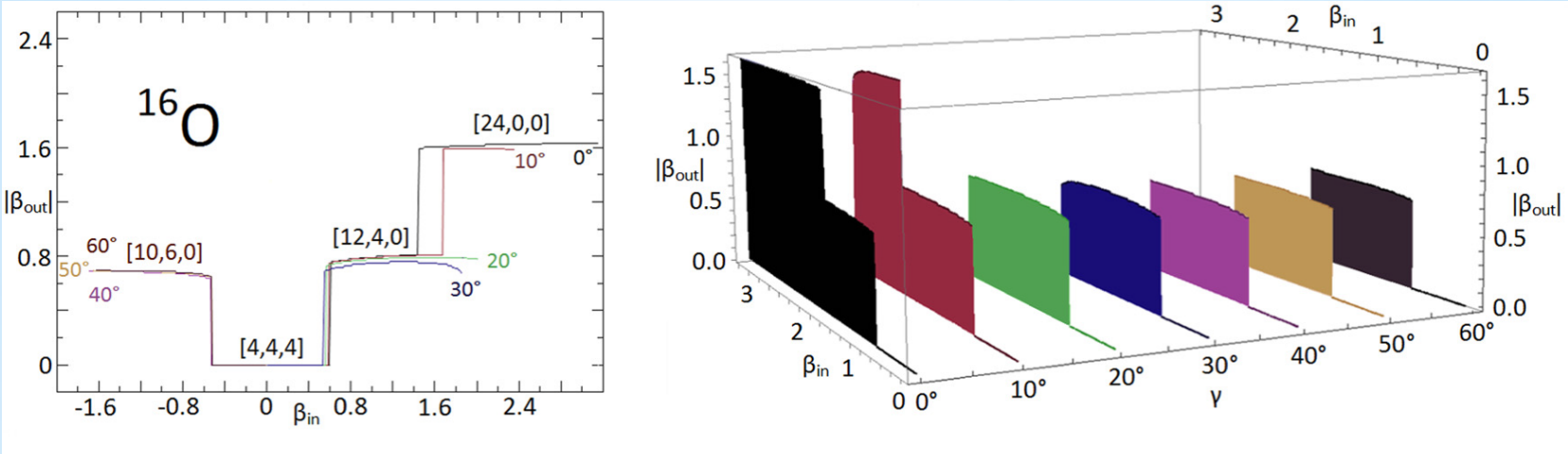
J. Cseh, Eur. Phys. J. Special Topic 229 (2020) 2543.

V. Applications

A) Shape isomers from the stability and selfconsistency of the (quasidynamiclal) U(3)

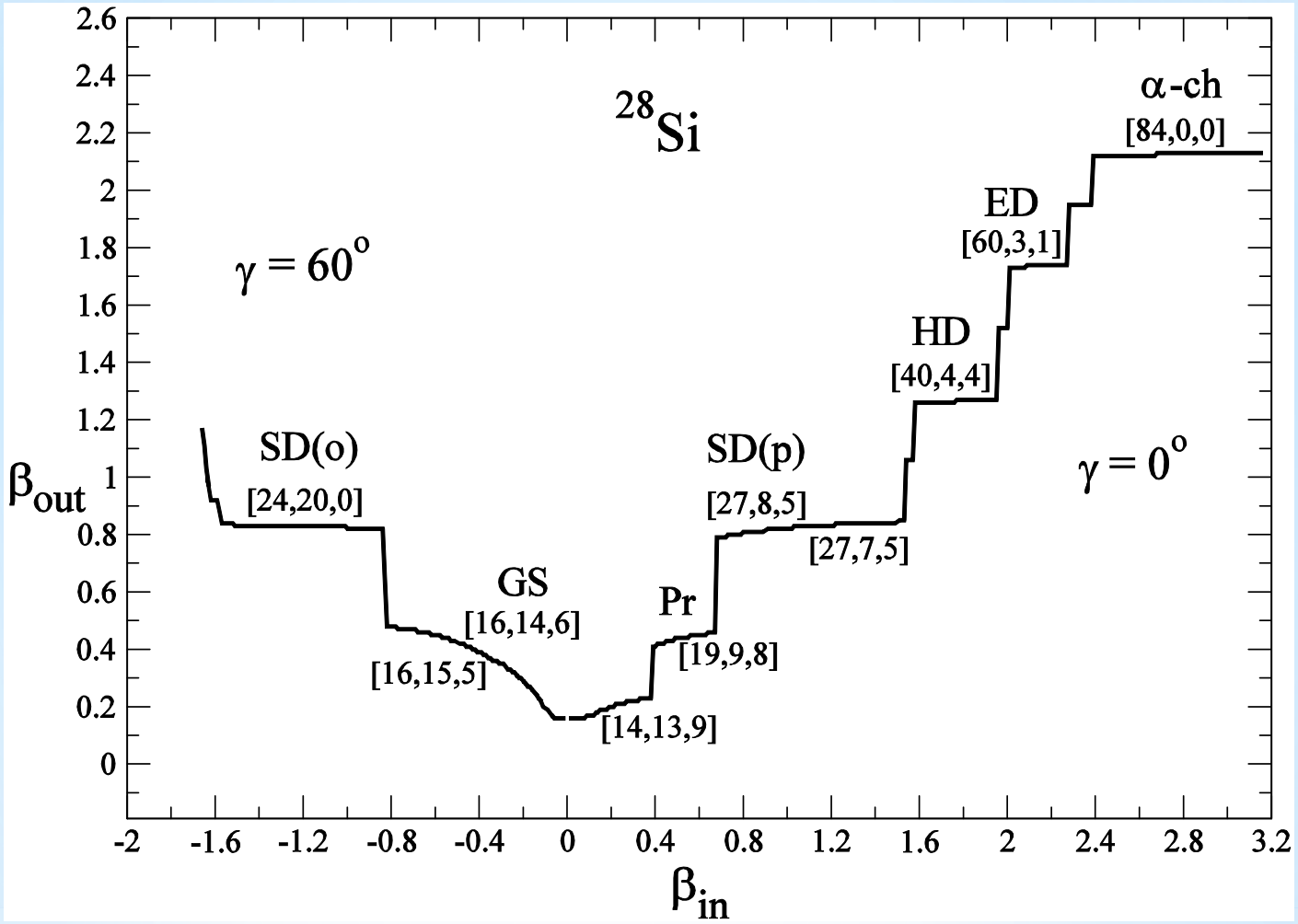




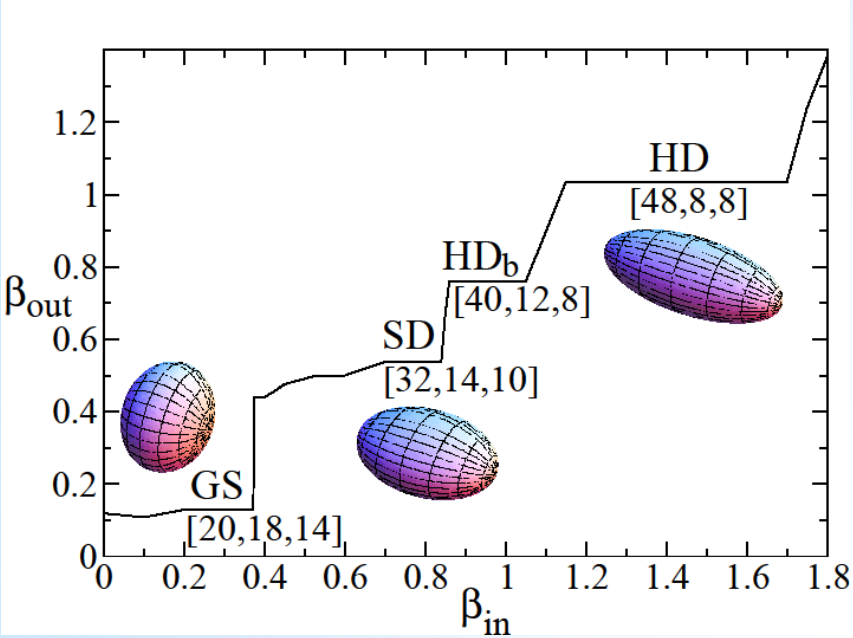


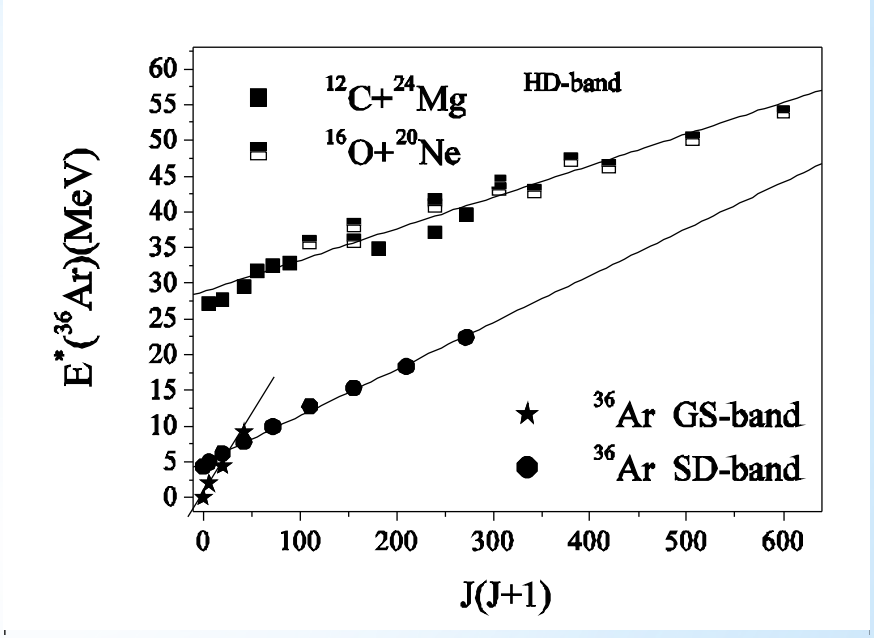
J. Cseh, G. Riczu, J. Darai, Phys. Lett. B 795 (2019) 160.

(Nilsson) Shell model calculation
SU(3)
Quadrupole shape
Shape isomers
Selection rule
Cluster configuration
Reaction channel



J. Darai, J. Cseh, D. Jenkins, Phys. Rev. C 86 (2012) 064309
D. Jenkins et al. Phys. Rev. C 86 (2012) 064308

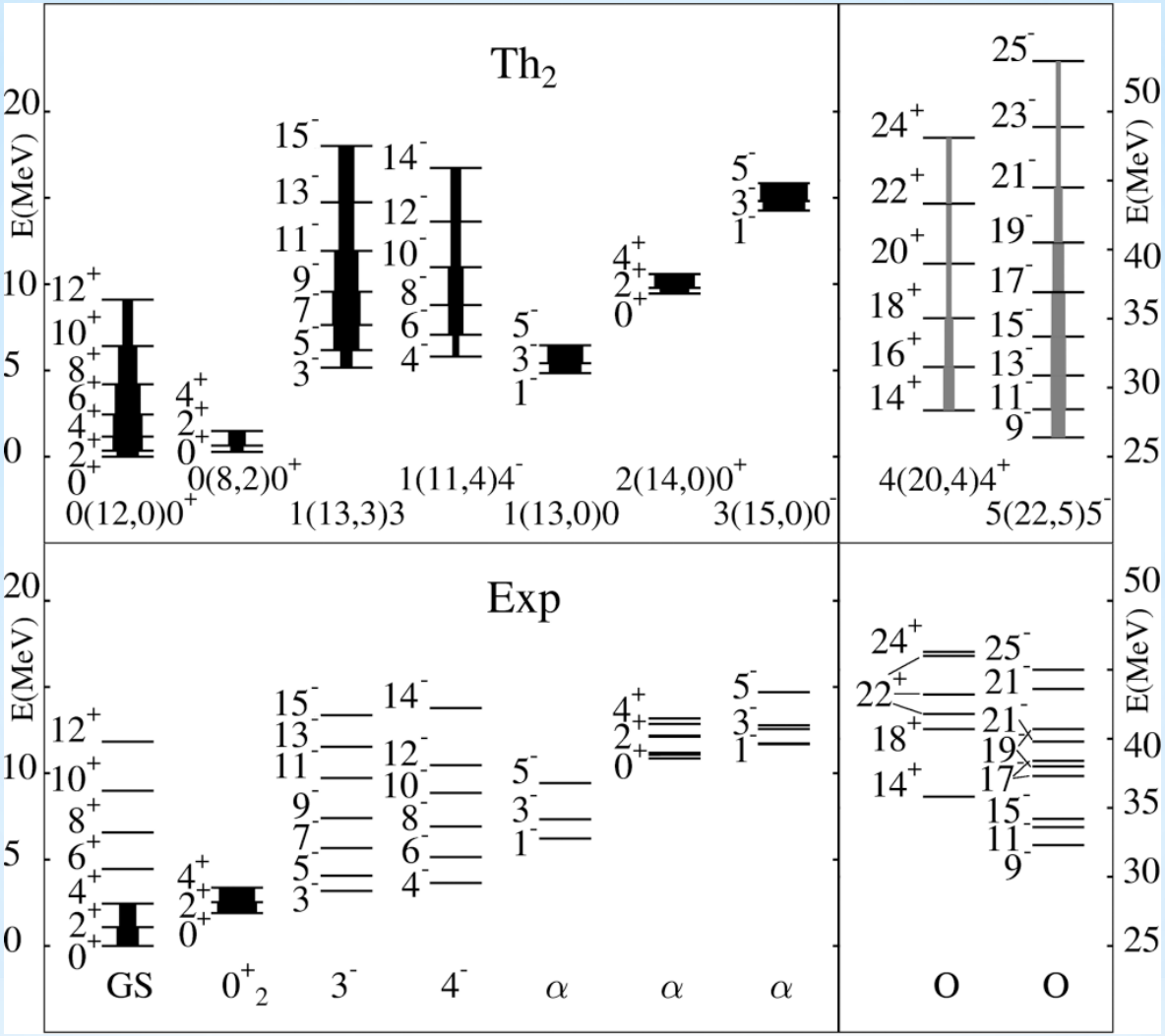




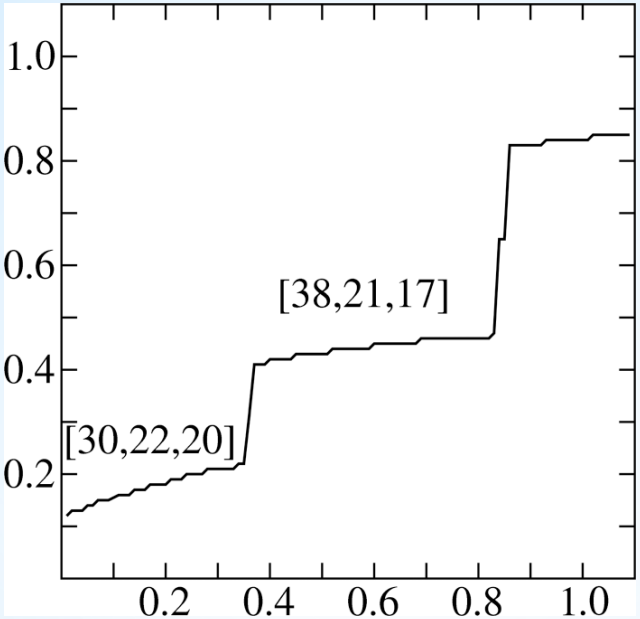
W. Sciani, Y. Otani, A. Lépine-Szily, et al, Phys. Rev. C 80 (2009) 034319
J. Cseh, J. Darai,et al. Phys Rev. C 80 (2009) 034320

Applications B)

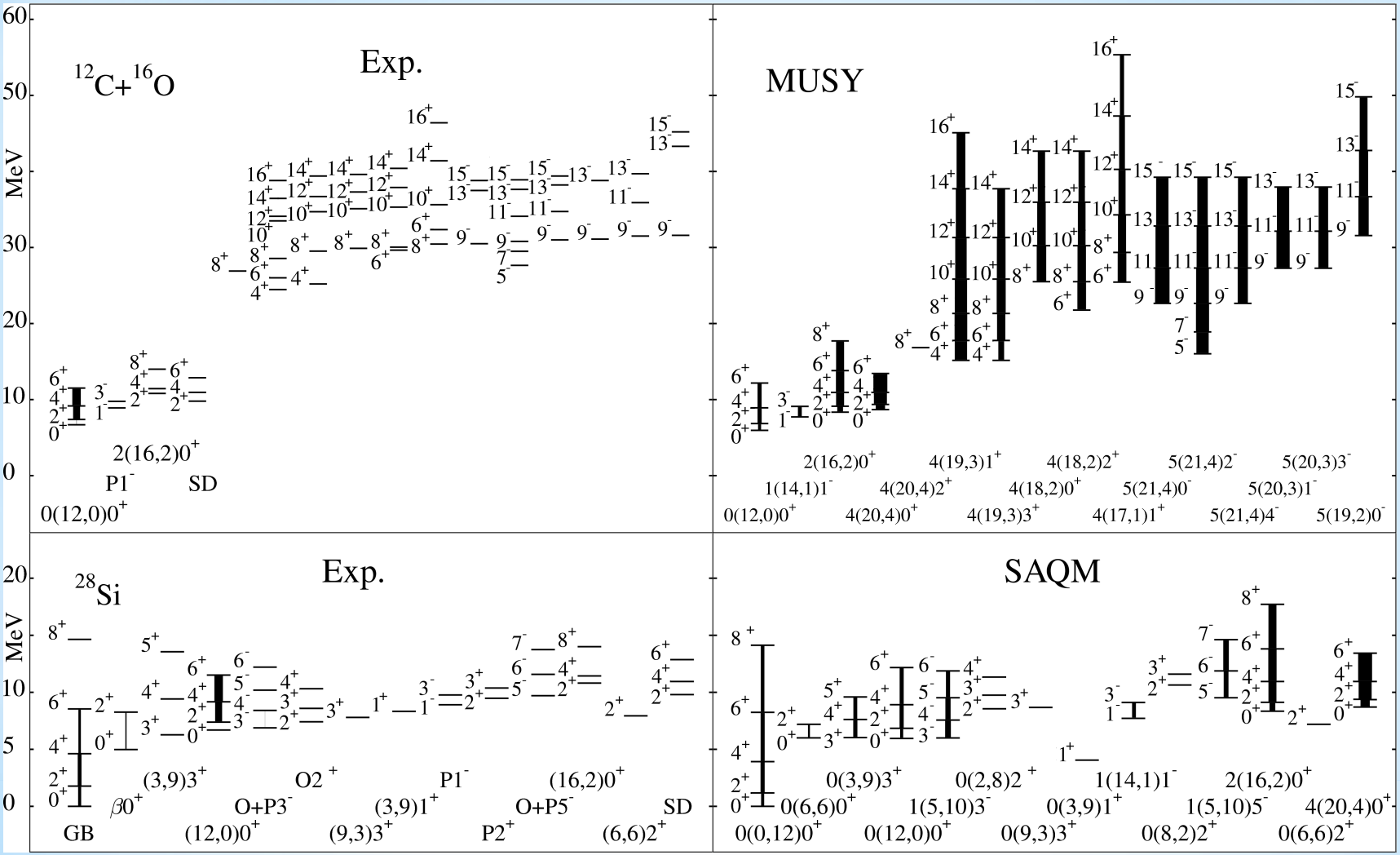
Unified description of spectra of different configurations in different regions of energy and deformation



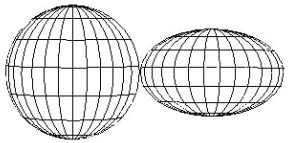
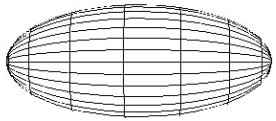
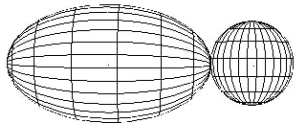
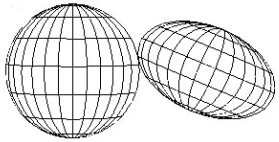
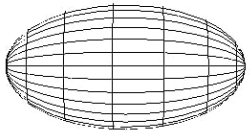
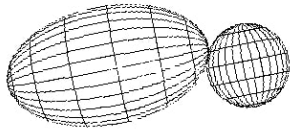
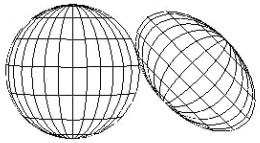
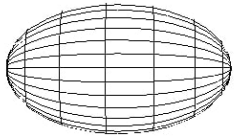
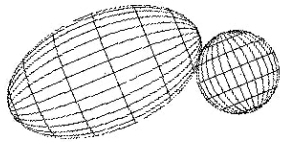
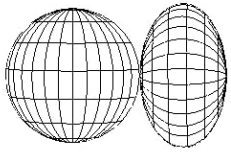
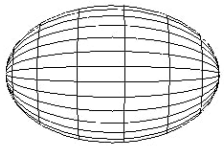
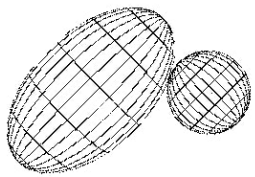
^{44}Ti: J. Cseh, Phys.Rev. C **103**, 064322 (2021)


1.0
0.8
0.6
0.4
0.2
[38,21,17]
[30,22,20]
0.2
0.4
0.6
0.8
1.0

$$H = \epsilon n + \alpha C_{SU3}^2 + \frac{1}{2\theta} L^2$$



J. Cseh, G Riczu, Phys. Lett B **757**, 312 (2016)

$^{16}O+^{12}C$	^{28}Si	$^{24}Mg+\alpha$	Quant. no.
			4[28,8,4]
			2[24,8,6]
			1[22,8,7]
			0[20,8,8]

V. Summary

MUSY: composite symmetry

dynamical symmetry in each configuration

+ transformations between them

1. Unified classification scheme for shell, collective and cluster models:

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

2. Transformations in the particle index space

Dual breaking:

- dynamical U(3), SU(3)
- spontaneous: SO(3) intrinsic

Unified description of spectra

Cluster-shell coexistence:

even 100 % overlap is possible
due to the antisymmetrization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Thank you for your attention!