

Study on the dineutron in the 2_1^+ resonance of ${}^6\text{He}$

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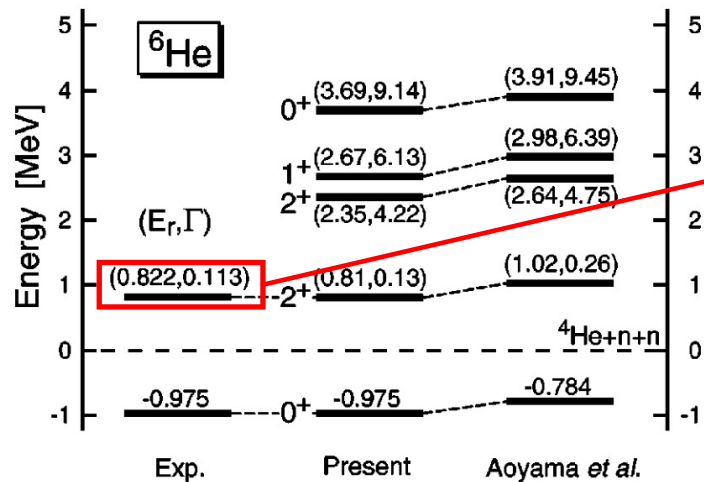
Phys. Rev. C **105**, L041601 (2022)

Resonance of ${}^6\text{He}$

Recently, by the development of experiments, resonant states of nuclei near the neutron dripline have been intensively pursued.

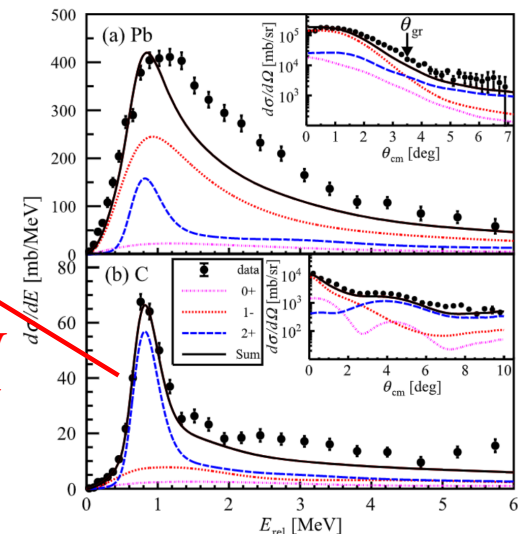
➤ Resonance in ${}^6\text{He}$

- 2_1^+ appears as a sharp peak in the breakup cross section.



T. Myo, et al., PRC **63**, 054313 (2001)

2_1^+
 $\epsilon_r = 0.822 \text{ MeV}$
 $\Gamma = 0.113 \text{ MeV}$



Y. L. Sun, et al., PLB **814**, 136072 (2021)

- The existence of the dineutron in the 2_1^+ state is discussed.

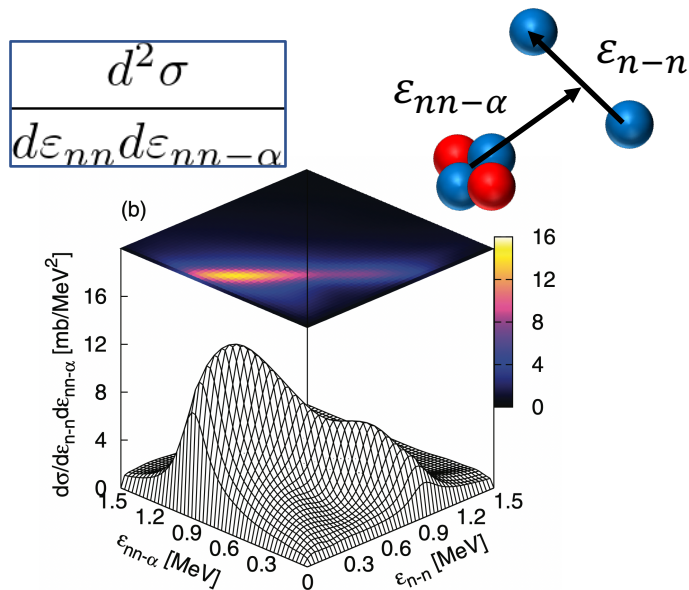
Y. Kikuchi, et al., PRC **88**, 021602(R) (2013)

A. Saito, et al., in preparation (2022)

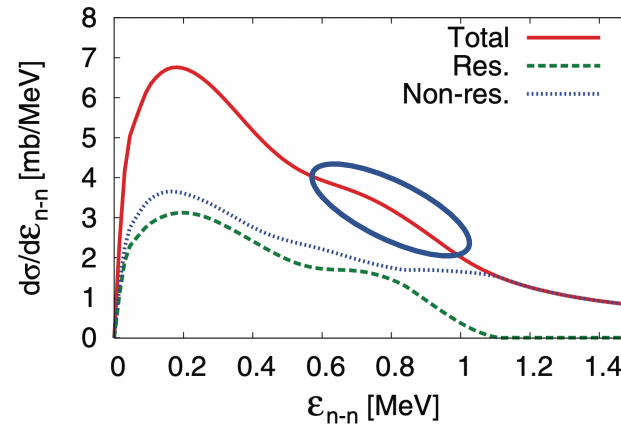
Dineutron in resonance

➤ Double Differential Breakup Cross Section (DDBUX)

${}^6\text{He} + {}^{12}\text{C} @ 240 \text{ MeV/A}$ Y. Kikuchi, et al., PRC 88, 021602(R) (2013)



$$\frac{d\sigma}{d\varepsilon_{n-n}} = \int \frac{d\sigma}{d\varepsilon_{n-n}d\varepsilon_{nn-\alpha}} d\varepsilon_{nn-\alpha}$$

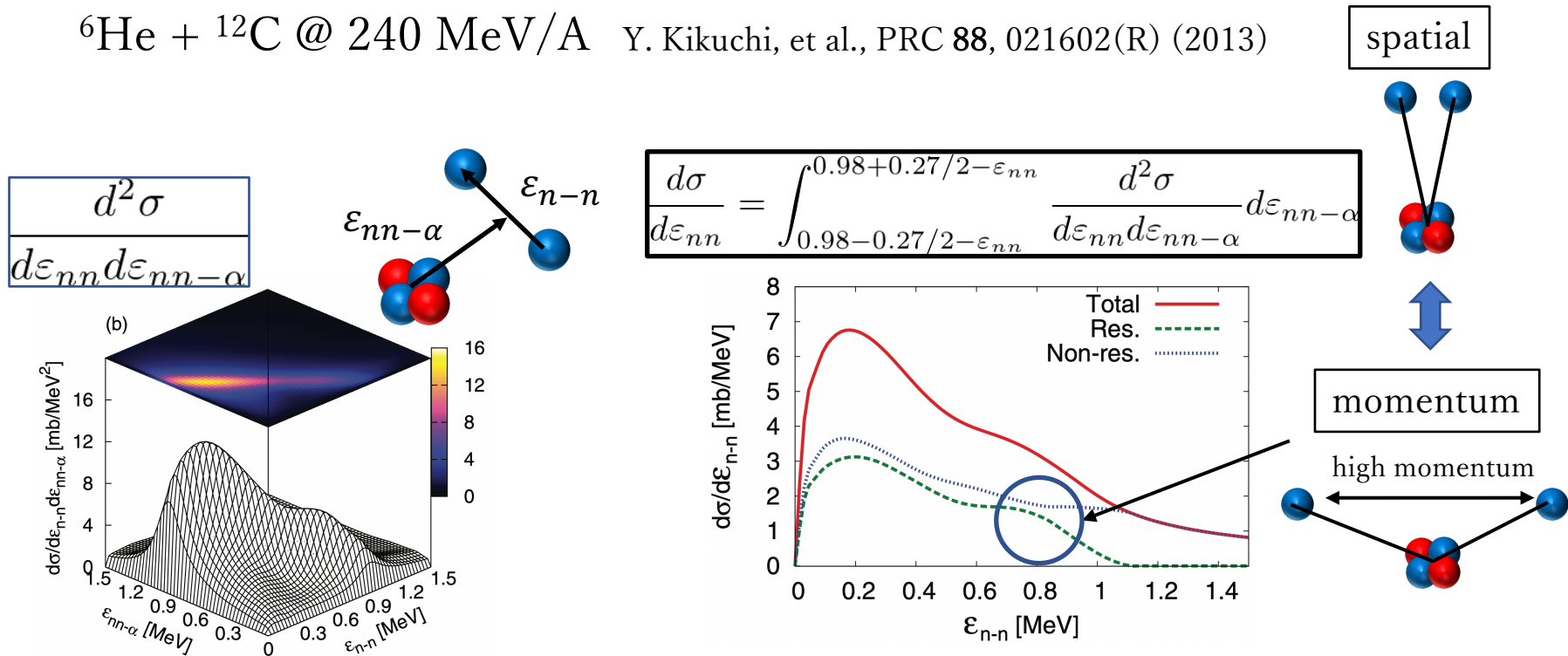


- Total (0^+ , 1^- , 2^+) : A peak around $\varepsilon_{n-n} = 0.7 \text{ MeV}$.

Dineutron in resonance

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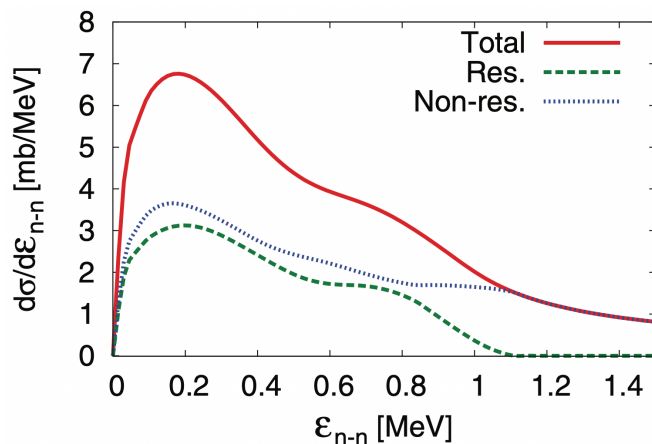
${}^6\text{He} + {}^{12}\text{C} @ 240 \text{ MeV/A}$ Y. Kikuchi, et al., PRC 88, 021602(R) (2013)



- Total ($0^+, 1^-, 2^+$) : A peak around $\varepsilon_{n-n} = 0.7 \text{ MeV}$.
- Res. : The cross section gated within the resonant energy of 2_1^+
Shoulder peak around $\varepsilon_{n-n} = 0.7 \text{ MeV} \rightarrow$ Dineutron in 2_1^+ ?

Motivation

- The shoulder peak comes from the 2_1^+ state?
 - Although the cross section gated within the resonant energy, it cannot completely exclude the nonresonant contributions from the cross section.
- The shoulder peak comes from the dineutron?
 - The previous study only shows the existence of the peak.
 - The dineutron consists of two neutrons with $S = 0$.What is the relationship between spin and peak?



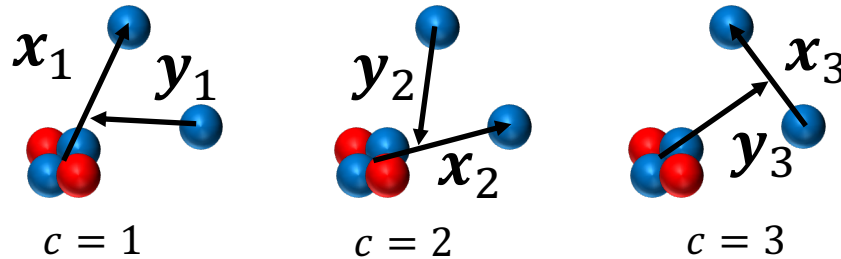
Purpose

We extract the resonant contribution from DDBUX and analyze the decay modes in more detail.

Complex Scaling Method (CSM)

CSM successfully describes resonances in the Few-body system.

- Complex scaling transformation : $U(\theta)\mathbf{x}U^{-1}(\theta) = \mathbf{x}e^{i\theta}$
- Gaussian Expansion Method



$$\Phi_{Im}(\mathbf{x}, \mathbf{y}) = \sum_{c=1}^3 \sum_{i,j,l,\lambda} \phi_{i,l}(x_c) \varphi_{j,\lambda}(y_c) [[Y_l(\hat{\mathbf{x}}_c) \otimes Y_\lambda(\hat{\mathbf{y}}_c)]_\Lambda \otimes [\eta_{1/2} \otimes \eta_{1/2}]_S]_{Im}$$

$$\phi_{i,l}(x_c) = x_c^l e^{-(x_c/x_i)^2} \quad \varphi_{j,\lambda}(y_c) = y_c^\lambda e^{-(y_c/y_j)^2}$$

→ General eigenvalue problem for complex symmetric matrices

$$\left[\begin{pmatrix} h_{i'j',ij}^\theta \end{pmatrix} - \varepsilon^\theta \begin{pmatrix} N_{i'j',ij} \end{pmatrix} \right] \begin{pmatrix} C_{ij}^\theta \end{pmatrix} = 0$$

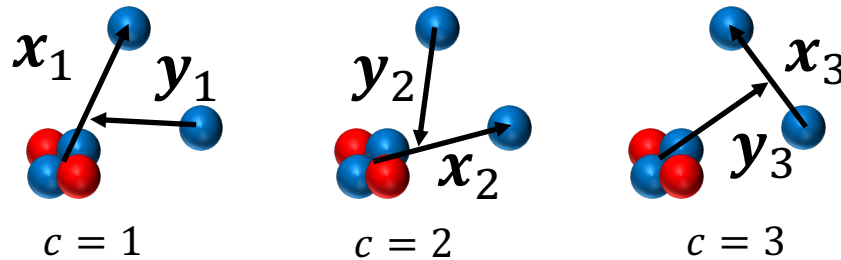
$$h_{i'j',ij}^\theta = \langle \tilde{\phi}_{i'} \tilde{\phi}_{j'} | h^\theta | \phi_i \phi_j \rangle$$

$$N_{i'j',ij} = \langle \tilde{\phi}_{i'} \tilde{\phi}_{j'} | \phi_i \phi_j \rangle$$

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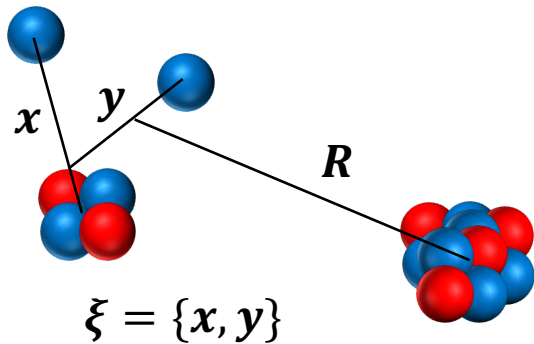
$$\phi_{i,l}(x_c) = x_c^l e^{-(x_c/x_i)^2} \quad \varphi_{j,\lambda}(y_c) = y_c^\lambda e^{-(y_c/y_j)^2}$$

→ General eigenvalue problem for complex symmetric matrices

	ε_r [MeV]	Γ [MeV]
Result	0.823	0.121

Continuum-Discretized Coupled Channels method (CDCC)

- ${}^6\text{He} (= \alpha + n + n) + \text{Target} \rightarrow 4\text{-body breakup reaction}$
- CDCC can describe 4-body breakup reaction precisely.



Schrödinger equation

$$\left[K_R + \sum_{i=nT, nT, \alpha T} U_i + V_C + h_{6\text{He}} - E \right] \Psi(\xi, \mathbf{R}) = 0$$

$$(h_{6\text{He}} - \varepsilon) \Phi_\varepsilon(\xi) = 0 \quad \Phi_\varepsilon(\xi): \text{eigenstates of } {}^6\text{He}$$

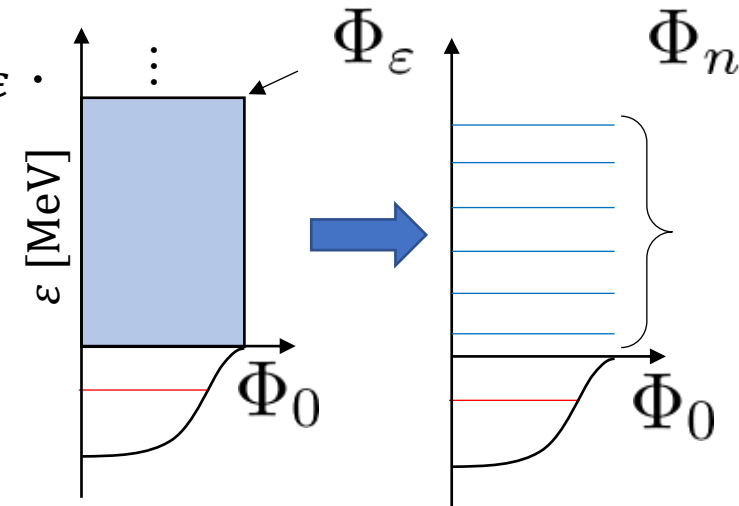
Ψ is expanded by eigenstates of ${}^6\text{He}$, Φ_ε .

$$\Psi(\xi, \mathbf{R}) = \Phi_0(\xi) \chi_0(\mathbf{R}) + \int d\varepsilon \Phi_\varepsilon(\xi) \chi_\varepsilon(\mathbf{R})$$



$$\Psi(\xi, \mathbf{R}) \simeq \boxed{\Phi_0(\xi)} \chi_0(\mathbf{R}) + \sum_n^{n_{\max}} \boxed{\Phi_n(\xi)} \chi_n(\mathbf{R})$$

bound discretized-continuum



Smoothing factor for CDCC

- The cross section obtained by CDCC is discretized with respect to energy.

- Continuous T-matrix

$$T_\varepsilon \simeq \sum_n \underbrace{\langle \Phi_\varepsilon^{(-)} | \Phi_n \rangle}_{\text{smoothing factor}} T_n^{\text{CDCC}}$$

wavefunction of ${}^6\text{He}$
 $\Phi_\varepsilon^{(-)}$: exact state
 Φ_n : discretized state

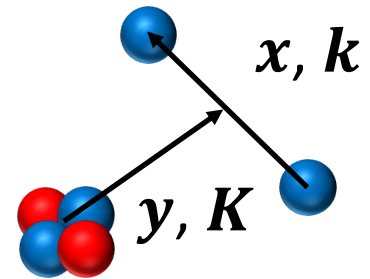
- Complex Scaled solutions of Lippmann-Schwinger eq. (CSLS)

Y. Kikuchi, et al., Phys. Rev. C **88** (2013), 021602(R)

$$\langle \Phi_\varepsilon^{(-)} | = \langle \phi_\varepsilon | + \langle \phi_\varepsilon | V \frac{1}{\varepsilon - h + i\eta}$$



$$\frac{1}{\varepsilon - h + i\eta} = U^{-1}(\theta) \sum_\nu |\Phi_\nu^\theta\rangle \frac{1}{\varepsilon - \varepsilon_\nu^\theta} \langle \tilde{\Phi}_\nu^\theta | U(\theta)$$



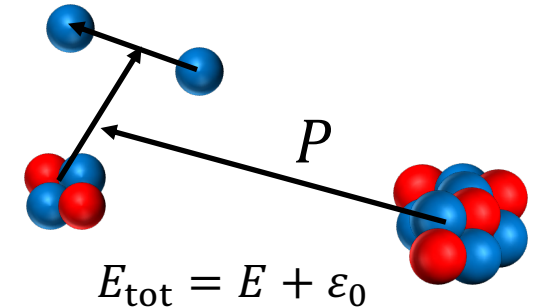
$$\langle \Phi_{\text{CSLS}}^{(-)} | = \langle \phi_\varepsilon | + \sum_\nu \langle \phi_\varepsilon | V U^{-1}(\theta) |\Phi_\nu^\theta\rangle \frac{1}{\varepsilon - \varepsilon_\nu^\theta} \langle \tilde{\Phi}_\nu^\theta | U(\theta)$$

plane wave : $\phi_\varepsilon = e^{i\mathbf{k}\cdot\mathbf{r}} e^{i\mathbf{K}\cdot\mathbf{R}}$

Double Differential Breakup Cross section (DDBUX)

- Continuous T-matrix with CSLS

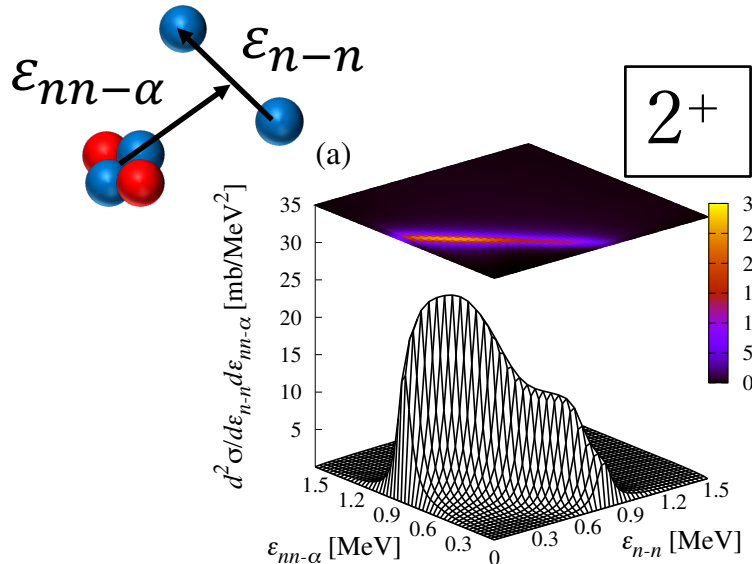
$$T(\mathbf{k}, \mathbf{K}) = \sum_n \langle \Phi_{\text{CSLS}}^{(-)} | \Phi_n \rangle T_n^{\text{CDCC}}$$



- DDBUX

$$\frac{d^2\sigma}{d\varepsilon_{n-n}d\varepsilon_{nn-\alpha}} = \sum_{nn'} T_n T_{n'}^\dagger \int d\mathbf{k}d\mathbf{K}d\mathbf{P} \langle \Phi_{\text{CSLS}}^{(-)} | \Phi_n \rangle \langle \Phi_{n'} | \Phi_{\text{CSLS}}^{(-)} \rangle$$

$$\times \delta \left(E_{\text{tot}} - \frac{\hbar^2 \mathbf{P}^2}{2\mu} - \varepsilon_{n-n} - \varepsilon_{nn-\alpha} \right) \delta \left(\varepsilon_{n-n} - \frac{\hbar^2 \mathbf{k}^2}{2\mu_{n-n}} \right) \delta \left(\varepsilon_{nn-\alpha} - \frac{\hbar^2 \mathbf{K}^2}{2\mu_{nn-\alpha}} \right)$$



${}^6\text{He} + {}^{12}\text{C} @ 240 \text{ MeV} / A$

- A peak structure at $\varepsilon_{n-n} + \varepsilon_{nn-\alpha} \sim 0.8 \text{ MeV}$.
- Both of resonant and nonresonant contributions are included.

Cross section for resonant state

➤ Rewrite T-matrix

$$\begin{aligned}
 T_\varepsilon &= \sum_n \langle \Phi_{\text{CSLS}}^{(-)} | \Phi_n \rangle T_n^{\text{CDCC}} \\
 &= \sum_n \langle \Phi_{\text{CSLS}}^{(-)} | \sum_\nu U^{-1}(\theta) |\Phi_\nu^\theta\rangle \langle \tilde{\Phi}_\nu^\theta | U(\theta) | \Phi_n \rangle T_n^{\text{CDCC}} \\
 &= \sum_\nu \langle \Phi_{\text{CSLS}}^{(-)} | U^{-1}(\theta) |\Phi_\nu^\theta\rangle \tilde{T}_\nu^\theta \quad (\tilde{T}_\nu^\theta = \sum_n \langle \tilde{\Phi}_\nu^\theta | U(\theta) | \Phi_n \rangle T_n^{\text{CDCC}})
 \end{aligned}$$

$$\begin{aligned}
 U^{-1}(\theta)U(\theta) &= 1 \\
 \text{Extended Complete Relation} \\
 \sum_\nu |\Phi_\nu^\theta\rangle \langle \tilde{\Phi}_\nu^\theta| &\simeq 1
 \end{aligned}$$

➤ DDBUX with the rewritten T-matrix

$$\begin{aligned}
 \frac{d^2\sigma}{d\varepsilon_{n-n}d\varepsilon_{nn-\alpha}} &= \sum_{\nu\nu'} \tilde{T}_\nu^\theta \tilde{T}_{\nu'}^{\theta\dagger} \int d\mathbf{k}d\mathbf{K}d\mathbf{P} \langle \Phi_{\text{CSLS}}^{(-)} | U^{-1}(\theta) |\Phi_\nu^\theta\rangle \langle \tilde{\Phi}_{\nu'}^{-\theta} | U^{-1}(\theta) | \Phi_{\text{CSLS}}^{(-)} \rangle \\
 &\times \delta \left(E_{\text{tot}} - \frac{\hbar^2 \mathbf{P}^2}{2\mu} - \varepsilon_{n-n} - \varepsilon_{nn-\alpha} \right) \delta \left(\varepsilon_{n-n} - \frac{\hbar^2 \mathbf{k}^2}{2\mu_{n-n}} \right) \delta \left(\varepsilon_{nn-\alpha} - \frac{\hbar^2 \mathbf{K}^2}{2\mu_{nn-\alpha}} \right)
 \end{aligned}$$

Cross section for resonant state

➤ Rewrite T-matrix

$$\begin{aligned}
 T_\varepsilon &= \sum_n \langle \Phi_{\text{CSLS}}^{(-)} | \Phi_n \rangle T_n^{\text{CDCC}} \\
 &= \sum_n \langle \Phi_{\text{CSLS}}^{(-)} | \sum_\nu U^{-1}(\theta) |\Phi_\nu^\theta\rangle \langle \tilde{\Phi}_\nu^\theta | U(\theta) | \Phi_n \rangle T_n^{\text{CDCC}} \\
 &= \sum_\nu \langle \Phi_{\text{CSLS}}^{(-)} | U^{-1}(\theta) |\Phi_\nu^\theta\rangle \tilde{T}_\nu^\theta \quad (\tilde{T}_\nu^\theta = \sum_n \langle \tilde{\Phi}_\nu^\theta | U(\theta) | \Phi_n \rangle T_n^{\text{CDCC}})
 \end{aligned}$$

$$U^{-1}(\theta)U(\theta) = 1$$

Extended Complete Relation

$$\sum_\nu |\Phi_\nu^\theta\rangle \langle \tilde{\Phi}_\nu^\theta| \simeq 1$$

➤ $\nu = \nu' = \text{Res.}$

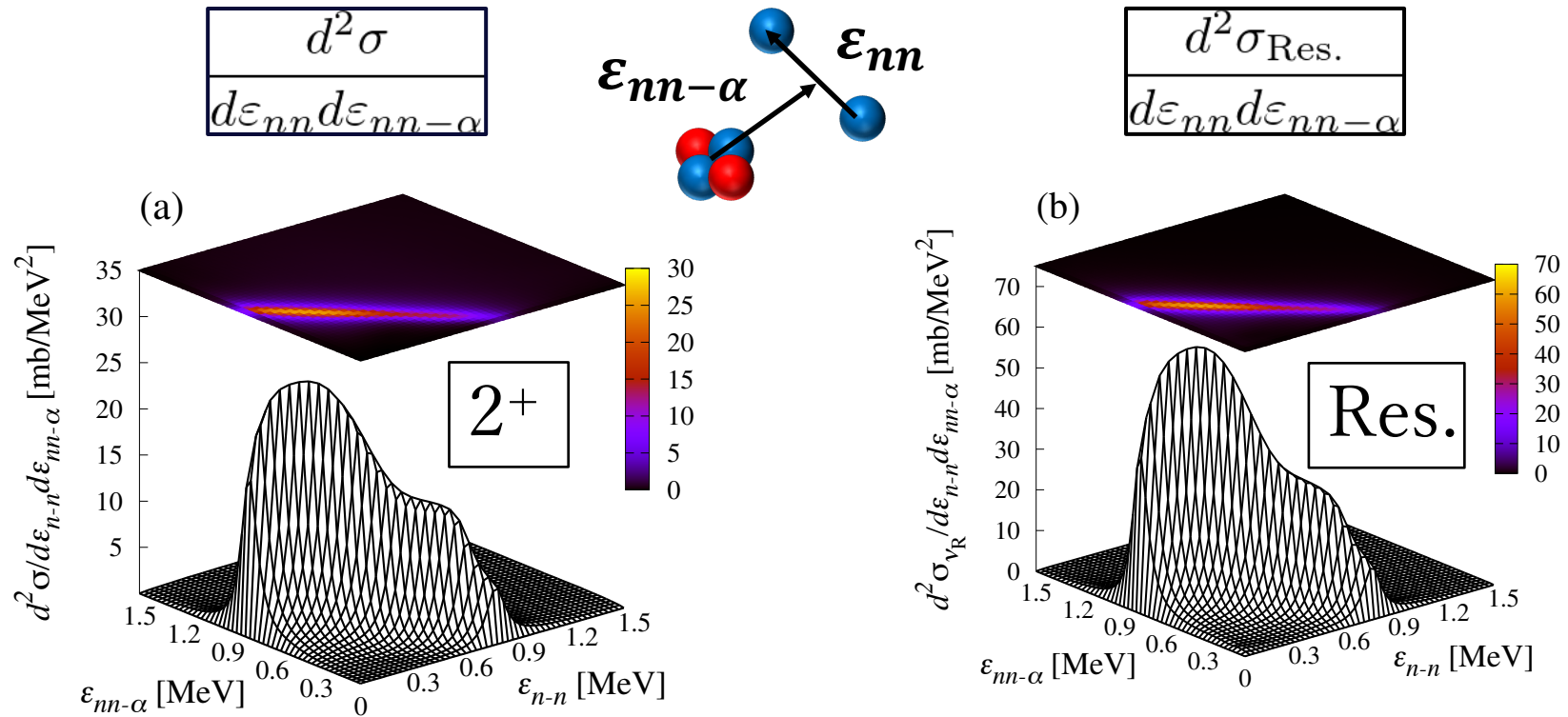
→ We define this term as a cross section for the resonance.

$$\begin{aligned}
 \frac{d^2\sigma_{\text{Res.}}}{d\varepsilon_{n-n}d\varepsilon_{nn-\alpha}} &\equiv |\tilde{T}_{\text{Res.}}^\theta|^2 \int d\mathbf{k}d\mathbf{K}d\mathbf{P} \left| \langle \Phi_{\text{CSLS}}^{(-)} | U^{-1}(\theta) | \Phi_{\text{Res.}}^\theta \rangle \right|^2 \\
 &\times \delta \left(E_{\text{tot}} - \frac{\hbar^2 \mathbf{P}^2}{2\mu} - \varepsilon_{n-n} - \varepsilon_{nn-\alpha} \right) \delta \left(\varepsilon_{n-n} - \frac{\hbar^2 \mathbf{k}^2}{2\mu_{n-n}} \right) \delta \left(\varepsilon_{nn-\alpha} - \frac{\hbar^2 \mathbf{K}^2}{2\mu_{nn-\alpha}} \right)
 \end{aligned}$$

- The shape of the cross section is reflected by $\left| \langle \Phi_{\text{CSLS}}^{(-)} | U^{-1}(\theta) | \Phi_{\text{Res.}}^\theta \rangle \right|^2$.

Double differential breakup cross section

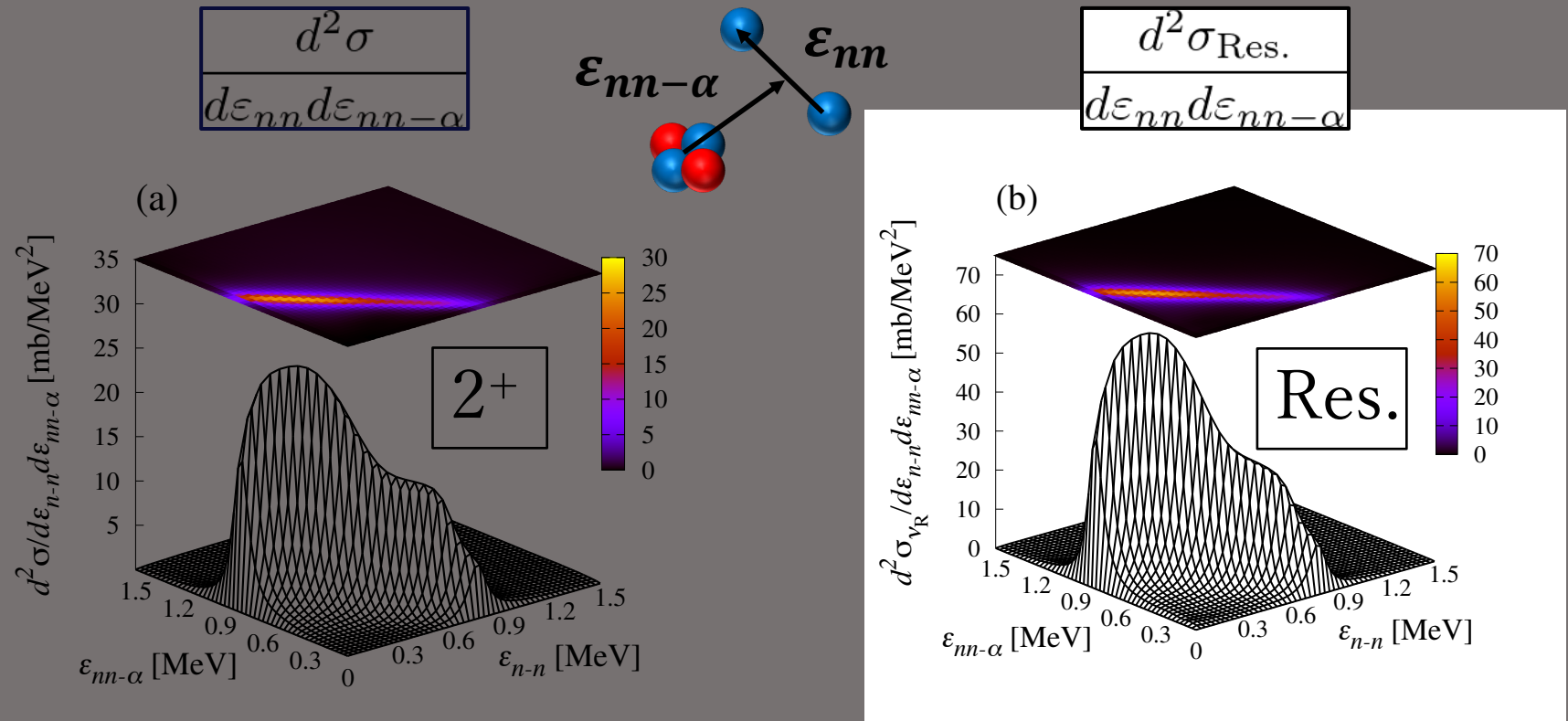
$${}^6\text{He} + {}^{12}\text{C} @ 240 \text{ MeV} / A$$



- DDBUX for the resonance has a peak structure at the same position as DDBUX for $I^\pi = 2^+$ state.

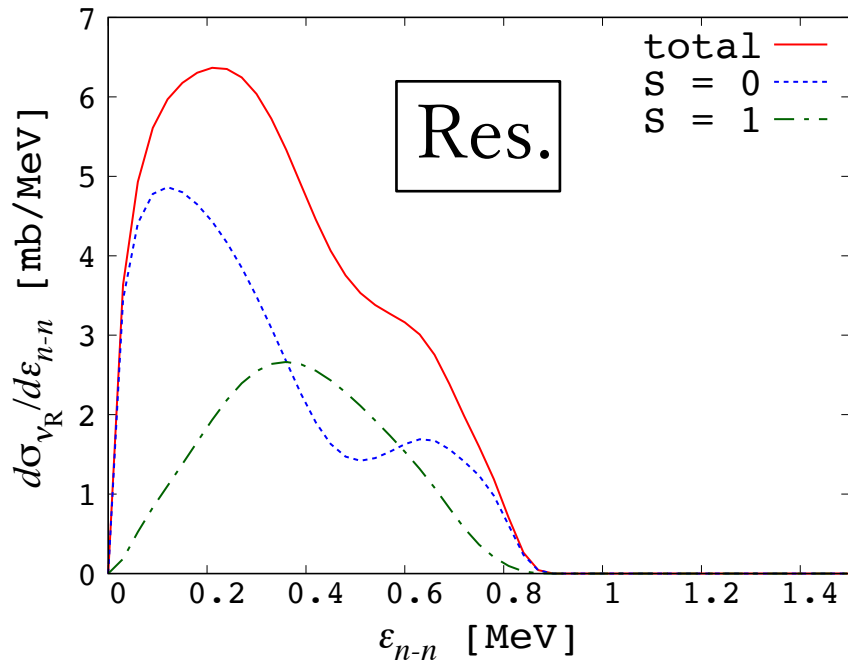
Double differential breakup cross section

${}^6\text{He} + {}^{12}\text{C} @ 240 \text{ MeV} / \text{A}$



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Contribution from 2_1^+



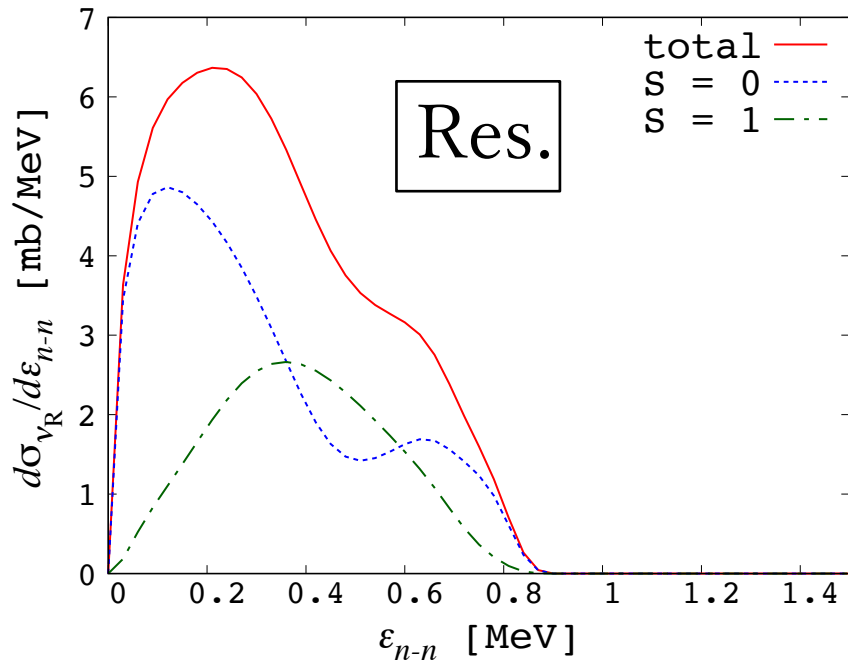
DDBUX is integrated over $\varepsilon_{nn-\alpha}$ in the region D .

$$\frac{d\sigma_{\text{Res.}}}{d\varepsilon_{n-n}} = \int_D \frac{d^2\sigma_{\text{Res.}}}{d\varepsilon_{n-n}d\varepsilon_{nn-\alpha}} d\varepsilon_{nn-\alpha}$$

$$\left(D : \varepsilon_r - \frac{\Gamma}{2} \leq \varepsilon_{n-n} + \varepsilon_{nn-\alpha} \leq \varepsilon_r + \frac{\Gamma}{2} \right)$$

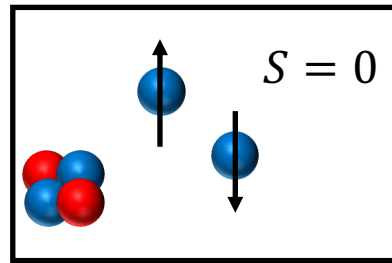
➤ **Solid line** : shoulder peak around $\varepsilon_{n-n} = 0.7$ MeV

Contribution from 2_1^+

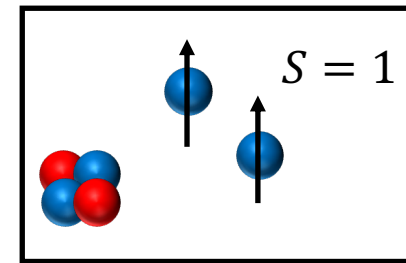


Separate the cross section into the $S = 0$ and 1 contribution.

$$\frac{d\sigma_{Res.}}{d\varepsilon_{n-n}} = \left(\frac{d\sigma_{Res.}}{d\varepsilon_{n-n}}\right)_{S=0} + \left(\frac{d\sigma_{Res.}}{d\varepsilon_{n-n}}\right)_{S=1}$$



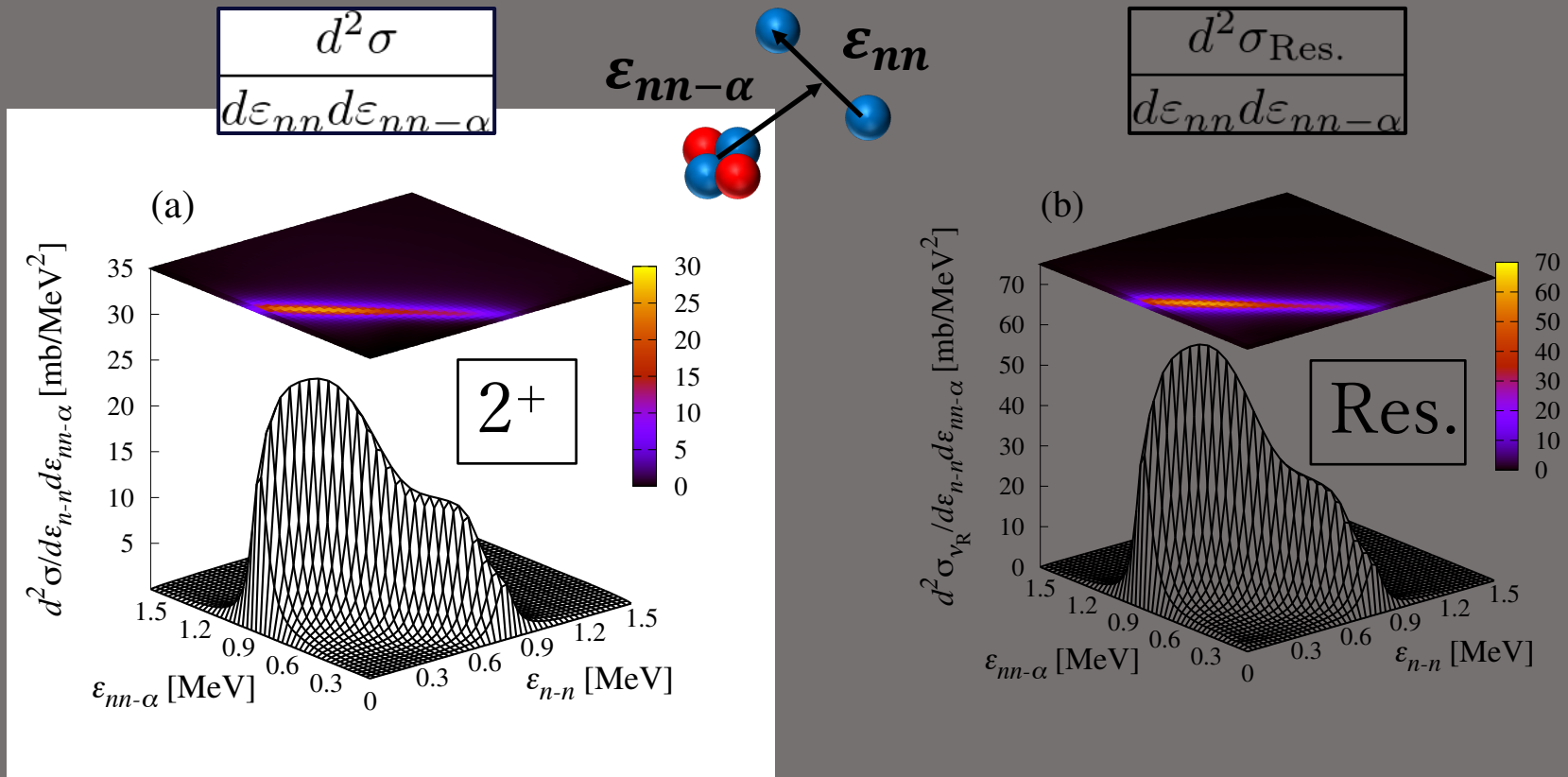
or



- **Solid line** : shoulder peak around $\varepsilon_{n-n} = 0.7$ MeV
 - **Dotted line** : $S = 0$ component has a peak at $\varepsilon_{n-n} = 0.7$ MeV.
- These results indicate existence of the dineutron in the 2_1^+ state.

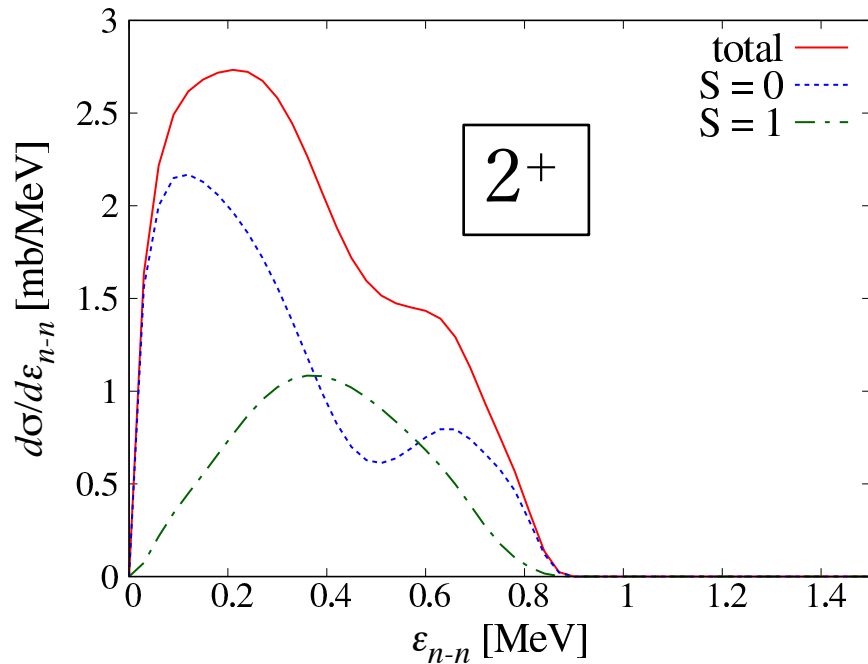
Double differential breakup cross section

${}^6\text{He} + {}^{12}\text{C} @ 240 \text{ MeV} / A$



- DDBUX for the resonance has a peak structure at the same position as DDBUX for $I^\pi = 2^+$ state.

Measurable cross section



The same definition as one, which discussed in the previous work.

$$\frac{d\sigma}{d\varepsilon_{n-n}} = \int_D \frac{d^2\sigma}{d\varepsilon_{n-n}d\varepsilon_{nn-\alpha}} d\varepsilon_{nn-\alpha}$$

$$\left(D : \varepsilon_r - \frac{\Gamma}{2} \leq \varepsilon_{n-n} + \varepsilon_{nn-\alpha} \leq \varepsilon_r + \frac{\Gamma}{2} \right)$$

- The same shoulder peak as in the case of the resonance.
- $S = 0$ component of the measurable cross section has also the peak at $\varepsilon_{n-n} = 0.7$ MeV.

The behavior of the resonant cross section and measurable cross section is same.

→ The shoulder peak can be expected to come from the dineutron in 2_1^+ .

Summary

We analyze the breakup reaction for ${}^6\text{He} + {}^{12}\text{C}$ system, with CDCC and CSLS.

- We define the DDBUX for the resonance.
- From the analysis of the cross sections of the resonant states, it is found that two neutrons with $S = 0$ contribute to the shoulder peak.
- Similar peaks from the $S=0$ component appears in the measurable cross sections.
- These results strongly support the existence of the dineutron in the 2_1^+ state of ${}^6\text{He}$.

Back up

Hamiltonian of ${}^6\text{He}$

$$h = \sum_{i=1}^3 t_i - T_G + \sum_{i=1}^2 V_{n\alpha}(r_i) + V_{nn}(r_3) + V_{nn\alpha}(r_1, r_2, r_3) + V_{\text{Pauli}}$$

➤ $V_{n\alpha}$: KKNN interaction

H. Kanada et al., Prog. Theor. Phys. **61**, 1327.

➤ V_{nn} : Minnesota interaction

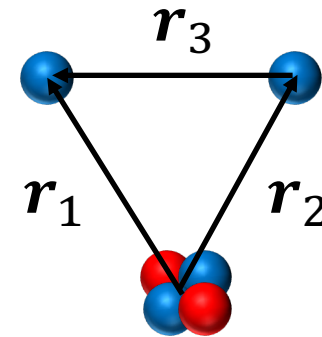
D. R. Thompson, et al., Nucl. Phys. A **286**, 53.

➤ $V_{nn\alpha}$: Phenomenological 3-body force

We introduce $V_{nn\alpha}$ to reproduce the binding energy of ground state and the resonant energy and decay width of the 2_1^+ state.

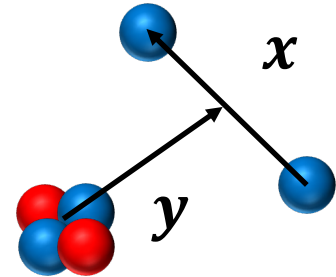
➤ V_{Pauli} : Orthogonal condition model

S. Saito, Prog. Theor. Phys. **41**, 705.

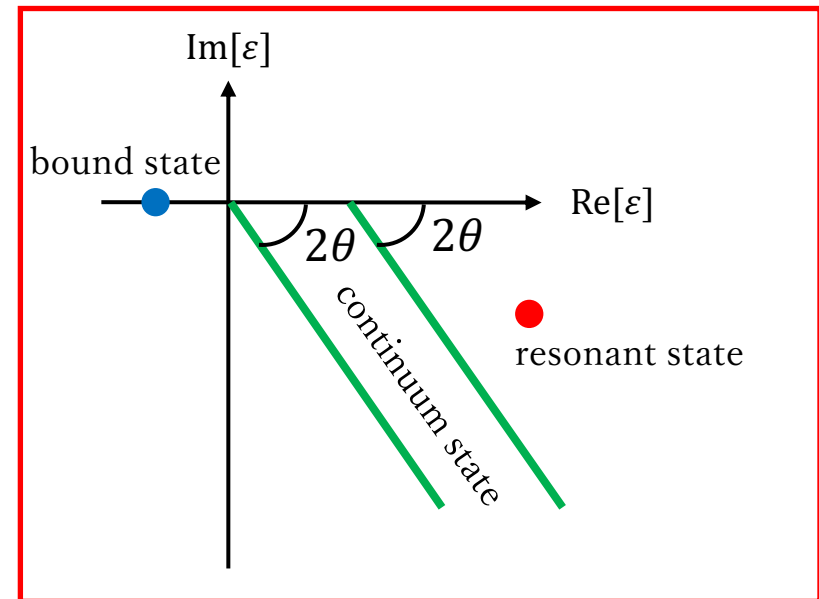
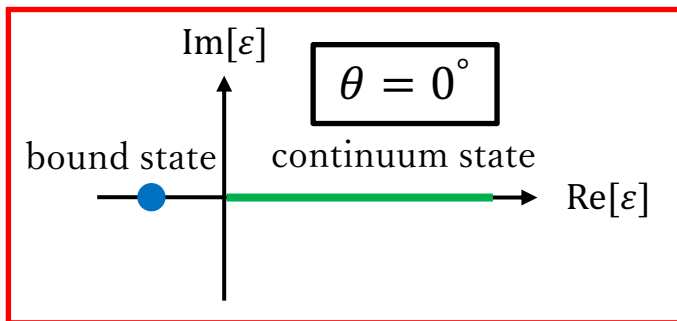


Complex Scaling Method (CSM)

Transform the internal coordinates \mathbf{x} , \mathbf{y} of ${}^6\text{He}$ as follow:



$$U(\theta)\mathbf{x}U^{-1}(\theta) = \mathbf{x}e^{i\theta}, \quad U(\theta)\mathbf{y}U^{-1}(\theta) = \mathbf{y}e^{i\theta}$$



Resonant states appears as isolated points.

$$E_{\text{res}} = E_r - i\frac{\Gamma}{2}$$

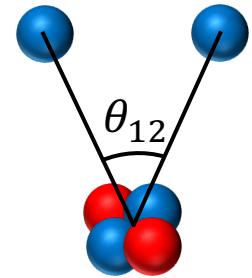
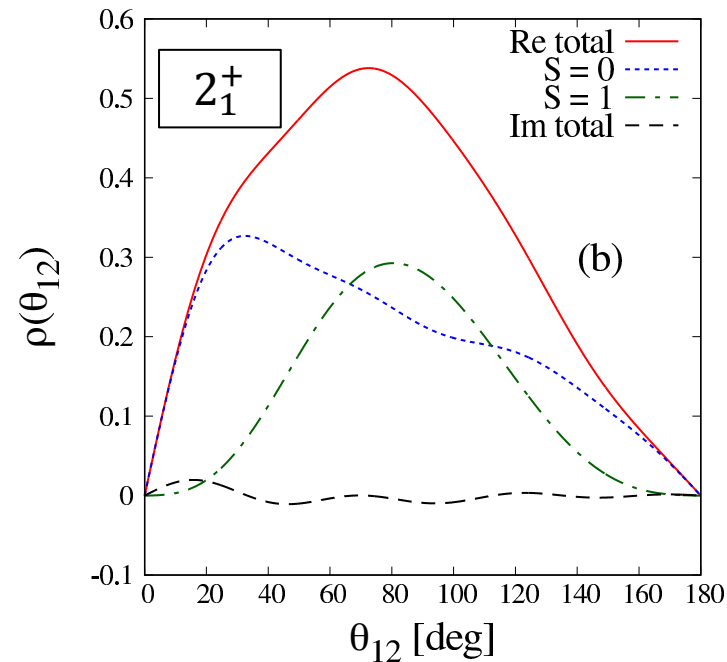
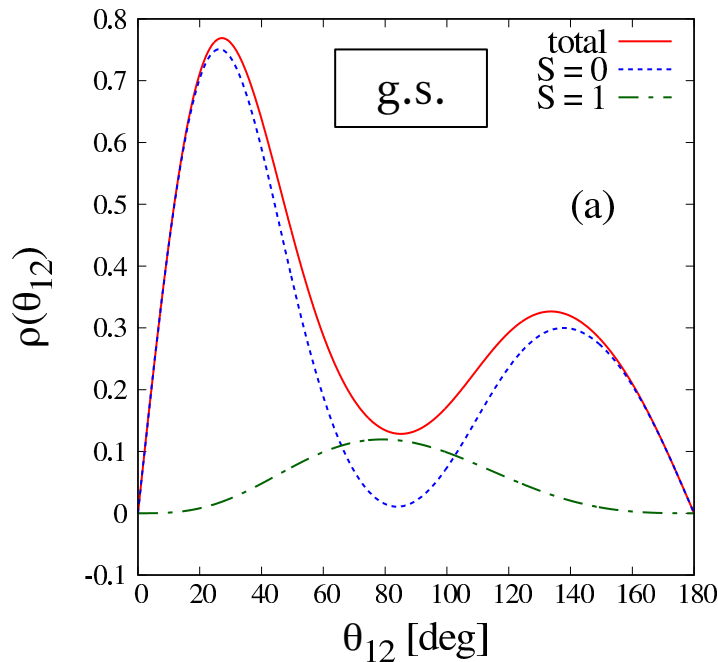
Resonant energy

Decay width

Angular density

$$\rho(\theta_{12}) = \langle \tilde{\Phi}_{\text{Res.}}^\theta | \delta(\theta_{12} - \omega) | \Phi_{\text{Res.}}^\theta \rangle$$

This density is defined in [A. T. Kruppa, et al., PRC **89**, 014330 (2013)].



➤ This density does not depend on the scaling angle θ .

➤ 2_1^+

- Total : A peak at $\theta_{12} \sim 80^\circ$.
- $S = 0$: A peak at $\theta_{12} \sim 30^\circ \rightarrow$ dineutron?

Complex scaling transformation

- Property of complex scaling transformation

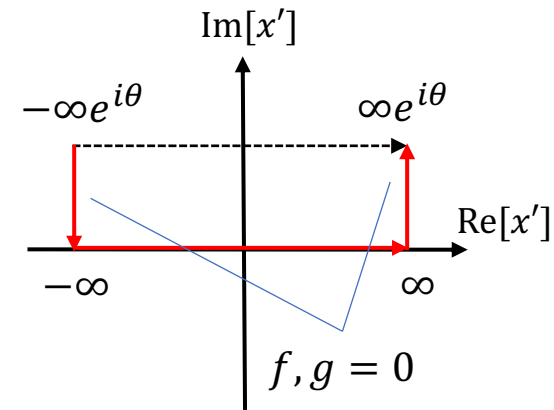
$$U^{-1}(\theta) = U(-\theta) = [U(\theta)]^*$$

$$\langle g|U(\theta)f\rangle = \langle U^\dagger(\theta)g|f\rangle$$

$$\begin{aligned}\langle g|U(\theta)f\rangle &= \int_{-\infty}^{\infty} dx g^*(x) e^{i\frac{\theta}{2}} f(xe^{i\theta}) \\ &= \int_{-\infty e^{i\theta}}^{\infty e^{i\theta}} dx' e^{-i\frac{\theta}{2}} g^*(x' e^{-i\theta}) f(x') \quad (x' = xe^{i\theta}) \\ &= \int_{-\infty}^{\infty} dx e^{-i\frac{\theta}{2}} g^*(xe^{-i\theta}) f(x) \\ &= \int_{-\infty}^{\infty} dx \left[e^{i\frac{\theta}{2}} g(xe^{i\theta}) \right]^* f(x) \\ &= \langle U(\theta)g|f\rangle \quad \longrightarrow \quad U(\theta) = U^\dagger(\theta)\end{aligned}$$

- Property of the scaled Hamiltonian

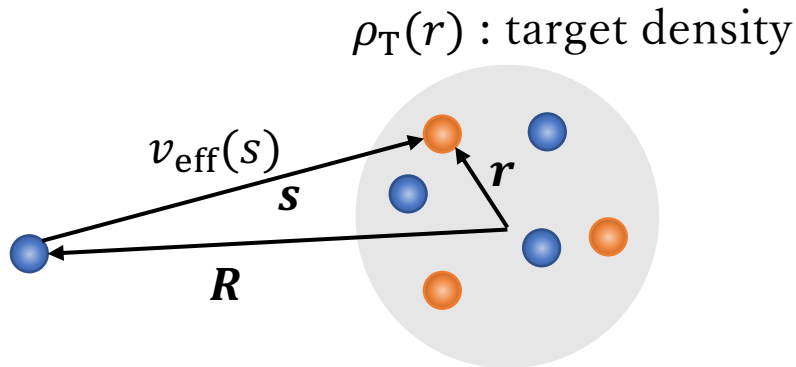
$$h^{\theta\dagger} = [U(\theta)hU^{-1}(\theta)]^\dagger = U^{-1}(\theta)hU(\theta) = h^{-\theta}$$



Optical potential

✓ Potential between n and T

Single Folding Model



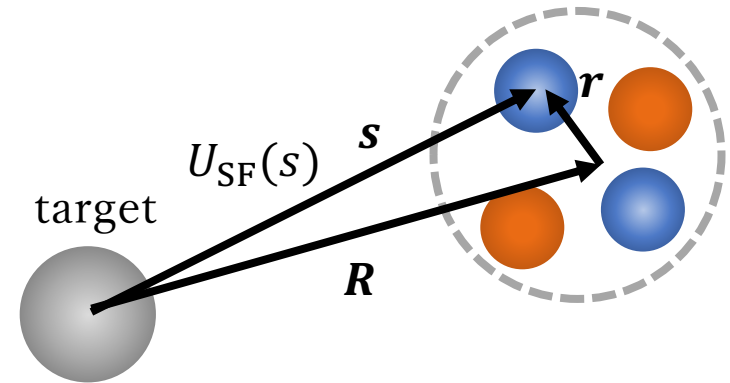
$$U_{\text{SF}}(R) = \int \rho^{(12\text{C})}(r) v_{\text{eff}}(s) d\mathbf{r}$$

$v_{\text{eff}}(s)$: Melbourne g-matrix

K. Amos, et al., in Advances in Nuclear Physics, **25**, 275 (2000).

✓ Potential between α and T

Double Single Folding Model



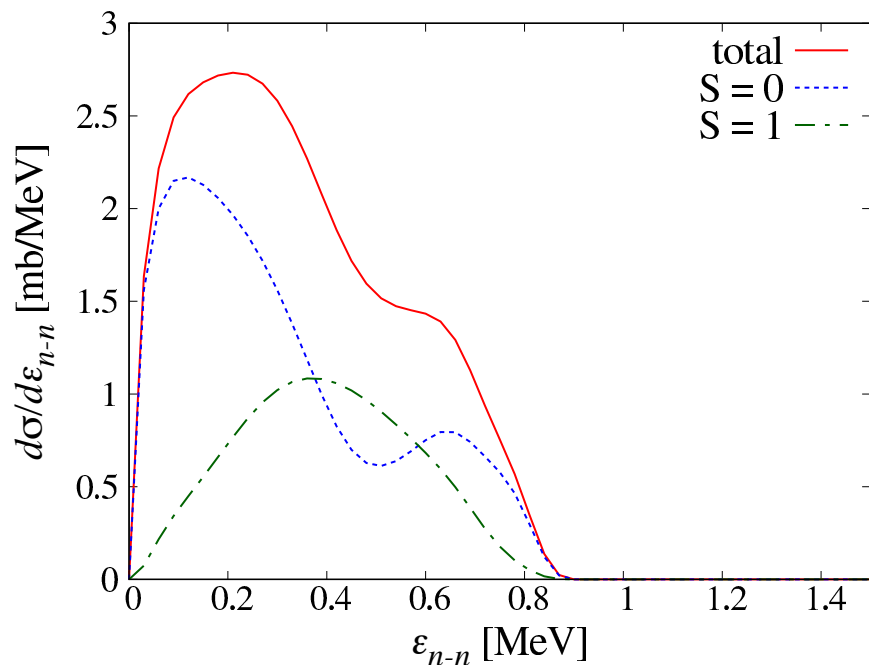
$$U_{\text{DSF}}(R) = \int d\mathbf{r} U_{\text{SF}}(s) \rho^{(\alpha)}(\mathbf{r})$$

NN interaction and the shape of the cross section

$$\frac{d\sigma}{d\varepsilon_{n-n}} = \int_D \frac{d^2\sigma}{d\varepsilon_{n-n}d\varepsilon_{nn-\alpha}} d\varepsilon_{nn-\alpha} \quad \left(D : \varepsilon_r - \frac{\Gamma}{2} \leq \varepsilon_{n-n} + \varepsilon_{nn-\alpha} \leq \varepsilon_r + \frac{\Gamma}{2} \right)$$

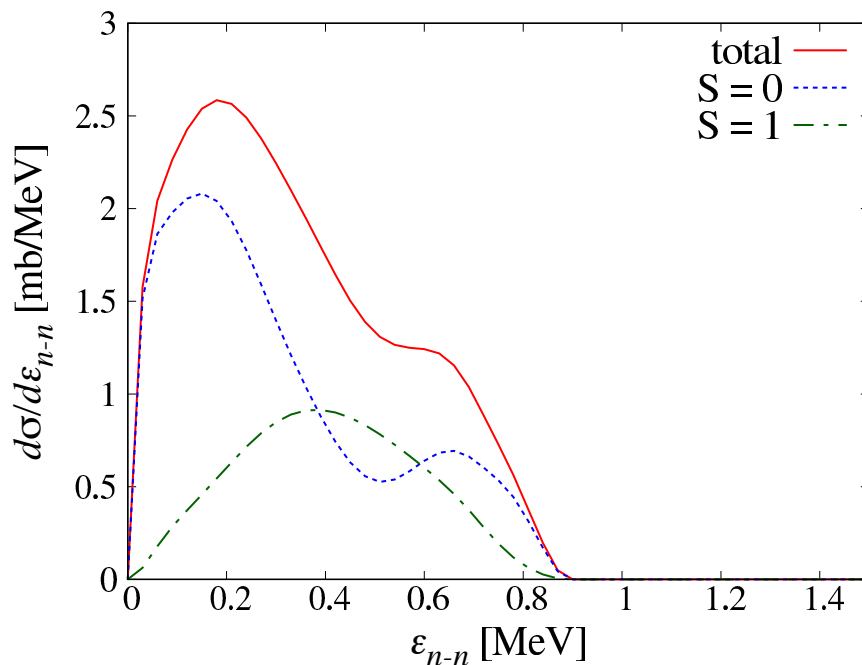
Minnesota interaction

D. R. Thompson, et al., Nucl. Phys. A 286, 53 (1977)



Gogny-Pires-Tourelle interaction

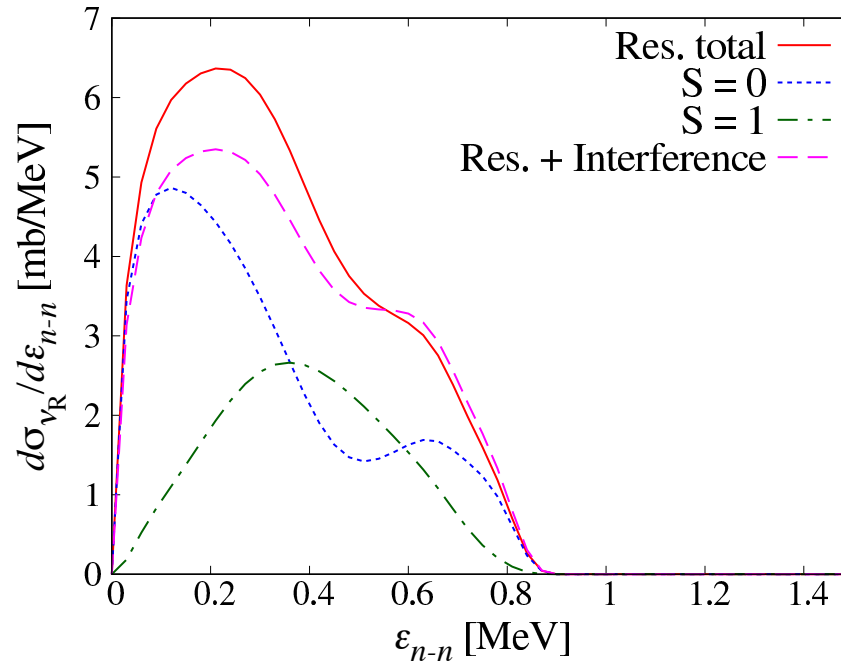
D. Gogny, et al., Phys. Lett. B 32, 591 (1970).



Interference between the resonant and nonresonant states

$$\left(\frac{d\sigma}{d\varepsilon_{n-n}}\right)_{\text{interference}} \equiv \int_D d\varepsilon_{n-n} 2\text{Re} \left[\sum_{\nu \in D'} T_\nu^\theta T_{\nu_R}^\theta \int d\mathbf{k} d\mathbf{K} d\mathbf{P} \langle \Phi_{\text{CSLS}}^{(-)} | U^{-1}(\theta) | \Phi_{\nu_R}^\theta \rangle \langle \Phi_\nu^{-\theta} | U^{-1}(\theta) | \Phi_{\text{CSLS}}^{(-)} \rangle \delta_{e.c.} \right]$$

$$(D' : \varepsilon_r - \Gamma/2 \leq \text{Re}[\varepsilon_\nu^\theta] \leq \varepsilon_r + \Gamma/2, \quad \nu \neq \nu_R).$$

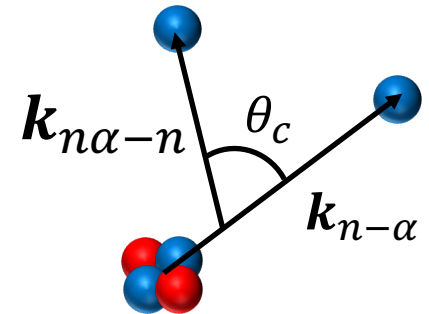


Future work

I will investigate the distribution of the correlation angle θ_c .

- Calculate the triple differential breakup cross section

$$\frac{d^3\sigma}{d\varepsilon_{n-\alpha}d\varepsilon_{n\alpha-n}d\theta_c}$$



- Discuss the following cross section

$$\frac{d^2\sigma}{d\varepsilon_{n-\alpha}d\theta_c} = \int_D d\varepsilon_{n\alpha-n} \frac{d^3\sigma}{d\varepsilon_{n-\alpha}d\varepsilon_{n\alpha-n}d\theta_c}$$
$$(\varepsilon_r - \Gamma/2 \leq \varepsilon_{n-\alpha} + \varepsilon_{n\alpha-n} \leq \varepsilon_r + \Gamma/2)$$