Toward *ab initio* charge symmetry breaking in nuclear energy density functionals

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RIKEN Interdisciplinary Theoretical and Mathematical Sciences Program

SCHOOL OF SCIENCE
THE UNIVERSITY OF TOKYO

Introduction

Nuclear Interaction and Isospin T

- Nuclear interaction: *almost* isospin symmetric

  $v_{pp}^{T=1} \simeq v_{pn}^{T=1} \simeq v_{nn}^{T=1}$ & $v_{pn}^{T=0} \neq v^{T=1}$
- ISB terms of bare nuclear int. is 0.1–$1\ \%$ of main (isospin sym.) part
- Atomic nuclei: also *almost* isospin symmetric
  - Atomic nuclei with the same T and A: *almost* the same properties
  - Nuclear properties: *almost* T_z-independent
- Most nuclear EDFs are constructred with assuming isospin symmetric

- Thanks to recent progress of precise measurements,
  isospin symmetry breaking (ISB) of atomic nuclei is highlighted
-

Introduction

An Example of Isospin Symmetry in Atomic Nuclei—Mirror Nuclei

- If neither Coulomb int. nor ISB terms of nuclear int. existed, the properties of mirror nuclei would be identical
  → In reality, the properties are not identical
- Mass difference of mirror nuclei ΔE_{tot} mainly originates from Coulomb int.
  → Coulomb int. is not enough "**Okamoto-Nolen-Schiffer anomaly**"
- Recently, differences of other properties have also been revealed

NSCL G.S. of ${}^{73}_{38}\mathrm{Sr}$ is $J^{\pi} = 5/2^{-}$, while G.S. of ${}^{73}_{35}\mathrm{Br}$ is $J^{\pi} = 1/2^{-}$

RIBF ${}^{70}_{36}\mathrm{Kr}$ has different shape to ${}^{70}_{34}\mathrm{Se}$

Okamoto. *Phys. Lett.* **11**, 150 (1964)

Nolen and Schiffer. *Annu. Rev. Nucl. Sci.* **19**, 471 (1969)

Hoff *et al. Nature* **580**, 52 (2020)

Wimmer *et al. Phys. Rev. Lett.* **126**, 072501 (2021)

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- Most nuclear EDFs are constructred with assuming isospin symmetric

- Thanks to recent progress of precise measurements, isospin symmetry breaking of (ISB) atomic nuclei is highlighted
- To understand such isospin symmetry breaking systematically, isospin symmetry breaking should be considerd in EDF (effective int.)

Isospin Symmetry Breaking (ISB) Terms of Nuclear Inteaction

- **Charge symmetry breaking (CSB)**
  - Difference between p-p int. and n-n int.

$$v_{\mathrm{CSB}} \equiv v_{nn}^{T=1} - v_{pp}^{T=1} \sim \tau_{zi} + \tau_{zj}$$

  - Originates from mass difference of nucleons ($m_p \neq m_n$) and π^0-η & ρ^0-ω mixings in meson-exchange process
- **Charge independence breaking (CIB)**
  - Difference between like-particle int. and diff.-particle int.

$$v_{\mathrm{CIB}} \equiv \frac{v_{nn}^{T=1} + v_{pp}^{T=1}}{2} - v_{np}^{T=1} \sim \tau_{zi}\tau_{zj}$$

  - Originates from mass difference of pions ($m_{\pi^0} \neq m_{\pi^\pm}$)
- In bare interaction, CIB is ≈ 10 times stronger than CSB

van Kolck. *Few-Body Syst. Suppl.* **9**, 444 (1995)
Wiringa *et al. Phys. Rev. C* **51**, 38 (1995)
Miller, Opper, and Stephenson. *Annu. Rev. Nucl. Part. Sci.* **56**, 253 (2006)

Isospin Symmetry Breaking in Skyrme Interaction

Skyrme-like s-wave ISB Interaction

$$v_{\text{Sky}}^{\text{CSB}}(\boldsymbol{r}) = s_0 (1 + y_0 P_\sigma) \delta(\boldsymbol{r}) \frac{\tau_{1z} + \tau_{2z}}{4}$$

$$v_{\text{Sky}}^{\text{CIB}}(\boldsymbol{r}) = u_0 (1 + z_0 P_\sigma) \delta(\boldsymbol{r}) \frac{\tau_{1z}\tau_{2z}}{2}$$

$$\mathcal{E}_{\text{CSB}}\left[\rho_p, \rho_n\right] = \frac{s_0 (1 - y_0)}{8} \left(\rho_n^2 - \rho_p^2\right)$$

$$\mathcal{E}_{\text{CIB}}\left[\rho_p, \rho_n\right] = \frac{u_0}{8} (1 - z_0) \left[\left(\rho_n^2 + \rho_p^2\right) - 2 (2 + z_0) \rho_n \rho_p\right]$$

- Parameters: s_0, u_0, y_0, and z_0

Sagawa, Van Giai, and Suzuki. *Phys. Lett. B* **353**, 7 (1995)
Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)
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- Parameters: s_0, u_0, y_0, and z_0
- Since ISB terms are tiny compared to isospin symmetric (main) part, it is better if these parameters can be fixed theoretically

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SAMi-ISB Interaction

- All the parameters including the main part are fitted altoghether
- $y_0 = z_0 = -1$ to select the spin-singlet ($S = 0$) channel
- s_0 and u_0 are fitting parameters

CSB $s_0 = -26.3\,\mathrm{MeV\,fm^3}$ Isobaric analog energy of $^{208}\mathrm{Pb}$

Fitted to exp. value

CIB $u_0 = +25.8\,\mathrm{MeV\,fm^3}$ CIB energy of symmetric nuclear matter

Ab initio!

Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

SkM*-ISB, SLy4-ISB, and SV-ISB Interaction

- ISB parts are introduced on top of conventional Skyrme int.
- CSB $s_0 \simeq -10\,\mathrm{MeV\,fm^3}$ and CIB u_0 are detemined to reproduce mirror displacement energy & triplet displacement energy

Fitted to exp. value

- CIB operator is different from that SAMi-ISB used

Bączyk, Dobaczewski *et al. Phys. Lett. B* **778**, 178 (2018)

Can we determine CSB s_0 theoretically??

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CSB is sensitive to ΔR_{np} and ΔE_{tot}

Isospin-Symmetric Term Dependence

- If different parameter set of $v_{\text{Sky}}^{\text{IS}}$ is used, how s_0-dependence on ΔR_{np} and ΔE_{tot} changes??

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)

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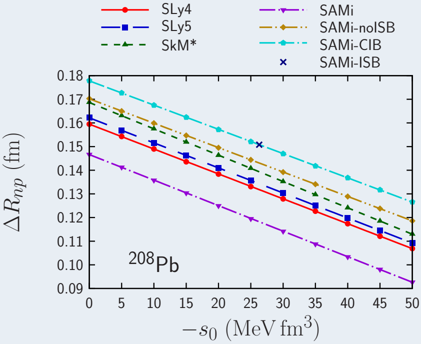
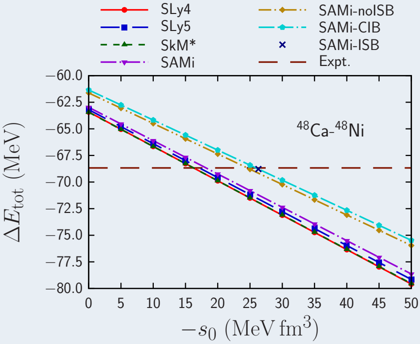


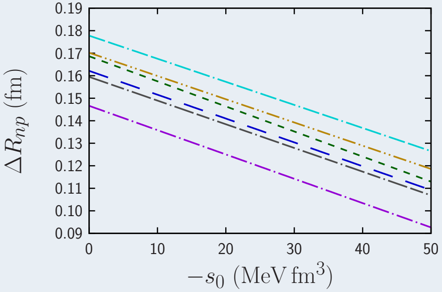


- $v_{\rm Sky}^{\rm IS}$ hardly affects the slope, although the absolute values are different

  Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)

Ab Initio Determination of CSB Strength s_0

A Method to Determine s_0



0. Calculate ΔR_{np} for various s_0 and fit to $\Delta R_{np} = a - bs_0$
1. Derive avaraged value $\overline{b}$

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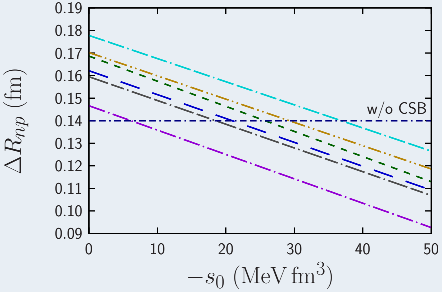

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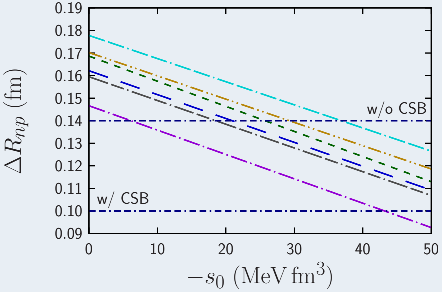

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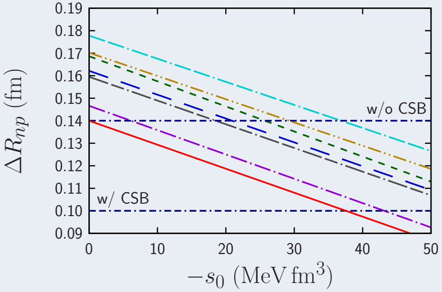

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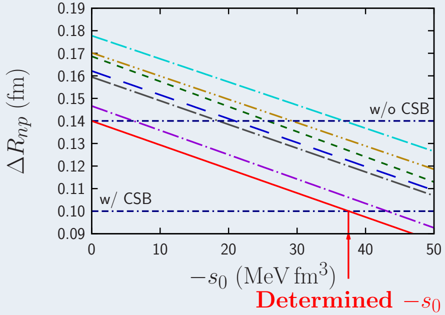

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4. s_0 can be determined by

$$s_0 = -\frac{\Delta R_{np}^{\text{w/ CSB}} - \Delta R_{np}^{\text{w/o CSB}}}{\bar{b}}$$

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)

Ab Initio Determination of CSB Strength s_0

Mysterious of CSB Strength

- ***Ab initio* determination**

  Combining with previous "slope" and theoretical calculation

  - $s_0 \simeq -2\,\mathrm{MeV\,fm^3}$ (ΔE_{tot} of $^{48}\mathrm{Ca}$-$^{48}\mathrm{Ni}$, CC & χEFT)
  - $s_0 \simeq -3\,\mathrm{MeV\,fm^3}$ (ΔE_{tot} of $^{10}\mathrm{Be}$-$^{10}\mathrm{C}$, VMC & AV18)

- **Phenomenological determination**—Referring experimental data

CC & χEFT: Novario, Lonardoni, Gandolfi, and Hagen. arXiv:2111.12775 [nucl-th]
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s_0-value: Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)
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- CSB effect in *ab initio* is $\times 0.1$ of that in DFT?!?!

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Open problem

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- *Ab initio* method to determine CSB strength is proposed
- Once ΔR_{np} or ΔE_{tot} with and without CSB are obtained, CSB strength can be determined
- Phenomenological value of s_0 is $\times 10$ of *ab initio* value
- CSB ctrb. to ΔR_{np} or ΔE_{tot} in DFT is $\times 10$ of those in *ab initio* calc

  Beyond MF Correction?? p-wave contribution??

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Thank you for attention!!