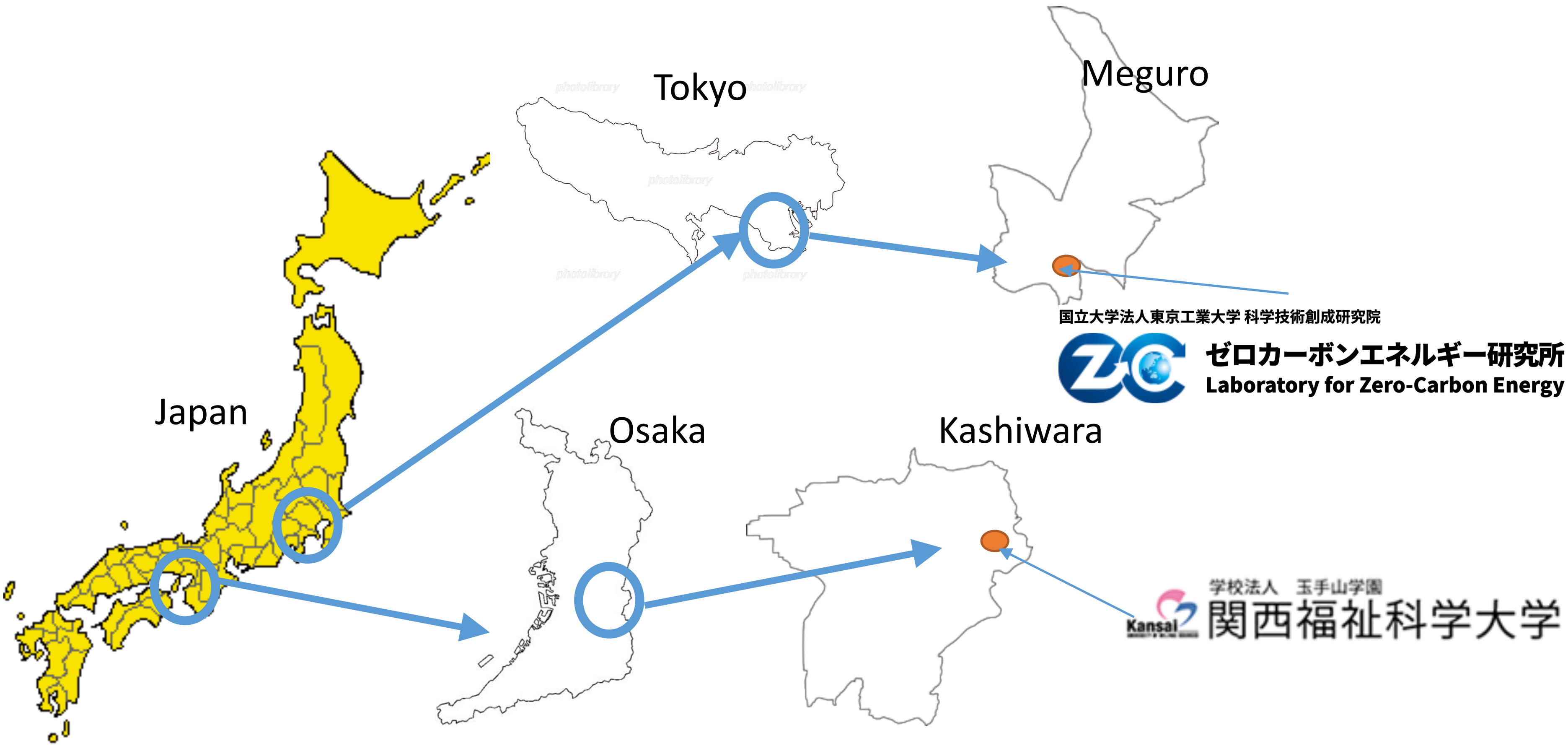
Single particle properties in the cranked rod-shaped potential

Shoujirou Mizutori (Kansai University of Welfare Sciences)
Tsunenori Inakura (Tokyo Institute of Technology)



Developments of Physics of Unstable Nuclei(YKIS2022b), May 23-May 27, Yukawa Institute, Kyoto Univ.

Extremely elongated shape of light N=Z nuclei

Exploring α linear-chain states:

Experimentally

Morinaga(1956), Chavallier et al.(1967), Wuasmaa et al. (1992), Rae et al. (1992), Freer (1995), Curtis et al. (2013)

Theoretically

Suzuki, Horiuchi, Ikeda (1972), Bauhoff et al. (1984), Marsh and Rae (1986) Itagaki et al. (2001)

Exploring elongated shape inspired by α linear-chain

Nilsson Strutinsky Method : Leander and Larsson (1975)

Skyrme Hartree Fock : Flocard, Heenen, Krieger and Weiss (1984)

Beyond mean field : Bender and Heenen(2003)

Finding out rod-shaped solution in full 3D cranked Skyrme DFT

T. Ichikawa, J.A. Maruhn, N. Itagaki, and S. Okubo, Phys. Rev. Lett. 107, 112501 (2011)

Calculations with DFT framework

J.M. Yao, N. Itagaki, and J. Meng. PRC 90, 054307(2014)

J.-P. Erban, E. Khan, T. Niksic, and D. Vretner, PRC90,054329(2014)

P.W. Zhao, N. Itagaki, and J. Meng. Phys. Rev. Lett. 115, 022501(2015)

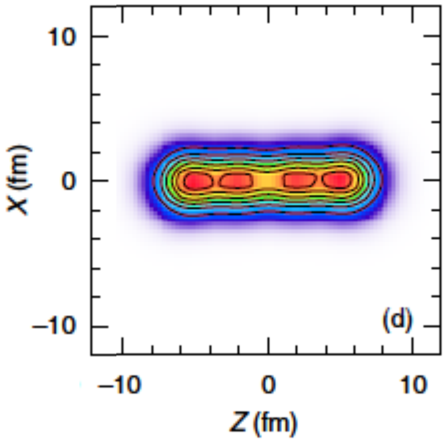
Y. Iwata, T. Ichikawa, N. Itagaki, J.A. Maruhn, and T. Otsuka, PRC 92, 011303(2015)

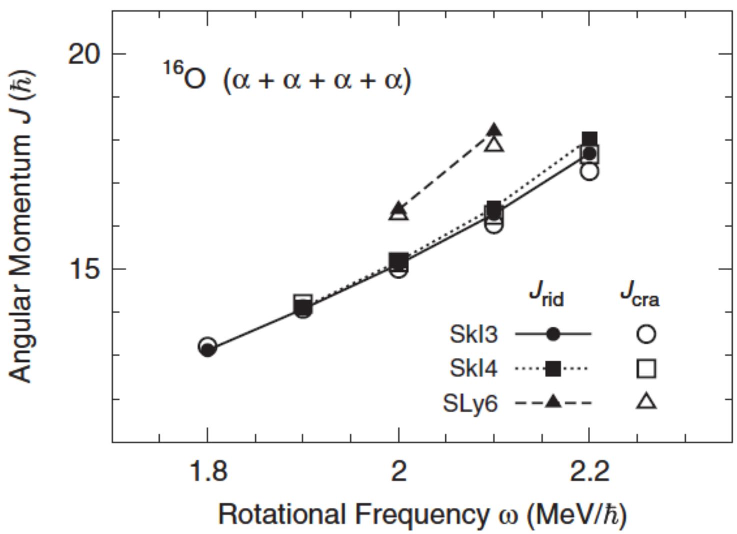
Z.X. Ren, S.Q. Zhang, P.W. Zhao, N. Itagaki, J.A. Marohn and J. Meng,
Sci. Chi.Phys.Mech.Astr. 62, 112062(2019)

Rod shaped solutions in DFT:

^{16}O

Ichikawa et. al.
Skyrme density functional.





Yao et. al.
covariant density functional.

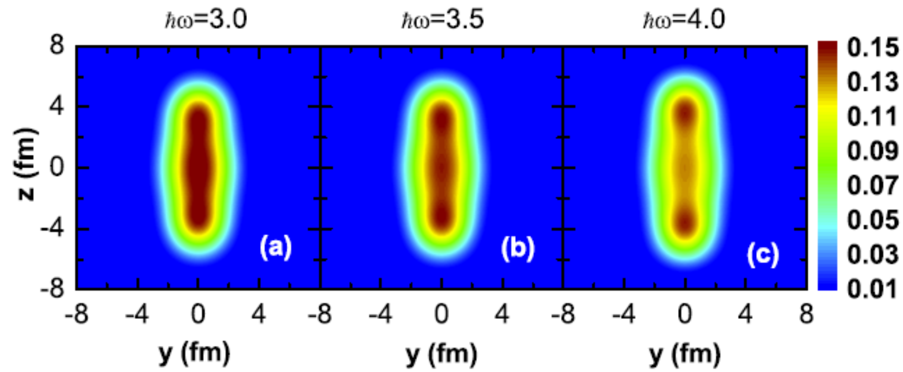


FIG. 9. (Color online) The total density distribution (in fm^{-3}) at $x = 0.3$ fm in ^{16}O corresponding to LCS by the cranking RMF calculation with rotational frequency $\hbar\omega = 3.0, 3.5, 4.0$ MeV, respectively. The rms radii of (long, medium, short) axes are (a) (3.0,1.3,1.1) fm, (b) (3.1, 1.4, 1.1) fm, and (c) (3.3, 1.4, 1.1) fm, respectively.

Rod shaped solutions in DFT

C isotopes

Zhao et. al.
covariant density functional.

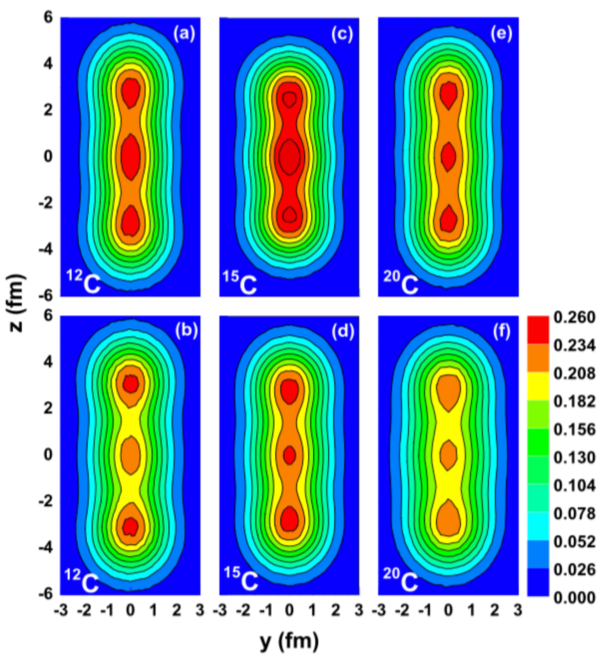
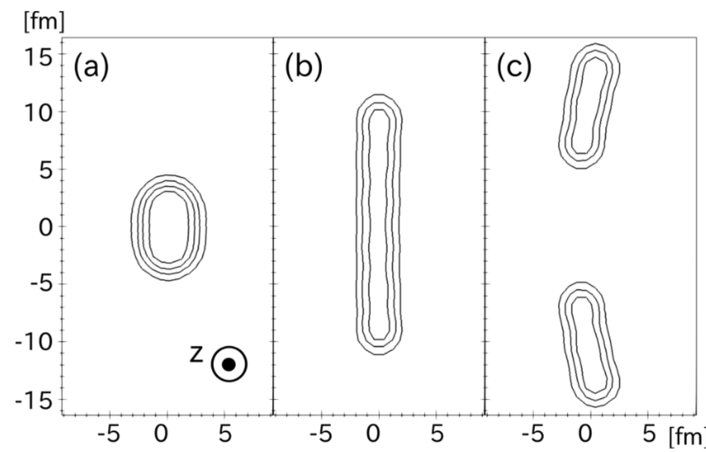
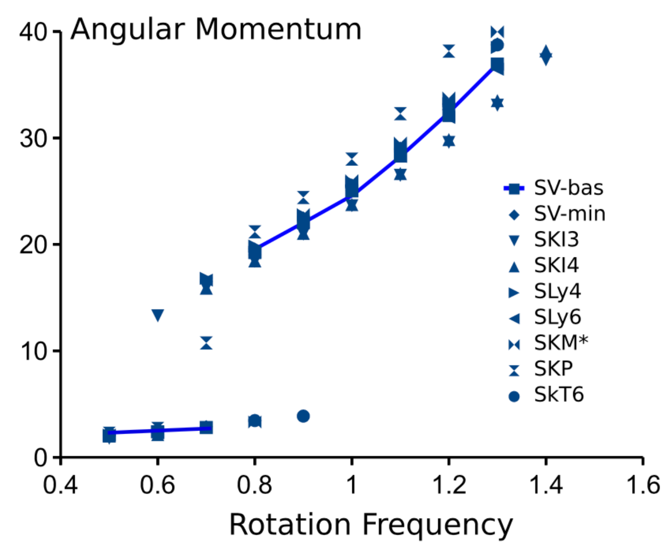


FIG. 2 (color online). Proton density distributions in the y-z plane (x direction is integrated) calculated by using the cranking covariant density functional theory for ^{12}C, ^{15}C, and ^{20}C at the rotational frequencies $\hbar\omega = 0.0$ MeV (a),(c),(e) and $\hbar\omega = 3.0$ MeV (b),(d),(f).

^{24}Mg

Iwata et. al.
Skyrme density functional.





Nucleon Localization Measure

Reinhard, Maruhn, Umar, and Oberacker, Phys. Rev. C 83, 034312(2011)

$$\mathcal{C}_{q\sigma}(\mathbf{r}) = \left[1 + \left(\frac{\tau_{q\sigma}\rho_{q\sigma} - \frac{1}{4}\left[\nabla\rho_{q\sigma}\right]^2 - \mathbf{j}^2_{q\sigma}}{\tau^{TF}_{q\sigma}\rho_{q\sigma}}\right)^2\right]^{-1}$$

$$\tau^{TF}_{q\sigma} = \frac{3}{5}(6\pi^2)^{2/3}\rho^{5/3}_{q\sigma}$$

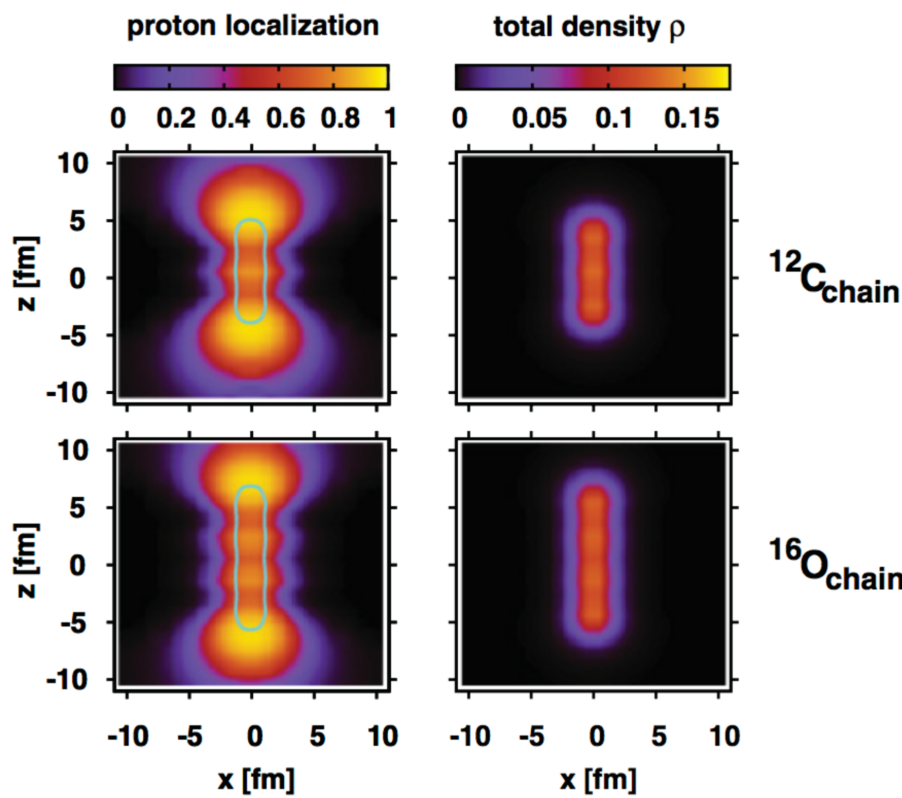
$$\tau_{q\sigma}(\mathbf{r}) = \sum_{\alpha\in q}|\nabla\phi_\alpha(\mathbf{r}_{q\sigma})|^2$$

$$\mathbf{j}_{q\sigma}(\mathbf{r}) = \sum_{\alpha\in q}\mathrm{Im}[\phi^*_\alpha(\mathbf{r}\sigma)\nabla\phi_\alpha(\mathbf{r}\sigma)]$$

$$\nabla\rho_{q\sigma}(\mathbf{r}) = 2\sum_{\alpha\in q}\mathrm{Re}[\phi^*_\alpha(\mathbf{r}\sigma)\nabla\phi_\alpha(\mathbf{r}\sigma)]$$

For uniform nuclear matter, $\tau_{q\sigma} = \tau^{TF}_{q\sigma}$

and derivative is 0, therefore $\mathcal{C} = \frac{1}{2}$



If a certain region is occupied by a wavefunction with specific $q\sigma$, $\mathcal{C} = 1$

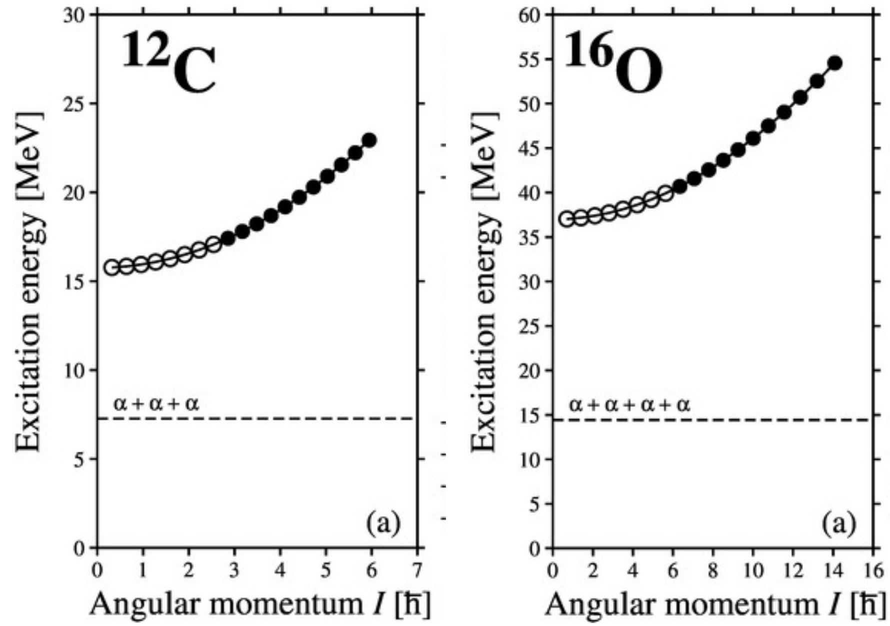
⟹ A typical example is $(0s)^4$ state

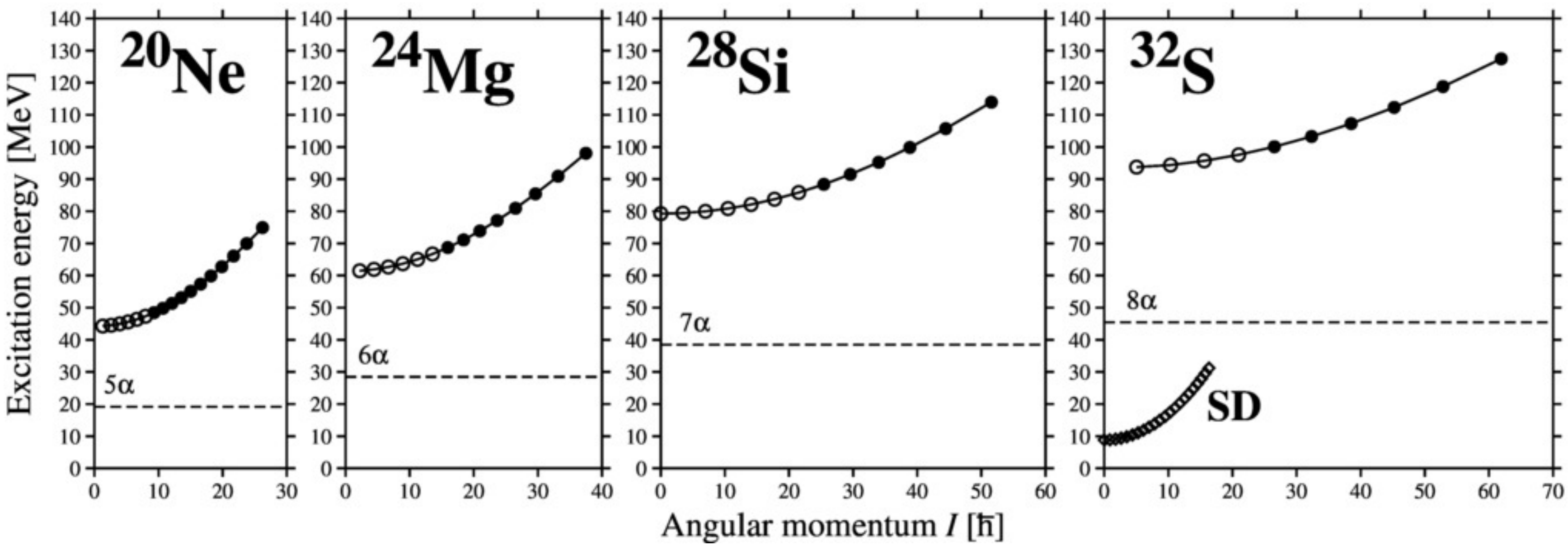
Can be regarded as a necessary condition to an alpha clustering state

Rod solutions for $^{12}C \sim ^{32}S$

Tsunenori Inakura and Shoujirou Mizutori,
Phys. Rev. C 98, 044312 (2018)

open circle : quasi-stable solution
filled circle : stable solution

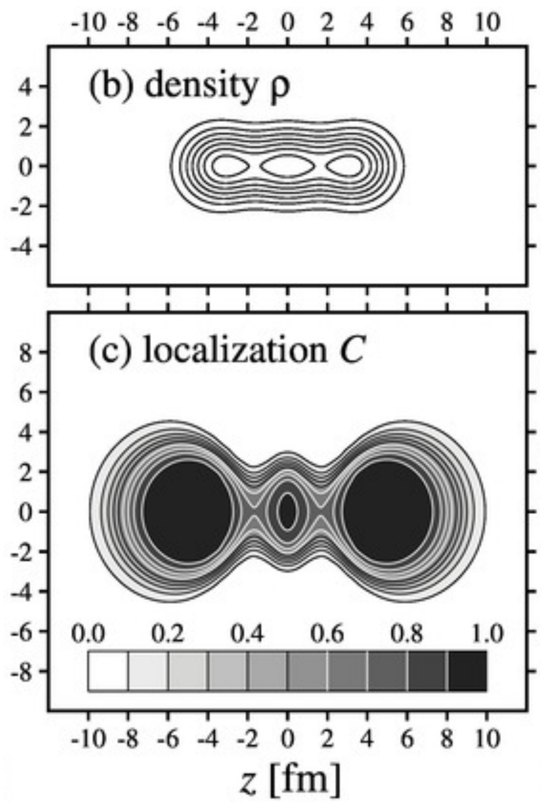




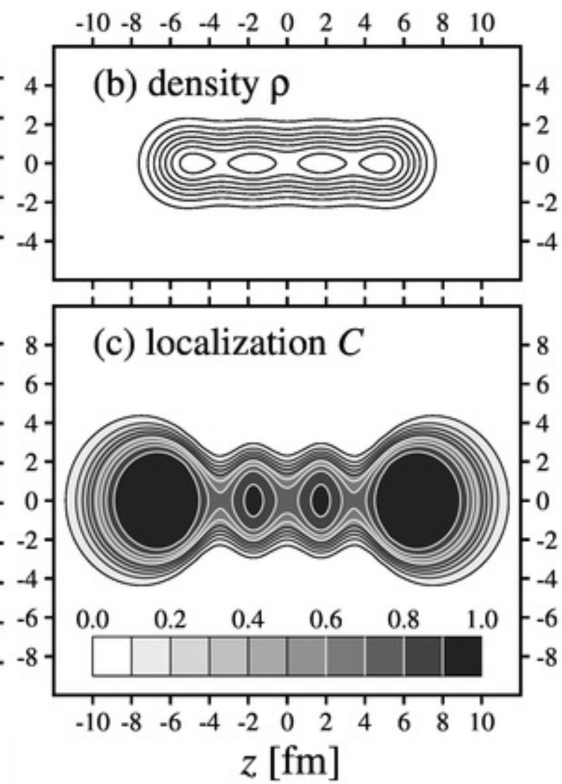
Nucleon localization measure at high spin

ω_{rot} = 1.0MeV

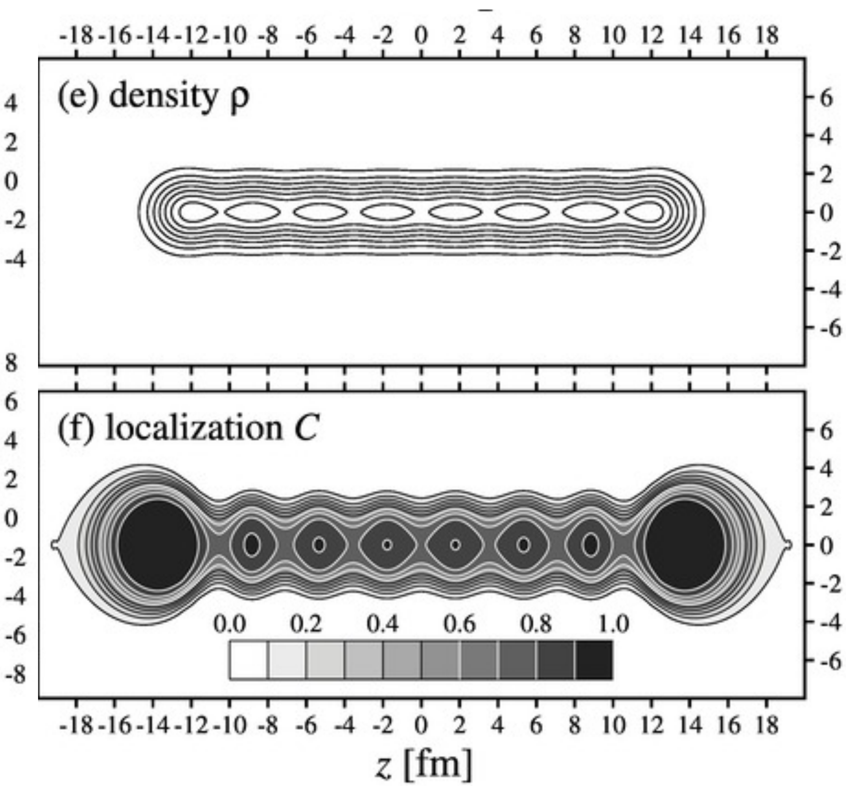
^{12}C



^{16}O



^{32}S



The potential model.

Cranked Rod-Shaped potential

$$h_{rod} = \frac{1}{2M}\boldsymbol{p}^2 + \frac{1}{2}M\omega^2(x^2+y^2) + \Theta(z-R\nu)\frac{1}{2}M\omega^2(z-R\nu)^2 + \Theta(-R\nu-z)\frac{1}{2}M\omega_\alpha^2(R\nu+z)^2$$

$$-\hbar\omega_{rot}J_x$$

$$\nu = \sqrt{\frac{\hbar}{M\omega}}$$

$$\tilde{l} = \tilde{r}\times p$$

$$\tilde{\boldsymbol{r}} = \begin{pmatrix} x \\ y \\ \Theta(z-R\nu)(z-R\nu)+\Theta(-z-R\nu)(z+R\nu) \end{pmatrix}$$

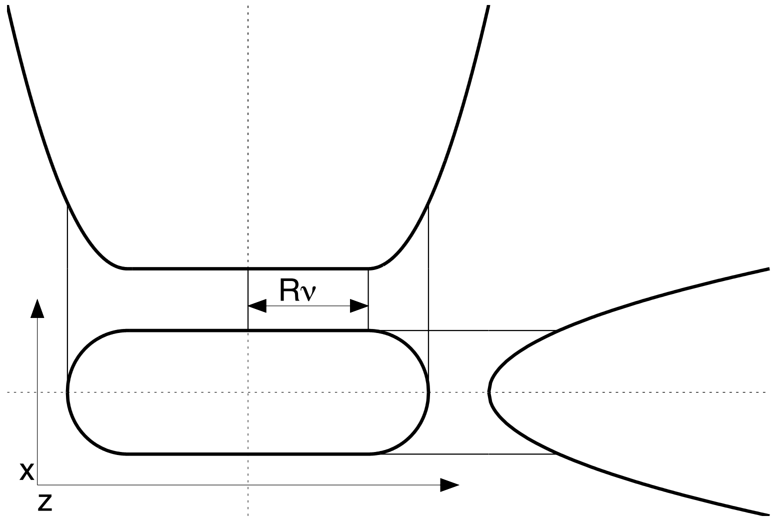


TABLE I. ω and R for each calculated nucleus.

nucleus	ω(MeV)	R
^{40}Ca, (ND)	13.33	0.6
^{32}S, (SD)	16.27	1.5
^{16}O, (rod)	25.83	3.5

κ=0.09

Indexing the solutions.

Basis states

$$\Psi_{\overline{n_3} n_\perp \Lambda\Omega} = \phi_{\overline{n_3}}(z)\varphi_{n_\perp\Lambda}(x,y)\chi(\Omega-\Lambda)$$

Solutions of the 2-dm H.O.potential

Solutions of the 1-dim hamiltonian

$$h(R) = \frac{1}{2M}p_z^2 + \frac{1}{2}\omega^2\tilde{z}^2$$

$$\tilde{z}(z;\nu R) = \Theta(z-\nu R)(z-\nu R) + \Theta(-z-\nu R)(z+\nu R)$$

$\overline{n_3}$ is the number of the nodes of the solution

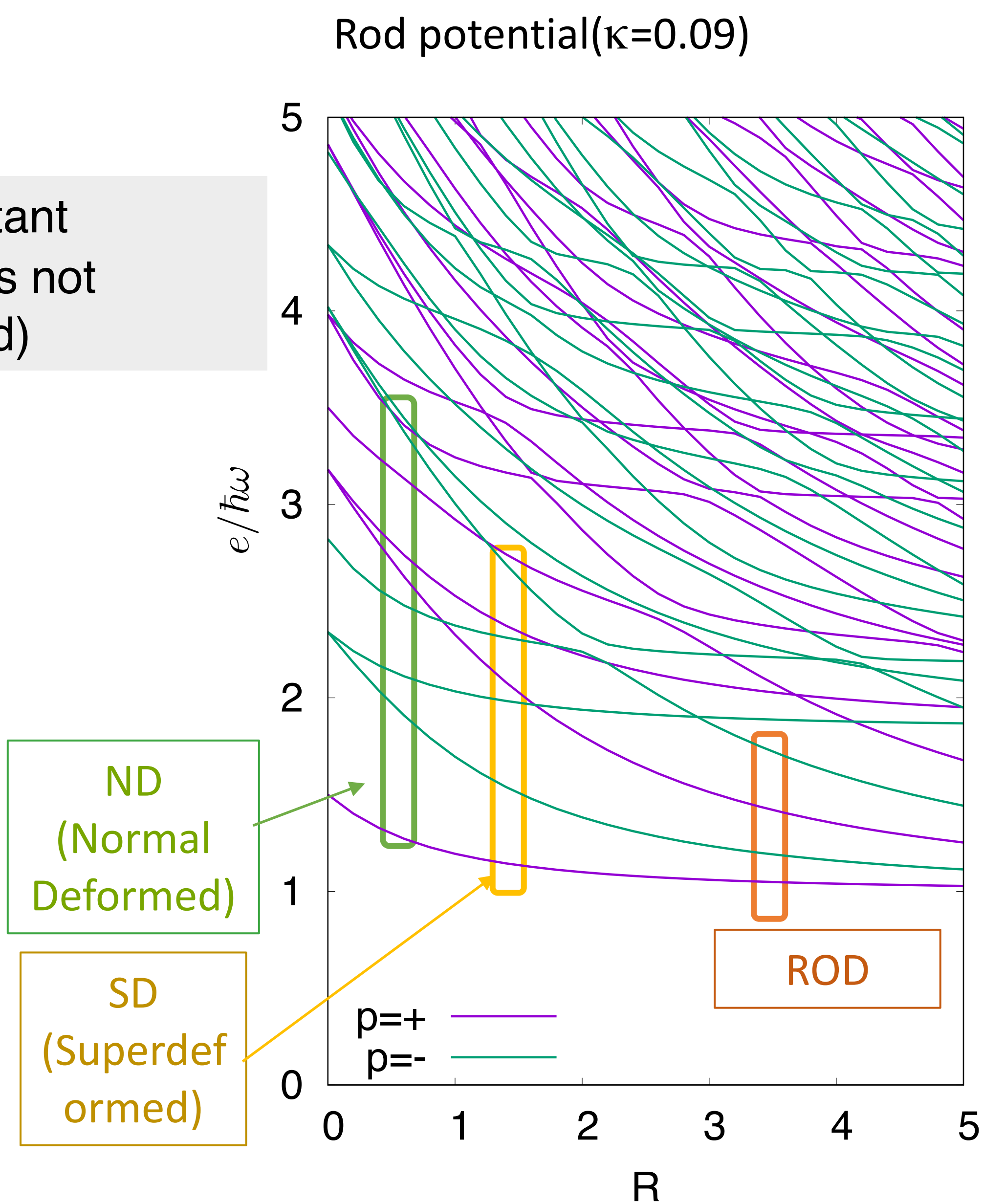
The basis states are labeled as $[\overline{N}\overline{n_3}\Lambda]\Omega$ here $\overline{N} = \overline{n_3} + \Lambda + 2\times n_\perp$

The single particle levels at ω=0 are also labeled by these index.

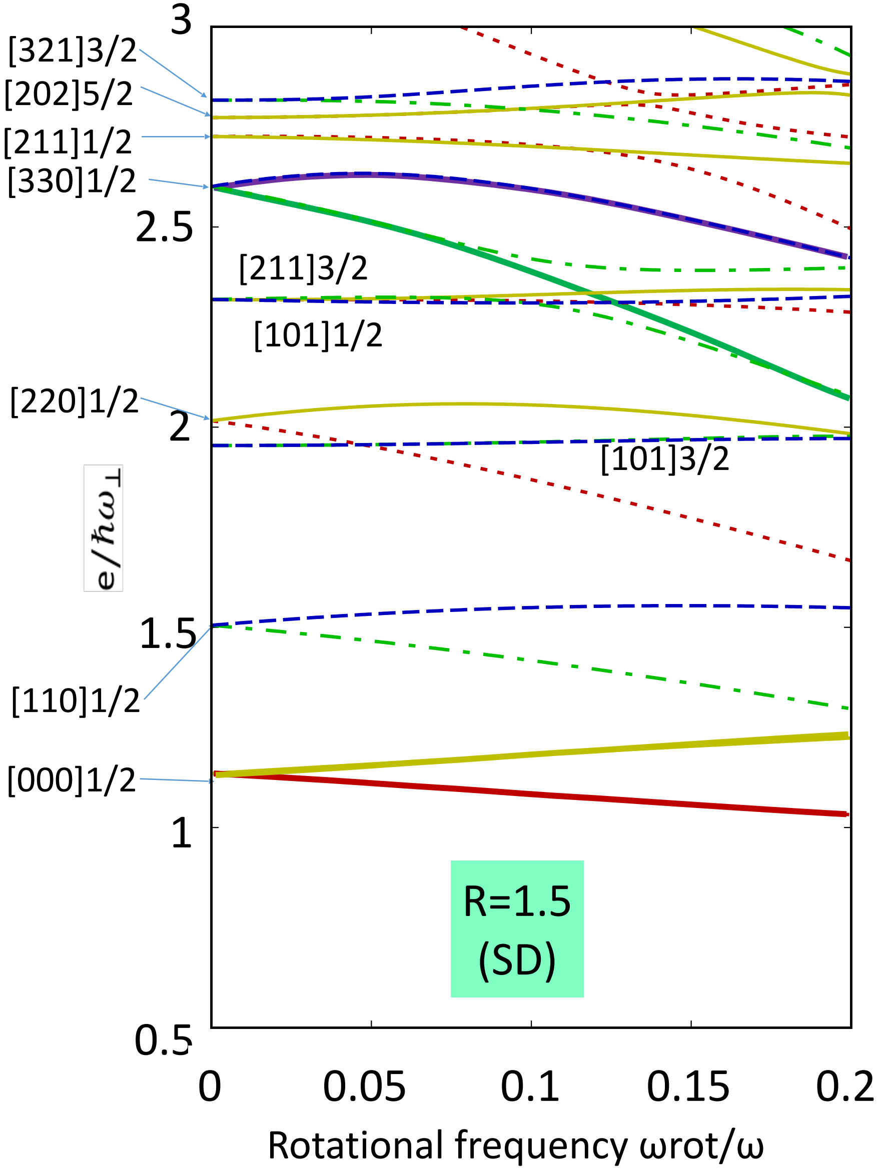
The routhians are also labeled by these index of the main component at ω=0.

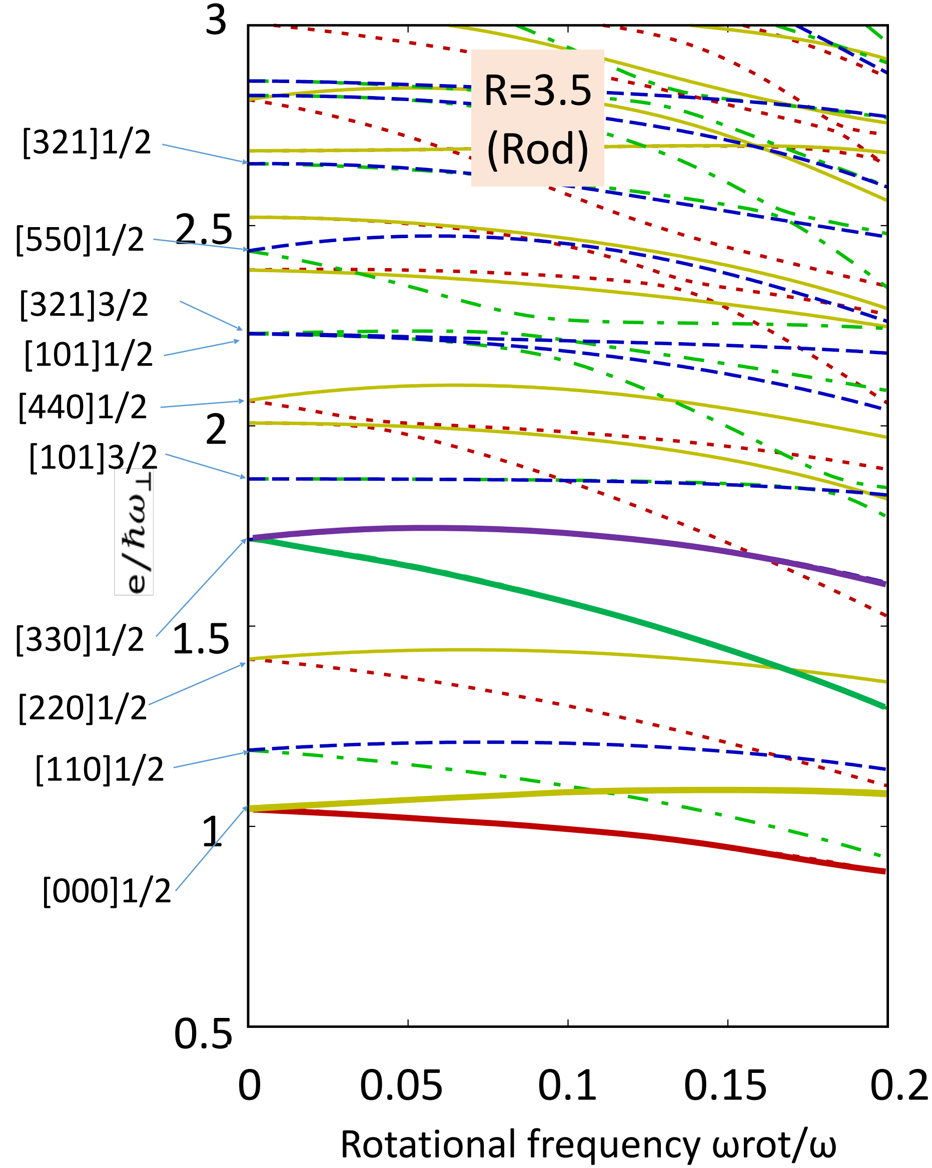
Nilsson Diagram

ω is constant
(Volume is not
conserved)



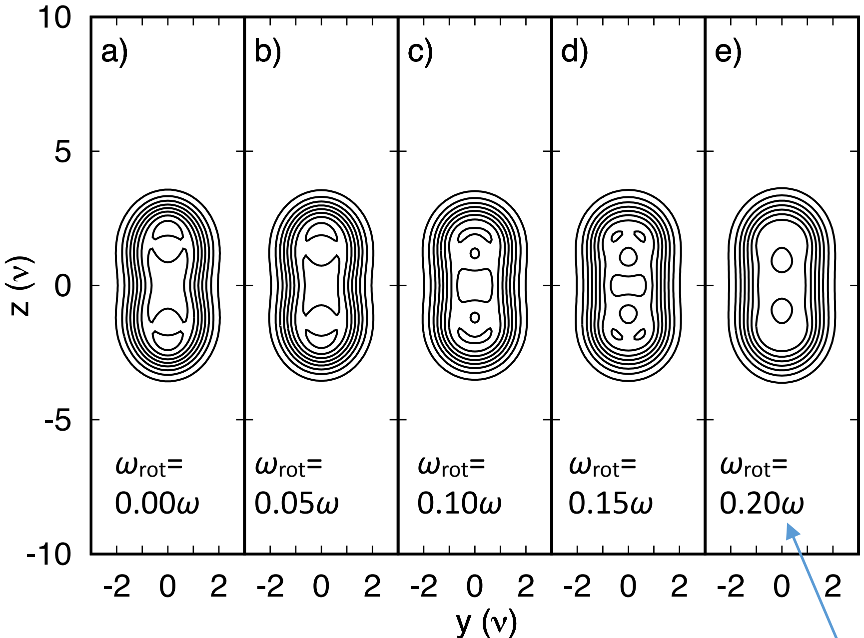
Single-particle routhian





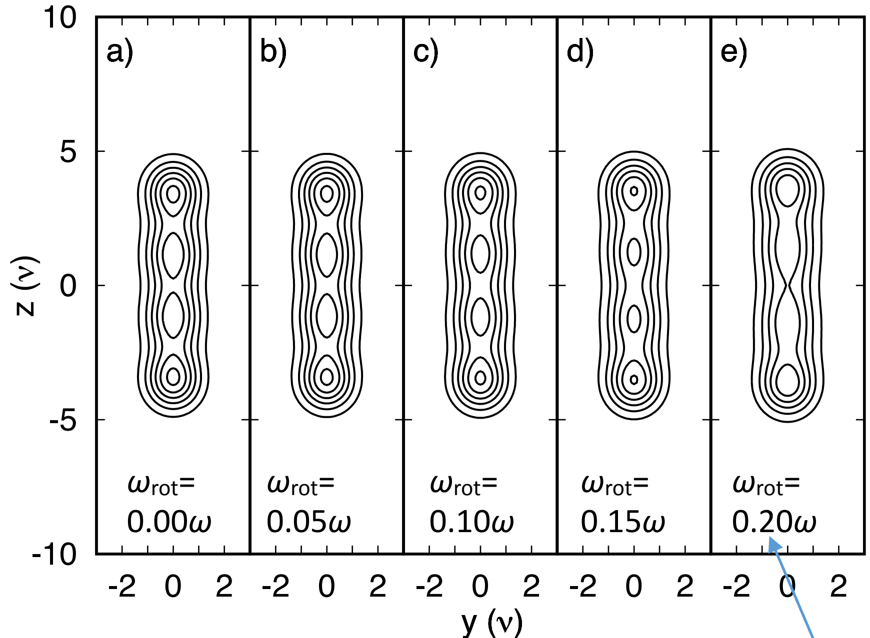
Density distribution of the nuclei

R=1.5 ,N=8 (SD)



~3.25MeV

R=3.5, N=4 (rod)



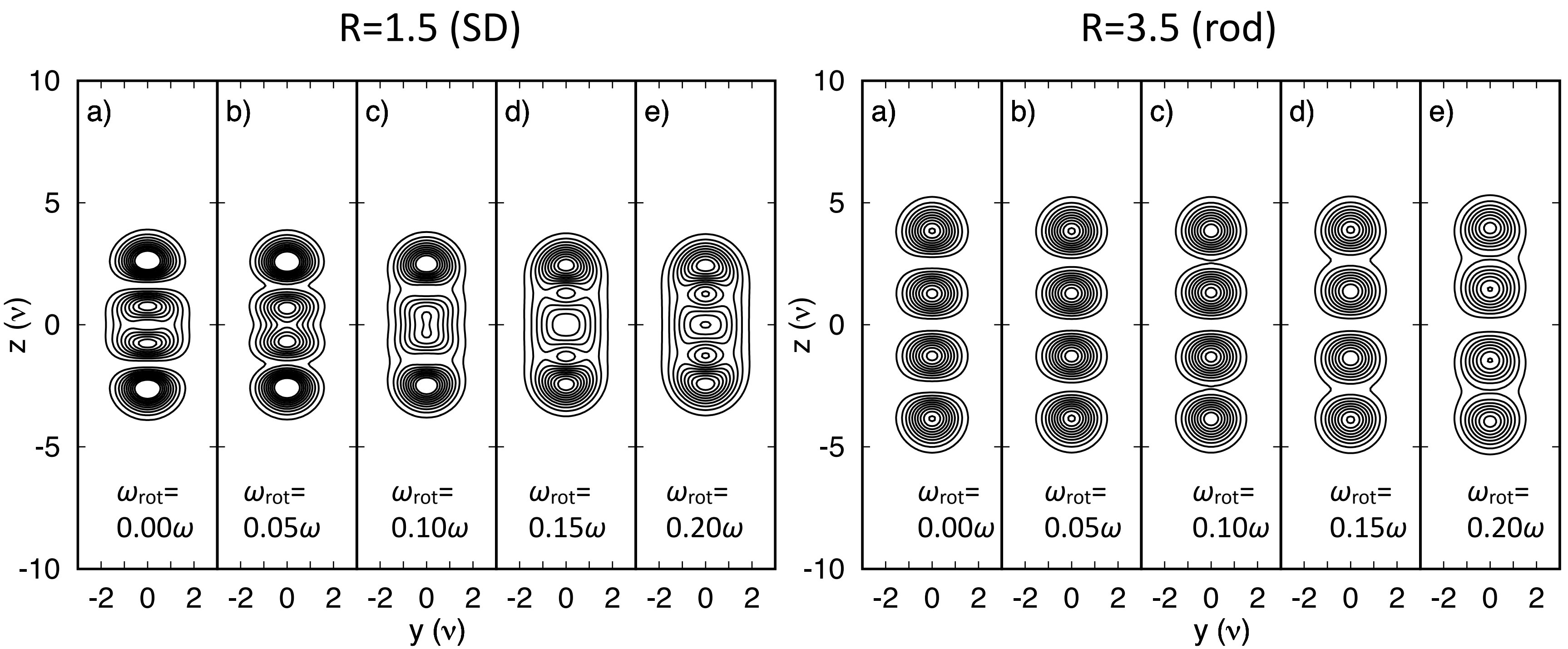
~5.17MeV

The structure looks changing

The structure looks stable

Wave functions

Wave function of unfavored $[\bar{3}\bar{3}0]1/2$ (Integrated over x-axis)



The structure looks changing

The structure looks stable

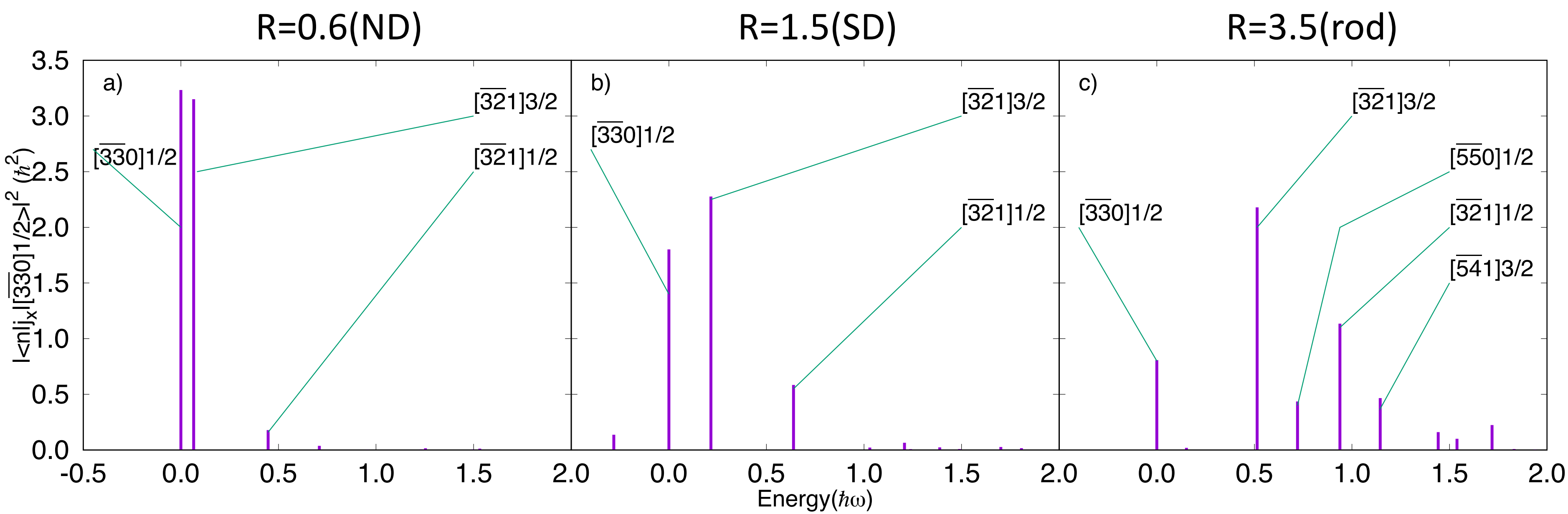
Components of the wave functions of [330]1/2(-i)

	R=1.5 (SD)			R=3.5 Rod		
basis	$\omega_{\rm rot}=0.0$	0.1ω	0.2ω	$\omega_{\rm rot}=0.0$	0.1ω	0.2ω
$[\overline{33}0]1/2$	0.781	0.430	0.280	0.962	0.873	0.547
$[\overline{32}1]3/2$	0.000	0.394	0.498	0.000	0.085	0.231
$[\overline{32}1]1/2$	0.174	0.071	0.047	0.030	0.002	0.000
$[\overline{31}2]5/2$	0.000	0.005	0.095	0.000	0.004	0.044
$[\overline{11}0]1/2$	0.000	0.000	0.000	0.000	0.003	0.068
$[\overline{54}1]1/2$	0.000	0.001	0.000	0.004	0.009	0.010
$[\overline{54}1]3/2$	0.000	0.000	0.001	0.000	0.003	0.027
$\langle j_x \rangle$	-1.342	1.256	2.037	-0.898	0.553	1.956

In the superdeformed states, alignment is caused by the admixture of [321]3/2 and eventually [312]5/2

In rod states, component of [330]1/2 remains the largest and admixture of other levels than N=3 levels are important

$|\langle\, i \,|\, Jx \,|\, [330]1/2 \rangle|^2$



couplings to [321]3/2 and [330]1/2 are dominant
Δe([330]1/2-[321]3/2)
is very small.

Coupling to [321]3/2 and [330]1/2 are large.
Δe([330]1/2-[321]3/2)
is small.

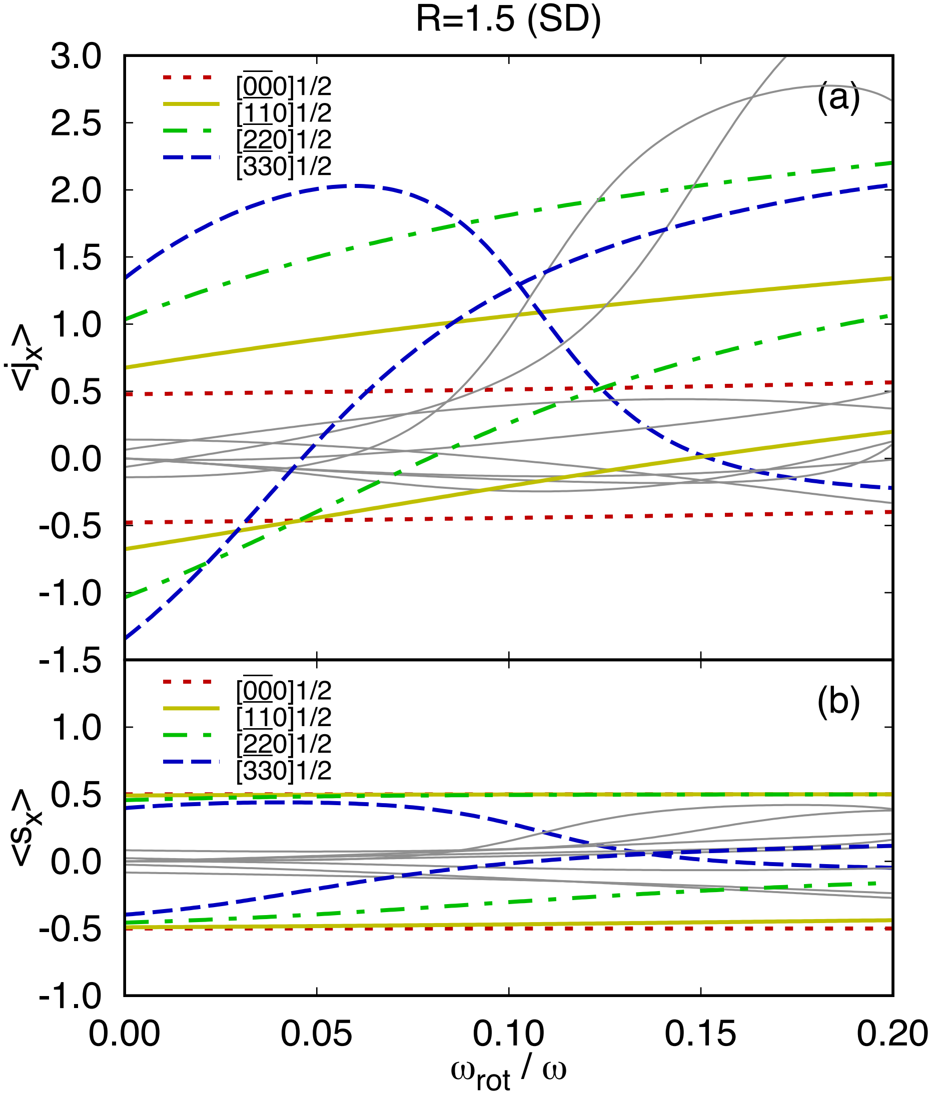
Couplings to other levels are sizable.
Δe([330]1/2-[321]3/2)
is very large.

Selection rule of the solutions of axial symmetric oscillator potential

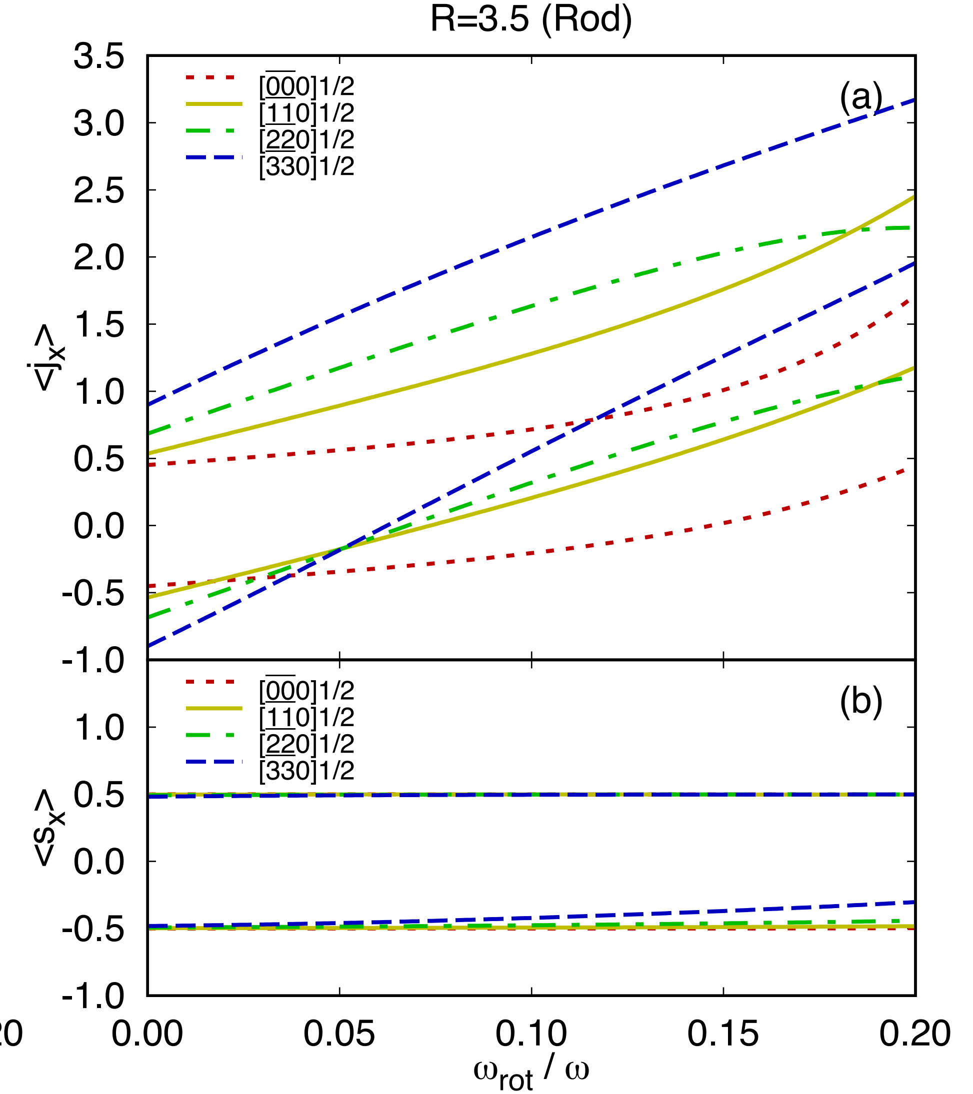
Essential feature : $$\left\langle n'|\frac{d}{dx}|n\right\rangle = -\langle n'|x|n\rangle \,,$$ Not valid for flat bottom Potential

Angular momentum of each levels

R=1.5 ,N=8 (SD)

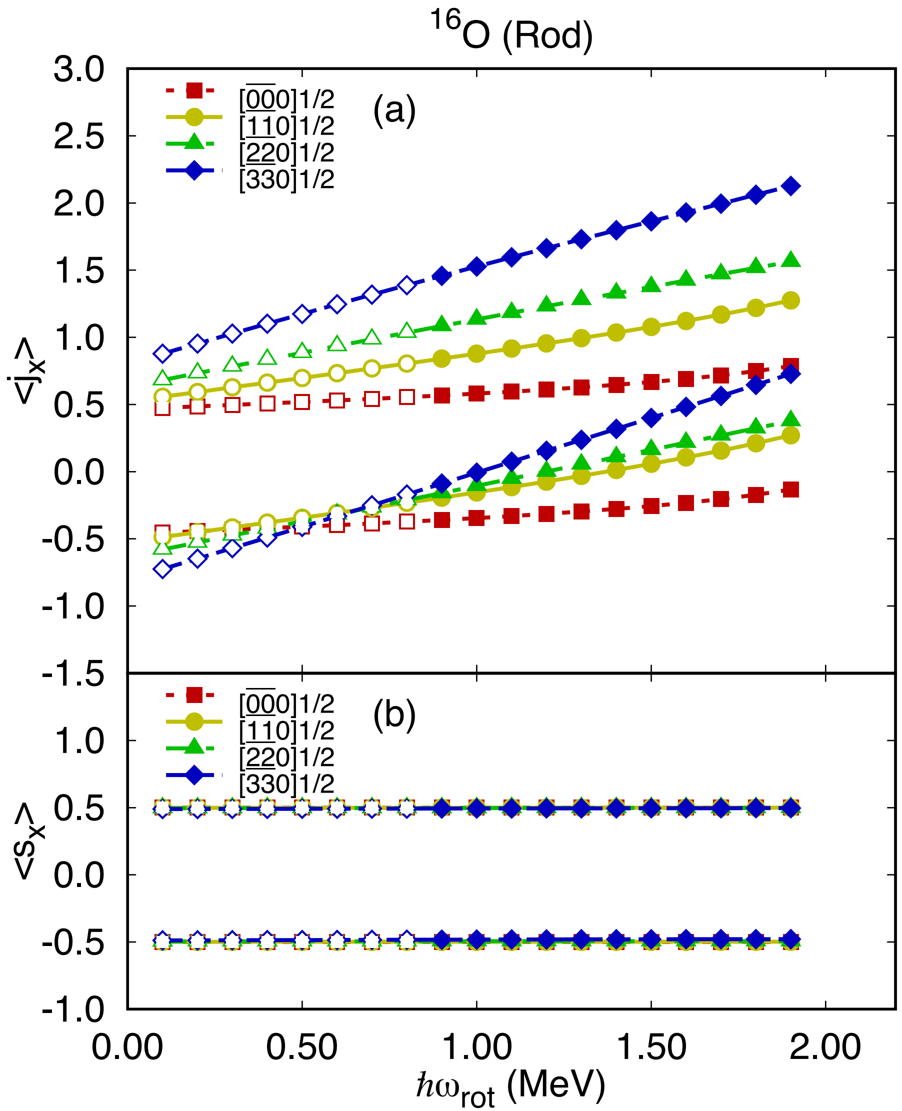


R=3.5, N=4 (rod)

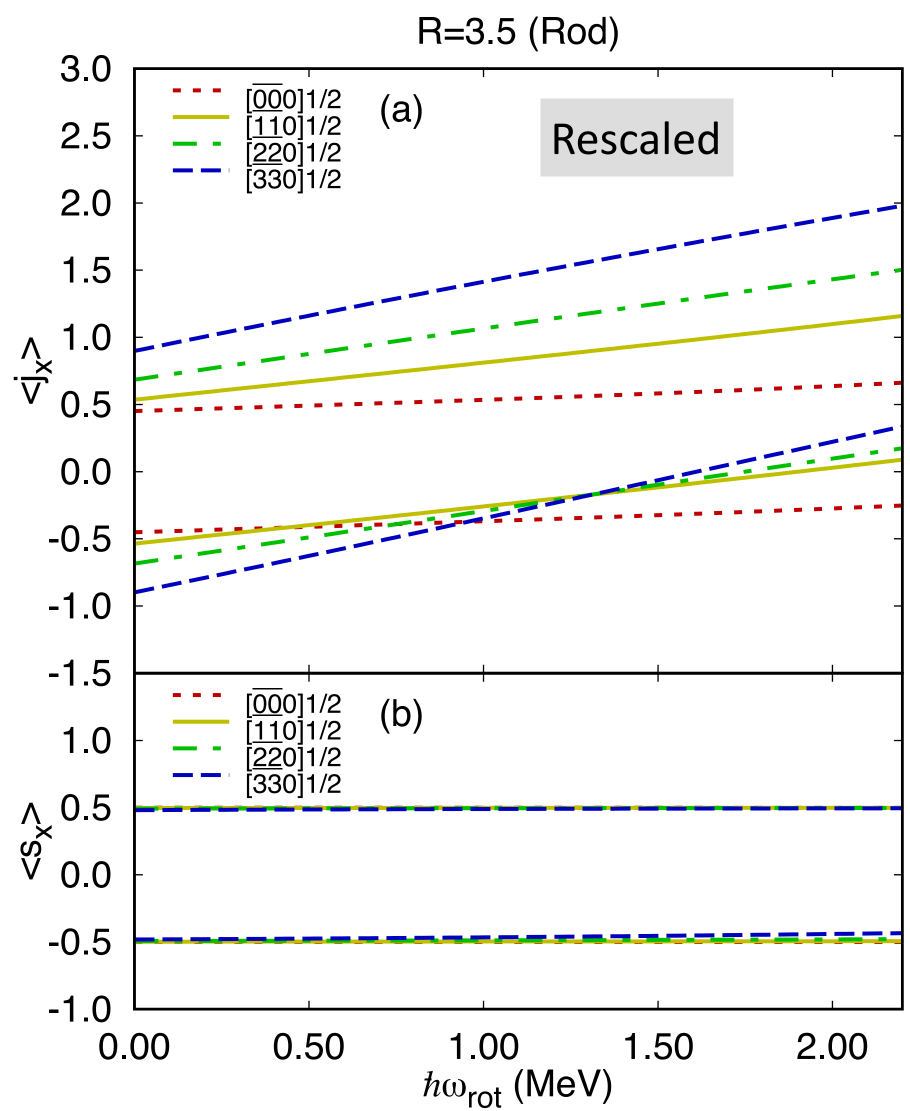


Angular momentum of each levels

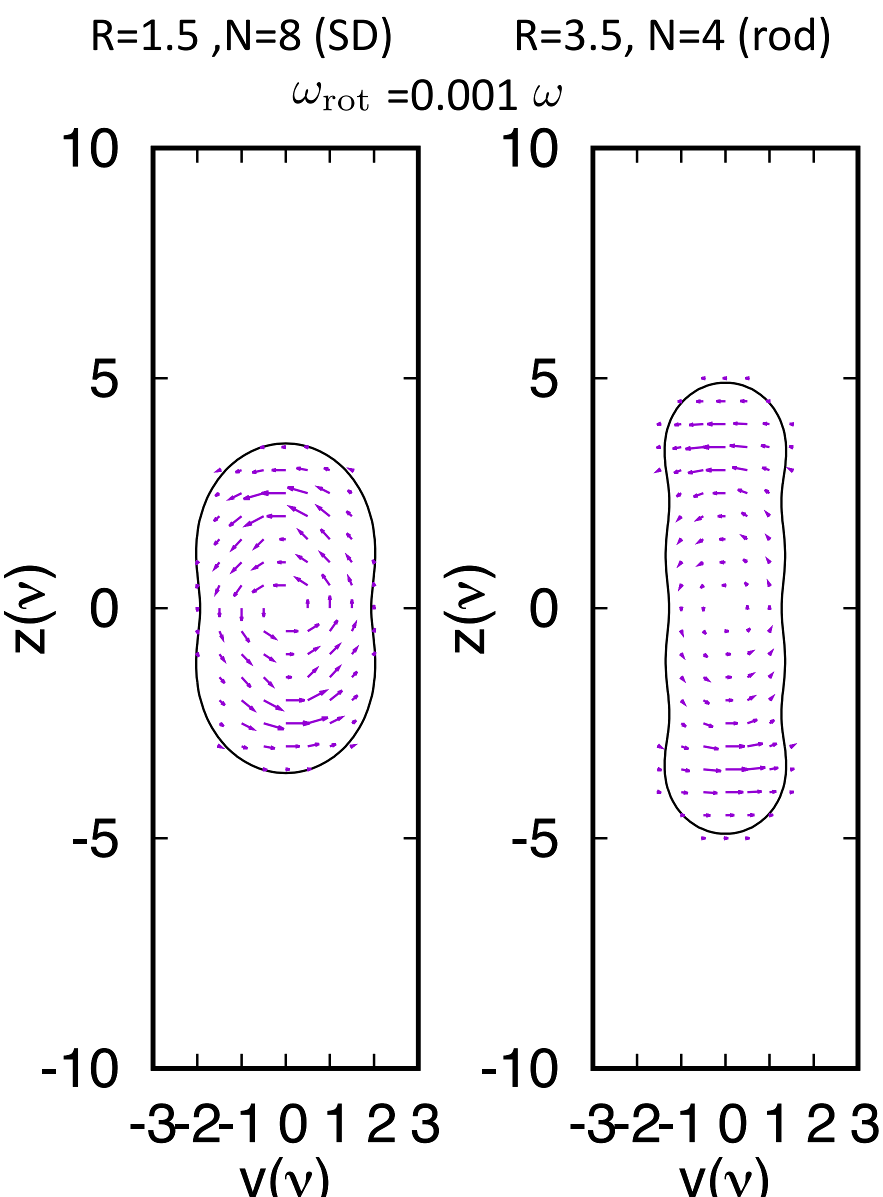
CSHF ^{16}O



Potential model



Current density

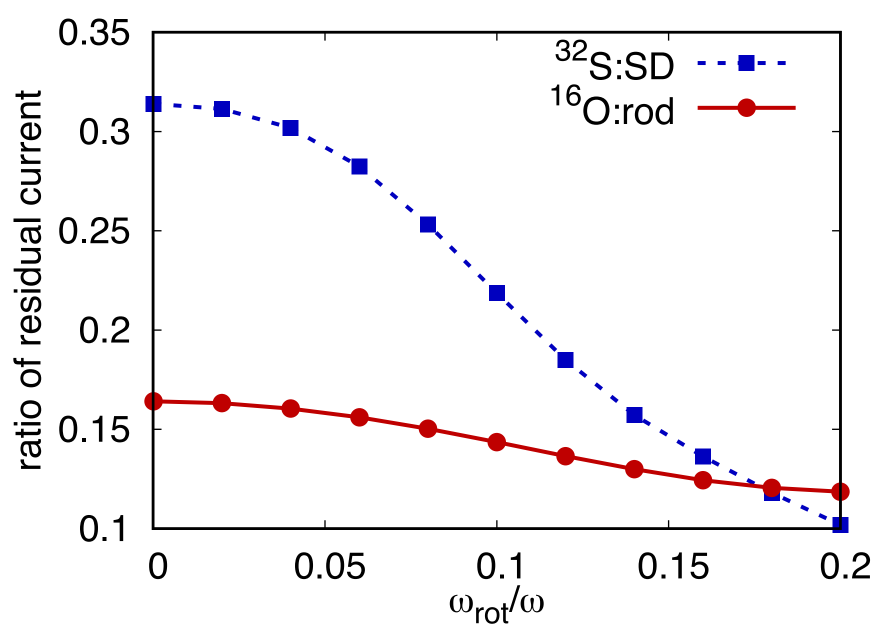


To extract characteristics
define fraction of the residual current

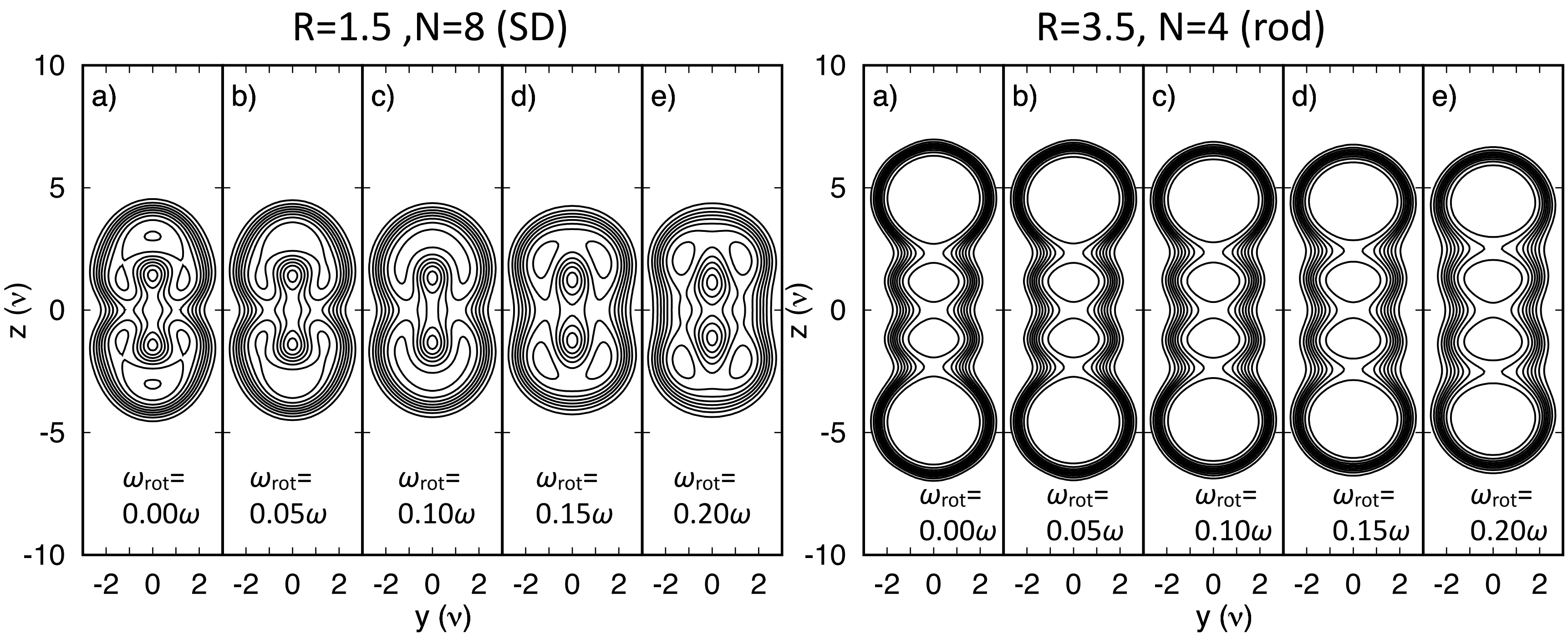
$$J_{res} = \sqrt{\frac{\int d\boldsymbol{r} |\boldsymbol{j}_{res}(\boldsymbol{r})|^2}{\int d\boldsymbol{r} |\boldsymbol{j}(\boldsymbol{r})|^2}}$$

Residual current is the difference between current and rigid body current

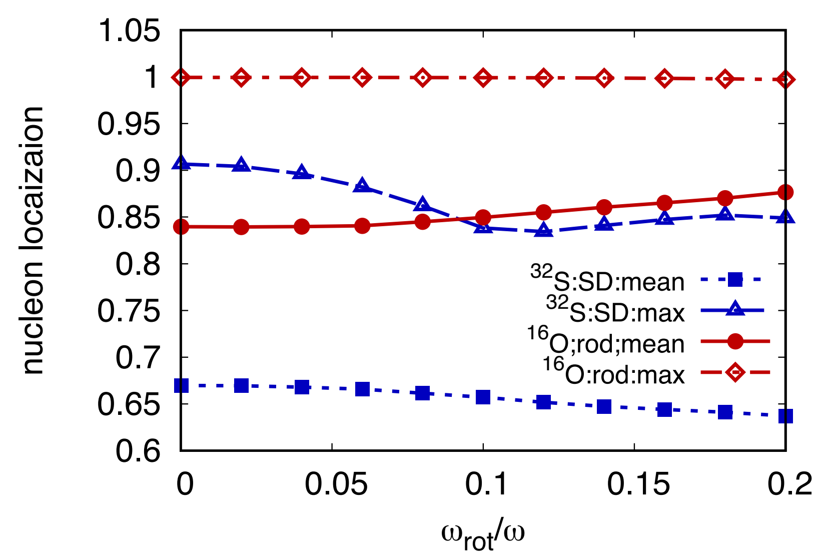
$$\boldsymbol{j}_{res}(\boldsymbol{r}) = \boldsymbol{j}(\boldsymbol{r}) - \rho(\boldsymbol{r}) \cdot \omega_{\mathrm{rot}} \times \boldsymbol{r}$$



Nucleon Localization Measure



Maximum and mean value of the nucleon localization measure



Conclusion

Very elongated shape hinders the selection rule of jx

Reduce the coupling by rotation within "$f_{7/2}$ partners".

The level emerged from [330]1/2 acquires the angular momentum by coupling with other levels than "$f_{7/2}$ partners".

It reserves [330]1/2 as the major component.

Due to the above features, rotational frequency dependence is very small in the rod nuclei.

<Jx> of all single-particle levels increase smoothly as increasing rotational frequency.

Very small deviation of current density from the rigid one.

Keeping very large nucleon localization measure at high rotational frequency.