HOKKAIDO UNIVERSITY

Cluster formation in light neutron-rich nuclei

YKIS2022b @ YITP + Zoom

May 25, 2022

Hideaki Motoki, Yoshiki Suzuki*, Tsuyoshi Kawai, and Masaaki Kimura
Hokkaido University
*RCNP

Background

○ Cluster formation is a universal phenomenon found in hierarchy layers of matter.

○ α cluster plays an important role in both finite and infinite nuclear systems.

- In dilute nuclear matter,
  α cluster formation is related to the nuclear density and symmetry energy

S. Typel *et al.*, PRC **81**, 015803 (2010)
K. Hagel *et al.*, PRL **108**, 062702 (2012)
S Typel *et al.*, EPJA **50**(2), 1 (2014)
Zhao-Wen Zhang *et al.*, PRC **95**, 064330 (2017)

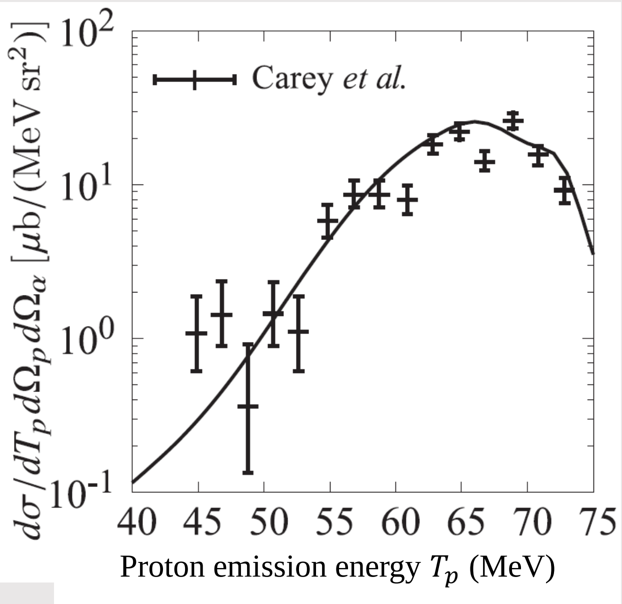
- In finite nuclei,
  the growth of neutron-skin hinders the α cluster formation

α cluster formation in the finite nuclei

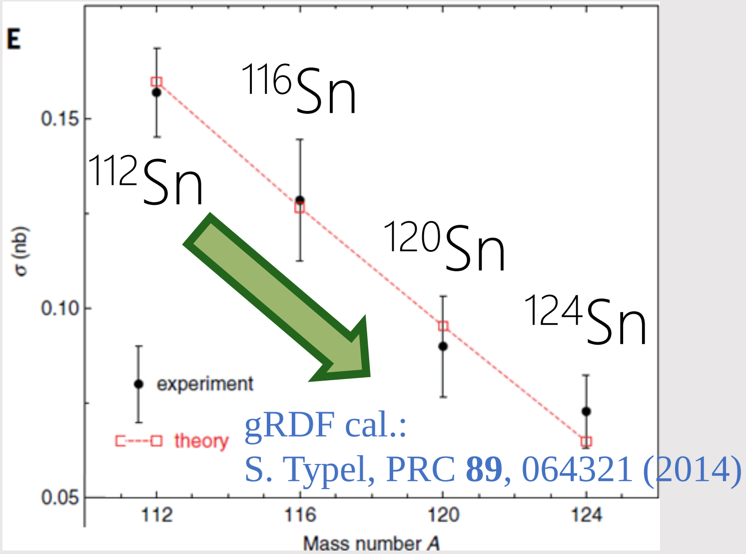
- (*p*, *p*α) is a useful measure for the <u>α cluster formation</u> at the nuclear surface
  - A negative correlation
    between the neutron-skin thickness and α cluster formation in Sn isotopes

- A negative correlation was also theoretically predicted in C isotopes

[20]Ne(*p*, *p*α)

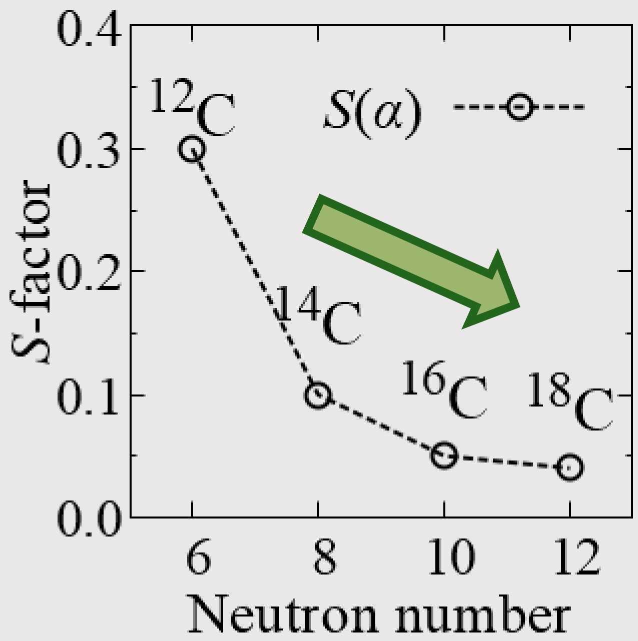


Sn(*p*, *p*α)



-> neutron-skin thickness

AMD+RWA



K. Yoshida *et al.*, PRC, **100**, 044601 (2019)　J. Tanaka *et al.*, Science, **371**(6526), 260 (2021)　Q. Zhao *et al.*, EPJA **57**, 157 (2021)

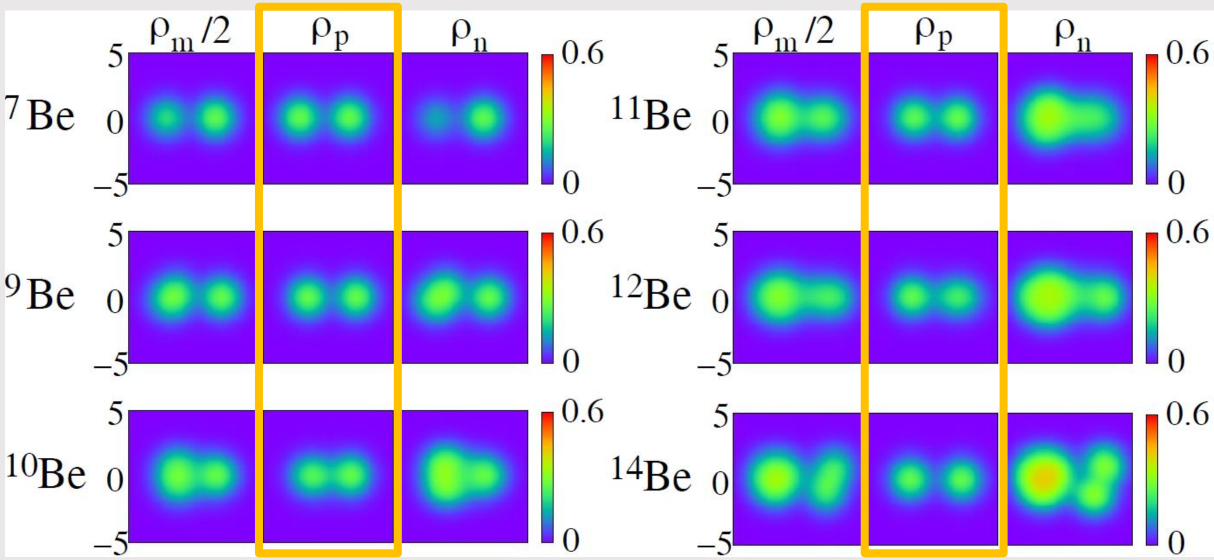
α cluster formation in Be and B isotopes

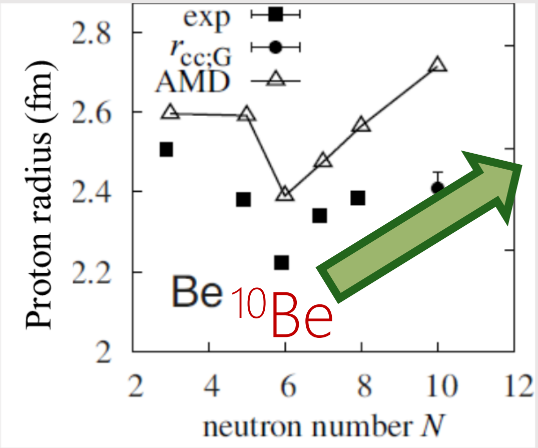
- α cluster formation is predicted to be enhanced toward the neutron drip-line based on the proton density and its radius

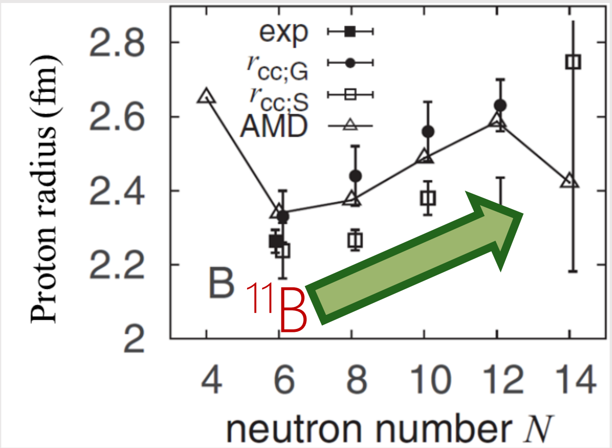
Y. Kanada-En'yo *et al.*, PRC **52**, 647 (1995)
Y. Kanada-En'yo *et al.*, PTPS **142**, 205 (2001)

Be isotopes

B isotopes





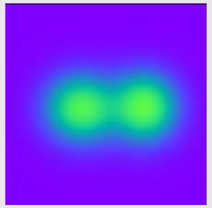


Y. Kanada-En'yo, PRC **91**, 014315 (2015)

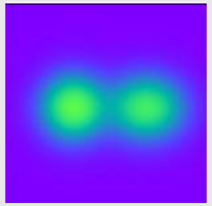
α cluster formation in Be and B isotopes

- α cluster formation is predicted to be enhanced toward the neutron drip-line based on the proton density and its radius

- In Be isotopes, the enhancement of clustering is explained by the **molecular orbit** occupied by the excess neutrons.
  - π-orbit bonds the 2α core
  - σ-orbit enhances the 2α core

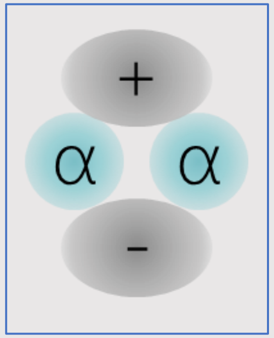


$^{10}Be(\pi^2)$

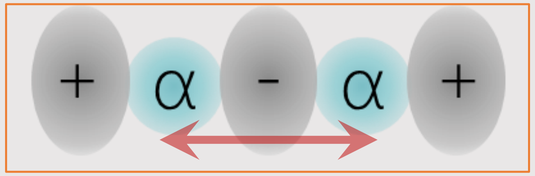


$^{12}Be(\pi^2\sigma^2)$

Proton density



π-orbit

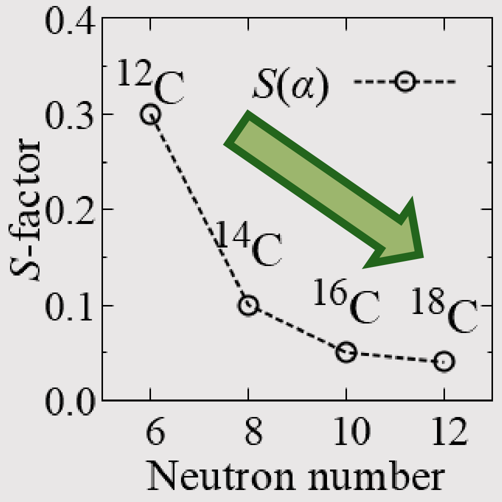


σ-orbit

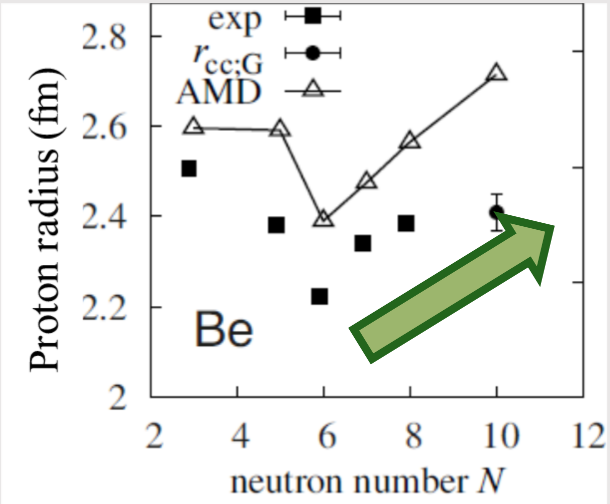
Y. Kanada-En'yo, PRC **91**, 014315 (2015)

Motivation

○ α cluster formation decreases



○ α cluster formation is enhanced



○ No direct evidence of the cluster development in Be and B isotopes has been found

– Physical quantities that can directly probe the cluster structure are desirable

Purpose of this talk

○ Investigate the cluster formation in Be and B isotopes with AMD.

– Not only α cluster but also ^{6}He and ^{8}He cluster formation probabilities

Antisymmetrized Molecular Dynamics (AMD)

Hamiltonian

$$\hat{H} = \sum_i^A \hat{t}_i - \hat{t}_{\mathrm{c.m.}} + \sum_{i<j}^A \hat{v}_{ij}^{\mathrm{Gogny\ D1S}} + \sum_{i<j\in\mathrm{proton}}^A \hat{v}_{ij}^{\mathrm{Coulomb}}$$

J. F. Berger, M. Girod and D. Gogny , CPC **63** , 365 (1991)

Intrinsic model wave function

- Single nucleon wave function

$$\phi_i(\boldsymbol{r}) = \exp\left\{-\sum_{\sigma=x,y,z} \nu_\sigma (r_\sigma - Z_{i\sigma})^2\right\}(a_i\chi_\uparrow + b_i\chi_\downarrow)\tau_i$$

- Parity projected wave function

$$\Phi^\pi = \hat{P}^\pi \mathcal{A}\{\phi_1, \phi_2, \cdots, \phi_A\}$$

Variational calculation

- The parameters $Z_{i\sigma}, \nu_\sigma, a_i, b_i$ are determined by variational calculation, minimizing the energy with a constraint on the nuclear quadrupole deformation parameters β
- We obtain the optimized intrinsic wave function $\Phi^\pi(\beta)$

Angular momentum projection

The eigenstate for total angular momentum J is described by projecting the basis wave function obtained by solving the variational calculation.

$$\Phi^{J\pi}_{MK}(\beta) = \hat{P}^J_{MK}\Phi^\pi(\beta) = \frac{2J+1}{8\pi^2}\int \mathrm{d}\Omega\, D^{J*}_{MK}(\Omega)R(\Omega)\Phi^\pi(\beta)$$

Generated Coordinate Method (GCM)

The coefficients $f_K(\beta_i)$ can be obtained by diagonalizing the Hamiltonian.
The quadrupole deformation parameter β is a generator coordinate.

► GCM wave function $\Psi^{J\pi}_{M,\alpha} = \sum_{iK} f_{K,\alpha}(\beta_i)\hat{P}^J_{MK}\Phi^\pi(\beta_i)$

► Hill-Wheeler Eq. $\sum_{jK'}(H_{iKjK'} - E_\alpha N_{iKjK'})f_{K',\alpha}(\beta_j) = 0$

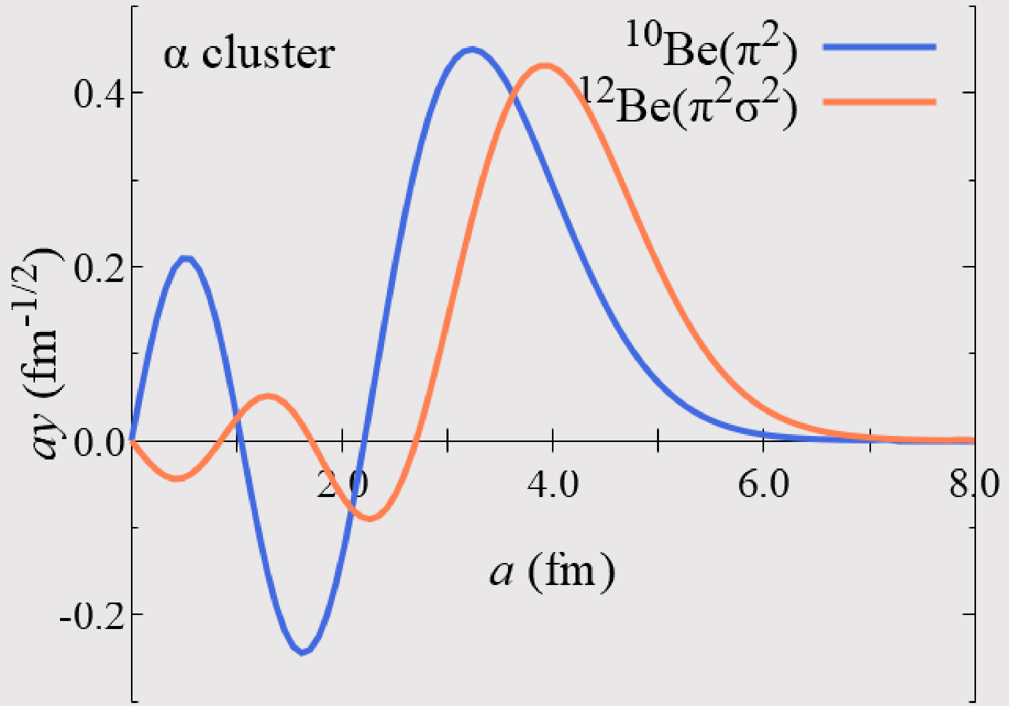
$$H_{iKjK'} = \langle \hat{P}^J_{MK}\Phi^\pi(\beta_i)|\,\hat{H}\,|\hat{P}^J_{MK'}\Phi^\pi(\beta_j)\rangle\,, \qquad N_{iKjK'} = \langle \hat{P}^J_{MK}\Phi^\pi(\beta_i)|\,\hat{P}^J_{MK'}\Phi^\pi(\beta_j)\rangle$$

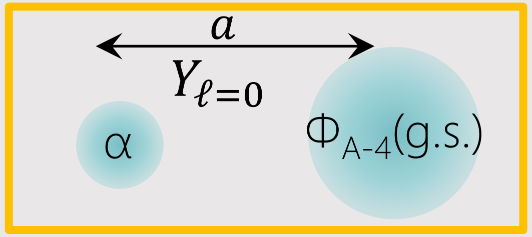
Evaluation of the cluster formation (e.g.: α cluster)

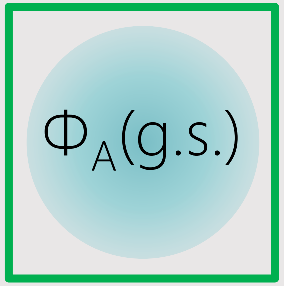
Reduced Width Amplitude (RWA)

Probability amplitude $\mathcal{Y}(a)$ at which α cluster Φ_α exists from the daughter nucleus Φ_{A-4}

$$\mathcal{Y}(a) = \sqrt{\binom{A}{4}} \langle \frac{\delta(r-a)}{ra} Y_\ell(\hat{\boldsymbol{r}})[\Phi_\alpha^{0+} \Phi_{A-4}(\mathrm{g.s.})] | \Phi_A(\mathrm{g.s.}) \rangle$$







Spectroscopic factor (S-factor)

$$S(\alpha) = \int_0^\infty \mathrm{d}a \, |a\mathcal{Y}(a)|^2$$

Y. Chiba *et al.*, PTEP **2017**, 053D01 (2017)

Evaluation of the cluster formation (e.g.: α cluster)

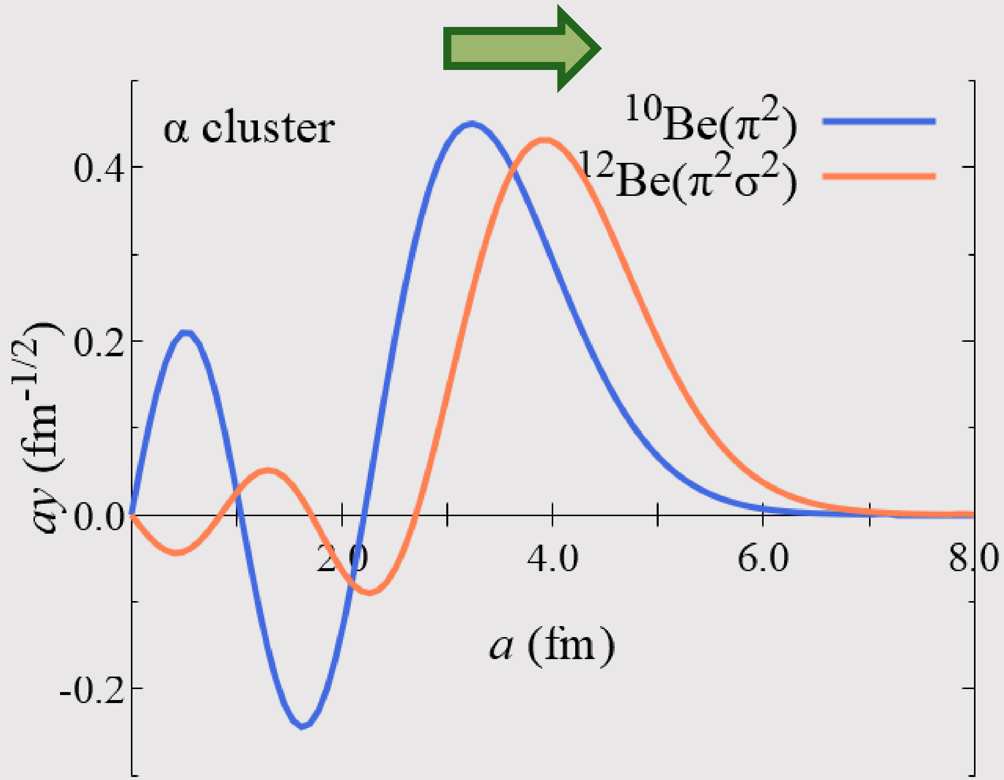
Reduced Width Amplitude (RWA)

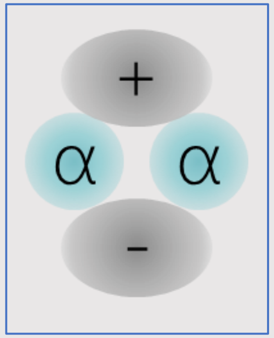
Probability amplitude $\mathcal{Y}(a)$ at which α cluster Φ_α exists from the daughter nucleus Φ_{A-4}

○ Be isotopes have the same molecular orbit configuration of previous AMD calculations

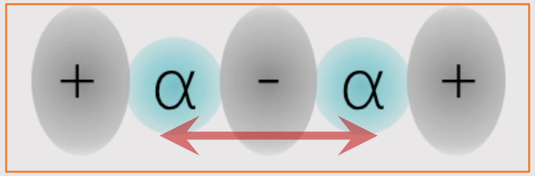
Y. Kanada-En'yo, PRC **91**, 014315 (2015)

σ-orbit enhances the 2α core





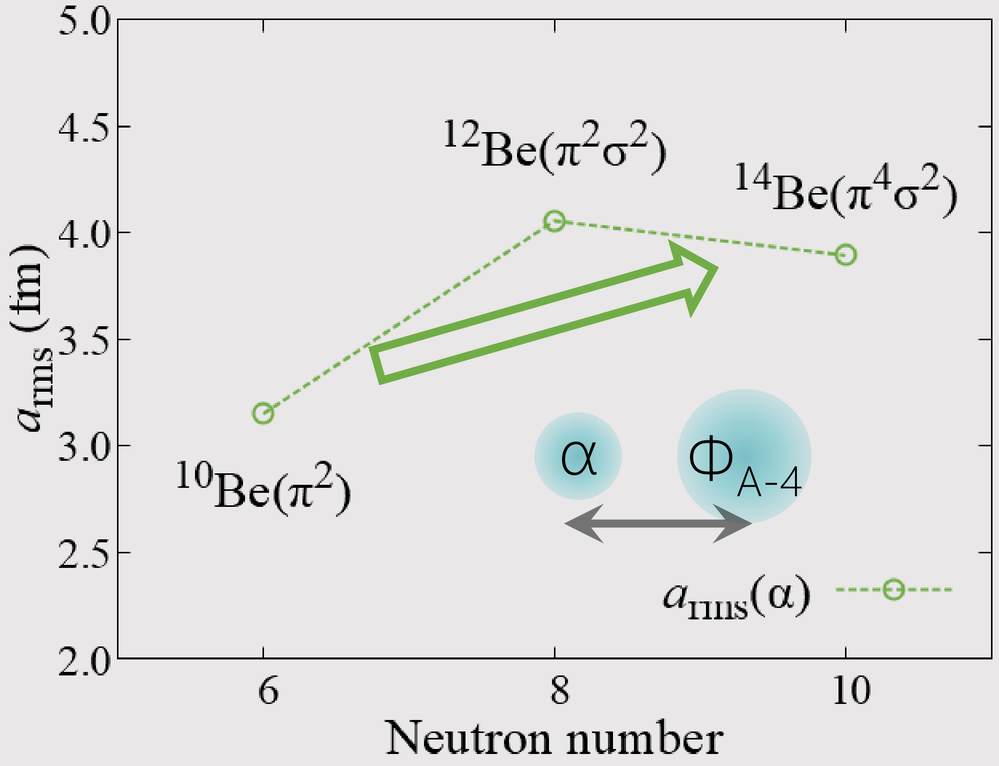
π-orbit

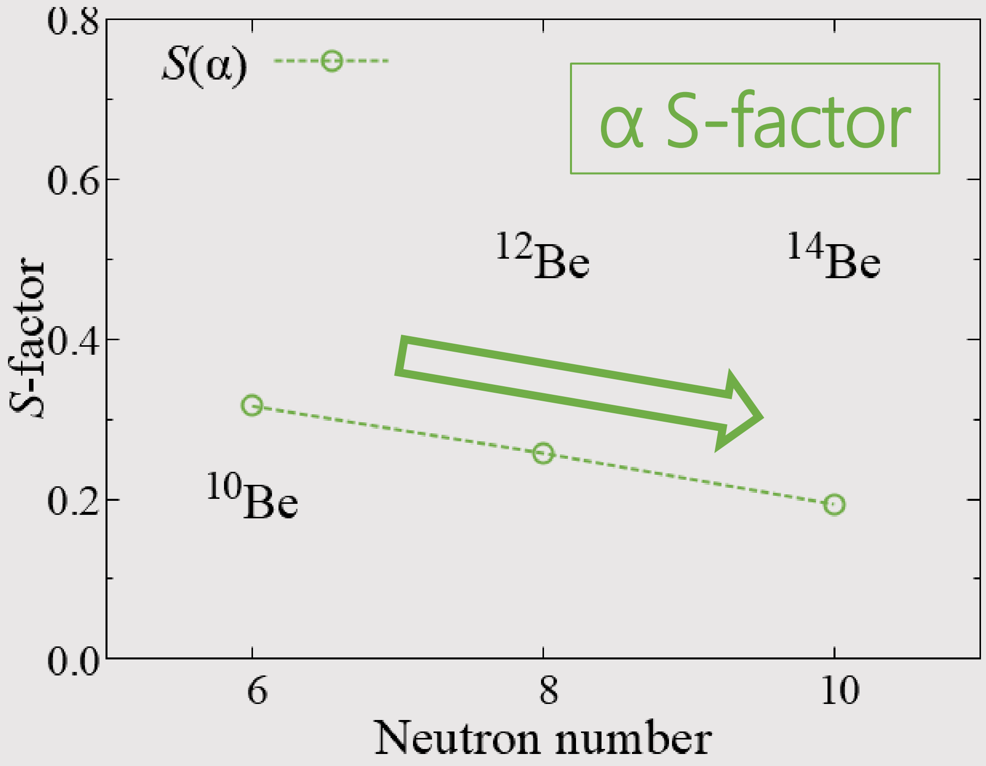


σ-orbit

① Be-isotope

- Cluster distance reflects the molecular orbit picture.
- The α cluster formation is decreasing as the number of neutrons increases.
  - S-factors of α cluster can not explain the enhancement of clustering.
  - There would be some clustering of ^{6}He.

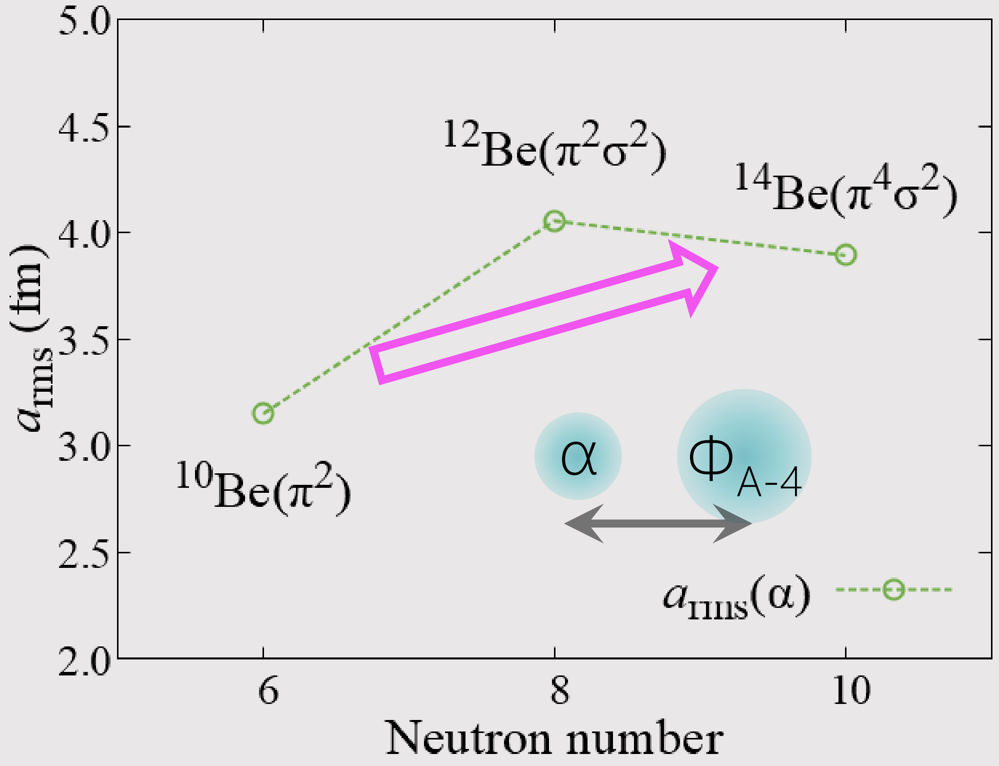


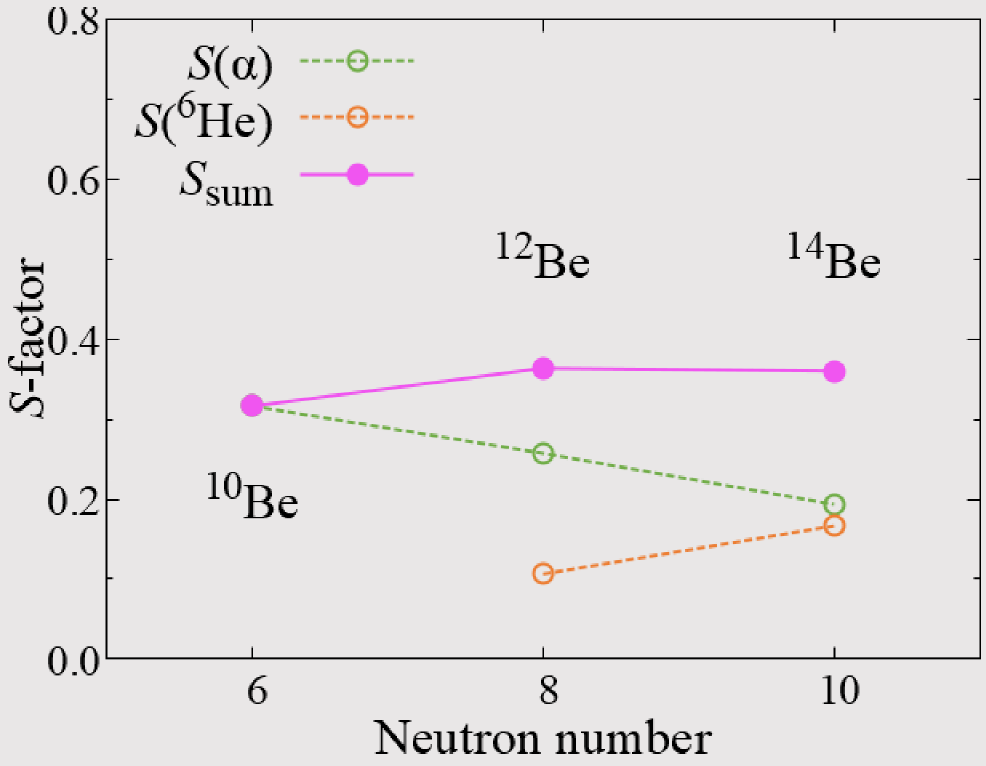


① Be-isotope

- The ^{6}He cluster formation is also considered in ^{12}Be and ^{14}Be.
  - S-factors of ^{6}He are comparable or not negligible to that of α.

- S_{sum}: Summation of α and ^{6}He S-factors
  - The structures of $|\alpha+\Phi_{A-4}\rangle$ and $|{}^{6}\mathrm{He}+\Phi_{A-6}\rangle$ are not very similar.

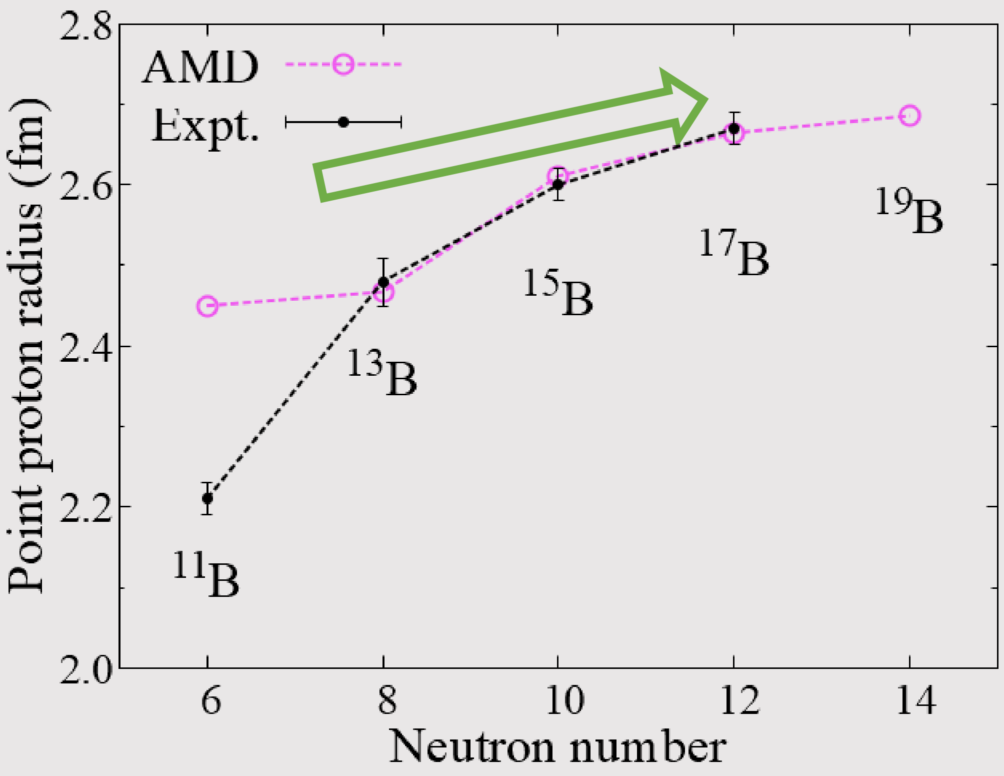


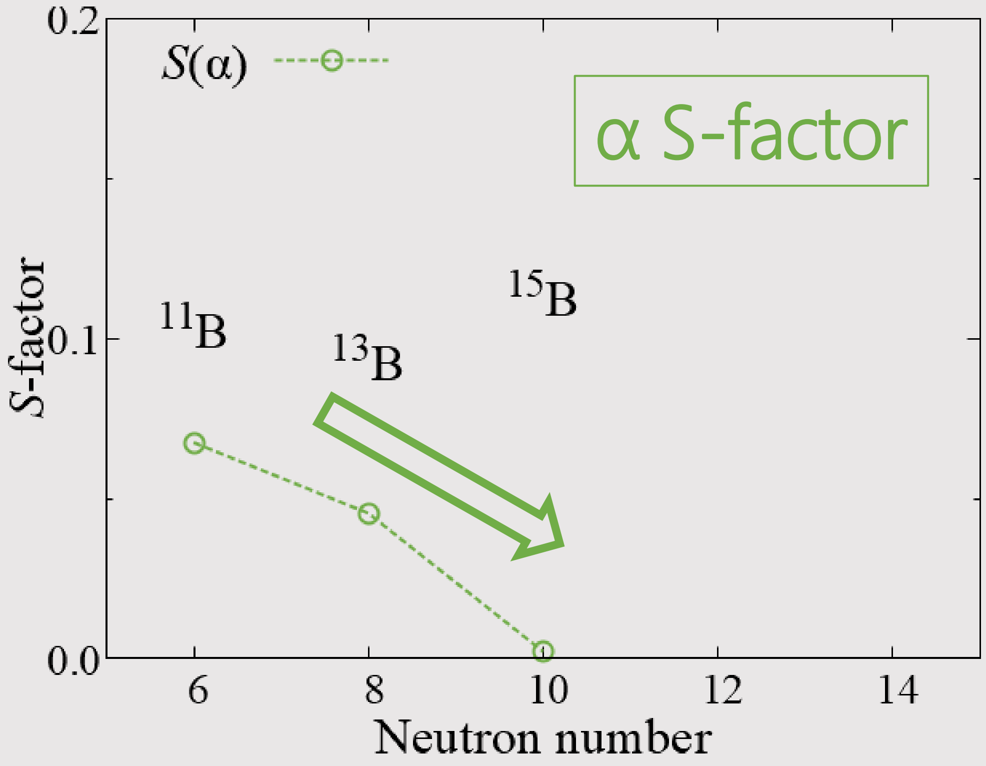


- Total S-factors of α and ^{6}He look correlated with the enhancement of clustering.

② B-isotope

- AMD reproduces the trend of the charge radius.
  - Cluster formation is expected to be enhanced toward the neutron drip-line
- The α cluster formation is decreasing as the number of neutrons increases.

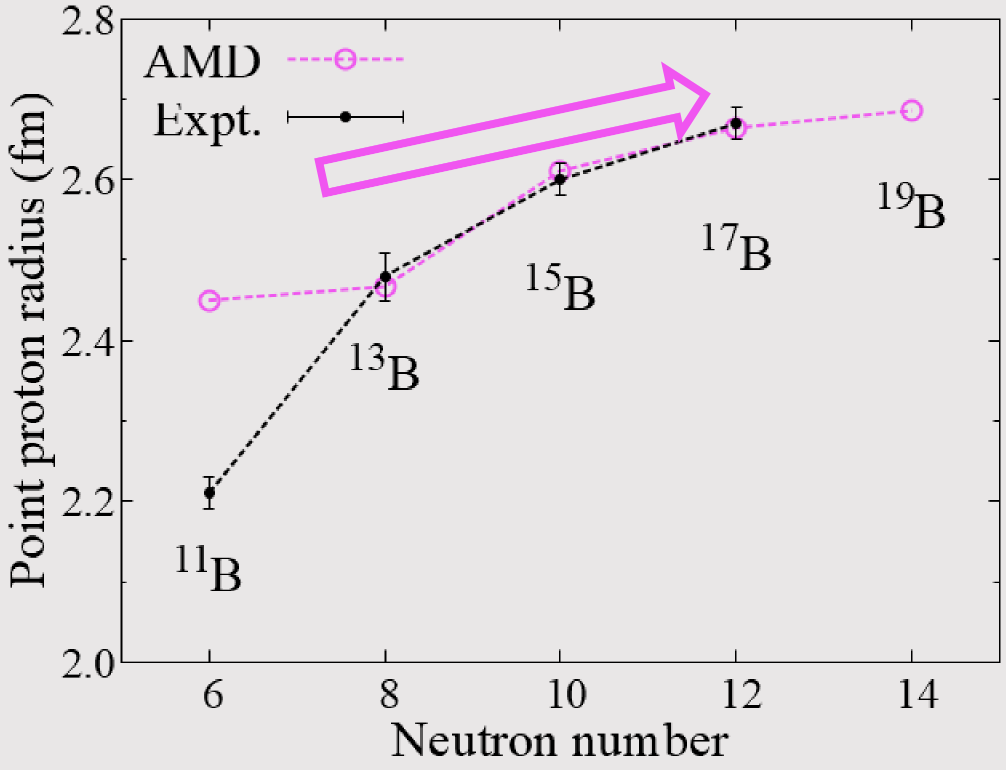


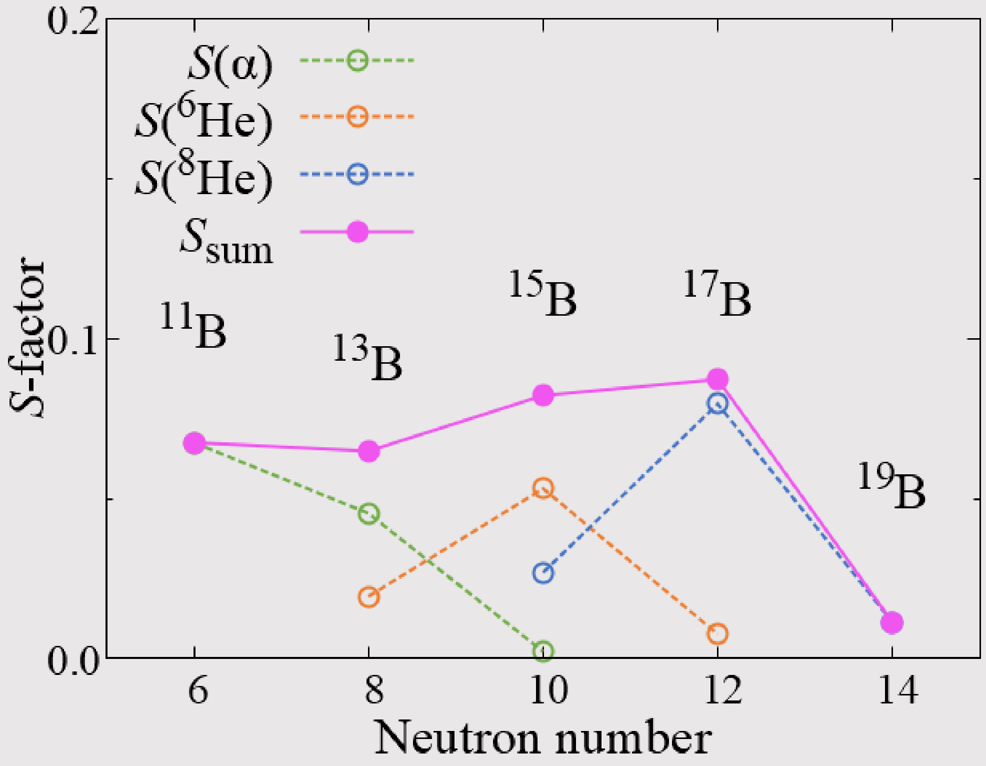


^{11}B: π^+ scattering
B. M. Barnett *et al*., Phys. Lett. **97B**, 45 (1980)
$^{13\text{-}17}$B: Charge-changing cross sections measurements
A. Estradé *et al*. PRL **113**, 132501 (2014)

② B-isotope

- The α, ^{6}He, and ^{8}He cluster formations are considered.





- Total S-factors of α, ^{6}He, and ^{8}He look correlated with the enhancement of clustering.
  - ^{8}He S-factor of ^{19}B is not large due to its oblate deformation.

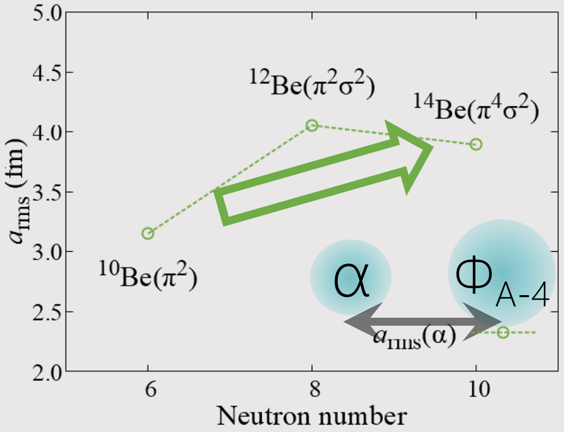
Summary

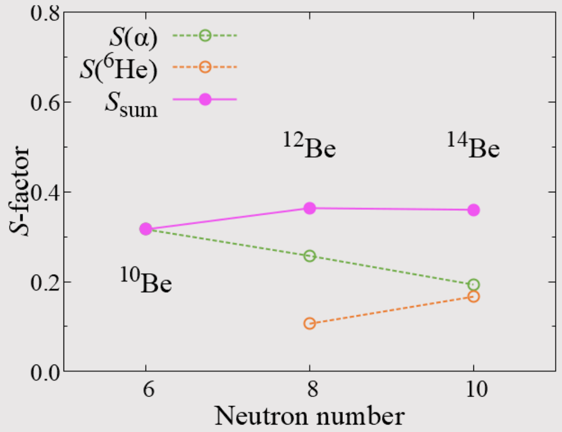
○ Cluster formation in light neutron-rich nuclei

- α cluster formation decreases in C isotopes
  but is expected to be enhanced in Be and B isotopes

○ Investigated the cluster formation in neutron-rich Be and B isotopes

- Not only α cluster but also ^{6}He and ^{8}He clusters were considered
- S_{sum} looks correlate to the enhancement of clustering





○ Plan

- Relation between the cluster formation and neutron-skin thickness
  to investigate the cluster formation in the neutron matter