



北京大学  
PEKING UNIVERSITY

Developments of Physics of Unstable Nuclei

(YKIS2022b) 23—27 May, 2022 Kyoto University

Search for triaxially deformed halo nuclei

Jie MENG (孟杰)

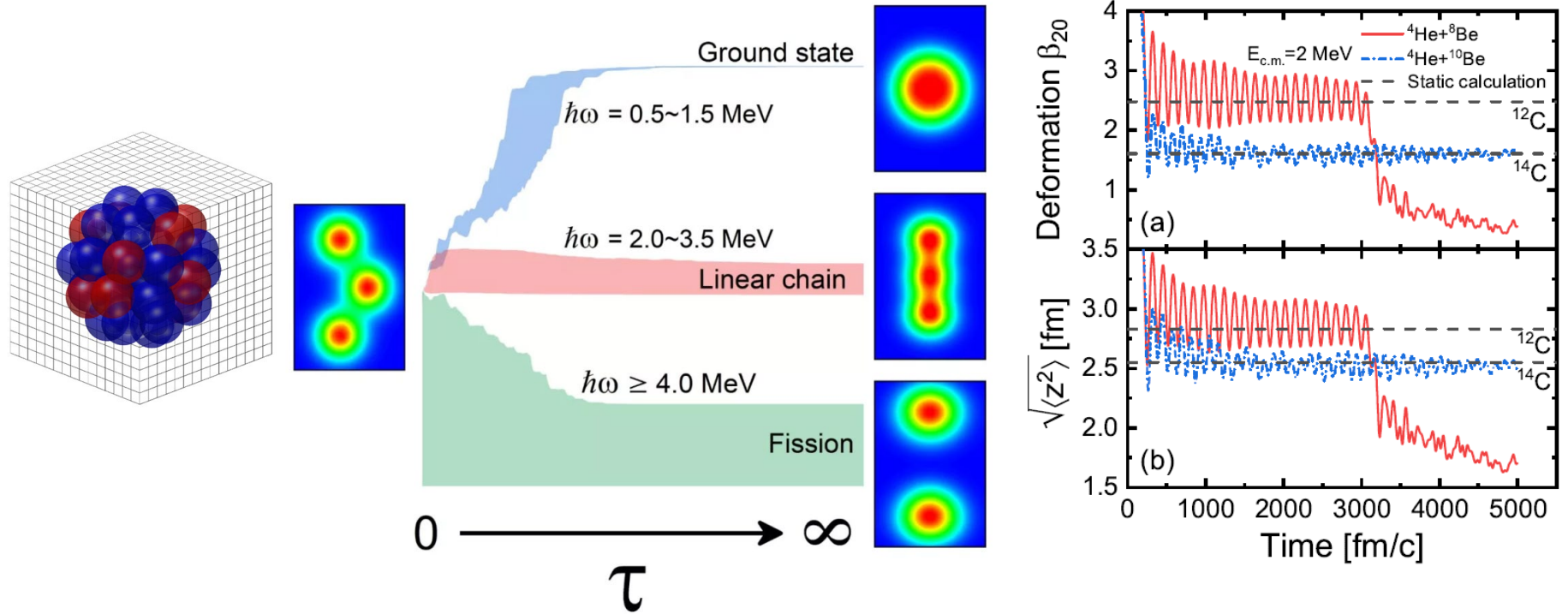
School of Physics, Peking University (北京大学物理学院)



# (Time-depend) CDFT in 3D lattice

EoM on 3D lattice with the inverse Hamiltonian and Fourier spectral methods.

Linear  $3\alpha$  clusters chain structure for  $^{12}\text{C}$  against the bending and fission in **cranking CDFT** and **TD CDFT** on a 3D lattice

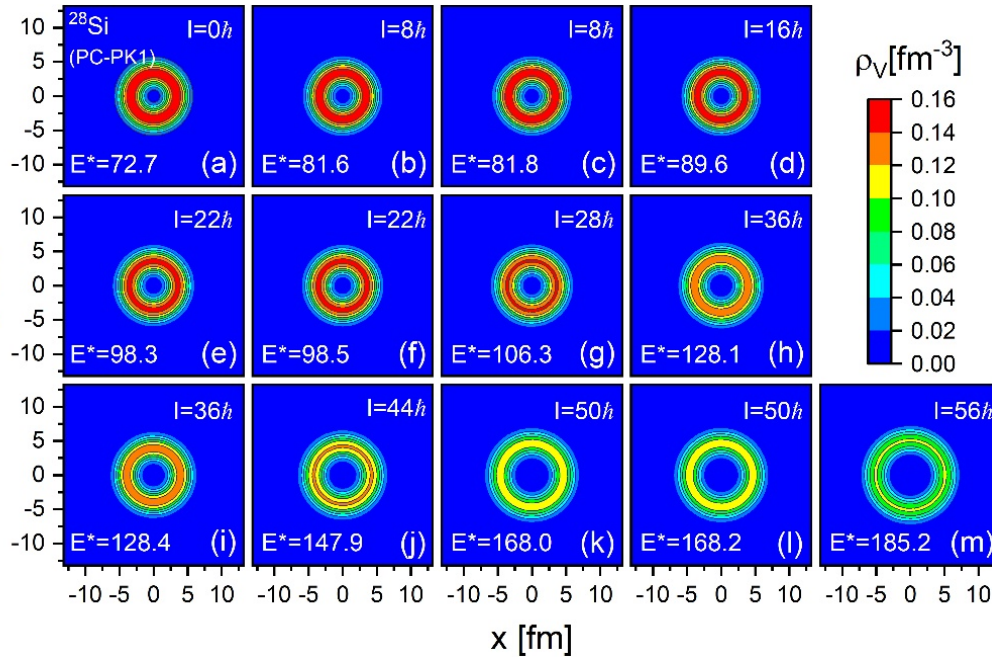
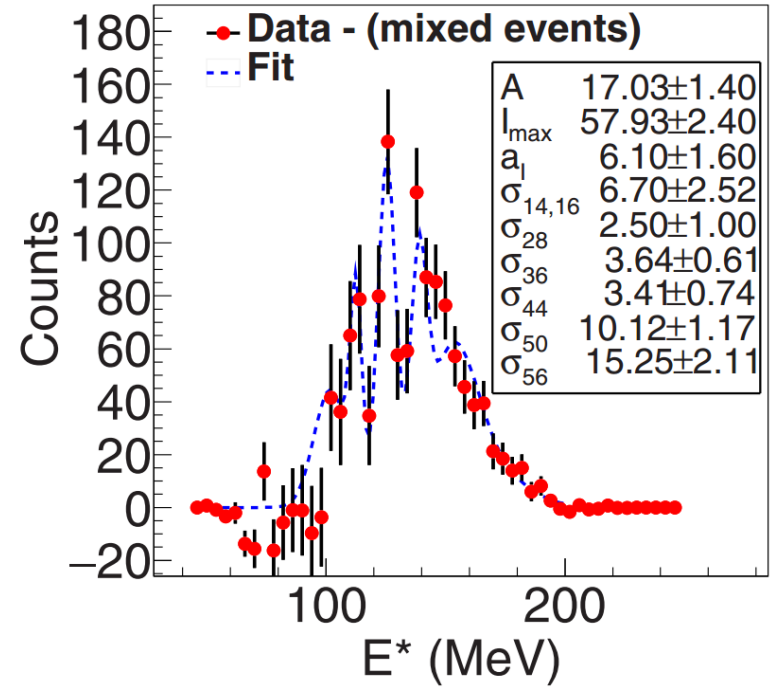


Ren, Zhang, Zhao, Itagaki, Maruhn, Meng, SCPMA 62, 112062 (2019)  
Ren, Zhao, Meng, Physics Letters B 801 (2020) 135194



## $7\alpha$ disassembly of $^{28}\text{Si}$

Cao et al., Phys. Rev. C 99, 014606 (2019)



Density distributions in the  $z = 0$  plane for the toroidal states in cranking CDFT on a 3D lattice



PHYSICAL REVIEW LETTERS **128**, 172501 (2022)

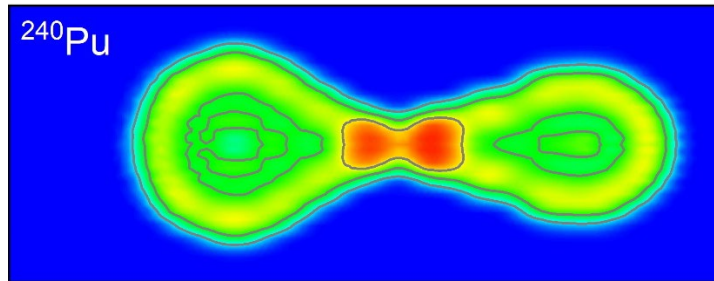
## Dynamical Synthesis of $^4\text{He}$ in the Scission Phase of Nuclear Fission

Z. X. Ren<sup>1</sup>, D. Vretenar<sup>2,1,\*</sup>, T. Nikšić<sup>2,1</sup>, P. W. Zhao<sup>1,†</sup>, J. Zhao<sup>3</sup>, and J. Meng<sup>1,‡</sup>

<sup>1</sup>State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China

<sup>2</sup>Physics Department, Faculty of Science, University of Zagreb, 10000 Zagreb, Croatia

<sup>3</sup>Center for Circuits and Systems, Peng Cheng Laboratory, Shenzhen 518055, China



裂变原子核在断裂点附近的局域化函数分布

- Z. X. Ren, J. Zhao, D. Vretenar, T. Nikšić, P. W. Zhao, and J. Meng, **Microscopic analysis of induced nuclear fission dynamics**, *Phys. Rev. C* **105**, 044313 (2022)
- Z. X. Ren, D. Vretenar, T. Nikšić, P. W. Zhao, J. Zhao, and J. Meng, **Dynamical synthesis of  $^4\text{He}$  in the scission phase of nuclear fission**, *Phys. Rev. Lett.* **128**, 172501 (2022)



PHYSICAL REVIEW C **105**, L011301 (2022)

Letter

## Dynamics of rotation in chiral nuclei

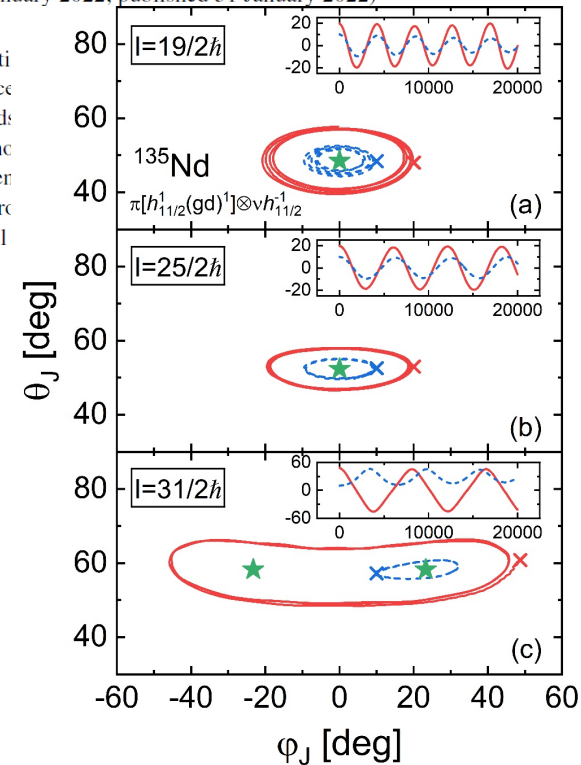
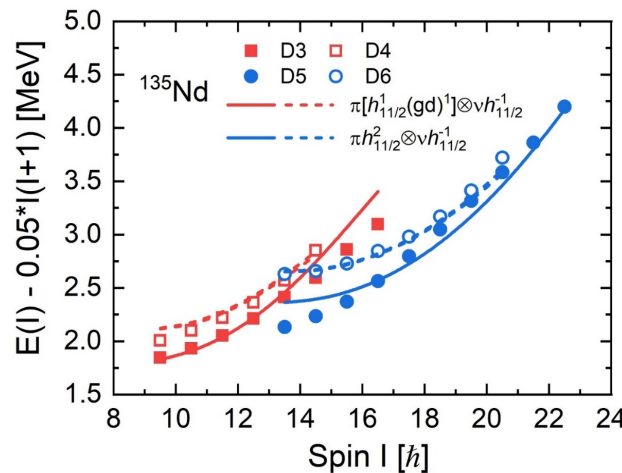
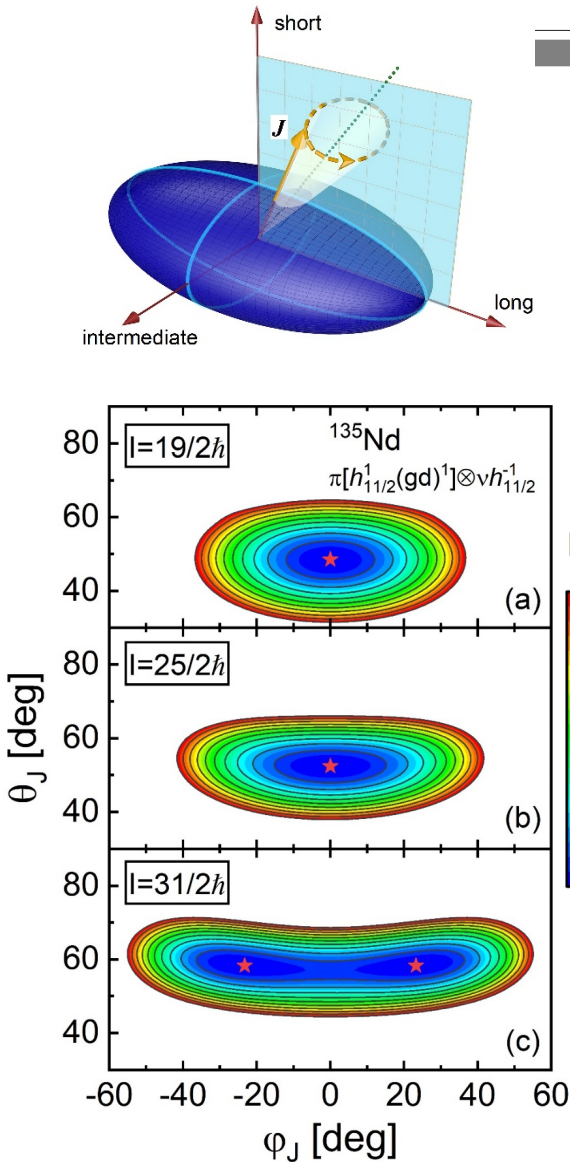
Z. X. Ren (任政学) , P. W. Zhao (赵鹏巍) ,\* and J. Meng (孟杰)

State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China

(Received 28 June 2021; revised 16 September 2021; accepted 14 January 2022; published 31 January 2022)

The dynamics of chiral nuclei is investigated for the first time with the time-dependent covariant density functional theories on a three-dimensional space lattice. The experimental energies of the two pairs of the chiral doublet bands [justable parameters beyond the well-defined density functional. A new feature revealed from the microscopic dynamics of the total angular momentum is associated with a transition from the planar into a planar configuration. This provides a fully microscopic and dynamical view to understand the chiral

[10.1103/PhysRevC.105.L011301](https://doi.org/10.1103/PhysRevC.105.L011301)





Article

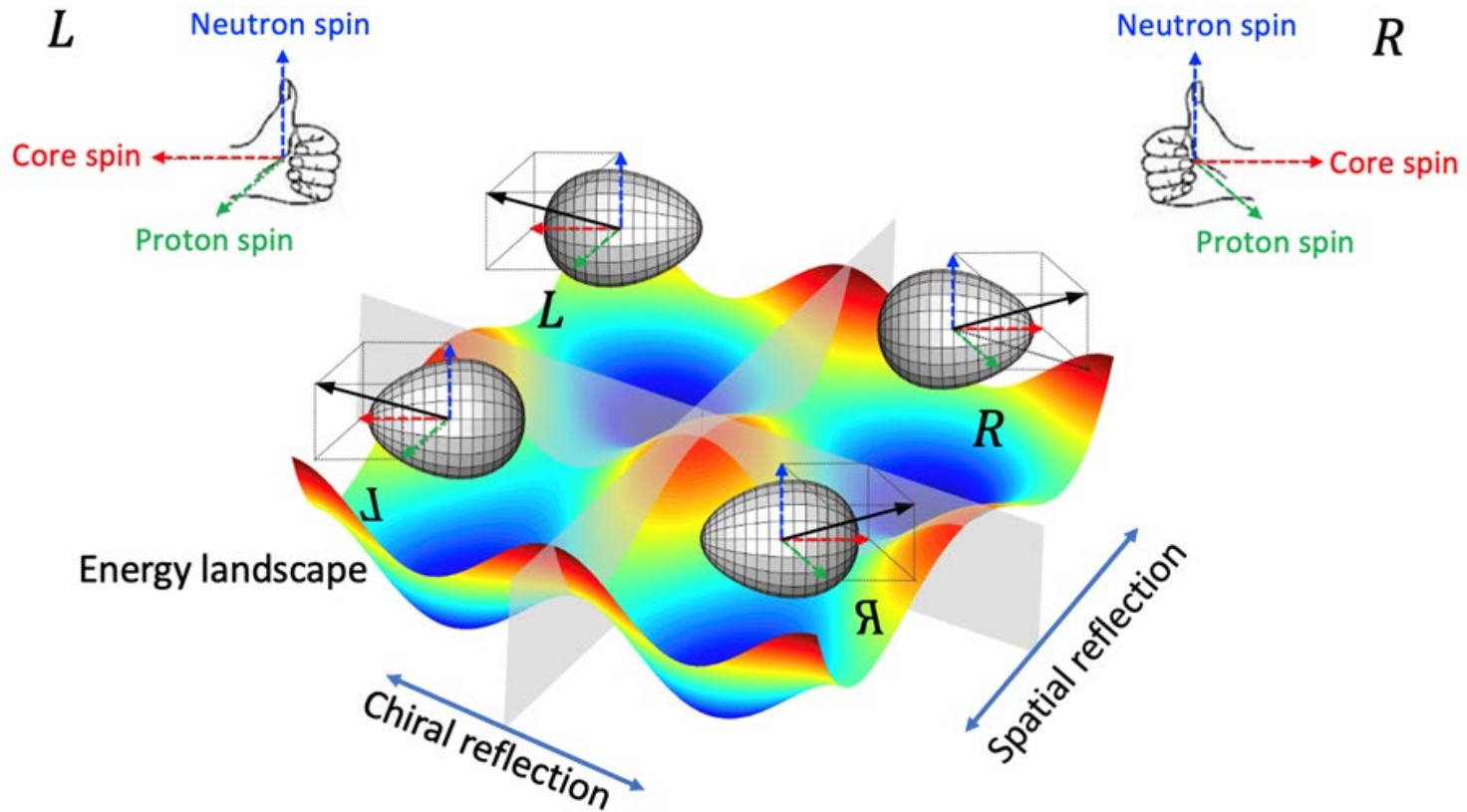
# Selection rules of electromagnetic transitions for chirality-parity violation in atomic nuclei

Yuanyuan Wang<sup>a</sup>, Xinhui Wu<sup>a</sup>, Shuangquan Zhang<sup>a,\*</sup>, Pengwei Zhao<sup>a</sup>, Jie Meng<sup>a,b,c</sup>

<sup>a</sup>State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China

<sup>b</sup>School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China

<sup>c</sup>Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

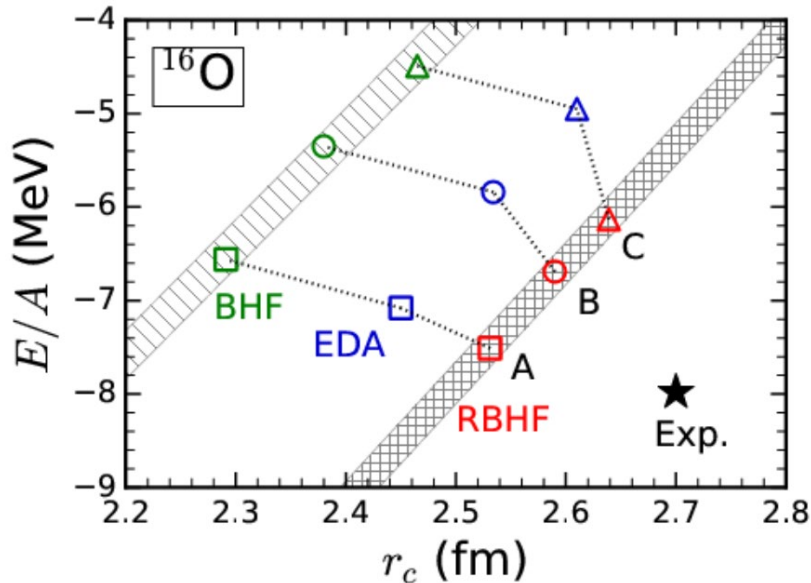


Nuclear Chirality-Parity (ChP) violation



# Relativistic Brueckner Hartree-Fock Theory

Shi-Hang Shen, Jin-Niu Hu, Hao-Zhao Liang, Jie Meng, Peter Ring, Shuang-Quan Zhang,  
Relativistic Brueckner–Hartree–Fock Theory for Finite Nuclei .



**Energies and charge radii of  $^{16}\text{O}$  in RBHF in comparison with EDA and BHF**

clear Physics 109 (2019) 103713



Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: [www.elsevier.com/locate/ppnp](http://www.elsevier.com/locate/ppnp)



Review

Towards an *ab initio* covariant density functional theory for nuclear structure

Shihang Shen<sup>a,b,c</sup>, Haozhao Liang<sup>d,e</sup>, Wen Hui Long<sup>f,g</sup>, Jie Meng<sup>a,h,i,\*</sup>, Peter Ring<sup>a,j</sup>

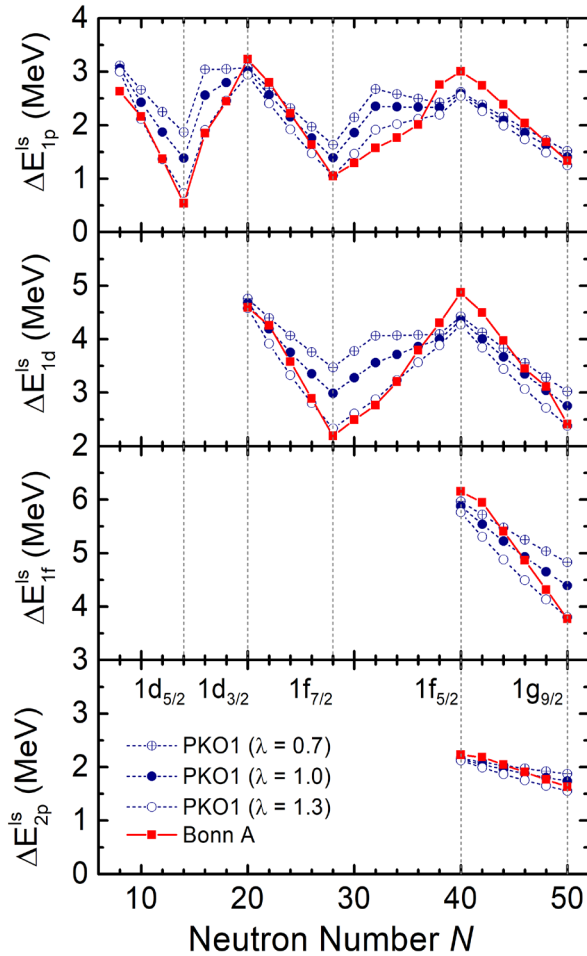
<sup>a</sup> State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China

<sup>b</sup> Dipartimento di Fisica, Università degli Studi di Milano, Italy

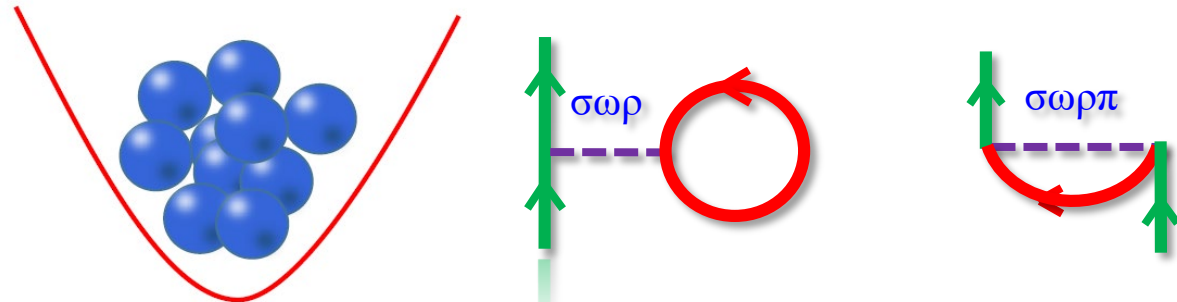
<sup>c</sup> INFN, Sezione di Milano, via Celoria 16, I-20133 Milano, Italy

<sup>d</sup> RIKEN Nishina Center, Wako 351-0198, Japan





- Neutron drop is a neutron system confined in an external field.
- A neutron drop provides also an ideal and simple system to investigate the effects of tensor forces.



Shi-Hang Shen, Hao-Zhao Liang, Jie Meng, Peter Ring, Shuang-Quan Zhang,

Effects of tensor forces in nuclear spin-orbit splittings from ab initio calculations.

**Phys. Lett. B** 778 (2018) 344–348

Relativistic Brueckner-Hartree-Fock theory for neutron drops

**Phys. Rev. C** 97, 054312 (2018)

- ❑ **RHF** shows similar pattern, mainly contributed by  **$\pi$ NN tensor interaction**.
- ❑ Neither RBHF nor CDFT includes **beyond-mean-field effects** ➔ a fair comparison!





Relativistic Brueckner-Hartree-Fock theory in nuclear matter without the average momentum approximation

Hui Tong (童辉),<sup>1</sup> Xiu-Lei Ren (任修磊),<sup>1,2</sup> Peter Ring,<sup>1,3</sup> Shi-Hang Shen (申时行),<sup>1</sup> Si-Bo Wang (王懿博),<sup>1</sup> and Jie Meng (孟杰)<sup>1,4,5,\*</sup>

<sup>1</sup>State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, C

<sup>2</sup>Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany

<sup>3</sup>Physik-Department der Technischen Universität München, D-85748 Garching, Germany

<sup>4</sup>Department of Physics, University of Stellenbosch, Stellenbosch, South Africa

Nuclear matter in relativistic Brueckner-Hartree-Fock theory with Bonn potential in the full Dirac space

Sibo Wang,<sup>1</sup> Qiang Zhao,<sup>1</sup> Peter Ring ,<sup>1,2</sup> and Jie Meng ,<sup>1,3,\*</sup>

<sup>1</sup>State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China

<sup>2</sup>Department of Physik, Technische Universität München, D-85747 Garching, Germany

<sup>3</sup>Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

(Received 25 March 2021; accepted 11 May 2021; published 24 May 2021)

Starting from the Bonn potential, the relativistic Brueckner-Hartree-Fock (RBHF) equations are solved for nuclear matter in the full Dirac space, which provides a unique way to determine the single-particle potentials and avoids the approximations applied in the RBHF calculations in the Dirac space with positive-energy states (PESs) only. The uncertainties of the RBHF calculations in the Dirac space with PESs only are investigated, and the importance of RBHF calculations in the full Dirac space is demonstrated. In the RBHF calculations in the full Dirac space, the empirical saturation properties of symmetric nuclear matter are reproduced, and the obtained equation of state agrees with the results based on the relativistic Green's function approach up to the saturation density.

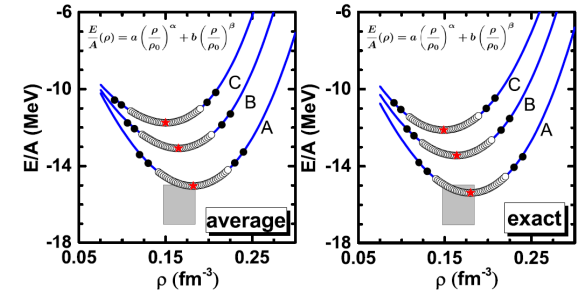
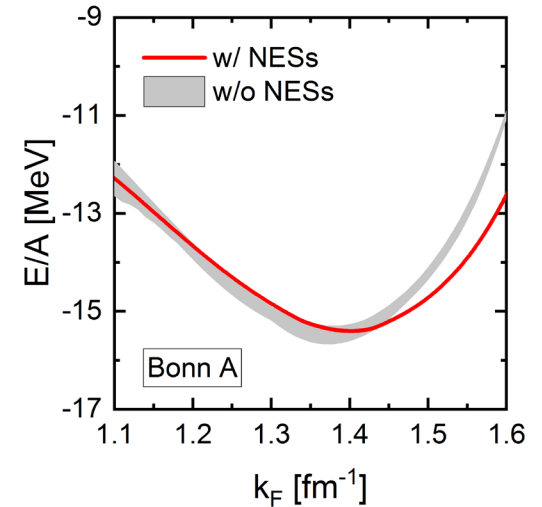


FIG. 1. Binding energy per nucleon for nuclear matter as a function of the total density  $\rho$ . Results for Bonn potentials A, B, C with (left panel) and without (right panel) c.m. momentum approximation are shown. The RBHF results are represented by open and solid circles, where open circles stand for the data used in the fit and solid circles indicate the validity of the results of the fit (solid curves). The red stars indicate the saturation points obtained from RBHF results.





## Chiral Nucleon-Nucleon Interaction

X. L. Ren, K. W. Li, L. S. Geng, B. W. Long, P. Ring, and J. Meng,

Leading order relativistic chiral nucleon-nucleon interaction,

*Chin. Phys. C* 42, 014103(2018) J

### PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press About Staff

Access by Beijing Peking Universit

#### Accurate Relativistic Chiral Nucleon-Nucleon Interaction up to Next-to-Next-to-Leading Order

Jun-Xu Lu, Chun-Xuan Wang, Yang Xiao, Li-Sheng Geng, Jie Meng, and Peter Ring  
*Phys. Rev. Lett.* **128**, 142002 – Published 6 April 2022

## Baryon Interaction by lattice QCD

### PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press About Staff

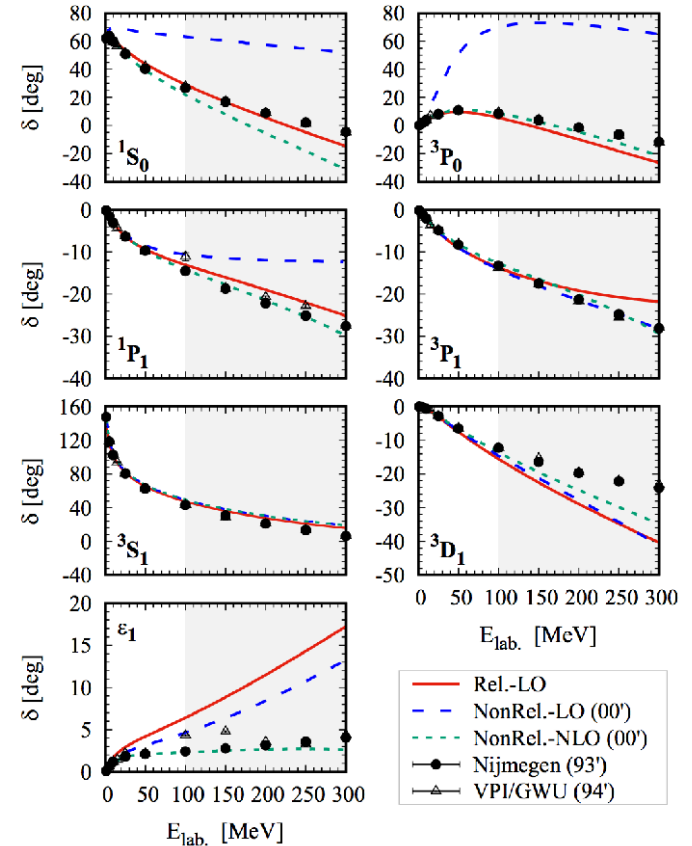
Accepted Paper

#### Dibaryon with highest charm number near unitarity from lattice QCD

*Phys. Rev. Lett.*

Yan Lyu, Hui Tong, Takuya Sugiura, Sinya Aoki, Takumi Doi, Tetsuo Hatsuda, Jie Meng, and Takaya Miyamoto

Accepted 2 July 2021





$$E[\rho] = T[\rho] + U[\rho] + \int V(\mathbf{r})\rho(\mathbf{r}) d^3\mathbf{r}$$

H-K Theorem proves the existence of a universal functional depends solely on density.

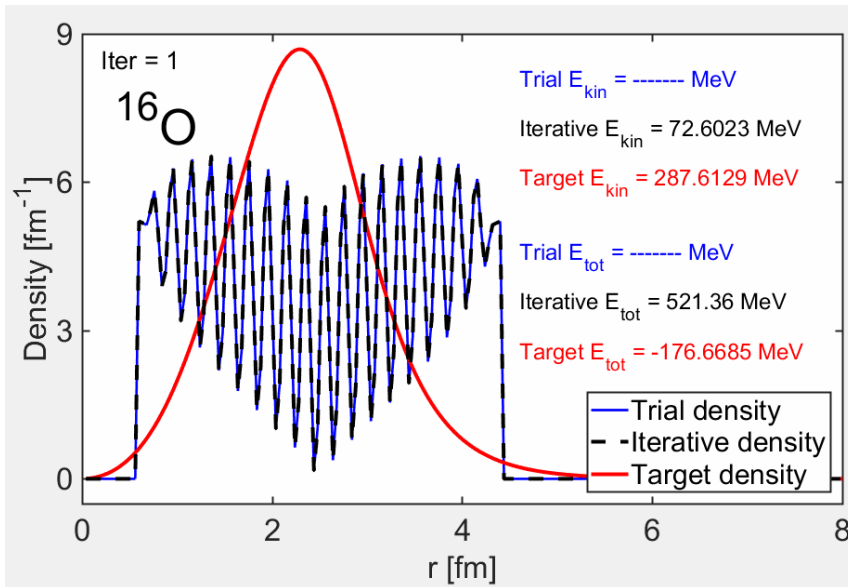
BUT, all previous attempts for a nuclear kinetic energy functional FAILS.

$$T[\rho] \doteq \sum_{i=1}^N \left\langle \varphi_i \left| -\frac{\hbar^2}{2m} \nabla^2 \right| \varphi_i \right\rangle$$

→ One has to introduce Kohn-Sham, i.e., a functional of orbits

By **Machine Learning**, a **robust and accurate orbital-free** density functional is established !

Robust self-consistent solution



Most accurate ever orbit-free DFT for nuclei

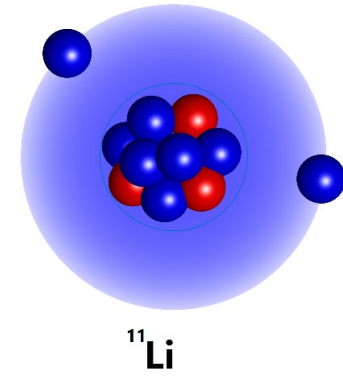
	Kohn-Sham	Machine-Learning	Experiment
${}^4\text{He}$ $E_{\text{tot}}$	-26.3700	-26.3931 (0.0012)	-28.2957
${}^4\text{He}$ $E_{\text{kin}}$	35.2138	35.2044 (0.0056)	/
${}^4\text{He}$ $\langle r^2 \rangle$	2.1626	2.1628 (0.0002)	1.6755
${}^{16}\text{O}$ $E_{\text{tot}}$	-127.3781	-127.1622 (0.1584)	-127.6193
${}^{16}\text{O}$ $E_{\text{kin}}$	219.2875	218.3458 (0.6882)	/
${}^{16}\text{O}$ $\langle r^2 \rangle$	2.8077	2.8113 (0.0047)	2.6991
${}^{40}\text{Ca}$ $E_{\text{tot}}$	-342.0645	-341.8027 (0.5724)	-342.0521
${}^{40}\text{Ca}$ $E_{\text{kin}}$	643.1100	642.9145 (1.6875)	/
${}^{40}\text{Ca}$ $\langle r^2 \rangle$	3.4677	3.4652 (0.0055)	3.4776



- ◆ Introduction
- ◆ Theoretical framework
- ◆ Results and discussion
- ◆ Summary



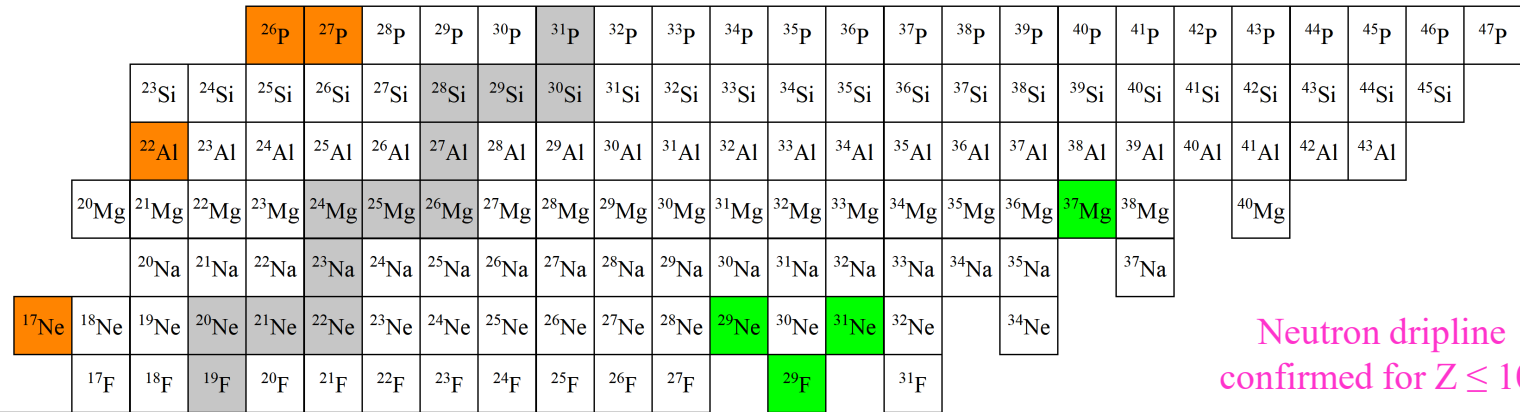
# Halo nuclei



$^{11}\text{Li}$

- Halo nuclei have attracted lots of attention since the discovery of the halo phenomenon in  $^{11}\text{Li}$ .

Tanihata *et al.*, PRL 55, 2676 (1985)  
Tanihata *et al.*, PPNP 68, 215 (2013)



Neutron dripline  
confirmed for  $Z \leq 10$

- Stable nuclei
- Proton-halo candidates
- Neutron-halo candidates

$^{22}\text{Al}$ : Lee *et al.*, PRL 125, 192503 (2020)  
 $^{29}\text{F}$ : Bagchi *et al.*, PRL 124, 222504 (2020)  
 $^{37}\text{Mg}$ : Kobayashi *et al.*, PRL 112, 242501 (2014)





- Few-body model Zhukov *et al.*, Phys. Rep. 231, 151 (1993)
- Shell model Otsuka *et al.*, PRL 70, 1385 (1993)
- Antisymmetrized molecular dynamics model Horiuchi *et al.*, ZPA 349, 279 (1994)
- Halo effective field theory Bertulani *et al.*, NPA 712, 37 (2002)
- Non-relativistic density functional theory Terasaki *et al.*, NPA 600, 371 (1996)
- **Covariant density functional theory (CDFT)** Koepf *et al.*, ZPA 340, 119 (1991)
  
- Advantages of the CDFT:
  - ✓ Spin-orbit automatically included Ring, PPNP 37, 193 (1996)
  - ✓ Lorentz covariance restricts parameters Vretenar *et al.*, Phys. Rep. 409, 101 (2005)
  - ✓ Pseudo-spin Symmetry Meng *et al.*, PPNP 57, 470 (2006)
  - ✓ Consistent treatment of time-odd fields Niksic *et al.*, PPNP 66, 519 (2011)
  - ✓ Relativistic saturation mechanism Meng and Zhou, JPG 42, 093101 (2015)
  - ✓ ... Liang *et al.*, Phys. Rep. 570, 1 (2015)  
Shen *et al.*, PPNP 109, 103713 (2019)



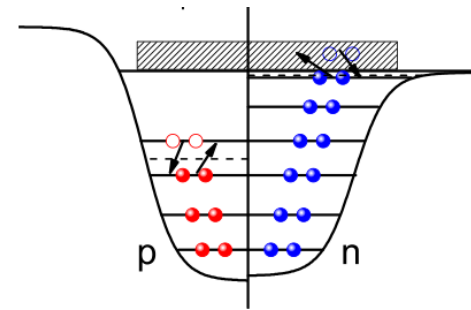
- ✓ Based on the CDFT, a self-consistent description of the neutron halo in  $^{11}\text{Li}$  is achieved by **the relativistic continuum Hartree-Bogoliubov (RCHB) theory**.

Meng and Ring, PRL 77, 3963 (1996)

- ✓ Pairing correlations and continuum effects are very important for the description of halos.

Meng and Ring, PRL 77, 3963 (1996)

Meng *et al.*, PPNP 57, 470 (2006)



- ✓ The RCHB theory also predicts giant halos in Zr isotopes.

Meng and Ring, PRL 80, 460 (1998)

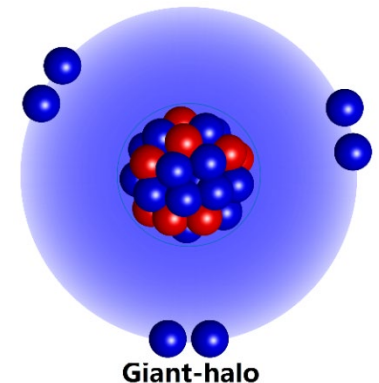
- ✓ Later studies based on DFT and CDFT support the prediction of giant halos in Zr and Ca isotopes.

Meng, Toki, Zeng, Zhang, and Zhou, PRC 65, 041302(R) (2002)

Sandulescu, Geng, and Hillhouse, PRC 68, 054323 (2003)

Terasaki, Zhang, Zhou, Meng, PRC 74, 054318 (2006)

Grasso, Yoshida, Sandulescu, and Van Giai, PRC 74, 064317 (2006)





- ✓ Based on the RCHB theory, a nuclear mass table including continuum effects has been constructed.

Times Cited: 97 (Web of Science)

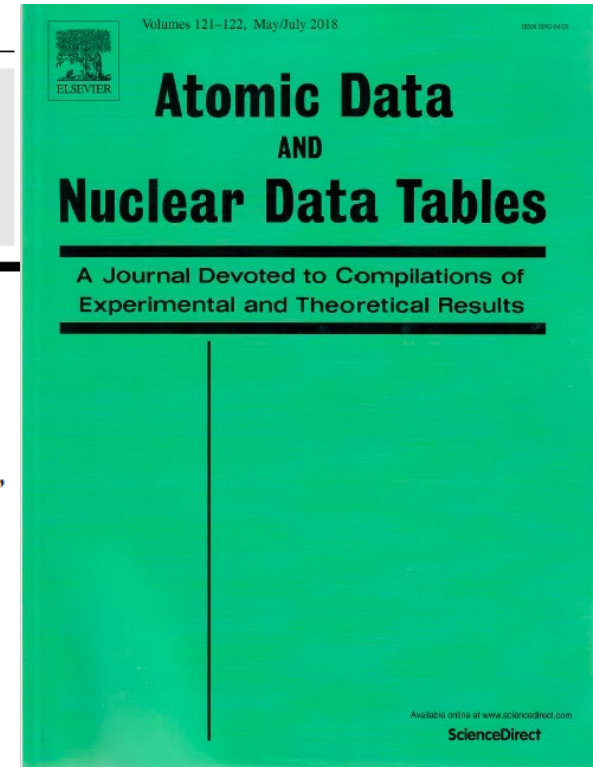
Atomic Data and Nuclear Data Tables 121–122 (2018) 1–215



Contents lists available at ScienceDirect

## Atomic Data and Nuclear Data Tables

journal homepage: [www.elsevier.com/locate/adt](http://www.elsevier.com/locate/adt)



### The limits of the nuclear landscape explored by the relativistic continuum Hartree–Bogoliubov theory

X.W. Xia<sup>a</sup>, Y. Lim<sup>b,c</sup>, P.W. Zhao<sup>d,e</sup>, H.Z. Liang<sup>f</sup>, X.Y. Qu<sup>a,g</sup>, Y. Chen<sup>d,h</sup>, H. Liu<sup>d</sup>, L.F. Zhang<sup>d</sup>, S.Q. Zhang<sup>d</sup>, Y. Kim<sup>c</sup>, J. Meng<sup>d,a,i,\*</sup>

<sup>a</sup> School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China

<sup>b</sup> Cyclotron Institute, Texas A&M University, College Station, TX 77843, USA

<sup>c</sup> Rare Isotope Science Project, Institute for Basic Science, Daejeon 305-811, Republic of Korea

<sup>d</sup> State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China

<sup>e</sup> Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA

<sup>f</sup> RIKEN Nishina Center, Wako 351-0198, Japan

<sup>g</sup> School of Mechatronics Engineering, Guizhou Minzu University, China

<sup>h</sup> Institute of materials, China Academy of Engineering Physics, Sichuan, 621907, China

<sup>i</sup> Department of Physics, University of Stellenbosch, Stellenbosch, South Africa

#### ARTICLE INFO

##### Article history:

Received 2 May 2017

Received in revised form 12 August 2017

Accepted 5 September 2017

Available online 1 November 2017

#### ABSTRACT

The ground-state properties of nuclei with  $8 \leq Z \leq 120$  from the proton drip line to the neutron drip line have been investigated using the spherical relativistic continuum Hartree–Bogoliubov (RCHB) theory with the relativistic density functional PC-PK1. With the effects of the continuum included, there are totally 9035 nuclei predicted to be bound, which largely extends the existing nuclear landscapes predicted with other methods. The calculated binding energies, separation energies, neutron and proton Fermi surfaces,



# Deformed halos

✓ There has been controversy over the existence of deformed halo nuclei.

Otsuka, Muta, Yokoyama, Fukunishi, and Suzuki, NPA 588, 113c (1995)

Misu, Nazarewicz, and Aberg, NPA 614, 44 (1997)

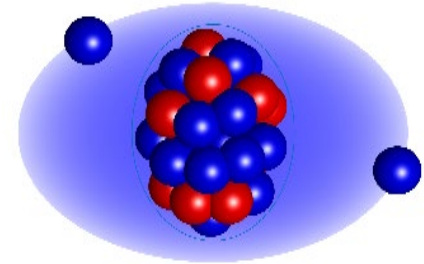
Tanihata, Hirata, and Toki, NPA 583, 769 (1995)

Nunes, NPA 757, 349 (2005)

✓ Considering axial deformation, pairing, and continuum effects, **the deformed relativistic Hartree-Bogoliubov theory in continuum (DRHBc)** predicts deformed halo nuclei.

Zhou, Meng, Ring, and Zhao, PRC 82, 011301(R) (2010)

Li, Meng, Ring, Zhao, and Zhou, PRC 85, 024312 (2012)



✓ Recently, several candidates of deformed halo nuclei have been suggested in experiment, such as  $^{31}\text{Ne}$  and  $^{37}\text{Mg}$ .

Nakamura *et al.*, PRL 112, 142501 (2014)

Kobayashi *et al.*, PRL 112, 242501 (2014)

✓ The DRHBc theory has been applied to study halo and other exotic phenomena.

Sun, Zhao, and Zhou, PLB 785, 530 (2018)

Zhang, Wang, and Zhang PRC 100, 034312 (2019)

Sun, Zhao, and Zhou, NPA 1003, 122011 (2020)

Sun, PRC 103, 054315 (2021)

Zhang *et al.*, PRC 104, L021301 (2021)

Pan *et al.*, PRC 104, 024331 (2021)

He *et al.*, CPC 45, 101001 (2021)





北京大學  
PEKING UNIVERSITY

# DRHBc Mass Table

- ✓ To construct a nuclear mass table including both deformation and continuum effects, the DRHBc Mass Table Collaboration was established.

Zhang *et al.* (DRHBc Mass Table Collaboration), PRC 102, 024314 (2020)



- ✓ Recently, the DRHBc mass table for even-even nuclei has been constructed, and it for odd nuclei is in progress.

Zhang *et al.* (DRHBc Mass Table Collaboration), ADNDT 144, 101488 (2022)

Pan *et al.* (DRHBc Mass Table Collaboration), arXiv:2205.01329





- **PC-PK1 + DRHBc, 2583 even-even nuclei with  $8 \leq Z \leq 120$ , first mass table including both deformation and continuum,  $\sigma = 1.5$  MeV**

K. Y. Zhang *et al.* (DRHBc Mass Table Collaboration), Nuclear mass table in deformed relativistic Hartree–Bogoliubov theory in continuum, I: Even–even nuclei, **At. Data Nucl. Data Tables** 144, 101488 (2022)

Atomic Data and Nuclear Data Tables 144 (2022) 101488



ELSEVIER

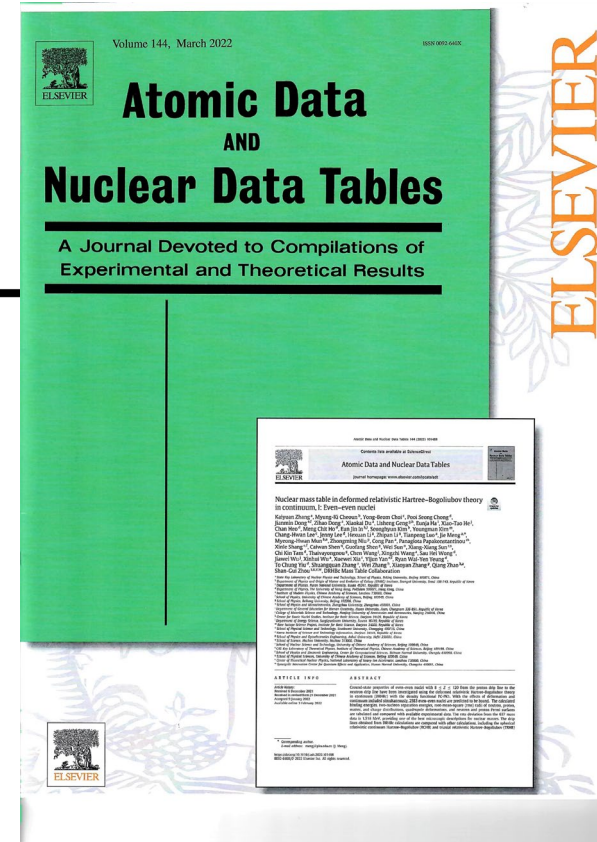
Contents lists available at ScienceDirect

Atomic Data and Nuclear Data Tables

journal homepage: [www.elsevier.com/locate/adt](http://www.elsevier.com/locate/adt)

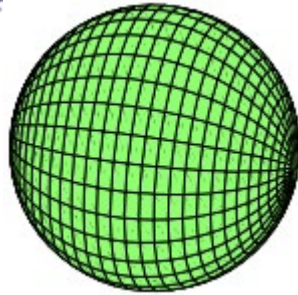
## Nuclear mass table in deformed relativistic Hartree–Bogoliubov theory in continuum, I: Even–even nuclei

Kaiyuan Zhang<sup>a</sup>, Myung-Ki Cheoun<sup>b</sup>, Yong-Beom Choi<sup>c</sup>, Pooi Seong Chong<sup>d</sup>, Jianmin Dong<sup>e,f</sup>, Zihao Dong<sup>a</sup>, Xiaokai Du<sup>a</sup>, Lisheng Geng<sup>g,h</sup>, Eunja Ha<sup>i</sup>, Xiao-Tao He<sup>j</sup>, Chan Heo<sup>d</sup>, Meng Chit Ho<sup>d</sup>, Eun Jin In<sup>k,l</sup>, Seonghyun Kim<sup>b</sup>, Youngman Kim<sup>m</sup>, Chang-Hwan Lee<sup>c</sup>, Jenny Lee<sup>d</sup>, Hexuan Li<sup>a</sup>, Zhipan Li<sup>n</sup>, Tianpeng Luo<sup>a</sup>, Jie Meng<sup>a,\*</sup>, Myeong-Hwan Mun<sup>b,o</sup>, Zhongming Niu<sup>p</sup>, Cong Pan<sup>a</sup>, Panagiota Papakonstantinou<sup>m</sup>, Xinle Shang<sup>e,f</sup>, Caiwan Shen<sup>q</sup>, Guofang Shen<sup>g</sup>, Wei Sun<sup>n</sup>, Xiang-Xiang Sun<sup>r,s</sup>, Chi Kin Tam<sup>d</sup>, Thaivayongnou<sup>g</sup>, Chen Wang<sup>j</sup>, Xingzhi Wang<sup>a</sup>, Sau Hei Wong<sup>d</sup>, Jiawei Wu<sup>j</sup>, Xinhui Wu<sup>a</sup>, Xuewei Xia<sup>t</sup>, Yijun Yan<sup>e,f</sup>, Ryan Wai-Yen Yeung<sup>d</sup>, To Chung Yiu<sup>d</sup>, Shuangquan Zhang<sup>a</sup>, Wei Zhang<sup>h</sup>, Xiaoyan Zhang<sup>p</sup>, Qiang Zhao<sup>k,a</sup>, Shan-Gui Zhou<sup>s,u,v,w</sup>, DRHBc Mass Table Collaboration

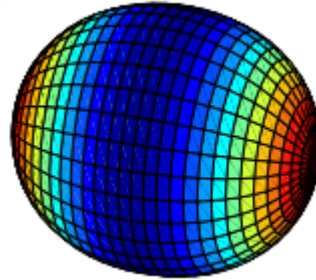




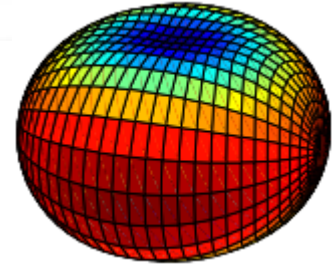
# Triaxially deformed halo nucleus?



Spherical



Axial



Triaxial

- ✓ **Triaxial deformation** plays a crucial role in nuclear chirality, wobbling motion, and fission.

Frauendorf and Meng, Nucl. Phys. A 617, 131 (1997)

Bohr and Mottelson, *Nuclear Structure* (1975)

Lu, Zhao, and Zhou, PRC 85, 011301(R) (2012)

- ✓ A study based on triaxial Woods-Saxon potential suggests that the triaxial deformation might extend the region of halo nuclei.

Uzawa, Hagino, and Yoshida, PRC 104, L011303 (2021)

Recently, a triaxial relativistic Hartree-Bogoliubov theory in continuum (TRHBc), which considers triaxial deformation, pairing, and continuum effects, is developed to search for triaxially deformed halo nuclei.



- ◆ Introduction
- ◆ Theoretical framework
- ◆ Results and discussion
- ◆ Summary



✓ The starting point is a effective Lagrangian density,

$$\mathcal{L} = \mathcal{L}^{\text{free}} + \mathcal{L}^{4\text{f}} + \mathcal{L}^{\text{der}} + \mathcal{L}^{\text{hot}} + \mathcal{L}^{\text{em}},$$

$$\mathcal{L}^{\text{free}} = \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - M)\psi.$$

$$\begin{aligned} \mathcal{L}^{4\text{f}} = & -\frac{1}{2}\alpha_S(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi) \\ & - \frac{1}{2}\alpha_{TS}(\bar{\psi}\vec{\tau}\psi)(\bar{\psi}\vec{\tau}\psi) - \frac{1}{2}\alpha_{TV}(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi)(\bar{\psi}\vec{\tau}\gamma^{\mu}\psi). \end{aligned}$$

$$\begin{aligned} \mathcal{L}^{\text{der}} = & -\frac{1}{2}\delta_S\partial_{\nu}(\bar{\psi}\psi)\partial^{\nu}(\bar{\psi}\psi) - \frac{1}{2}\delta_V\partial_{\nu}(\bar{\psi}\gamma_{\mu}\psi)\partial^{\nu}(\bar{\psi}\gamma^{\mu}\psi) \\ & - \frac{1}{2}\delta_{TS}\partial_{\nu}(\bar{\psi}\vec{\tau}\psi)\partial^{\nu}(\bar{\psi}\vec{\tau}\psi) - \frac{1}{2}\delta_{TV}\partial_{\nu}(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi)\partial^{\nu}(\bar{\psi}\vec{\tau}\gamma^{\mu}\psi) \end{aligned}$$

$$\mathcal{L}^{\text{hot}} = -\frac{1}{3}\beta_S(\bar{\psi}\psi)^3 - \frac{1}{4}\gamma_S(\bar{\psi}\psi)^4 - \frac{1}{4}\gamma_V[(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi)]^2$$

$$\mathcal{L}^{\text{em}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\bar{\psi}\gamma^{\mu}\frac{1-\tau_3}{2}A_{\mu}\psi$$

$M$ : nucleon mass

$\alpha_S, \alpha_V, \alpha_{TV}, \delta_S, \delta_V, \delta_{TV}, \beta_S, \gamma_S, \gamma_V$ : coupling constants



- ✓ Relativistic Hartree-Bogoliubov (RHB) equations for nucleons read

$$\begin{pmatrix} \hat{h}_D - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D^* + \lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix} .$$

- $\lambda$  is the Fermi energy,  $E_k$  is the quasi-particle energy, and  $(U_k, V_k)^T$  is the quasi-particle wave function.
- The Dirac Hamiltonian  $h_D(\mathbf{r}) = \boldsymbol{\alpha} \cdot [\mathbf{p} - \mathbf{V}(\mathbf{r})] + V^0(\mathbf{r}) + \beta[M + S(\mathbf{r})]$

$$\begin{aligned} S(\mathbf{r}) &= \alpha_S \rho_S + \beta_S \rho_S^2 + \gamma_S \rho_S^3 + \delta_S \Delta \rho_S, \\ V^\mu(\mathbf{r}) &= \alpha_V j^\mu + \gamma_V (j_\mu j^\mu) j^\mu + \delta_V \Delta j^\mu + e A^\mu \\ &\quad + \alpha_{TV} j_3^\mu + \delta_{TV} \Delta j_3^\mu. \end{aligned}$$

$$\begin{aligned} \rho_S(\mathbf{r}) &= \sum_k \bar{V}_k(\mathbf{r}) V_k(\mathbf{r}), \\ j^\mu(\mathbf{r}) &= \sum_k \bar{V}_k(\mathbf{r}) \gamma^\mu V_k(\mathbf{r}), \\ j_3^\mu(\mathbf{r}) &= \sum_k \bar{V}_k(\mathbf{r}) \gamma^\mu \tau_3 V_k(\mathbf{r}), \\ j_c^\mu(\mathbf{r}) &= \sum_k \bar{V}_k(\mathbf{r}) \gamma^\mu \frac{1 - \tau_3}{2} V_k(\mathbf{r}). \end{aligned}$$

- The pairing potential  $\Delta(\mathbf{r}_1, \mathbf{r}_2) = V^{PP}(\mathbf{r}_1, \mathbf{r}_2) \kappa(\mathbf{r}_1, \mathbf{r}_2)$





- ✓ To include triaxial deformation, densities and potentials are expanded in terms of spherical harmonic functions,

$$f(\mathbf{r}) = \sum_{\lambda\mu} f_{\lambda\mu}(r) Y_{\lambda\mu}(\theta, \varphi)$$

- ✓ According to spatial reflection symmetry and reflection symmetries with respect to  $xy$ ,  $xz$ , and  $yz$  plane,

$$\hat{P} f(\mathbf{r}) = \sum_{\lambda\mu} f_{\lambda\mu}(r) Y_{\lambda\mu}(\pi - \theta, \pi + \varphi) = \sum_{\lambda\mu} f_{\lambda\mu}(r) (-1)^\lambda Y_{\lambda\mu}(\theta, \varphi)$$

$$\hat{P}_z f(\mathbf{r}) = \sum_{\lambda\mu} f_{\lambda\mu}(r) Y_{\lambda\mu}(\pi - \theta, \varphi) = \sum_{\lambda\mu} f_{\lambda\mu}(r) (-1)^{\lambda+\mu} Y_{\lambda\mu}(\theta, \varphi)$$

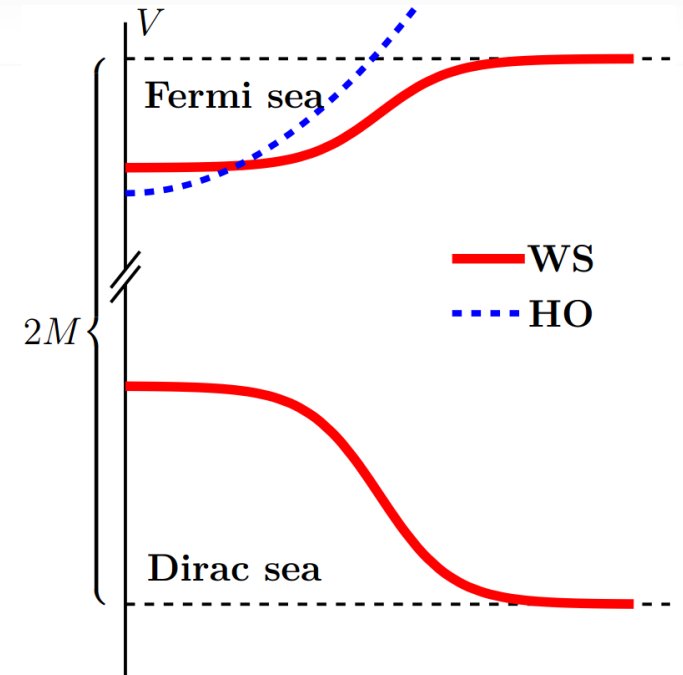
$$\hat{P}_x f(\mathbf{r}) = \sum_{\lambda\mu} f_{\lambda\mu}(r) Y_{\lambda\mu}(\theta, \pi - \varphi) = \sum_{\lambda\mu} f_{\lambda\mu}(r) Y_{\lambda-\mu}(\theta, \varphi)$$

$\lambda, \mu$  are restricted to be even numbers, and  $-\mu$  component is equal to  $\mu$  component.



# Dirac Woods-Saxon basis

- ✓ In the TRHBc theory, the RHB equations are solved in a **Dirac Woods-Saxon (DWS) basis**, whose wave function has a more realistic behavior at large  $r$  compared with the HO basis.



- ✓ The DWS basis is obtained by solving a Dirac equation containing the spherical Woods-Saxon potential. Because the basis function itself is a Dirac spinor, there is no need to expand upper and lower components separately.

Zhou, Meng, and Ring, PRC 68, 034323 (2003)

$$\hat{h}_0 |\varphi_{n\kappa m}\rangle = \epsilon_{n\kappa} |\varphi_{n\kappa m}\rangle, \quad \varphi_{n\kappa m}(\mathbf{r}\sigma) = \frac{1}{r} \begin{pmatrix} iG_{n\kappa}(r) Y_{jm}^l(\Omega\sigma) \\ -F_{n\kappa}(r) Y_{jm}^{\tilde{l}}(\Omega\sigma) \end{pmatrix}$$



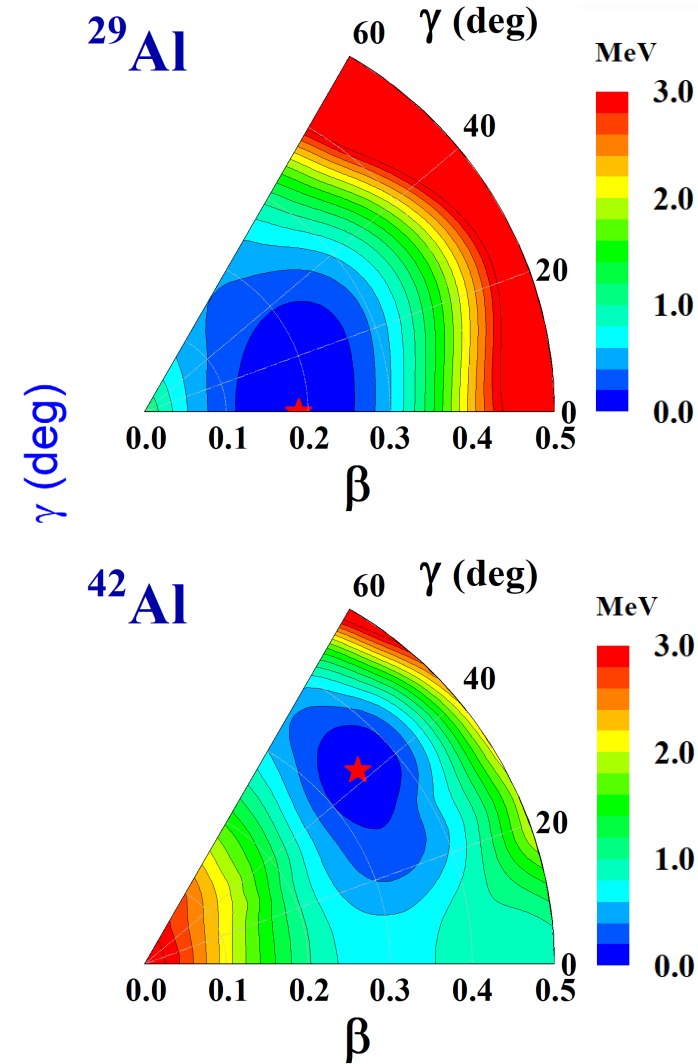
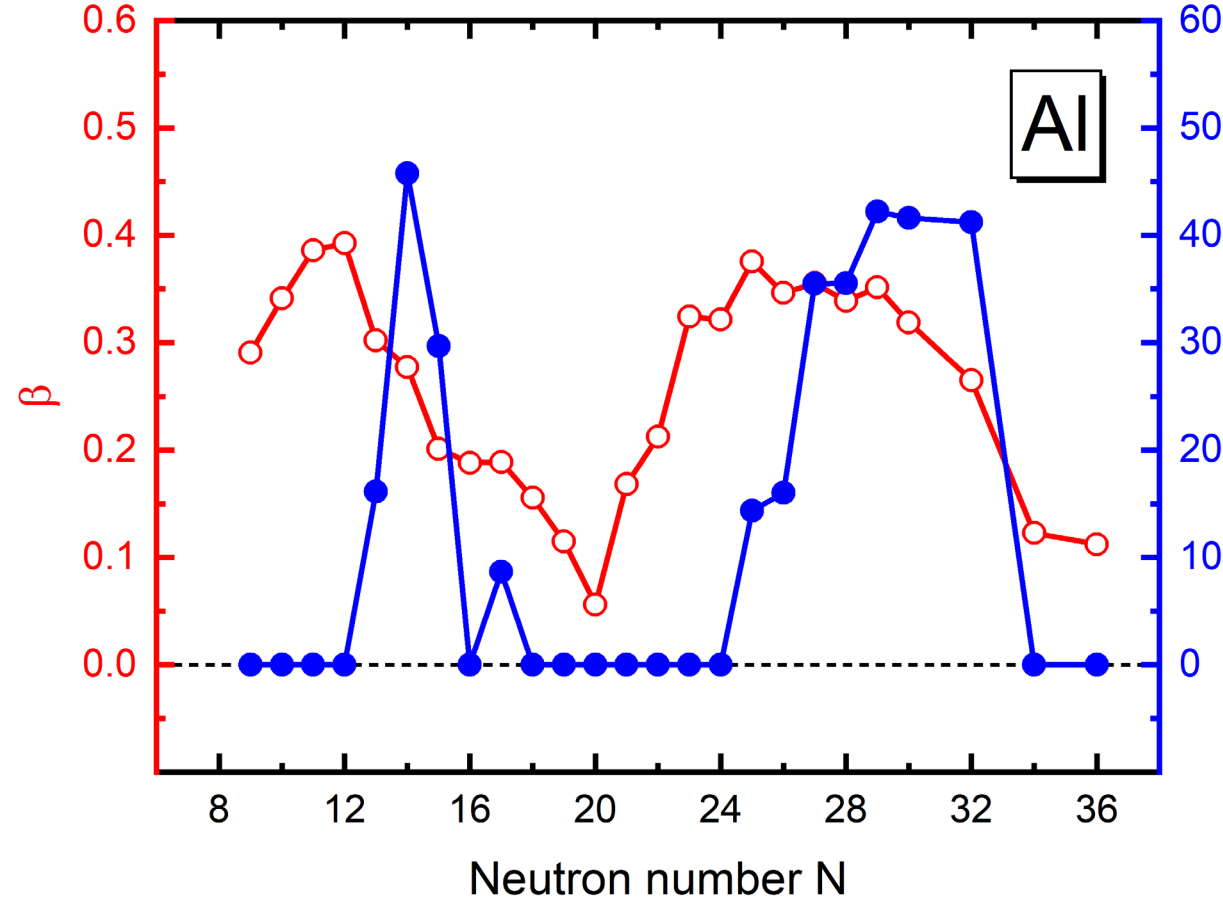
- ◆ Introduction
- ◆ Theoretical framework
- ◆ Results and discussion
- ◆ Summary



- ✓ Density functional: **PC-PK1**, NL3\*, NLSH, PK1
- ✓ Pairing interaction: Density-dependent zero-range pairing force
- ✓ Box size:  $R_{\text{box}} = 20 \text{ fm}$
- ✓ Mesh size:  $\Delta r = 0.1 \text{ fm}$
- ✓ Energy cutoff:  $E_{\text{cut}} = 300 \text{ MeV}$
- ✓ Angular momentum cutoff:  $J_{\text{max}} = 19/2 \hbar$
- ✓ Pairing strength:  $V_0 = -342.5 \text{ MeV fm}^3$   
*Xia et al., ADNDT 121–122, 1 (2018)*
- ✓ Spherical harmonic expansion order:  $\lambda_{\text{max}} = 6$

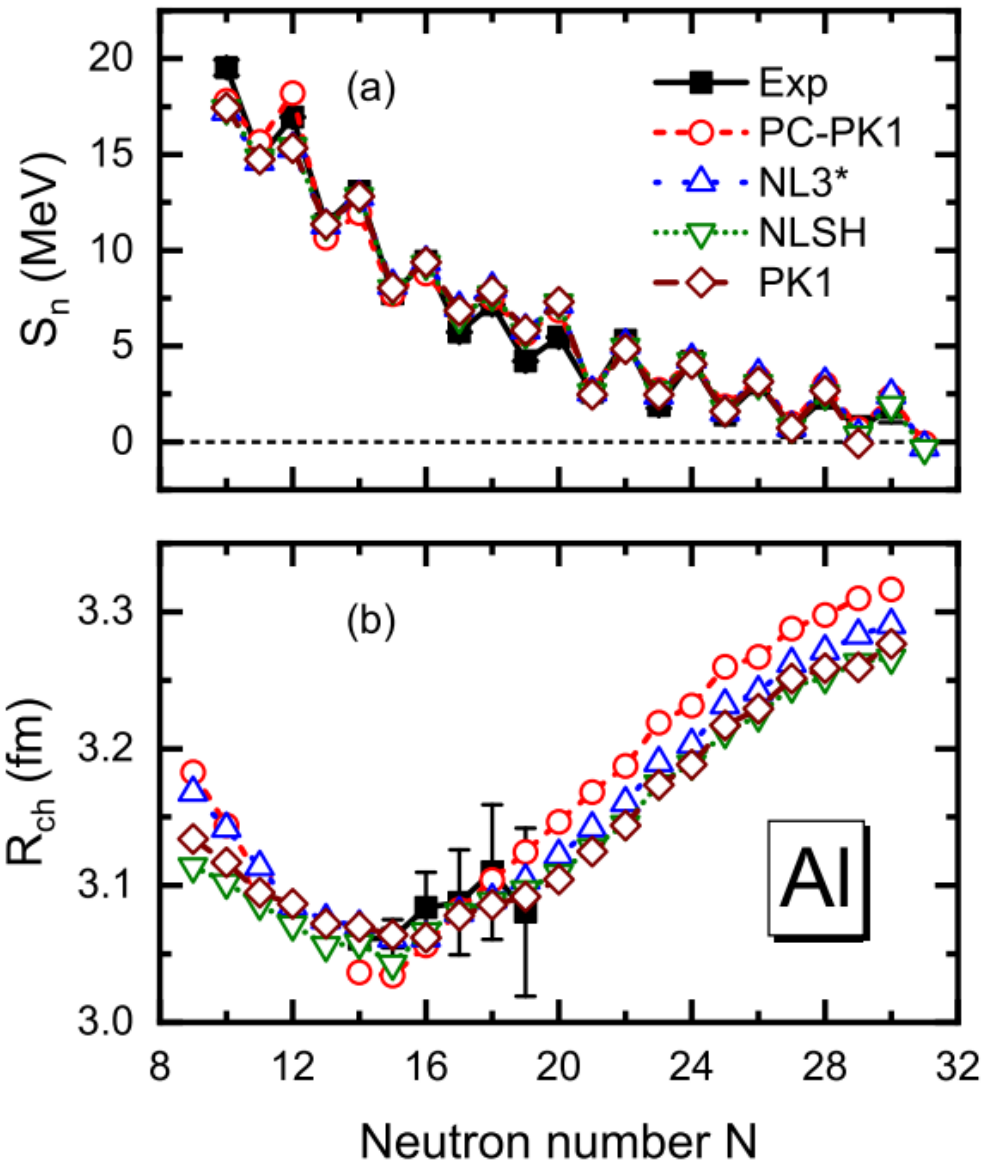


# Deformation of Al isotopes



✓ Light neutron-rich nuclei are calculated, and several Al isotopes are found to be triaxially deformed.





$S_n$  data taken from CPC 45, 030003 (2021)  
[Experimental values for  $11 \leq N \leq 25$ ]

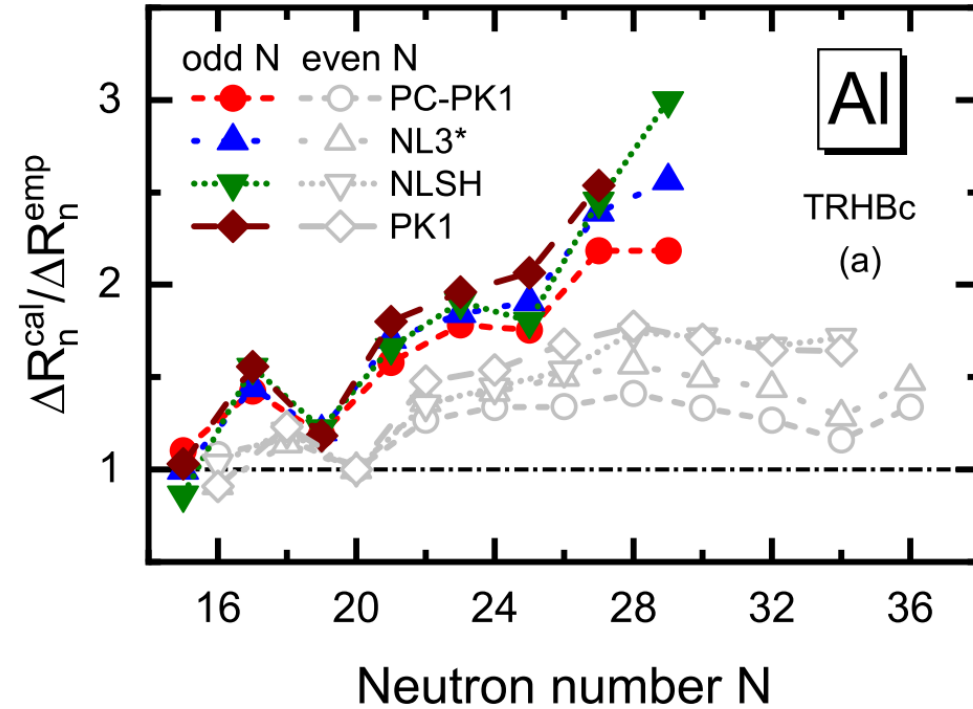
$R_{ch}$  data taken from PRC 103, 014318 (2021)

✓ AME2020 data and experimental charge radii are reproduced well.

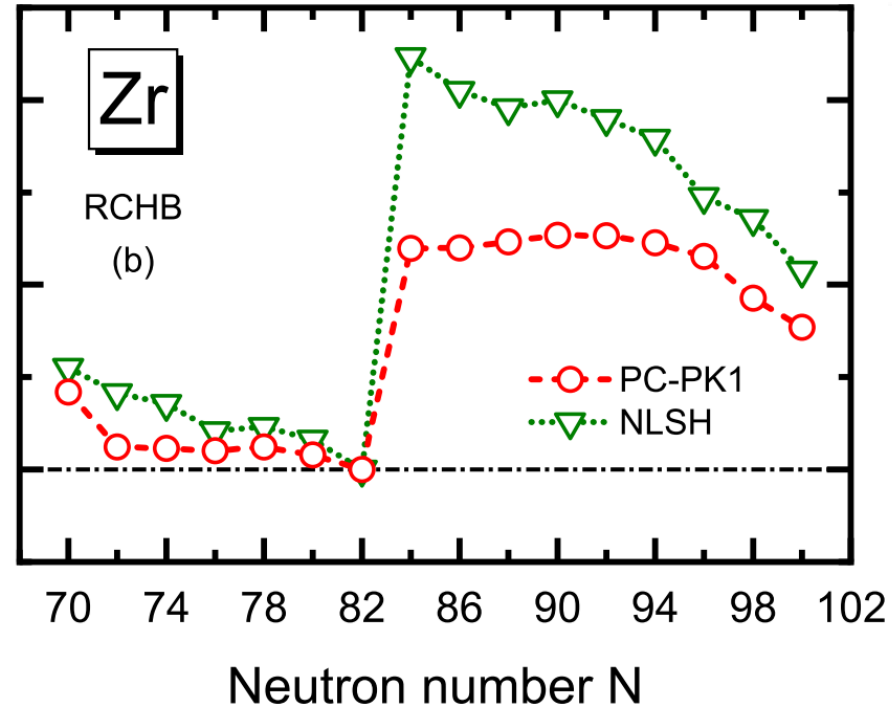
✓  $^{42}\text{Al}$  is triaxially deformed, and it has a small  $S_n = 0.67(64)$  MeV.



# Increase of neutron radii



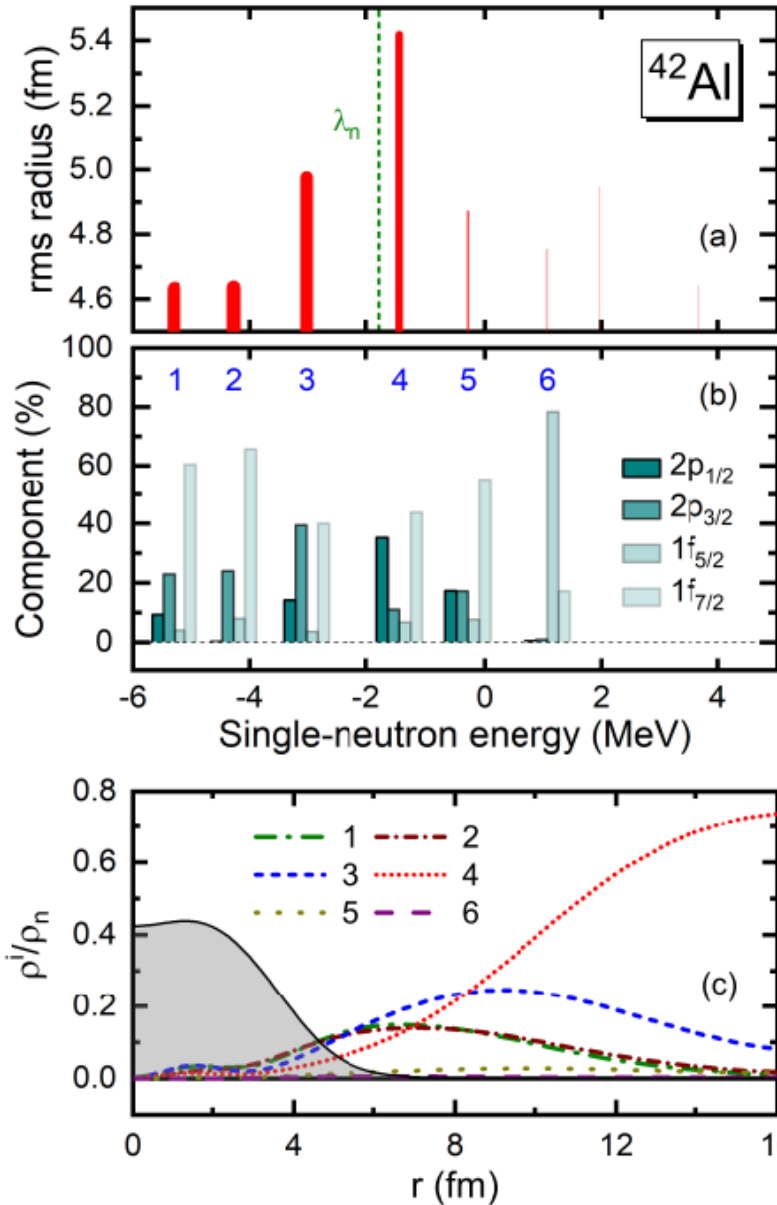
$$R_n^{\text{emp}} = r_0 N^{1/3}$$



RCHB + PC-PK1: ADNDT 121–122, 1 (2018)

RCHB + NLSH: PRL 80, 460 (1998)

- ✓ The ratio of TRHBc calculated neutron radius increment to that from empirical formula increases significantly at  $^{40,42}\text{Al}$ , which is similar to the increase in giant halo Zr isotopes.



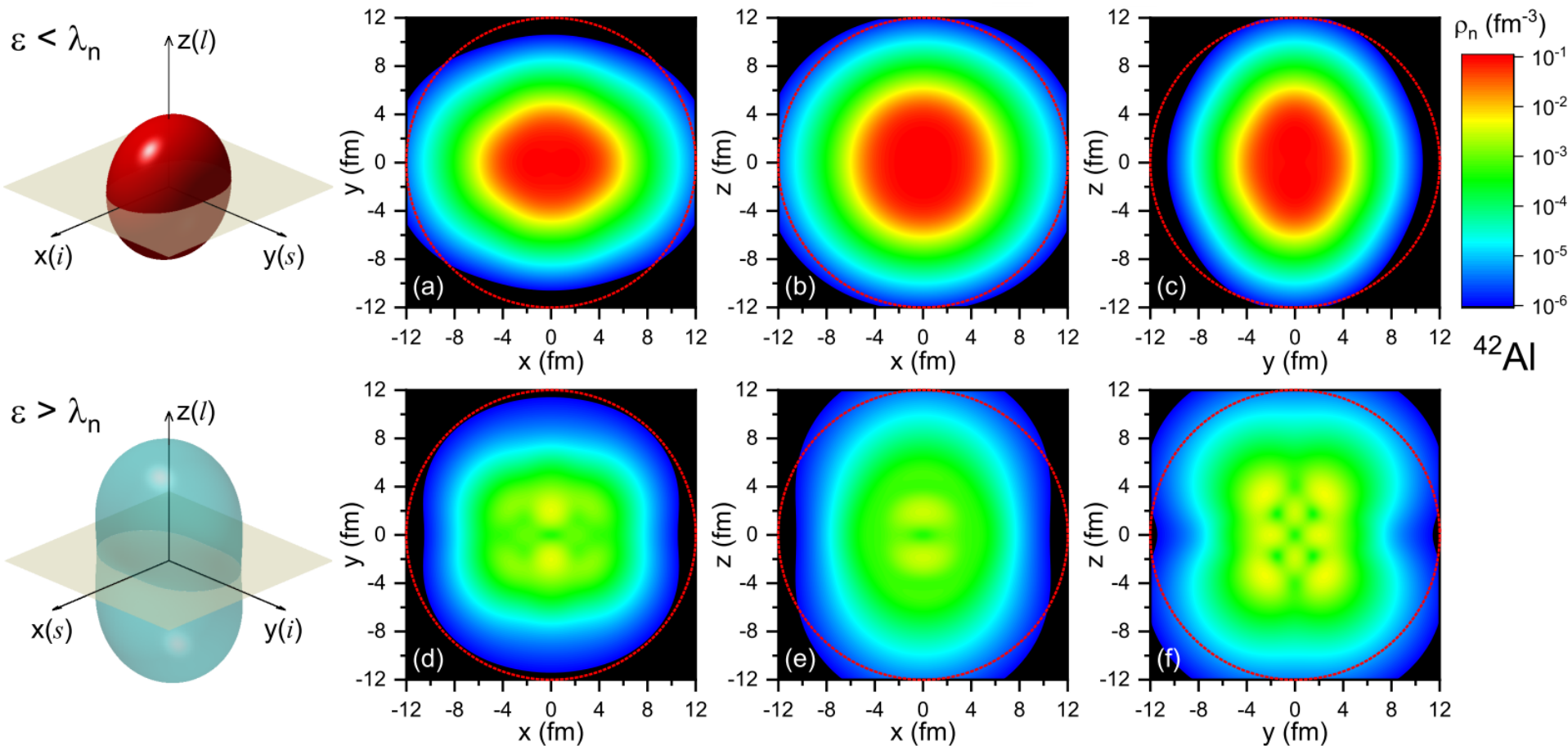
✓ Weakly bound level 4 occupied by the valance neutron has a remarkably large rms radius.

✓ Level 4 has considerable  $2p$  components ( $\geq 45\%$ ).

✓ Level 4 contributes dominantly to the total neutron density in the region far from center.



# Density in different planes



✓ Core:  $r = 3.85 \text{ fm}$ ,  $\beta = 0.38$ ,  $\gamma = 50^\circ$ ,  $z > x > y$

✓ Halo:  $r = 5.26 \text{ fm}$ ,  $\beta = 0.79$ ,  $\gamma = -23^\circ$ ,  $z > y > x$



- ◆ Introduction
- ◆ Theoretical framework
- ◆ Results and discussion
- ◆ Summary





- ✓ RCHB and DRHBc theories based on the CDFT have been successful for spherical and deformed halo nuclei.
- ✓ Triaxial relativistic Hartree-Bogoliubov theory in continuum (TRHBc) is developed to explore triaxially deformed halo nuclei.
- ✓ TRHBc theory reproduces available experimental data for Al isotopes.  $^{42}\text{Al}$  has triaxially deformed ground state, small  $S_n$ , and suddenly increased neutron radius.
- ✓ The orbital occupied by the valance odd neutron of  $^{42}\text{Al}$  has a significantly large radius, due to its weak binding and considerable  $2p$  components, and it contributes dominantly to neutron density at large  $r$ .
- ✓ Decomposed neutron densities suggest that  $^{42}\text{Al}$  is a triaxially deformed halo nucleus with a triaxial shape decoupling between the core and the halo.



北京大学  
PEKING UNIVERSITY

*Thank you for your attention!*