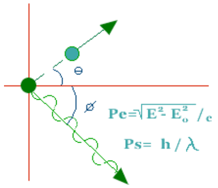
Unified description of nuclear structure and reactions employing the dispersive optical model (DOM)

YKIS2022b workshop
5/23/2022

Washington
University in St. Louis



DOM activities: Wim Dickhoff
Bob Charity
Lee Sobotka
Louk Lapikas (e,e'p)
Henk Blok (e,e'p)
Kazuyuki Ogata (p,2p)
Kazuki Yoshida (p,2p)
Hossein Mahzoon (Ph.D.2015)
Mack Atkinson (Ph.D. 2019)
Natalya Calleya (Grad)
Cole Pruitt (Ph.D. 2019)
Bob Wiringa
Maria Piarulli
Arnau Rios

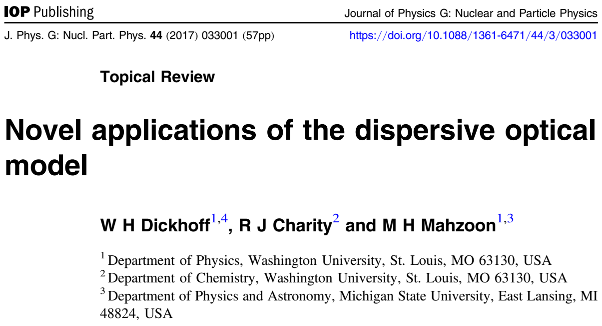
- Motivation: essential for studying unstable nuclei and link to neutron stars
- Optical potentials 2022
- Removal probabilities puzzle(s)
- Green's functions/propagator method
  - vehicle for ab initio calculations —> matter & finite nuclei
  - as a framework to link data at positive and negative energy (and to generate predictions for unstable nuclei as well as neutron skins)

-> **dispersive optical model** (DOM <- started by Claude Mahaux)

- DOM with non-local potentials ^{12}C, $^{16-18}$O, ^{40,48}Ca, ^{58,64}Ni, ^{112,124}Sn, ^{208}Pb
- Revisit (e,e'p) data from NIKHEF ^{40}Ca and ^{48}Ca —> N-Z dependence
- Neutron skin in ^{48}Ca and ^{208}Pb —> PREX II
- Conclusion and outlook
- Ground-state energy and EOS —> saturation properties

Optical potentials 2022

Reviews



IOP Publishing Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. **44** (2017) 033001 (57pp) https://doi.org/10.1088/1361-6471/44/3/033001

Topical Review

Novel applications of the dispersive optical model

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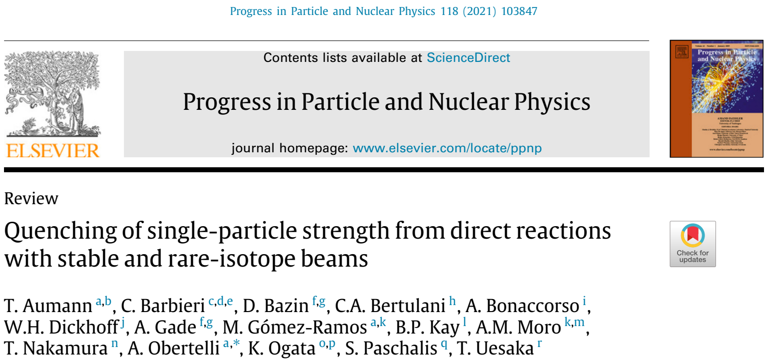
[2] Department of Chemistry, Washington University, St. Louis, MO 63130, USA
[3] Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA



Progress in Particle and Nuclear Physics 118 (2021) 103847

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

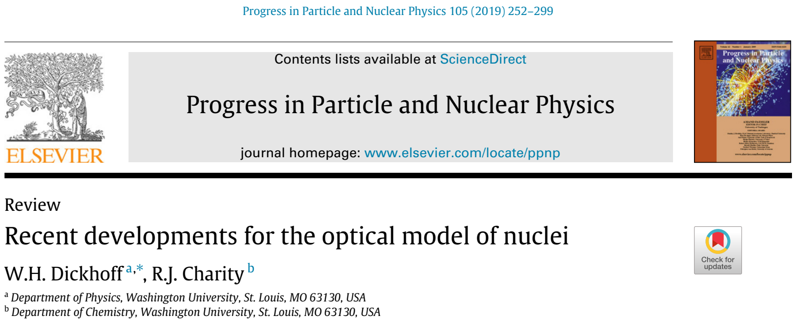
journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

Quenching of single-particle strength from direct reactions with stable and rare-isotope beams

T. Aumann[a,b], C. Barbieri[c,d,e], D. Bazin[f,g], C.A. Bertulani[h], A. Bonaccorso[i], W.H. Dickhoff[j], A. Gade[f,g], M. Gómez-Ramos[a,k], B.P. Kay[l], A.M. Moro[k,m], T. Nakamura[n], A. Obertelli[a,*], K. Ogata[o,p], S. Paschalis[q], T. Uesaka[r]



Progress in Particle and Nuclear Physics 105 (2019) 252–299

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

Recent developments for the optical model of nuclei

W.H. Dickhoff[a,*], R.J. Charity[b]

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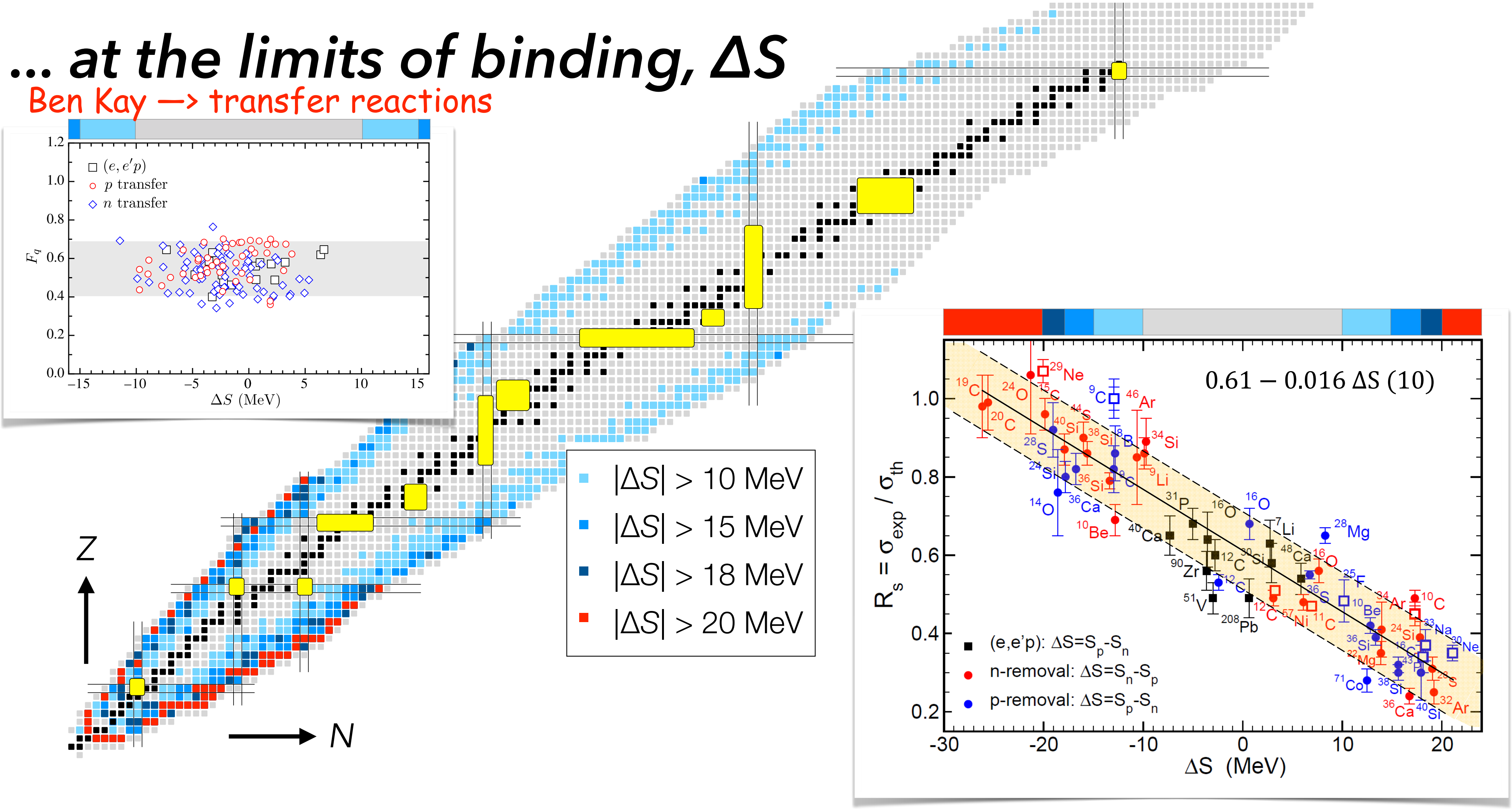
Unstable Nuclei and Optical Potentials

- Masses and electromagnetic properties —> **unambiguous**!

- All other properties depend on description of strongly interacting probe with target —> **ambiguous**! (or in other words model dependent)
- Optical potentials are an important ingredient in analyzing e.g. transfer reactions, knockout reactions like (e,e'p) and (p,2p), the latter available in inverse kinematics!
- **Status 2022**
  - Global potentials still local, non-dispersive and **not** constrained by scattering data for exotic nuclei
  - Ab initio methods have as yet **very** limited relevance e.g. for FRIB physics
  - DOM potentials available for a reasonable set of nuclei and can be **extrapolated**
  - DOM potentials are nonlocal, dispersive, and **describe structure and reaction domains simultaneously**

… at the limits of binding, ΔS

Ben Kay —> transfer reactions



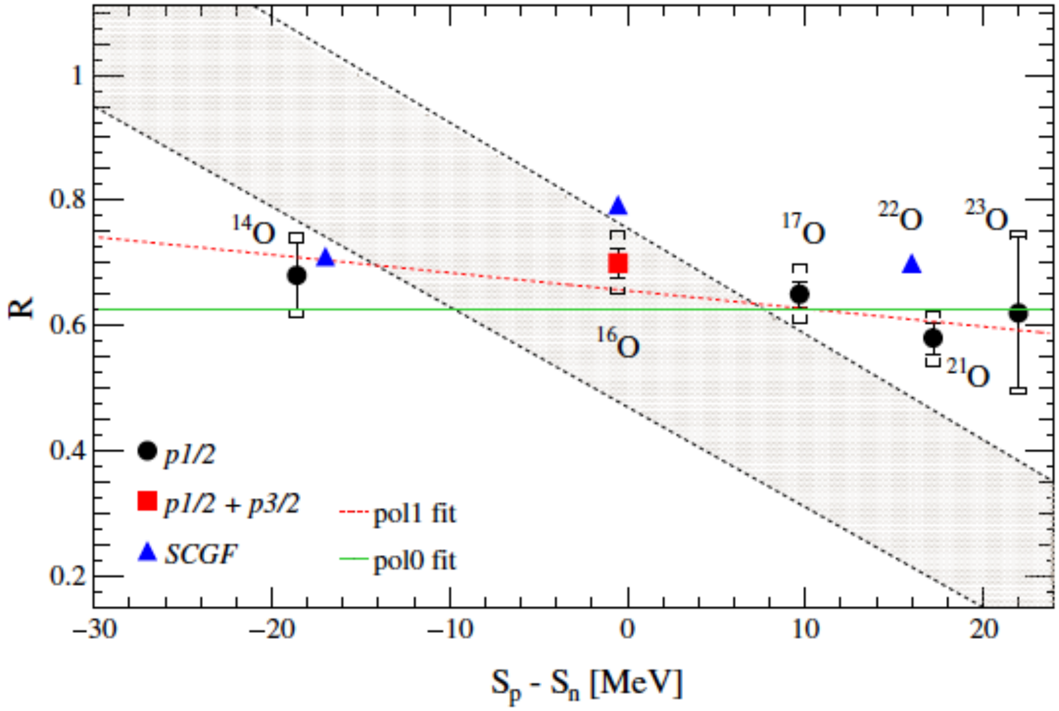
- |ΔS| > 10 MeV
- |ΔS| > 15 MeV
- |ΔS| > 18 MeV
- |ΔS| > 20 MeV

Chart modified from plot in T. Aumann et al., Prog. Part. Nucl. Phys. ***118****, 103847 (****2021****)*

J. A. Tostevin and A. Gade, Phys. Rev. C ***103****, 054610 (****2021****) [an update to Phys. Rev. C* ***90****, 057602 (****2014****)*

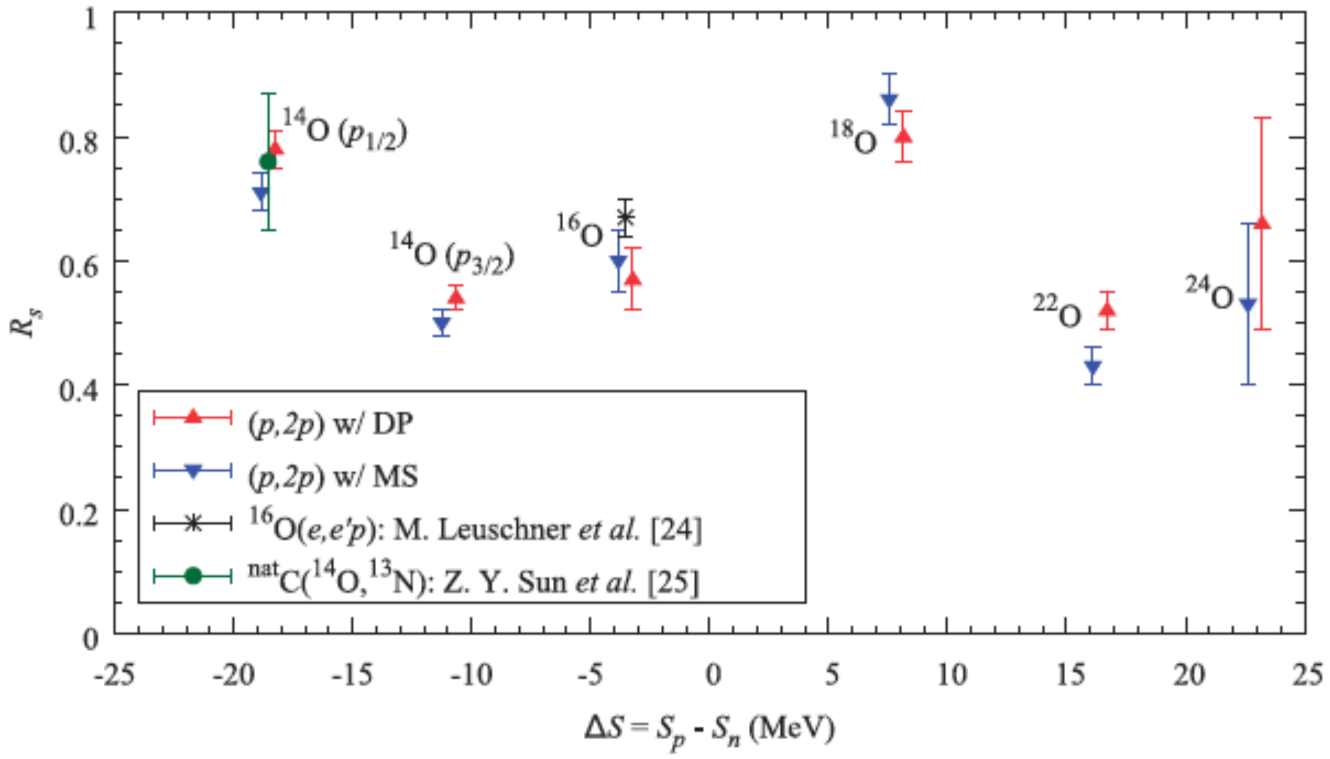
O(p,2p) L. Atar et al. Phys. Rev. Lett. 120, 052501 (2018)

- “Ab initio” interaction has “no” tensor force —> spectroscopic factors?
- Reaction model: distorted waves not constrained by experimental information as a function of nucleon asymmetry
- Inconsistent with np dominance observed in 2N knockout reactions (Or et al.)



O(p,2p)

- S. Kawase et al. Prog. Theor. Exp. Phys. 2018, 021D01
- DWIA uses optical potentials not constrained by scattering data for unstable nuclei



reactions and structure

Status of "reduction" factors/spectroscopic factors

T. Aumann, C. Barbieri, D. Bazin et al. *Progress in Particle and Nuclear Physics 118 (2021) 103847*

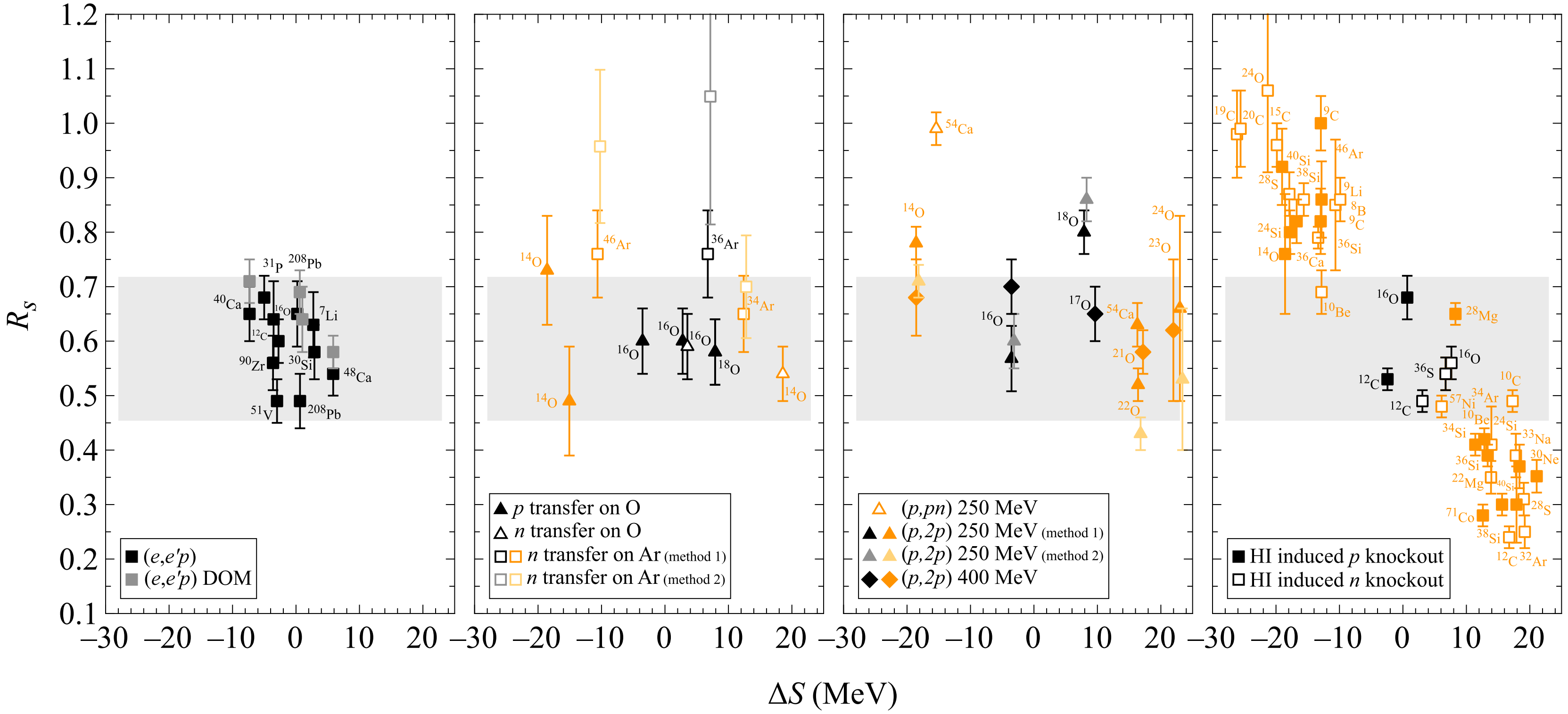
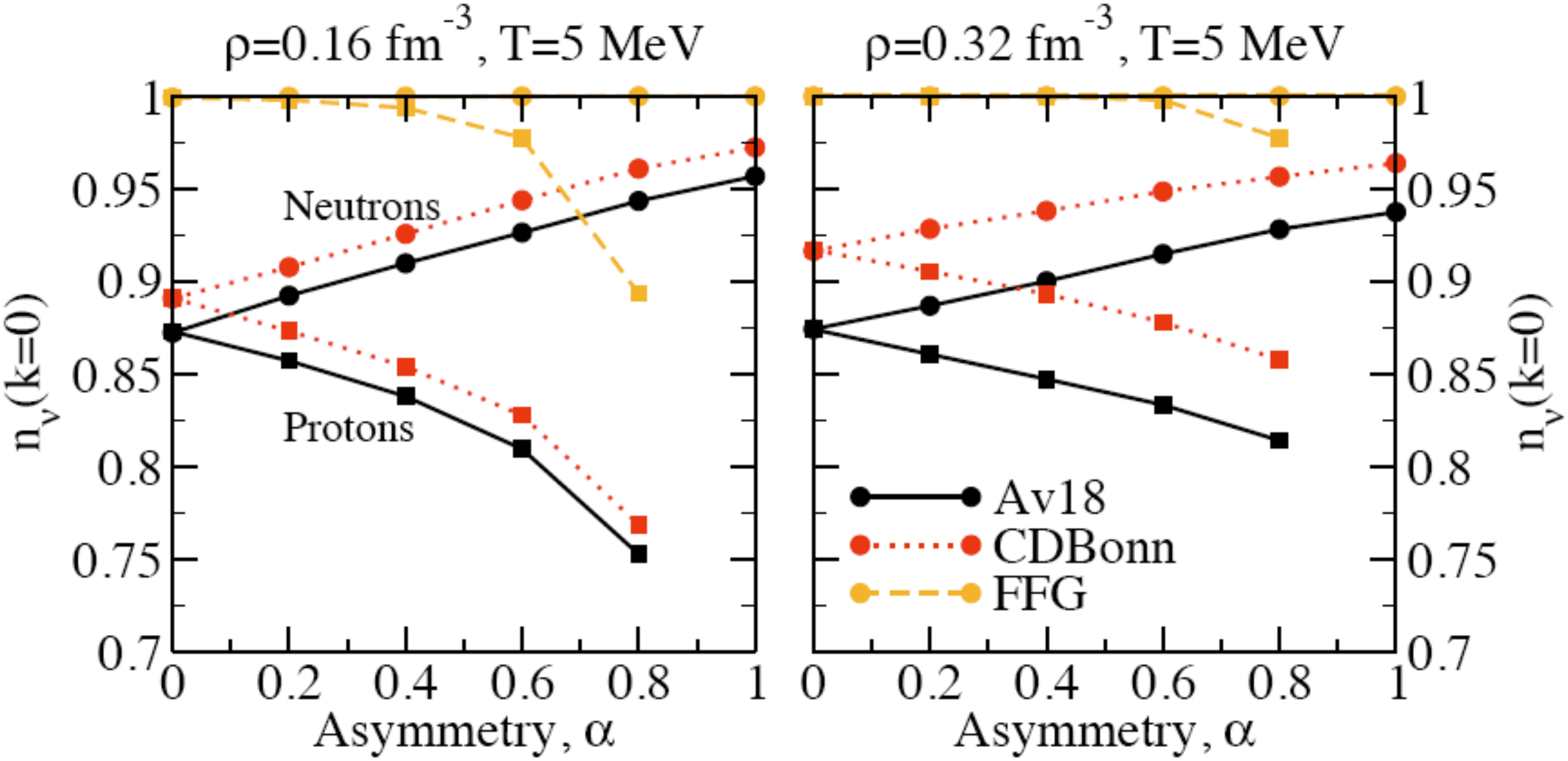


Fig. 56. The four panels of this plot show the quenching (reduction) factors for (a) electron-induced knockout reactions [87,172,237,376], (b) transfer reactions with radioactive ion beams [55,57,203], (c) quasifree (p, $2p$) proton knockout on stable nuclei (from the compilation in [239]) and radioactive nuclei [58,59], and (d) the inclusive intermediate-energy knockout data [46]. The measurements are compared to predictions based on effective-interaction shell-model SFs while, in the case of (e, $e'p$), the integrated strength is compared to the independent-particle expectation.

Unambiguous ingredient: Depletion as a function of asymmetry in matter treating short-range and tensor correlations



A. Rios, A. Polls, and W. H. Dickhoff

Depletion of the nuclear Fermi sea.

Phys. Rev. C79, 064308 (2009).

reactions and structure

Properties and relevance of optical potential or self-energy

- Describes **all** observables related to elastic scattering
- Generates wave functions inside the nucleus - distorted waves - used to describe other nuclear reactions aimed at extracting nuclear structure information
- Dispersive optical model describes additional observables related to the ground state
- Can be **extrapolated** to exotic nuclei [but uncertain]
- Astrophysics: relevant for the description of rapid neutron capture
- Lots of recent activity to generate the optical potential starting from the "NN" interaction
  - Nuclear matter approach [old and recent]
  - Multiple scattering using free NN interaction [T-matrix]
  - Green's function method for finite nuclei [old and recent]
  - Coupled cluster method
  - No core shell model [including symmetry-adapted version]
  - DFT + 2nd order/RPA contributions

Some problems with "ab initio" approaches

- Multiple scattering approaches useful at higher energy but hard to improve over spectator approximation (problems with polarization data)
- Methods that generate the propagator directly and then generate optical potential by inverting the Dyson equation
  - Coupled cluster
  - No core shell model
  - Both methods struggle with the η infinitesimal

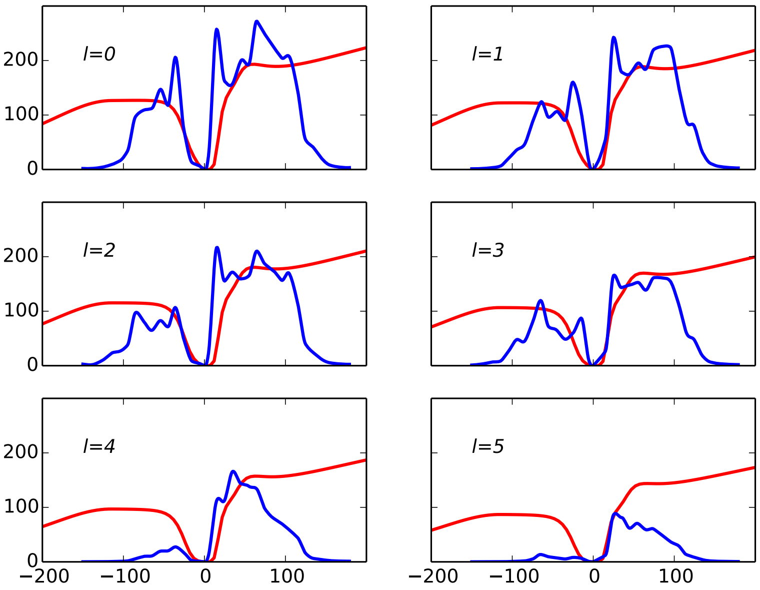
$$G_{\ell j}(r,r';E) = \sum_m \frac{\langle \Psi_0^A | \, a_{r\ell j} \, | \Psi_m^{A+1} \rangle \, \langle \Psi_m^{A+1} | \, a^\dagger_{r'\ell j} \, | \Psi_0^A \rangle}{E - (E_m^{A+1} - E_0^A) + i\eta} + \sum_n \frac{\langle \Psi_0^A | \, a^\dagger_{r'\ell j} \, | \Psi_n^{A-1} \rangle \, \langle \Psi_n^{A-1} | \, a_{r\ell j} \, | \Psi_0^A \rangle}{E - (E_0^A - E_n^{A-1}) - i\eta}$$

$$G_{\ell j}(r,r';E) = G^{(0)}_{\ell j}(r,r';E) + \int d\tilde{r} \; \tilde{r}^2 \int d\tilde{r}' \; \tilde{r}'^2 G^{(0)}_{\ell j}(r,\tilde{r};E)\Sigma_{\ell j}(\tilde{r},\tilde{r}';E)G_{\ell j}(\tilde{r}',r';E)$$

  - Methods that use HO basis cannot describe density of states and higher angular momentum

Illustrate: comparison with ab initio FRPA calculation

- Volume integrals of imaginary part of nonlocal ab initio (FRPA) self-energy compared with DOM result for ^{40}Ca



- Ab initio

S. J. Waldecker, C. Barbieri and W. H. Dickhoff
Microscopic self-energy calculations and dispersive-optical-model potentials.
Phys. Rev. C84, 034616 (2011), 1-11.

DOM initiated by Mahaux
St. Louis extensions (nonlocality, energy domain, isotope chains)

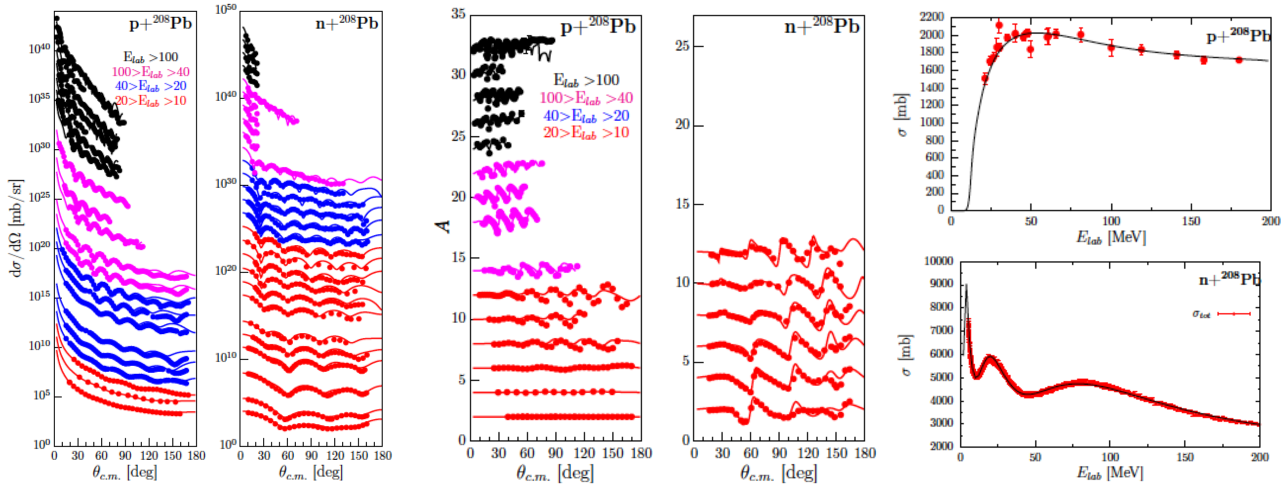
- Nonlocal and dispersive optical potential

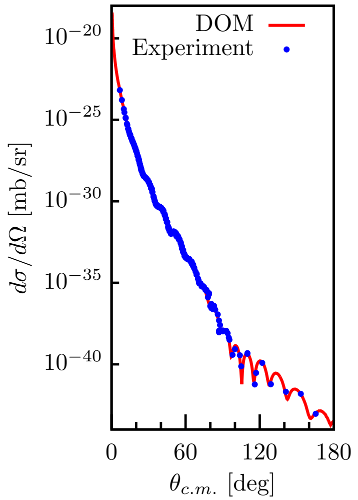
$$\begin{aligned}
\mathrm{Re}\ \Sigma_{\ell j}(r,r';E) = {} & \mathrm{Re}\ \Sigma_{\ell j}(r,r';\varepsilon_F) \\
- {} & \mathcal{P}\int_{\varepsilon_T^+}^{\infty} \frac{dE'}{\pi} \mathrm{Im}\ \Sigma_{\ell j}(r,r';E') \left[\frac{1}{E-E'} - \frac{1}{\varepsilon_F - E'} \right] \\
+ {} & \mathcal{P}\int_{-\infty}^{\varepsilon_T^-} \frac{dE'}{\pi} \mathrm{Im}\ \Sigma_{\ell j}(r,r';E') \left[\frac{1}{E-E'} - \frac{1}{\varepsilon_F - E'} \right]
\end{aligned}$$

- *Allows consideration of negative energy experimental information [charge density]*
- Subtracted dispersion relation emphasizes influence of energies close to the Fermi energy
- Empirical information constrains binding potential at the Fermi energy as well as volume integrals of the imaginary part at positive energy

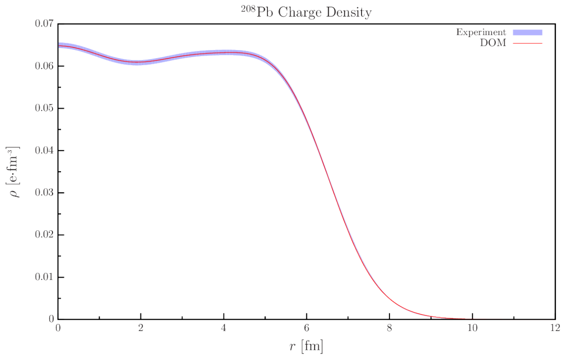
Dispersive Optical Model (St. Louis group)

- Mahaux & Sartor 1991 —> Washington University group since 2006
- Use experimental data to constrain the nucleon self-energy while linking structure and reaction domain using dispersion relations





E<0 —>



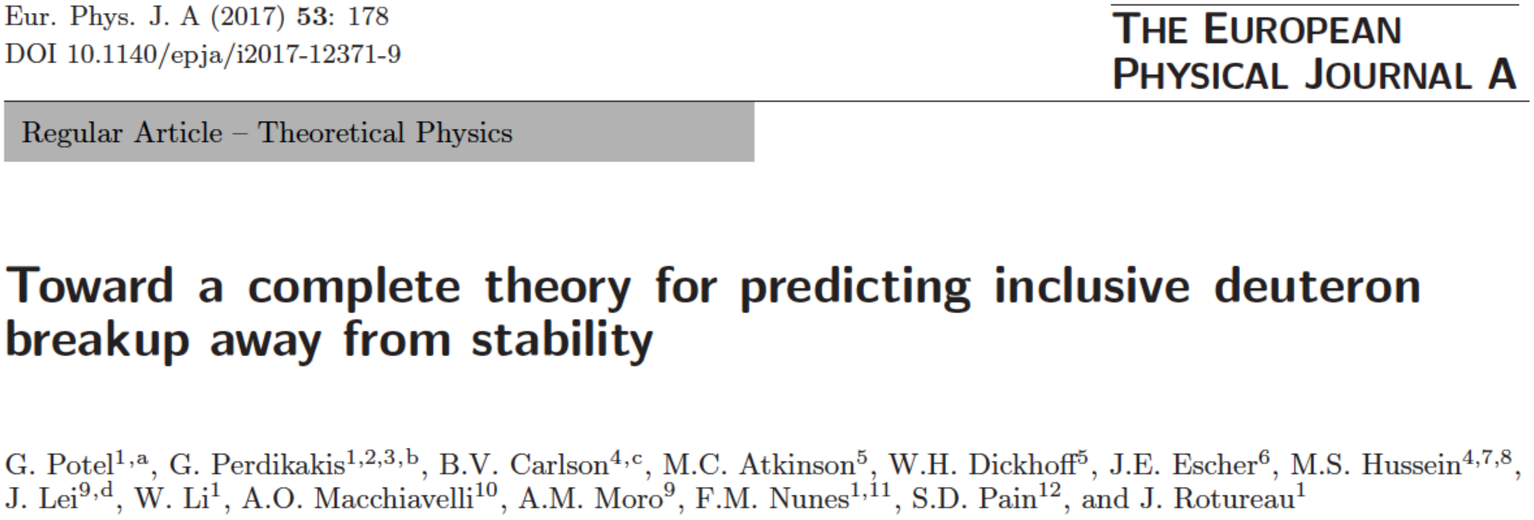
M. C. Atkinson, M. H. Mahzoon, M. A. Keim, B. A. Bordelon, C. D. Pruitt, R. J. Charity, and W. H. Dickhoff
Phys. Rev. C 101, 044303 (2020), 1-15. [arXiv:1911.09020]

Indirectly:

- Generates proton/neutron distorted waves
- Overlap functions with their normalization (spectroscopic factors)

Mack Atkinson —> TRIUMF —> LLNL

(d,p) reaction



Eur. Phys. J. A (2017) **53**: 178
DOI 10.1140/epja/i2017-12371-9

THE EUROPEAN
PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Toward a complete theory for predicting inclusive deuteron breakup away from stability

G. Potel[1,a], G. Perdikakis[1,2,3,b], B.V. Carlson[4,c], M.C. Atkinson[5], W.H. Dickhoff[5], J.E. Escher[6], M.S. Hussein[4,7,8], J. Lei[9,d], W. Li[1], A.O. Macchiavelli[10], A.M. Moro[9], F.M. Nunes[1,11], S.D. Pain[12], and J. Rotureau[1]

- State of the art inclusive (d,p)
- Employs local DOM potentials constrained for ^{40}Ca and ^{48}Ca and extrapolated to ^{60}Ca
- Explores link with (n,γ) process

Why DOM?

- Compare standard optical potential with DOM

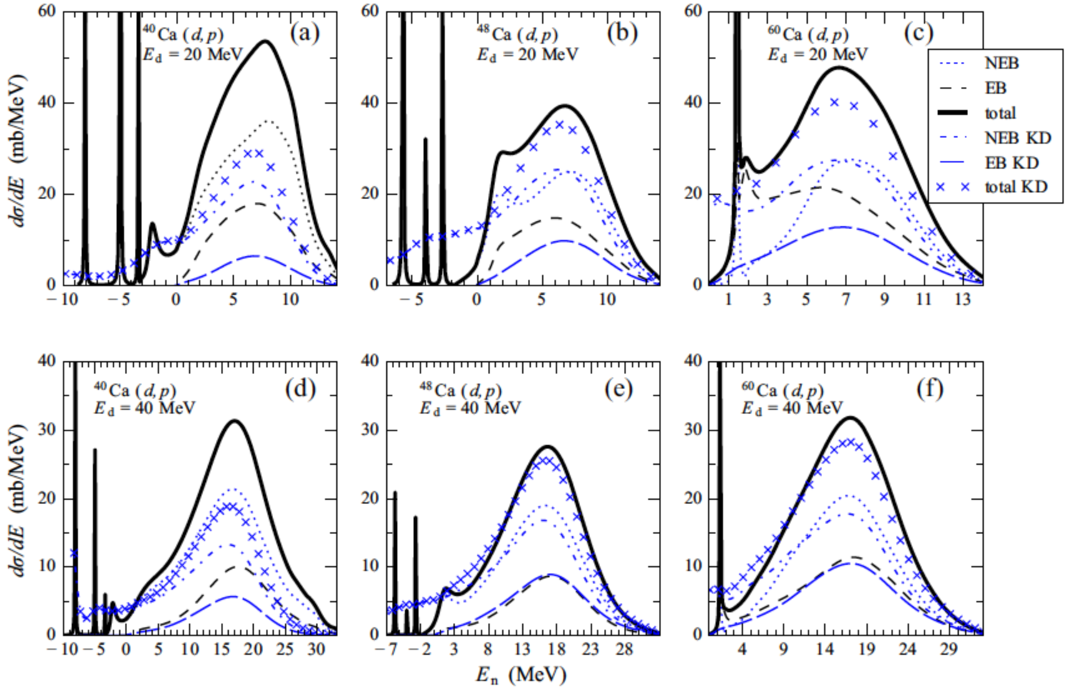
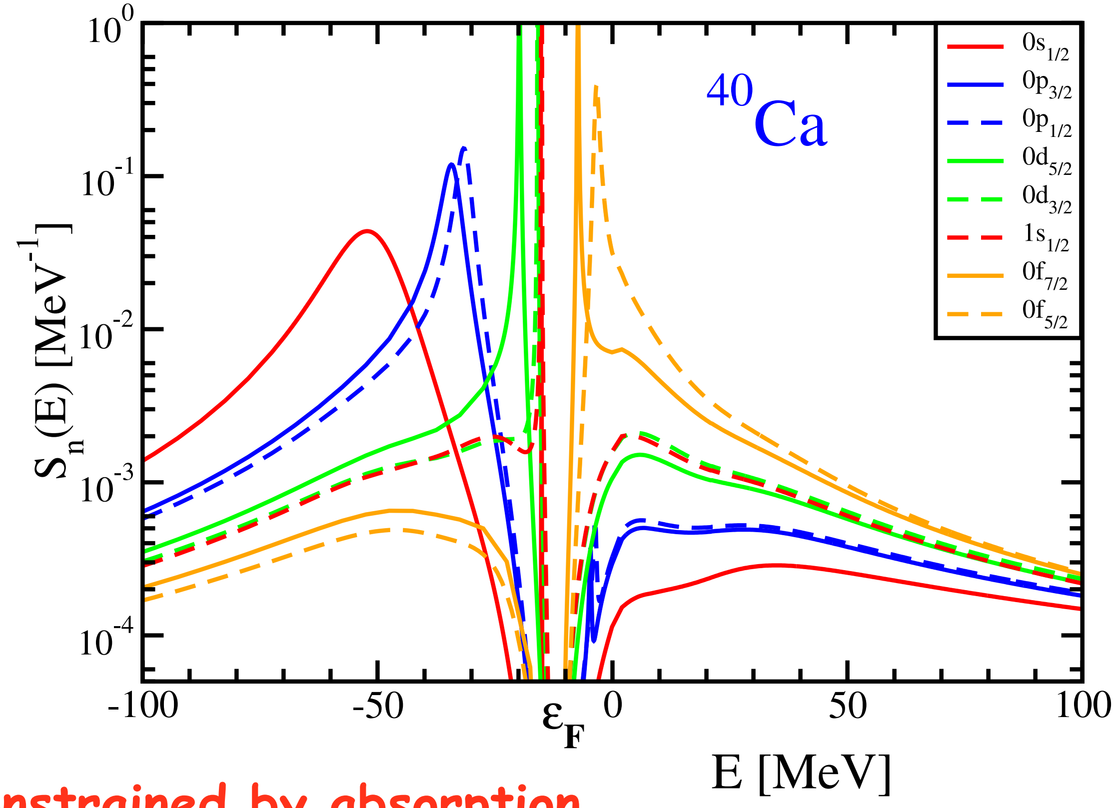


Fig. 8. Comparison of KD phenomenological optical potential and the DOM: elastic breakup (EB) and non-elastic breakup (NEB) proton spectra for the reactions ^{40}Ca(d,p), ^{48}Ca(d,p), and ^{60}Ca(d,p), at $E_d = 20$ MeV and $E_d = 40$ MeV.

- Global potentials do not generate relevant information at negative energies —> dispersive approach essential for unstable nuclei

Spectral function for bound states from DOM analysis

- [0,200] MeV —> constrained by elastic scattering data

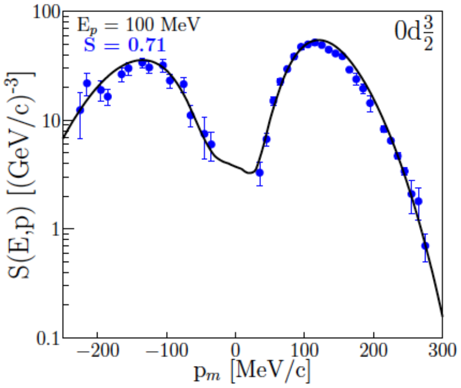


Emptiness constrained by absorption necessary to describe elastic scattering!

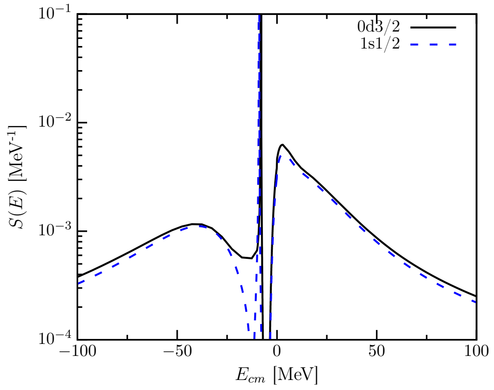
PRC90, 061603(R) (2014)

Check with (e,e'p) cross sections (Mack Atkinson)

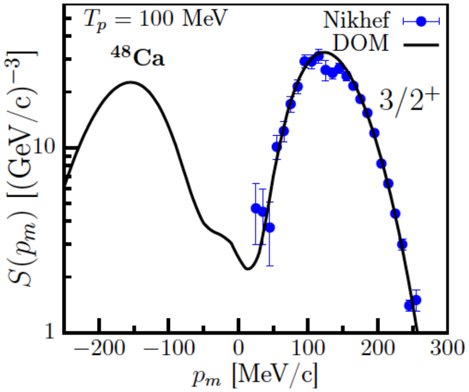
- ^{40}Ca



[Phys. Rev. C98, 044627 (2018)]



- ^{48}Ca

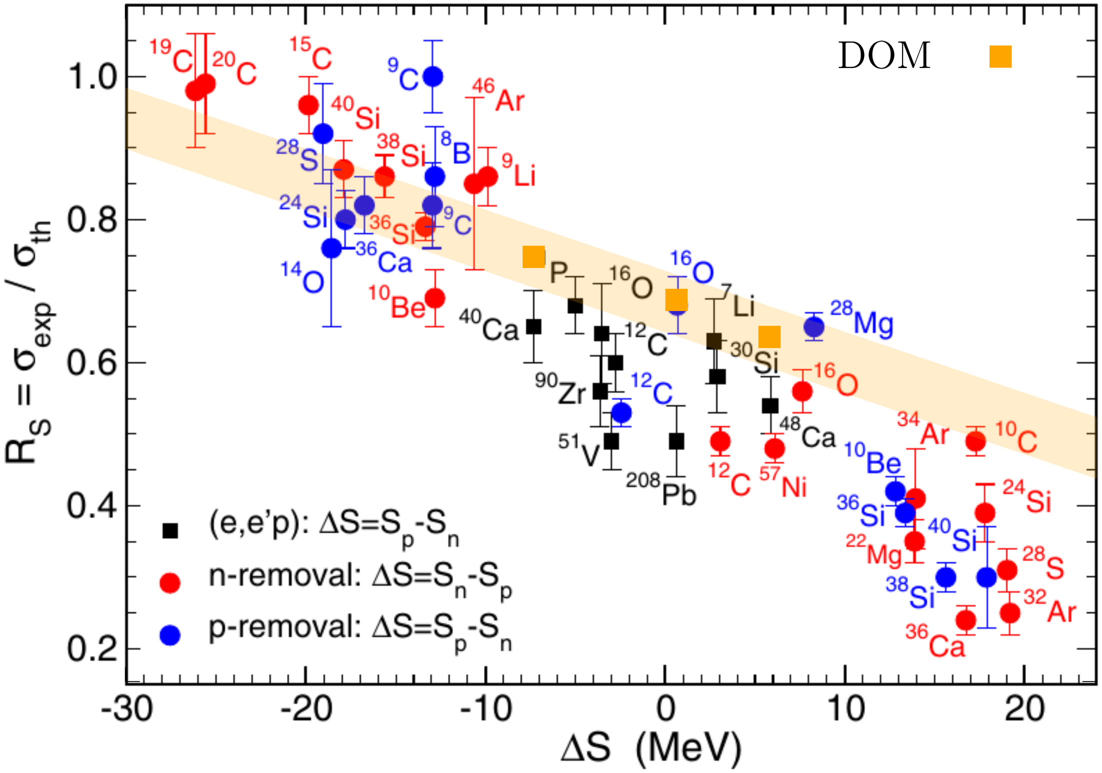


[Phys. Lett. B 798, 135027 (2019)]

- No further adjustments!
- **Both structure and reaction properties allowed to change when 8 n added**

Compare with Gade plot

Very near the Fermi energy in ^{40}Ca and ^{48}Ca from (e,e'p) —> error band

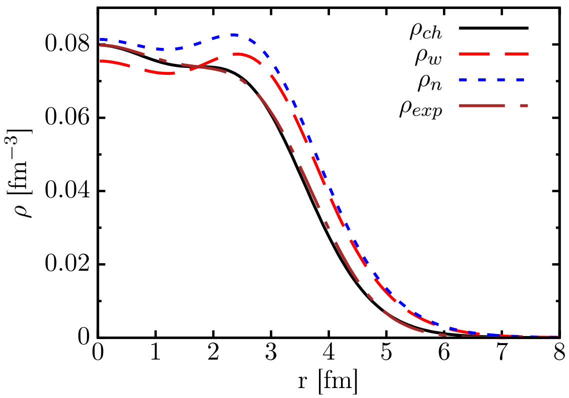


Quenching sp strength review: Aumann et al, Prog. Part. Nucl. Phys. 118, 103847 (2021)

DOM

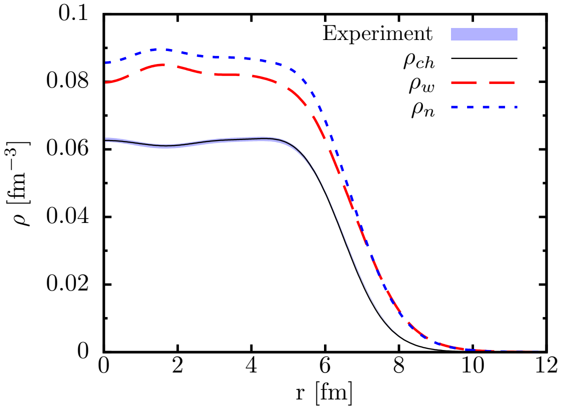
Neutron skins in ^{48}Ca and ^{208}Pb from DOM predictions

- DOM 2017



M. H. Mahzoon, M. C. Atkinson, R. J. Charity, and W. H. Dickhoff
Phys. Rev. Lett. **119,** 222503 (2017), 1-5.

- DOM 2020



M. C. Atkinson, M. H. Mahzoon, M. A. Keim, B. A. Bordelon, C. D. Pruitt, R. J. Charity, and W. H. Dickhoff
Phys. Rev. C 101, 044303 (2020), 1-15.

MCMC DOM prediction of neutron skins

- Markov Chain Monte Carlo

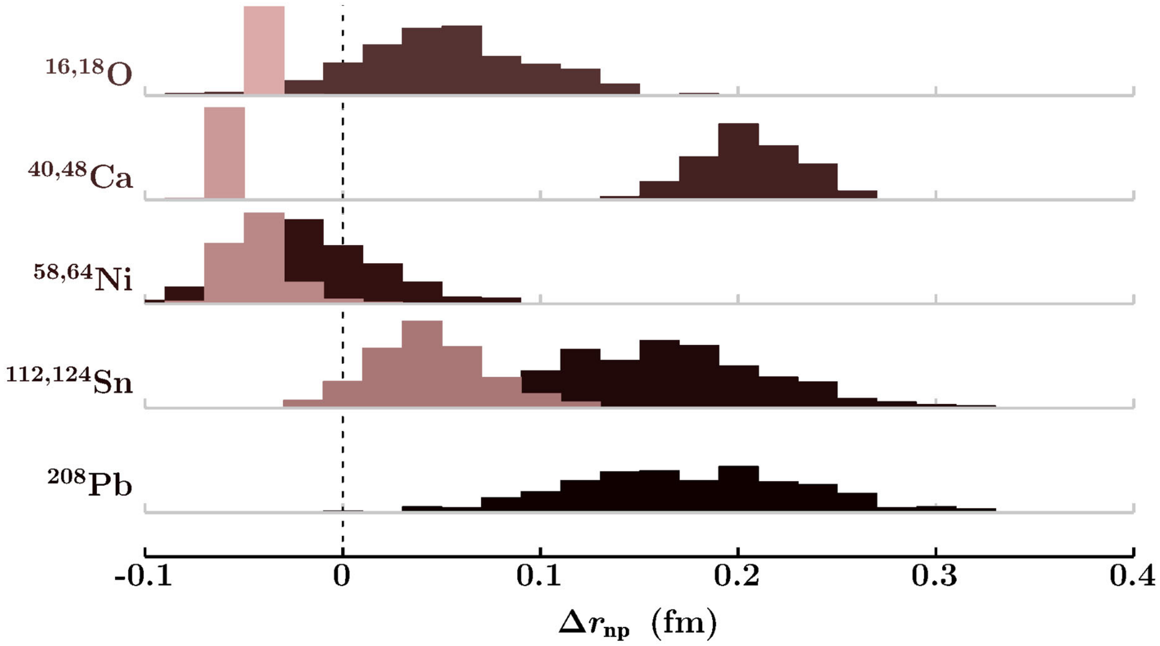


TABLE I. Neutron skins (Δr_{np}), in fm, from this work. The 16th, 50th, and 84th percentile values of the skin distribution are reported as 50_{16}^{84}.

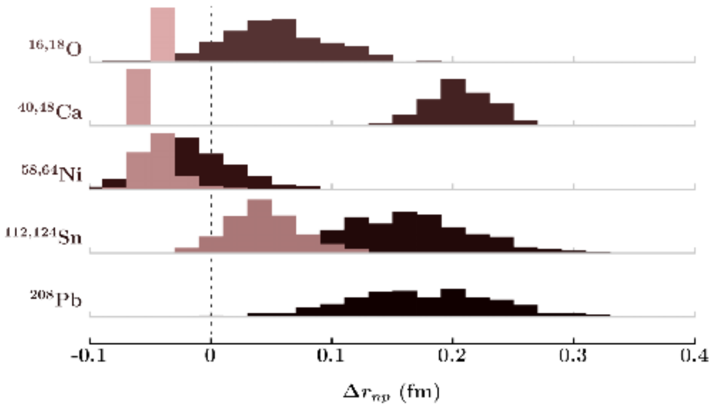
^{16}O	^{18}O	^{40}Ca	^{48}Ca	^{58}Ni	^{64}Ni	^{112}Sn	^{124}Sn	^{208}Pb
$-0.025_{-0.027}^{-0.023}$	$0.06_{0.02}^{0.11}$	$-0.051_{-0.055}^{-0.048}$	$0.22_{0.19}^{0.24}$	$-0.03_{-0.05}^{-0.02}$	$-0.01_{-0.04}^{0.03}$	$0.05_{0.02}^{0.08}$	$0.17_{0.12}^{0.23}$	$0.18_{0.12}^{0.25}$

C. D. Pruitt, R. J. Charity, L. G. Sobotka, M. C. Atkinson, and W. H. Dickhoff
Phys. Rev. Lett. 125, 102501 (2020), 1-6.

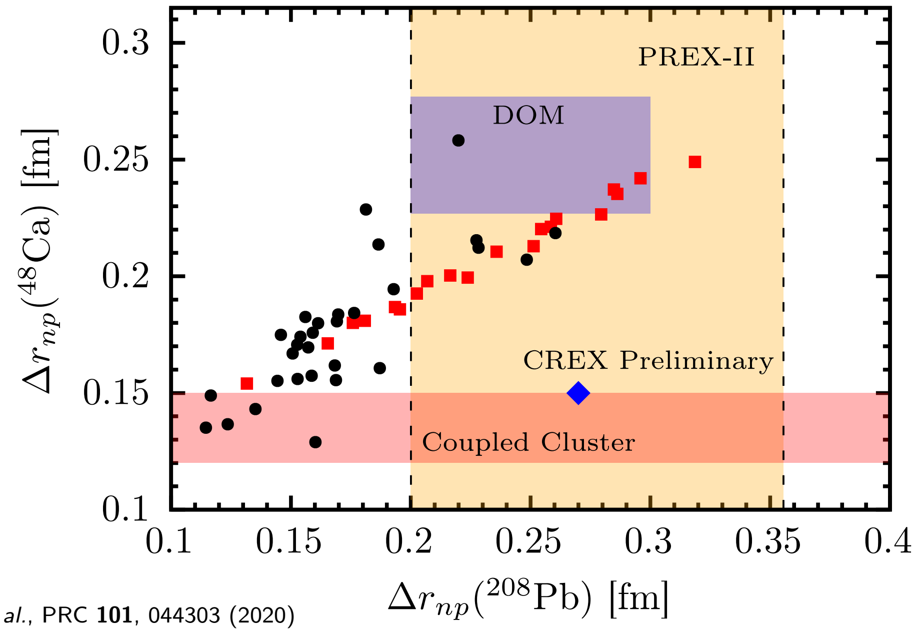
Neutron skin puzzle & DOM (slide Mack Atkinson now at LLNL)

Neutron Skin: $\Delta r_{np} = r_n - r_p$

- r_n can be measured through parity-violating electron scattering (weak)
- PREX-II at Jefferson Lab measured ^{208}Pb skin
  - Preliminary CREX results for ^{48}Ca released at DNP meeting 2021
  - Very surpising ^{48}Ca skin!



C.D. Pruiit *et al.*, PRL **125**, 102501 (2020)



M.C. Atkinson *et al.*, PRC **101**, 044303 (2020)

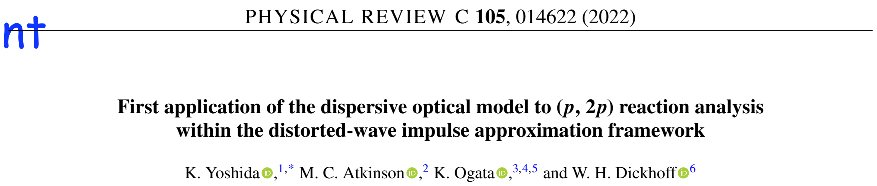
Outlook FRIB era

- Accurate global optical potential requires a lot of theoretical effort
  - Topical Collaboration: Holt, Dickhoff, Elster, Lovell, Nunes, Potel, Stroberg (DOE proposal)
- Accurate global optical potential requires a lot of experimental effort as well!

- Removal probability puzzle only slow progress
- Neutron skins very interesting

- **Conceptual paradigm shift: need to treat reactions and structure simultaneously**

- (p,2p) good starting point
- First DOM analysis —>



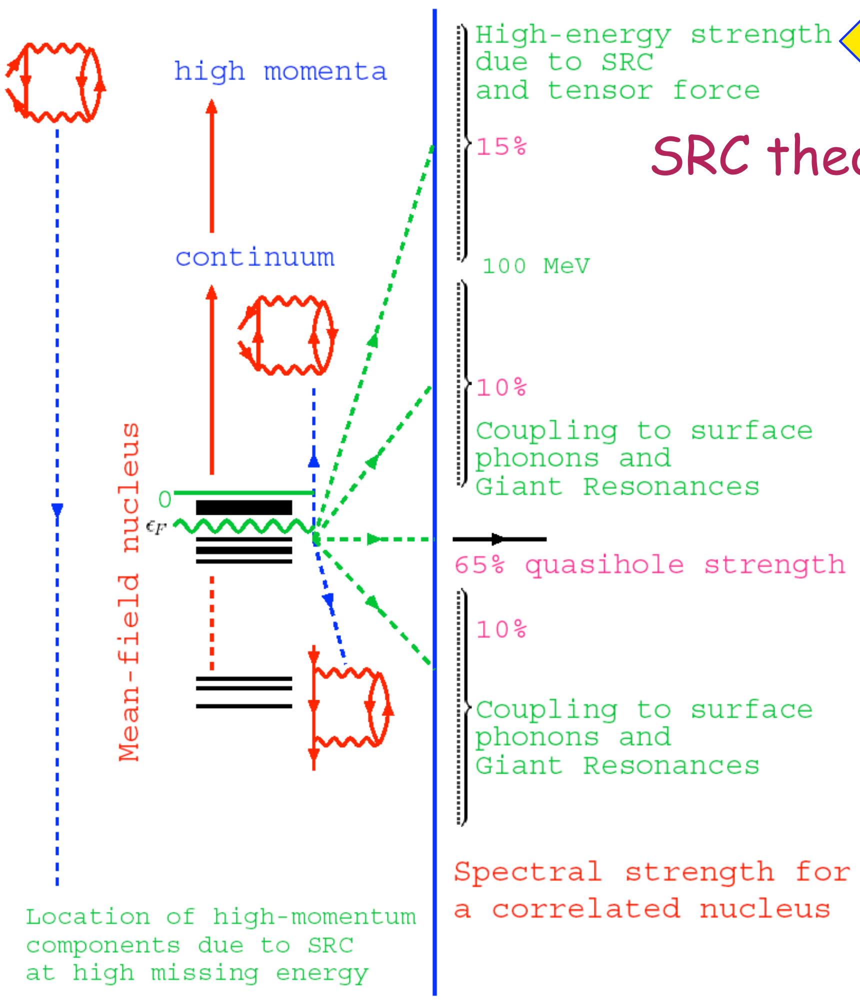
PHYSICAL REVIEW C **105**, 014622 (2022)

First application of the dispersive optical model to $(p, 2p)$ reaction analysis within the distorted-wave impulse approximation framework

K. Yoshida[1,*] M. C. Atkinson[2] K. Ogata[3,4,5] and W. H. Dickhoff[6]

Reviewed in Prog. Part. Nucl. Phys. 52 (2004) 377-496

Location of single-particle strength in closed-shell (stable) nuclei



High-energy strength due to SRC and tensor force

Coupling to surface phonons and Giant Resonances

Coupling to surface phonons and Giant Resonances

Location of high-momentum components due to SRC at high missing energy

Spectral strength for a correlated nucleus

SRC theory

Elastic nucleon scattering

NIKHEF (e,e'p) data

L. Lapikás

Nucl. Phys. A553,297c (1993)

For example: protons in ^{208}Pb

SRC

JLab E97-006

Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.

High-momentum predictions & relation to ground-state energy

Ground-state energy can be included in the DOM

$$E/A = \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^{\infty} dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^{\infty} dk k^2 \int_{-\infty}^{\varepsilon_F} dE\, E\, S_{\ell j}(k;E)$$

Succeeds

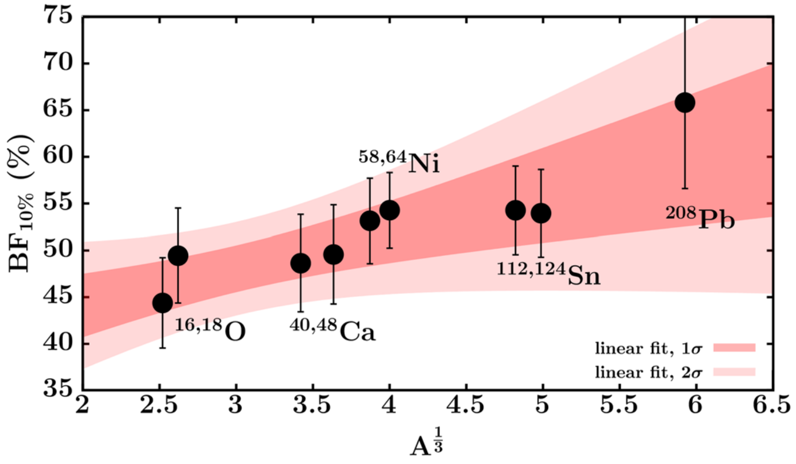
A	DOM E_0^A/A	Mass equation	Expt. E_0^A/A
^{12}C	−7.85	−7.29	−7.68
^{40}Ca	−8.46	−8.50	−8.55
^{48}Ca	−8.66	−8.59	−8.66
^{208}Pb	−7.76	−7.81	−7.87

Phys. Rev. C 102, 044333 (2020)

Because fraction of binding energy from 10% most deeply bound nucleons includes the high-momentum contribution

Phys. Rev. Lett. 125, 102501 (2020)

Predicted in Phys. Rev. C51, 3040 (1995)

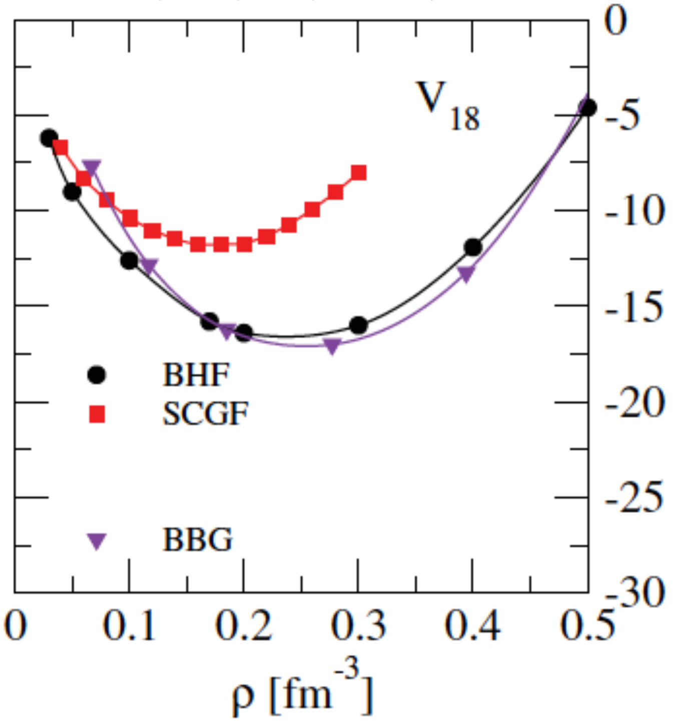


SCGF & SRC compared to BHF and BBG

PHYSICAL REVIEW C **86**, 064001 (2012)

Comparative study of neutron and nuclear matter with simplified Argonne nucleon-nucleon potentials

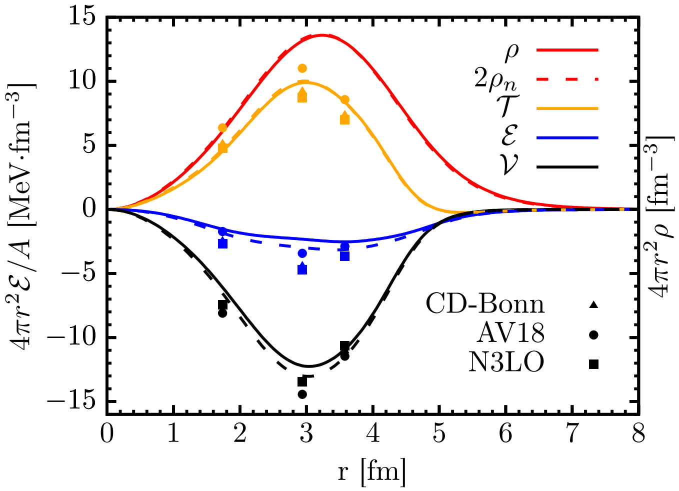
M. Baldo,[1] A. Polls,[2] A. Rios,[3] H.-J. Schulze,[1] and I. Vidaña[4]

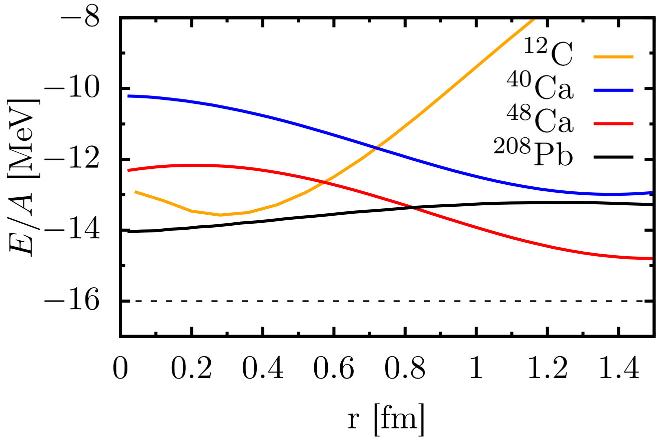


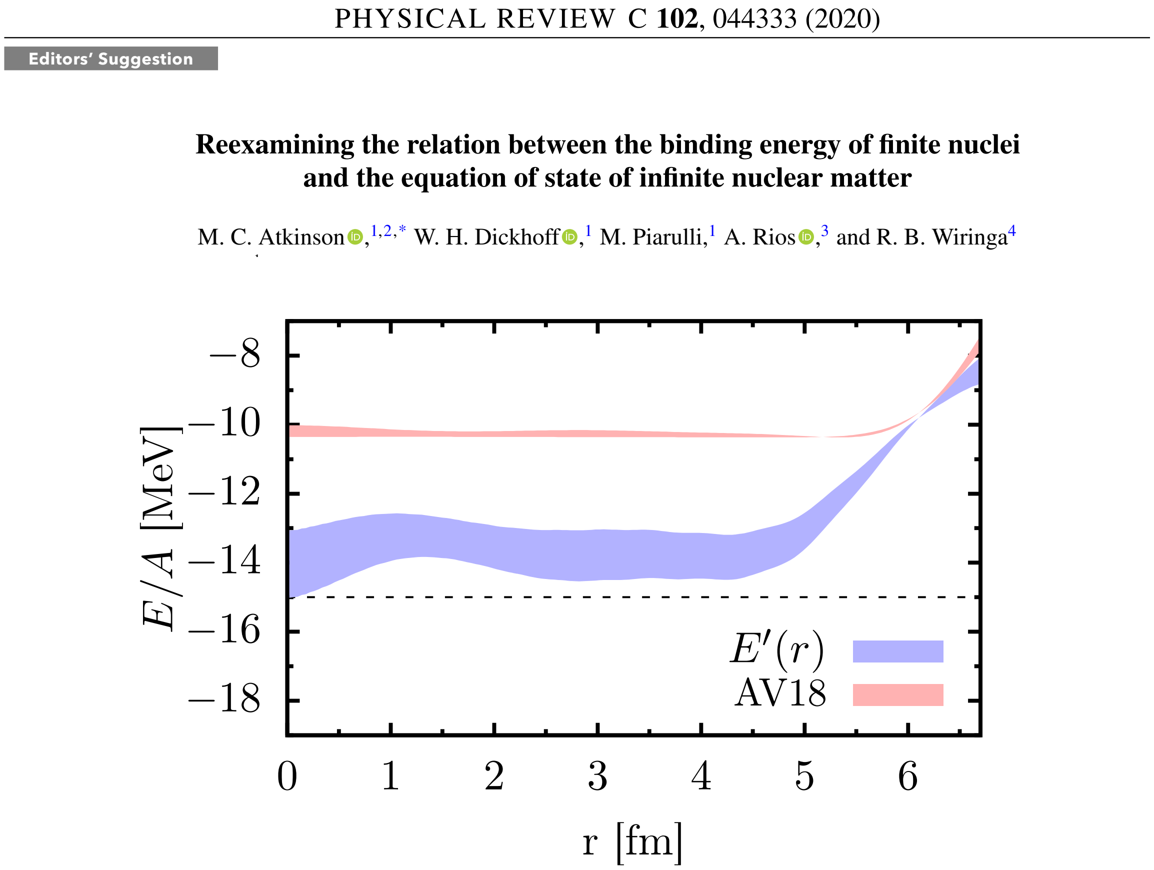
- BBG requires a repulsive NNN at high density to improve density

Consequence

- Maybe 16 MeV binding is not needed!







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Editors' Suggestion

Reexamining the relation between the binding energy of finite nuclei and the equation of state of infinite nuclear matter

M. C. Atkinson,[1,2,*] W. H. Dickhoff,[1] M. Piarulli,[1] A. Rios,[3] and R. B. Wiringa[4]

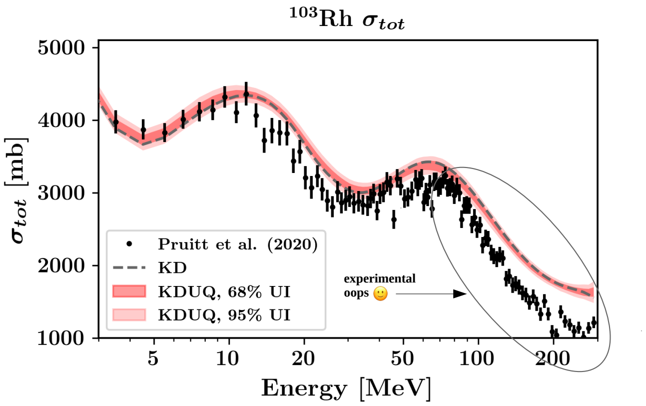
Cole Pruitt (LLNL)

“OMP uncertainty – characterized empirically for KD and CH89 – is larger than assumed:

Global KD OMP has systematic over/underestimation for σ_{rxn}

Roughly 20% std. dev. for (n,*)/(p,*) cross sections on stable targets KDUQ/CHUQ help pinpoint where data and model are grossly inconsistent“

- Total neutron cross sections using an iterative procedure using KD potential



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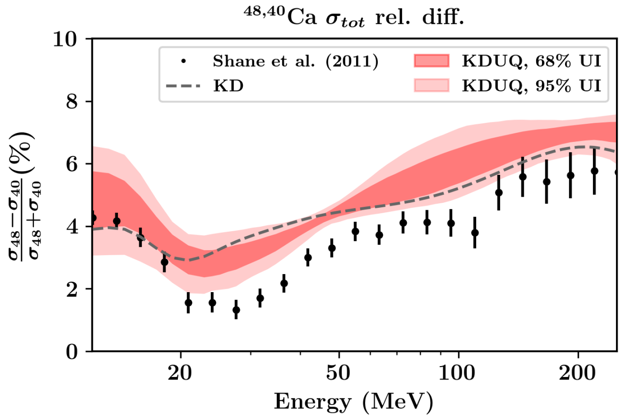

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- Problem at high energy but relative OK —> another look at the neutron skin of ^{48}Ca?

Propagator / Green's function and spectral functions & spectroscopic factors

- Lehmann representation

$$\begin{aligned} G_{\ell j}(k,k';E) &= \sum_m \frac{\langle \Psi_0^A | \, a_{k\ell j} \, | \Psi_m^{A+1} \rangle \langle \Psi_m^{A+1} | \, a^\dagger_{k'\ell j} \, | \Psi_0^A \rangle}{E - (E_m^{A+1} - E_0^A) + i\eta} \\ &+ \sum_n \frac{\langle \Psi_0^A | \, a^\dagger_{k'\ell j} \, | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | \, a_{k\ell j} \, | \Psi_0^A \rangle}{E - (E_0^A - E_n^{A-1}) - i\eta} \end{aligned}$$

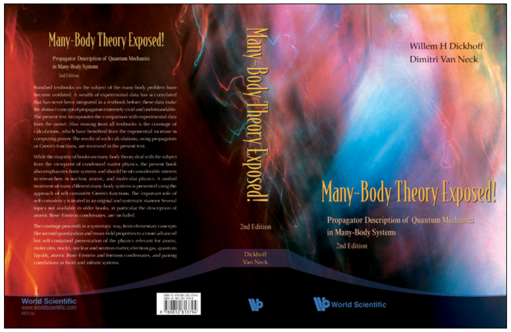
- Any other single-particle basis can be used & continuum integrals implied
- Overlap functions --> numerator Corresponding eigenvalues --> denominator
- Spectral function

$$\begin{aligned} S_{\ell j}(k;E) &= \frac{1}{\pi} \, \mathrm{Im} \, G_{\ell j}(k,k;E) \qquad\qquad E \le \varepsilon_F^- \\ &= \sum_n \left| \langle \Psi_n^{A-1} | \, a_{k\ell j} \, | \Psi_0^A \rangle \right|^2 \delta(E - (E_0^A - E_n^{A-1})) \end{aligned}$$

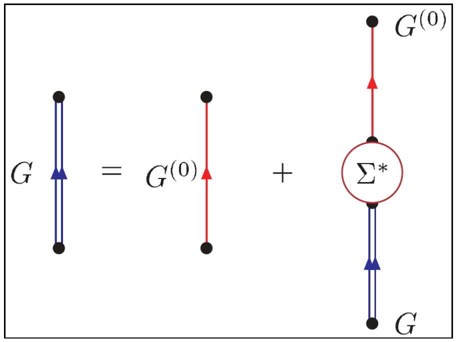
- Discrete transitions

$$\sqrt{S^n_{\ell j}} \; \phi^n_{\ell j}(k) = \langle \Psi_n^{A-1} | \, a_{k\ell j} \, | \Psi_0^A \rangle$$

- Momentum distribution: integrate spectral function to ε_F^-
- Positive energy —> see later



Propagator from Dyson Equation and "experiment"



Equivalent to …

Schrödinger-like equation with: $E_n^- = E_0^A - E_n^{A-1}$

Self-energy: non-local, energy-dependent potential

With energy dependence: spectroscopic factors < 1

⇒ as extracted from (e,e′p) reaction

$$\frac{k^2}{2m}\phi_{\ell j}^n(k) + \int dq\; q^2\; \Sigma_{\ell j}^*(k,q;E_n^-)\;\phi_{\ell j}^n(q) = E_n^-\;\phi_{\ell j}^n(k)$$

Spectroscopic factor $\mathcal{Z}_{\ell j}^n = \int dk\; k^2\; \left|\langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle\right|^2 < 1$

Dyson equation also yields $\left[\chi_{\ell j}^{elE}(r)\right]^* = \langle\Psi_{elE}^{A+1}|\, a_{r\ell j}^\dagger\, |\Psi_0^A\rangle$ for positive energies

Elastic scattering wave function for protons or neutrons

Dyson equation therefore provides:

Link between scattering and structure data from **dispersion relations**

DOM