Resonance states in light nuclei studied by analytic continuation in the coupling constant

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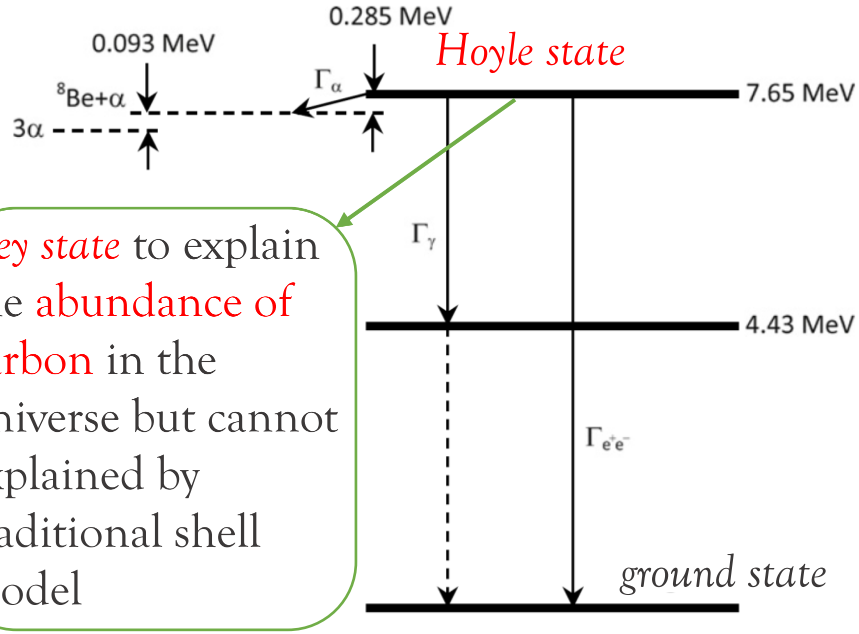
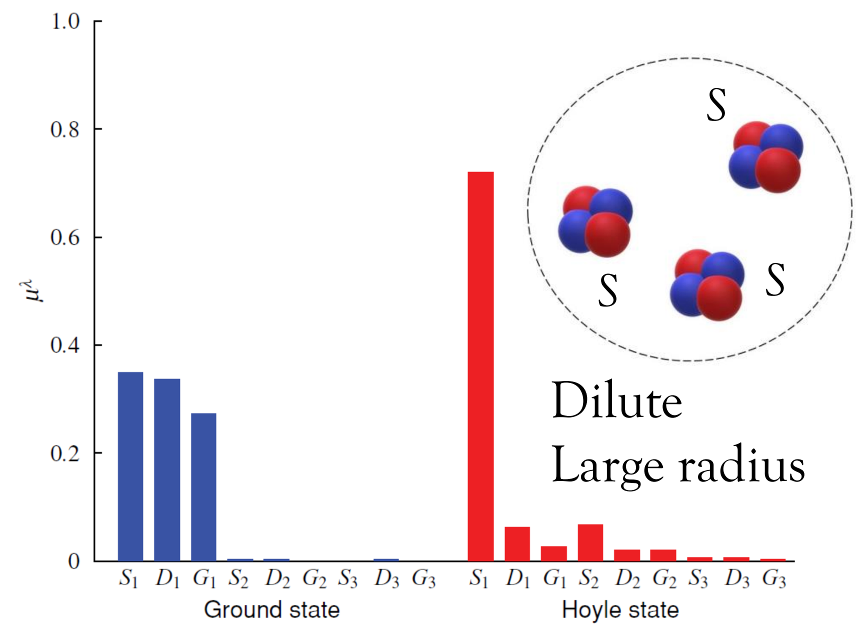
Hoyle state

- The second excited (0_2^+) state of ^{12}C at E = 7.65 MeV is one of the most famous α-clustered system referred to as the Hoyle state.
- It was found at the beginning of the 2000s that the Hoyle state could be Bose-Einstein condensate of α particles occupying the same S-state.

A. Tohsaki *et al.*, Phys. Rev. Lett. **87**, 192501 (2001)
M. Freer *et al.*, Prog Part Nucl Phys. **78**, 1-23 (2014)



Key state to explain the abundance of carbon in the universe but cannot explained by traditional shell model



Hoyle-analog state in ^{13}C

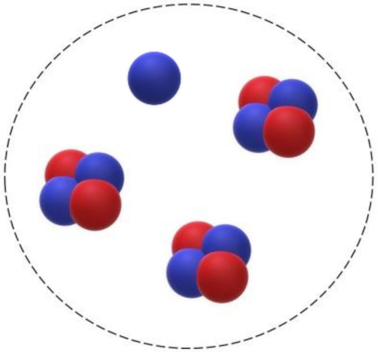
- As a simple extension of the Hoyle state, we consider that ^{13}C is one promising candidate of the $3\alpha + n$ condensation.

- It would show the condensation characteristics of the Hoyle state with a valence neutron as a fermion impurity, which is called Hoyle-analog state.

- We will search for the Hoyle-analog state in ^{13}C, where it is expected with the lowest orbits of the valence neutron:

$$\text{Hoyle state } (0^+) \otimes \begin{cases} s\text{-wave neutron} \rightarrow 1/2^+ \text{ state} \\ p\text{-wave neutron} \rightarrow 1/2^-, 3/2^- \text{ state} \end{cases}$$

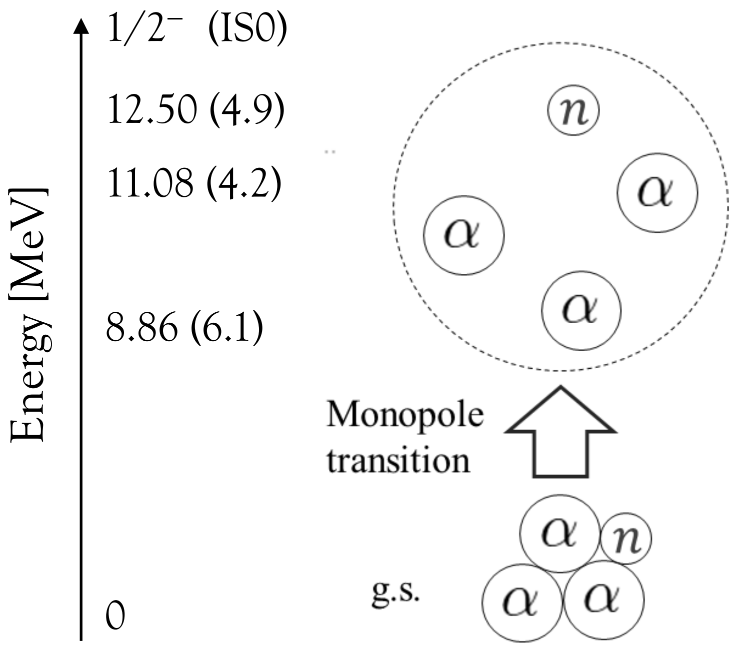
Candidate states of ^{13}C that Hoyle-analog state can be found



Experimental implication

- A decade ago, Kawabata *et al.* performed an α scattering experiment to measure the isoscalar monopole excitations to confirm the Hoyle-analog state in ^{13}C. T. Kawabata *et al.*, Int. J. Mod. Phys. E **17**, 2071 (2008)

$\rightarrow$ several excited $1/2^-$ states have the $3\alpha + n$ cluster configuration. Thus, we focus on these states.



> Isoscalar monopole transition

$$M(\mathrm{IS0}) = \langle 1/2_n^- \mid \sum_{i=1}^{A} r_i^2 \mid 1/2_{\mathrm{g.s.}}^- \rangle$$

- Single particle excitation does not show the large IS0 transition.
- Large value: spatially distributed α particles $\rightarrow$ large radius with dilute signature

Real-time evolution method

- In order to generate the configurations of resonance, we introduce the real-time evolution method (REM), which uses the equation-of-motion.

$$i\hbar \sum_{j=1}^{N} \sum_{\sigma=x,y,z} C_{i\rho j\sigma} \frac{dZ_{j\sigma}}{dt} = \frac{\partial \mathcal{H}_{int}}{\partial Z^*_{i\rho}}$$

R. Imai ***et al.***, PRC 99, 064327 (2019)

- Brink-Bloch wave function

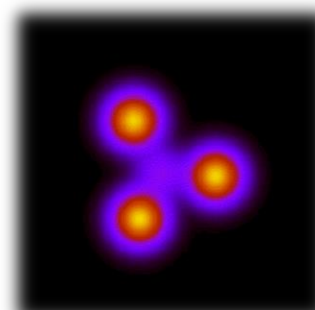
$$\Phi(\boldsymbol{Z}_1, ..., \boldsymbol{Z}_N) = \mathcal{A}[\Phi_\alpha(\boldsymbol{Z}_1) \cdots \Phi_\nu(\boldsymbol{Z}_N)]$$

$$\Phi_\alpha(\boldsymbol{Z}) = \mathcal{A}[\varphi(\boldsymbol{r}_1, \boldsymbol{Z})\chi_{p\uparrow} \cdots \varphi(\boldsymbol{r}_4, \boldsymbol{Z})\chi_{n\downarrow}]$$

$$\Phi_\nu(\boldsymbol{Z}) = \varphi(\boldsymbol{r}, \boldsymbol{Z})\chi_{\nu\tau}$$

$$\varphi(\boldsymbol{r}, \boldsymbol{Z}) = \left(\frac{2\nu}{\pi}\right)^{3/4} \exp\left\{-\nu\left(\boldsymbol{r} - \boldsymbol{Z}\right)^2\right\}$$

^{13}C (3α + n)



$$\Phi(\mathbf{Z}_1, ..., \mathbf{Z}_4) \rightarrow \Phi(\mathbf{Z}_1(t), ..., \mathbf{Z}_4(t))$$

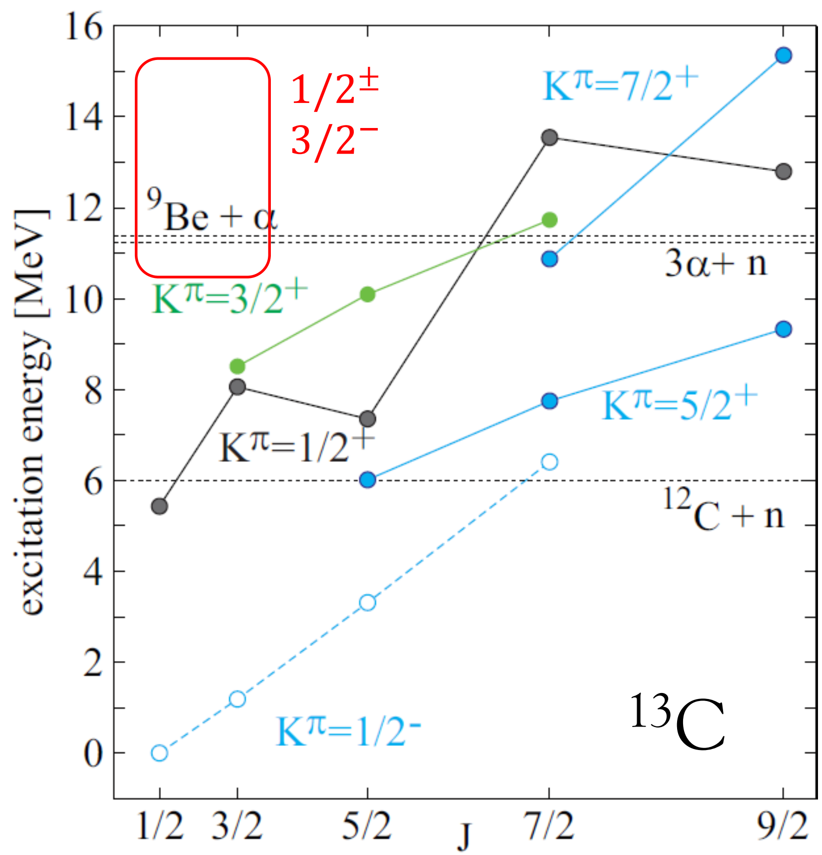
$$\Psi^{J\pi}_M = \int_0^{T_{\max}} dt \sum_{K=-J}^{J} \hat{P}^{J\pi}_{MK} f_K(t) \Phi(\boldsymbol{Z}_1(t), ..., \boldsymbol{Z}_4(t))$$

Resonance states

- Resonance states above thresholds including the Hoyle-analog state are difficult to identify from the continuum.

- We studied the $3\alpha + n$ cluster structures in ^{13}C, but we could not investigate the possible Hoyle-analog state.

- Thus, we introduced the analytic continuation in the coupling constant (ACCC) to identify the resonance states.



S. Shin, B. Zhou, M. Kimura, Phys. Rev. C 103, 054313 (2021)

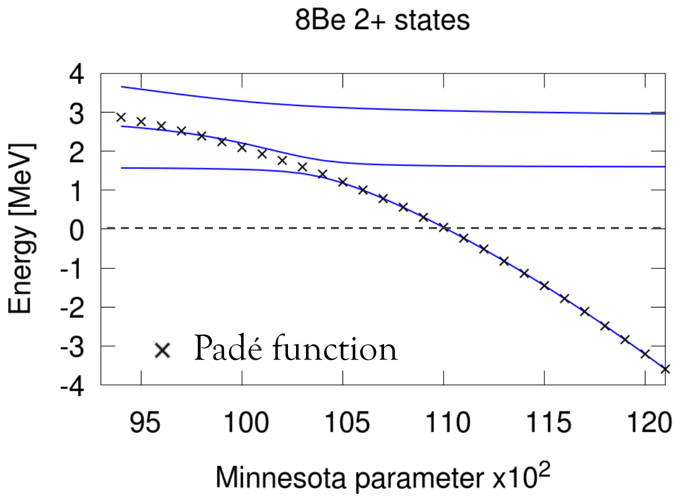
ACCC

- ACCC extrapolates the resonance states from their bound states with an artificial attractive potential. V. I. Kukulin et al., Theory of Resonances (1989)

$$H_{\mathrm{ACCC}} = H_{\mathrm{original}} + \lambda H_{\mathrm{attractive}}$$

$$k_l(x) = i\,\frac{c_0 + c_1 x + c_2 x^2 + \cdots + c_M x^M}{1 + d_1 x + d_2 x^2 + \cdots + d_N x^N}$$

$$E_R = \frac{\hbar^2}{2m}(k_R^2 - k_I^2), \quad \Gamma = \frac{2\hbar^2}{m} k_R k_I > 0$$

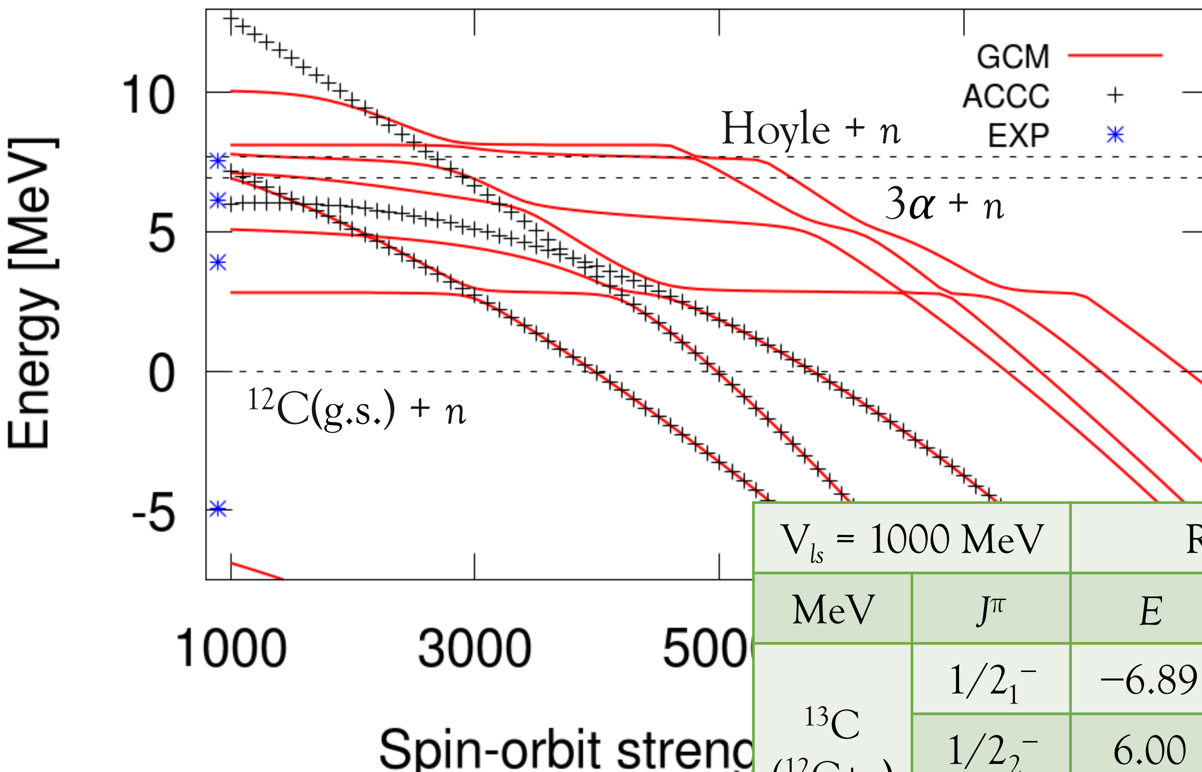


	J^π	REM E	REM Γ	Tanaka et al. E	Tanaka et al. Γ
^{8}Be	0^+	0.224	0.001	0.208	0.003
	2^+	2.87	1.42	2.85	1.44
	4^+	11.77	4.82		
^{5}He	$3/2^-$	0.78	0.66	0.77	0.64
	$1/2^-$	1.98	5.62	1.98	5.4
	$1/2^+$	12.7	163	12	180

N. Tanaka et al., PRC 59, 1319 (1999)

- NN and spin-orbit interactions are well described within ACCC.

$1/2^-$ states of ^{13}C

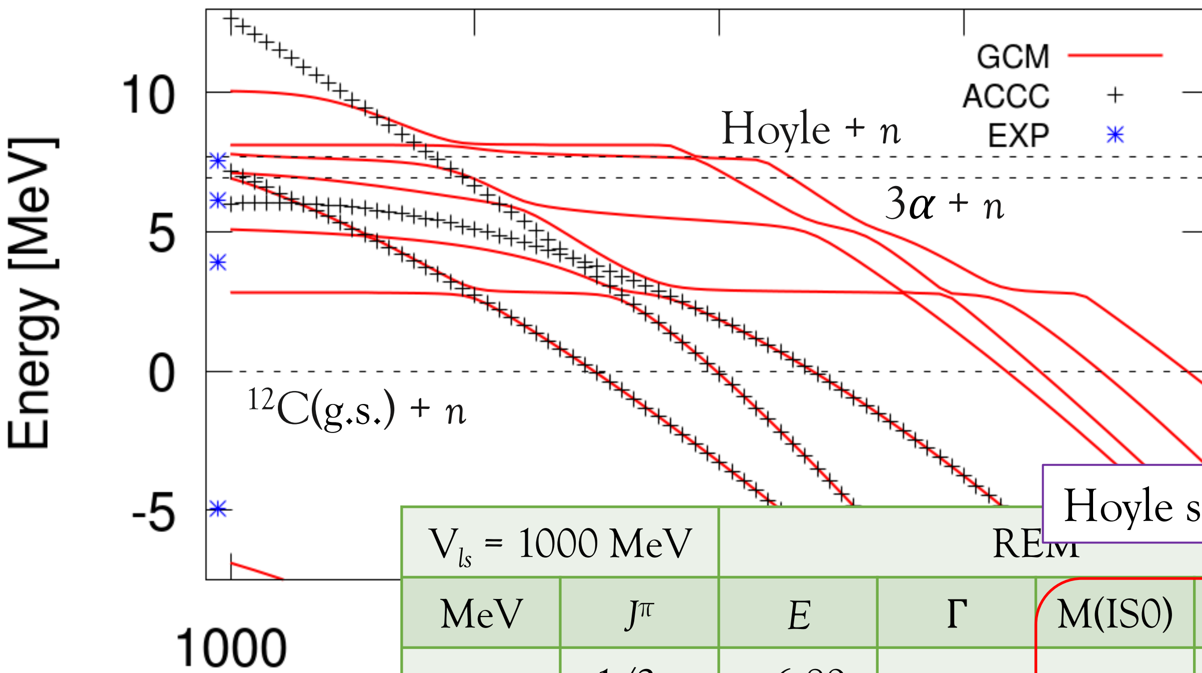


Volkov No.2
Majorana para. = 0.592
Hoyle state 7.68 MeV (7.65 exp)

Want to search for the Hoyle-analog state

V_{ls} = 1000 MeV		REM		EXP	
MeV	J^π	E	Γ	E	Γ
^{13}C ($^{12}C+n$)	$1/2_1^-$	−6.89	-	−4.95	-
	$1/2_2^-$	6.00	1.26	3.91	0.15
	$1/2_3^-$	7.17	0.57	6.13	<0.004

1/2$^-$ states of ^{13}C



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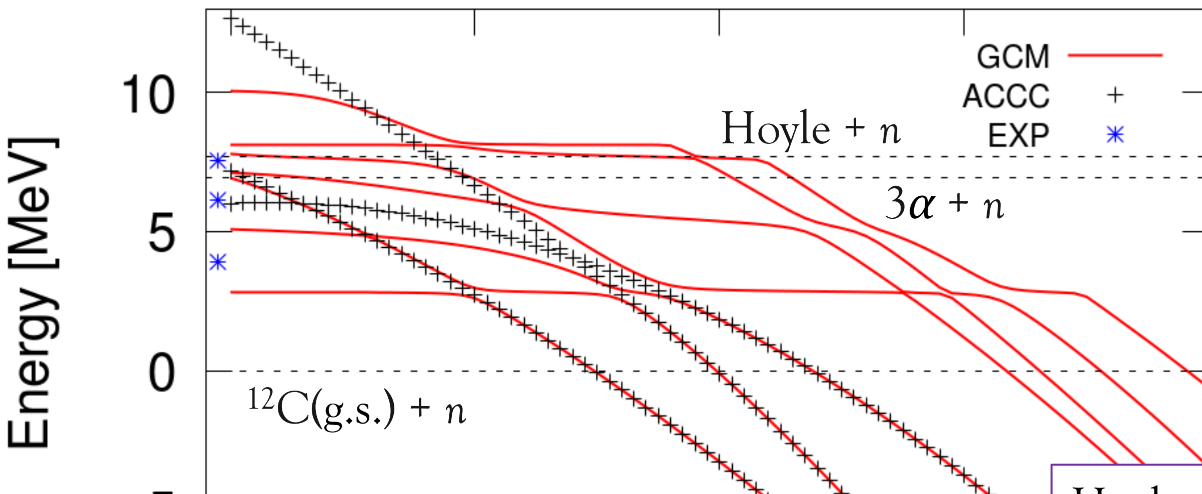
Want to search for the Hoyle-analog state

Hoyle state (IS0; $0^+_{\text{g.s.}} \to 0^+_2$) : 5.4 fm^2

V_{ls} = 1000 MeV		REM					EXP		
MeV	J^π	E	Γ	M(IS0)	M(E0)	E	Γ	M(IS0)
^{13}C (^{12}C+n)	$1/2_1^-$	−6.89	-	-	-	−4.95	-	-
	$1/2_2^-$	6.00	1.26	11.58	4.98	3.91	0.15	6.1
	$1/2_3^-$	7.17	0.57	7.18	3.61	6.13	<0.004	4.2

$$(\Phi|\,\hat{O}\,|\psi\rangle = \underset{k\to k_R}{\text{Cont}} \int_0^\infty \Phi(k,r)\hat{O}\psi(r)dr$$

$1/2^-$ states of ^{13}C



Volkov No.2

Majorana para. = 0.592

Hoyle state 7.68 MeV (7.65 exp)

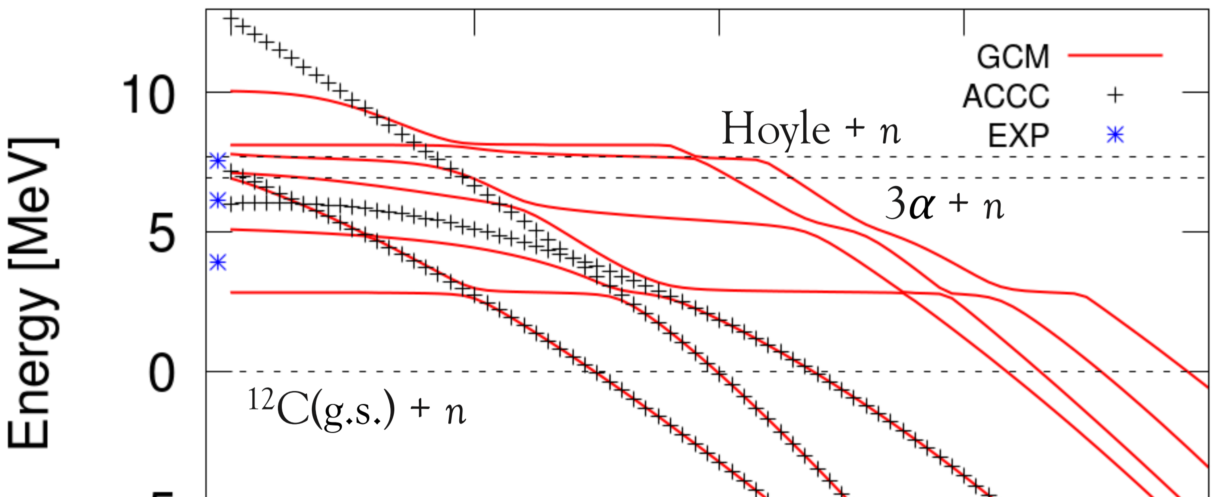
Want to search for the Hoyle-analog state

Hoyle state : ≥ 3.4 fm

V_{ls} = 1000 MeV		REM						EXP		
MeV	J^π	E	Γ	M(IS0)	M(E0)	R_m	R_p	E	Γ	M(IS0)
^{13}C (^{12}C+n)	$1/2_1^-$	−6.89	-	-	-	2.42	2.37	−4.95	-	-
	$1/2_2^-$	6.00	1.26	11.58	4.98	2.78	2.73	3.91	0.15	6.1
	$1/2_3^-$	7.17	0.57	7.18	3.61	2.65	2.62	6.13	<0.004	4.2

$$(\Phi|\,\hat{O}\,|\psi\rangle = \underset{k\to k_R}{\mathrm{Cont}} \int_0^\infty \Phi(k,r)\hat{O}\psi(r)dr$$

$1/2^-$ states of ^{13}C



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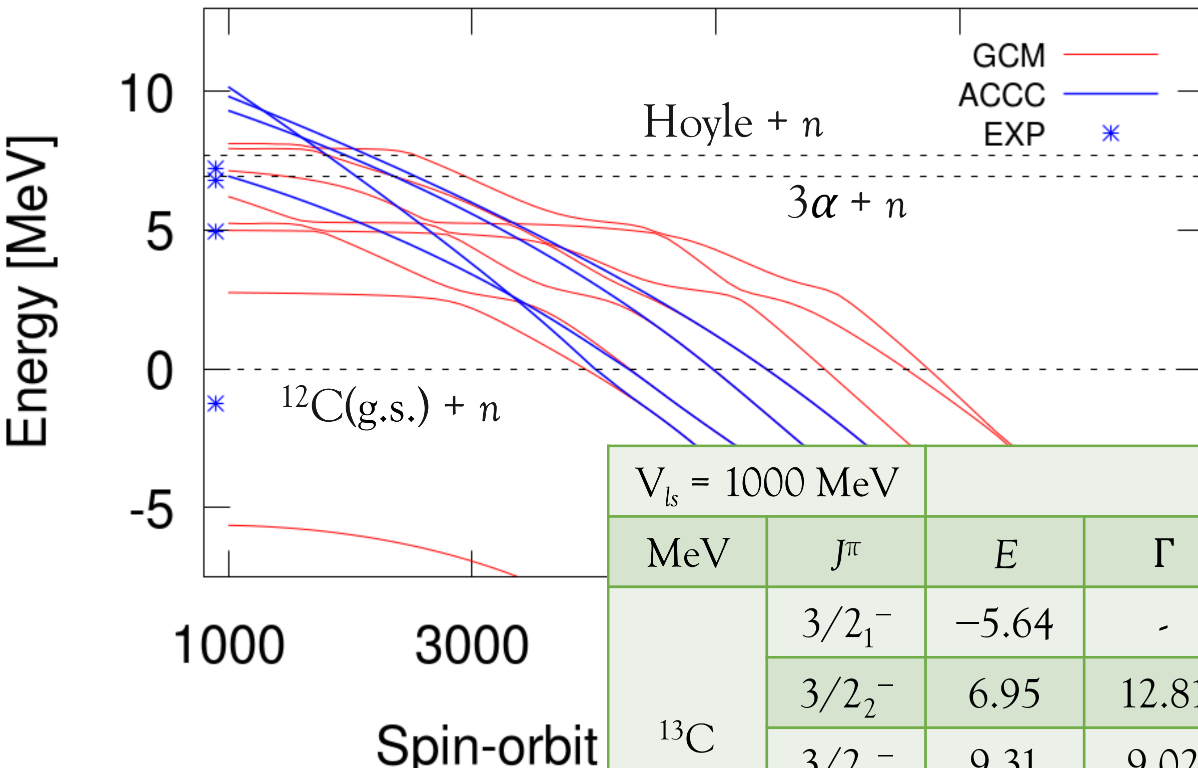
V_{ls} = 1000 MeV		REM						EXP		
MeV	J^π	E	Γ	M(IS0)	M(E0)	R_m	R_p	E	Γ	M(IS0)
^{13}C (^{12}C+n)	$1/2_1^-$	−6.89	-	-	-	2.42	2.37	−4.95	-	-
	$1/2_2^-$	6.00	1.26	11.58	4.98	2.78	2.73	3.91	0.15	6.1
	$1/2_3^-$	7.17	0.57	7.18	3.61	2.65	2.62	6.13	<0.004	4.2

We propose as the candidate of the Hoyle-analog state

3/2$^-$ states of ^{13}C



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Want to search for the Hoyle-analog state

V_{ls} = 1000 MeV		REM				EXP
MeV	J^π	E	Γ	R_m	R_p	E
^{13}C (^{12}C+n)	$3/2_1^-$	−5.64	-	2.45	2.40	−1.26
	$3/2_2^-$	6.95	12.81	2.64	2.59	4.95
	$3/2_3^-$	9.31	9.02	2.67	2.64	6.80
	$3/2_4^-$	9.82	3.65	2.80	2.75	7.24
	$3/2_5^-$	10.17	4.09	2.72	2.68	-

⇨ need further investigation

Summary

- We have applied REM + ACCC framework to investigate resonances in light nuclei.
- The resonances in ^{8}Be and ^{5}He were well reproduced compared with the previous ACCC calculation.
- Its application to ^{13}C with strengthening the spin-orbit strength identified the resonance states in ^{13}C.
- We suggest the $1/2_2^-$ state could be the Hoyle-analog state.