Monopole transition as a probe for nuclear shape and clusters

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Collaborators: Y. Taniguchi, Y. Kanada-Enyo, Y. Chiba, Y. Suzuki, W. Horiuchi

Topic #1 : Monopole transition as a probe for nuclear shape and its fluctuation

Topic #2 : Monopole transition as a probe for nuclear clustering

monopole operator and monopole excitation

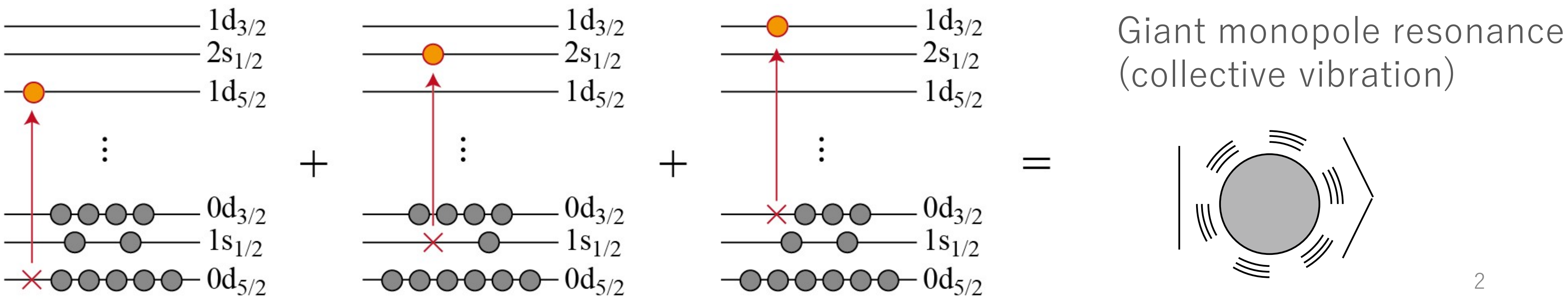
○ Electric/isoscalar monopole operators (definitions)

Electric : $$\mathcal{M}_{E0} = \sum_{i=1}^{A} (\boldsymbol{r}_i - \boldsymbol{r}_{\rm cm})^2 \frac{1+\tau_z}{2}$$

Isoscalar : $$\mathcal{M}_{IS0} = \sum_{i=1}^{A} (\boldsymbol{r}_i - \boldsymbol{r}_{\rm cm})^2$$

($\boldsymbol{r}_i$: single-particle coordinate
$\boldsymbol{r}_{\rm cm}$: center-of-mass coordinate)

- Squared protons/matter radius operators
- Induces $2\hbar\omega$ (1p1h) excitation without angular momentum transfer ($\Delta\ell = 0$)
- Coherent superposition gives rise to the Giant monopole resonance (GMR)

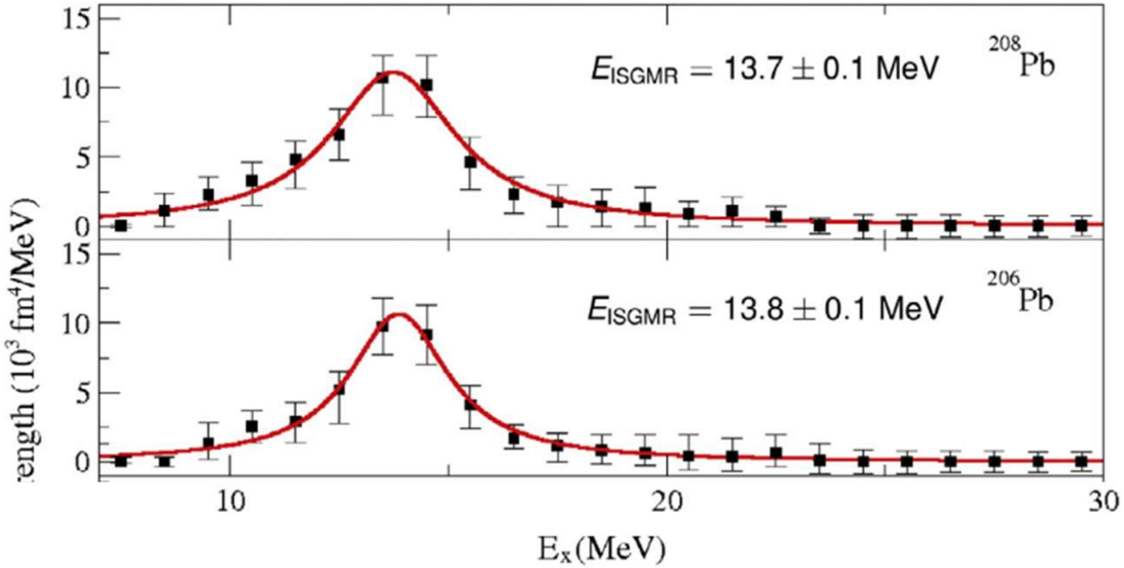


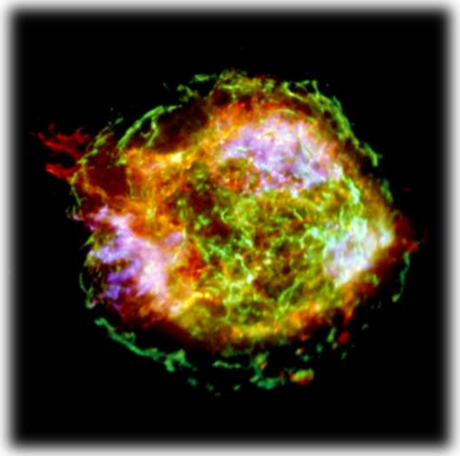
monopole transition and nuclear incompressibility

○ GMR as a probe for the nuclear incompressibility

- GMR is the vibration of nuclear density (breathing mode)

  ⇨ Energy of GMR should be closely related to the incompressibility of finite nucleus and infinite nuclear matter





Core collapse supernova

e.g.) J.P. Blaizot, Phys. Rep. 64, 171 (1980). U. Garg and G. Colò, PPNP101, 55 (2018).

○ This is not all the story for monopole transitions

The other side of the monopole transition

○ Plan of this talk

In this talk, I'll focus on the different aspects of the monopole transition

Topic #1 : Monopole transition as a probe for nuclear shape and its fluctuation

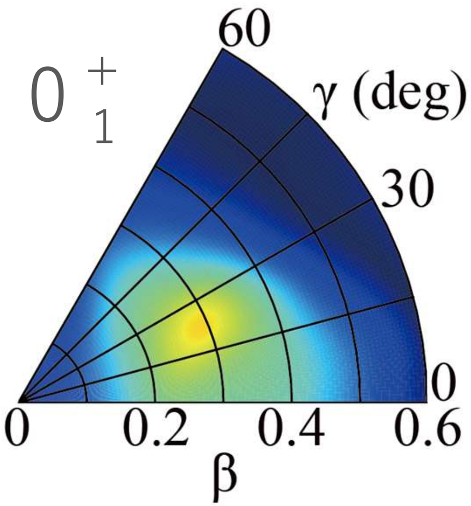
Topic #2 : Monopole transition as a probe for nuclear clustering

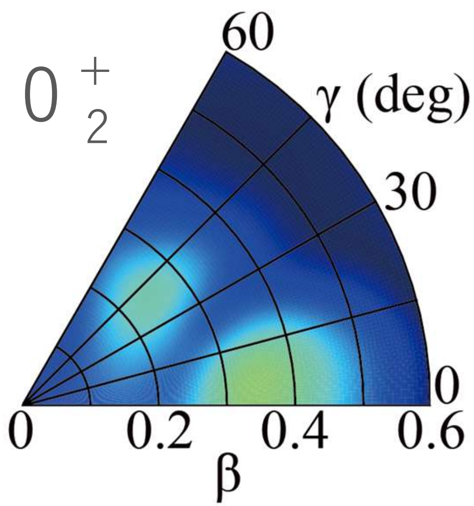
Let me introduce basic idea for these topics and show you some examples

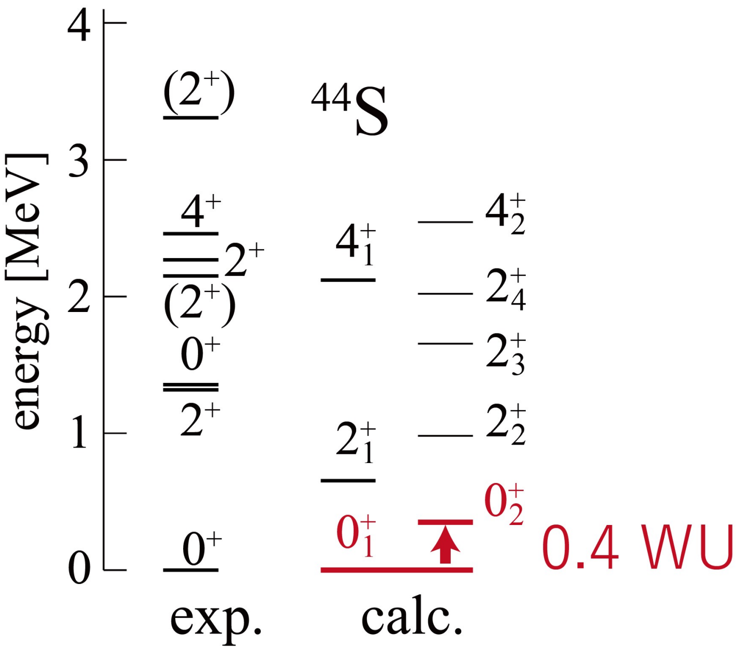
For more detail, you are directed to

- Oral presentations by Y. Suzuki (Friday afternoon, Shape coexistence)
  by Y. Taniguchi (This afternoon, $^{12}C+^{12}C$ Cluster)

Topic #1
Monopole transition as a probe for nuclear shape and its fluctuation







Monopole transition and nuclear shape, its fluctuation

○ Monopole transition is a probe for the shape coexistence and shape fluctuation

K. Heyde and J. L. Wood, Rev. Mod. Phys. 83, 16555 (2011)

Suppose that the ground state and an excited states are described by the superposition of two different state vectors (different nuclear shapes) denoted by $|A\rangle$ and $|B\rangle$

$$|0_1^+\rangle = a\,|A\rangle + b\,|B\rangle\,,$$
$$|0_2^+\rangle = -b\,|A\rangle + a\,|B\rangle\,,$$

(a and b are the mixing amplitudes)

Monopole matrix element

$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle = ab\,\{\langle B|\mathcal{M}|B\rangle - \langle A|\mathcal{M}|A\rangle\} + (a^2 - b^2)\,\langle B|\mathcal{M}|A\rangle$$
$$\simeq\ ab\,\{\langle B|\mathcal{M}|B\rangle - \langle A|\mathcal{M}|A\rangle\}$$

This vanishes when $|A\rangle$ and $|B\rangle$ are quite different or $a = b = 1/\sqrt{2}$

Size difference of $|A\rangle$ and $|B\rangle$

○ Matrix element is given by the size differences (no particle-hole excitations)

○ A good example of the quantum superposition

An example for the shape coexistence and monopole transition

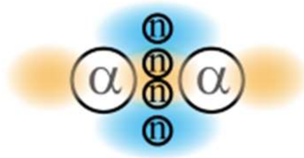
○ One of a beautiful example is ^{12}Be ground state and isomeric state

2.2 MeV

$$|0_2^+\rangle = -b\,|A\rangle + a\,|B\rangle\,,$$

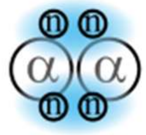
$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle \simeq ab\,\{\langle B|\mathcal{M}|B\rangle - \langle A|\mathcal{M}|A\rangle\}$$

$$|0_1^+\rangle = a\,|A\rangle + b\,|B\rangle\,,$$

$|A\rangle$ = 

$2\alpha + (0p)^2(sd)^2$

- Larger radius
- Broken magic # N=8

$|B\rangle$ = 

$2\alpha + (0p)^4$

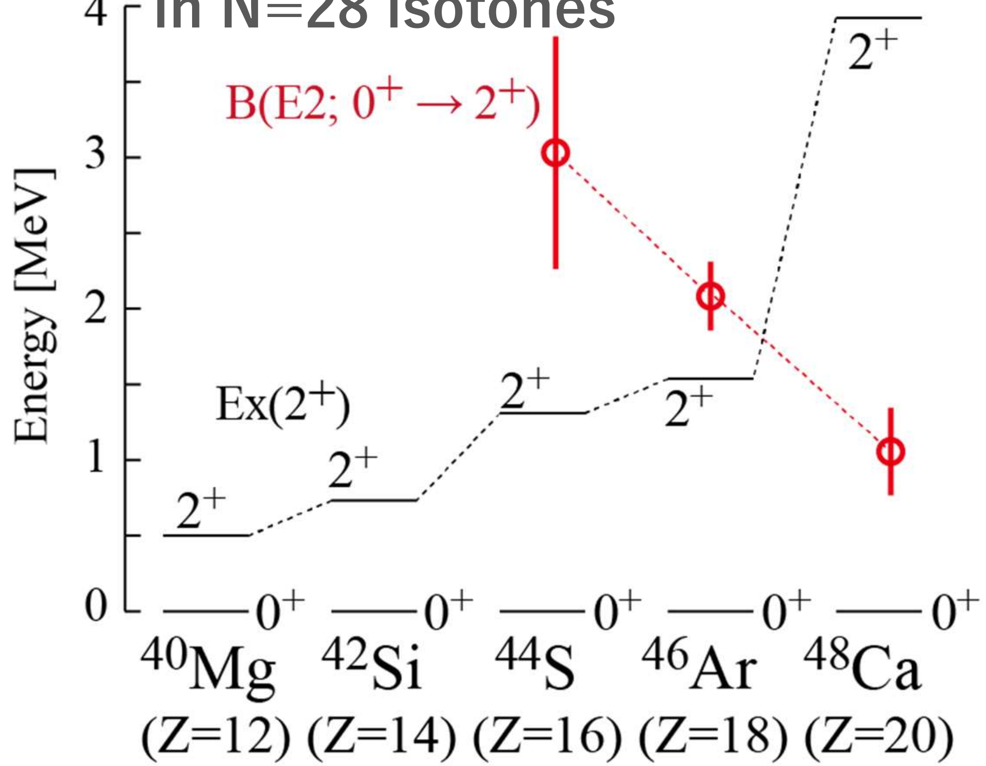
- Smaller radius
- Closed N=8 shell

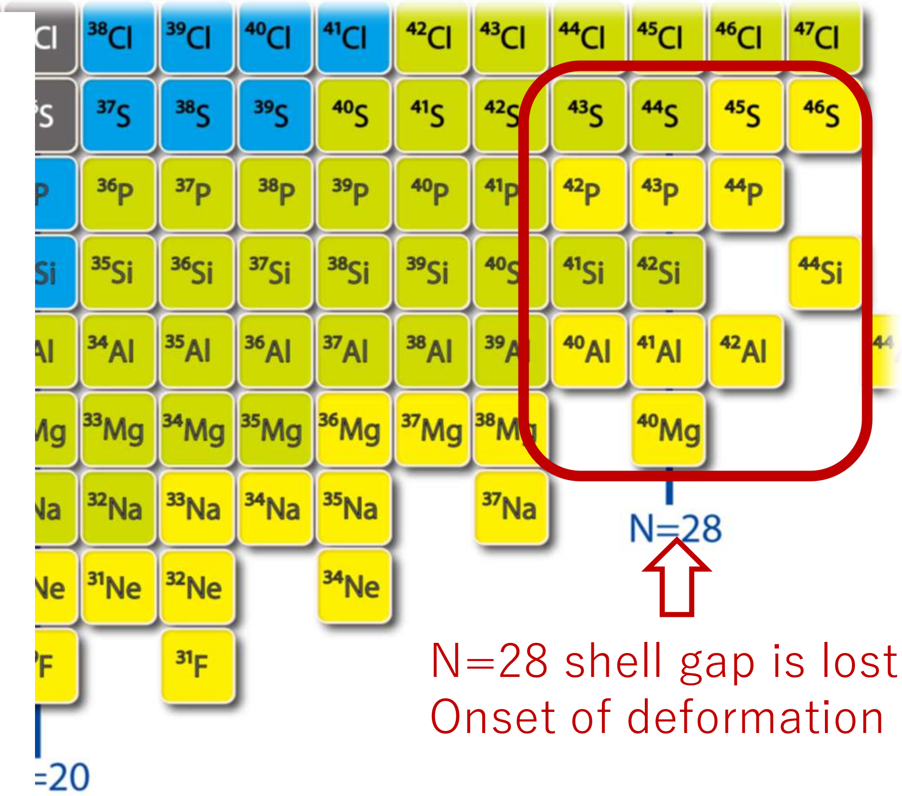
○ Electric monopole transition rate (the lifetime of 2nd 0+ state) has been measured and its enhancement has been reported

This example indicates an interesting relationship between nuclear shape (deformation), magic number and monopole transition

A region of shape coexistence : neutron-rich N≃28 nuclei

Onset of the deformation
in N=28 isotones





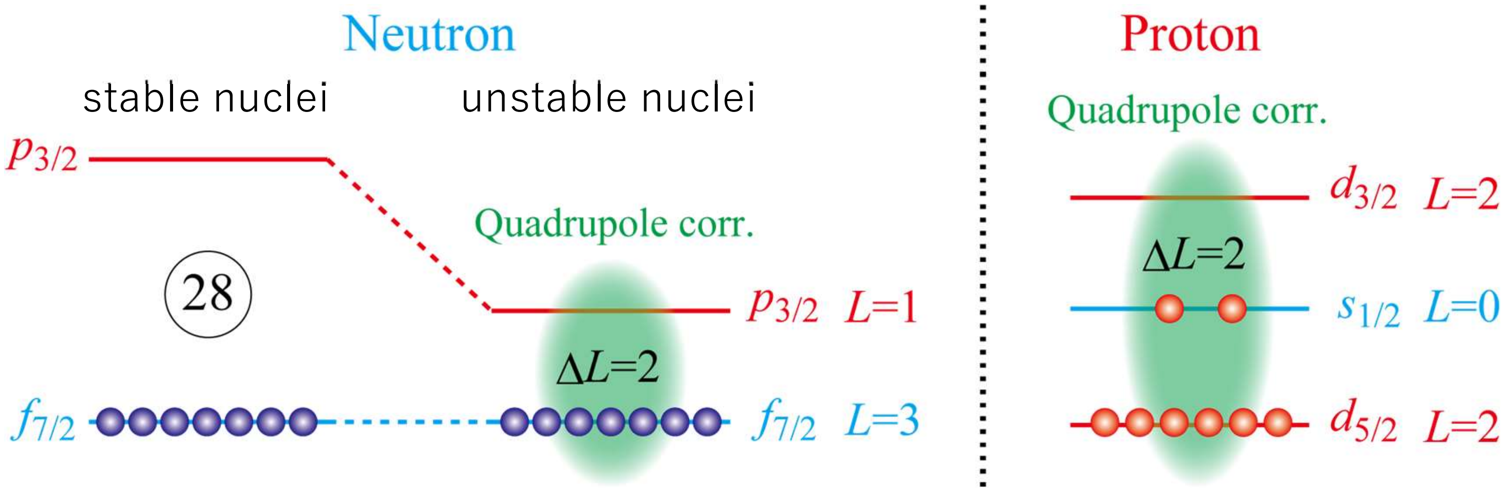
N=28 shell gap is lost
Onset of deformation

S. Takeuchi, et al., PRL109, 1 (2012).
H. Scheit, et al., 77, 3967 (1996)

A region of shape coexistence : neutron-rich N≃28 nuclei

- Quenching of N=28 shell gap leads to the degeneracy of p- and f-wave
- Mg, Si and S (Z=12,14 and 16) have proton half-filling of sd-shell
  ⇨ Quadrupole correlations between protons and neutrons

⇨ **Shape coexistence:** various deformed shapes coexist at small energies



Numerical Method: Antisymmetrized Molecular Dynamics (AMD)

○ **Hamiltonian** Gogny D1S density functional J. F. Berger et al., CPC 63, 365 (1991)

$$\hat{H} = \sum_{i}^{A} \hat{t}_i - \hat{t}_{c.m.} + \sum_{i<j}^{A} \hat{v}_{\mathrm{GognyD1S}}(r_{ij}) + \sum_{i<j}^{Z} \hat{v}_{\mathrm{Coulomb}}(r_{ij})$$

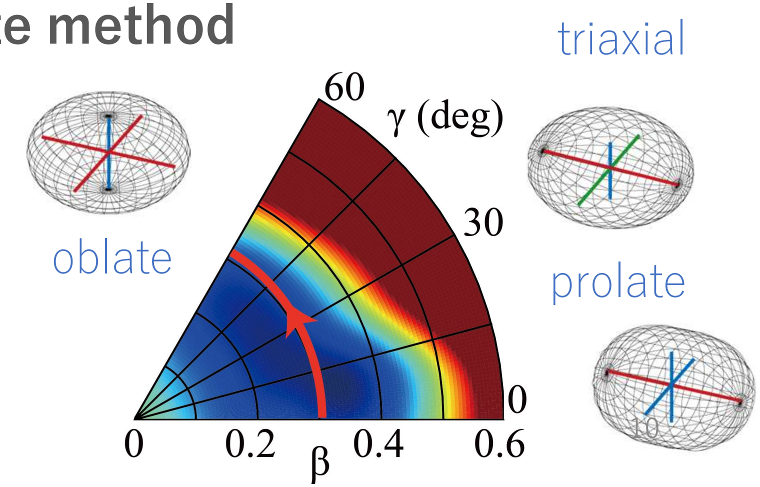
○ **Model wave function** Antisymmetrized product of nucleon wave packets

$$\Psi^{\pi} = \frac{1+\pi\hat{P}_r}{2}\mathcal{A}\{\varphi_1, \varphi_2, ..., \varphi_A\}\,, \qquad \varphi_i(\boldsymbol{r}) = \exp\left\{-\nu(\boldsymbol{r}-\boldsymbol{Z}_i)^2\right\}\cdot(a_i\,|\uparrow\rangle + b_i\,|\uparrow\rangle)$$

○ **β, γ-constraint, J-projection, generator coordinate method**

$$\Psi^{J\pi}_{M\alpha} = \sum_{iK} g_{iK\alpha} \underline{P^J_{MK}}\, \underline{\Phi^{\pi}(\beta_i, \gamma_i)},$$

J-projection Constrained wave function



Shape coexistence in N=28 nuclei

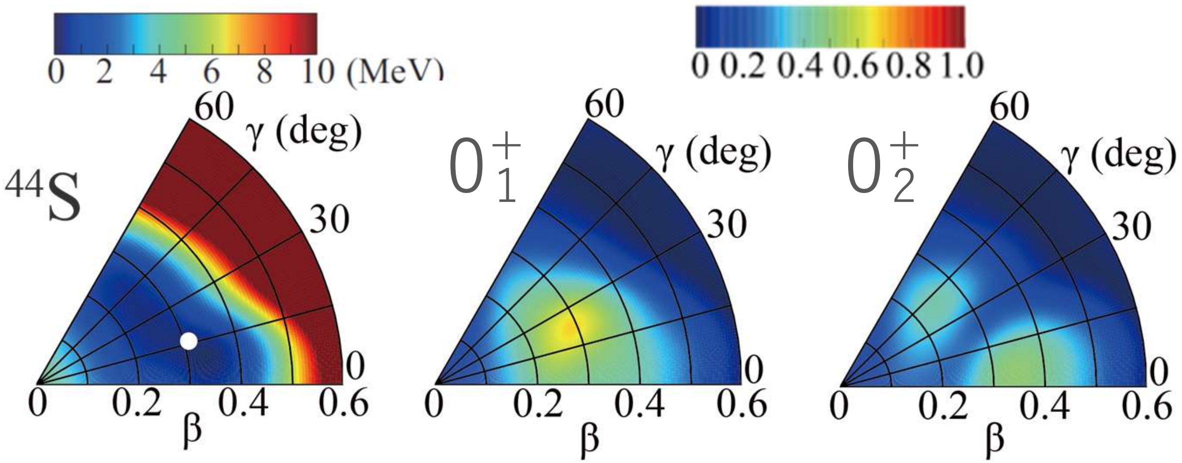
^{44}S exhibits “large amplitude collective motion”

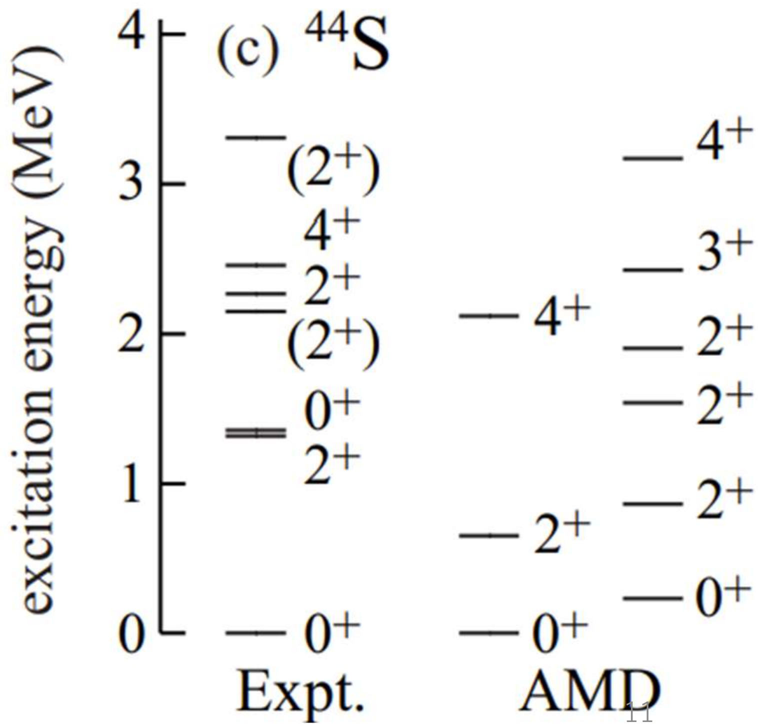
- Y. Suzuki, W. Horiuchi, M.K. PTEP2022 in print.
- Y. Suzuki, M.K. PRC104, 024327 (2021)

○ ^{44}Si has soft energy surface against γ deformation

○ GCM amplitude has broad and non-localized distribution ⇒ Large shape fluctuation

energy surface GCM amplitude

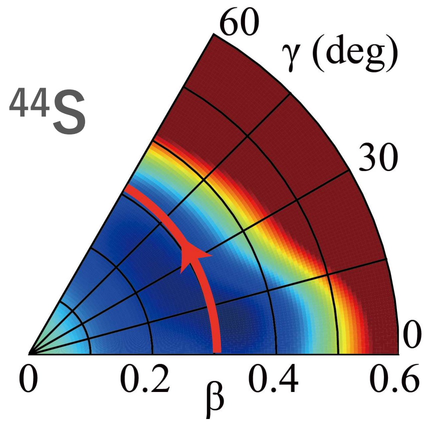


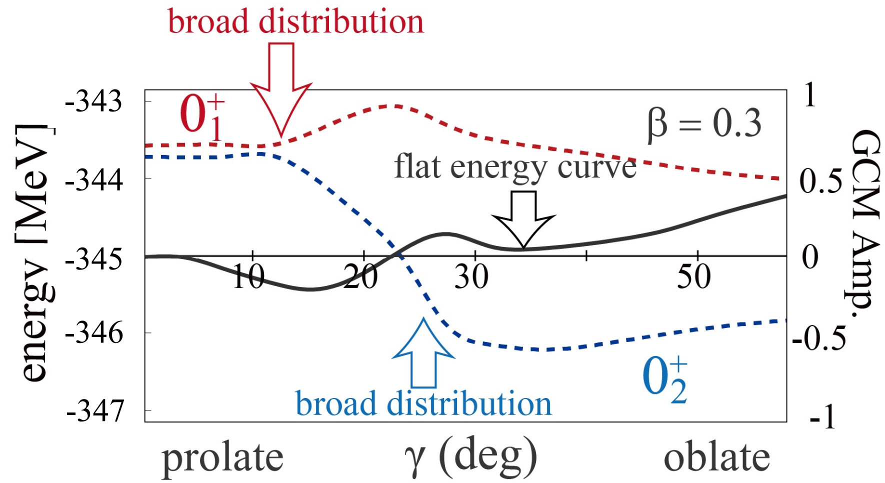


Shape coexistence in N=28 nuclei

^{44}S: Large amplitude collective motion

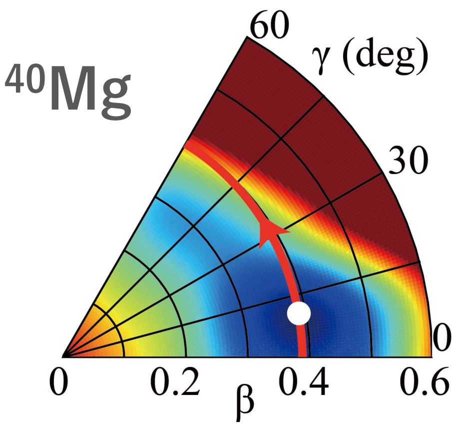
- Flat energy surface
- Non-localized GCM
  ⇨ Large amp. collectivity

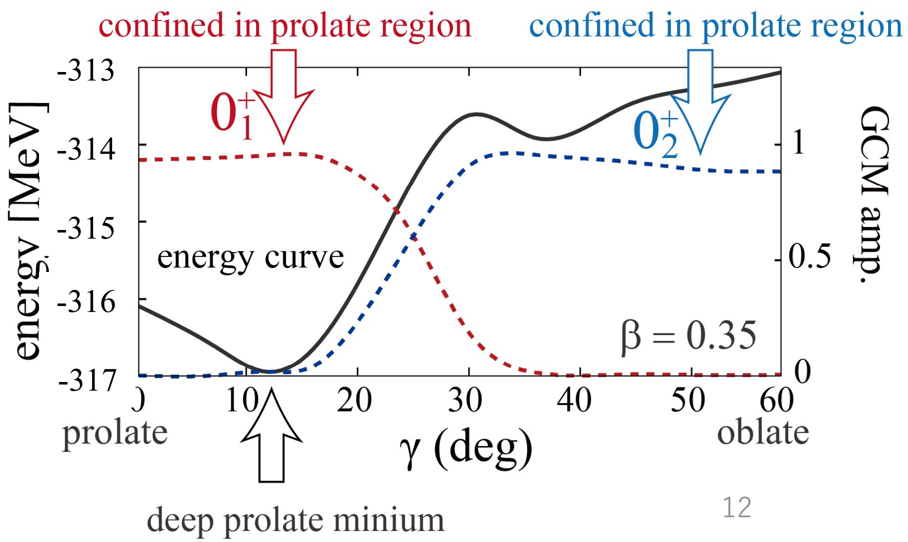




^{40}Mg (^{42}Si): Rigid rotors

- Deep energy minimum
- GCM amp are localized
  ⇨ Rigid rotor





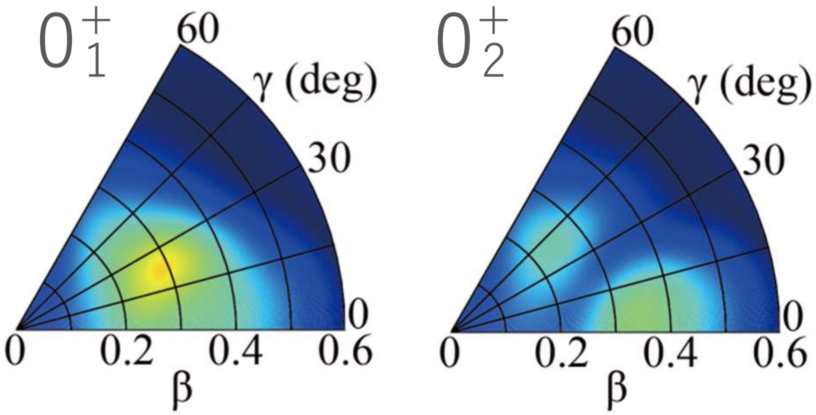
Monopole transition in N=28 nuclei

Monopole transition is enhanced in ^{44}S (B(IS0) ~ 0.4 Wu)

$$|A\rangle = |\text{prolate}\rangle,\ |B\rangle = |\text{oblate}\rangle,\ a = b = 1/\sqrt{2}$$

$$|0_1^+\rangle = a\,|A\rangle + b\,|B\rangle = 1/\sqrt{2}\,\left(|\text{prolate}\rangle + |\text{oblate}\rangle\right)$$

$$|0_2^+\rangle = -b\,|A\rangle + a\,|B\rangle = 1/\sqrt{2}\,\left(|\text{prolate}\rangle - |\text{oblate}\rangle\right)$$



Monopole matrix element

$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle = ab\,\{\langle B|\mathcal{M}|B\rangle - \langle A|\mathcal{M}|A\rangle\} + (a^2 - b^2)\,\langle B|\mathcal{M}|A\rangle$$

$$= \frac{1}{2}\,\{\langle \text{oblate}|\mathcal{M}|\text{oblate}\rangle - \langle \text{prolate}|\mathcal{M}|\text{prolate}\rangle\} \sim 0.4\ \text{Wu}$$

Proportional to the size difference of prolate and oblate shapes

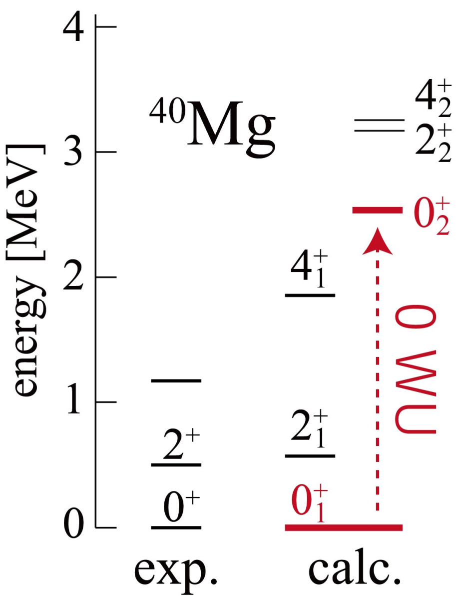
○ Monopole transition in ^{44}S is a fingerprint of the large amplitude collective motion

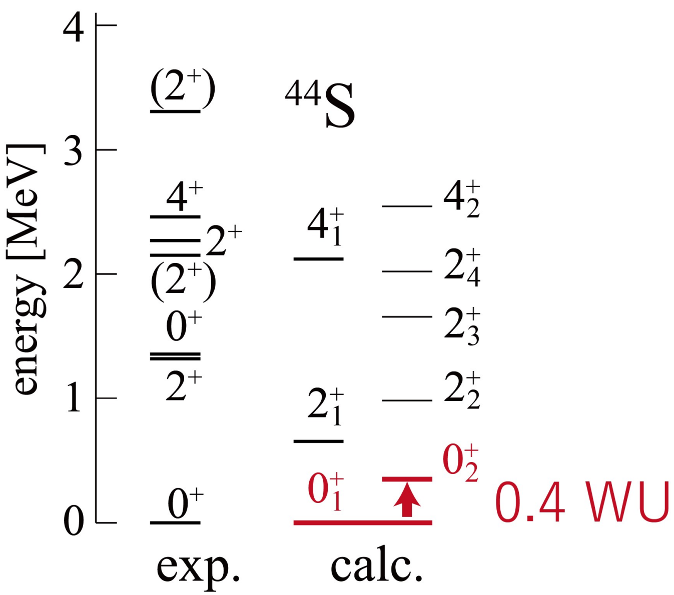
Monopole transition in N=28 nuclei

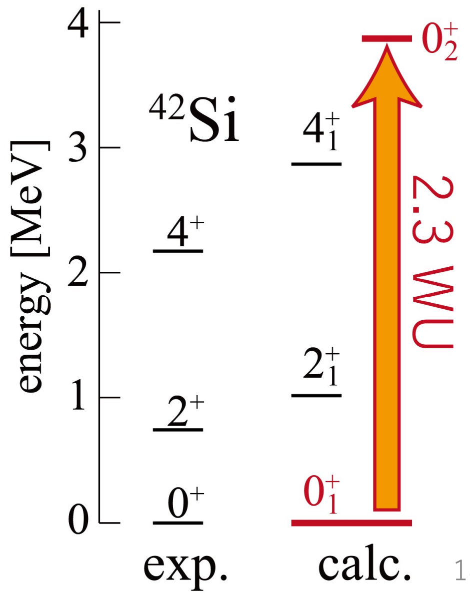
Monopole transition as a probe for shape coexistence in N=28 nuclei

Monopole transition strengths are different in order of magnitudes
⇨ This reflects the shape and structure of individual nuclei

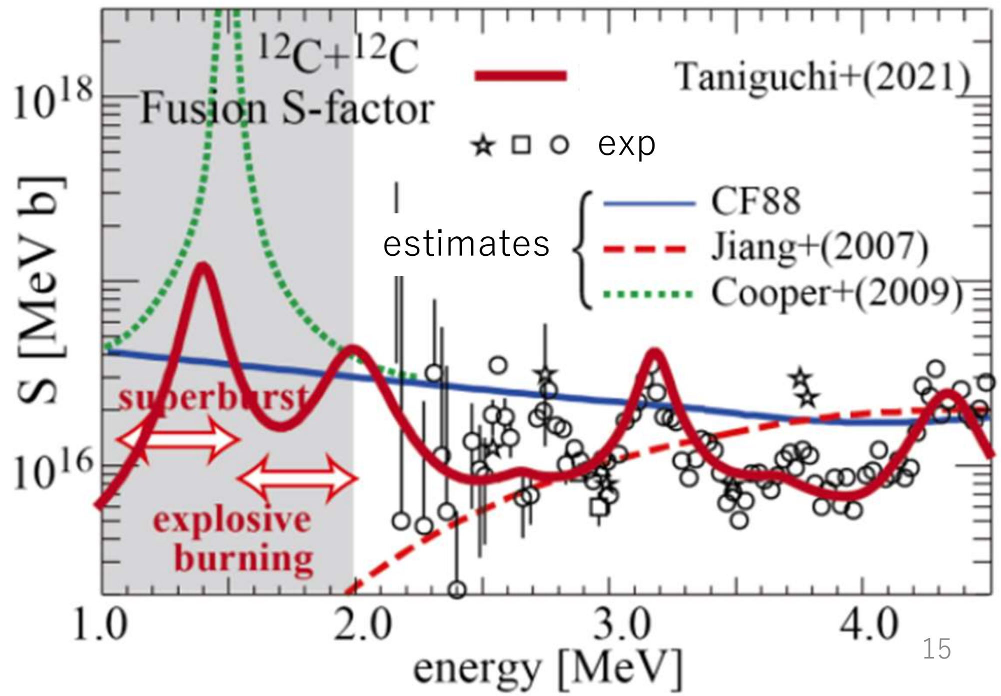
Talk by Y. Suzuki (Friday afternoon)







Topic #2
Monopole transition as a probe for clustering



Monopole transition as a probe for clustering

Enhancement of the monopole transition to the cluster states in ^{11}B

T. Kawabata & Y. Kanada-Enyo et al., PLB 646, 6 (2007), PRC 75, 024302 (2007)

The mechanism of the enhancement was explained by Yamada et al. (PTP120, 1139)

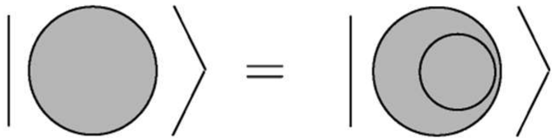
○ Bayman-Bohr theorem [Nucl. Phys. 9, 596 (1958/1959)]
An SU(3) shell model wave function is mathematically equivalent to a cluster wave function

$$\Phi_{g.s.}(^{20}\mathrm{Ne}) \simeq \mathcal{A}\{\underbrace{(0s)^4(0p)^{12}}_{^{16}\mathrm{O}\ \text{core}}(0d1s)^4\} = n\mathcal{A}\{\underbrace{R_{80}(r)Y_{00}(\hat{r})}_{\text{overlapping } \alpha \text{ and } ^{16}\mathrm{O} \text{ clusters}}\phi_\alpha\phi_{^{16}\mathrm{O}}\}\phi_{cm}(\boldsymbol{r}_{cm})$$

^{16}O core

overlapping α and ^{16}O clusters

shell model wave function
= completely overlapping clusters



The interpretation of this theorem is that
"The degrees-of-freedom of cluster excitation is embedded
even in a pure shell model ground state"

Monopole transition as a probe for clustering

The ordinary idea : Monopole operator gives rise to GMR

○ Shell model wave function

$$\Phi_{g.s.}(^{20}\mathrm{Ne}) = \mathcal{A}\{\,(0s)^4(0p)^{12}(0d1s)^4\,\}$$

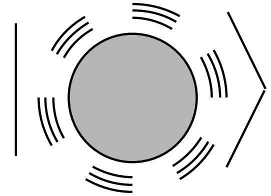
○ Monopole operator

$$\mathcal{M}^{IS0}_{\mu} = \sum_{i=1}^{A}(\boldsymbol{r}_i - \boldsymbol{r}_{\mathrm{cm}})^2$$

○ Monopole operator yields 1p1h configurations

GMR

$$\mathcal{M}\Phi_{g.s.}(^{20}\mathrm{Ne}) = \mathcal{A}\{\,(0s)^4(0p)^{12}(1s0d)^3(\mathbf{2s})\,\} + \mathcal{A}\{\,(0s)^4(0p)^{12}(1s0d)^3(\mathbf{1d})\,\} + \cdots = $$



This is true, but let us think different

Monopole transition as a probe for clustering

Another aspect : Monopole operator gives rise to clusters

○ Bayman-Bohr theorem

$$\Phi_{g.s.}(^{20}\mathrm{Ne}) \simeq \mathcal{A}\{(0s)^4(0p)^{12}(0d1s)^4\} = n\mathcal{A}\{R_{80}(r)Y_{00}(\hat{r})\phi_\alpha\phi_{^{16}\mathrm{O}}\}\phi_{cm}(\boldsymbol{r}_{cm})$$

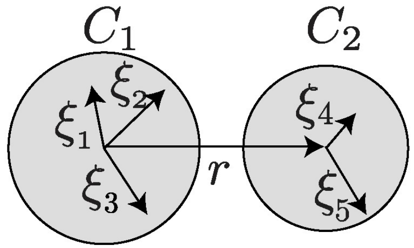
○ Cluster coordinate representation of the monopole operator

$$\mathcal{M}^{IS0}_\mu = \sum_{i=1}^{A}(\boldsymbol{r}_i - \boldsymbol{r}_{\mathrm{cm}})^2 = \sum_{i\in C_1}\xi_i^2 + \sum_{i\in C_2}\xi_i^2 + \frac{C_1C_2}{C_1+C_2}r^2$$

C_1, C_2 : masses of clusters

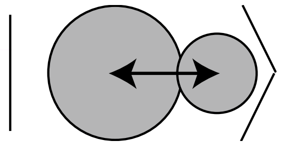
r : inter-cluster coordinate

ξ_i : internal coordinates of clusters



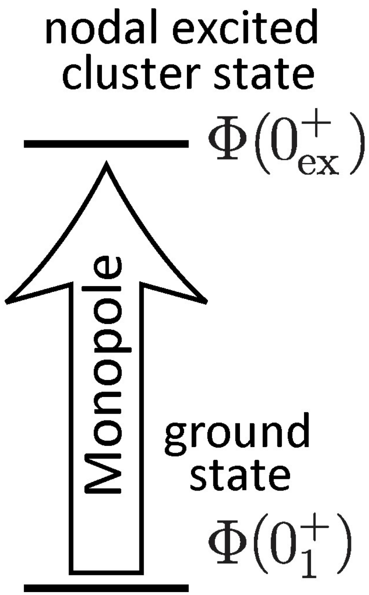
○ Monopole operator excites the inter-cluster motion

$$\Rightarrow \mathcal{M}^{IS0}_\mu \Phi_{g.s.}(^{20}\mathrm{Ne}) \simeq \sum_{N=N_0+2}^{\infty} f_N n_N \mathcal{A}\{R_{\boldsymbol{N}0}(r)Y_{00}(\hat{r})\phi_\alpha\phi_{^{16}\mathrm{O}}\}$$

Excited 0^+ state 

Monopole transition as a probe for clustering

Assuming that the ground state is a simple shell model state, the transition matrix can be estimated analytically (PTP120, 1139 (2008)).



Cluster estimate for ^{20}Ne (analytical)

$$M^{IS0} = \langle \Phi(0^+_{\rm ex}) | \mathcal{M}^{IS0} | \Phi(0^+_1) \rangle$$

$$= f_{N_0+2} \sqrt{\frac{\mu_{N_0}}{\mu_{N_0+2}}} \langle R_{N_0 0} | r^2 | R_{N_0+2 0} \rangle$$

$$\simeq 7.67\, f_{N_0+2} = 5.48 \text{ fm}^2$$

Single particle estimate

$$M^{IS0}_{\rm WU} = \frac{3}{5}(1.2 A^{1/3})^2 \simeq 6.37 \text{ fm}^2$$

We note that the same argument also applies to the isoscalar dipole transition

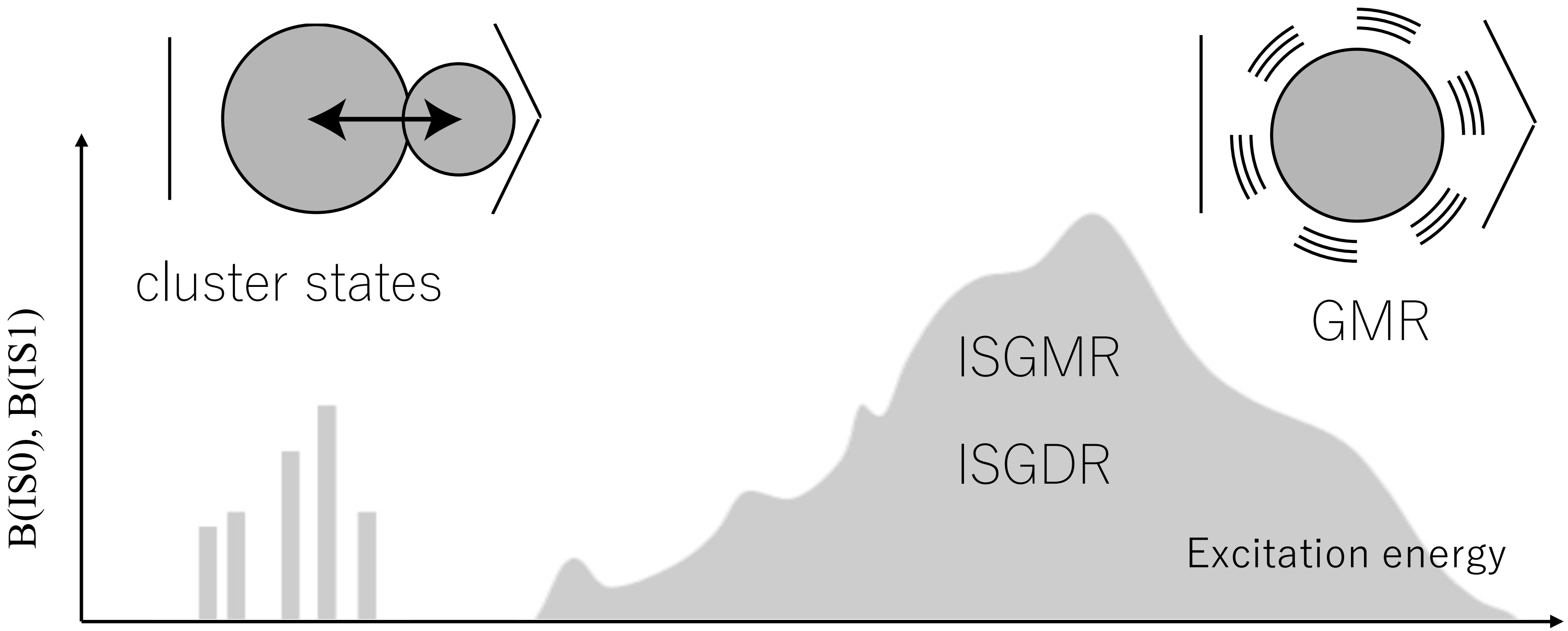
Y. Chiba, M. K., Y. Taniguchi, Phys. Rev. C 93, 034319 (2016)

Monopole transition as a probe for clustering

Now, we understood that the monopole(dipole) transition to the cluster state can be as strong as the single particle estimate

◎ Collective excitation: GMR, ISGDR $E_x > 15$ MeV

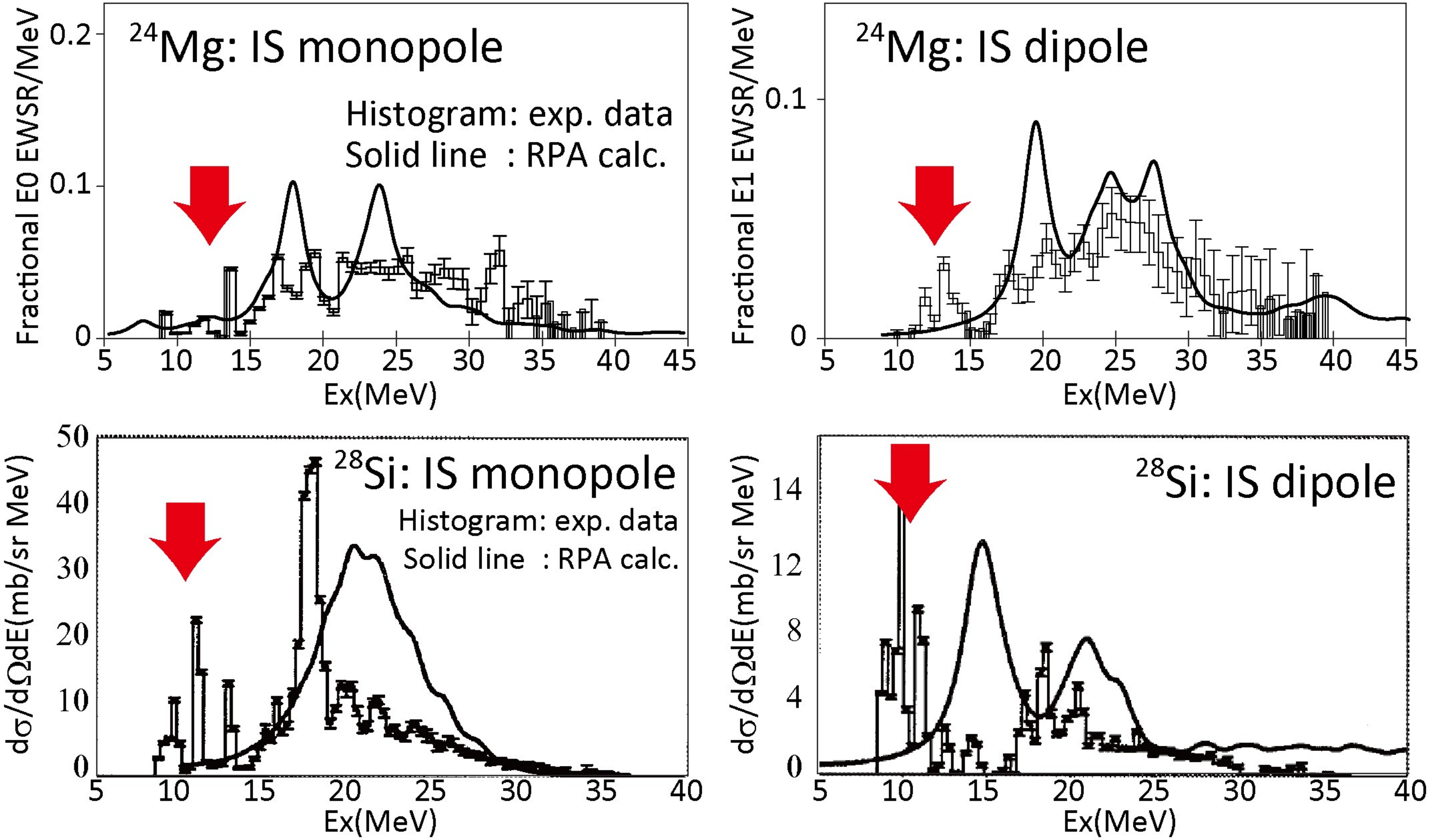
◎ Cluster excitation: $E_x < 15$ MeV (clusters may appear close to the decay threshold)



Monopole transition as a probe for clustering

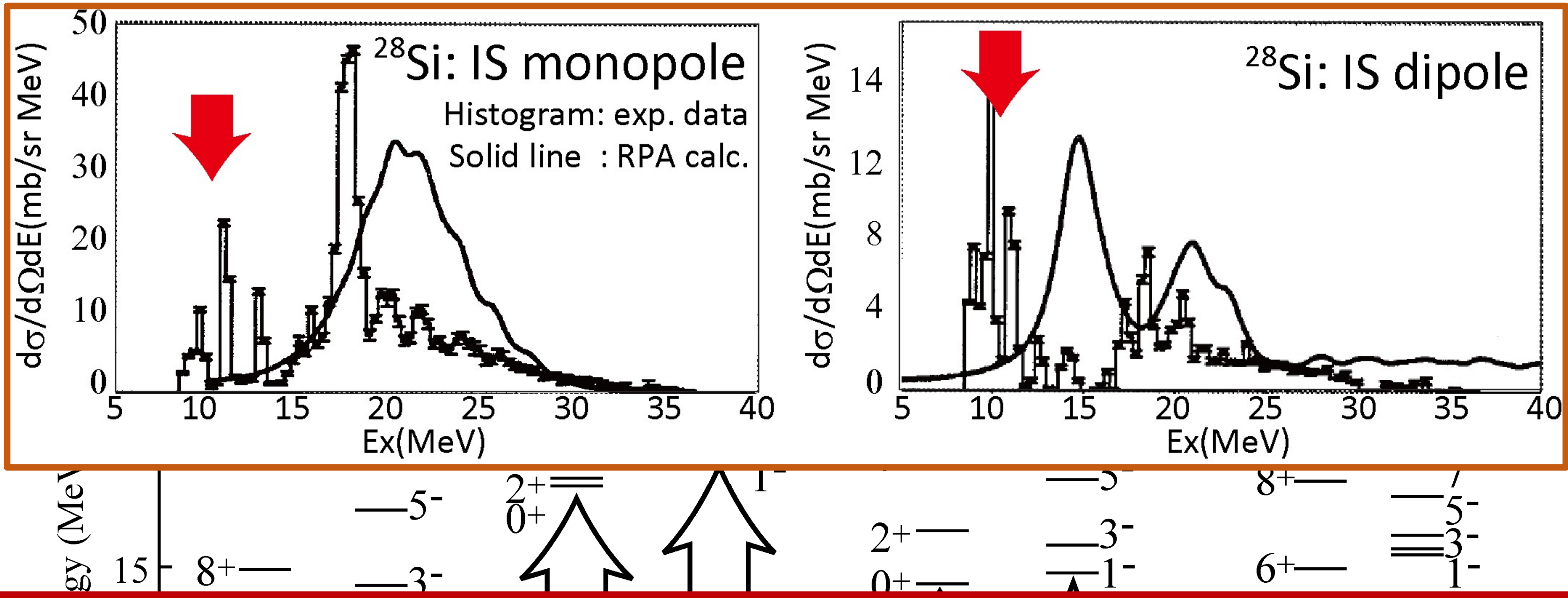
Experimental data

X. Chen et al.,PRC80, 014312 (2009). D. H. Young-Blood et al.,PRC65, 034302 (2002).



3.2 Result for ^{28}Si (α+^{24}Mg and ^{8}Be+^{20}Ne clustering)

Y. Taniguchi, Y. Kanada-En'yo and M.K. PRC80, 044316 (2009).
Y. Chiba, M.K., and Y. Taniguchi, PRC (2019)



This has motivated us to explore various clusters using monopole transition

- Hoyle state and its analogous states in ^{12}C, ^{13}C, ^{11}B ···
- Exotic cluster states in neutron-rich nuclei ^{10}Be, ^{12}Be ···

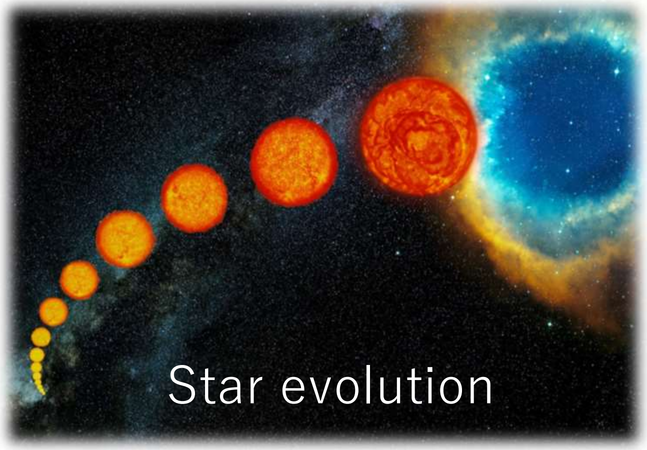
Monopole transition as a probe for clustering

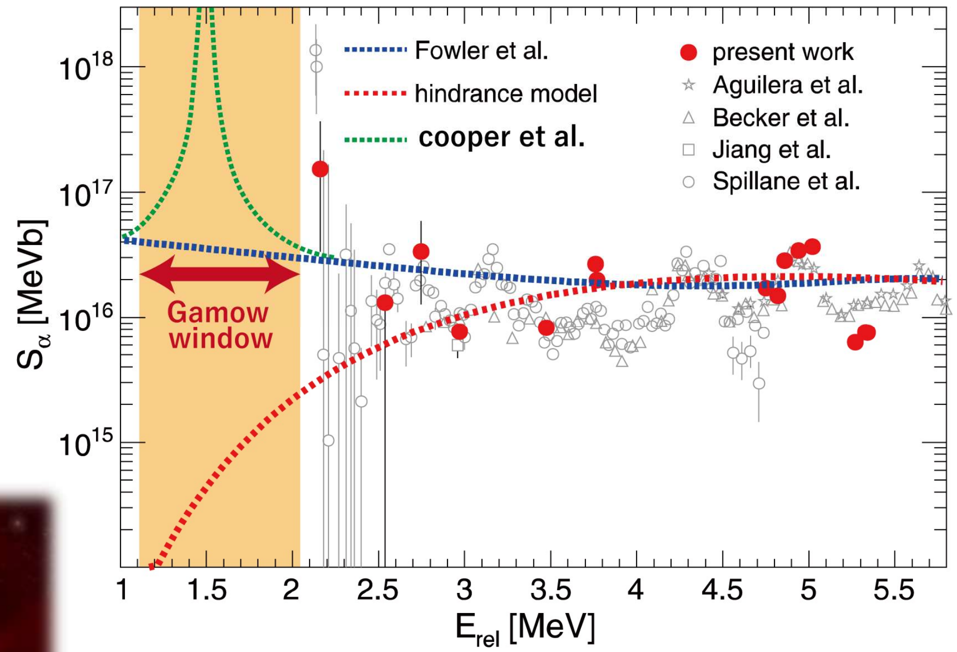
An important application of this idea is $^{12}C+^{12}C$ stellar fusion reaction

One of the most important reaction

$$^{12}\mathrm{C} + ^{12}\mathrm{C} \to p + ^{23}\mathrm{Na} + 2.2\mathrm{MeV}$$
$$\to \alpha + ^{20}\mathrm{Ne} + 4.6\mathrm{MeV}$$

A. Tumino et al., Nature557, 687 (2018).
G. Fruet et al., PRL124, 192701 (2020).
W.P. Tan et al., PRL124, 192702 (2020).





Reaction rate at low-energy cannot be measured

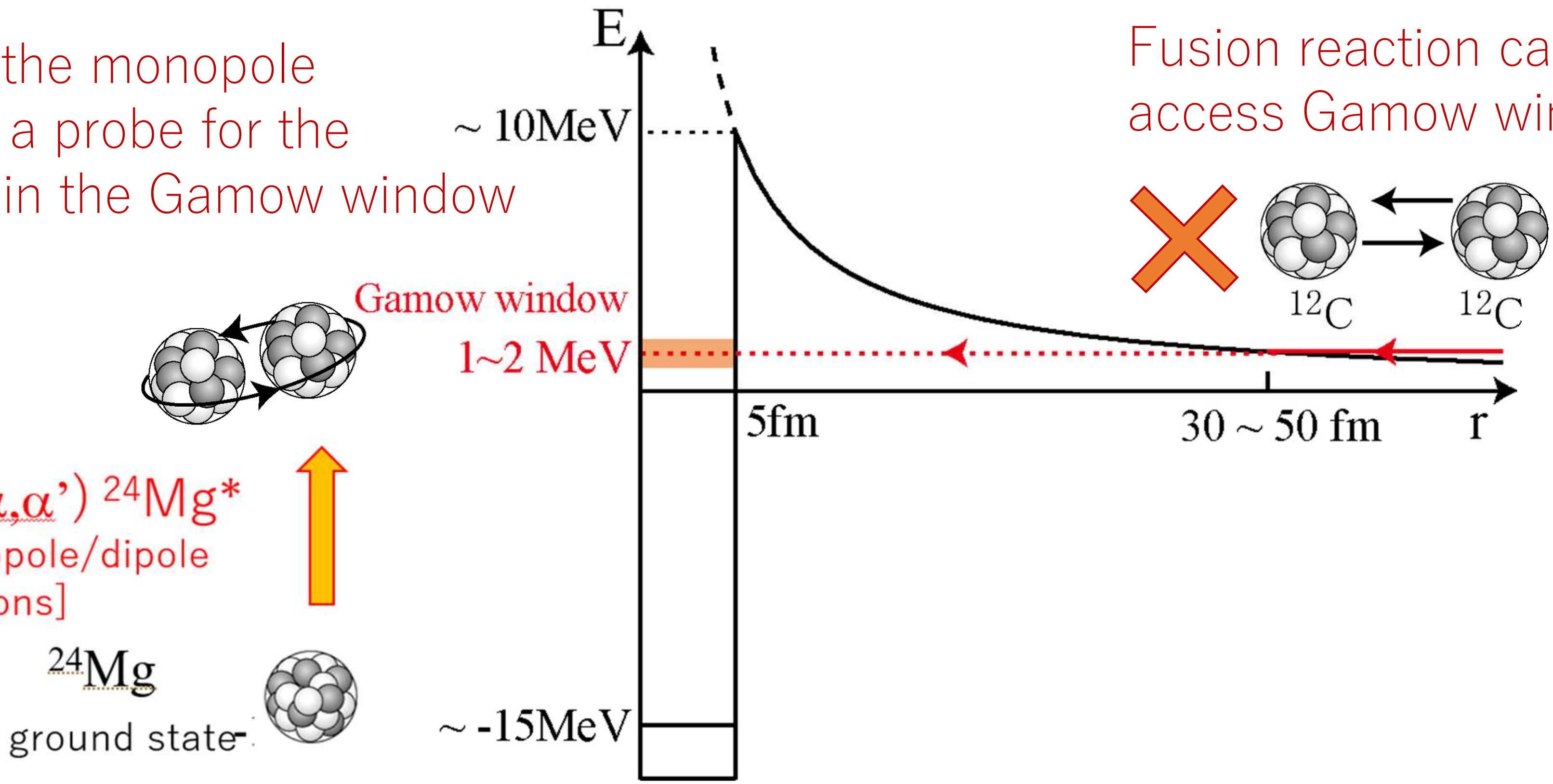
$^{12}C + ^{12}C$ fusion reaction and molecular resonances

We want to know if there is cluster resonances exist within Gamow window as they increase the reaction rate in order of magnitude

We propose the monopole transition as a probe for the clusters within the Gamow window

Fusion reaction cannot access Gamow window

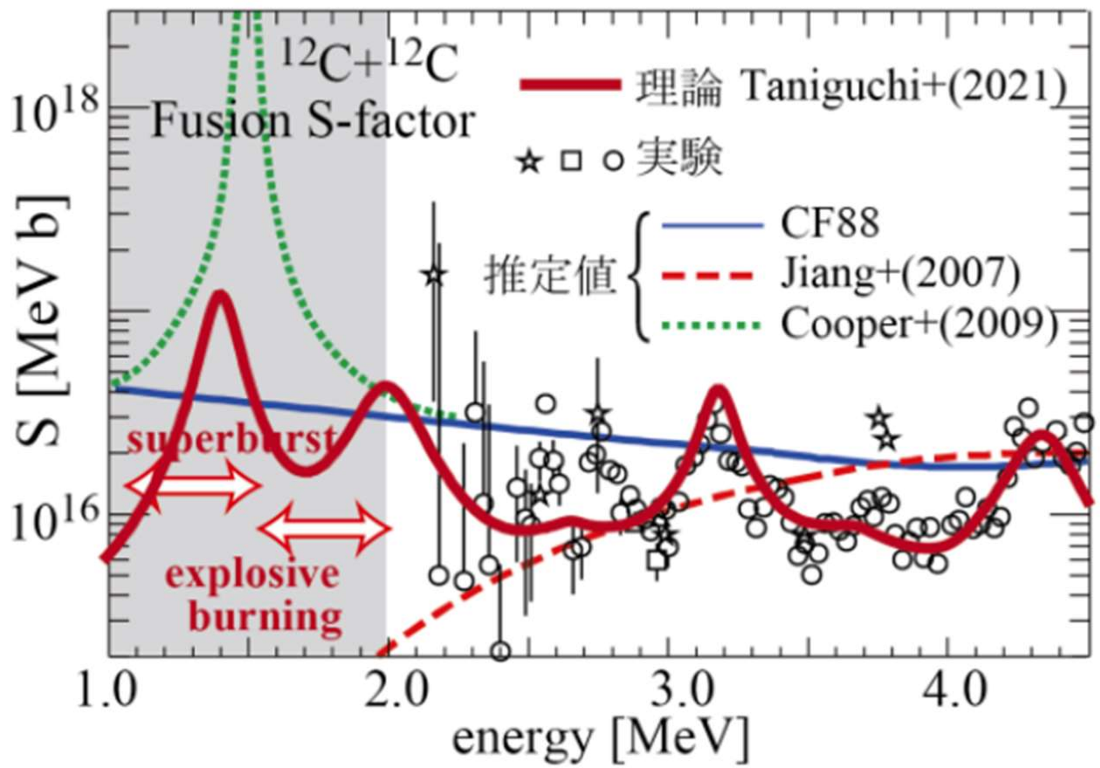
$^{24}Mg(\alpha,\alpha')\,^{24}Mg^*$
[IS monopole/dipole transitions]



$^{12}C+^{12}C$ fusion reaction and molecular resonances

In fact, a microscopic calculation predicts the $^{12}C+^{12}C$ resonance in the GW which enhances the reaction rate at stellar temperature

Y. Taniguchi, M.K. Phys. Lett B. 823, 136790 (2021)　　Talk by Y. Taniguchi in this afternoon



$^{12}C+^{12}C$ resonances have enhanced monopole (quadrupole) strengths

J^π	E_R	M_{IS}	$\theta^2_C \times 10^2$ $l=J$	$\theta^2_{\alpha_0} \times 10^2$ $l=0$	2
2^+	0.93	1.56	1.4	—	3.5
0^+	0.94	0.59	7.3	0.20	—
2^+	1.50	1.04	2.9	—	1.1
2^+	2.18	0.51	3.4	—	1.0
0^+	3.02	1.05	11	0.26	—
2^+	3.56	0.23	1.2	—	0.038
2^+	3.73	0.41	8.3	—	0.10

Summary

I have focued on the different aspects of the monopole transition

Topic #1 : probe for nuclear shape and its fluctuation

Quenching of N=28 shell gap induces the interesting features of N=28 isotones

^{44}S manifests “large amplitude collective motion”, which have large shape fluctuation

The monopole transition will provide us deeper understanding

Topic #2 : probe for nuclear clustering

Monopole transition to the cluster states is enhanced, which serve as a new probe for clusters

An interesting and important application is ^{12}C+^{12}C cluster resonance