Halo effects in the $^{11}Li(p,t)^{9}Li$ reaction

P. Descouvemont

Université Libre de Bruxelles, Brussels, Belgium

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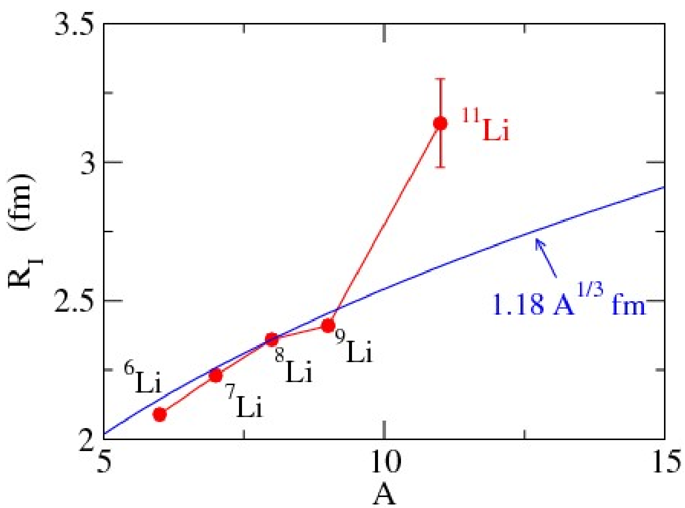
*P. Descouvemont, Phys. Rev. C **104**, 024613 (2021).*

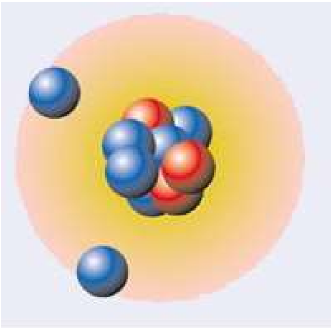
1. Introduction

- ^{11}Li is a well known halo nucleus: low separation energy of the 2 external neutrons (S_{2n}=0.34 MeV, τ=9 ms)

  I. Tanihata et al. Phys. Rev. Lett. 55 (1985) 2676



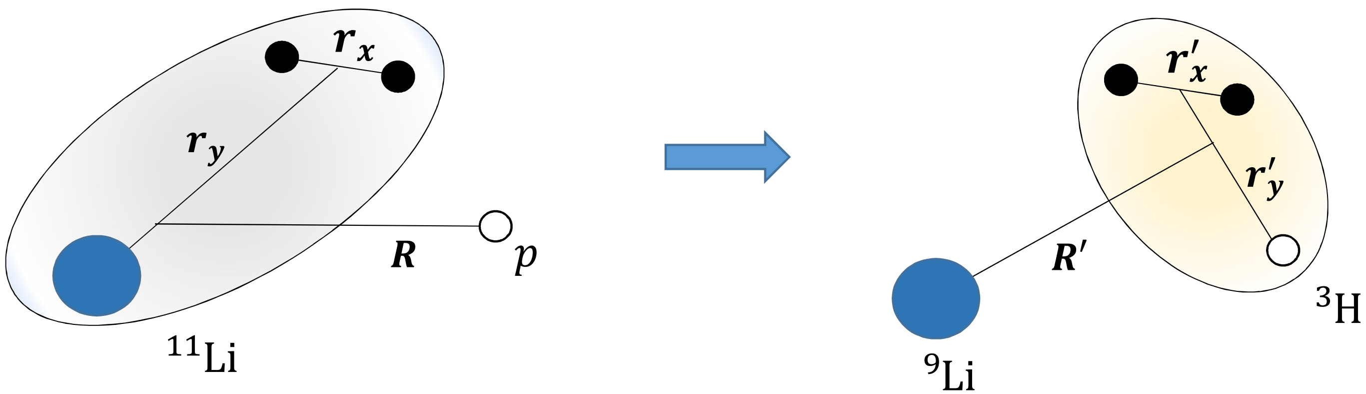
Interpretation as a halo structure



- Many experimental data on ^{11}Li (reactions)
  - Elastic scattering
  - Breakup
  - **2n transfer ^{11}Li(p,t)^{9}Li**: I. Tanihata et al, Phys. Rev. Lett. **100**, 192502 (2008).

- Many calculations on ^{11}Li
  - Spectroscopy
  - Reactions: breakup effects are important (low binding energy) → CDCC well adapted

Four-body model



^{11}Li(=^{9}Li+n+n) + p → ^{9}Li+t(=p+n+n)

a) Description of ^{11}Li, ^{3}H: three-body model in the hyperspherical formalism
b) ^{11}Li+p and ^{9}Li+t scattering states: CDCC (ground-state and breakup) → equivalent potentials
c) Transfer reaction ^{11}Li(p,t)^{9}Li: DWBA

$$\frac{d\sigma}{d\Omega} \sim \left| < \boldsymbol{\Phi}(^{11}\mathbf{Li})\chi_i(E,R)|\Delta V|\Phi(^3\mathrm{H})\chi_f(E-Q,R') > \right|^2$$

with $\chi_i(E,R)$=^{11}Li+p scattering wave function
$\chi_f(E-Q,R')$=^{9}Li+t scattering wave function

→ sensitive to the **^{11}Li wave function**
→ 2 neutron transfer : more difficult than 1-neutron transfer

2. Three-body model of ^{11}Li and ^{3}H

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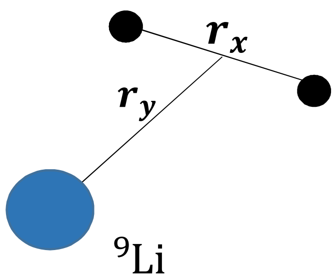
Hyperspherical formalism: ^{11}Li=^{9}Li+n+n



- Approximations on ^{9}Li: structure neglected, spin=0
- $H = T_1 + T_2 + T_3 + V_{12} + V_{13} + V_{23}$
- Hyperradius $\rho = \sqrt{x^2 + y^2}$ with $x = \sqrt{\mu_x} r_x,\ y = \sqrt{\mu} r_y$
- Hyperangle: $\tan\alpha = \frac{y}{x}$

  → 1 length: ρ

  → 5 angles: $\Omega_5 = (\alpha, \Omega_x, \Omega_y)$

In hyperspherical coordinates: $H = T_\rho + V(\rho, \alpha, \Omega_x, \Omega_y)$

Eigenstates of T_ρ: **hyperspherical functions** $\mathcal{Y}^L_{K l_x l_y}(\alpha, \Omega_x, \Omega_y) = \mathcal{Y}^L_{K\gamma}(\Omega_5)$

known functions (analytical)

extension of spherical harmonics $Y_l^m(\Omega)$ in 2-body problems

K=hypermoment

Hyperspherical formalism

- Schrödinger equation : $H\Psi^{LM} = E\Psi^{LM}$

- The wave function is expanded in hyperspherical harmonics (with $\gamma = (\ell_x, \ell_y)$, $K = \ell_x + \ell_y + 2n, n \geq 0$)

$$\Psi^{LM}(\rho, \Omega_5) = \sum_{K=0}^{\infty} \sum_{\gamma} \mathcal{Y}_{K\gamma}^{L}(\Omega_5)\, \chi_{K\gamma}^{L}(\rho)$$

Known functions | To be determined

- The radial functions are obtained from a set of coupled differential equations

$$-\frac{\hbar^2}{2\mathrm{m}_N}\left(\frac{d^2}{d\rho^2} - \frac{K(K+4)}{\rho^2}\right)\chi_{K\gamma}^{L}(\rho) + \sum_{K',\gamma'} V_{K\gamma,K'\gamma'}(\rho)\chi_{K\gamma}^{L}(\rho) = E\chi_{K\gamma}^{L}(\rho)$$

  - Potentials $V_{K\gamma,K'\gamma'}(\rho)$ are determined from $V_{12} + V_{13} + V_{23}$
  - Two-body potentials V_{ij} contains spurious Pauli forbidden states → must be removed
  - Equivalent to a standard coupled-channel problem (up to ~100-200 channels)
  - In practice: summation over K is limited to K_{max}
    $\chi_{K\gamma}^{L}(\rho)$ are expanded over a basis (Lagrange basis here)

- For ^{3}H=p+n+n: more complicated (3 spins 1/2)

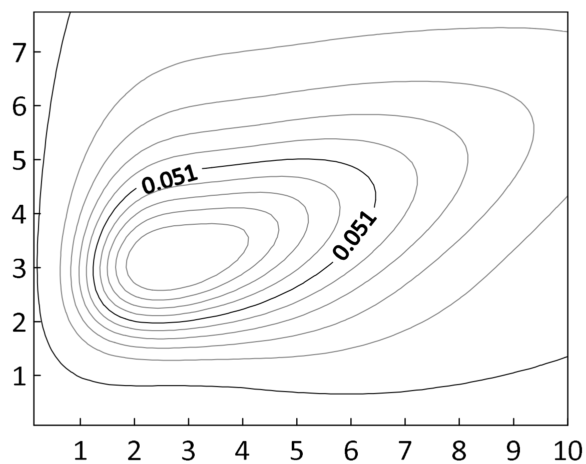
2. Three-body model of ^{11}Li and ^{3}H

- N-n and n-p interactions: Minnesota potential (reproduces the deuteron binding energy)
- ^{9}Li+n : H. Esbensen, G. F. Bertsch, and K. Hencken, Phys. Rev. C 56, 3054 (1997) renormalized by 1.0051
  Pauli forbidden states $0s_{1/2}$, $0p_{3/2}$ removed by a supersymmetric transformation

→ S_{2n} (^{11}Li)=0.378 (fitted), $\sqrt{<r^2>} = 3.12$ fm, exp=3.16 ± 0.11 fm

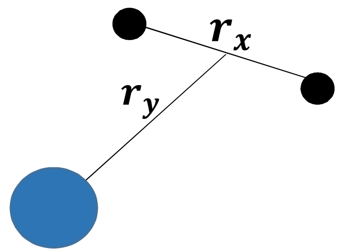
→ B(^{3}H)=8.38 MeV, exp=8.48 MeV

^{11}Li wave function

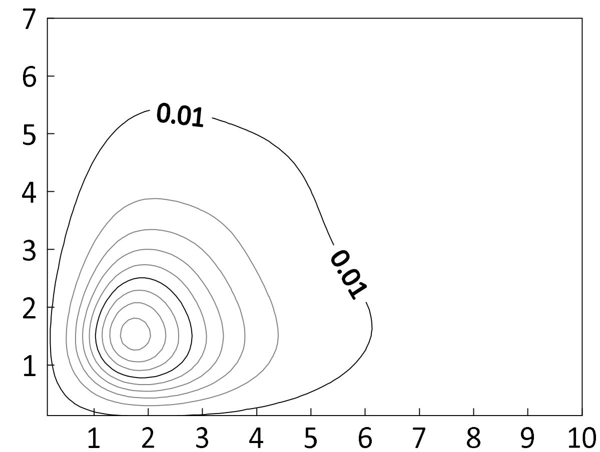


r_y (fm)

r_x (fm)



^{3}H wave function



r_y (fm)

r_x (fm)

3. ^{11}Li+p and ^{9}Li+t scattering states

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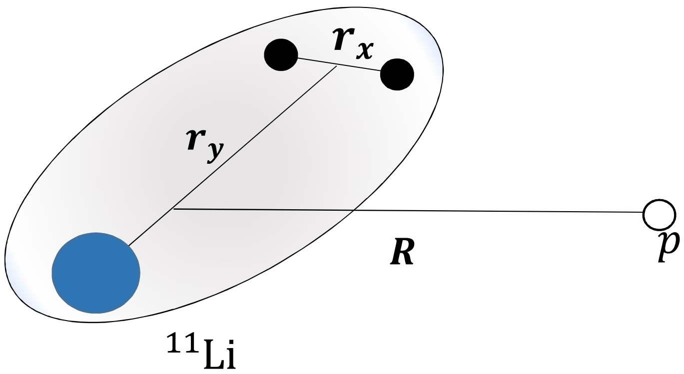
CDCC calculation for ^{11}Li+p : T. Matsumoto et al., PTEP 2019, 126
P. Descouvemont, Phys. Rev. C **101**, 64611 (2020).



Total hamiltonian: $H = H_0(\boldsymbol{x},\boldsymbol{y}) + T_R + \sum_{ij} U_{i-t}(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y})$

With H_0=internal hamiltonian of ^{11}Li (or ^{3}H)
T_R =relative kinetic energy
$U_{i-t}(\boldsymbol{s})$ =optical potential between fragments i and the target (^{9}Li+p: KD03, n+p: Minnesota)

Then: standard CDCC procedure

3. ^{11}Li+p and ^{9}Li+t scattering states

Step 1: solve $H_0 \Phi_{0k}^{jm} = E_{0k}^{j} \Phi_{0k}^{jm}$ for ^{11}Li (hyperspherical coordinates)

With Φ_{0k}^{jm} expanded on a basis (Lagrange functions: matrix elements are simple)

→ negative energies = physical states

positive energies = pseudostates=(discrete) approximations of the continuum

Step 2: Define channel functions: $\varphi_c(\boldsymbol{x},\boldsymbol{y},\Omega_R) = \left[\left[\Phi_{0k}^{j}(\boldsymbol{x},\boldsymbol{y}) \otimes \chi_p\right]^{I} \otimes Y_L(\Omega_R)\right]^{JM}$

with index $c = (j,k,I,L)$

I = channel spin

L =angular momentum between p and ^{11}Li

and expand the total wave function as $\Psi^{JM\pi} = \sum_c u_c^{J\pi}(R)\varphi_c(\boldsymbol{x},\boldsymbol{y},\Omega_R)$ with $u_c^{J\pi}(R)$ to be determined

(truncation parameters: jmax, Emax)

Step 3

Compute matrix elements of the potential $\sum_{ij} U_{ij}(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y},\boldsymbol{r})$

$$V_{cc'}^{J}(R) = \langle \varphi_c \left| \sum_i U_{i-t}(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y}) \right| \varphi_{c'} \rangle$$

4. Step 4: Solve the coupled-channel system

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dR^2}-\frac{L(L+1)}{R^2}\right)+E_c-E\right]u_c^{J\pi}(R)+\sum_{c'}V_{cc'}^{J\pi}(R)u_{c'}^{J\pi}(R)=0$$

- Standard coupled-channel system (general form common to most scattering theories)
- At large distances (only Coulomb) : $u_c^{J\pi}(R) \to I_c(R)\delta_{c\omega} - O_c(R)U_{c\omega}^{J\pi}$ (ω =entrance channel)
  $U_{c\omega}^{J\pi}$ = scattering matrix: provides the cross sections (elastic, inelastic, breakup, etc.)
- Solved with the R-matrix method (space divided in an internal and an external regions)
- The system must be solved for each $J\pi$
- Problems:
  - Many channels c
  - Many $J\pi$ values (depends on energy)
  - Long range of the potentials $V_{cc'}^{J\pi}(R)$ (due to Coulomb)

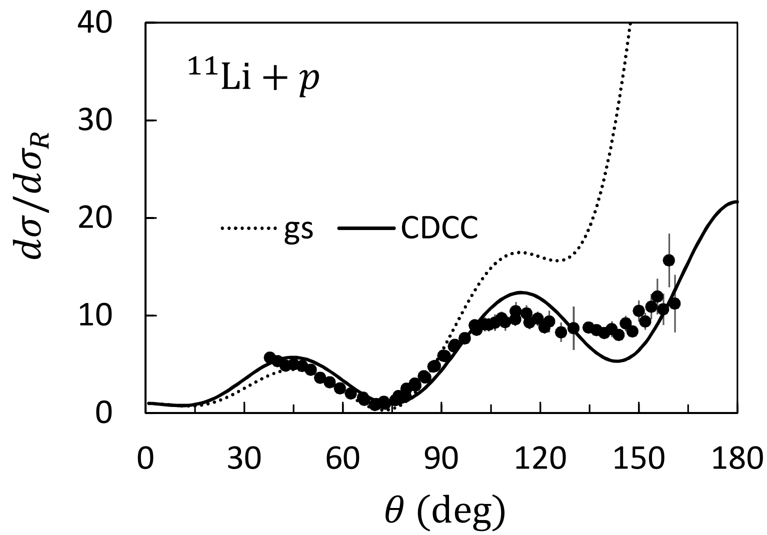
  ➔ Long calculations + many (convergence) tests

5. Step 5
   Determing the cross sections from the scattering matrices (elastic scattering, breakup)

3. ^{11}Li+p and ^{9}Li+t scattering states

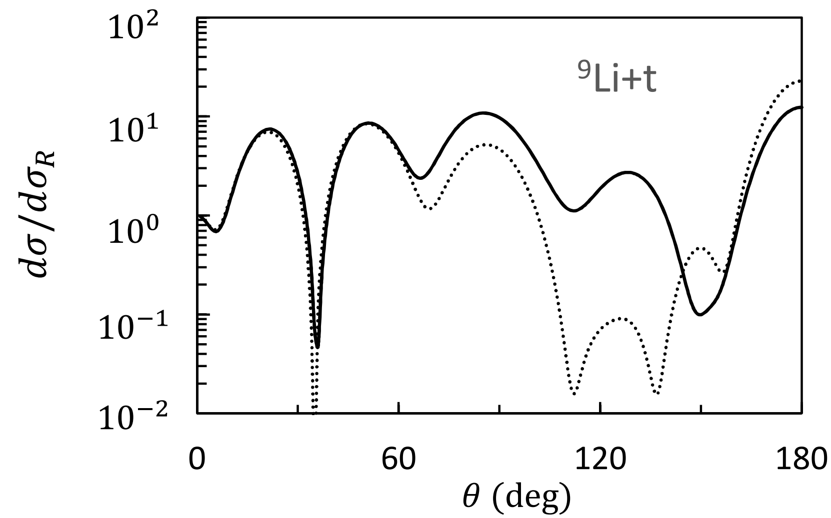
^{11}Li+p

- Calculation at $E_{lab}(^{11}Li)$=66 MeV
- Optical potential ^{9}Li+p: Koning-Delaroche 2003
- j_{max}=3, E_{max}=10 MeV
- data from J. Tanaka et al., PLB **774**, 268 (2017).



^{9}Li+t

- no data
- systematics from D. Y. Pang et al., Phys. Rev. C 79, 024615 (2009).



4. The $^{11}Li(p,t)^{9}Li$ cross section

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Cross section computed from the scattering matrices

$$U_{if}^{J\pi} = -\frac{i}{\hbar}\sum_{\alpha\beta}\iint u_{\alpha i}^{J\pi}(R) K_{\alpha i,\beta f}^{J\pi}(R,R') u_{\beta f}^{J\pi}(R') dR\, dR'$$

^{11}Li+p scattering wave functions

^{9}Li+t scattering wave functions

Two-neutron transfer kernel

- Computed from the ^{11}Li and ^{3}H wf
- α= gs and PS of ^{11}Li
- β= gs and PS of ^{3}H
- See details in P. D., PRC **104**, 024613 (2021).

Many terms, slow convergence of the integrals (PS)

→ simplification with equivalent potentials: replace the full CDCC problem by an equivalent single-channel problem

4. The ^{11}Li(p,t)^{9}Li cross section

Equivalent potentials: I. Thompson, M. Nagarajan, J. Lilley, and M. Smithson, Nucl. Phys. A 505, 84 (1989)

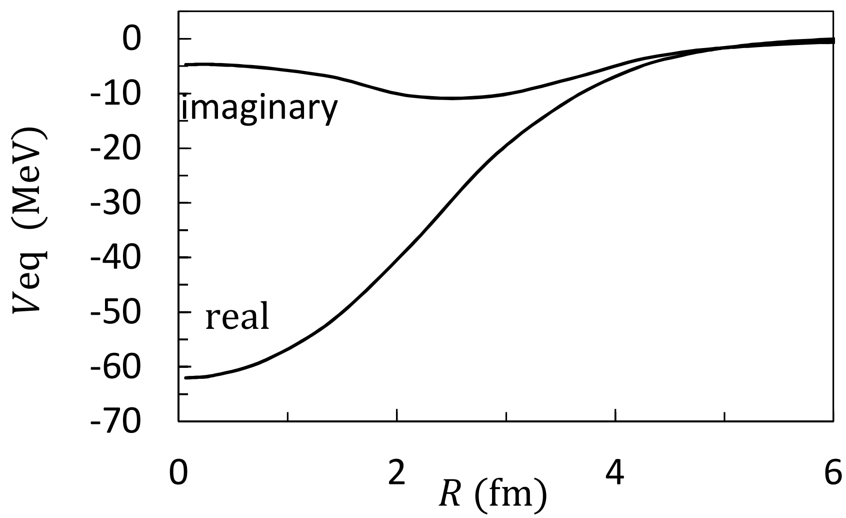
The goal is to replace the multichannel system

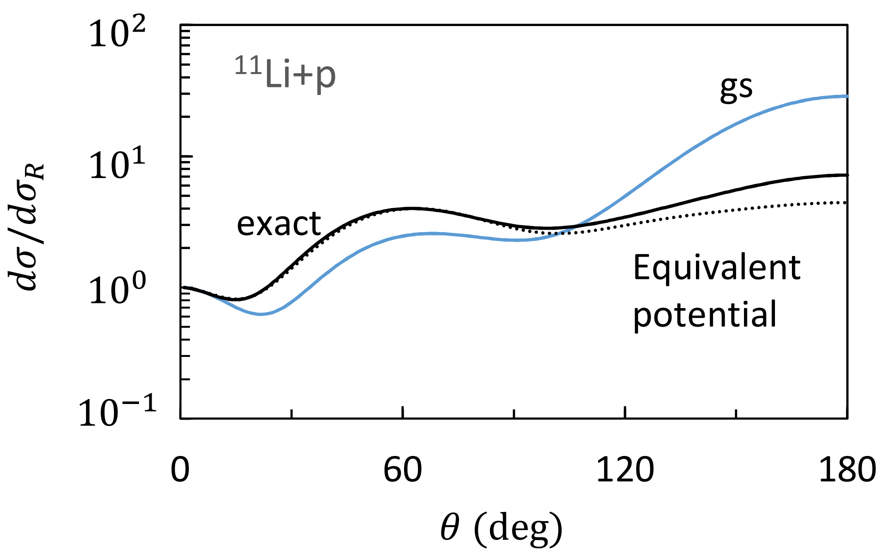
$$[T_L + E_c - E]u_c^{J\pi}(R) + \sum_{c'} V_{cc'}^{J\pi}(R)u_{c'}^{J\pi}(R) = 0$$

By a simpler (single-channel) equation

$$[T_L + V_{eq}(R) - E]\tilde{u}_0^{J\pi}(R) = 0$$

- The equivalent potentials can be determined from the original potentials $V_{cc'}^{J\pi}(R)$ and wave functions $u_c^{J\pi}(R)$
- Not strictly equivalent → tests with the scattering cross section





4. The $^{11}Li(p,t)^{9}Li$ cross section

With this approximation

$$U_{if}^{J\pi} = -\frac{i}{\hbar}\sum_{\alpha\beta}\iint u_{\alpha i}^{J\pi}(R)K_{\alpha i,\beta f}^{J\pi}(R,R')u_{\beta f}^{J\pi}(R')dR\,dR'$$

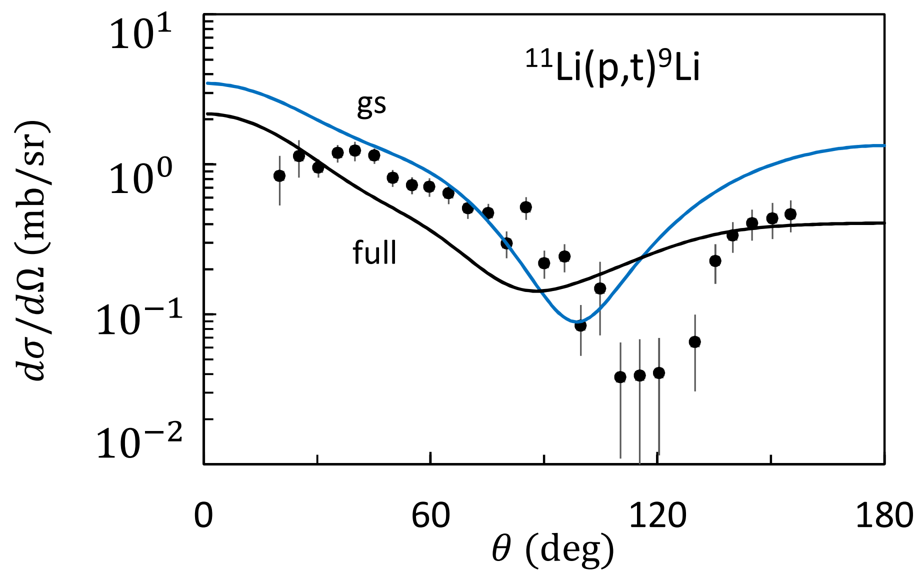
becomes

$$U_{if}^{J\pi} \approx -\frac{i}{\hbar}\iint \tilde{u}_{0i}^{J\pi}(R)K_{0i,0f}^{J\pi}(R,R')\tilde{u}_{0f}^{J\pi}(R')dR\,dR'$$

(no more summation over the PS, only the ^{11}Li and ^{3}H ground states are involved)

→ Tests with smaller j_{max}, E_{max}

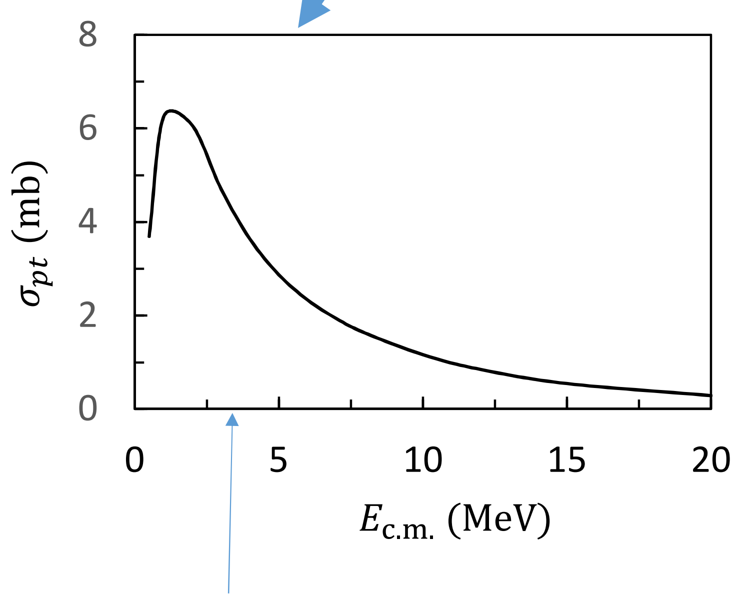
E_{lab}=33 MeV (E_{cm}=2.75 MeV)
Data: I. Tanihata et al, Phys. Rev. Lett. **100**, 192502 (2008).



4. The ^{11}Li(p,t)^{9}Li cross section

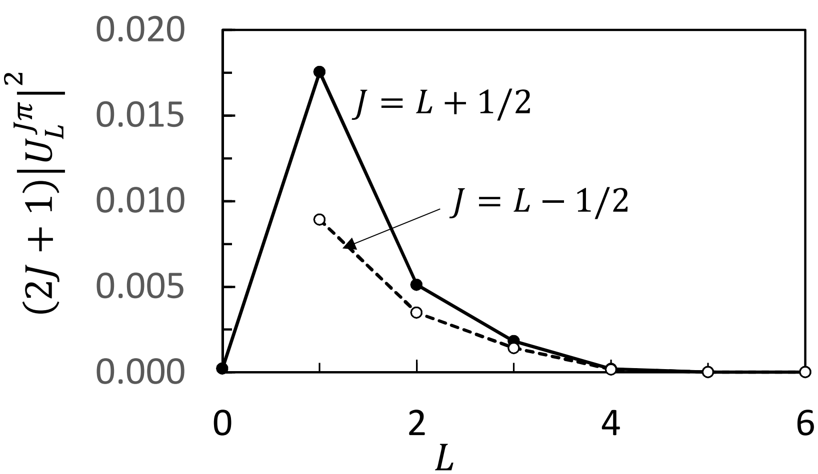
Analysis of halo effects

Integrated cross section: $\sigma_{pt}(E) = \frac{\pi}{2k^2}\sum_{J\pi}(2J+1)\left|U_L^{J\pi}(E)\right|^2$



Experiment: Ecm=2.75 MeV

Ecm=2.75 MeV



→ Dominant J=3/2,5/2

4. The ^{11}Li(p,t)^{9}Li cross section

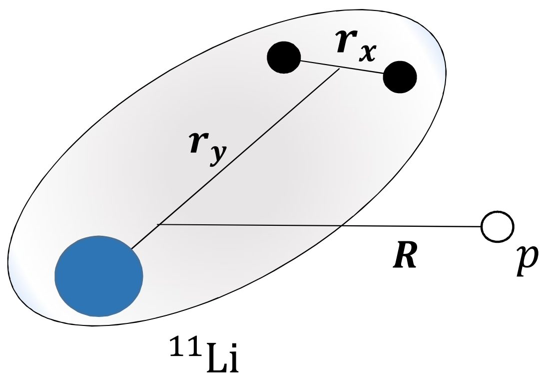
Analysis of the scattering matrix for J=3/2 and 5/2

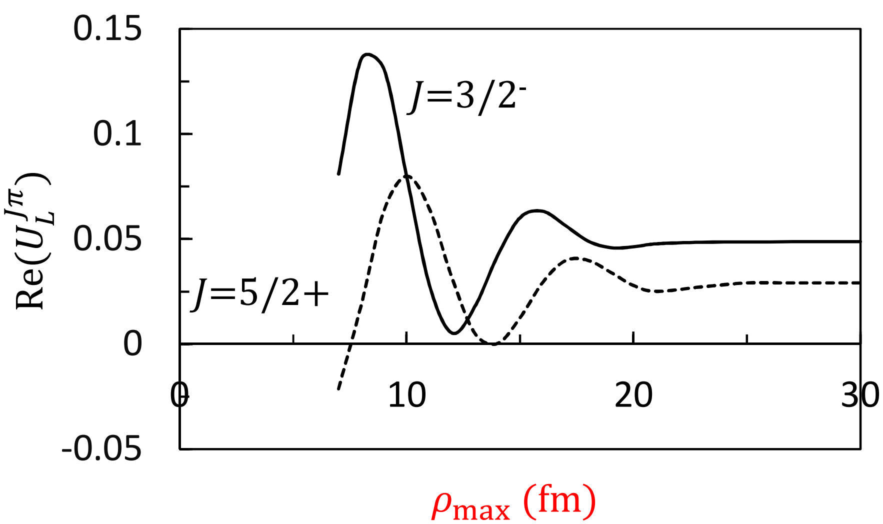
$$U_{if}^{J\pi} = -\frac{i}{\hbar}\iint \tilde{u}_{0i}^{J\pi}(R) K_{0i,0f}^{J\pi}(R,R') \tilde{u}_{0f}^{J\pi}(R') dR\, dR'$$

Depends on the ^{11}Li wave function

→ depends on ρ

→ Cut-off value $\rho_{\max}$



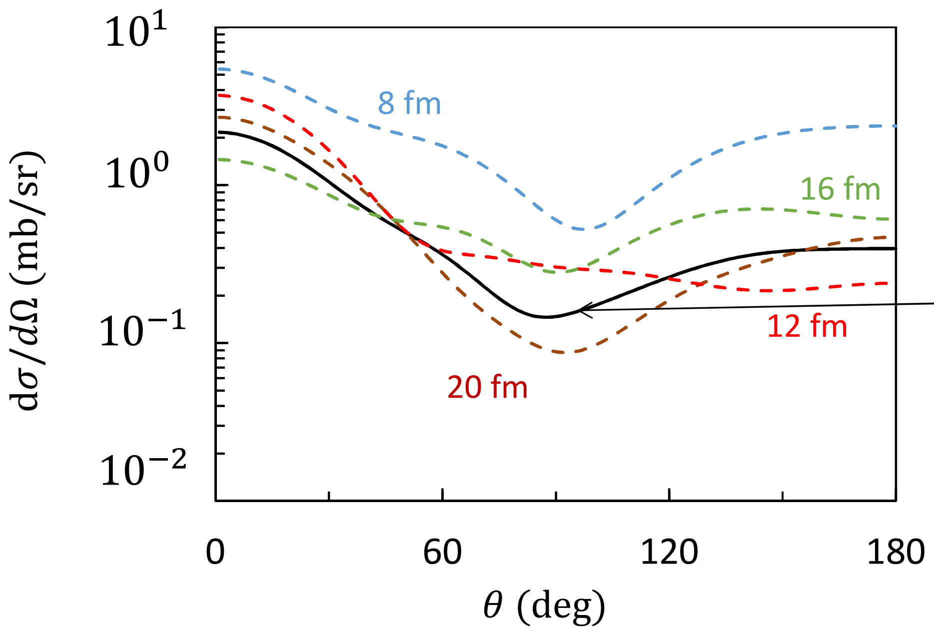


➔ Sensitive to large ρ values

➔ Sensitive to the long-range part of the wave function

4. The $^{11}Li(p,t)^{9}Li$ cross section

Convergence of the cross section with $\rho_{\max}$



- Convergence for $\rho_{max} \geq 25$ fm
- Probes the ^{11}Li wf at large distances
  → test of the halo structure

5. Conclusion

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- Description of ^{11}Li: ^{9}Li+n+n in hyperspherical coordinates, ^{9}Li+n Pauli forbidden states removed by a supersymmetry transformation
- ^{11}Li+p elastic scattering with CDCC (Elab=66 MeV, Ecm=5.5 MeV): sensitive to the ^{11}Li breakup at large angles
- ^{11}Li(p,t)^{9}Li
    - Two-neutron transfer reaction
    - 3-body wave functions for ^{11}Li and ^{3}H
    - Breakup effects are simulated through ^{11}Li+p and ^{9}Li+t equivalent potentials

Open questions

- Influence of core excitations in ^{11}Li?

- ^{9}Li+p low energy

→Optical potential valid?
→Possible resonances?
→Role of Pauli forbidden states?

- Need for experimental data on

    ^{9}Li+t elastic scattering
    ^{11}Li(p,t)^{9}Li at higher energies
    ^{11}Li(p,t)^{9}Li and ^{11}Li+p elastic scattering at the same energy

Thank you!