The variety of nuclear shapes in Super Heavy Nuclei

J. Luis Egido in collab. with Andrea Jungclaus

Aim of the talk

I am going to talk about the theoretical description of superheavy elements using the Finite Range density dependent Gogny interaction and sophisticated microscopical approaches namely the symmetry conserving configuration mixing theories.

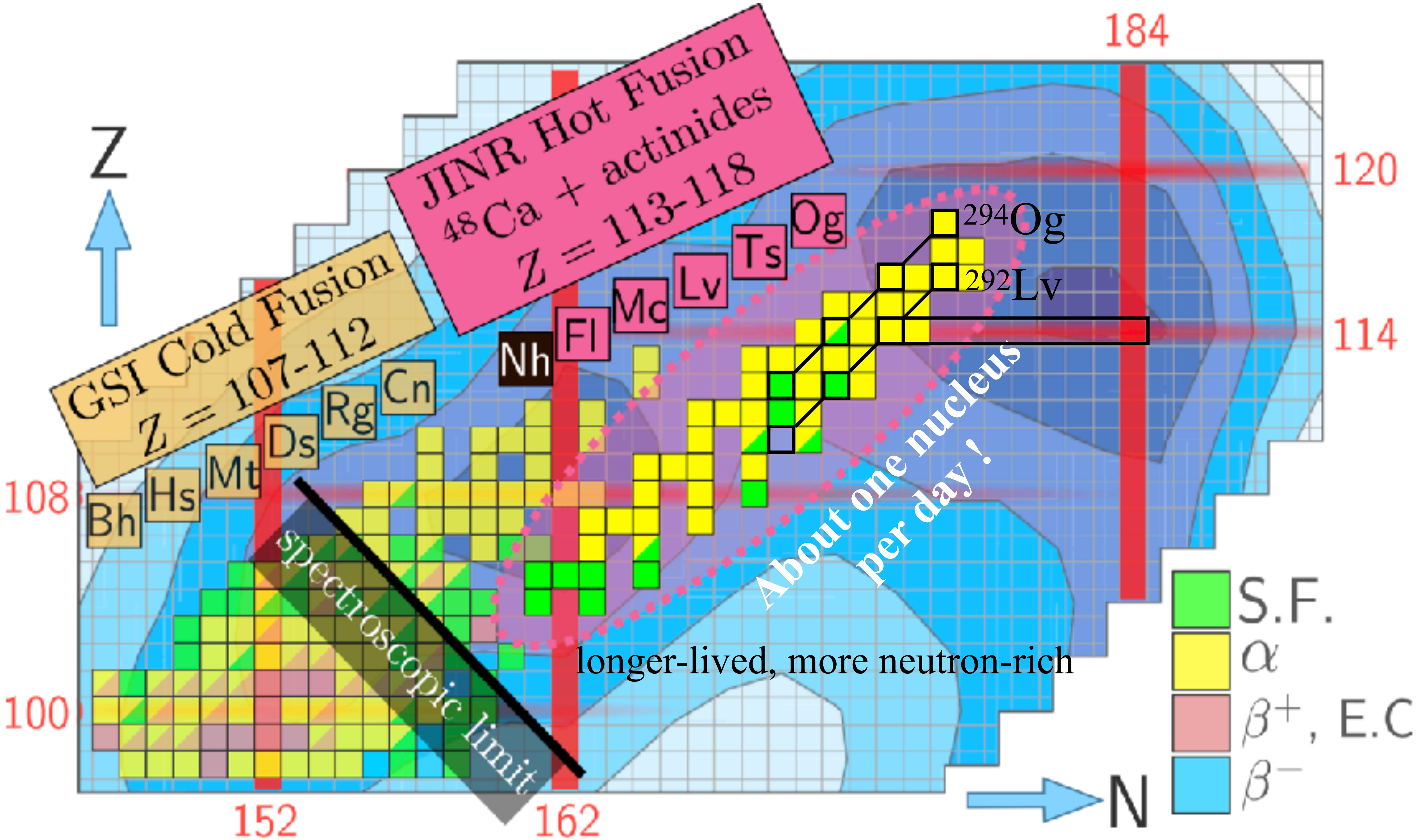
Outline of the talk:

1.- **Short description of the theory**

2.- **Ground state deformations & shape coexistence in the Flerovium isotopes**.

3.- **Low-Energy excited states in the alpha decay chains of ^{292}Lv & ^{294}Og**

The upper end of the chart of nuclides



^{208}Pb/^{209}Bi targets

A. Samark-Roth, PhD thesis, Lund University, 2021

Overview of calculations in SHN

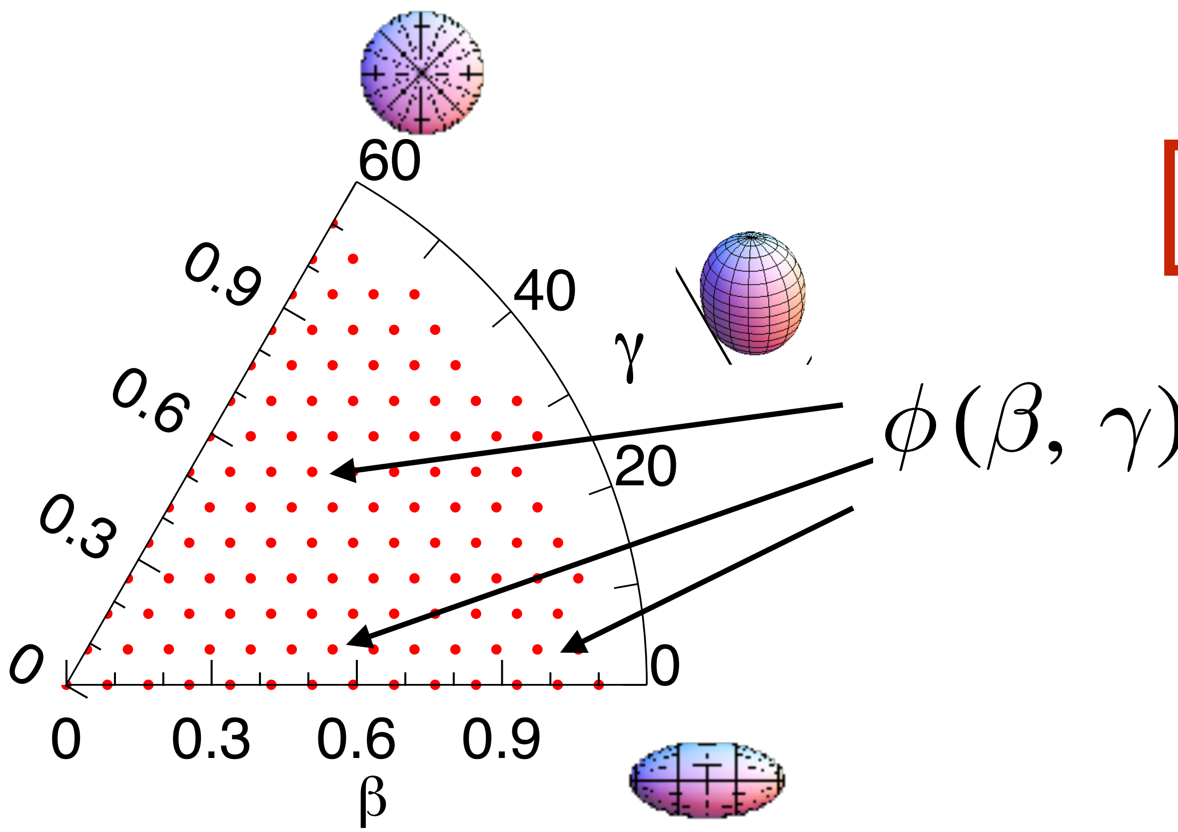
Macro-Micro (see, for example, the review of A. Sobiczewski & K. Pomorski Prog. Part. Nucl. Phys. **58** (2007)292-349)

Self-Consistent Theories: With Skyrme, Gogny or relativistic interactions

MFA (HFB) → Axial (Many calc.)
MFA (HFB) → Triaxial (Some calc.)

BMFA (Shape mixing & PN+AMP) → Axial (One calc.)
BMFA (Shape mixing & PN+AMP) → Triaxial (No calc.)

Theories: Mean field and Beyond

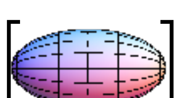
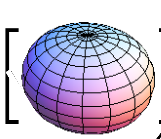
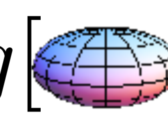


AXIAL APPROX: $\gamma = 0^o$ and 60^o

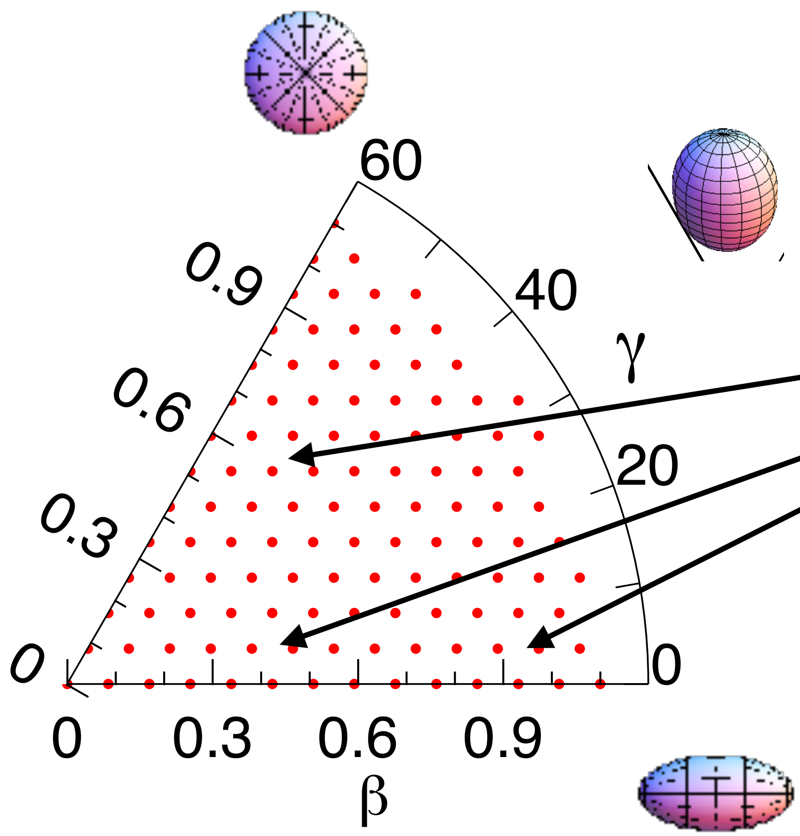
TRIAXIAL APPROX: $0^o \leq \gamma \leq 60^o$

The symmetry-conserving configuration mixing (SCCM)

$$|\Psi^{N,I}_{M,\sigma}\rangle = \sum_{\beta,\gamma,K} f^{I}_{\sigma}(\beta, \gamma, K) P^{N} P^{I}_{MK} \, |\phi(\beta, \gamma)\rangle$$

$= e[$$]^{I}_{M} + \ldots + f[$$]^{I}_{M} + \ldots + g[$$]^{I}_{M}$

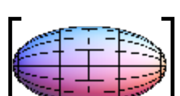
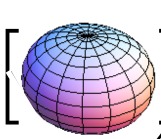
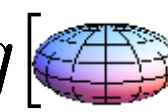
The symmetry-conserving configuration mixing (SCCM)



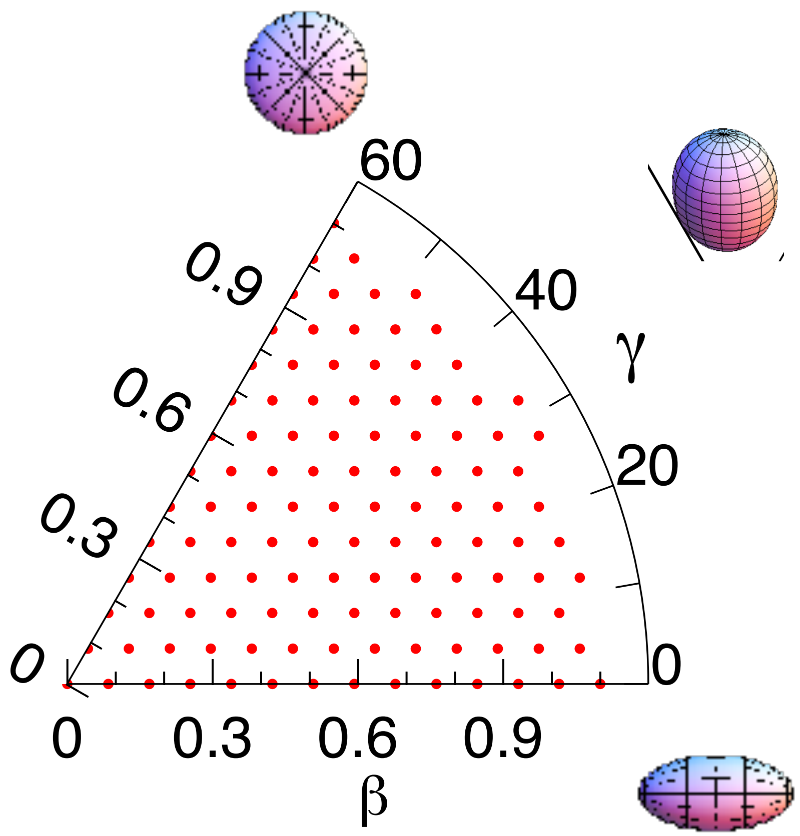
$\phi(\beta,\gamma)$ is determined in the PNVAP approach

PNVAP-PES & PNAMP-PES

$$|\Psi^{N,I}_{M,\sigma}\rangle = \sum_{\beta,\gamma,K} f^{I}_{\sigma}(\beta,\gamma,K) P^{N} P^{I}_{MK} |\phi(\beta,\gamma)\rangle$$

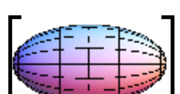
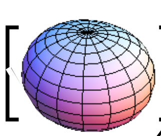
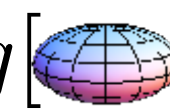
$= e[$  $]^{I}_{M} + \ldots + f[$  $]^{I}_{M} + \ldots + g[$  $]^{I}_{M}$

The symmetry-conserving configuration mixing (SCCM)



The weights $f_\sigma^I(\beta,\gamma,K)$ are determined solving the Hill-Wheeler equation

$$|\Psi^{N,I}_{M,\sigma}\rangle = \sum_{\beta,\gamma,K} f_\sigma^I(\beta,\gamma,K) P^N P^I_{MK} |\phi(\beta,\gamma)\rangle$$

$= \quad e[$  $]^I_M + \ldots + f[$  $]^I_M + \ldots + g[$  $]^I_M$

The Gogny Interaction

J. Dechargé, D. Gogny, Phys. Rev. C 21, 1568 (1980)

In the calculations we use large configuration spaces (13 Mayor Oscillator shells, tests have been done with 17).Therefore no effective charges are needed. We use the D1S parametrisation of the Gogny force:

$$V(1,2) = \sum_{i=1}^{2} e^{-(\vec{r}_1-\vec{r}_2)^2/\mu_i^2} \left(W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau\right) \quad \text{central term}$$

$$+iW_0(\sigma_1+\sigma_2)\vec{k}\times\delta(\vec{r}_1-\vec{r}_2)\vec{k} \quad \text{Spin-orbit term}$$

$$+t_3(1+x_0P^\sigma)\delta(\vec{r}_1-\vec{r}_2)\rho^\alpha((\vec{r}_1+\vec{r}_2)/2) \quad \text{density-dependent term}$$

$$+V_{\text{Coulomb}}(\vec{r}_1,\vec{r}_2) \quad \text{Coulomb term}$$

D1S Parametrization (Berger et al. 1984)

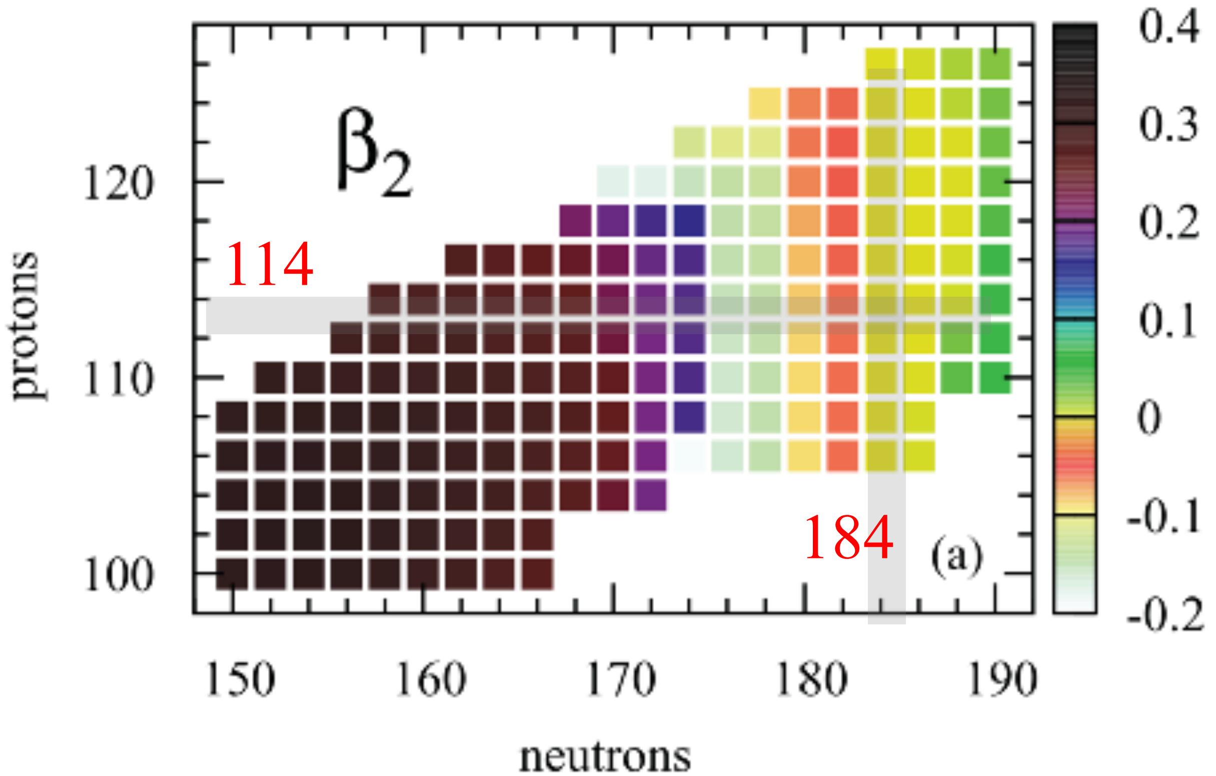
i	$\mu(\text{fm})^2$	W	B	H	M
1	0,7	-1720,3	1300	-1813,53	1397,6
2	1,2	103,638	-163,48	162,81	-223,93

W_0=130 MeV fm^5
x_0=1.0, α =1/3
t_3=1390.6 MeV fm^4

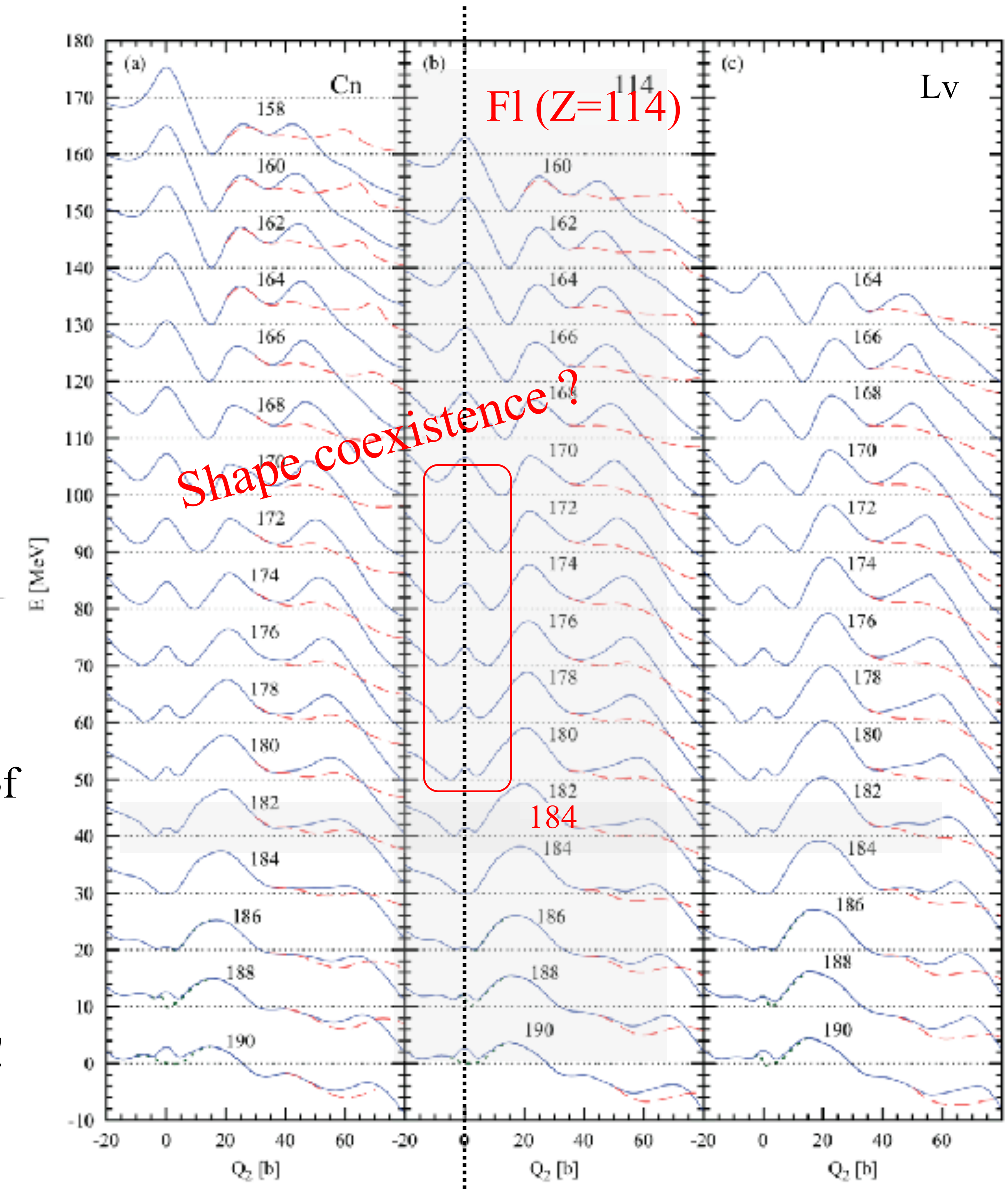
The Flerovium Isotopes

$^{288-298}Fl$

The simplest approach: Axial-symmetric mean-field studies



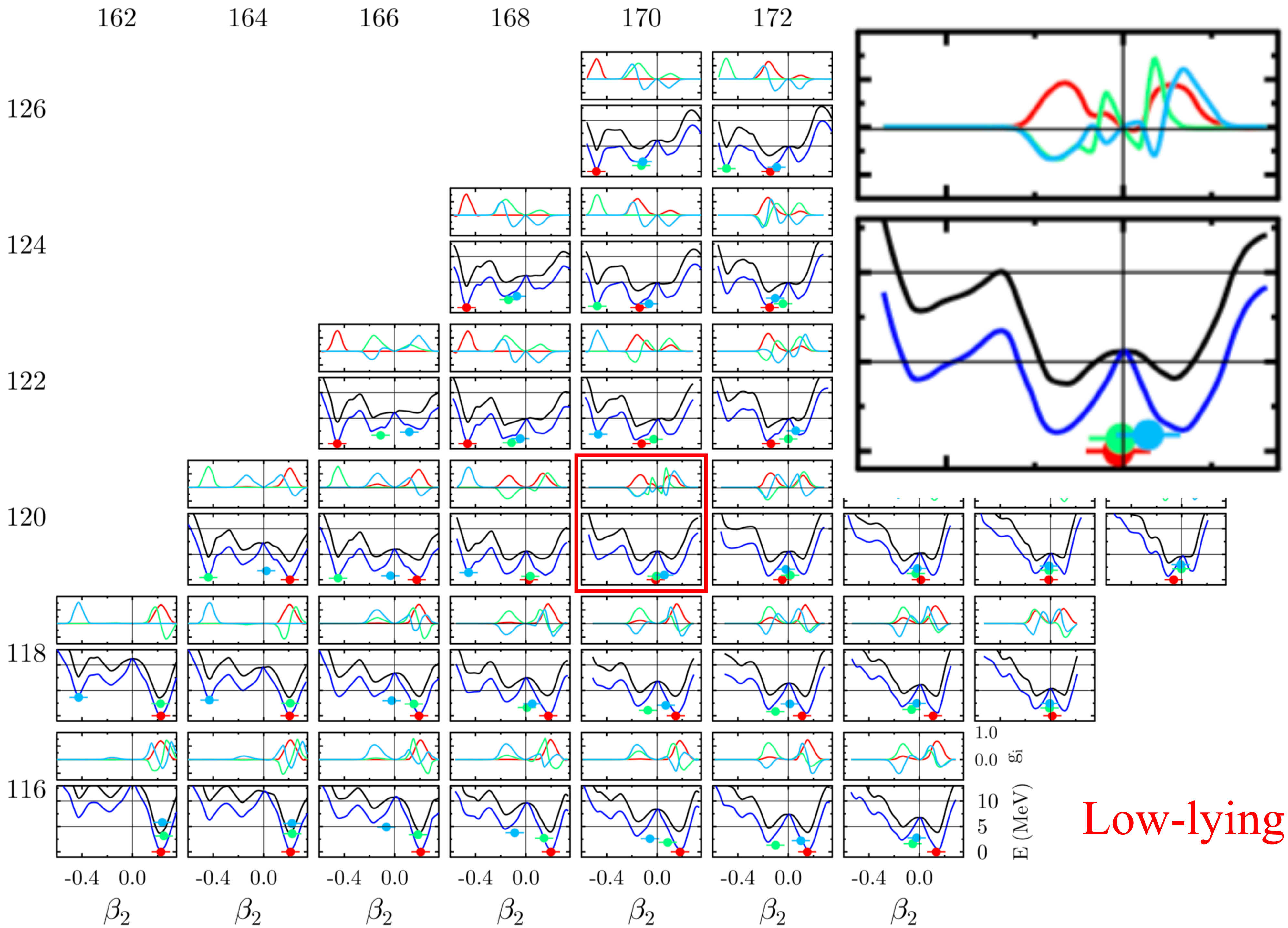
M. Warda and J.L. Egido, Phys. Rev. C 86, 014322 (2012)



Similar MF studies using other interactions led to different predictions for the position of shell gaps !

Several minima in potential energy curve:
Beyond-mean-field effects may be important !

First axial beyond-mean-field (BMF) study of super-heavy nuclei



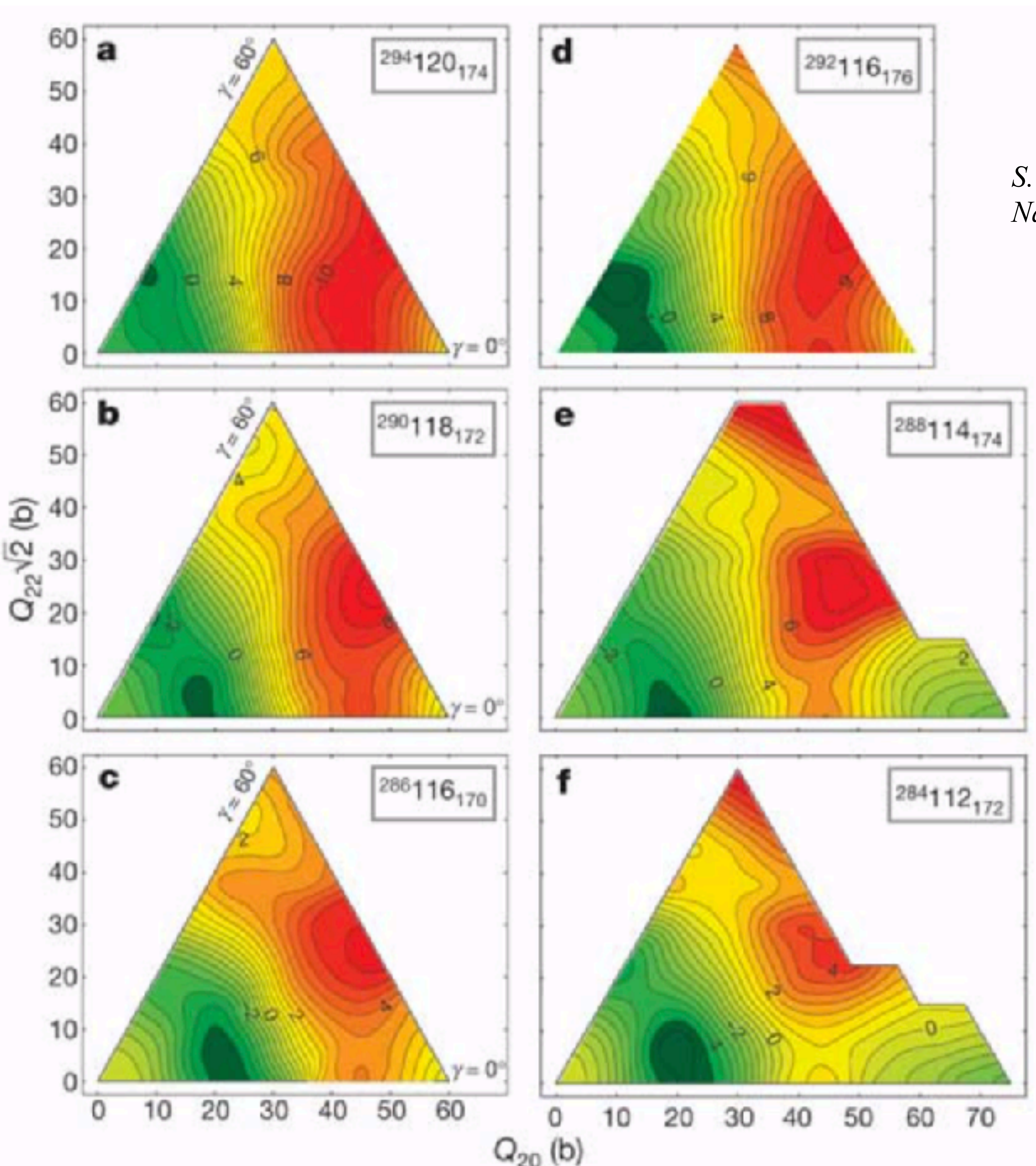
Collective wave functions of the first three 0^+ states

Projection on:
- particle number only
- angular momentum

Low-lying excited 0^+ states !

P.-H. Heenen, J. Skalski, A. Staszczak and D. Vretenar, Nucl. Phys. A 944, 415 (2015)

First triaxial mean-field study of super-heavy nuclei



S. Cwiok, P.-H. Heenen and W. Nazarewicz
Nature 433, 705 (2005)

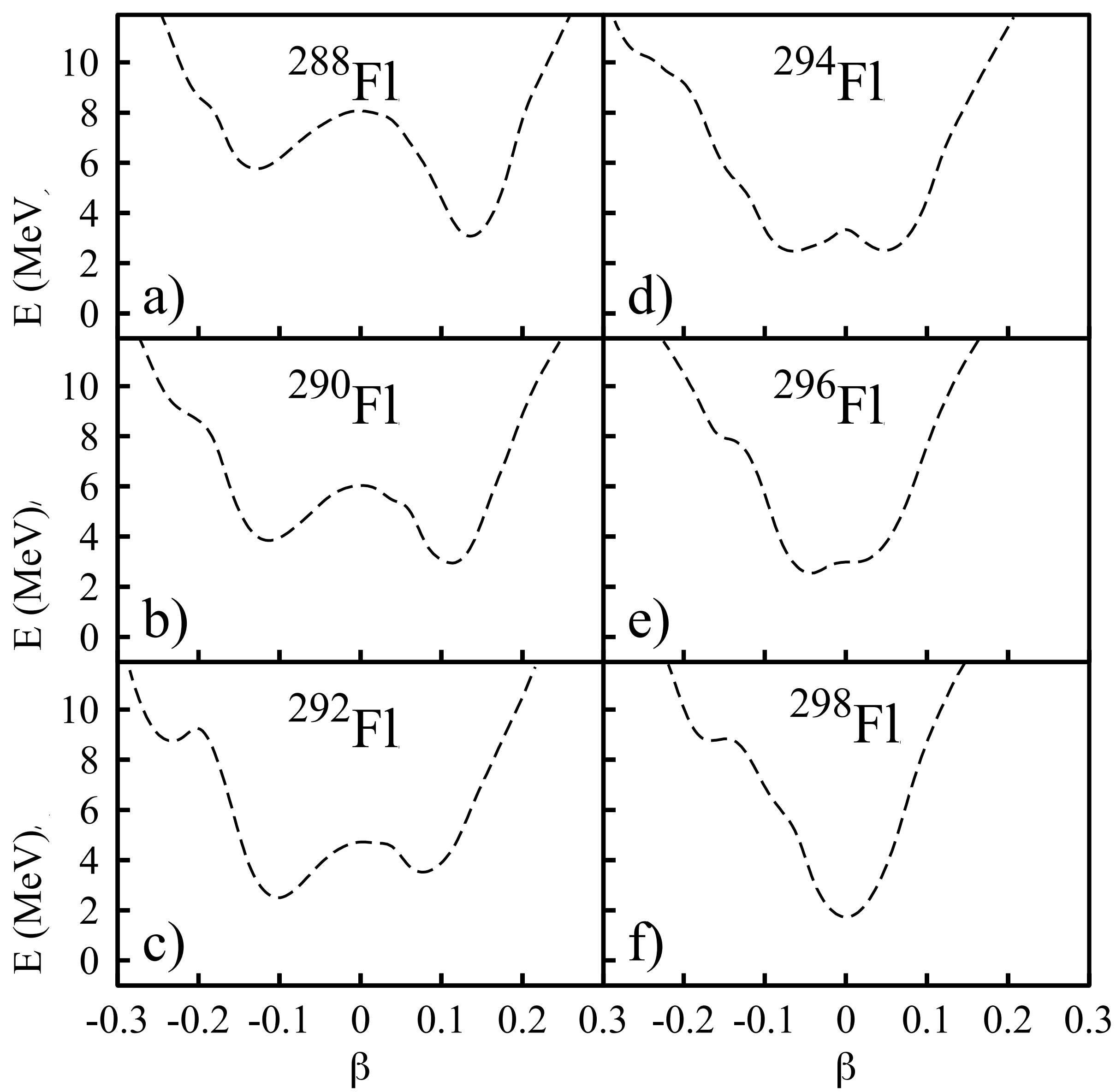
Part I

PHYSICAL REVIEW LETTERS **125**, 192504 (2020)

Predominance of Triaxial Shapes in Transitional Super-Heavy Nuclei: Ground-State Deformation and Shape Coexistence along the Flerovium (Z = 114) Chain of Isotopes

J. Luis Egido[1,*] and Andrea Jungclaus[2,†]
[1]*Departamento de Física Teórica and CIAFF, Universidad Autónoma de Madrid, E-28049 Madrid, Spain*
[2]*Instituto de Estructura de la Materia, CSIC, E-28006 Madrid, Spain*

(Received 9 September 2020; revised 30 September 2020; accepted 12 October 2020; published 5 November 2020)



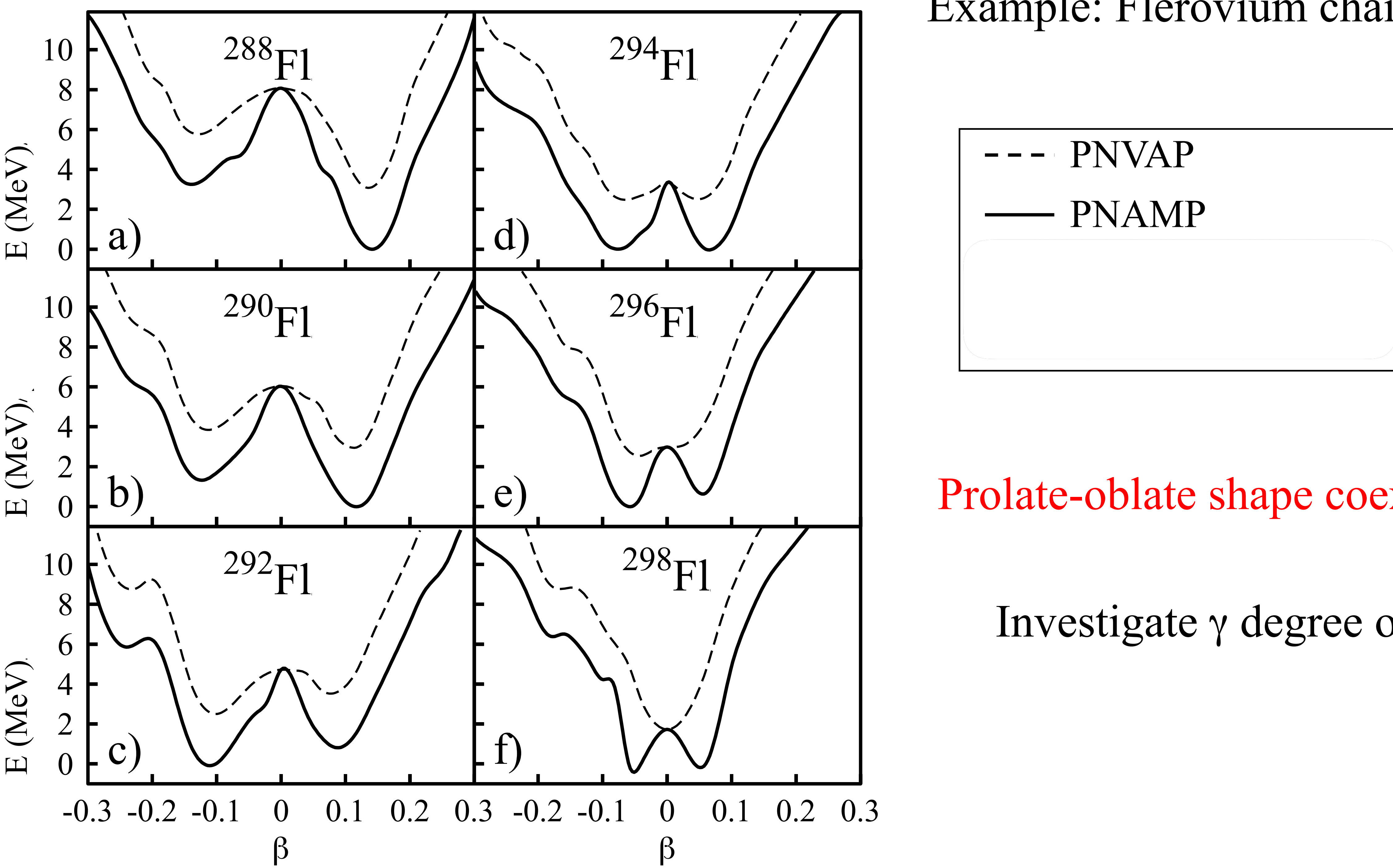
Example: Flerovium chain (Z=114)

PNVAP

Prolate-oblate shape coexistence ?

Investigate γ degree of freedom !

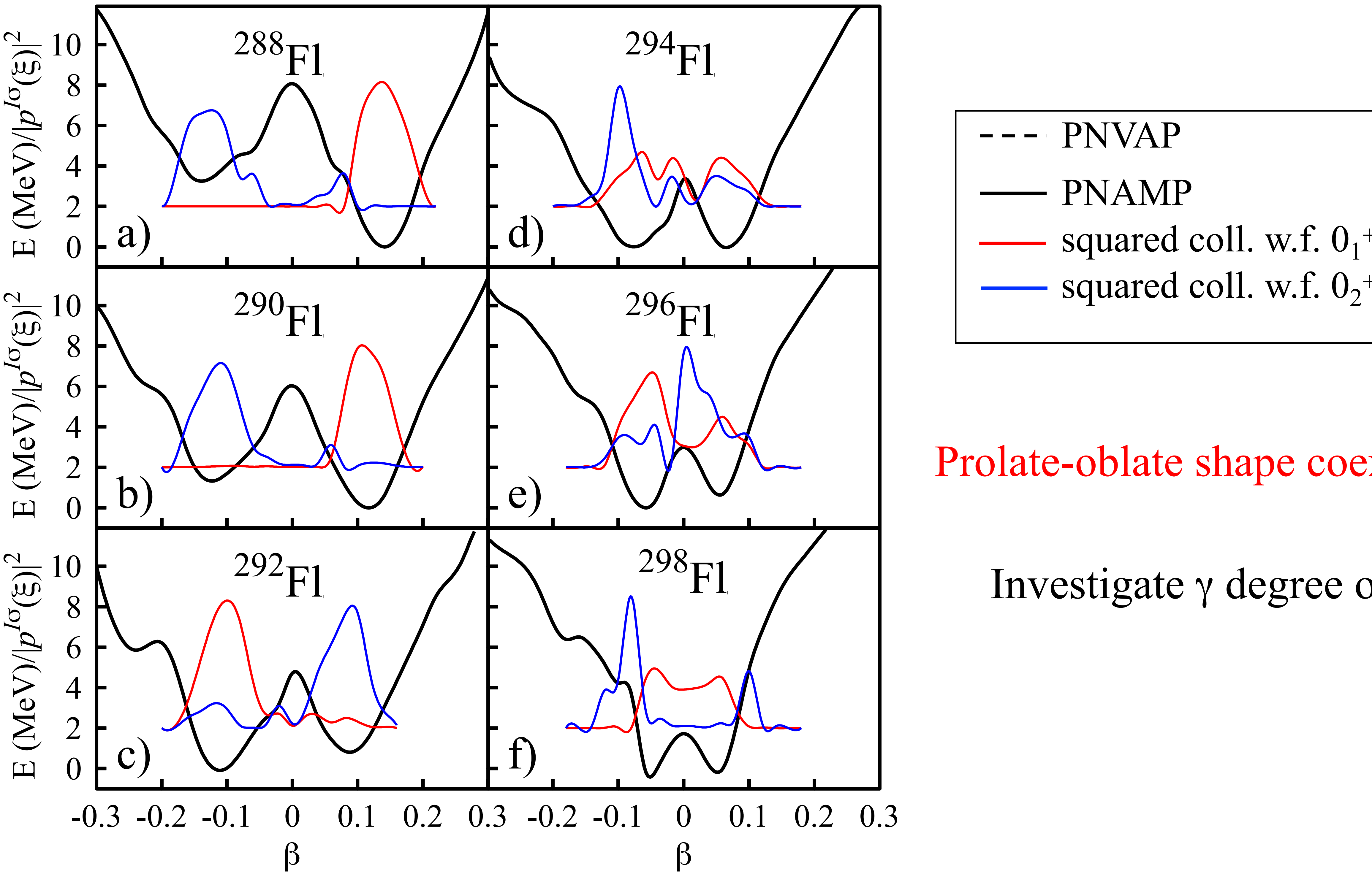
Example: Flerovium chain (Z=114)



Prolate-oblate shape coexistence ?

Investigate γ degree of freedom !

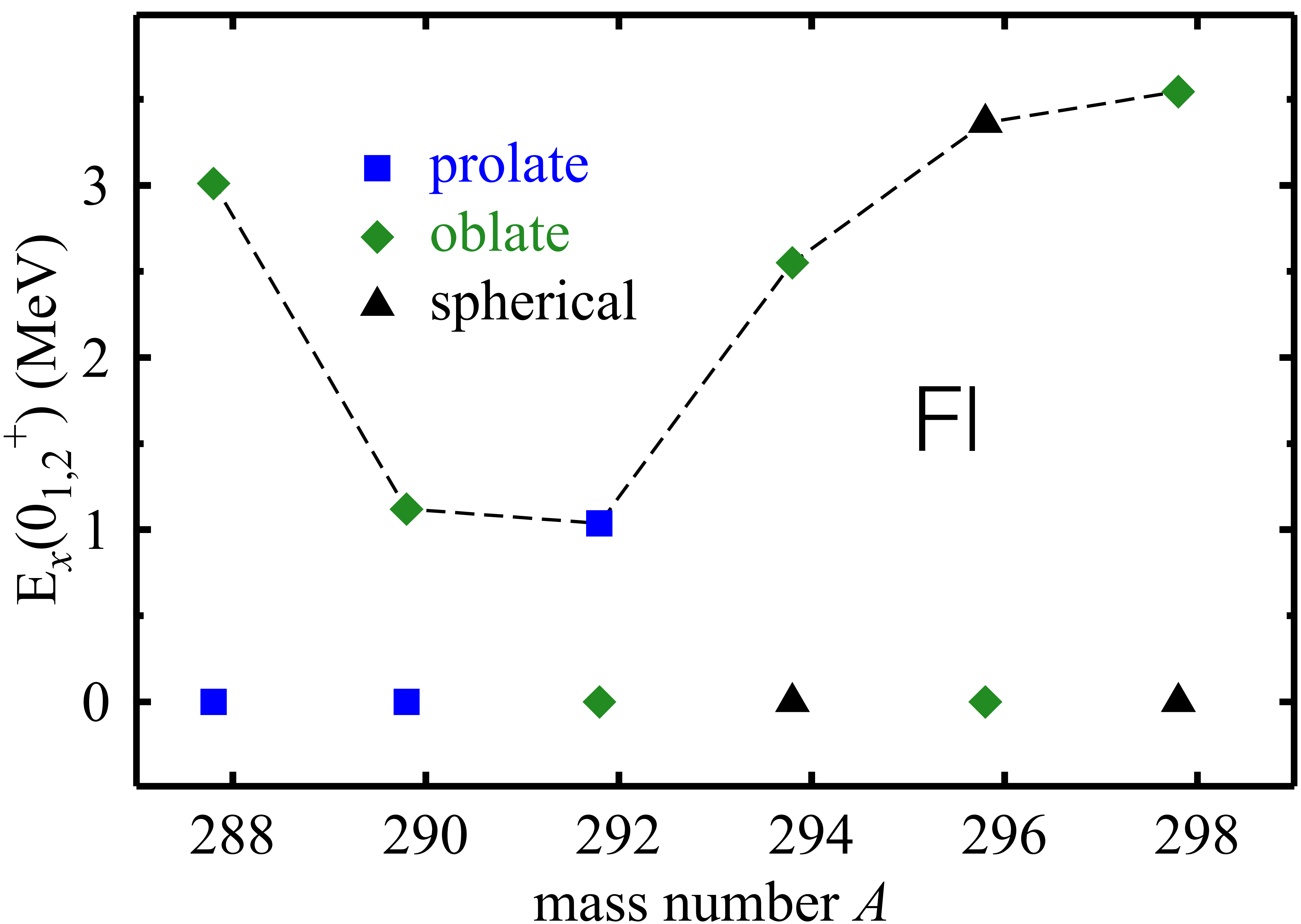
Example: Flerovium chain (Z=114)



Prolate-oblate shape coexistence ?

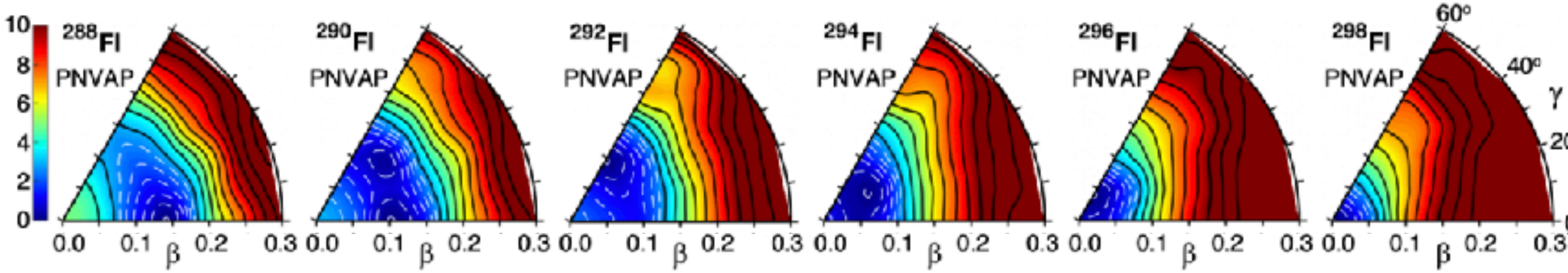
Investigate γ degree of freedom !

Excitation energies of the 0^+_2 states and their shapes (AXIAL)



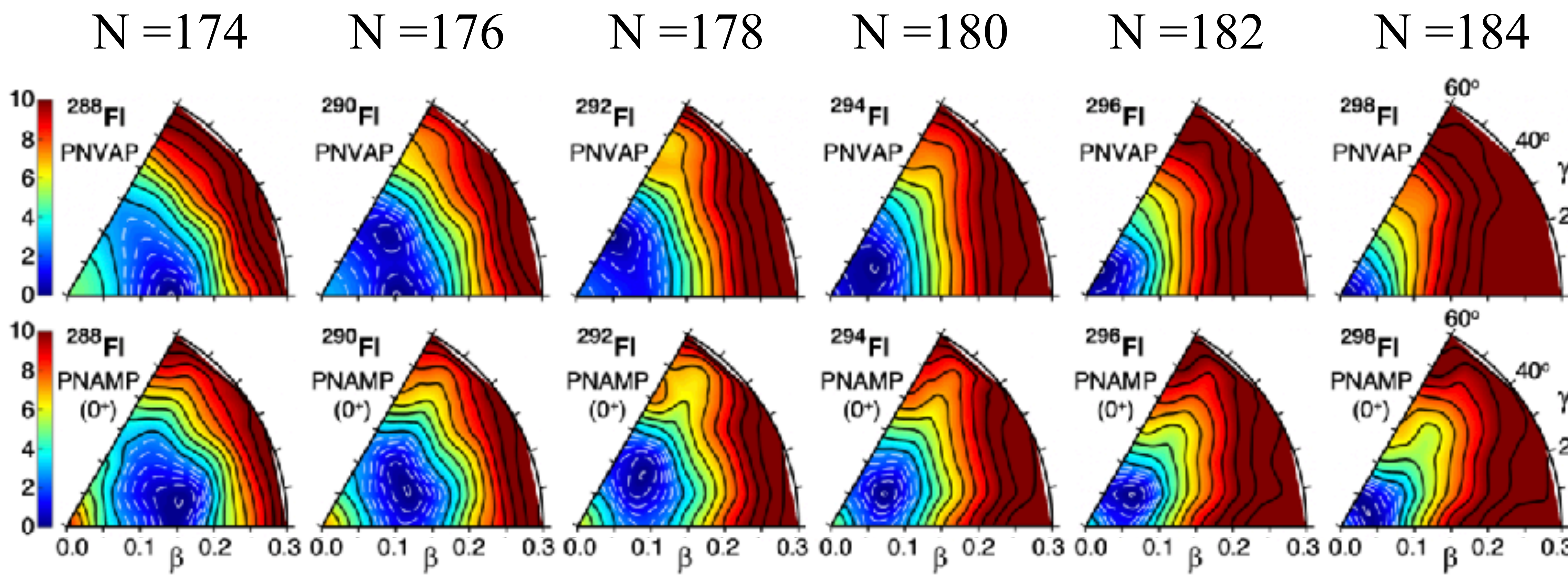
PNVAP

N =174 N =176 N =178 N =180 N =182 N =184



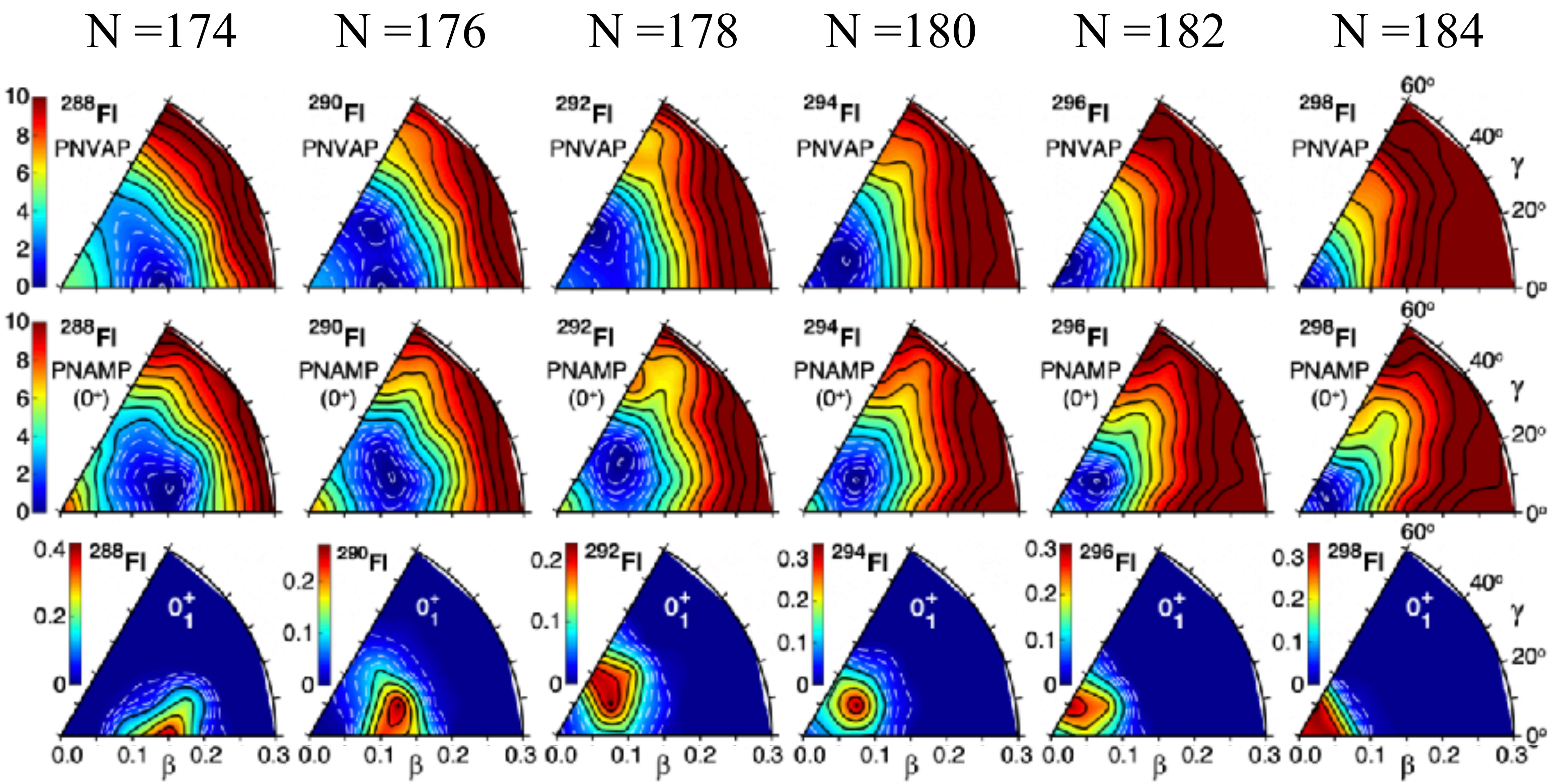
J.L. Egido and A. Jungclaus,
Phys. Rev. Lett. 125, 192504 (2020)

PNAMP



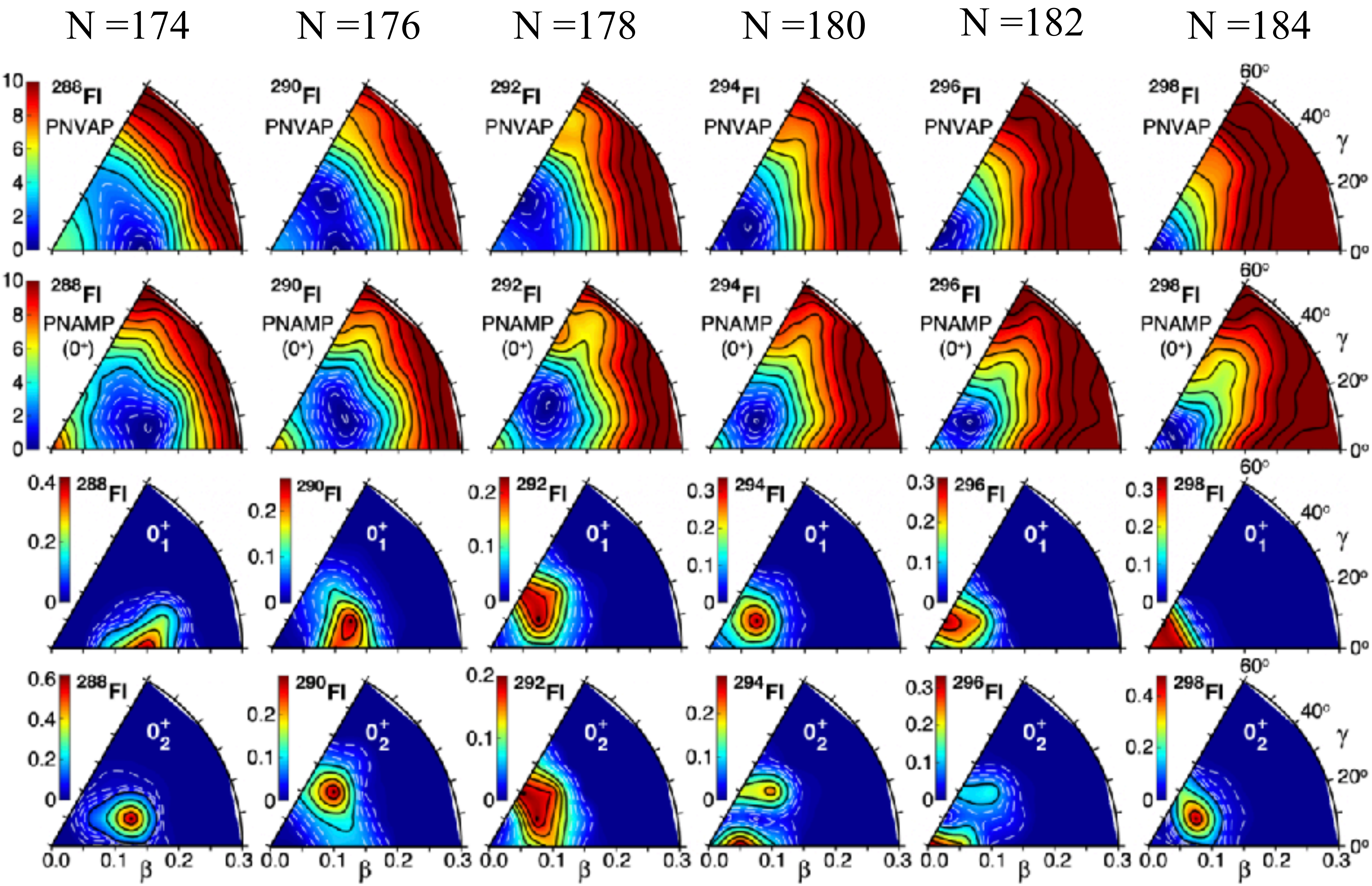
J.L. Egido and A. Jungclaus,
Phys. Rev. Lett. 125, 192504 (2020)

Collective wave functions



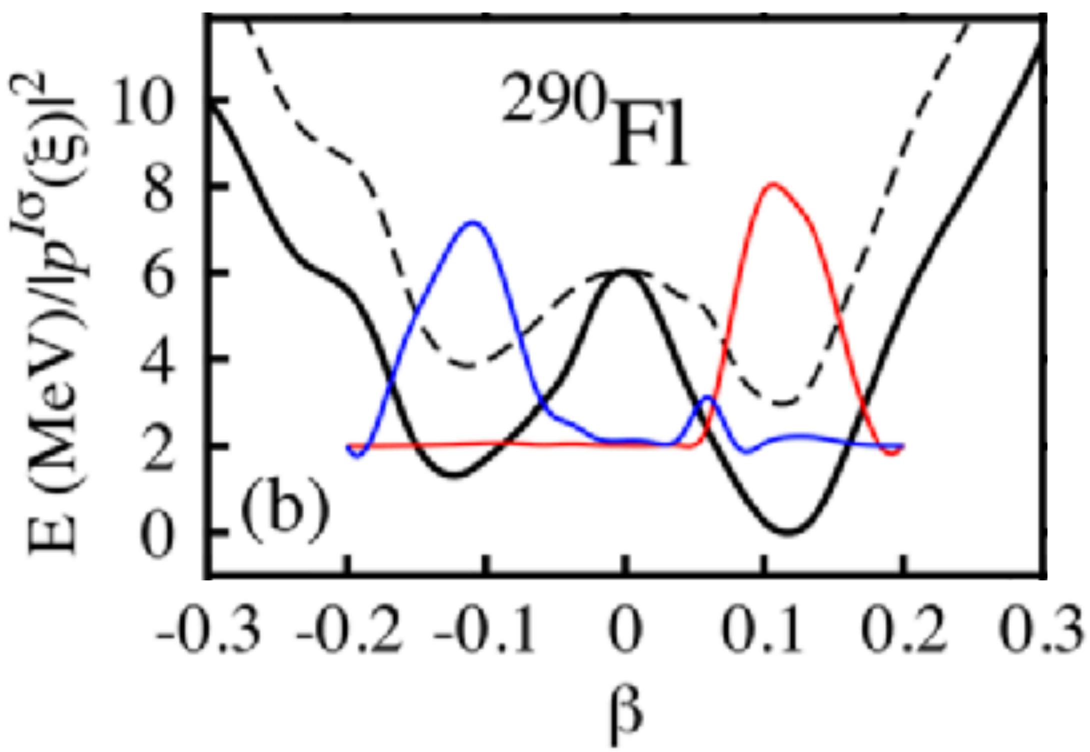
J.L. Egido and A. Jungclaus,
Phys. Rev. Lett. 125, 192504 (2020)

Collective wave functions

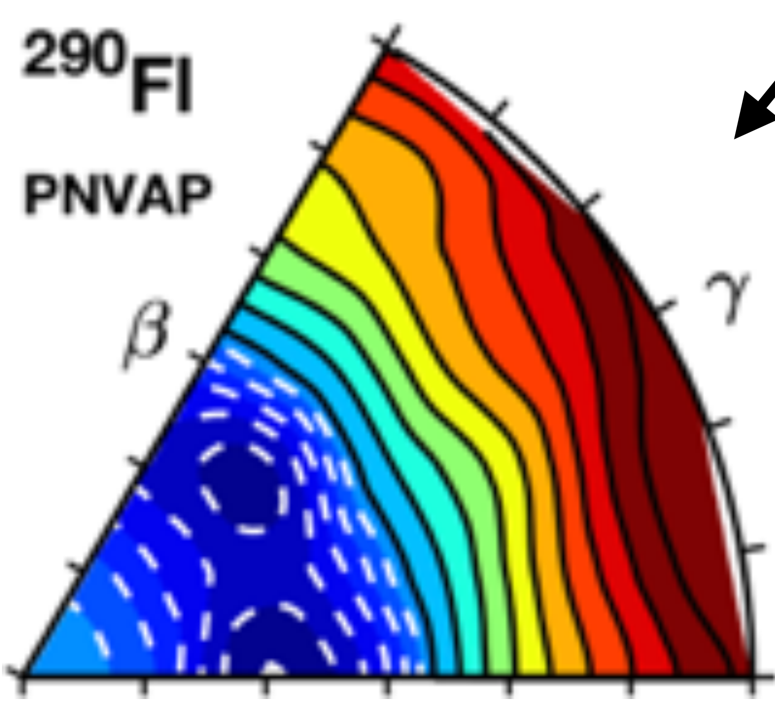


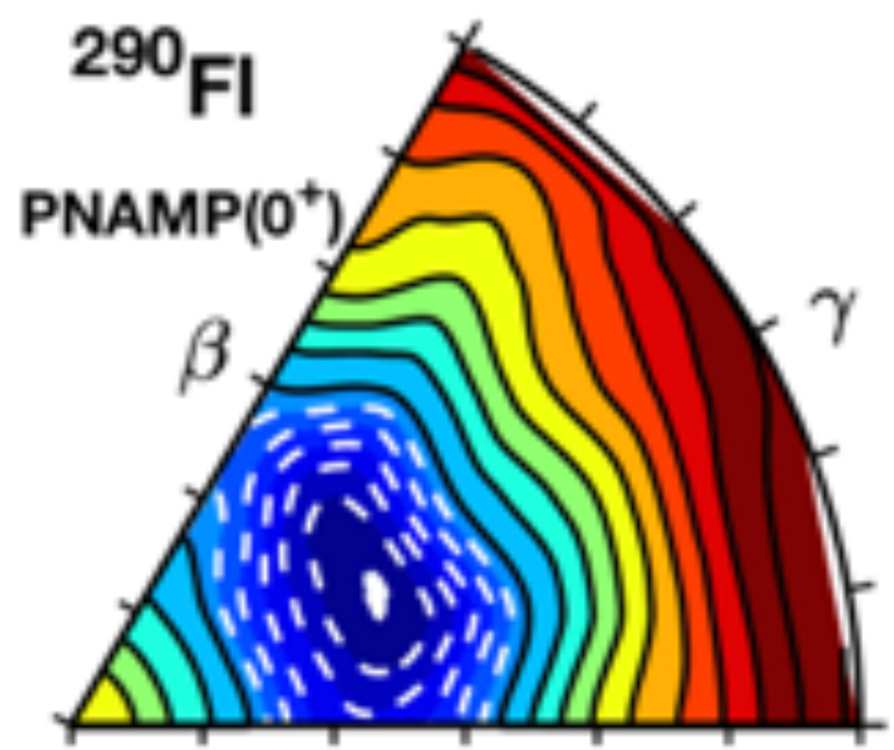
J.L. Egido and A. Jungclaus, Phys. Rev. Lett. 125, 192504 (2020)

From prolate-oblate to triaxial-triaxial shape coexistence

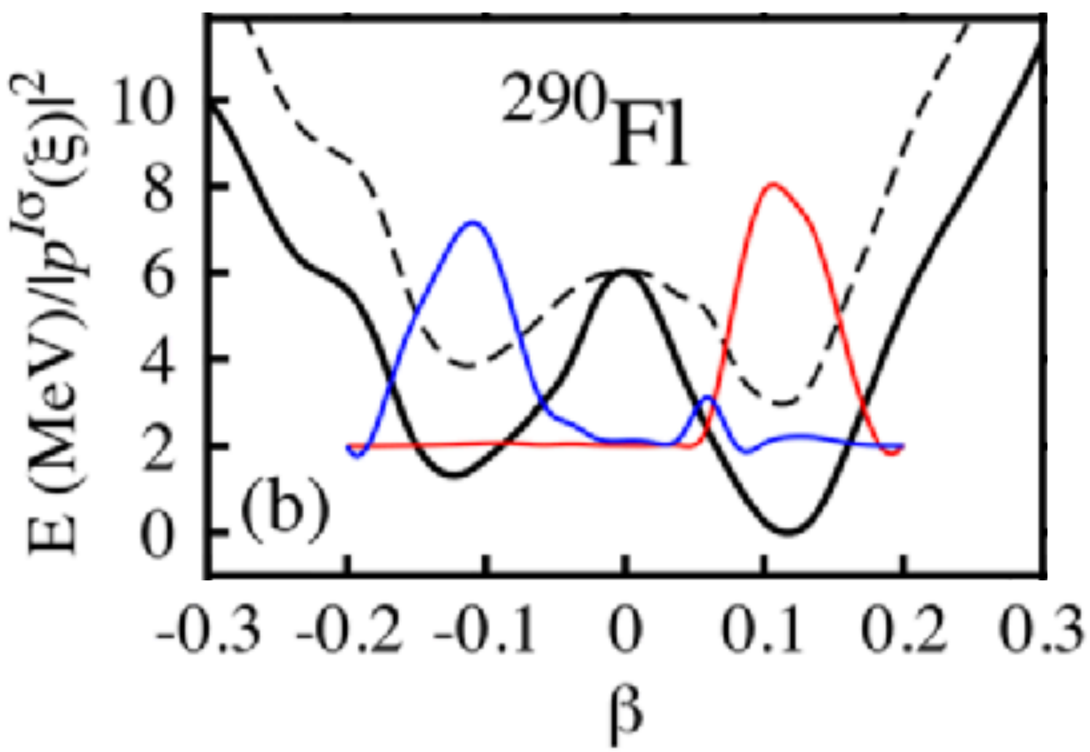


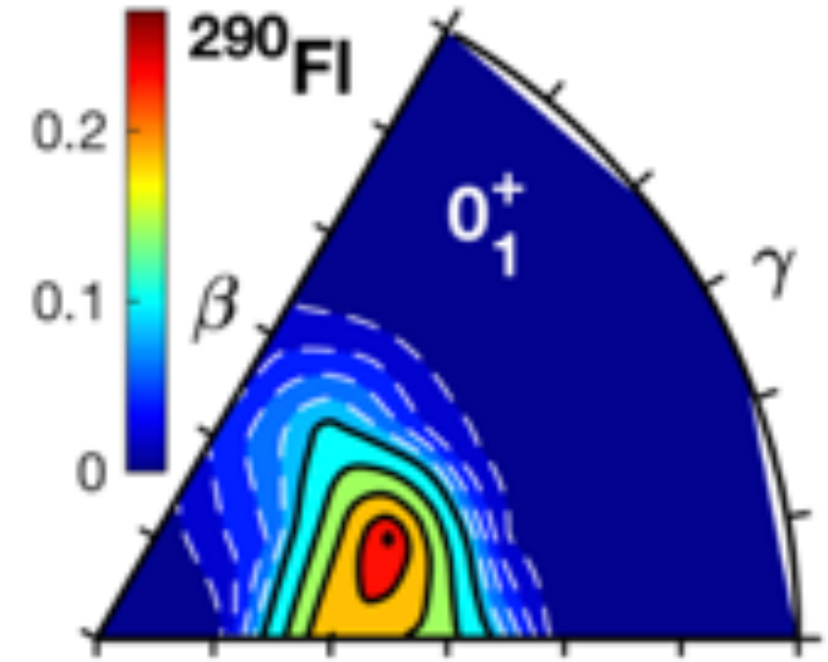
Axial-symmetric minima are saddle points in the β-γ plane !

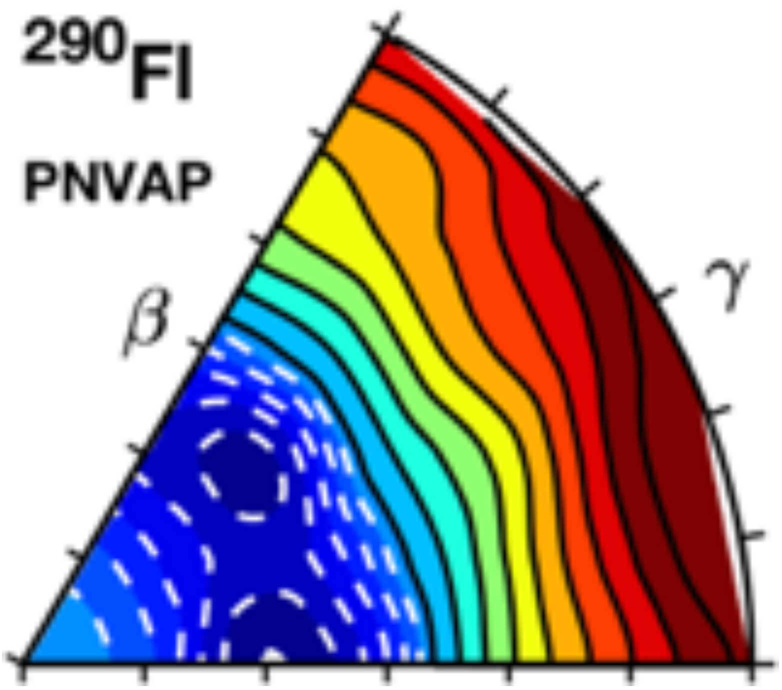


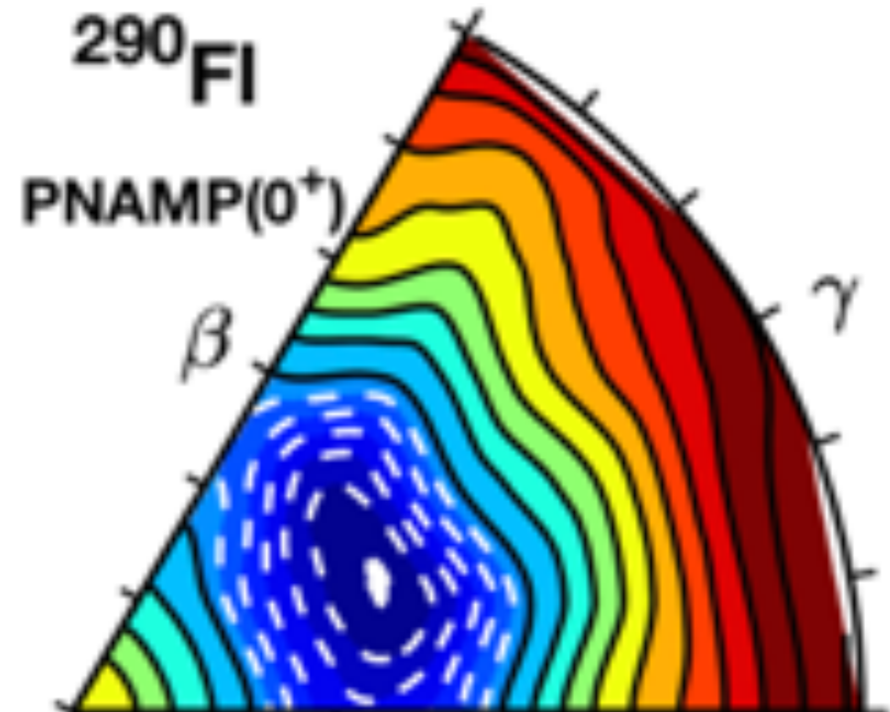


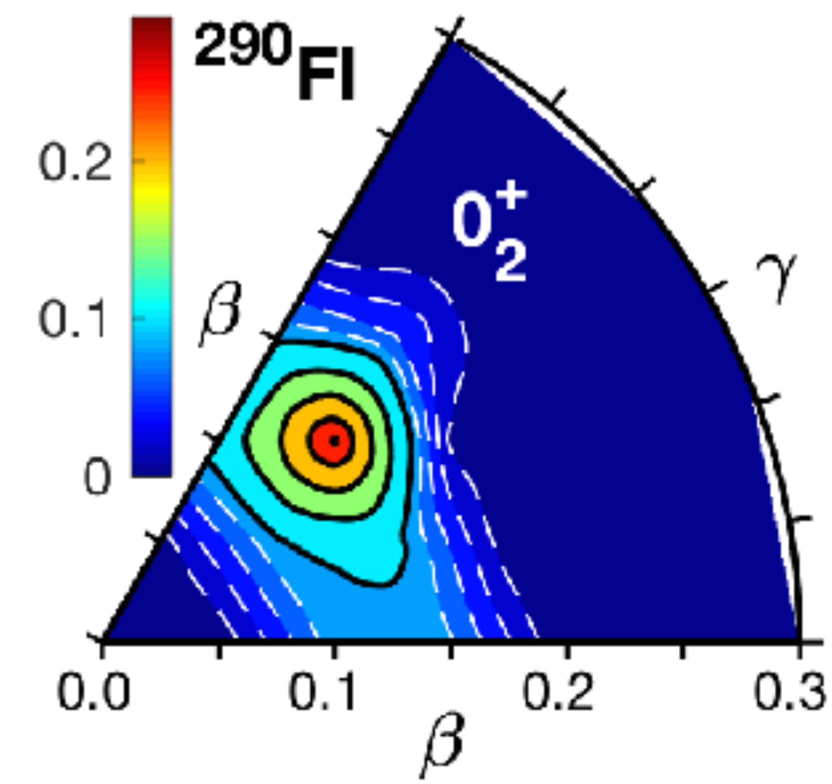
From prolate-oblate to triaxial-triaxial shape coexistence



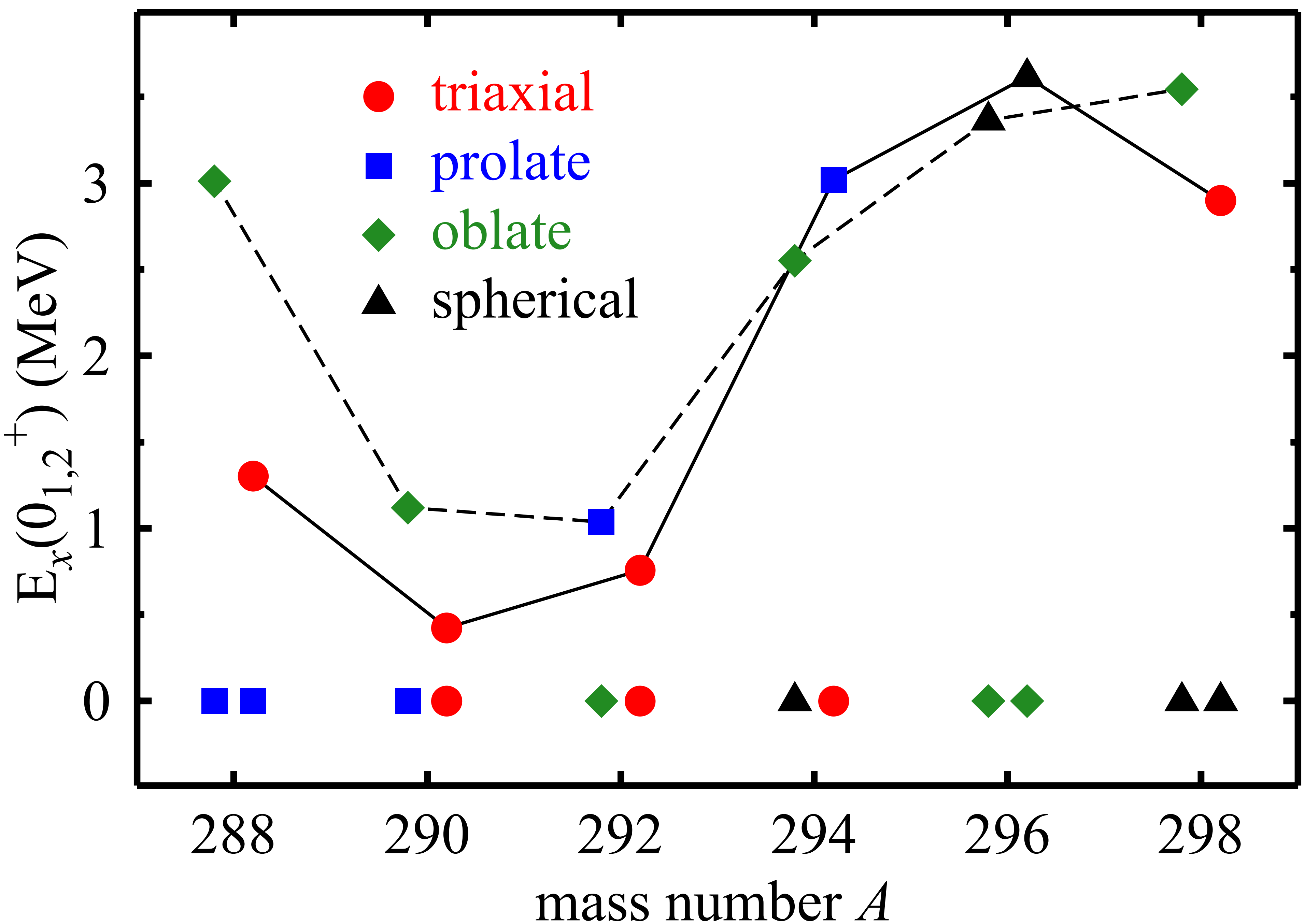




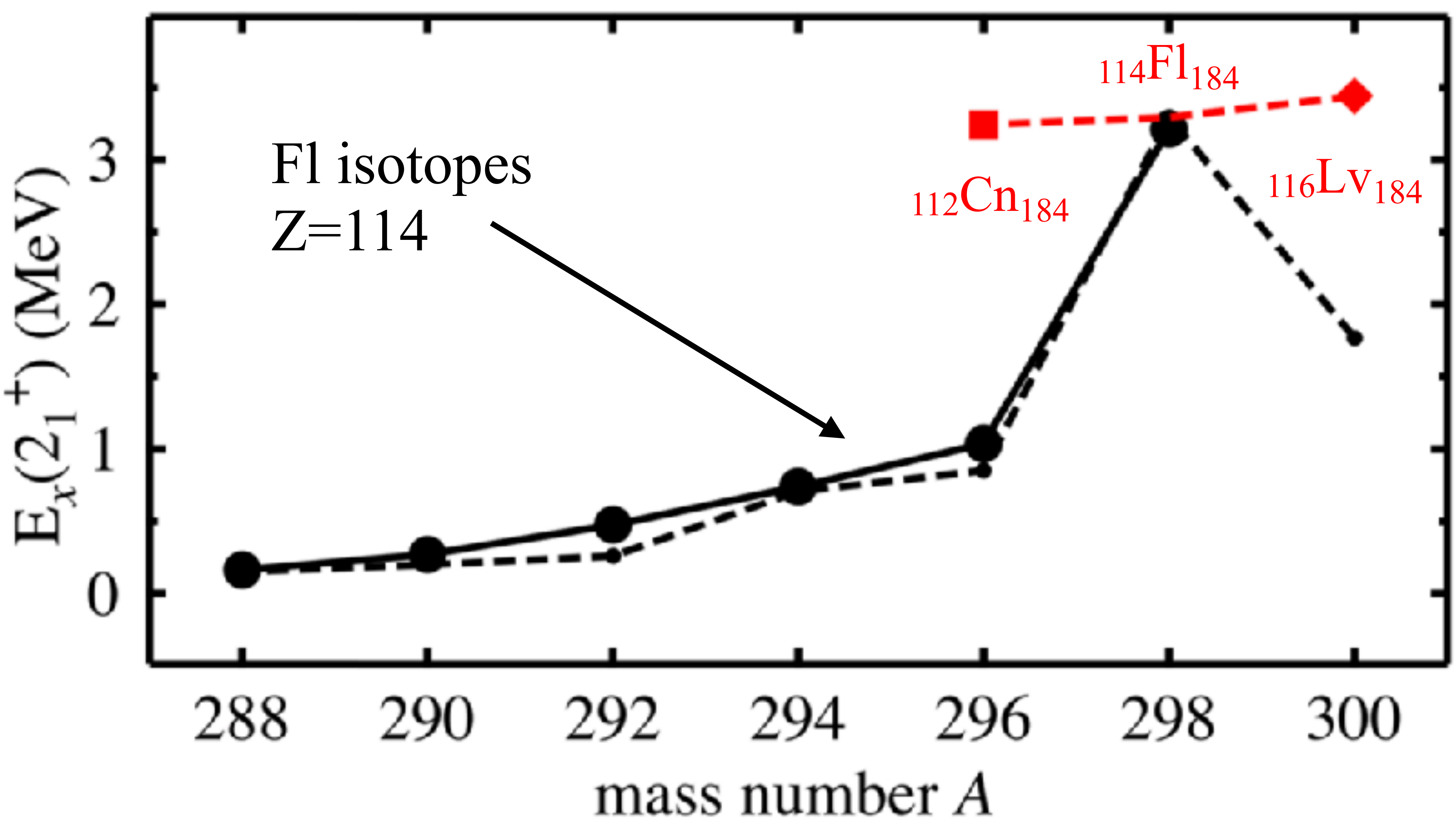




Excitation energies of the 0^+_2 states and their shapes (AXIAL & TRIAXIAL)

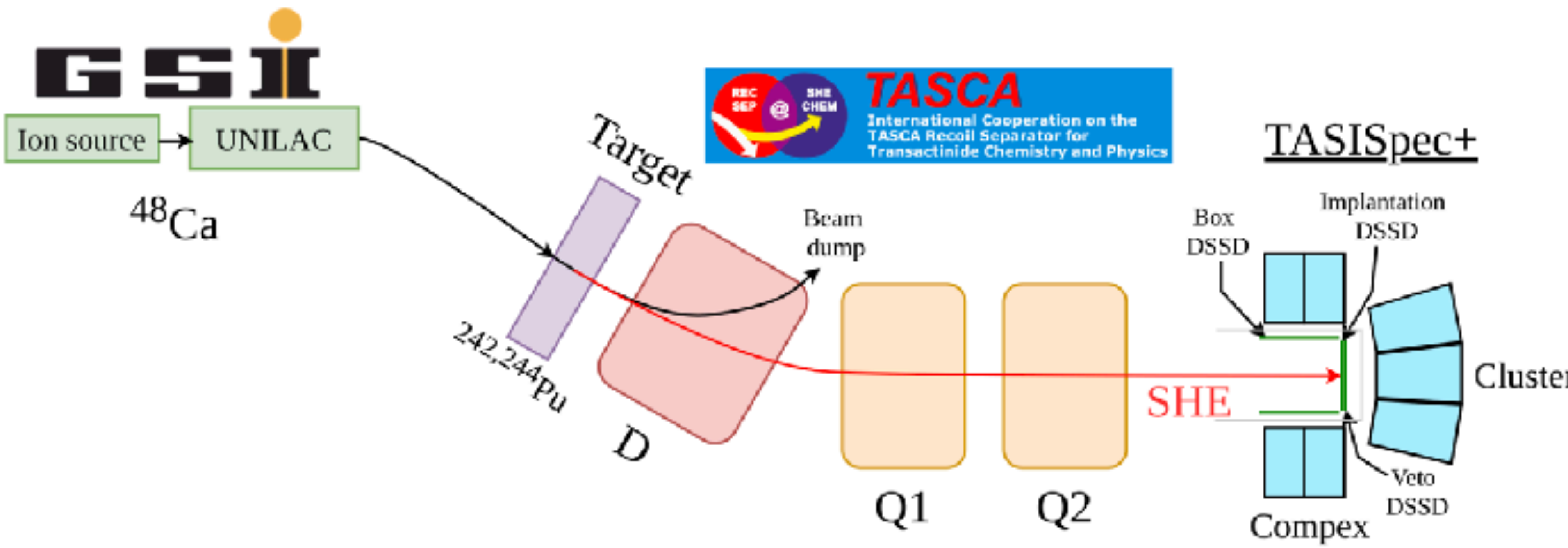


2_1^+ excitation energy as indicator for shell gaps



With the Gogny force also the triaxial calculations predict a shell closure at N=184, but not at Z=114 !

Low-Energy excited states in the alpha decay chains of ^{292}Lv & ^{294}Og



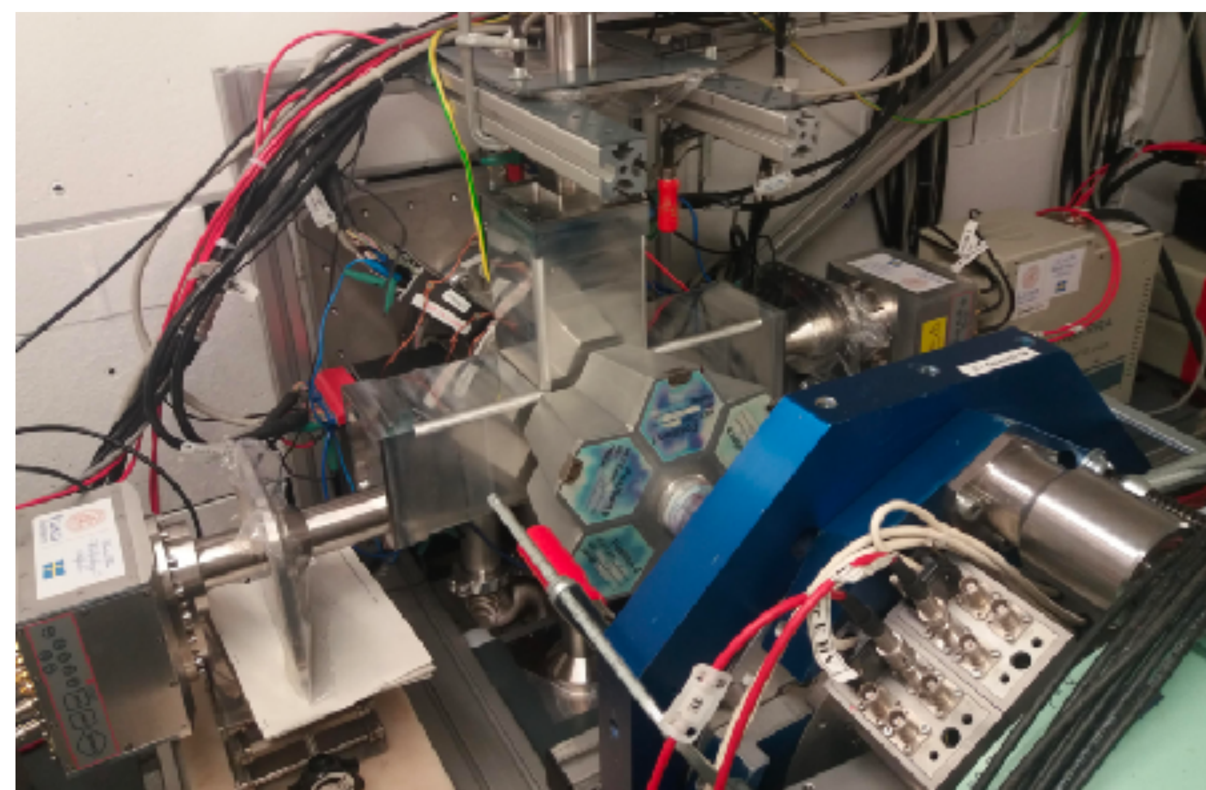
PHYSICAL REVIEW LETTERS **126**, 032503 (2021)

Editors' Suggestion | Featured in Physics

Spectroscopy along Flerovium Decay Chains: Discovery of ^{280}Ds and an Excited State in ^{282}Cn

A. Såmark-Roth,[1,*] D. M. Cox,[1] D. Rudolph,[1] L. G. Sarmiento,[1] B. G. Carlsson,[1] J. L. Egido,[2] P. Golubev,[1] J. Heery,[3] A. Yakushev,[4] S. Åberg,[1] H. M. Albers,[4] M. Albertsson,[1] M. Block,[4,5,6] H. Brand,[4] T. Calverley,[3] R. Cantemir,[4] R. M. Clark,[7] Ch. E. Düllmann,[4,5,6] J. Eberth,[8] C. Fahlander,[1] U. Forsberg,[1] J. M. Gates,[7] F. Giacoppo,[4,5] M. Götz,[4,5,6] S. Götz,[4,5,6] R.-D. Herzberg,[3] Y. Hrabar,[1] E. Jäger,[4] D. Judson,[3] J. Khuyagbaatar,[4,5] B. Kindler,[4] I. Kojouharov,[4] J. V. Kratz,[6] J. Krier,[4] N. Kurz,[4] L. Lens,[4,6,†] J. Ljungberg,[1] B. Lommel,[4] J. Louko,[9] C.-C. Meyer,[5,6] A. Mistry,[10,4] C. Mokry,[5,6] P. Papadakis,[3,‡] E. Parr,[4] J. L. Pore,[7] I. Ragnarsson,[1] J. Runke,[4,6] M. Schädel,[4] H. Schaffner,[4] B. Schausten,[4] D. A. Shaughnessy,[11] P. Thörle-Pospiech,[5,6] N. Trautmann,[6] and J. Uusitalo[9]

[1]*Department of Physics, Lund University, 22100 Lund, Sweden*
[2]*Departamento de Física Teórica and CIAFF, Universidad Autónoma de Madrid, 28049 Madrid, Spain*
[3]*Department of Physics, University of Liverpool, Liverpool L69 7ZE, United Kingdom*
[4]*GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany*
[5]*Helmholtz Institute Mainz, 55099 Mainz, Germany*
[6]*Department Chemie–Standort TRIGA, Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany*
[7]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[8]*Institut für Kernphysik, Universität zu Köln, 50937 Köln, Germany*
[9]*Department of Physics, University of Jyväskylä, 40014 Jyväskylä, Finland*
[10]*Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany*
[11]*Nuclear and Chemical Sciences Division, Lawrence Livermore National Laboratory, Livermore, California 94550, USA*



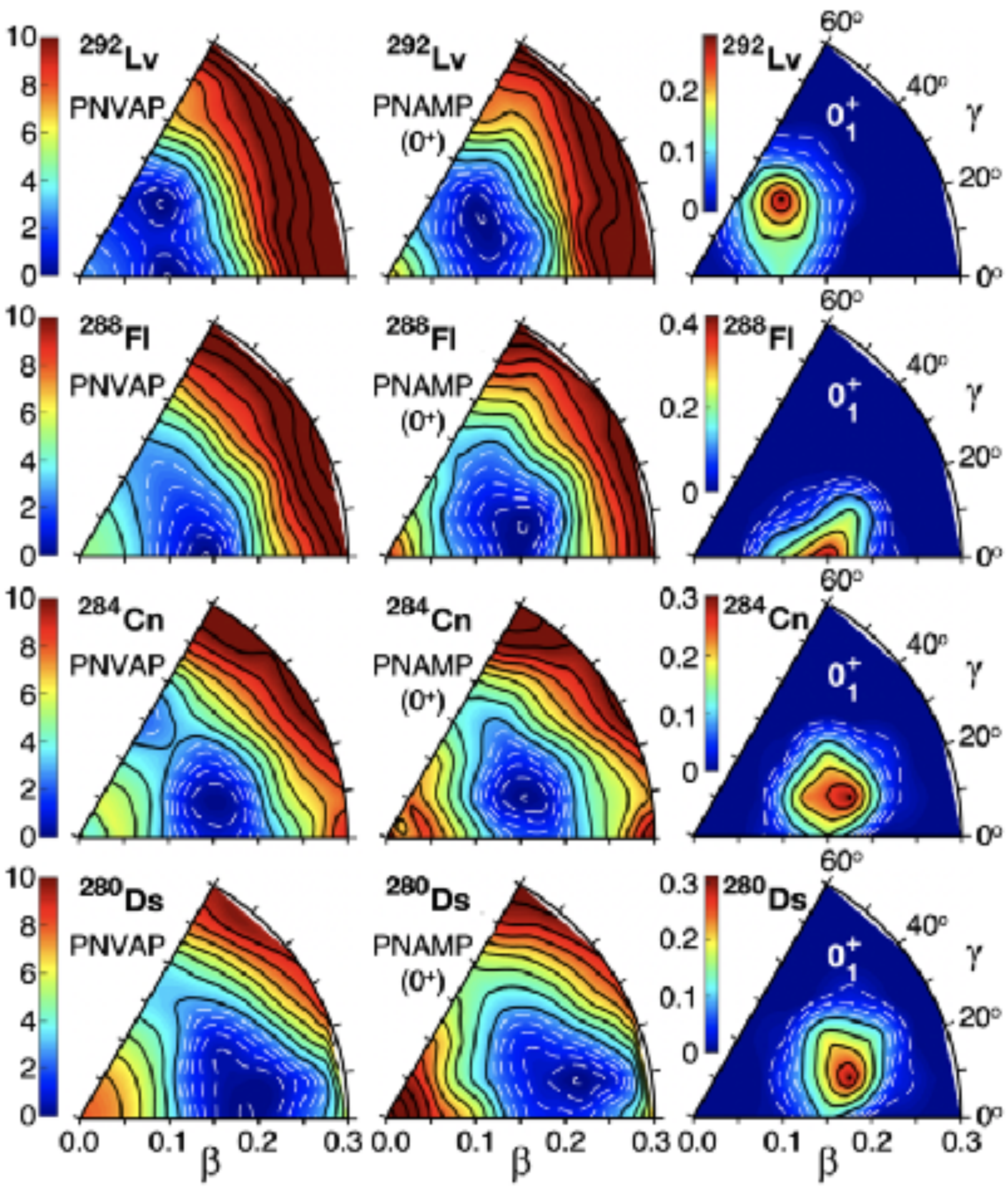
Part II

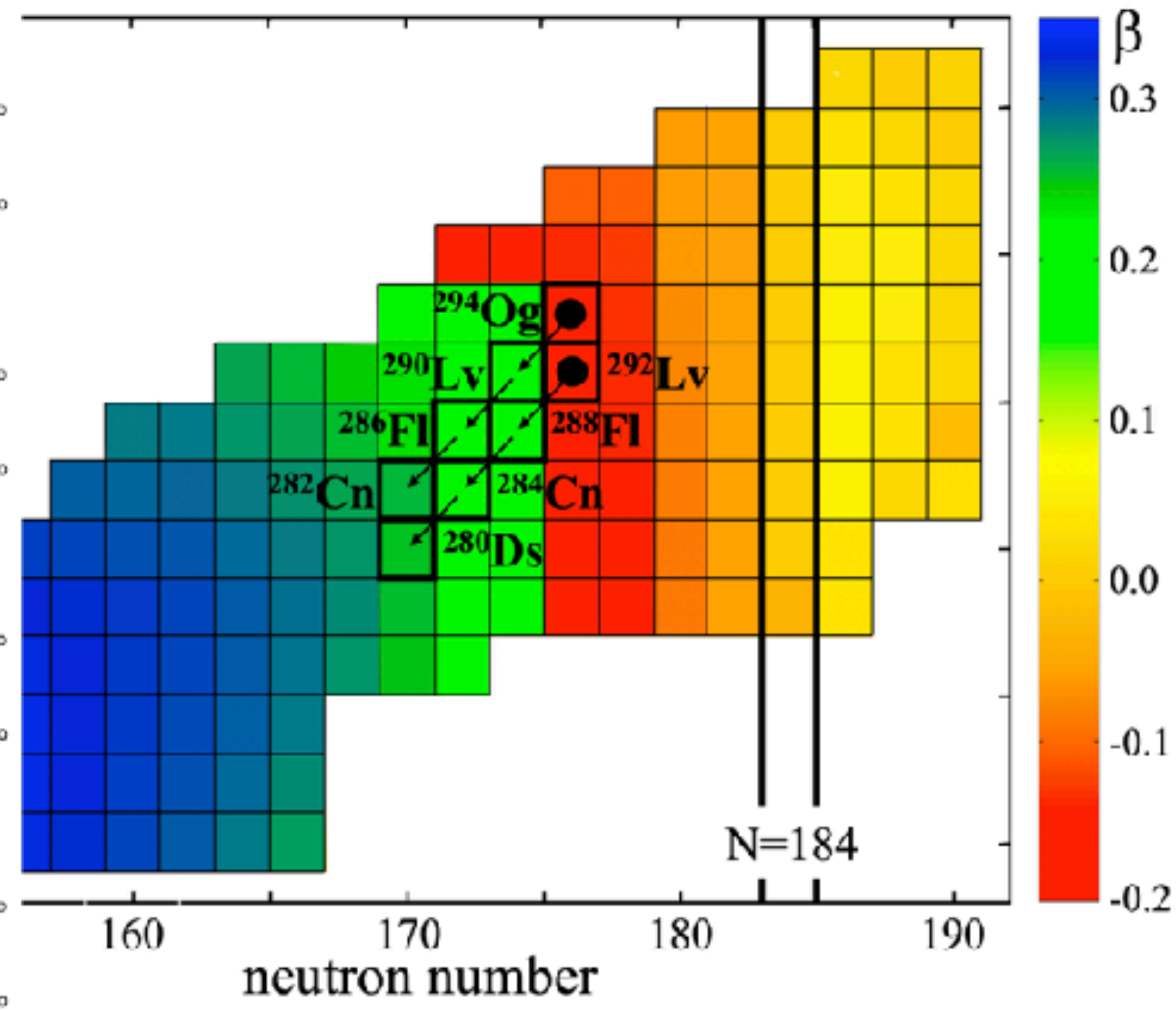
PHYSICAL REVIEW LETTERS **126,** 192501 (2021)

Low-Energy Nuclear Excitations along the α-Decay Chains of Superheavy ^{292}Lv and ^{294}Og

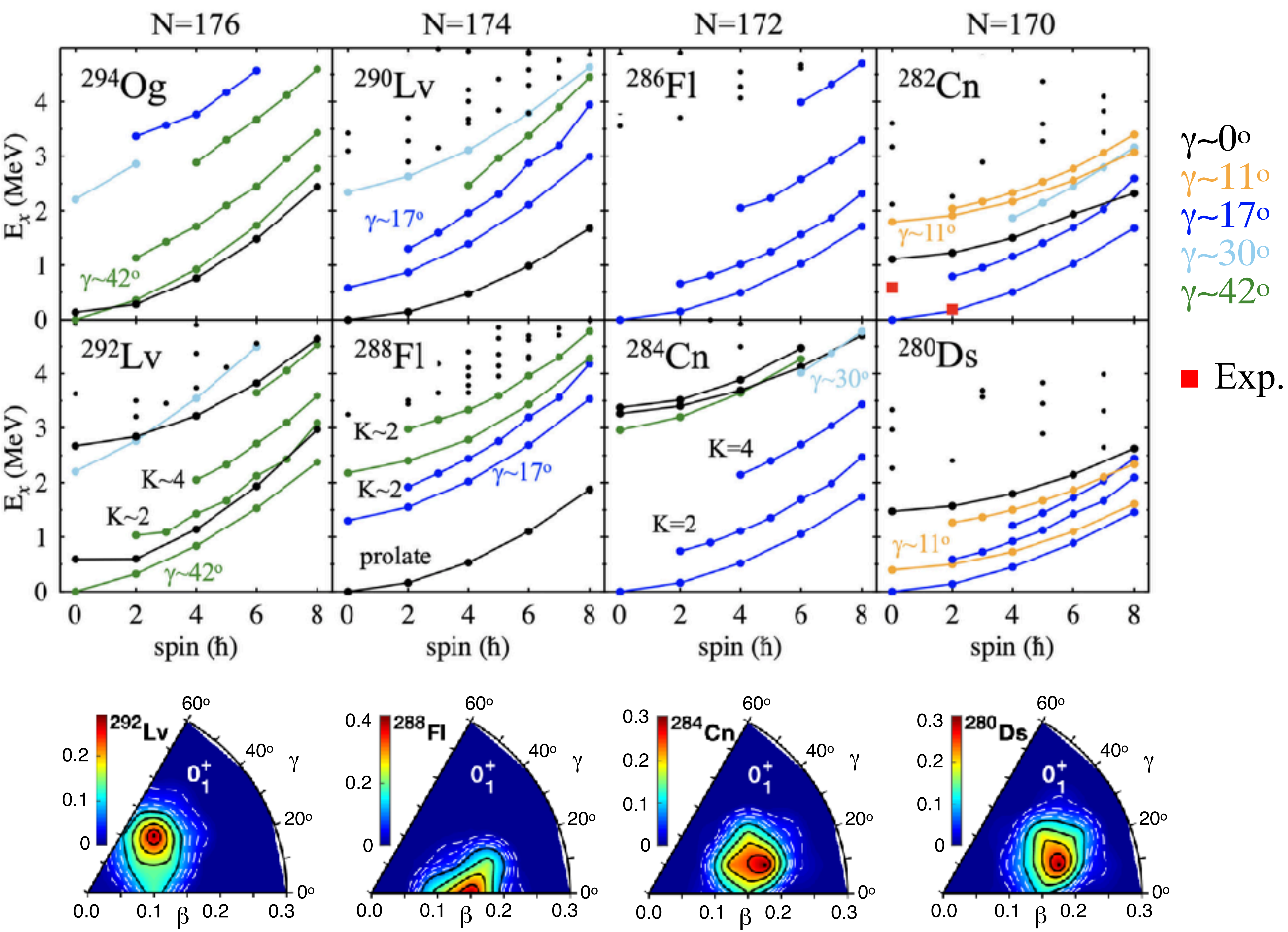
J. Luis Egido[1,*] and Andrea Jungclaus[2,†]
[1]*Departamento de Física Teórica and CIAFF, Universidad Autónoma de Madrid, E-28049 Madrid, Spain*
[2]*Instituto de Estructura de la Materia, CSIC, E-28006 Madrid, Spain*

(Received 8 February 2021; revised 5 April 2021; accepted 15 April 2021; published 10 May 2021)

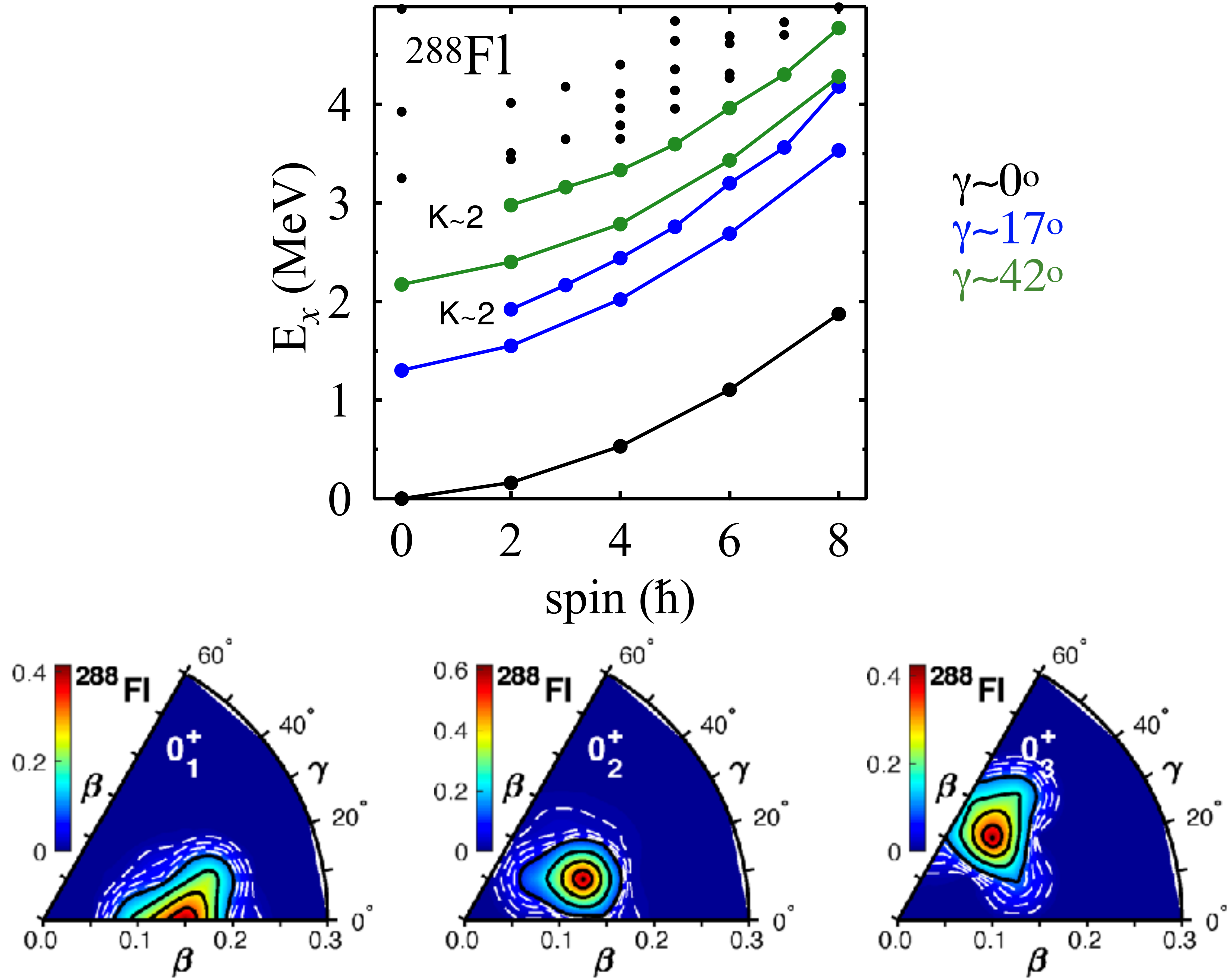




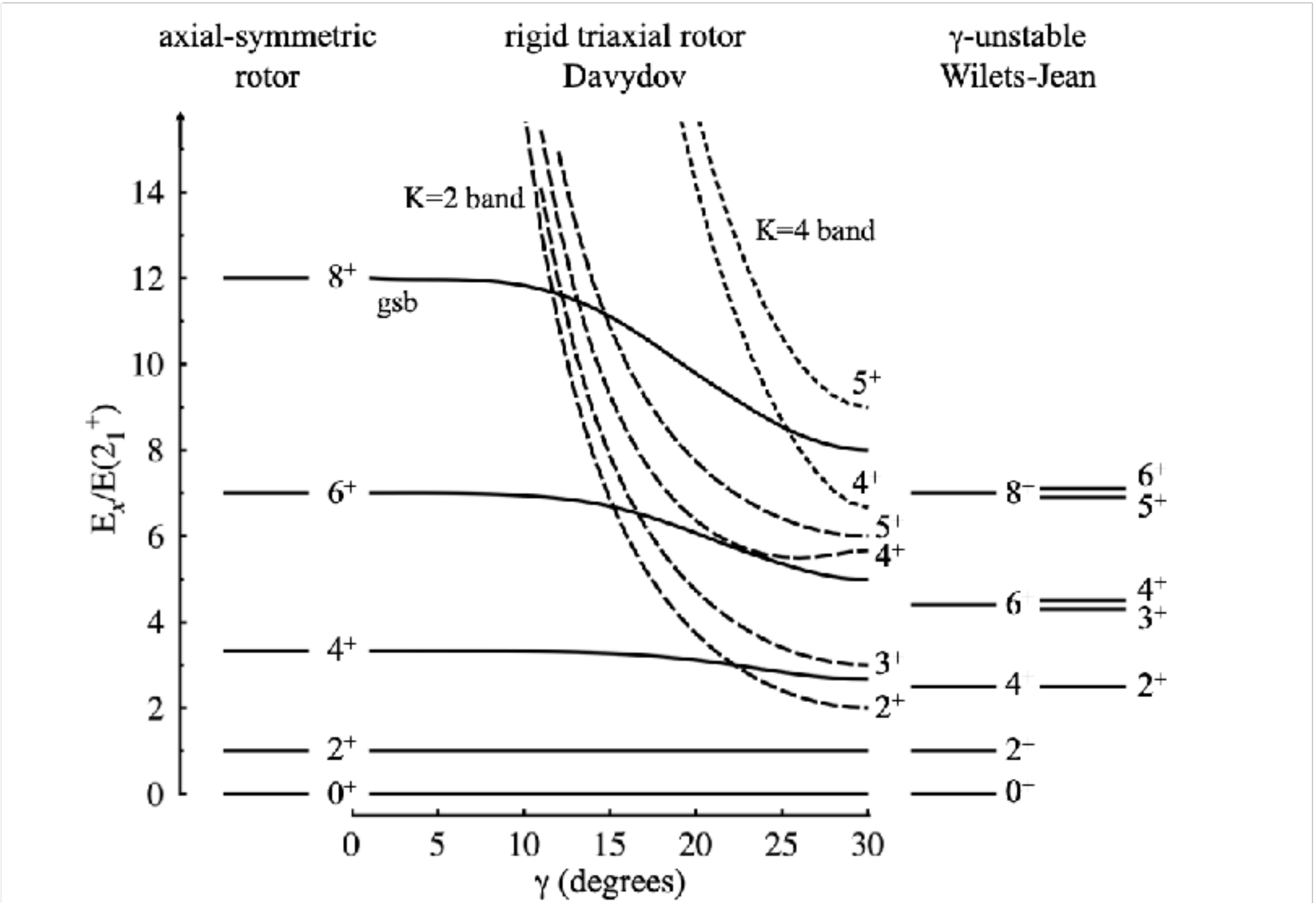
Band structures along the ^{292}Lv and ^{294}Og decay chains

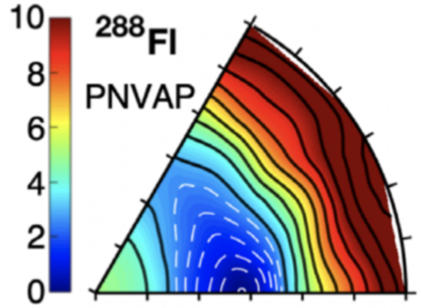


Excited bands in ^{288}Fl

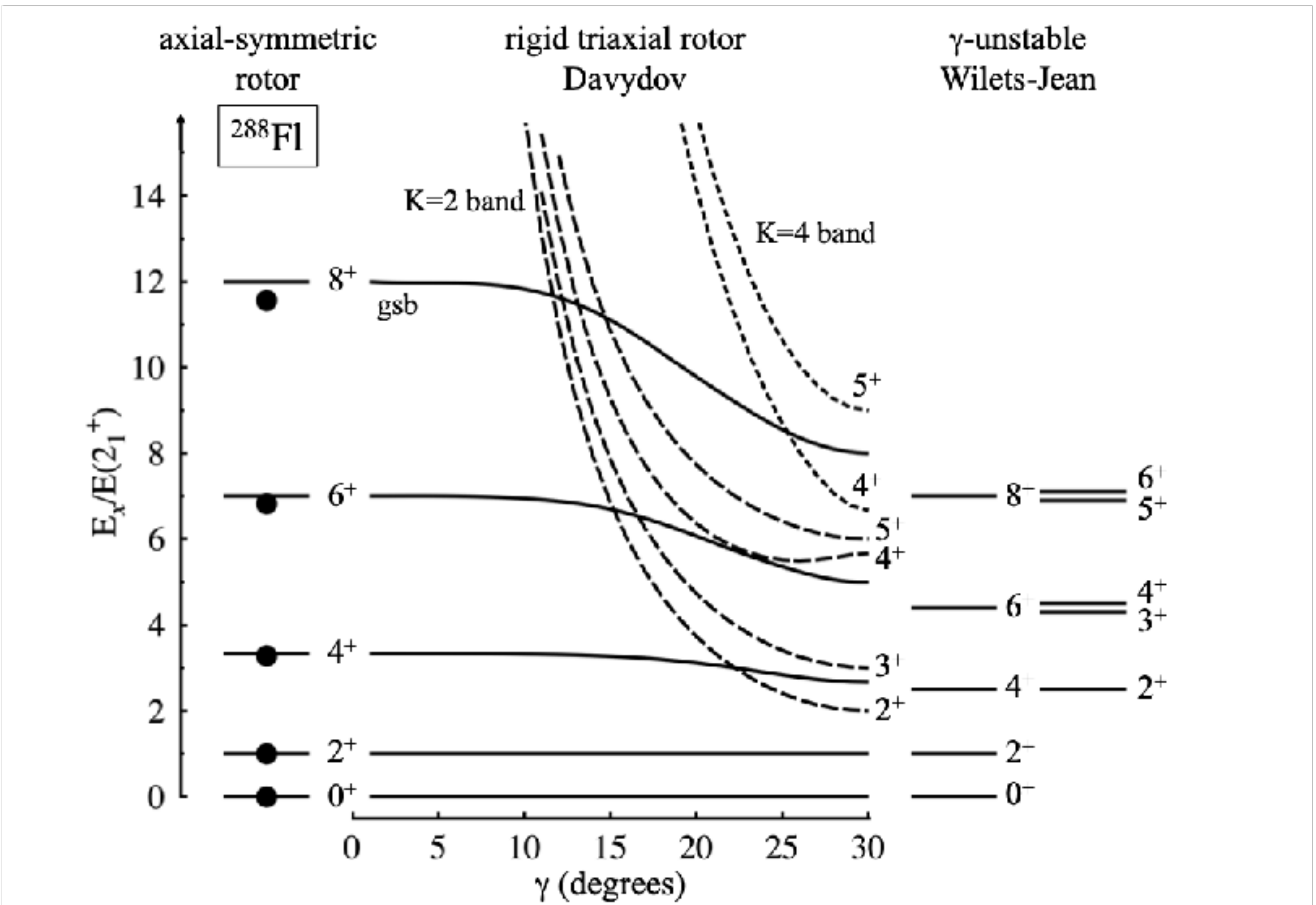


γ~0°
γ~17°
γ~42°

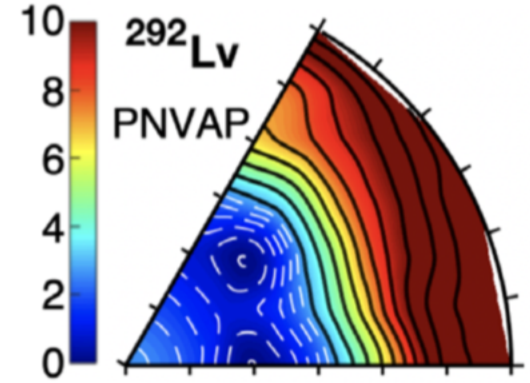

axial-symmetric rotor
rigid triaxial rotor Davydov
γ-unstable Wilets-Jean
K=2 band
K=4 band
gsb
$E_x/E(2_1^+)$
γ (degrees)
8+
6+
4+
2+
0+
5+
4+
5+
4+
3+
2+
8−
6+
4+
2−
0−
6+
5+
4+
3+
2+



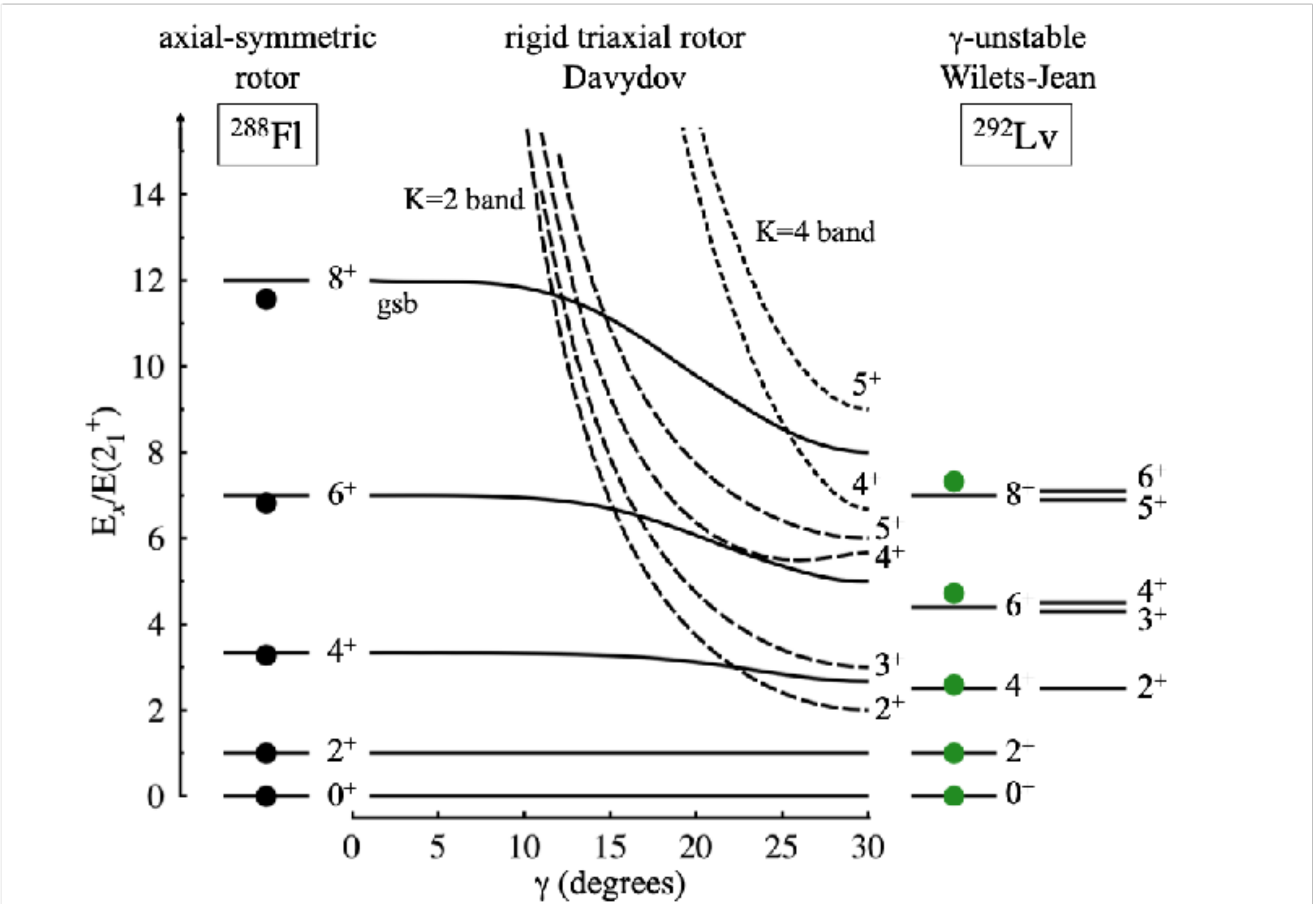
10
8
6
4
2
0
288Fl
PNVAP



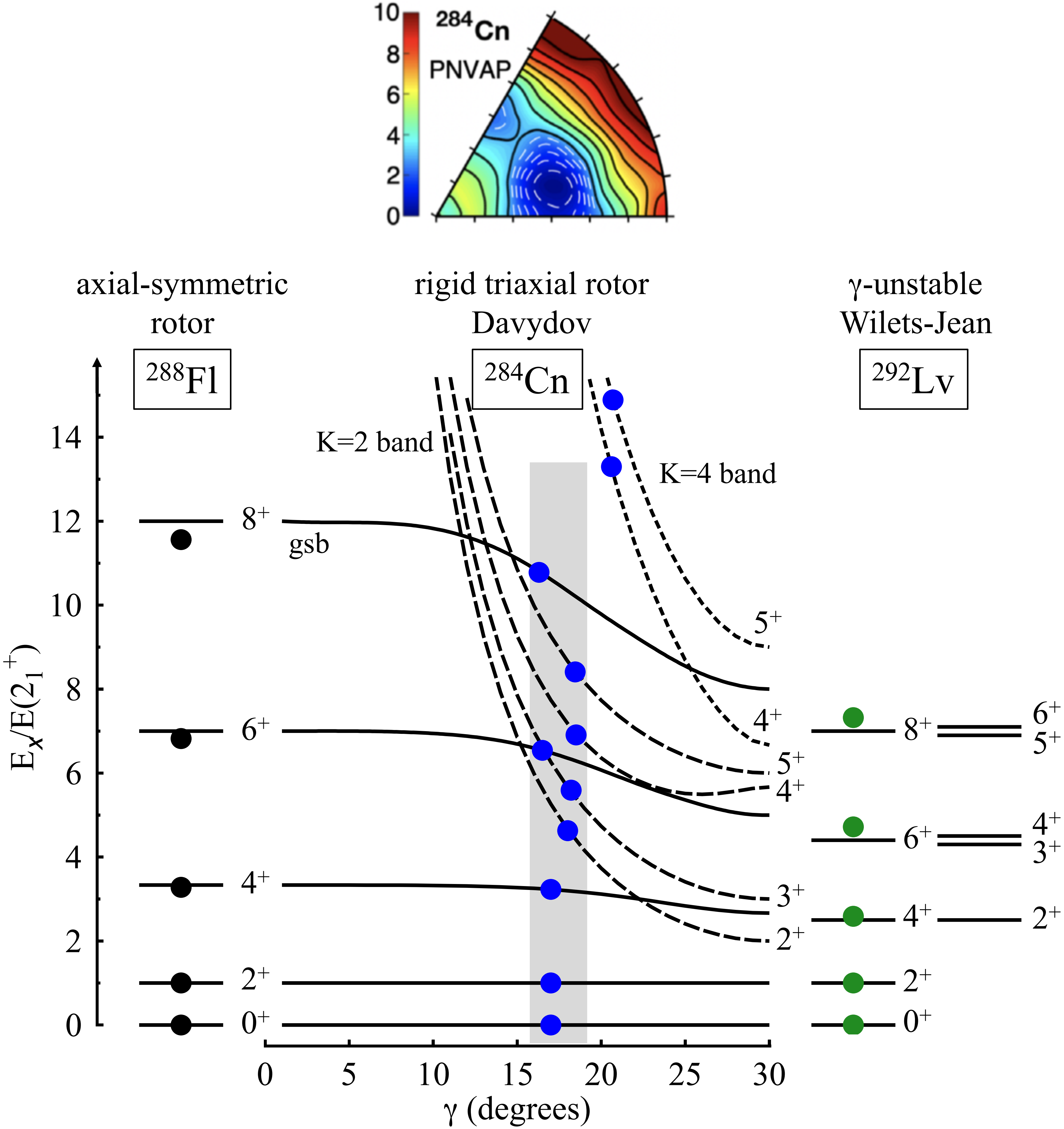
axial-symmetric rotor
rigid triaxial rotor Davydov
γ-unstable Wilets-Jean
288Fl
K=2 band
K=4 band
gsb
E_x/E(2_1^+)
γ (degrees)
0
5
10
15
20
25
30
0
2
4
6
8
10
12
14
8+
6+
4+
2+
0+
5+
4+
3+
2+
8−
6+
4+
2−
0−
6+
5+
4+
3+
2+



292Lv
PNVAP
10
8
6
4
2
0



axial-symmetric rotor
rigid triaxial rotor Davydov
γ-unstable Wilets-Jean
288Fl
292Lv
K=2 band
K=4 band
gsb
8+
6+
4+
2+
0+
5+
4+
5+
4+
3+
2+
8−
6+
4+
2−
0−
6+
5+
4+
3+
2+
E_x/E(2_1^+)
γ (degrees)
0
5
10
15
20
25
30
0
2
4
6
8
10
12
14



284Cn
PNVAP
10
8
6
4
2
0
axial-symmetric rotor
288Fl
rigid triaxial rotor Davydov
284Cn
γ-unstable Wilets-Jean
292Lv
K=2 band
K=4 band
gsb
8+
6+
4+
2+
0+
5+
4+
3+
2+
14
12
10
8
6
4
2
0
Ex/E(2_1+)
0
5
10
15
20
25
30
γ (degrees)

Conclusions

- State-of-the-art symmetry conserving configuration mixing calculations provide a rich variety of nuclear shapes in SHN. We predict six different ground state deformations for the six Flerovium isotopes studied at variance with axial calculations.

- We predict a new shape coexistence in ^{290}Fl. Two 0^+ triaxial states are predicted to coexist within less than 500 keV.

- We have calculated the first excited states for the decay chains of 292Lv and 294Og. The predicted values are in agreement with the experimental available values

- The comparison with the classical collective models shows the richness of shapes of the SHN.