Cluster formation in light neutron-rich nuclei

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Hideaki Motoki, Yoshiki Suzuki*, Tsuyoshi Kawai, and Masaaki Kimura
Hokkaido University
*RCNP
Cluster formation is a universal phenomenon found in hierarchy layers of matter.

α cluster plays an important role in both finite and infinite nuclear systems.

- In dilute nuclear matter, α cluster formation is related to the nuclear density and symmetry energy.
- In finite nuclei, the growth of neutron-skin hinders the α cluster formation.

S. Typel et al., PRC 81, 015803 (2010)
K. Hagel et al., PRL 108, 062702 (2012)
S. Typel et al., EPJA 50(2), 1 (2014)
Zhao-Wen Zhang et al., PRC 95, 064330 (2017)
α cluster formation in the finite nuclei

- $(p, p\alpha)$ is a useful measure for the α cluster formation at the nuclear surface
  - A negative correlation
    - between the neutron-skin thickness and α cluster formation in Sn isotopes

- A negative correlation was also theoretically predicted in C isotopes

$^{20}\text{Ne}(p, p\alpha)$

$\text{Sn}(p, p\alpha)$

AMD+RWA

K. Yoshida et al., PRC, 100, 044601 (2019)
Q. Zhao et al., EPJA 57, 157 (2021)
α cluster formation in Be and B isotopes

α cluster formation is predicted to be enhanced toward the neutron drip-line based on the proton density and its radius.

Y. Kanada-En’yo et al., PRC 52, 647 (1995)
Y. Kanada-En’yo et al., PTPS 142, 205 (2001)

Y. Kanada-En’yo, PRC 91, 014315 (2015)
\( \alpha \) cluster formation in Be and B isotopes

- \( \alpha \) cluster formation is predicted to be enhanced toward the neutron drip-line based on the proton density and its radius.

- In Be isotopes, the enhancement of clustering is explained by the molecular orbit occupied by the excess neutrons.
  - \( \pi \)-orbit bonds the 2\( \alpha \) core
  - \( \sigma \)-orbit enhances the 2\( \alpha \) core

\[ ^{10}\text{Be}(\pi^2) \]
\[ ^{12}\text{Be}(\pi^2\sigma^2) \]

Proton density

Y. Kanada-En'yo, PRC 91, 014315 (2015)
**Motivation**

- **α cluster formation decreases**
- **α cluster formation is enhanced**

![Graph showing the decrease in S-factor with increasing neutron number for various isotopes like C, 14C, 16C, 18C, etc.](image1)

- No direct evidence of the cluster development in Be and B isotopes has been found
  - Physical quantities that can directly probe the cluster structure are desirable

**Purpose of this talk**

- Investigate the cluster formation in Be and B isotopes with AMD.
  - Not only α cluster but also $^6$He and $^8$He cluster formation probabilities
Antisymmetrized Molecular Dynamics (AMD)

Hamiltonian
\[ \hat{H} = \sum_{i} \hat{t}_{i} - \hat{t}_{\text{c.m.}} + \sum_{i<j}^{A} \hat{v}_{ij}^{\text{Gogny D1S}} + \sum_{i<j \in \text{proton}}^{A} \hat{v}_{ij}^{\text{Coulomb}} \]

Intrinsic model wave function
- Single nucleon wave function
  \[ \phi_{i}(r) = \exp \left\{ - \sum_{\sigma=x,y,z} v_{\sigma} (r_{\sigma} - Z_{i\sigma})^2 \right\} \left( a_{i} \chi_{\uparrow} + b_{i} \chi_{\downarrow} \right) \tau_{i} \]
- Parity projected wave function
  \[ \Phi^{\pi} = \hat{P}^{\pi} A \{ \phi_{1}, \phi_{2}, \cdots, \phi_{A} \} \]

Variational calculation
- The parameters \( Z_{i\sigma}, v_{\sigma}, a_{i}, b_{i} \) are determined by variational calculation, minimizing the energy with a constraint on the nuclear quadrupole deformation parameters \( \beta \)
- We obtain the optimized intrinsic wave function \( \Phi^{\pi}(\beta) \)

Angular momentum projection

The eigenstate for total angular momentum $J$ is described by projecting the basis wave function obtained by solving the variational calculation.

$$\Phi^{J\pi}_{MK}(\beta) = \hat{P}^{J}_{MK} \Phi^{\pi}(\beta) = \frac{2J + 1}{8\pi^2} \int d\Omega \ D^{J\pi}_{MK}(\Omega) R(\Omega) \Phi^{\pi}(\beta)$$

Generated Coordinate Method (GCM)

The coefficients $f_K(\beta_i)$ can be obtained by diagonalizing the Hamiltonian. The quadrupole deformation parameter $\beta$ is a generator coordinate.

- GCM wave function
  $$\Psi^{J\pi}_{MK,\alpha} = \sum_{iK} f_{K,\alpha}(\beta_i) \hat{P}^{J}_{MK} \Phi^{\pi}(\beta_i)$$

- Hill-Wheeler Eq.
  $$\sum_{jK'} (H_{iKjK'} - E_{\alpha} N_{iKjK'}) f_{K',\alpha}(\beta_j) = 0$$

$$H_{iKjK'} = \langle \hat{P}^{J}_{MK} \Phi^{\pi}(\beta_i) | \hat{H} | \hat{P}^{J}_{MK'} \Phi^{\pi}(\beta_j) \rangle, \quad N_{iKjK'} = \langle \hat{P}^{J}_{MK} \Phi^{\pi}(\beta_i) | \hat{P}^{J}_{MK'} \Phi^{\pi}(\beta_j) \rangle$$
Evaluation of the cluster formation (e.g.: α cluster)

**Reduced Width Amplitude (RWA)**
Probability amplitude $\mathcal{Y}(a)$ at which α cluster $\Phi_\alpha$ exists from the daughter nucleus $\Phi_{A-4}$

\[ \mathcal{Y}(a) = \sqrt{\left(\frac{A}{4}\right)} \left\langle \frac{\delta(r-a)}{r_\alpha} \right\rangle Y_\ell(\hat{r}) [\Phi_\alpha^{0+} + \Phi_{A-4}(\text{g.s.})] |\Phi_A(\text{g.s.})]\]

**Spectroscopic factor (S-factor)**

\[ S(\alpha) = \int_0^\infty da \ |a\mathcal{Y}(a)|^2 \]

Y. Chiba *et al.*, PTEP 2017, 053D01 (2017)
Evaluation of the cluster formation (e.g.: α cluster)

**Reduced Width Amplitude (RWA)**
Probability amplitude $\mathcal{Y}(a)$ at which α cluster $\Phi_\alpha$ exists from the daughter nucleus $\Phi_{A-4}$

- Be isotopes have the same molecular orbit configuration of previous AMD calculations

Y. Kanada-En’yo, PRC 91, 014315 (2015)

\[ \sigma\text{-orbit enhances the } 2\alpha \text{ core} \]

![Graph showing probability amplitude vs. a (fm) for $\alpha$ cluster, $^{10}$Be($\pi^2$), and $^{12}$Be($\pi^2\sigma^2$).](image)
Be-isotope

- Cluster distance reflects the molecular orbit picture.
- The $\alpha$ cluster formation is decreasing as the number of neutrons increases.
  - S-factors of $\alpha$ cluster can not explain the enhancement of clustering.
  - There would be some clustering of $^6$He.
The \( ^6\text{He} \) cluster formation is also considered in \(^{12}\text{Be} \) and \(^{14}\text{Be} \).

- S-factors of \(^6\text{He} \) are comparable or not negligible to that of \( \alpha \).

S\(_{\text{sum}} \): Summation of \( \alpha \) and \(^6\text{He} \) S-factors

- The structures of \(|\alpha + \Phi_{A-4}\rangle \) and \(|^6\text{He} + \Phi_{A-6}\rangle \) are not very similar.

Total S-factors of \( \alpha \) and \(^6\text{He} \) look correlated with the enhancement of clustering.
B-isotope

- AMD reproduces the trend of the charge radius.
  
- Cluster formation is expected to be enhanced toward the neutron drip-line

- The $\alpha$ cluster formation is decreasing as the number of neutrons increases.

$^{11}\text{B}$: $\pi^+$ scattering

$^{13-17}\text{B}$: Charge-changing cross sections measurements
A. Estradé et al. PRL 113, 132501 (2014)
The $\alpha$, $^6\text{He}$, and $^8\text{He}$ cluster formations are considered.

Total $S$-factors of $\alpha$, $^6\text{He}$, and $^8\text{He}$ look correlated with the enhancement of clustering.

- $^8\text{He}$ $S$-factor of $^{19}\text{B}$ is not large due to its oblate deformation.
Summary

- Cluster formation in light neutron-rich nuclei
  - $\alpha$ cluster formation decreases in C isotopes but is expected to be enhanced in Be and B isotopes

- Investigated the cluster formation in neutron-rich Be and B isotopes
  - Not only $\alpha$ cluster but also $^6$He and $^8$He clusters were considered
  - $S_{\text{sum}}$ looks correlate to the enhancement of clustering

Plan

- Relation between the cluster formation and neutron-skin thickness to investigate the cluster formation in the neutron matter