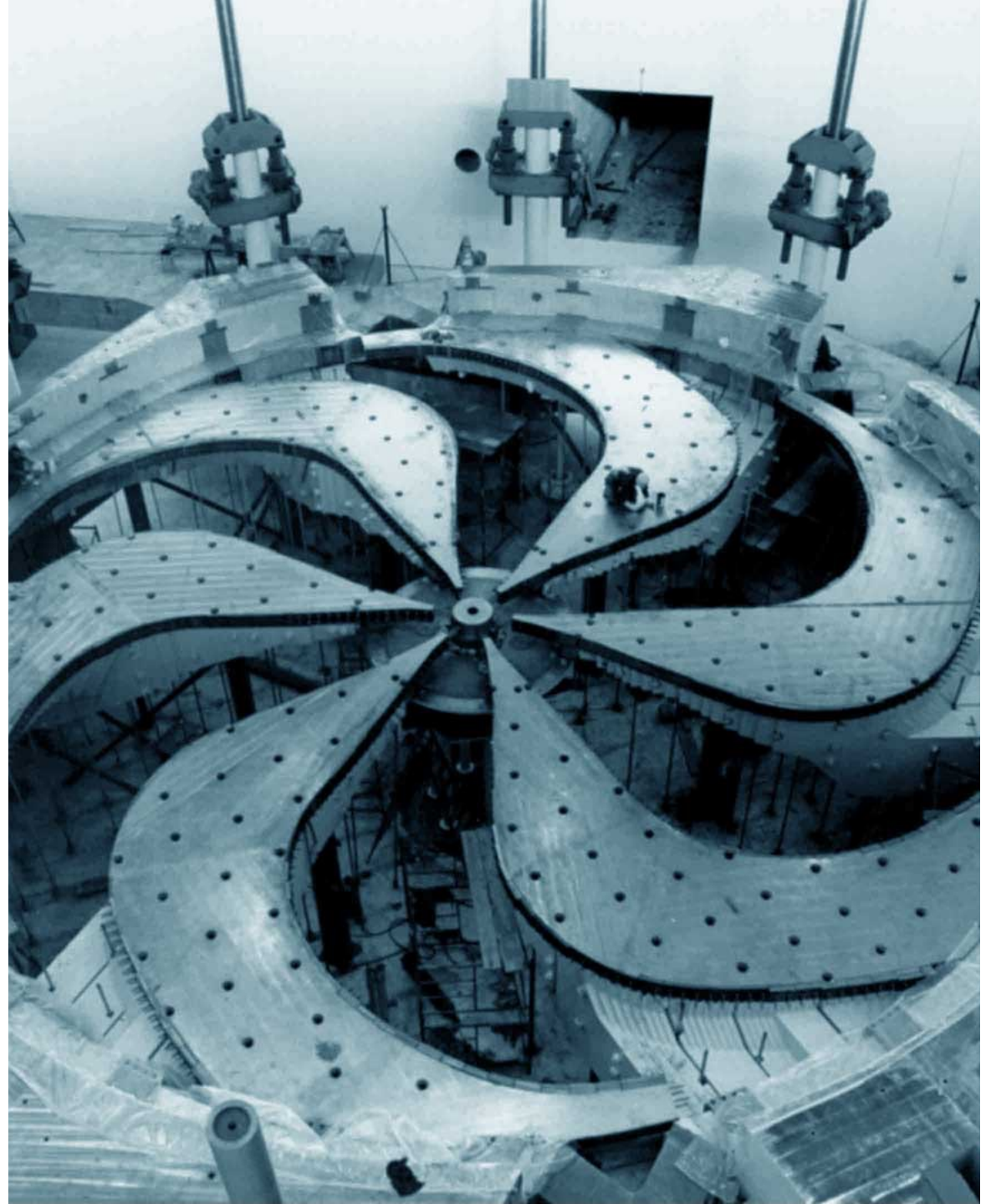


No-core shell model with continuum applications to nuclear astrophysics

SYMPOSIUM
“Developments of Physics of Unstable Nuclei (YKIS2022b)”
23-27 May 2022

Petr Navratil
TRIUMF

2022-06-09



Nuclear reactions in astrophysics

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- Nuclear reactions play important role in astrophysics and cosmology
 - Light nuclei synthesized in Big Bang
 - Heavier nuclei up to iron produced by fusion, transfer, radioactive capture in the stars
 - Still heavier nuclei produced by s-, i-, r-, rp-processes

- Thermonuclear reaction rate $r_{12} = \langle \sigma v \rangle n_1 n_2$

$$\langle \sigma v \rangle = \left(\frac{8}{\mu \pi} \right)^{1/2} (kT)^{-3/2} \int_0^{\infty} E \sigma(E) \exp(-E/kT) dE$$

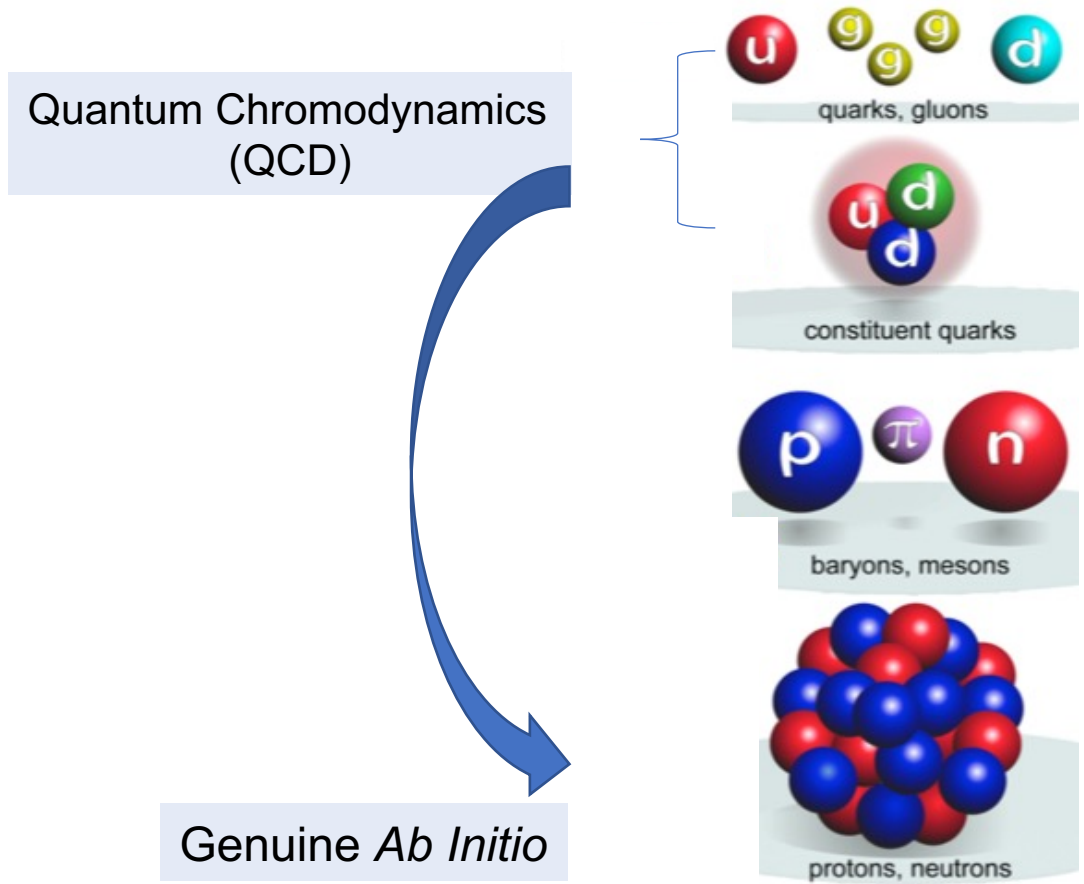
- S-factor

$$S(E) = \sigma(E) E \exp(2\pi\eta) \quad \text{Sommerfeld parameter} \quad \eta = Z_1 Z_2 e^2 / \hbar v$$

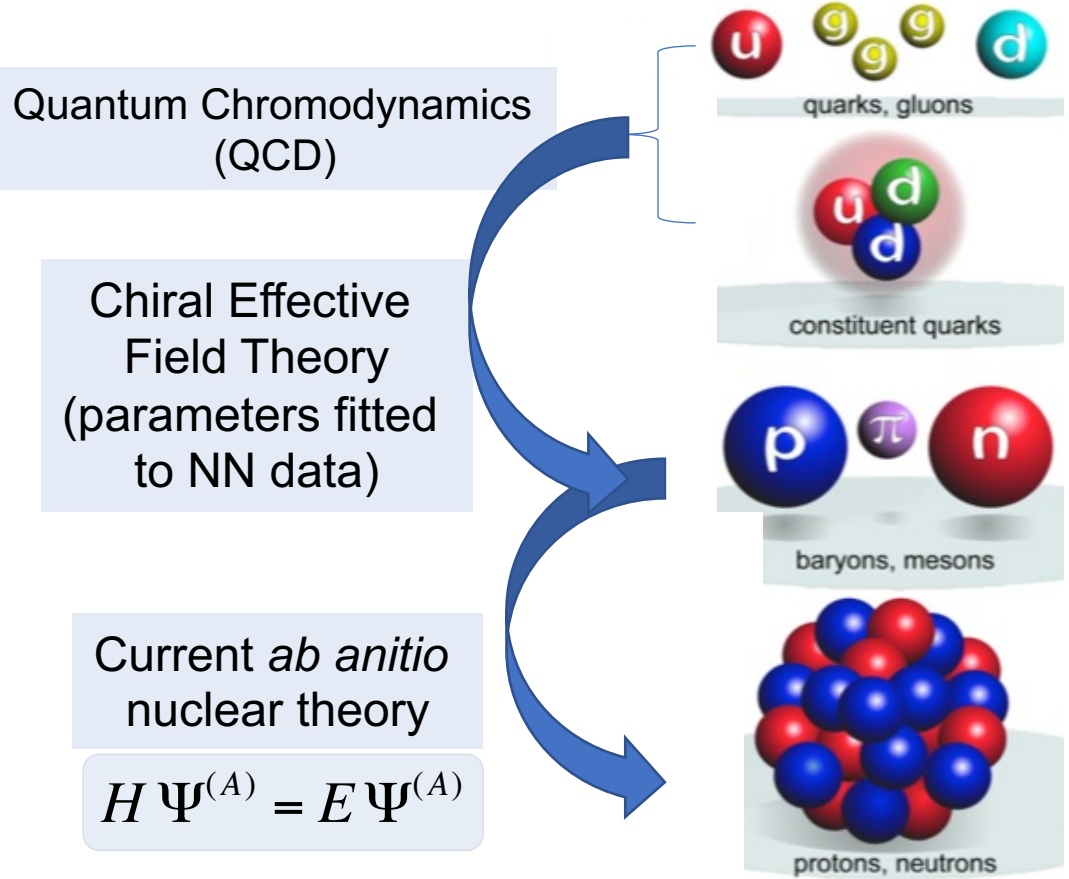
$$\langle \sigma v \rangle = \left(\frac{8}{\mu \pi} \right)^{1/2} (kT)^{-3/2} \int_0^{\infty} S(E) \exp(-E/kT - d/E^{1/2}) dE \quad d = (2\mu)^{1/2} \pi Z_1 Z_2 e^2 / \hbar$$

- Gamow window – narrow energy range; frequently inaccessible to experiments – **theory important**

First principles or *ab initio* nuclear theory



First principles or *ab initio* nuclear theory – what we do at present



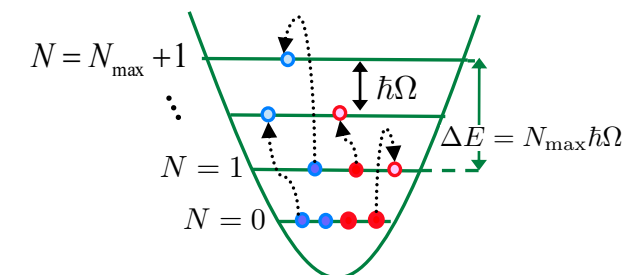


Conceptually simplest ab initio method: No-Core Shell Model (NCSM)

- Basis expansion method
 - Harmonic oscillator (HO) basis truncated in a particular way (N_{\max})
 - Why HO basis?
 - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 – ^4He , ^{16}O , ^{40}Ca)
 - Equivalent description in relative-coordinate and Slater determinant basis
- Short- and medium range correlations
- Bound-states, narrow resonances

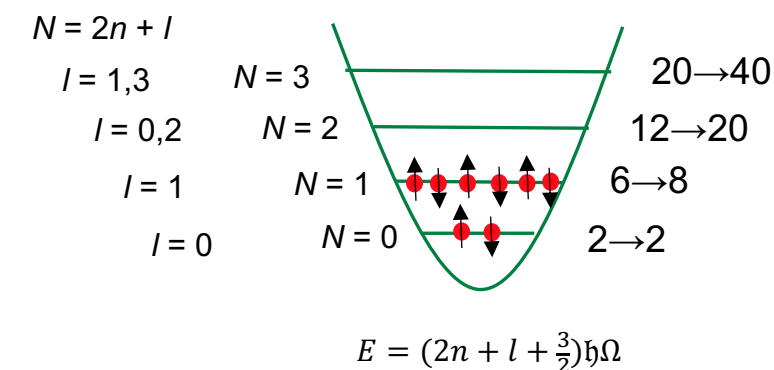


NCSM



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{HO}(\vec{\eta}_1, \vec{\eta}_2, \dots, \vec{\eta}_{A-1})$$

$$\Psi_{SD}^A = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj}^{SD} \Phi_{SDNj}^{HO}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$



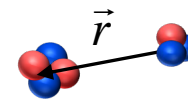
Ab Initio Calculations of Reactions Important for Astrophysics

Unified approach to bound & continuum states: NCSMC

- No-core shell model (NCSM)
 - A -nucleon wave function expansion in the harmonic-oscillator (HO) basis
 - short- and medium range correlations
 - Bound-states, narrow resonances
- NCSM with Resonating Group Method (NCSM/RGM)
 - cluster expansion, clusters described by NCSM
 - proper asymptotic behavior
 - long-range correlations
- Combine the above: *ab initio* no-core shell model with continuum (NCSMC)



NCSM



NCSM/RGM

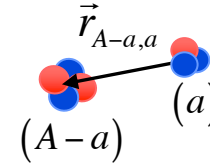
NCSMC

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{Nucleus} \\ \lambda \end{matrix} \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & (a) \\ \text{Cluster} & \text{Cluster} \\ \nu \end{matrix} \right\rangle$$

Unknowns

S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

Binary cluster basis



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- Working in partial waves ($\nu \equiv \{A-a \alpha_1 I_1^{\pi_1} T_1; a \alpha_2 I_2^{\pi_2} T_2; s\ell\}$)

$$|\psi^{J^{\pi T}}\rangle = \sum_{\nu} \hat{A}_{\nu} \left[\underbrace{\left(|A-a \alpha_1 I_1^{\pi_1} T_1\rangle \right)}_{\text{Target}} \underbrace{\left(|a \alpha_2 I_2^{\pi_2} T_2\rangle \right)}_{\text{Projectile}} \right]^{(sT)} Y_{\ell}(\hat{r}_{A-a,a}) \frac{g_{\nu}^{J^{\pi T}}(r_{A-a,a})}{r_{A-a,a}}$$

- Introduce a dummy variable \vec{r} with the help of the delta function

$$|\psi^{J^{\pi T}}\rangle = \sum_{\nu} \int \frac{g_{\nu}^{J^{\pi T}}(r)}{r} \hat{A}_{\nu} \left[\left(|A-a \alpha_1 I_1^{\pi_1} T_1\rangle \right) \left(|a \alpha_2 I_2^{\pi_2} T_2\rangle \right) \right]^{(sT)} Y_{\ell}(\hat{r}) \delta(\vec{r} - \vec{r}_{A-a,a}) r^2 dr d\hat{r}$$

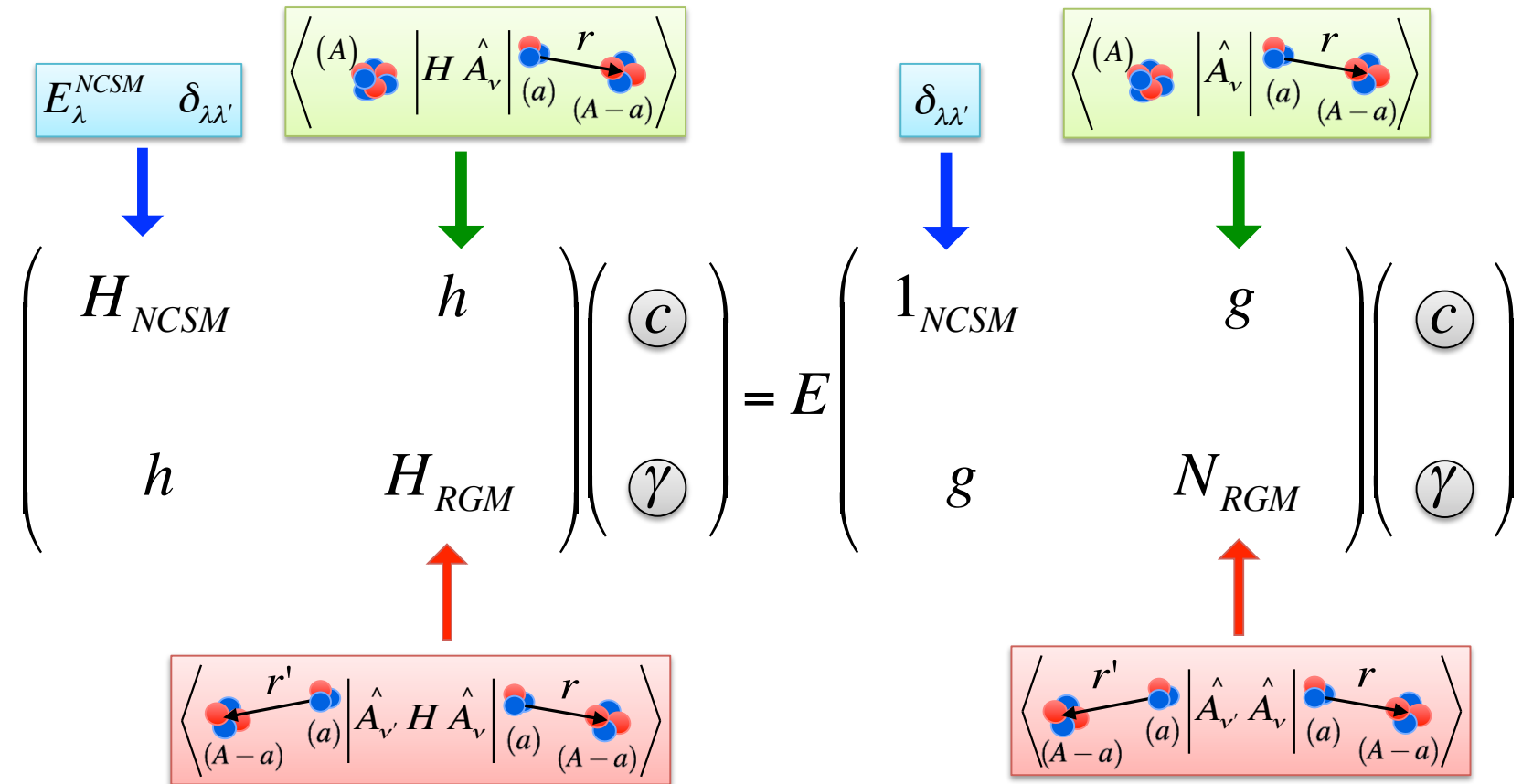
- Allows to bring the wave function of the relative motion in front of the antisymmetrizer

$$\sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} \vec{r} \\ (A-a) \quad (a) \end{array}, \nu \right\rangle$$

Coupled NCSMC equations

$$H \Psi^{(A)} = E \Psi^{(A)}$$

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{cluster} \end{matrix}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & (a) \\ \text{cluster} & \text{cluster} \end{matrix}, \nu \right\rangle$$



Radiative capture calculations in NCSMC

- Typically, E1 transitions the most important

- Operator
$$\hat{D} = e \sum_{i=1}^A \frac{1 + \tau_i^z}{2} (\mathbf{r}_i - \mathbf{R}_{\text{c.m.}}^{(A)})$$

- NCSMC wave function
$$|\Psi_A^{J^\pi T}\rangle = \sum_{\lambda} c_{\lambda}^{J^\pi T} |AJ^\pi T\rangle + \sum_{\nu} \int dr r^2 \frac{\gamma_{\nu}^{J^\pi T}(r)}{r} \mathcal{A}_{\nu} |\Phi_{\nu r}^{J^\pi T}\rangle$$

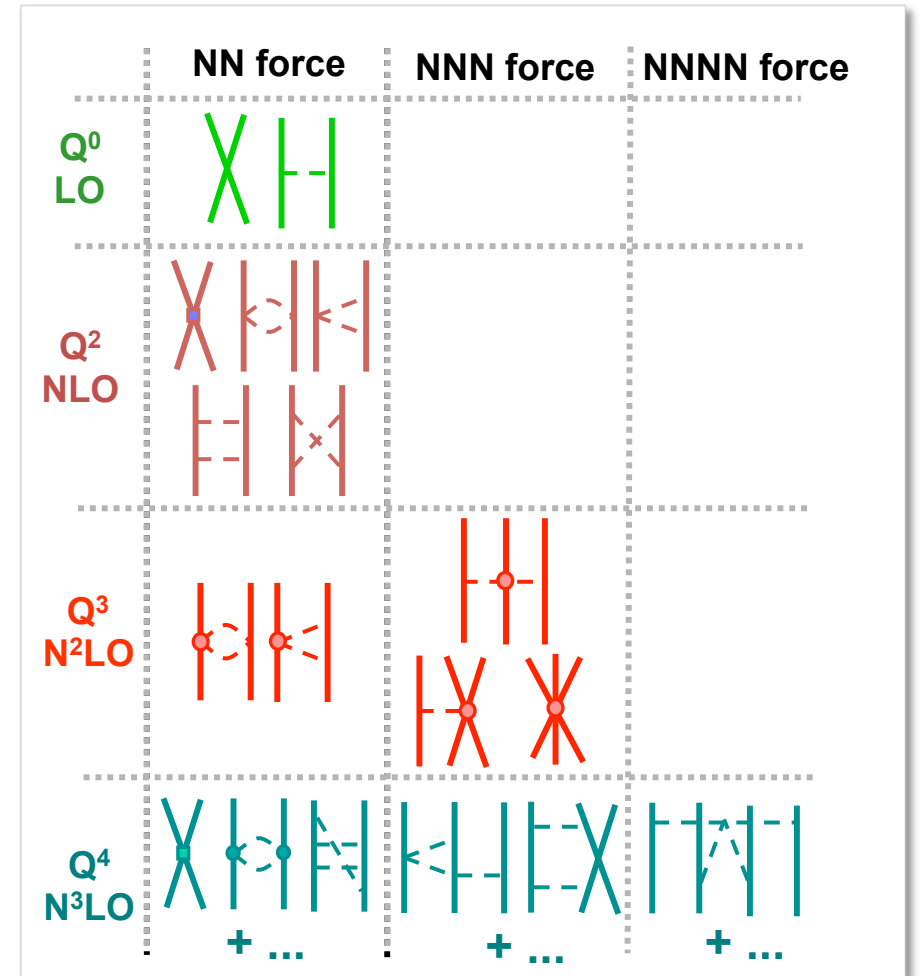
- Matrix element
$$\begin{aligned} \langle \Psi_f^{J_f^\pi T_f} M_{T_f} | | \hat{O}_{\mu\tau}^{(\kappa\tau)} | | \Psi_i^{J_i^\pi T_i} M_{T_i} \rangle &= \sum_{\lambda_i \lambda_f} \tilde{c}_{\lambda_f}^* \tilde{c}_{\lambda_i} \langle A\lambda_f J_f T_f M_{T_f} | | \hat{O}_{\mu\tau}^{(\kappa\tau)} | | A\lambda_i J_i T_i M_{T_i} \rangle \\ &+ \sum_{\lambda_f \nu_i} \tilde{c}_{\lambda_f}^* \int_0^{\infty} dr r^2 \tilde{\chi}_{\nu_i}(r) \frac{1}{r} \langle A\lambda_f J_f T_f M_{T_f} | | \hat{O}_{\mu\tau}^{(\kappa\tau)} \hat{\mathcal{A}}_{\nu_i} | | \Phi_{\nu_i r}^{J_i T_i M_{T_i}} \rangle \\ &+ \sum_{\lambda_i \nu_f} \tilde{c}_{\lambda_i} \int_0^{\infty} dr r^2 \tilde{\chi}_{\nu_f}^*(r) \frac{1}{r} \langle \Phi_{\nu_f r}^{J_f T_f M_{T_f}} | | \hat{\mathcal{A}}_{\nu_f} \hat{O}_{\mu\tau}^{(\kappa\tau)} | | A\lambda_i J_i T_i M_{T_i} \rangle \\ &+ \sum_{\nu_f \nu_i} \int_0^{\infty} dr' r'^2 \tilde{\chi}_{\nu_f}^*(r') \frac{1}{r'} \int_0^{\infty} dr r^2 \tilde{\chi}_{\nu_i}(r) \frac{1}{r} \langle \Phi_{\nu_f r'}^{J_f T_f M_{T_f}} | | \hat{\mathcal{A}}_{\nu_f} \hat{O}_{\mu\tau}^{(\kappa\tau)} \hat{\mathcal{A}}_{\nu_i} | | \Phi_{\nu_i r}^{J_i T_i M_{T_i}} \rangle \end{aligned}$$

- Express operator as target, projectile, and relative motion parts

$$\hat{D} = e \sum_{i=1}^{A-a} \frac{1 + \tau_i^z}{2} (\mathbf{r}_i - \mathbf{R}_{\text{c.m.}}^{(A-a)}) + e \sum_{i=A-a+1}^A \frac{1 + \tau_i^z}{2} (\mathbf{r}_i - \mathbf{R}_{\text{c.m.}}^{(a)}) + e \frac{Z_{(A-a)}a - Z_{(a)}(A-a)}{A} \mathbf{r}_{A-a,a}$$

Input for NCSMC calculations: Nuclear forces from chiral Effective Field Theory

- Approach taking advantage of the separation of scales
 - Based on the symmetries of QCD
 - Chiral symmetry of QCD ($m_u \approx m_d \approx 0$), spontaneously broken with pion as the Goldstone boson
 - Degrees of freedom: nucleons + pions
 - Systematic low-momentum expansion to a given order (Q/Λ_χ)
 - Hierarchy
 - Consistency
 - Low energy constants (LEC)
 - Fitted to data
 - Can be calculated by lattice QCD



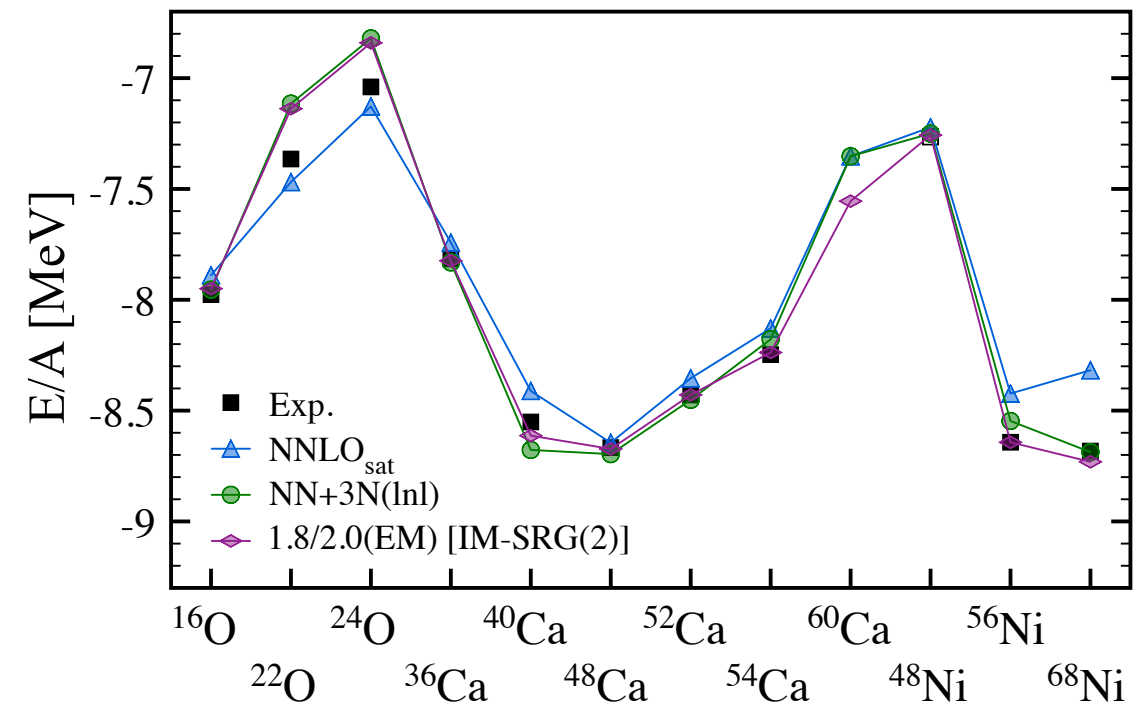
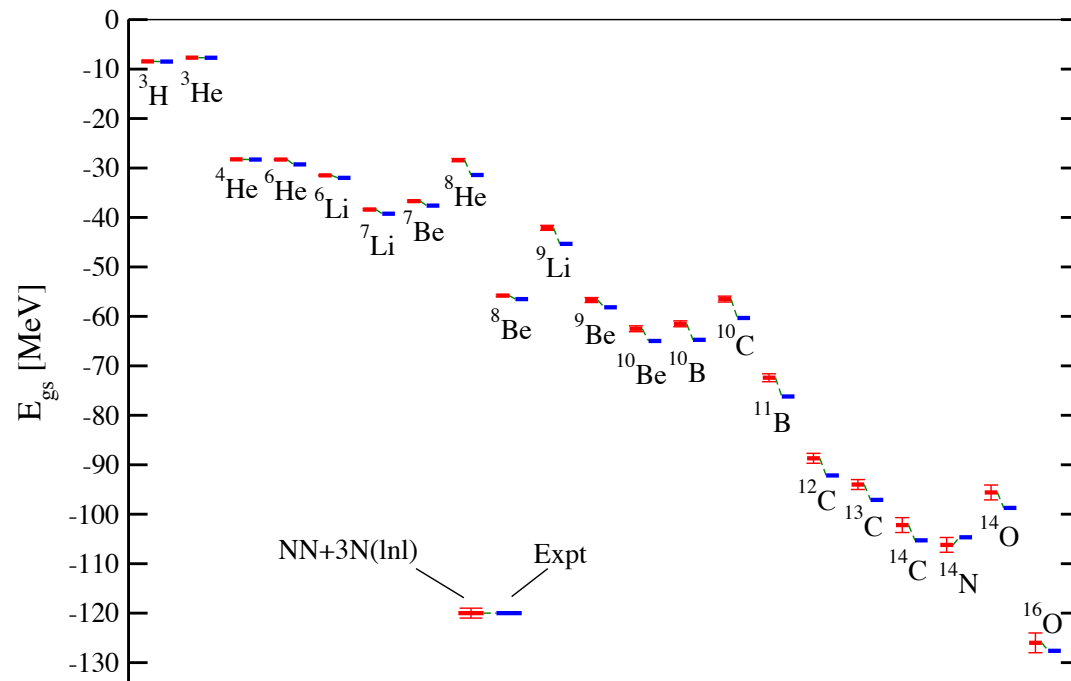
$\Lambda_\chi \sim 1 \text{ GeV}$:
Chiral symmetry breaking scale

Binding energies of light and selected medium mass nuclei from chiral NN+3N forces

11

- Quite reasonable description of binding energies across the nuclear charts becomes feasible
 - The Hamiltonian fully determined in $A=2$ and $A=3,4$ systems**
 - Nucleon–nucleon scattering, deuteron properties, ^3H and ^4He binding energy, ^3H half life
 - Light nuclei – NCSM
 - Medium mass nuclei – Self-Consistent Green’s Function method

NN N³LO (Entem-Machleidt 2003)
3N N²LO w local/non-local regulator

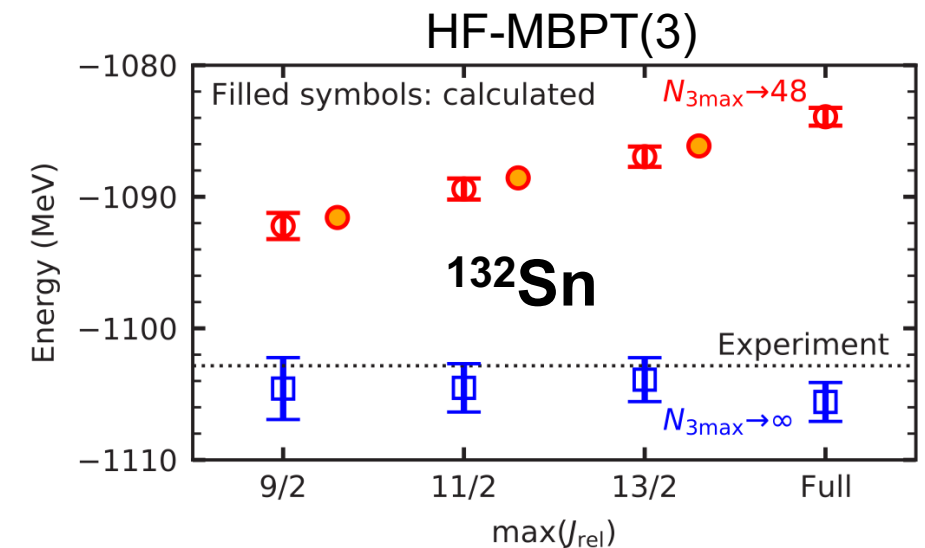
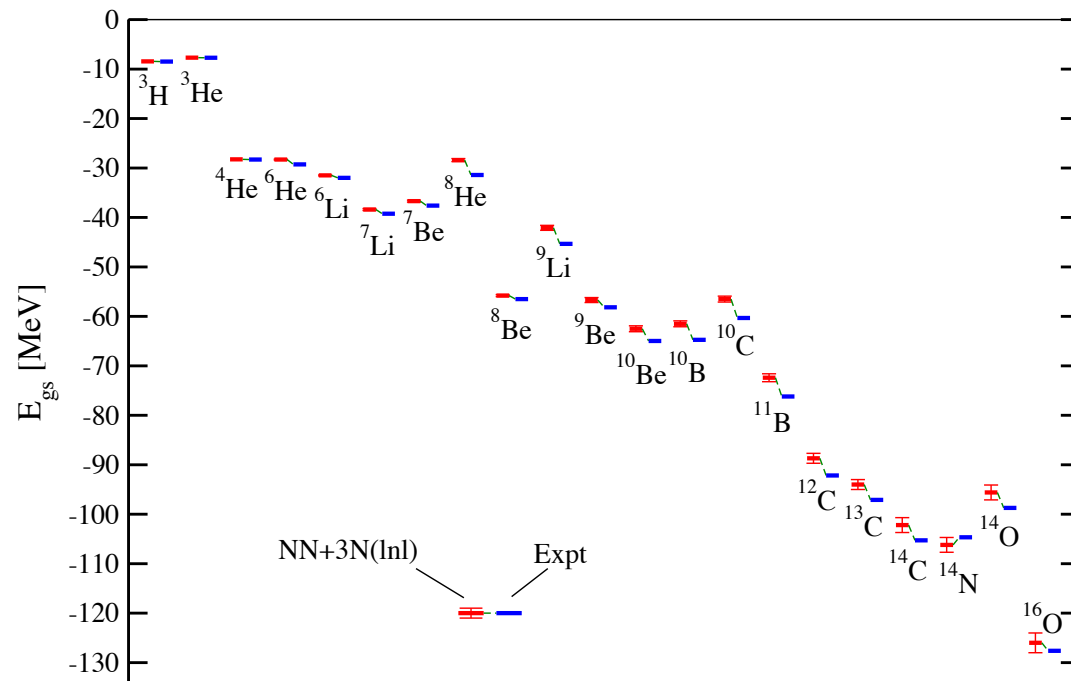


Binding energies of light and selected medium mass nuclei from chiral NN+3N forces

12

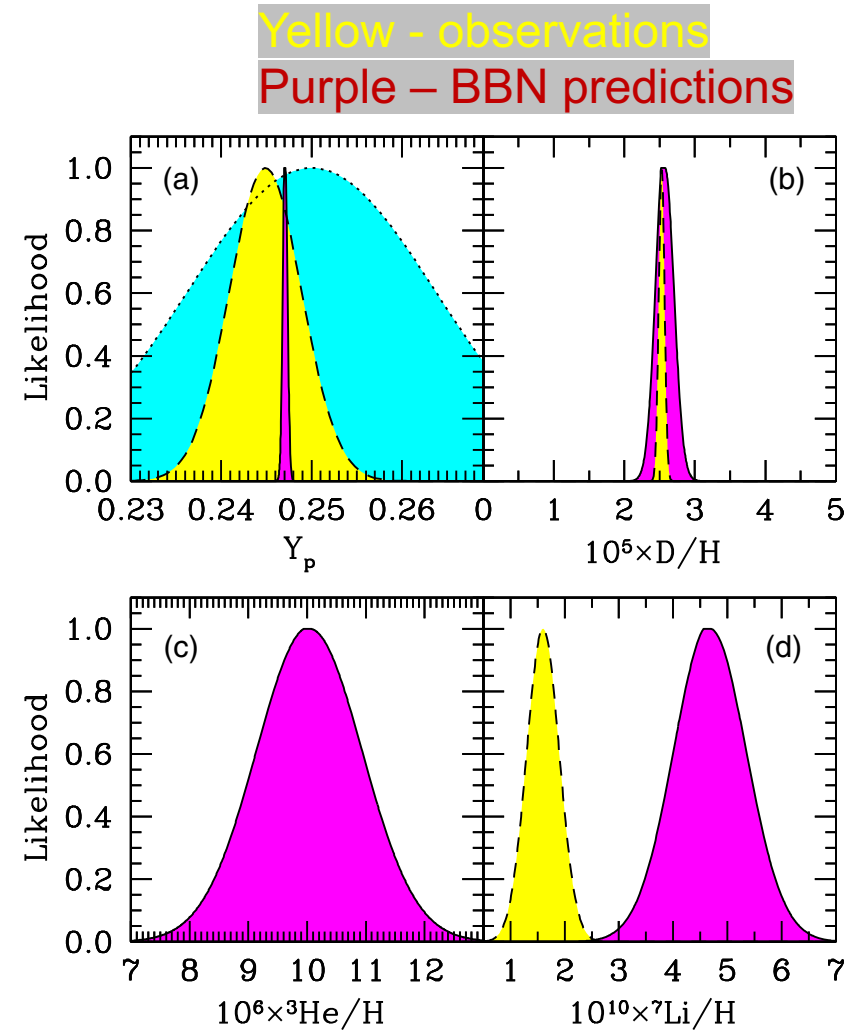
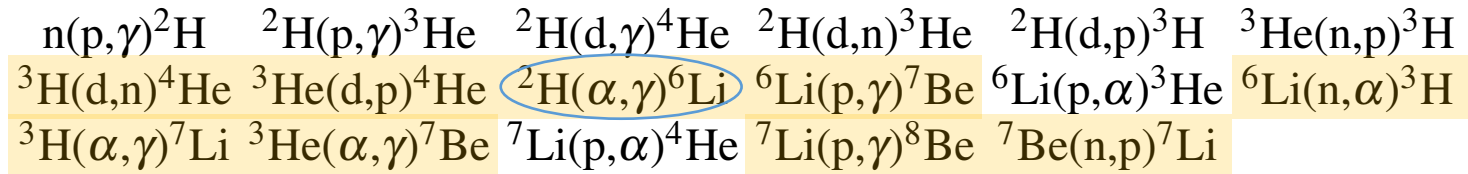
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NN N³LO (Entem-Machleidt 2003)
3N N²LO w local/non-local regulator



Big Bang Nucleosynthesis

- Primordial nucleosynthesis – 10 s to 20 min after the Big Bang
 - Helium 25% of the mass of the Universe
 - Prediction depends on
 - Baryon-to-photon ratio – CMB Planck data
 - Nuclear reaction rates



- D and ^4He in agreement with observations
- ^3He – reliable measurements do not exist
- ^7Li – predictions three times higher than observations
- ^6Li ?

Radiative capture of deuterons on ^4He

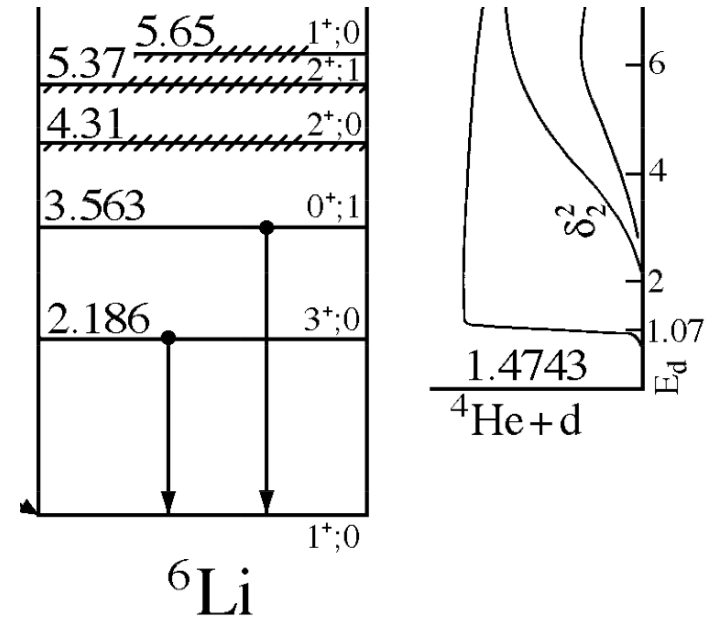
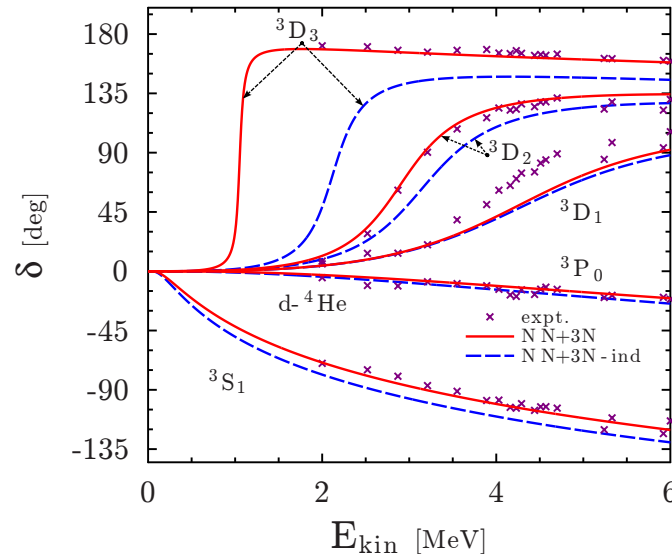
- Reaction $^4\text{He}(d,\gamma)^6\text{Li}$ responsible for ^6Li production in BBN
- Three orders of magnitude discrepancy between BBN predictions and observations
 - Problem with our understanding of the reaction rate?
 - New physics?
 - Problem with astronomical observations?

$^6\text{Li} \leftrightarrow ^4\text{He}+d$ investigated within NCSMC in the past

PRL 114, 212502 (2015) PHYSICAL REVIEW LETTERS week ending 29 MAY 2015

Unified Description of ^6Li Structure and Deuterium- ^4He Dynamics with Chiral Two- and Three-Nucleon Forces

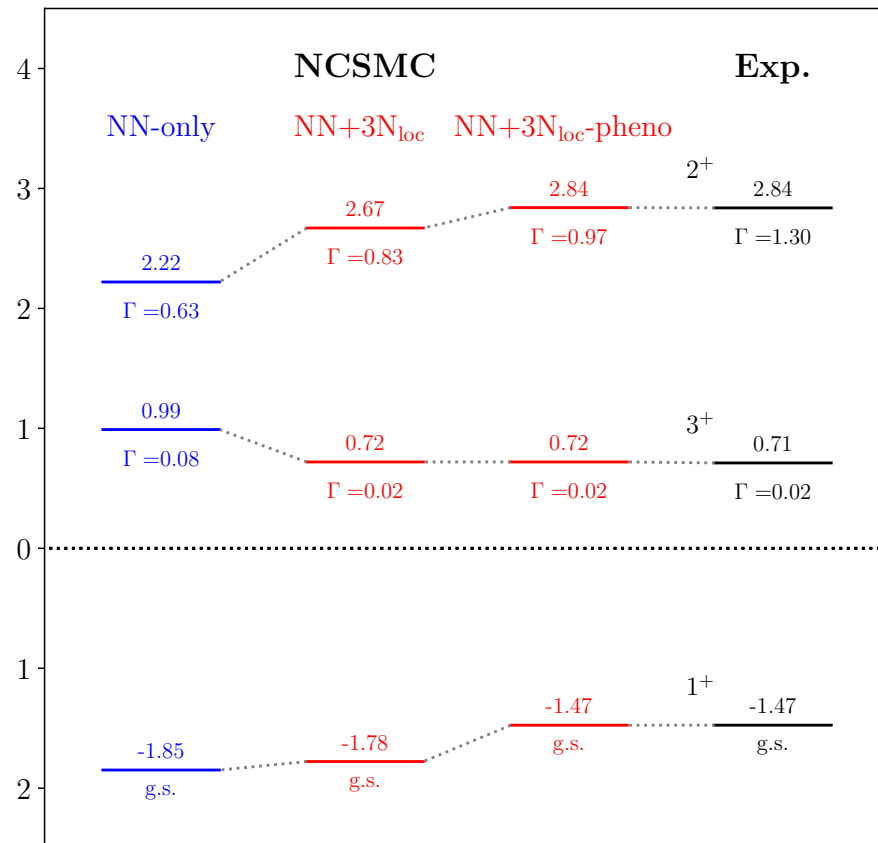
Guillaume Hupin,^{1*} Sofia Quaglioni,^{1,3} and Petr Navrátil^{2,3}



Radiative capture of deuterons on ${}^4\text{He}$

- NCSMC calculations with chiral NN+3N interaction

Structure of ${}^6\text{Li}$



${}^4\text{He}+d$ threshold

Ground state properties:
Energy

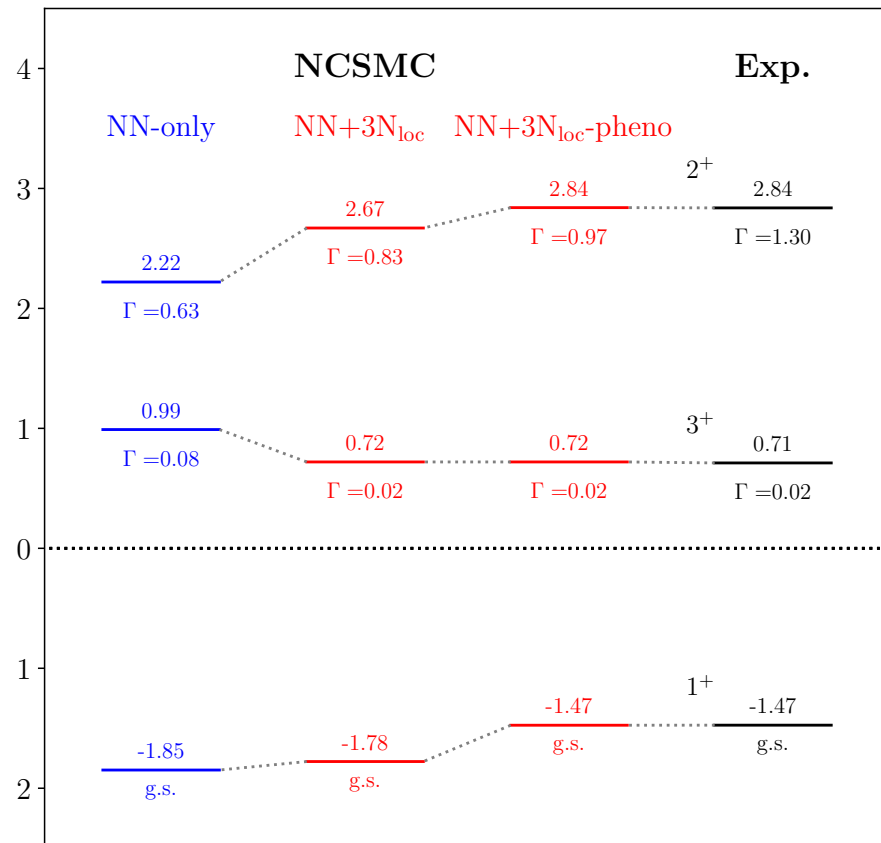
Asymptotic normalization constants
Magnetic moment

	NN-only	NCSMC	NCSMC-pheno	Exp. or Eval.
$E_{\text{g.s.}}$ [MeV]	-1.848	-1.778	-1.474	-1.4743
C_0 [$\text{fm}^{-1/2}$]	2.95	2.89	2.62	2.28(7)
C_2 [$\text{fm}^{-1/2}$]	-0.0369	-0.0642	-0.0554	-0.077(18)
C_2/C_0	-0.013	-0.022	-0.021	-0.025(6)(10)
μ [μ_N]	0.85	0.84	0.84	0.8220473(6)

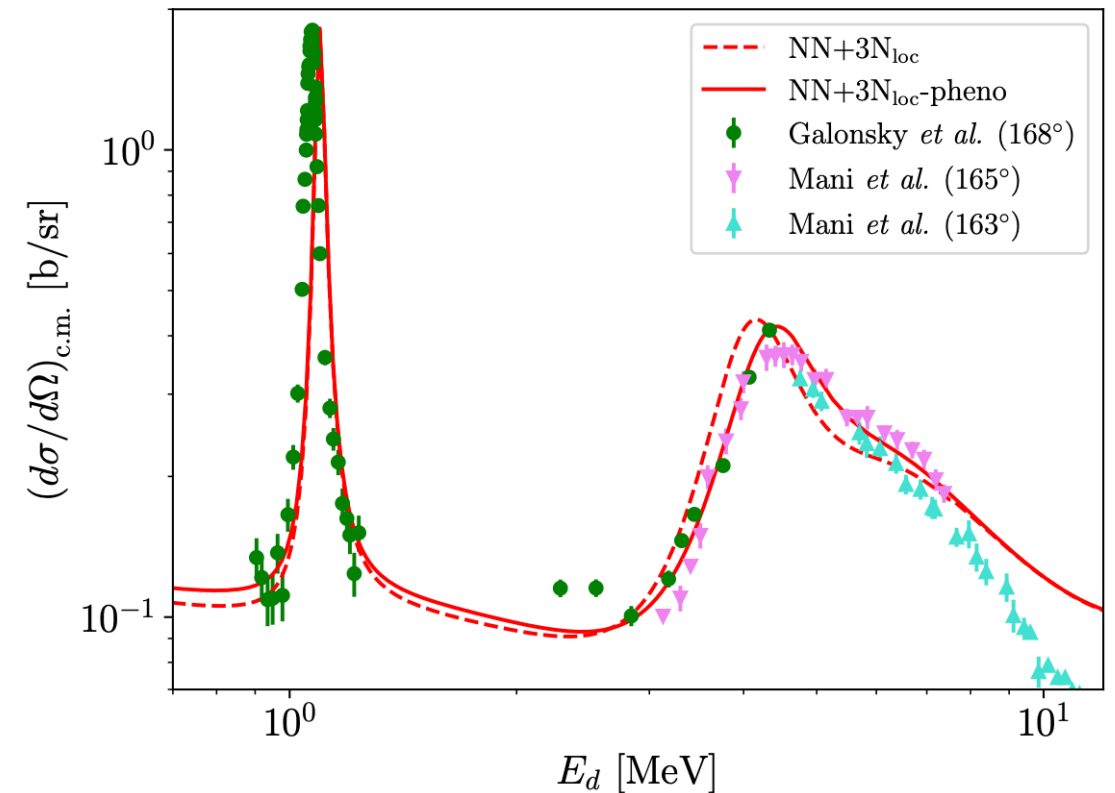
Radiative capture of deuterons on ${}^4\text{He}$

- NCSMC calculations with chiral NN+3N interaction

Structure of ${}^6\text{Li}$



Elastic scattering ${}^4\text{He}(d,d){}^4\text{He}$ cross section at the deuteron back scattered angle 164°



Radiative capture of deuterons on ${}^4\text{He}$

- NCSMC calculations with chiral NN+3N interaction

- Capture S-factor

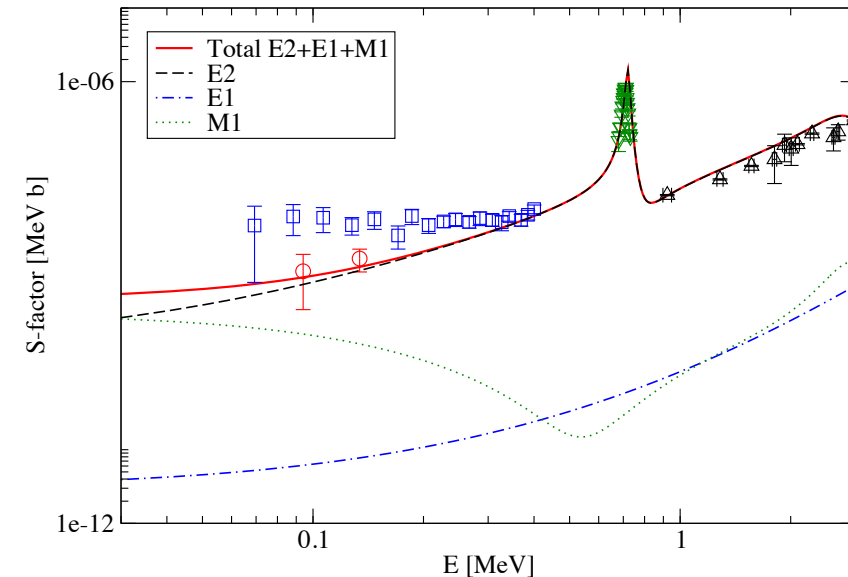
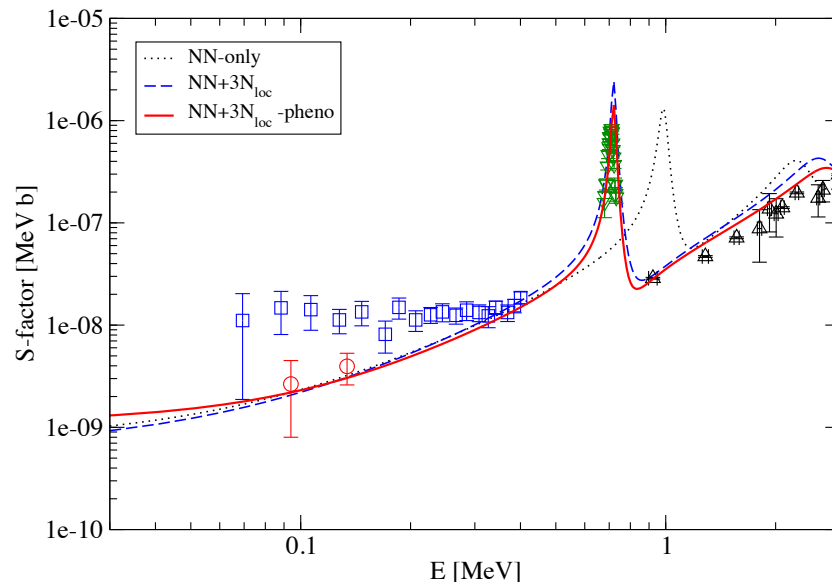
Dominated by E2

M1 significant at low energy

E1 negligible – isospin suppressed ($T=0 \rightarrow T=0$)

$$\mathcal{M}^{E\lambda} = e \sum_{j=1}^A \frac{1 + \tau_{jz}}{2} [\mathbf{r}_j - \mathbf{R}_{cm}^{(A)}]^\lambda$$

$$\mathcal{M}^{M1} = \frac{\mu_N}{\hbar c} \sqrt{\frac{3}{4\pi}} \sum_{j=1}^A (g_{lj} L_j + g_{sj} S_j)$$

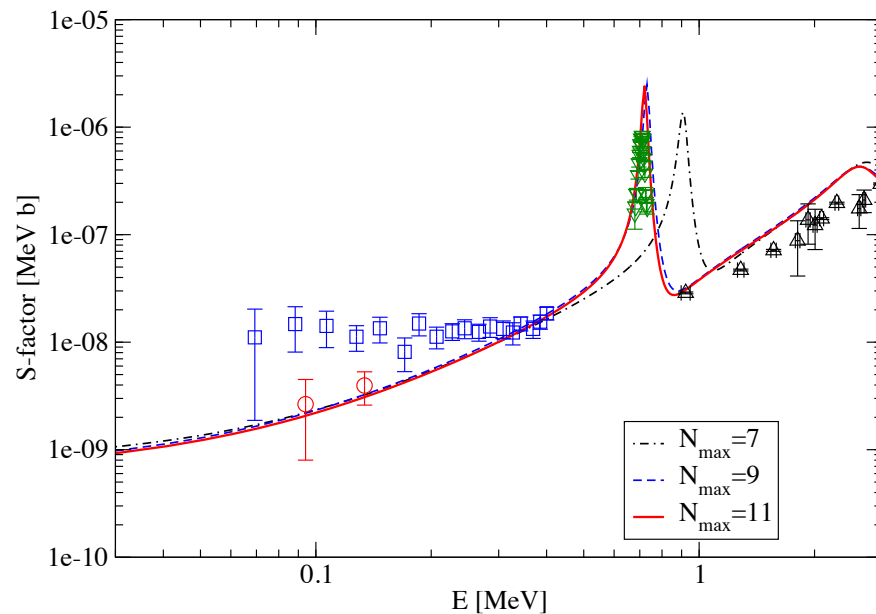


Low energy S-factor consistent with LUNA data, below the ${}^6\text{Li}$ Coulomb breakup data

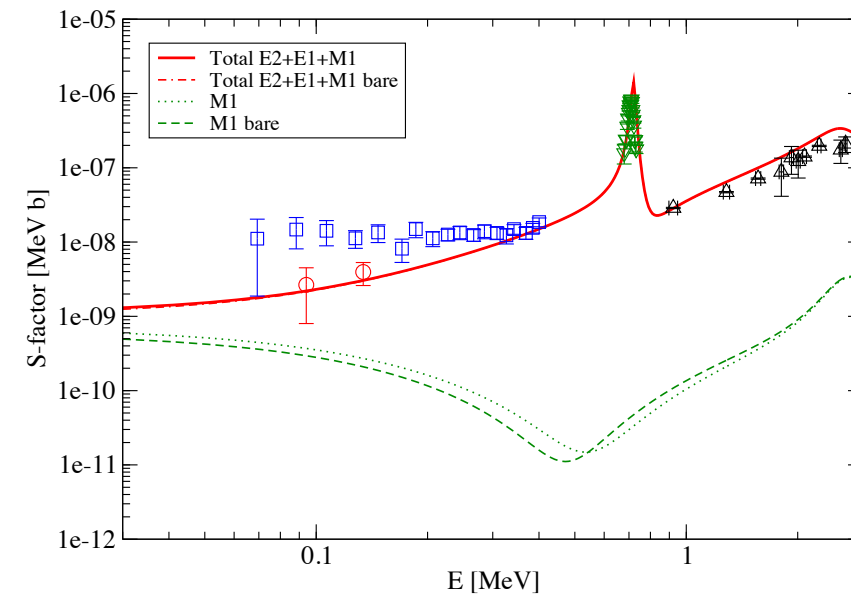
Radiative capture of deuterons on ${}^4\text{He}$

- NCSMC calculations with chiral NN+3N interaction
 - Capture S-factor

Convergence of the S-factor with N_{max}



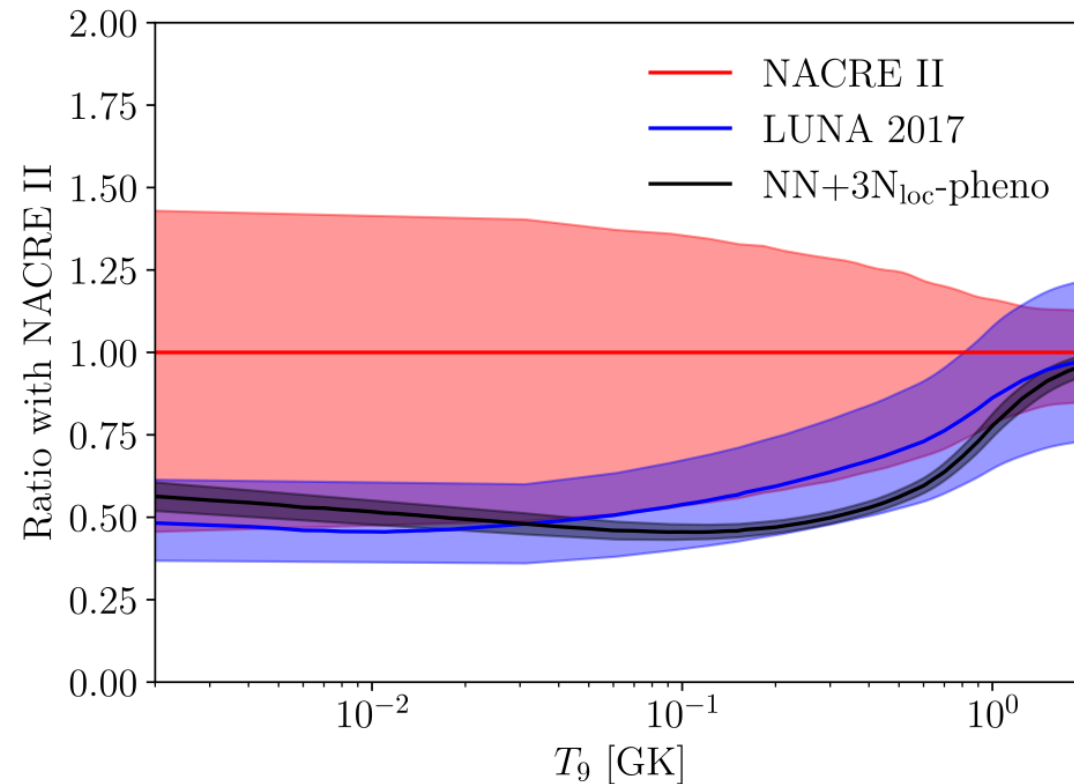
SRG renormalization of the M1 operator



Low energy S-factor consistent with LUNA data, below the ${}^6\text{Li}$ Coulomb breakup data

Radiative capture of deuterons on ${}^4\text{He}$

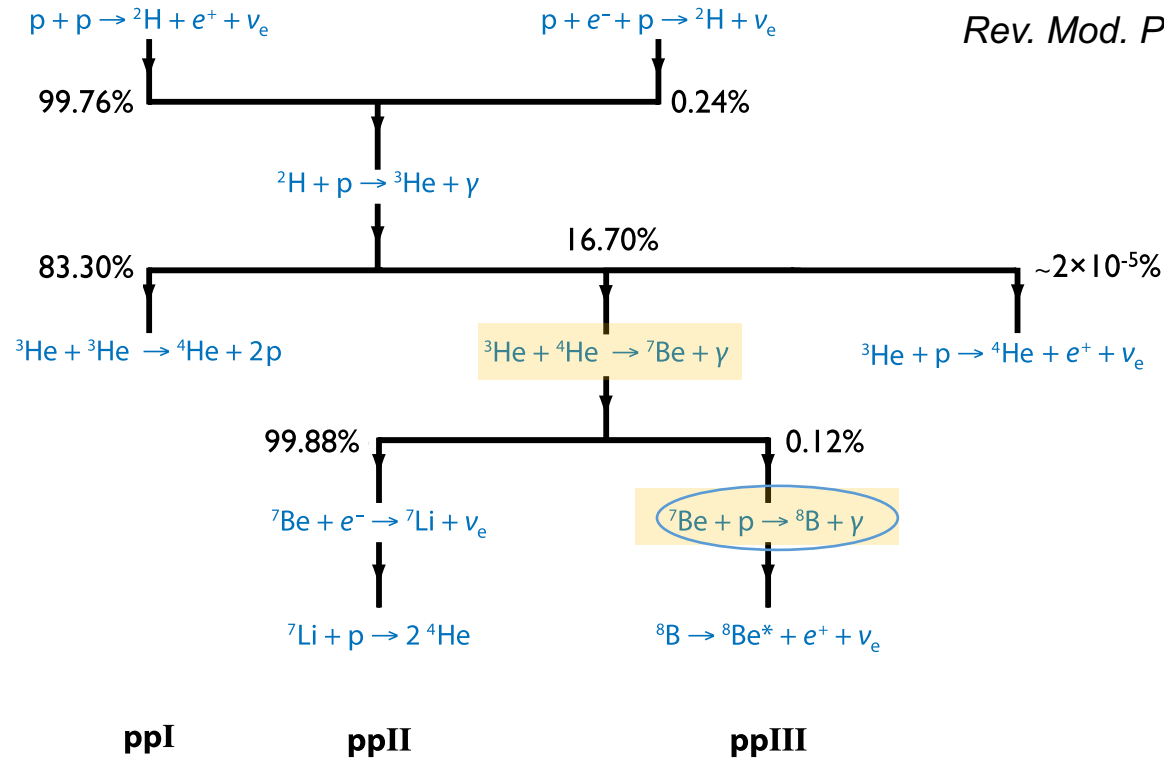
- NCSMC calculations with chiral NN+3N interaction
 - Thermonuclear reaction rate



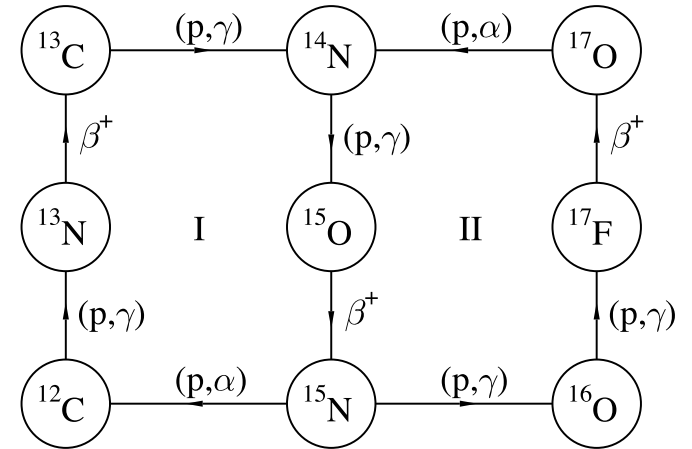
Thermonuclear reaction rate smaller than NACRE II evaluation, agreement with LUNA result with less uncertainty

Stellar burning

- Hydrogen burning – converting 4 protons to ^4He
 - pp chain
 - CNO cycles



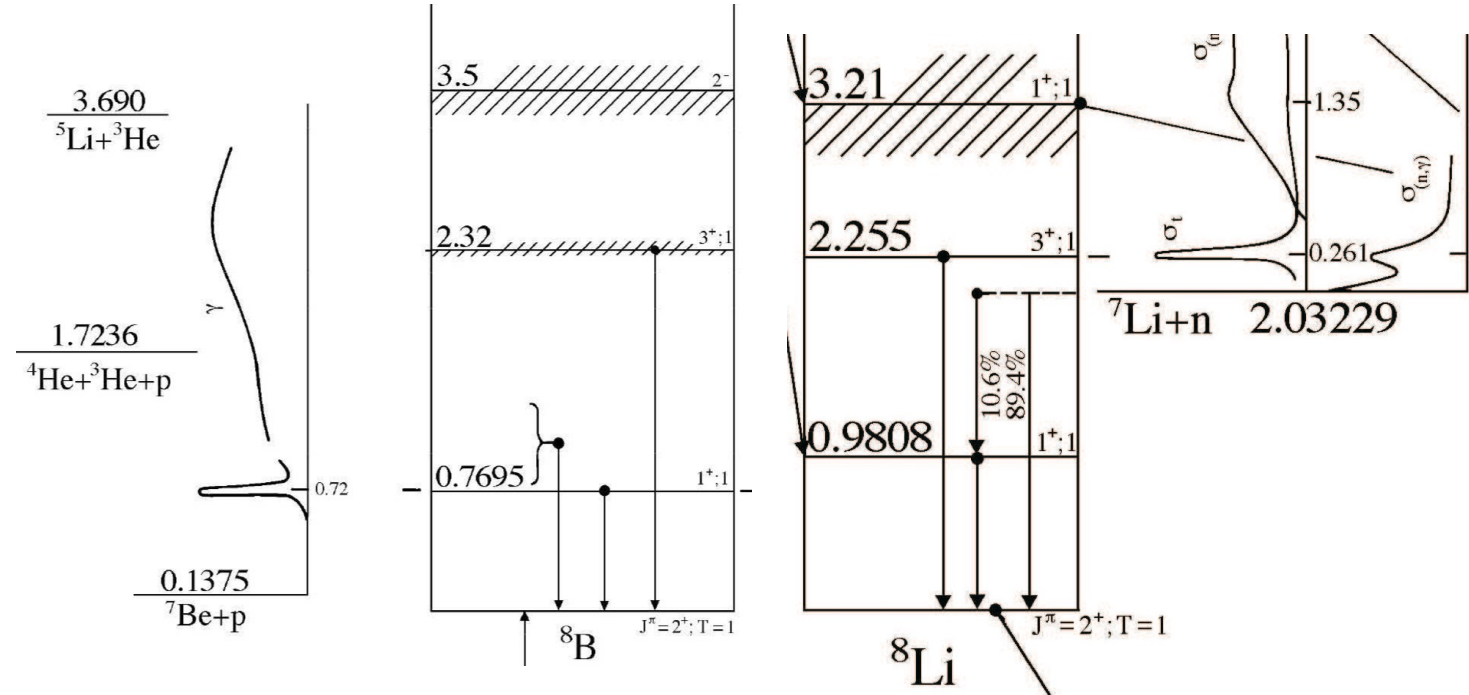
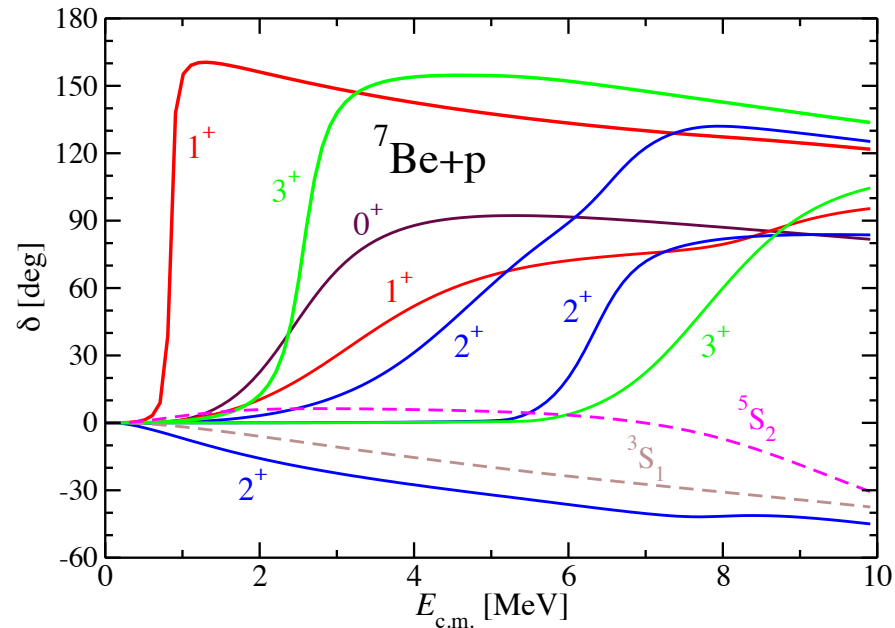
Rev. Mod. Phys. **83**,195–245 (2011)



- Helium burning $\alpha + \alpha \rightarrow ^8\text{Be}$ $^8\text{Be}(\alpha, \gamma) ^{12}\text{C}$ $^{12}\text{C}(\alpha, \gamma) ^{16}\text{O}$

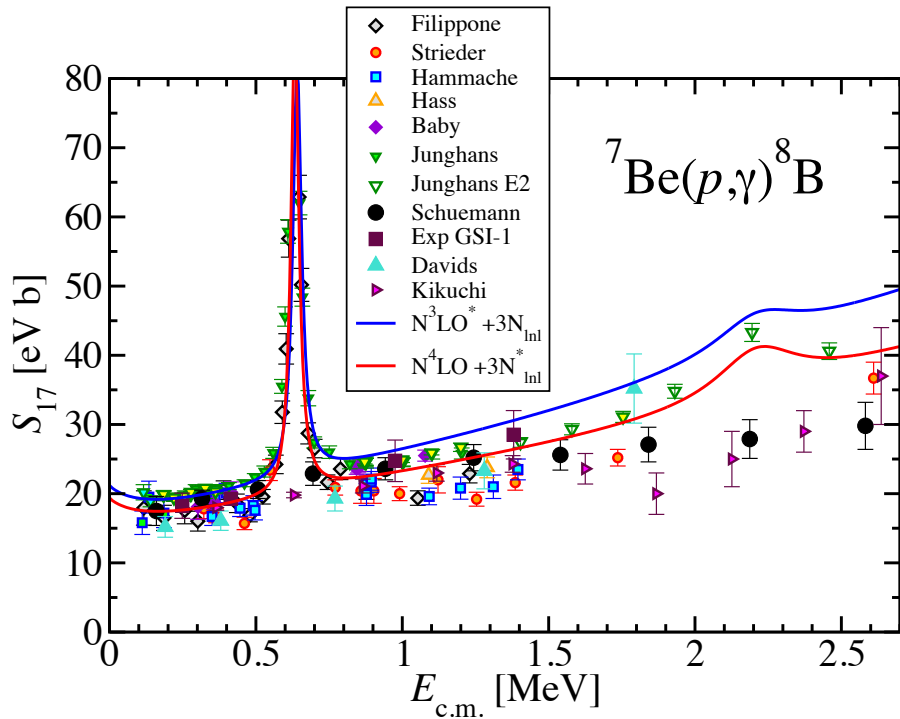
Radiative capture of protons on ${}^7\text{Be}$

- Solar pp chain reaction, solar ${}^8\text{B}$ neutrinos
- NCSMC calculations with a set of chiral NN+3N interactions as input
- Example of ${}^8\text{B}$ structure results



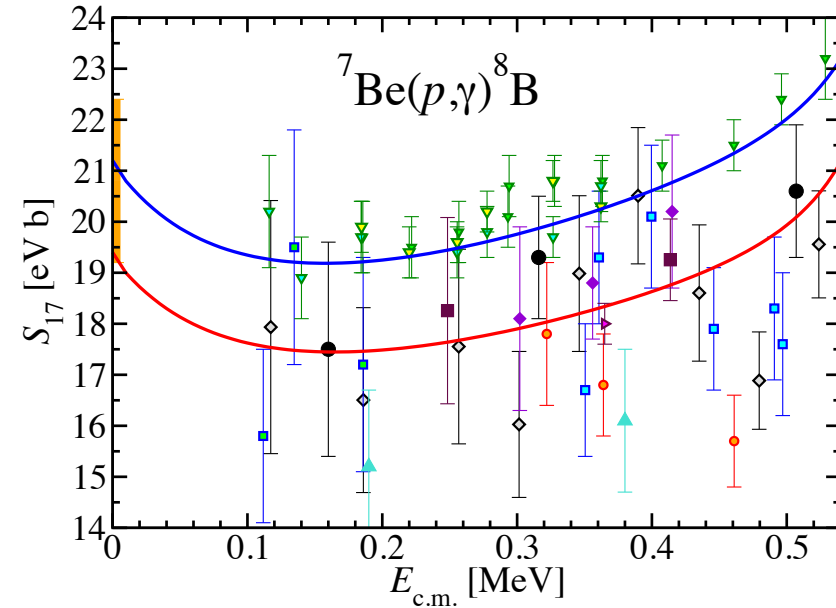
Radiative capture of protons on ${}^7\text{Be}$

■ NCSMC S-factor results



E1 non-resonant, M1/E2 at 1^+ and 3^+ resonances

	$C_{p_{1/2}}$	$C_{p_{3/2}}$	a_1	a_2	$S_{17}(0)$
$N^2\text{LO} + 3N_{\text{lnl}}$	0.384	0.691	4.4(1)	-0.5(1)	23.9
$N^3\text{LO} + 3N_{\text{lnl}}$	0.390	0.678	1.3(1)	-4.7(1)	23.5
$N^4\text{LO} + 3N_{\text{lnl}}$	0.354	0.669	1.6(1)	-4.4(1)	22.0
$N^4\text{LO} + 3N_{\text{lnl}}^*$	0.343	0.621	1.3(1)	-5.0(1)	19.3
$N^3\text{LO}^* + 3N_{\text{lnl}}$	0.334	0.663	0.1(1)	-7.7(1)	21.1
$N^3\text{LO}^* + 3N_{\text{loc}}$	0.308	0.584	2.5(1)	-3.6(2)	16.8
Ref. [41]	0.315(9)	0.66(2)	$17.34^{+1.11}_{-1.33}$	$-3.18^{+0.55}_{-0.50}$	



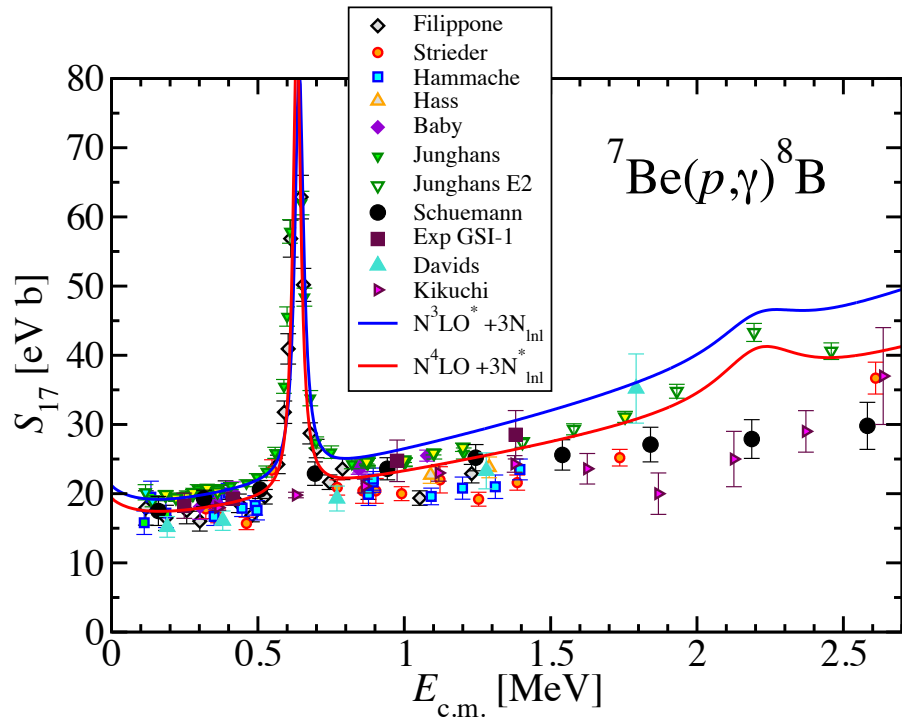
Ab initio prediction for the radiative capture of protons on ${}^7\text{Be}$

K. Kravvaris,¹ P. Navrátil,² S. Quaglioni,¹ C. Hebborn,^{3,1} and G. Hupin⁴

arXiv: 2202.11759

Radiative capture of protons on ${}^7\text{Be}$

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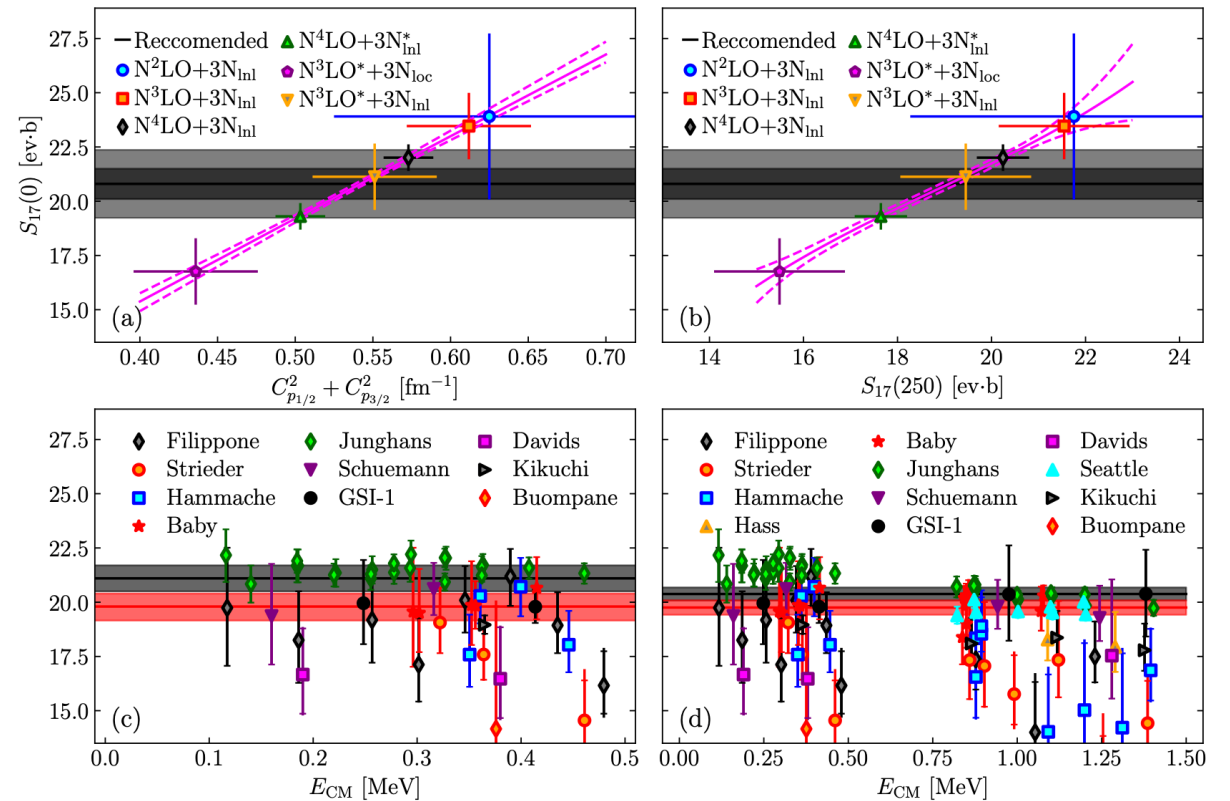


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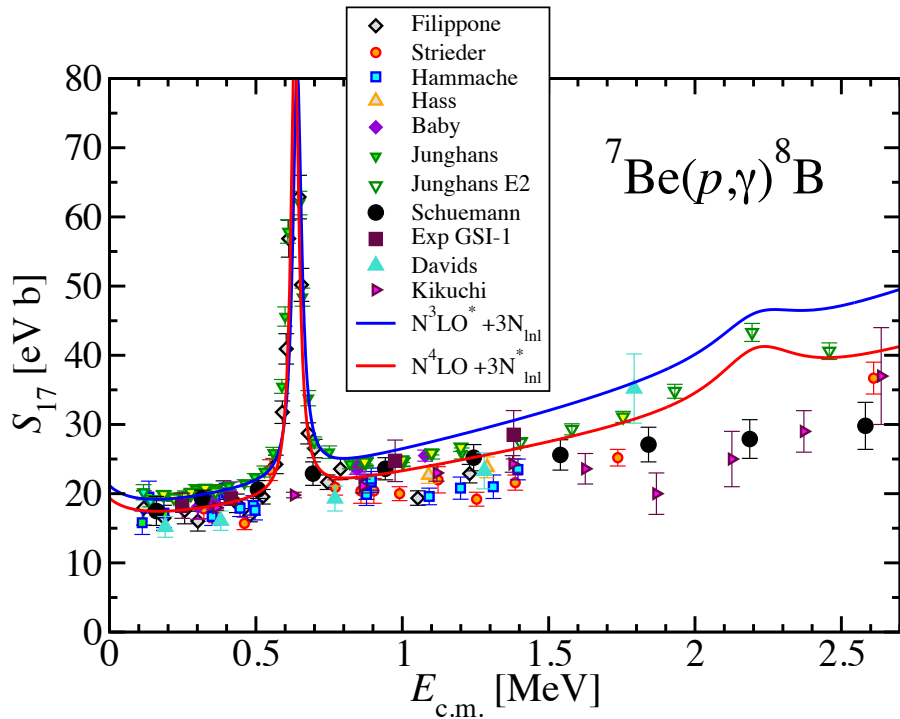
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Radiative capture of protons on ${}^7\text{Be}$

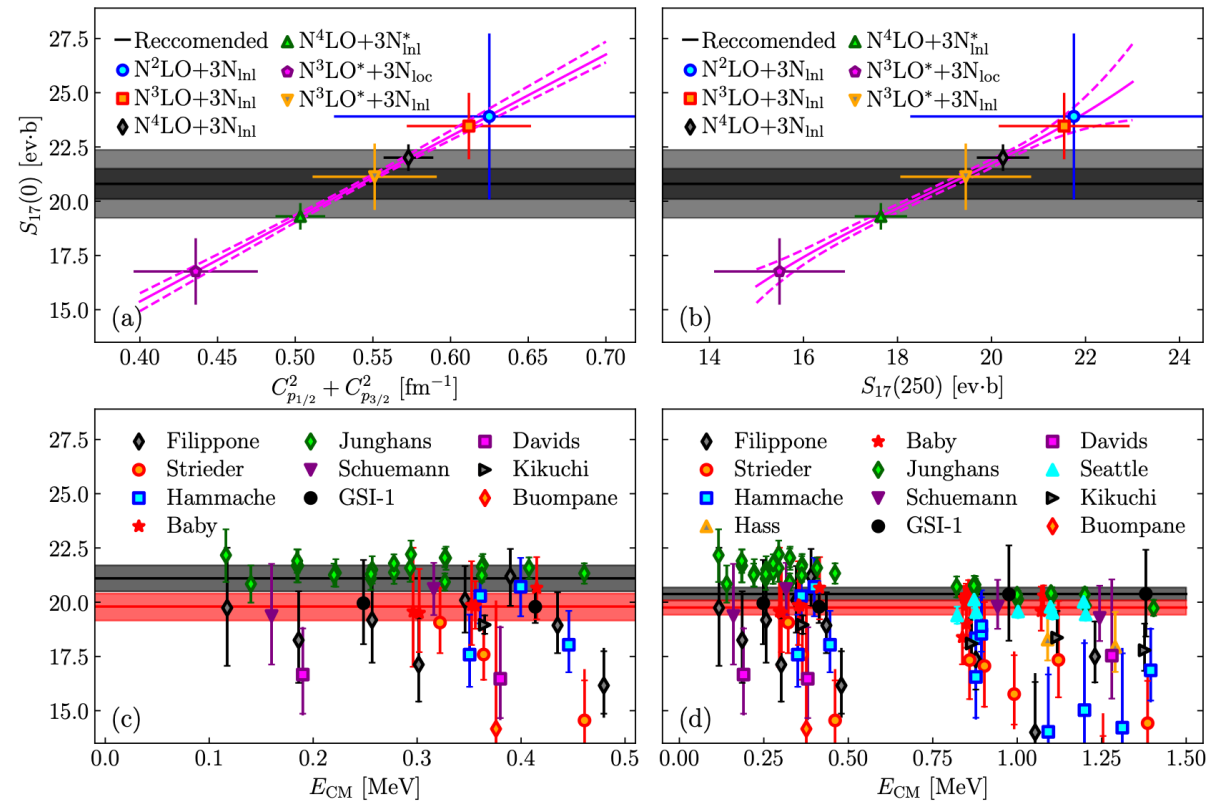
NCSMC S-factor results



Recommended value $S_{17}(0) \sim 19.8(3)$ eV b

Latest evaluation in *Rev. Mod. Phys.* **83**,195–245 (2011):
 $S_{17}(0) = 20.8 \pm 0.7(\text{expt}) \pm 1.4(\text{theory})$ eV b

	$C_{p_{1/2}}$	$C_{p_{3/2}}$	a_1	a_2	$S_{17}(0)$
$\text{N}^2\text{LO}+3\text{N}_{\text{lnl}}$	0.384	0.691	4.4(1)	-0.5(1)	23.9
$\text{N}^3\text{LO}+3\text{N}_{\text{lnl}}$	0.390	0.678	1.3(1)	-4.7(1)	23.5
$\text{N}^4\text{LO}+3\text{N}_{\text{lnl}}$	0.354	0.669	1.6(1)	-4.4(1)	22.0
$\text{N}^4\text{LO}+3\text{N}_{\text{lnl}}^*$	0.343	0.621	1.3(1)	-5.0(1)	19.3
$\text{N}^3\text{LO}^*+3\text{N}_{\text{lnl}}$	0.334	0.663	0.1(1)	-7.7(1)	21.1
$\text{N}^3\text{LO}^*+3\text{N}_{\text{loc}}$	0.308	0.584	2.5(1)	-3.6(2)	16.8
Ref. [41]	0.315(9)	0.66(2)	$17.34^{+1.11}_{-1.33}$	$-3.18^{+0.55}_{-0.50}$	



Conclusions

- *Ab initio* calculations of nuclear structure and reactions becoming feasible beyond the lightest nuclei
 - Make connections between the low-energy QCD, many-body systems, and nuclear astrophysics
- Applications of *ab initio* theory to nuclear reactions important for astrophysics
 - ${}^4\text{He}(d,\gamma){}^6\text{Li}$ – importance of M1 contribution at very low energies
 - ${}^7\text{Be}(p,\gamma){}^8\text{B}$ – new evaluation with significantly reduced uncertainty
- *Ab initio* nuclear theory essential for precision applications such as tests of fundamental symmetries
 - Quenching of g_A
 - Double beta decay matrix elements
 - Isospin mixing correction δ_C
 - Nuclear anapole moment, electric dipole moment
 - ...

In synergy with experiments, *ab initio* nuclear theory is the right approach to understand low-energy properties of atomic nuclei

Thank you!
Merci!
ありがとうございました

