Subtracted Second RPA method and nuclear collective states

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Effects of the Skyrme tensor force on 0^+, 2^+, and 3^- states in ^{16}O and ^{40}Ca nuclei with second random phase approximation

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PHYSICAL REVIEW C 103, 054308 (2021)

Second RPA and Extended RPA

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SRPA (Second RPA based on density functional theory)

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SSRPA (subtracted SRPA): subtract double-counting contribution)

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RPA ground state is defined as

$$|\Psi\rangle = e^{\hat{S}}|\Phi\rangle,$$

where

$$\hat{S} = \sum_{ph} C_{ph}(t) a_p^\dagger a_h,$$

SRPA operator is

$$\hat{S} = \sum_{ph} C_{ph}(t) a_p^\dagger a_h + \frac{1}{2} \sum_{php'h'} \hat{C}_{pp'hh'}(t) a_p^\dagger a_{p'}^\dagger a_h a_{h'}.$$

The basic idea is the same as the coupled cluster model with s- and d- pairs.

Hohenberg- Kohn (HK) theorem [29] and the Kohn-Sham (KS) procedure

the energy-density functional $E[\rho]$ is universal: in the presence of an additional local Hermitian operator $\lambda Q(\mathbf{r})$, with λ an arbitrary constant, $E[\rho]$ is modified in a simple way,

$$E[\rho] \to E_\lambda[\rho] = E[\rho] + \lambda \int d\mathbf{r}\, Q(\mathbf{r})\rho(\mathbf{r}). \qquad (1)$$

system's density changes from the unperturbed ground-state density ρ_0 to a new one, ρ_λ, given by

$$\rho_\lambda = \rho_0 + \lambda \int d\mathbf{r}\, R(\omega = 0, \mathbf{r}, \mathbf{r}')Q(\mathbf{r}), \qquad (2)$$

In the KS approach, the fundamental philosophy or the essential assumption is EDF(in our case Skyrme interaction) is exact to produce the binding energy and the ground state density

In the adiabatic limit, $\omega \to 0$ $\qquad R(\omega = 0) = R_{\mathrm{KS}}^{\mathrm{RPA}},$

since a small amplitude limit of TDHF is RPA.

By using a time-dependent Kohn-Sham procedure as a time dependent Hohenberg Kohn (HK) theorem (known as Runge-Gross theorem) gives the energy dependent response function

$$R(\omega,\mathbf{r},\mathbf{r}') = R^0_{\mathrm{KS}}(\omega,\mathbf{r},\mathbf{r}') + \int d\mathbf{r}_1 d\mathbf{r}_2 \times R^0_{\mathrm{KS}}(\omega,\mathbf{r},\mathbf{r}_1) V(\omega,\mathbf{r}_1,\mathbf{r}_2) R(\omega,\mathbf{r}_2,\mathbf{r}'),$$

where R^0_{KS} is the bare Kohn-Sham (mean-field) response and $V(\omega)$ is a frequency-dependent effective interaction obtained from the time-dependent energy-density functional $\mathcal{E}[\rho(t),t]$.

But since $R^{\mathrm{RPA}}_{\mathrm{KS}}$ is correct (as correct as the Skyrme functional, anyway) in the adiabatic limit, we must modify the SRPA so that it gives the RPA response at $\omega = 0$.

SRPA and RPA effective interactions by $U(\omega)$,

$$U(\omega) \equiv V^{\mathrm{SRPA}}(\omega) - V^{\mathrm{RPA}}(\omega) \rightarrow 0 \qquad \omega \rightarrow 0$$

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^\nu a_p^\dagger a_h - Y_{ph}^\nu a_h^\dagger a_p) + \sum_{\substack{p_1<p_2 \\ h_1<h_2}} (X_{p_1p_2h_1h_2}^\nu a_{p_1}^\dagger a_{p_2}^\dagger a_{h_2} a_{h_1} - Y_{p_1p_2h_1h_2}^\nu a_{h_1}^\dagger a_{h_2}^\dagger a_{p_2} a_{p_1})$$

RPA equation.

$$\begin{bmatrix} A & B \\ -B^* & -A^* \end{bmatrix} \begin{bmatrix} X^\nu \\ Y^\nu \end{bmatrix} = \hbar\omega_\nu \begin{bmatrix} X^\nu \\ Y^\nu \end{bmatrix}$$

Where

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

$$X = \begin{pmatrix} X_1^\nu \\ X_2^\nu \end{pmatrix}, Y = \begin{pmatrix} Y_1^\nu \\ Y_2^\nu \end{pmatrix}$$

$$\begin{aligned} A_{11} &= A_{ph;p'h'} \\ &= <HF|[a_h^\dagger a_p, [H, a_{p'}^\dagger a_{h'}]]|HF> \\ &= (E_p - E_h)\delta_{pp'}\delta_{hh'} + \bar{V}_{ph'hp'} \end{aligned}$$

$$\begin{aligned} B_{11} &= B_{ph;p'h'} \\ &= - <HF|[a_h^\dagger a_p, [H, a_{h'}^\dagger a_{p'}]]|HF> \\ &= \bar{V}_{pp'hh'} \end{aligned}$$

$$\begin{aligned} A_{12} &= A_{ph;p_1p_2h_1h_2} \\ &= <HF|[a_h^\dagger a_p, [H, a_{p_1}^\dagger a_{p_2}^\dagger a_{h_2} a_{h_1}]]|HF> \\ &= U(h_1h_2)\bar{V}_{p_1p_2ph_2}\delta_{hh_1} - U(p_1p_2)\bar{V}_{hp_2h_1h_2}\delta_{pp_1} \end{aligned}$$

$U(h_1h_2)$ is an anti-symmtrizer.

$$\begin{aligned} A_{22} &= A_{p_1p_2h_1h_2;p_1'p_2'h_1'h_2'} \\ &= <HF|[a_{h_1}^\dagger a_{h_2}^\dagger a_{p_2} a_{p_1}, [H, a_{p_1'}^\dagger a_{p_2'}^\dagger a_{h_2'} a_{h_1'}]]|HF> \\ &= (E_{p_1} + E_{p_2} - E_{h_1} - E_{h_2})U(p_1p_2)U(h_1h_2) \\ &\quad \times \delta_{p_1p_1'}\delta_{p_2p_2'}\delta_{h_1h_1'}\delta_{h_2h_2'} \\ &\quad + U(h_1h_2)\bar{V}_{p_1p_2p_1'p_2'}\delta_{h_1h_1'}\delta_{h_2h_2'} \\ &\quad + U(p_1p_2)\bar{V}_{h_1h_2h_1'h_2'}\delta_{p_1p_1'}\delta_{p_2p_2'} \\ &\quad - U(p_1p_2)U(h_1h_2)U(p_1'p_2')U(h_1'h_2') \\ &\quad \times \bar{V}_{p_1h_1'p_1'h_1}\delta_{p_2p_2'}\delta_{h_2h_2'} \end{aligned}$$

In SRPA with subtraction procedure (SSRPA), A_{11} and B_{11} are modified.

$$A^S_{11'} = A_{11'} + \sum_2 A_{12}(A_{22})^{-1}A_{21'} + \sum_2 B_{12}(A_{22})^{-1}B_{21'}$$
$$B^S_{11'} = B_{11'} + \sum_2 A_{12}(A_{22})^{-1}B_{21'} + \sum_2 B_{12}(A_{22})^{-1}A_{21'}$$

$$\mathcal{A}^S_F = \begin{pmatrix} A_{11'} + \sum_{2,2'} A_{12}(A_{22'})^{-1}A_{2'1'} + \sum_{2,2'} B_{12}(A_{22'})^{-1}B_{2'1'} & A_{12} \\ A_{21} & A_{22'} \end{pmatrix},$$
$$\mathcal{B}^S_F = \begin{pmatrix} B_{11'} + \sum_{2,2'} A_{12}(A_{22'})^{-1}B_{2'1'} + \sum_{2,2'} B_{12}(A_{22'})^{-1}A_{2'1'} & B_{12} \\ B_{21} & 0 \end{pmatrix}.$$

If the coupling amongst the 2p-2h configuration is neglected, A_{22} will becomes diagonal, this approximation is denoted by SRPAD. In SRPAD, A_{22} is calculated by:

$$A^D_{22} = \delta_{p_1p'_1}\delta_{p_2p'_2}\delta_{h_1h'_1}\delta_{h_2h'_2}(E_{p_1} + E_{p_2} - E_{h_1} - E_{h_2}) \quad (8)$$

The transition operators of the spin-independent modes are:

$$F_\lambda^{IS} = \sum r_i^n Y_{\lambda 0}(r_i)$$
$$F_\lambda^{IV} = \sum r_i^n Y_{\lambda 0}(r_i)\tau_z(i) \qquad (10)$$

$$B(E_\lambda) = |\sum_{ph} b_{ph}(E_\lambda)|^2 = |\sum_{ph}(X_{ph}^\lambda + (-1)^J Y_{ph}^\lambda)F_{ph}^\lambda|^2$$

$$\sum_{ph}(|X_{ph}^\nu|^2 - |Y_{ph}^\nu|^2) + \sum_{p_1p_2h_1h_2}(|X_{p_1p_2h_1h_2}^\nu|^2 - |Y_{p_1p_2h_1h_2}^\nu|^2)$$
$$= n_1 + n_2 = 1$$

$$m_1 = \sum_\nu \hbar\omega_\nu |<\nu|F|0>|^2,$$

can be calculated analytically [86],

$$m_1 = \begin{cases} \frac{4}{4\pi}\frac{\hbar^2}{2m}A<r^2>, & \lambda = 0 \\ \frac{50}{4\pi}\frac{\hbar^2}{2m}A<r^2>, & \lambda = 2 \end{cases}$$

TABLE I: Isoscalar EWSR m_1 obtained in the analytic formula (15), RPA and SRPA calculations for ^{16}O and ^{40}Ca with SGII interaction.

^{16}O			
State	analytic one	RPA	SRPA
0^+	676.37	673.87	673.87
2^+	8454.65	8375.43	8375.43

^{40}Ca			
State	analytic one	RPA	SRPA
0^+	2889.16	2879.92	2879.92
2^+	36114.5	35934.4	35934.4

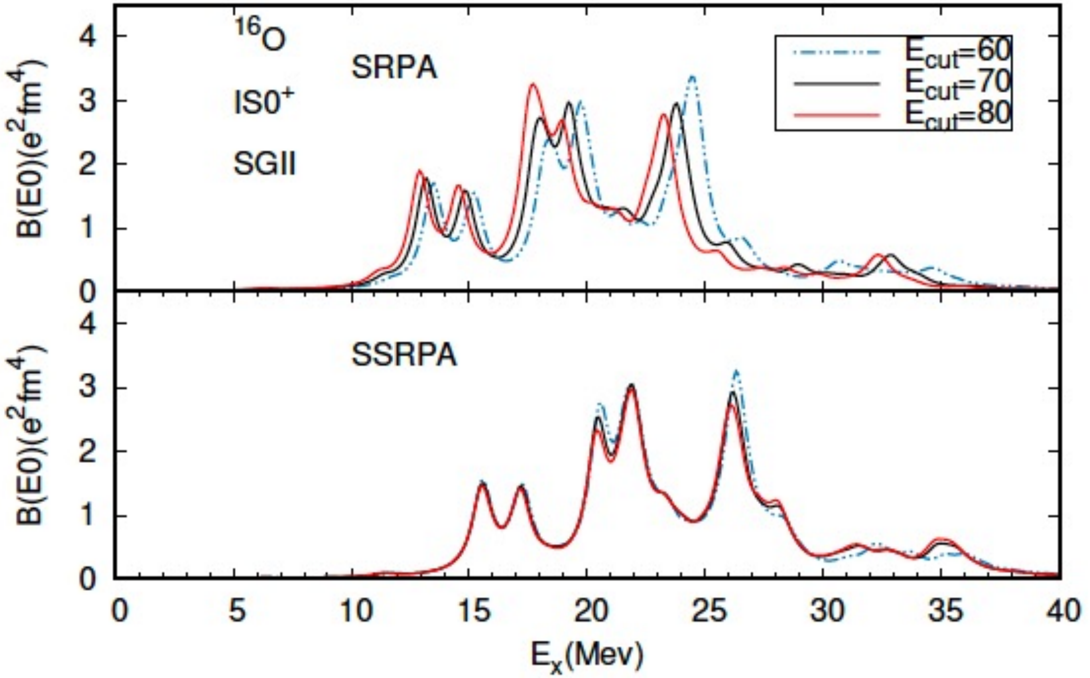

FIG. 1: IS 0^+ strength distributions for ^{16}O calculated by SRPA (upper panel)and SSRPA (lower panel) by SGII interaction with 2p-2h energy cutoff 60, 70, and 80 MeV. See the text for more details.

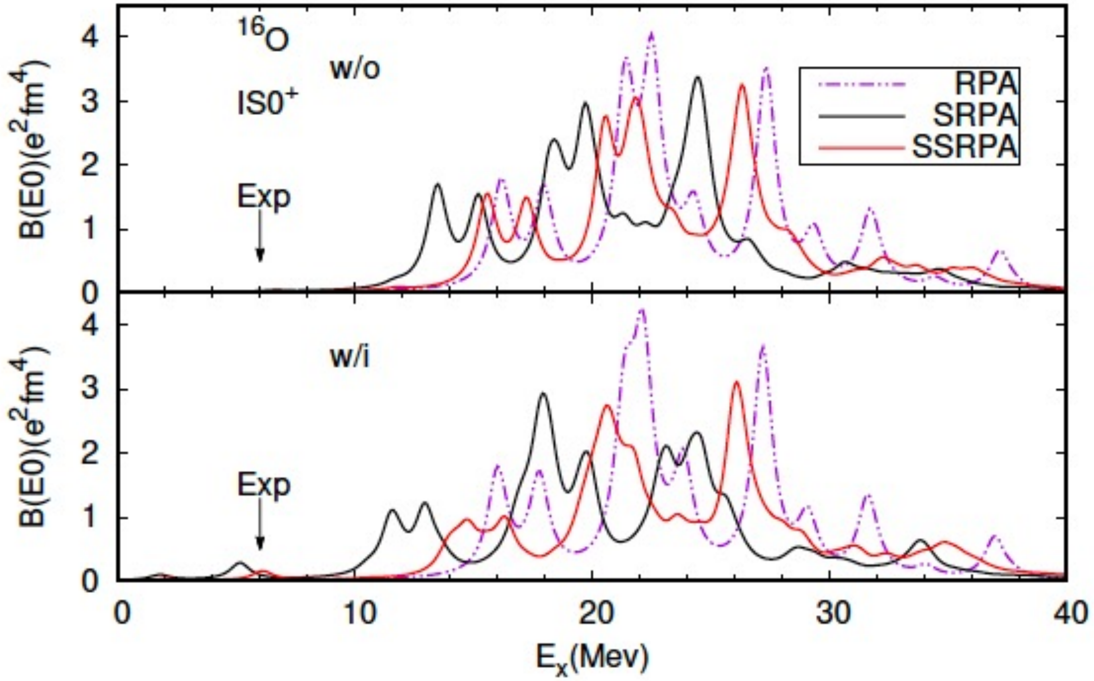

FIG. 2: IS 0^+ strength distributions in ^{16}O. The results calculated without and with tensor interaction are shown in the upper and lower panels, respectively. Results of RPA, SRPA, and SSRPA are labelled by purple dash lines, black solid lines, and red solid lines, respectively. The lowest state measured by experiment [3] is represented by an arrow.

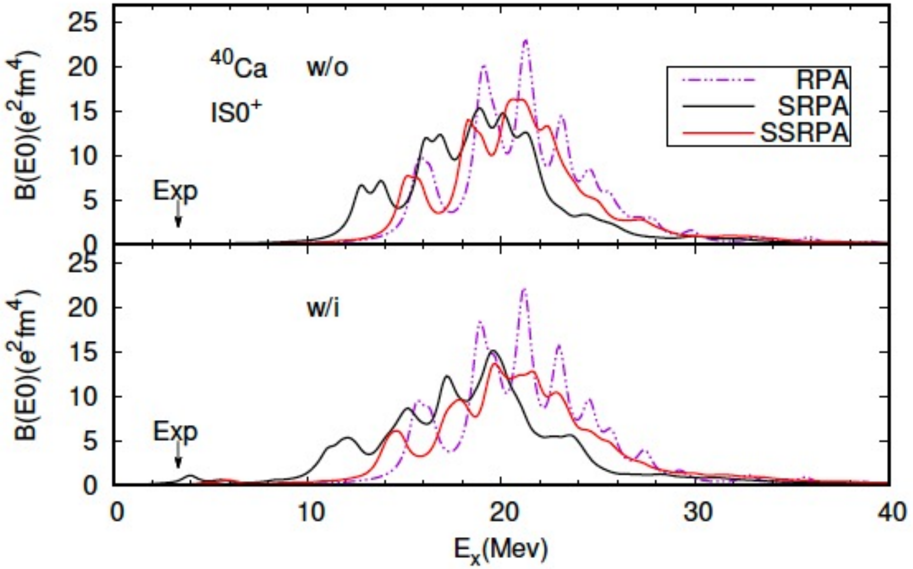

FIG. 4: The same as Fig. 2, but for IS 0^+ in ^{40}Ca nucleus, The experimental data is taken from Ref.[5].

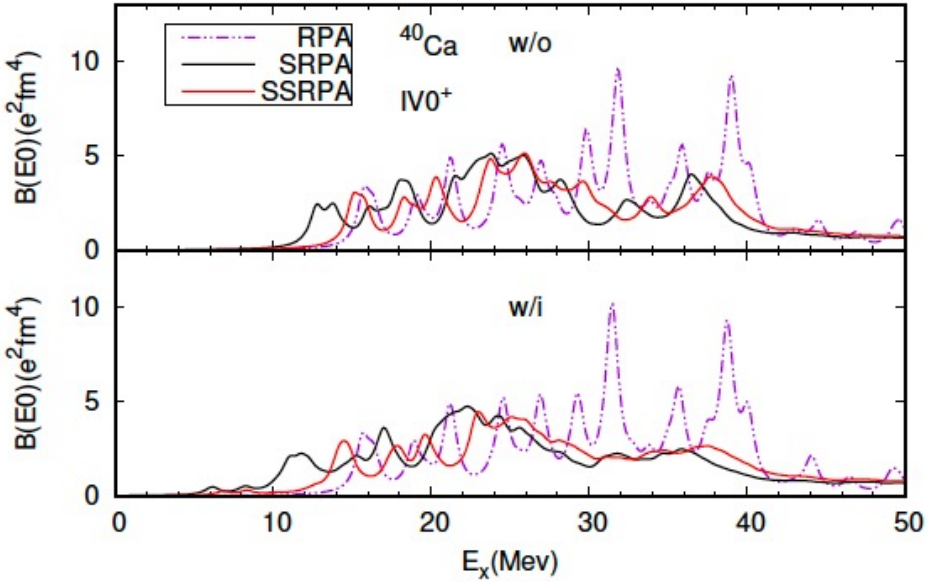

FIG. 5: The same as Fig. 4, but for IV 0^+ in ^{40}Ca.

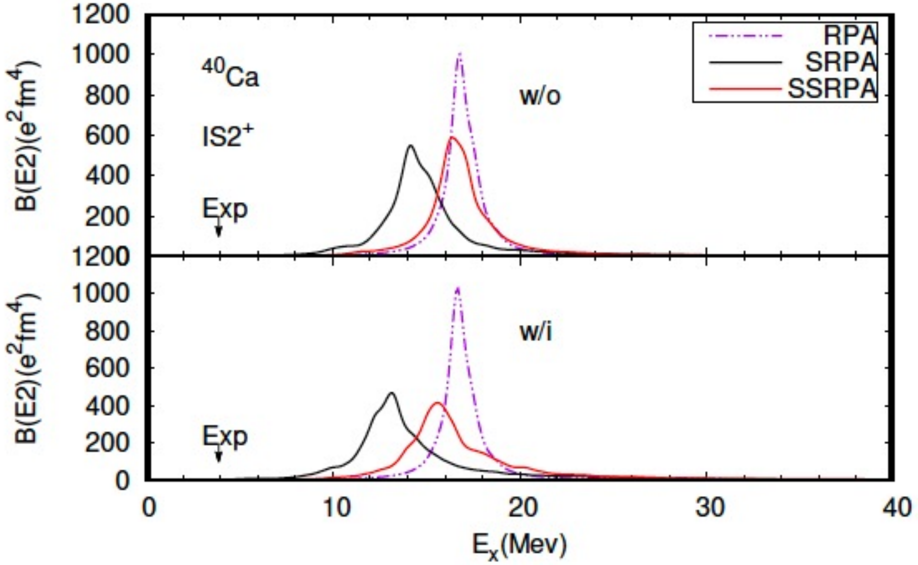

FIG. 9: The same as Fig. 4, but for IS 2^+ in ^{40}Ca. Experimental data is taken from Ref.[8].

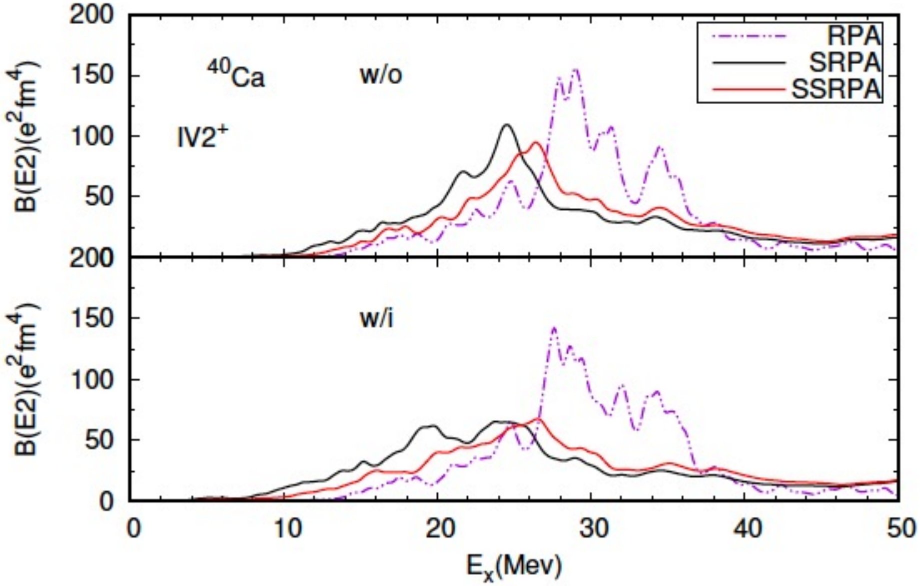

FIG. 11: The same as Fig. 10, but for IV 2^+ in ^{40}Ca.

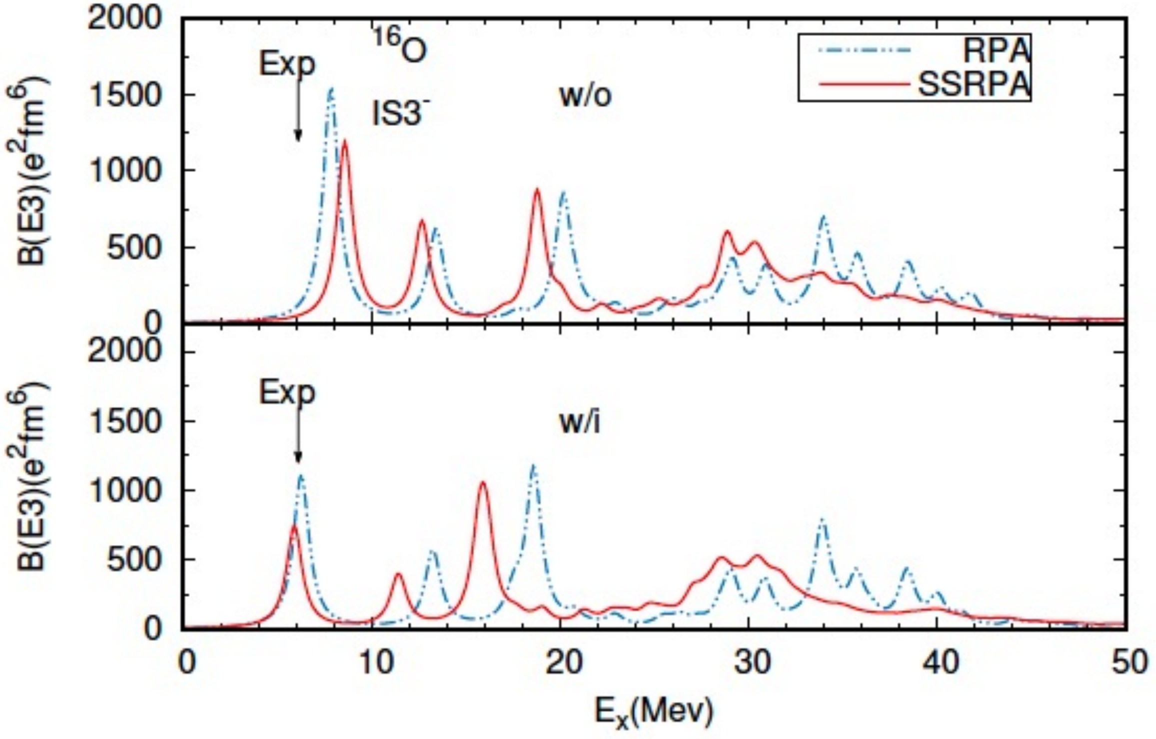

FIG. 12: Isoscalar octupole strength distributions in ^{16}O calculated without tensor(upper panel) and with tensor(lower panel). Blue dash lines are RPA result, while red solid lines are SSRPA result. Experimental data is taken from *****.

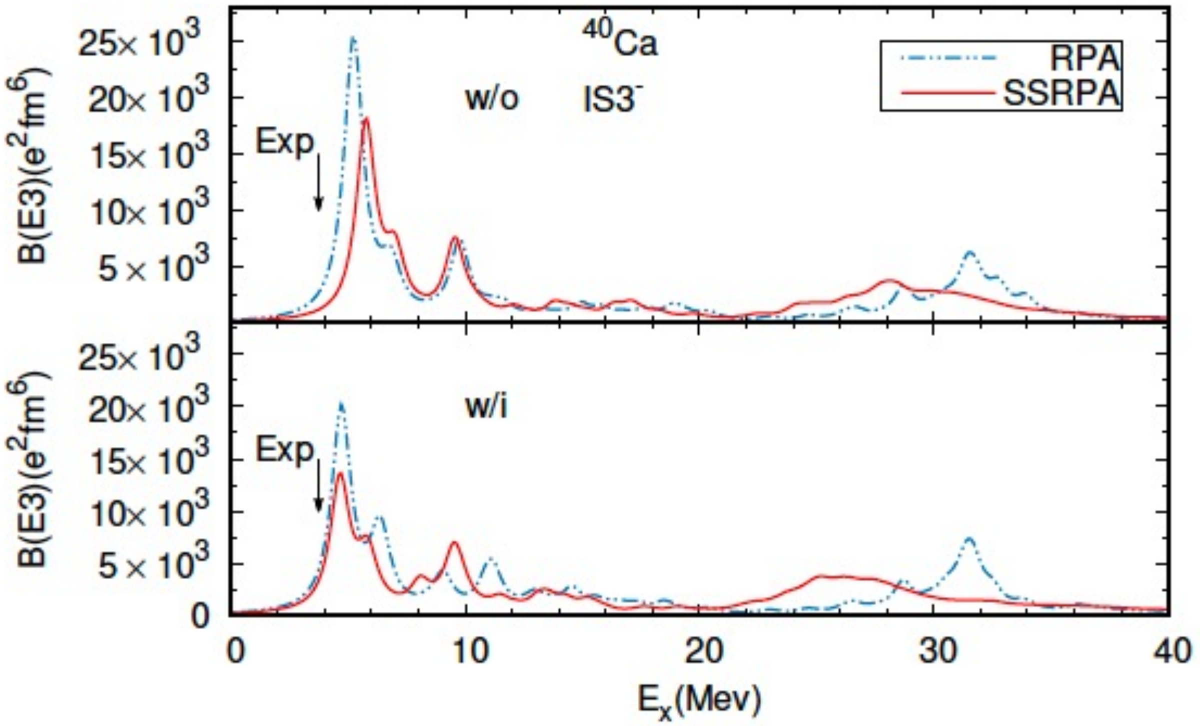

FIG. 14: IS3$^-$ in ^{40}Ca without(upper panel) with(lower panel) tensor force. Blue lines are RPA results, while red lines are SSRPA results. Experimental data is taken from *****.

Gamow-Teller states and 2particle-2hole configurations

D. F. Bertsch and I.Hamamoto, Phys. Rev. C 26, 132[illegible]
(1982).

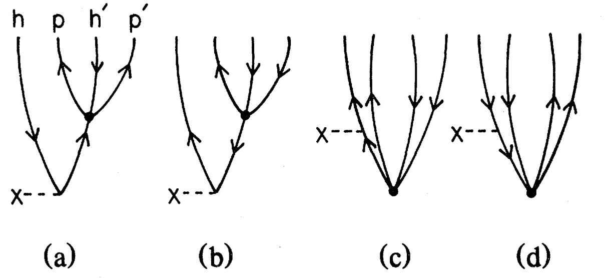

FIG. 3. Four types of amplitude included in the actual calculation. (a) should of course also include the graph with *h* and *h′* interchanged.

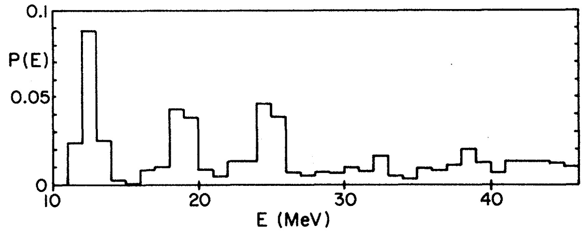

FIG. 4. Calculated strength distribution $P(E)$ for the Gamow-Teller operator in ^{90}Zr. Energies are measured with respect to the ground state of ^{90}Nb.

TABLE I. Contributions to Gamow-Teller strength in the region 10–45 MeV excitation in ^{90}Zr, $\int_{10}^{45} P(E)\,dE$, with $P(E)$ defined in Eq. (4). The partial sums need not add to the total because of possible coherence of amplitudes.

$\int P$	Graphs (a) + (b)	Graphs (c) + (d)	Total
Tensor	0.13	0.06	0.20
Central	0.25	0.15	0.36
Total	0.38	0.20	0.56

Spreading of the Gamow-Teller Resonance in ^{90}Nb and ^{208}Bi
Nguyen Dinh Dang, Akito Arima, Toshio Suzuki, and Shuhei Yamaji, PRL79, 1638 (1997)

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Gamow-Teller Strength in ^{48}Ca and ^{78}Ni with the Charge-Exchange Subtracted Second Random-Phase Approximation

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The Gamow-Teller transitions in magic nuclei calculated by the charge-exchange subtracted second random phase approximation

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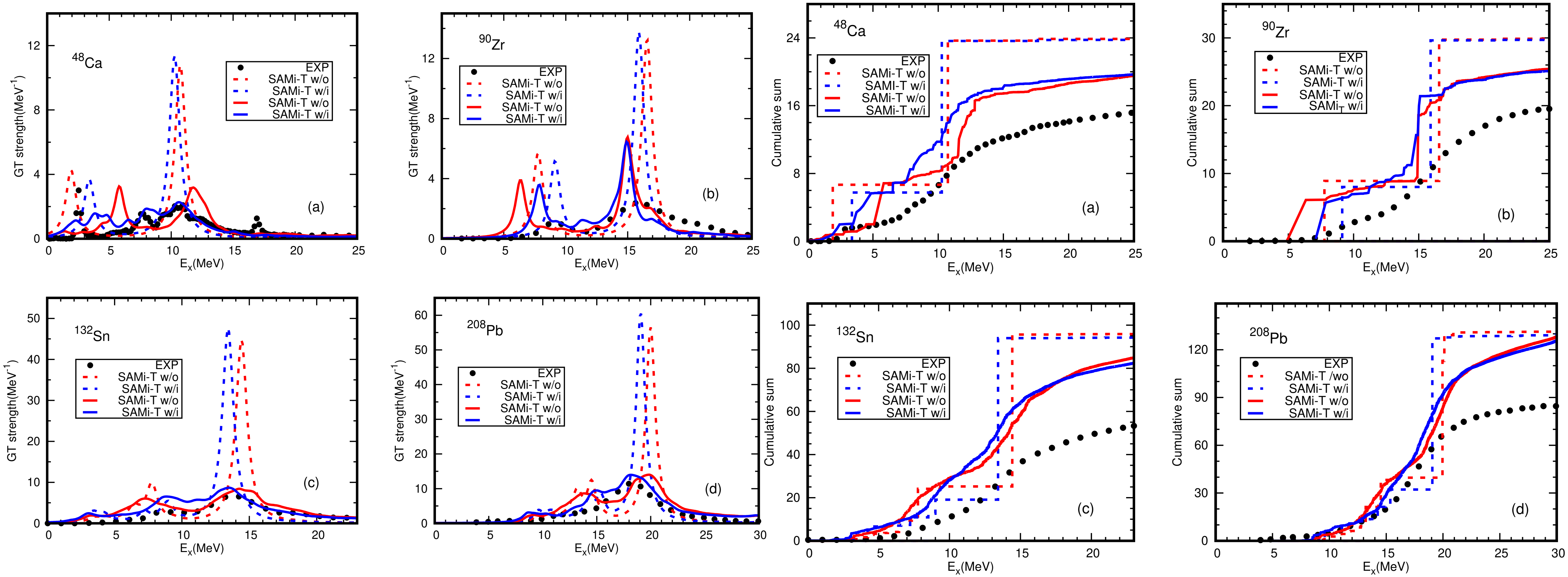
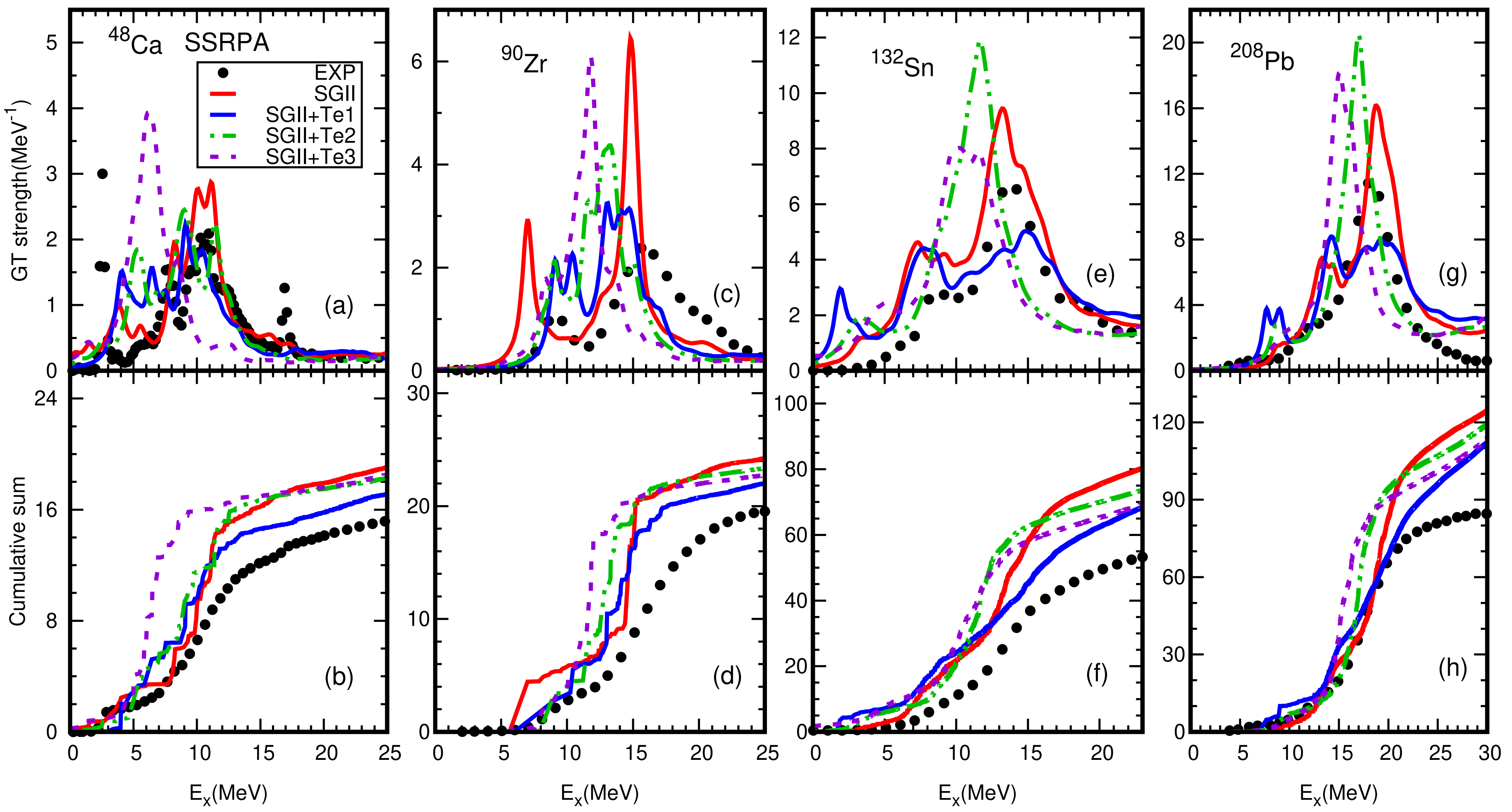
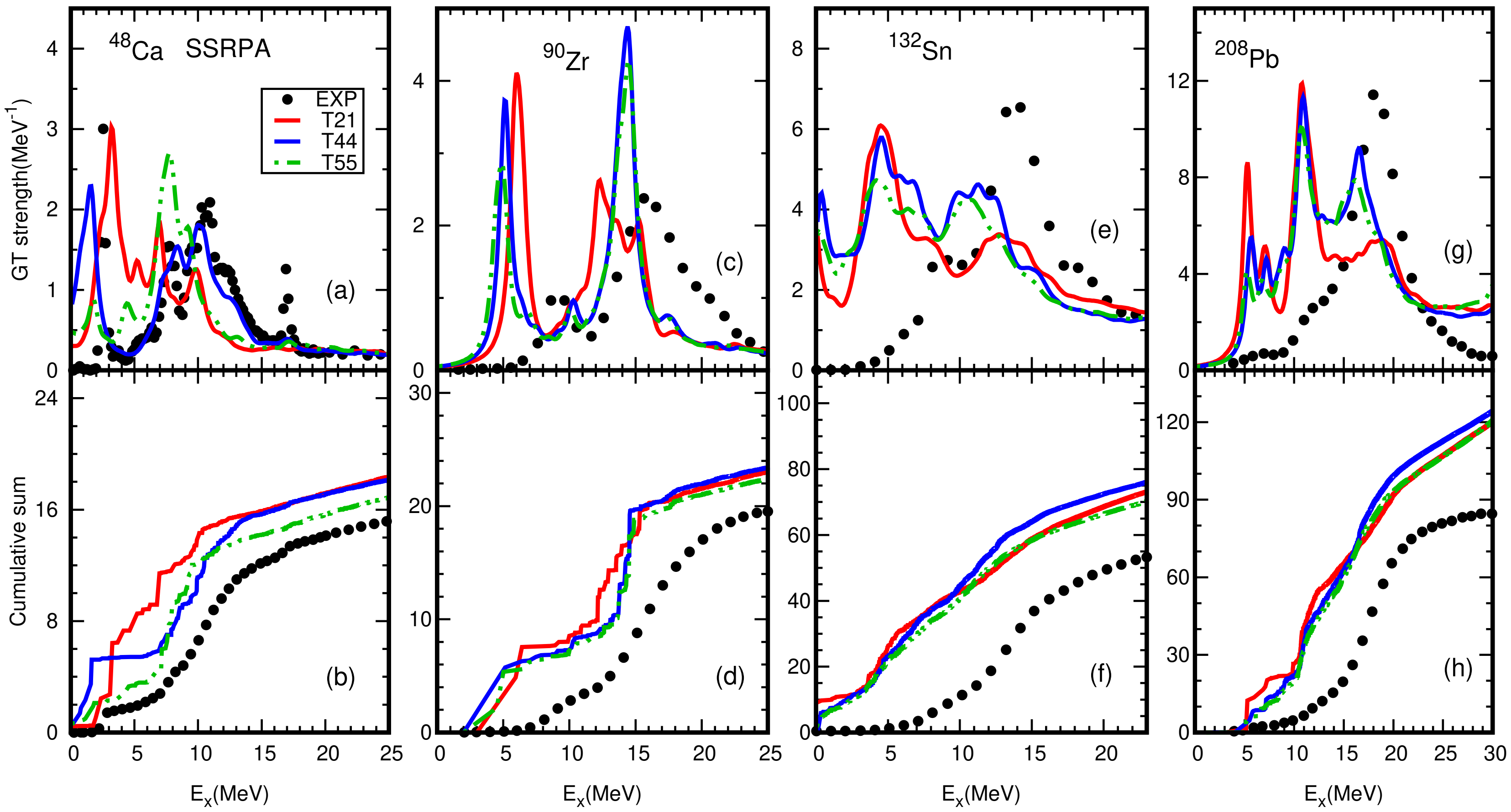


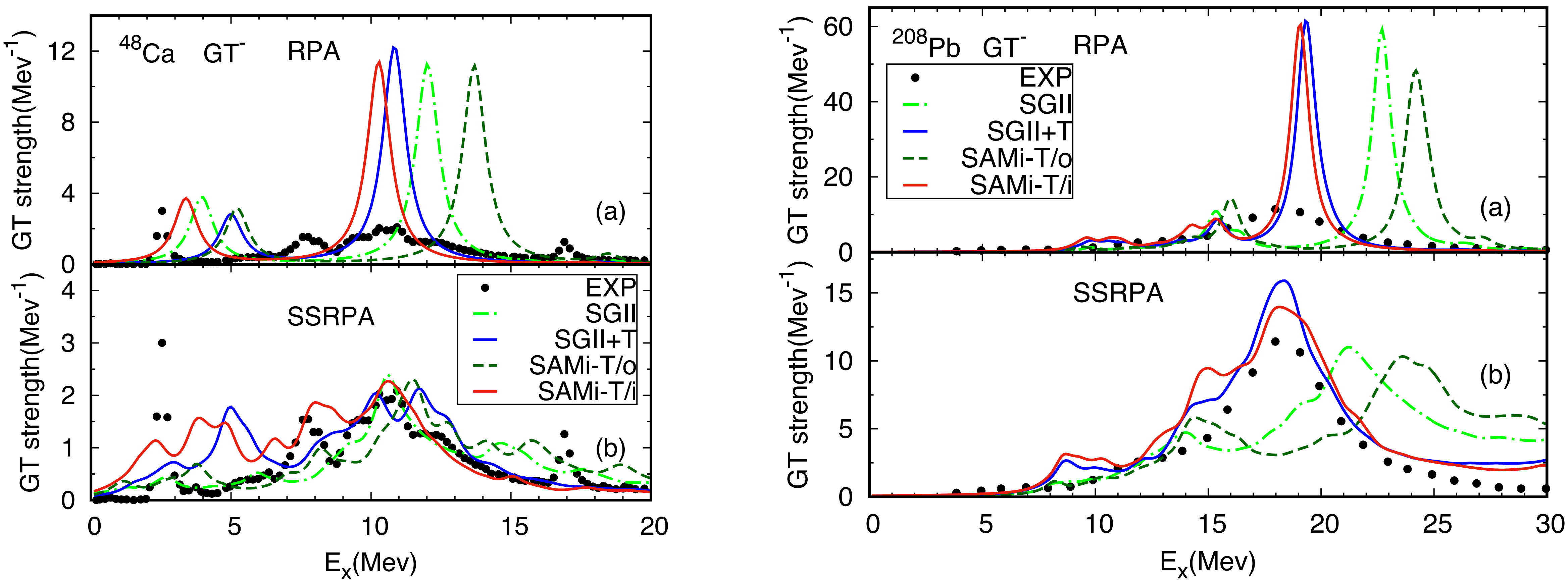
^{48}Ca
EXP
SAMi-T w/o
SAMi-T w/i
SAMi-T w/o
SAMi-T w/i
(a)
GT strength(MeV^{-1})
E_x(MeV)
^{90}Zr
EXP
SAMi-T w/o
SAMi-T w/i
SAMi-T w/o
SAMi-T w/i
(b)
GT strength(MeV^{-1})
E_x(MeV)
^{48}Ca
EXP
SAMi-T w/o
SAMi-T w/i
SAMi-T w/o
SAMi-T w/i
(a)
Cumulative sum
E_x(MeV)
^{90}Zr
EXP
SAMi-T w/o
SAMi-T w/i
SAMi-T w/o
SAMi$_T$ w/i
(b)
Cumulative sum
E_x(MeV)
^{132}Sn
EXP
SAMi-T w/o
SAMi-T w/i
SAMi-T w/o
SAMi-T w/i
(c)
GT strength(MeV^{-1})
E_x(MeV)
^{208}Pb
EXP
SAMi-T w/o
SAMi-T w/i
SAMi-T w/o
SAMi-T w/i
(d)
GT strength(MeV^{-1})
E_x(MeV)
^{132}Sn
EXP
SAMi-T w/o
SAMi-T w/i
SAMi-T w/o
SAMi-T w/i
(c)
Cumulative sum
E_x(MeV)
^{208}Pb
EXP
SAMi-T /wo
SAMi-T w/i
SAMi-T w/o
SAMi-T w/i
(d)
Cumulative sum
E_x(MeV)



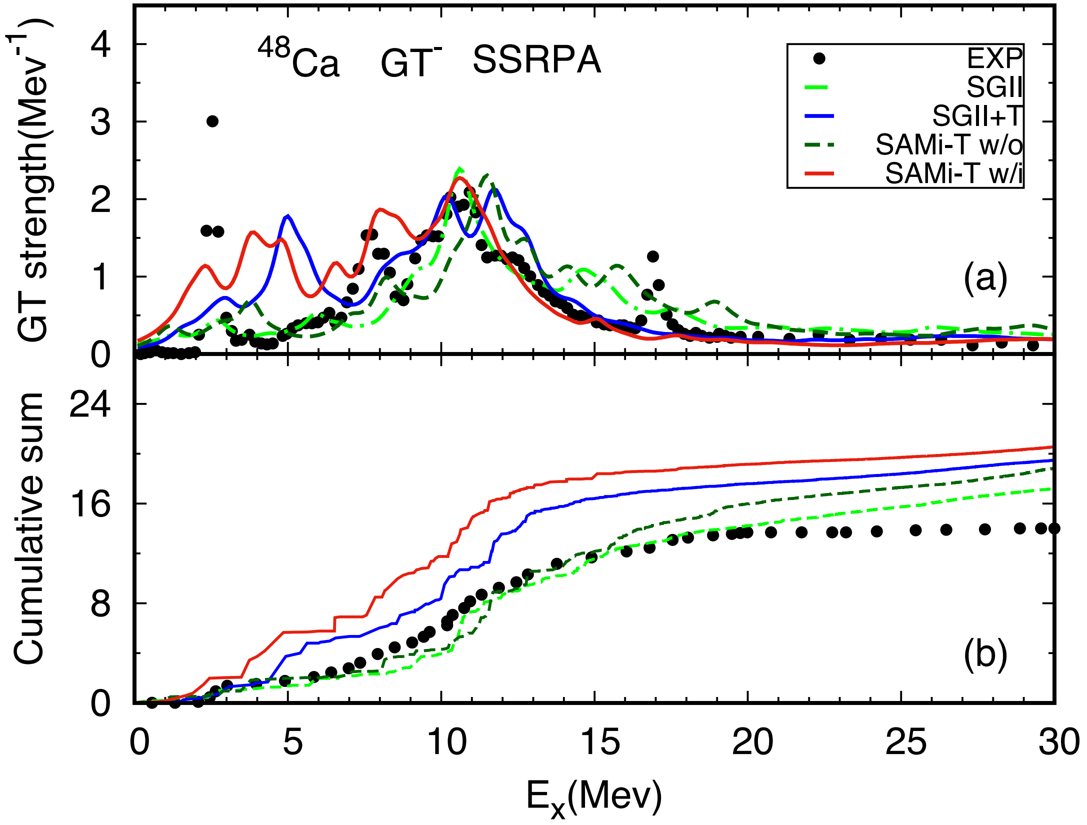
48Ca SSRPA
EXP
SGII
SGII+Te1
SGII+Te2
SGII+Te3
90Zr
132Sn
208Pb
GT strength(MeV-1)
Cumulative sum
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
Ex(MeV)
Ex(MeV)
Ex(MeV)
Ex(MeV)

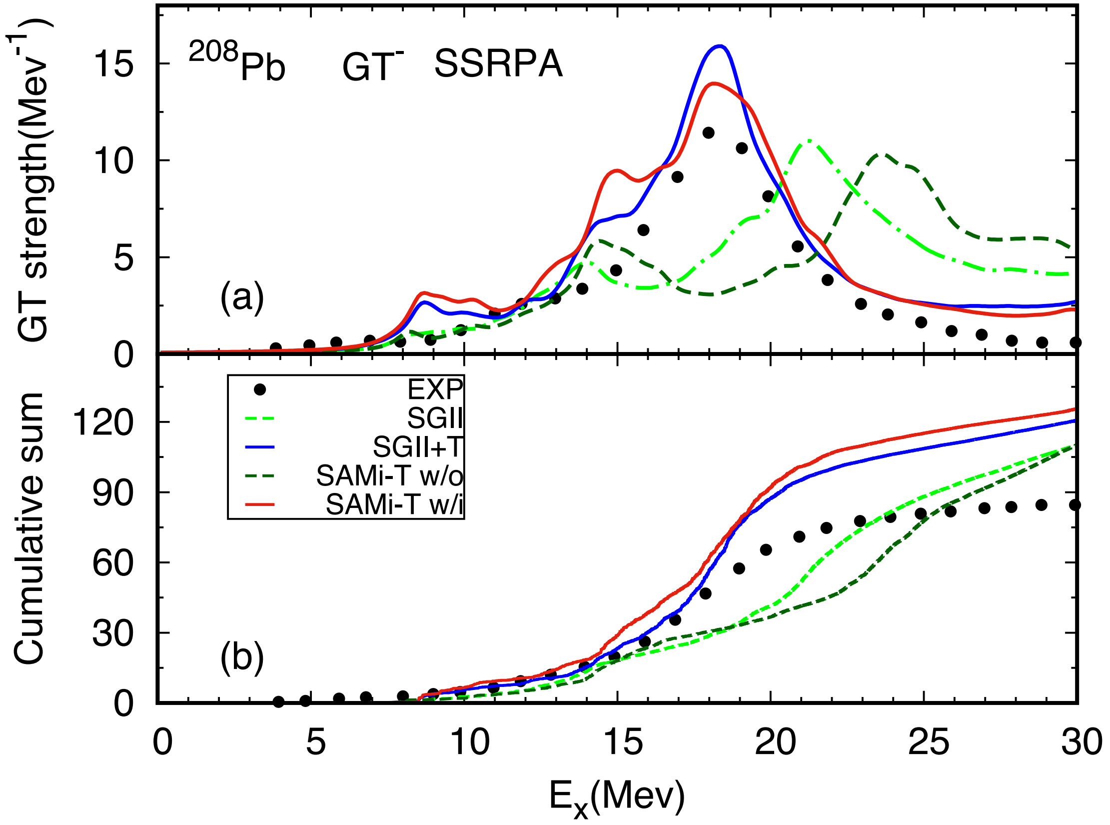


48Ca SSRPA
90Zr
132Sn
208Pb
EXP
T21
T44
T55
GT strength(MeV-1)
Cumulative sum
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
Ex(MeV)
Ex(MeV)
Ex(MeV)
Ex(MeV)



^{48}Ca GT$^-$ RPA
(a)
SSRPA
(b)
EXP
SGII
SGII+T
SAMi-T/o
SAMi-T/i
GT strength(Mev^{-1})
E$_x$(Mev)
^{208}Pb GT$^-$ RPA
(a)
SSRPA
(b)



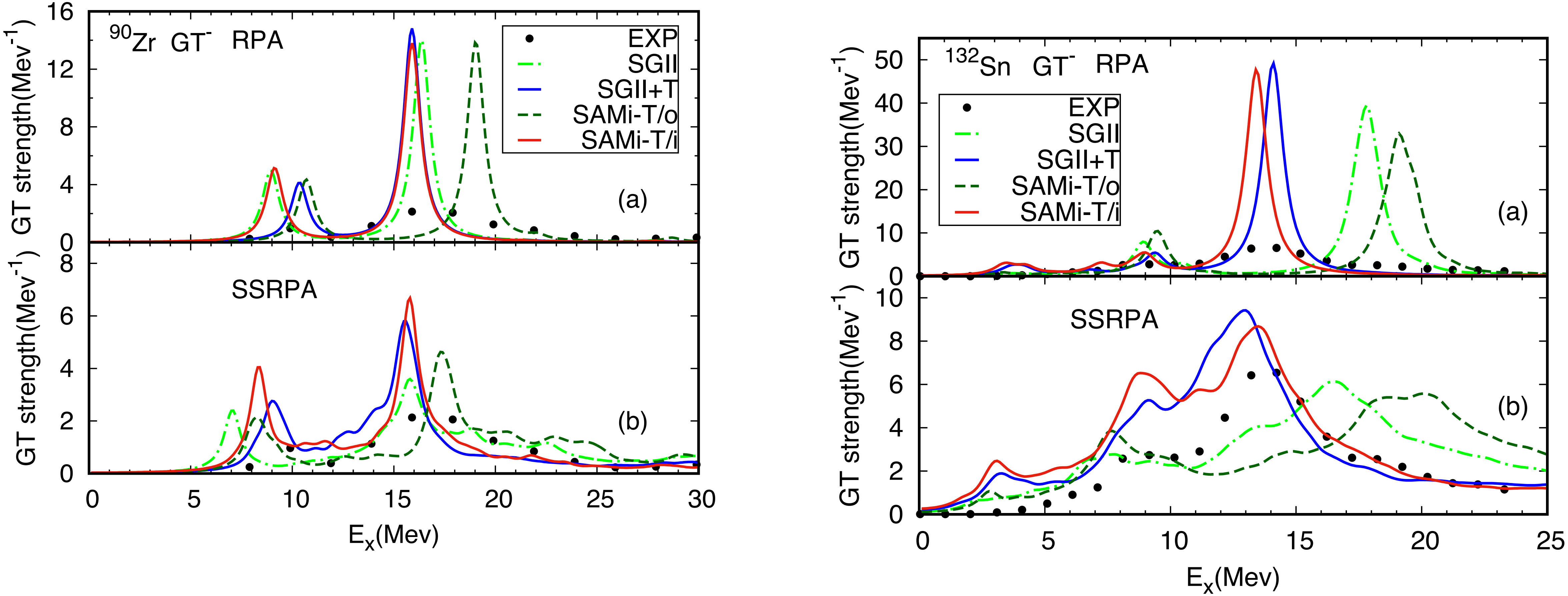


Ex<20 MeV

Quenching 35% SGII or SAMi-T no tensor
20% with Tensor terms

Ex<25 MeV

Quenching 30% SGII or SAMi-T no tensor
20% tensor terms



^{90}Zr GT$^-$ RPA
EXP
SGII
SGII+T
SAMi-T/o
SAMi-T/i
(a)
SSRPA
(b)
GT strength(Mev^{-1})
E$_x$(Mev)
^{132}Sn GT$^-$ RPA

Summary

SS RPA(subtracted second RPA) model is applied to describe collective states of medium-heavy and heavy nuclei.

Low-monopole states are affected by the tensor force and get a better agreement with experimental data.

Gamow-Teller states of ^{90}Zr and ^{208}Pb are also studied by SSRPA and 2p-2states make a larger spreading width on top of the proper excitation energies compared with experimental ones.

Quenching: ^{48}Ca 20-35% Ex<20 MeV
^{208}Pb 20-30% Ex<25 MeV

Future perspectives

Ab initio EDF to apply SSRPA