Shape coexistence in *N*=28 neutron-rich nuclei

arXiv:2201.05731, PTEP in print

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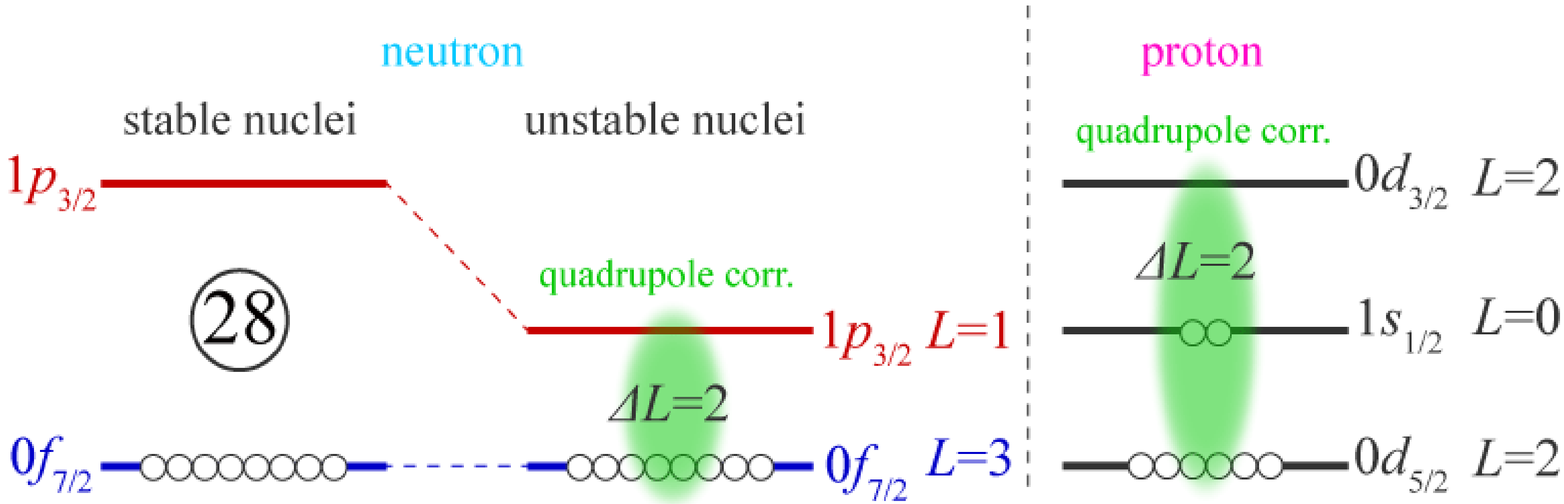
Introduction: Erosion of N=28 Shell Closure



○ N=28 shell quenching causes degeneracy of p and f wave
⇒ Neutron excitations induces strong quadrupole correlations

○ Mg, Si and S (Z=12, 14 and 16) are mid sd-shell (proton) nuclei
⇒ Protons also have strong quadrupole correlations

Various nuclear shapes will appear and coexist!

Abstract of this talk

We investigate the shapes of the low-lying states of the N=28 nuclei (^{40}Mg, ^{42}Si, ^{44}S) using the antisymmetrized molecular dynamics (AMD)

"What types of shapes coexist?"

⇒ Completely different shape coexistence occurs depending on the nuclide (proton number)

"What observables does this difference affect?"

⇒ Monopole transition between 0^+ states

Framework: Antisymmetrized Molecular Dynamics (AMD)

Hamiltonian: Gogny D1S density functional

J.F. Berger *et al*. CPC63, 365 (1991).

$$\hat{H} = \sum_i^A \hat{t}_i - \hat{t}_{\rm cm} + \frac{1}{2}\sum_{ij\in{\rm proton}}^Z \hat{v}_{ij}^{\rm C} + \frac{1}{2}\sum_{ij}^A \hat{v}_{ij}^{\rm NN}$$

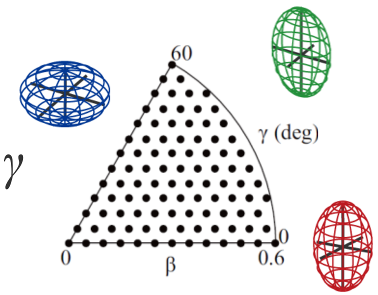
Model wave function: antisymmetrized product of nucleon Gaussian wave packets

$$\Phi^\pi = \hat{P}^\pi \mathcal{A}\{\varphi_1, \varphi_2, \dots, \varphi_A\} \qquad \varphi_i(\vec{r}) = \exp\{-\sum_{\sigma=x,y,z} \nu_\sigma (r_\sigma - Z_{i\sigma})^2\}(a_i\chi_\uparrow + b_i\chi_\downarrow)\tau_i$$

Variational calculation

Minimize the sum of expectation energy and constraint potential putting constraint on the quadrupole deformation parameters β and γ

$$\widetilde{E}^\pi(\beta,\gamma) = \langle\Phi^\pi(\beta,\gamma)|\hat{H}|\Phi^\pi(\beta,\gamma)\rangle + v_\beta(\langle\beta\rangle - \beta)^2 + v_\gamma(\langle\gamma\rangle - \gamma)^2$$



Generator coordinate method (GCM)

The wave functions with various shapes are superposed
The amplitude for each shape is determined by the diagonalization of Hamiltonian

$$\Psi^{J\pi}_{M\alpha} = \sum_{iK} g_{iK\alpha} P^J_{MK} \Phi^\pi(\beta_i, \gamma_i)$$

Shape of ^{40}Mg, ^{42}Si and ^{44}S (ground state)

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^{40}Mg (Z=12, N=28)
Prolate rigid rotor

^{42}Si (Z=14, N=28)
Oblate rigid rotor

^{44}S (Z=16, N=28)
Deformed,
but no rigid shape

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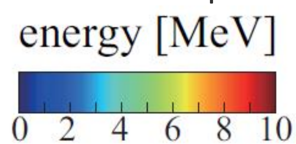
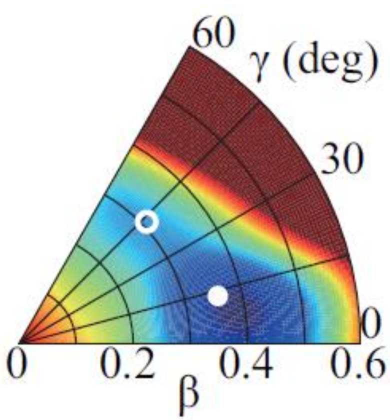
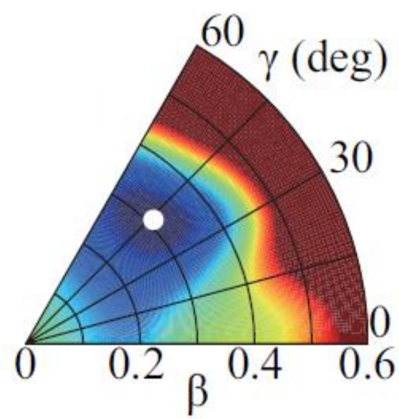
○Energy surface: energy as a function of deformation parameters β and γ

$$\langle \Phi^{J\pi}_{MK}(\beta,\gamma)|\hat{H}|\Phi^{J\pi}_{MK}(\beta,\gamma)\rangle$$

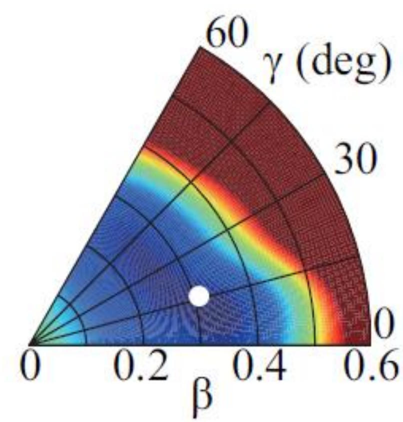




Minimum at
prolate deformation



Minimum at
oblate deformation



γ-soft at
large β deformation

Shape of ^{40}Mg, ^{42}Si and ^{44}S (ground state)

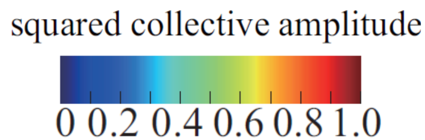
^{40}Mg (Z=12, N=28)
Prolate rigid rotor

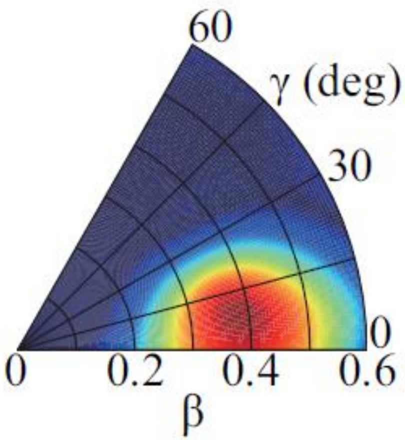
^{42}Si (Z=14, N=28)
Oblate rigid rotor

^{44}S (Z=16, N=28)
Deformed,
but **no rigid shape**

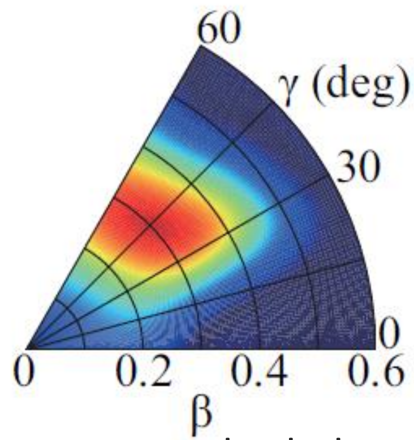
○Collective amplitude: probability to observe a definite nuclear shape

$$\langle \Phi^{J\pi}_{MK}(\beta,\gamma) | \Psi^{J\pi}_{M\alpha} \rangle$$

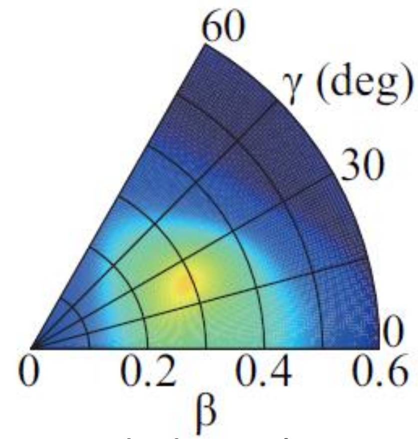




Large probability of prolate deformation



Large probability of oblate deformation



Broad distribution
⇒ No rigid shape,
Fluctuating γ def.

Shape Coexistence in ^{40}Mg, ^{42}Si and ^{44}S

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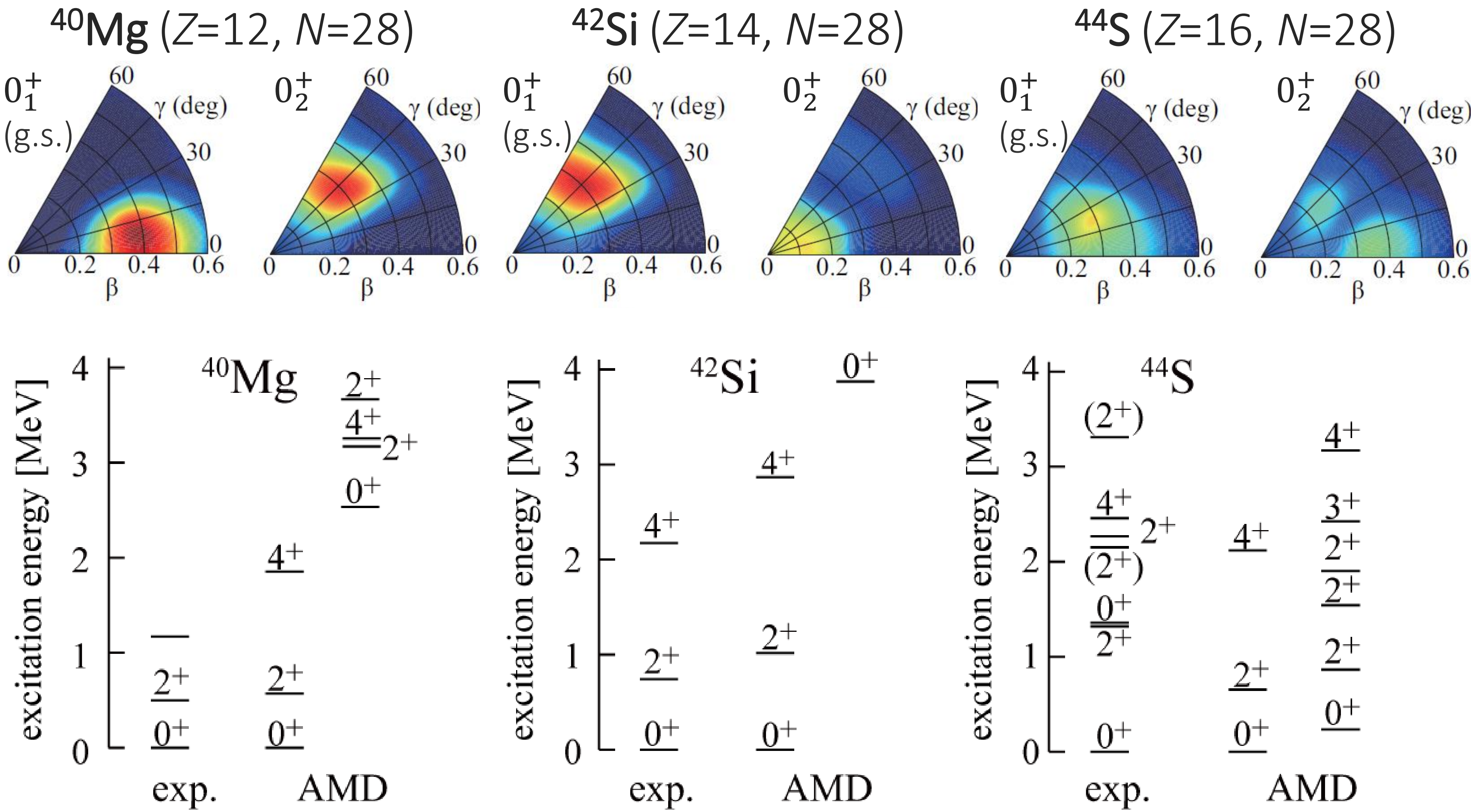
Collective amp. suggests completely different shape coexistence!



H.L. Crawford *et al.* PRL122, 52501 (2019), S. Takeuchi *et al*. PRL109, 182501 (2012), D. Sohler *et al*. PRC66, 054302(2002).

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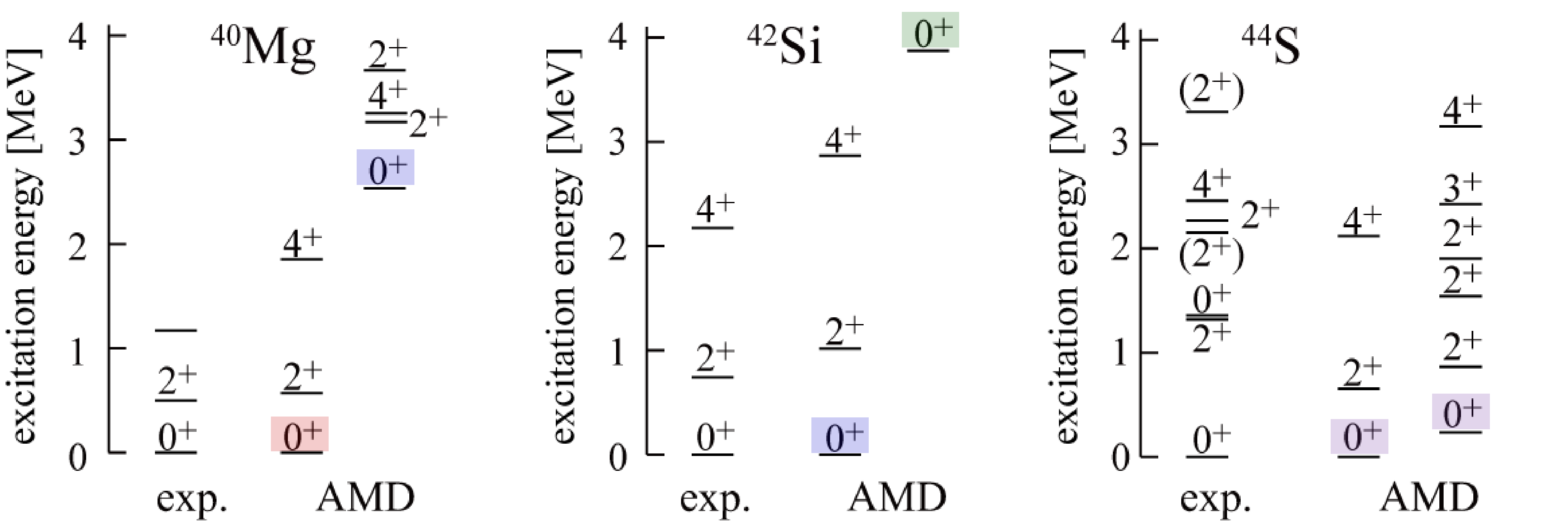
^{40}Mg (Z=12, N=28)	^{42}Si (Z=14, N=28)	^{44}S (Z=16, N=28)
0_1^+ state : prolate	0_1^+ state : oblate	0_1^+ state : fluctuation
0_2^+ state : oblate	0_2^+ state : spherical	0_2^+ state : fluctuation



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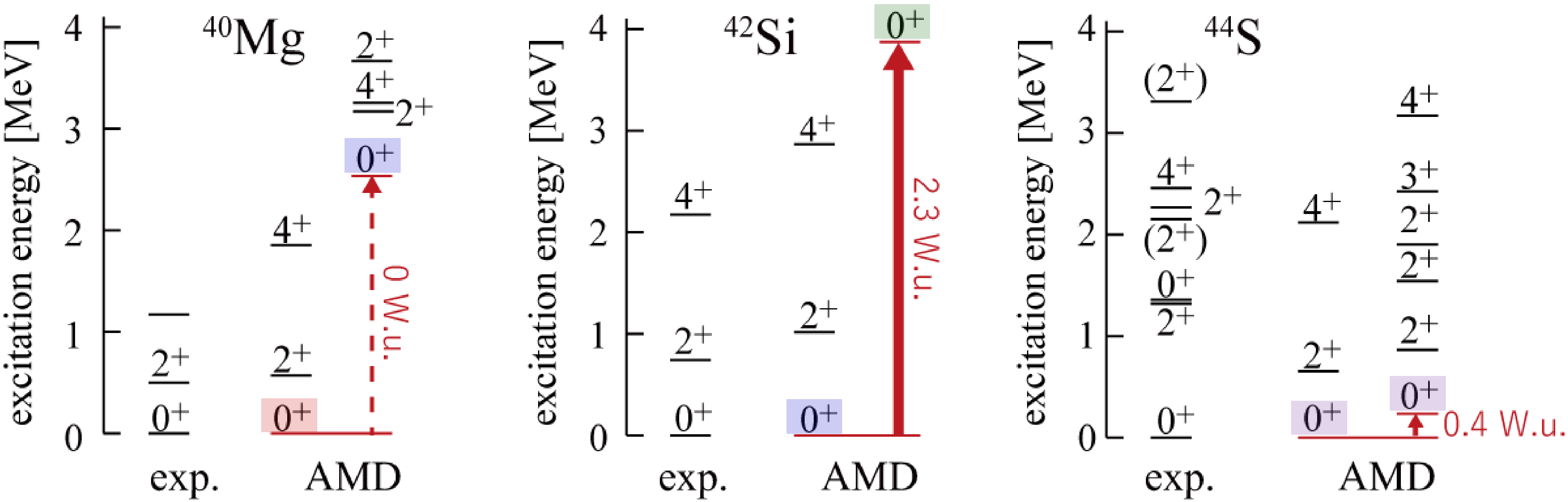
Shape Coexistence in ^{40}Mg, ^{42}Si and ^{44}S

What observables does the different aspects of shape coexistence affect?

⇒ Monopole transition strengths between 0^+ states!

Very weak transition | **Very strong** transition | **Strong** transition



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Shape Coexistence and Monopole Transition

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J.L. Wood *et al.* NPA651, 323 (1999)

Suppose that there are two state vectors $|A\rangle$ and $|B\rangle$ with different nuclear shapes, and the 0_1^+ and 0_2^+ states are described by their linear combinations

$$|0_1^+\rangle = a|A\rangle + b|B\rangle, \quad |0_2^+\rangle = b|A\rangle - a|B\rangle.$$

The operator of the isoscalar monopole transition corresponds to the squared radius

$$\mathcal{M}(\mathrm{IS0}) = \sum_i^A \hat{r}_i^2.$$

$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle = ab(\langle A|\mathcal{M}|A\rangle - \langle B|\mathcal{M}|B\rangle) + (b^2 - a^2)\langle A|\mathcal{M}|B\rangle.$$

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- ○ 1st term becomes large when the radii of two shapes $|A\rangle$ and $|B\rangle$ are different and the mixing is strong ($a \approx b \approx 1/\sqrt{2}$)
- ○ 2nd term vanishes when the particle-hole configurations of $|A\rangle$ and $|B\rangle$ differ by 2p2h or more than that because M is a one-body operator

The strength of the monopole transition varies depending on **the degree of mixing** and **the particle-hole configuration**

Comparison of rigid rotors (^{40}Mg v.s. ^{42}Si)

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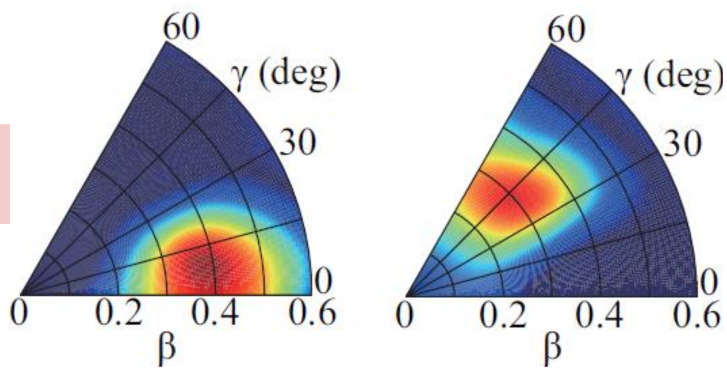
In the case of ^{40}Mg (very weak transition, $B(\mathrm{IS0}) = 0.0$ W.u.)

0_1^+ state : prolate



0_2^+ state : oblate

$$|0_1^+\rangle = a|A\rangle + b|B\rangle, \quad |0_2^+\rangle = b|A\rangle - a|B\rangle.$$

The prolately- and oblately-deformed rigid rotors coexist, and their mixing is rather small

$$|A\rangle = |\mathrm{prolate}\rangle, |B\rangle = |\mathrm{oblate}\rangle, a = 1, b = 0$$

The monopole transition matrix is given by the transition between the prolately- and oblately-deformed states

$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle = \langle \mathrm{oblate}|\mathcal{M}|\mathrm{prolate}\rangle$$

Comparison of rigid rotors (^{40}Mg v.s. ^{42}Si)

In the case of ^{40}Mg (very weak transition, $B(\mathrm{IS0}) = 0.0$ W.u.)

0_1^+ state : prolate

the s.p. configuration of
8 valence neutrons

$|\Omega = \pm 1/2\rangle^2 | \pm 3/2\rangle^2 | \pm 5/2\rangle^2 | \pm 1/2\rangle^2$

0_2^+ state : oblate

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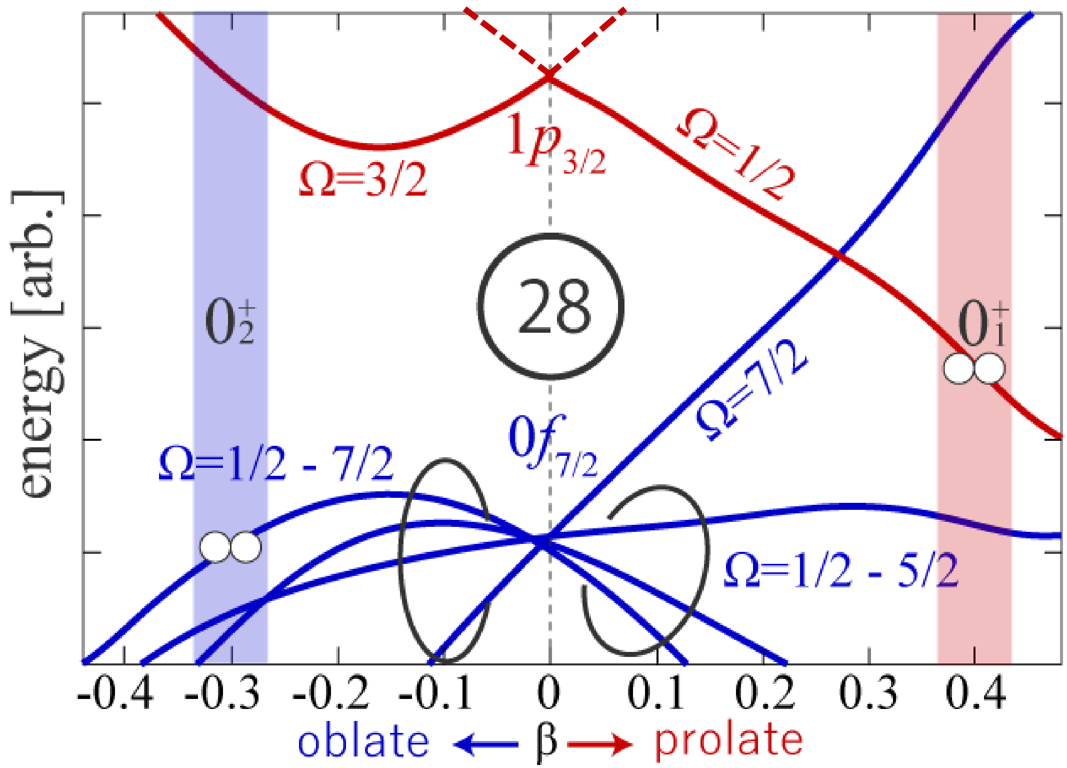
$|\Omega = \pm 1/2\rangle^2 | \pm 3/2\rangle^2 | \pm 5/2\rangle^2 | \pm 7/2\rangle^2$



Neutron configuration is different between the 0_1^+ and 0_2^+ states

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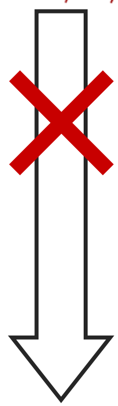

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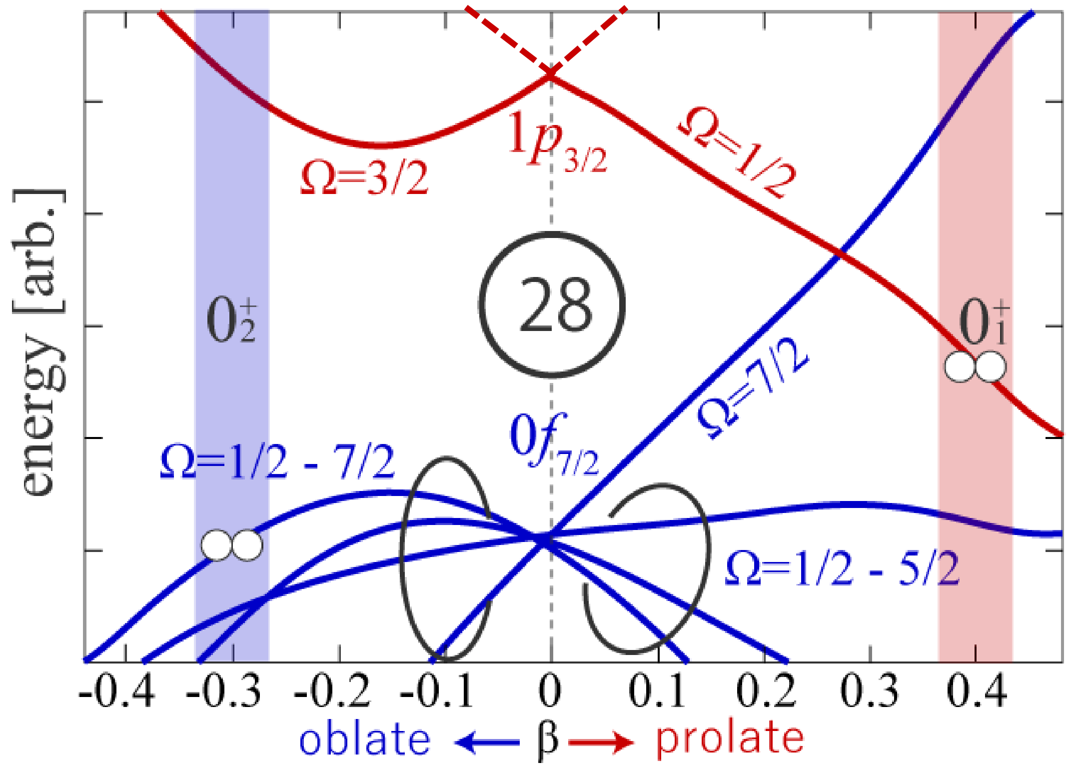


Neutron configuration is different between the 0_1^+ and 0_2^+ states

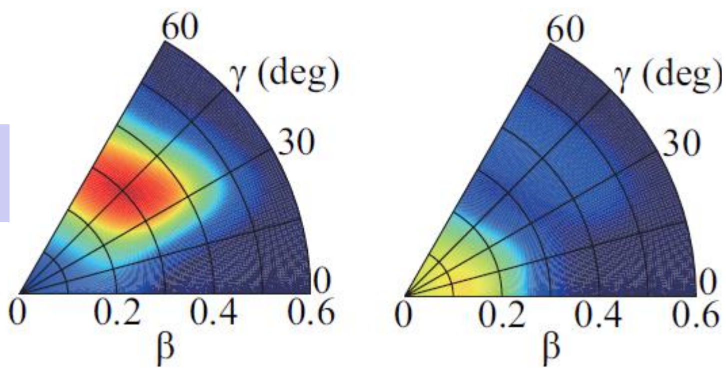
⇒ **Monopole transition is hindered and the matrix element becomes 0**

$$\langle 0_2^+ | \mathcal{M} | 0_1^+ \rangle = \langle \mathrm{oblate} | \mathcal{M} | \mathrm{prolate} \rangle = 0$$

Comparison of rigid rotors (^{40}Mg v.s. ^{42}Si)

In the case of ^{42}Si (very strong transition, $B(\mathrm{IS0}) = 2.3$ W.u.)

0_1^+ state : oblate



0_2^+ state : spherical

$$|0_1^+\rangle = a|A\rangle + b|B\rangle, \quad |0_2^+\rangle = b|A\rangle - a|B\rangle.$$

The oblately-deformed rigid rotor and spherical state coexist, and their mixing is rather small

$$|A\rangle = |\text{oblate}\rangle, |B\rangle = |\text{spherical}\rangle, a = 1, b = 0$$

The monopole transition matrix is given by the transition between the oblately-deformed state and spherical state

$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle = \langle \text{spherical}|\mathcal{M}|\text{oblate}\rangle$$

Comparison of rigid rotors (^{40}Mg v.s. ^{42}Si)

In the case of ^{42}Si (very strong transition, $B(\mathrm{IS0}) = 2.3$ W. u.)

0_1^+ state : oblate

the s.p. configuration of
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$|\Omega = \pm 1/2\rangle^2 | \pm 3/2\rangle^2 | \pm 5/2\rangle^2 | \pm 7/2\rangle^2$

0_2^+ state : spherical

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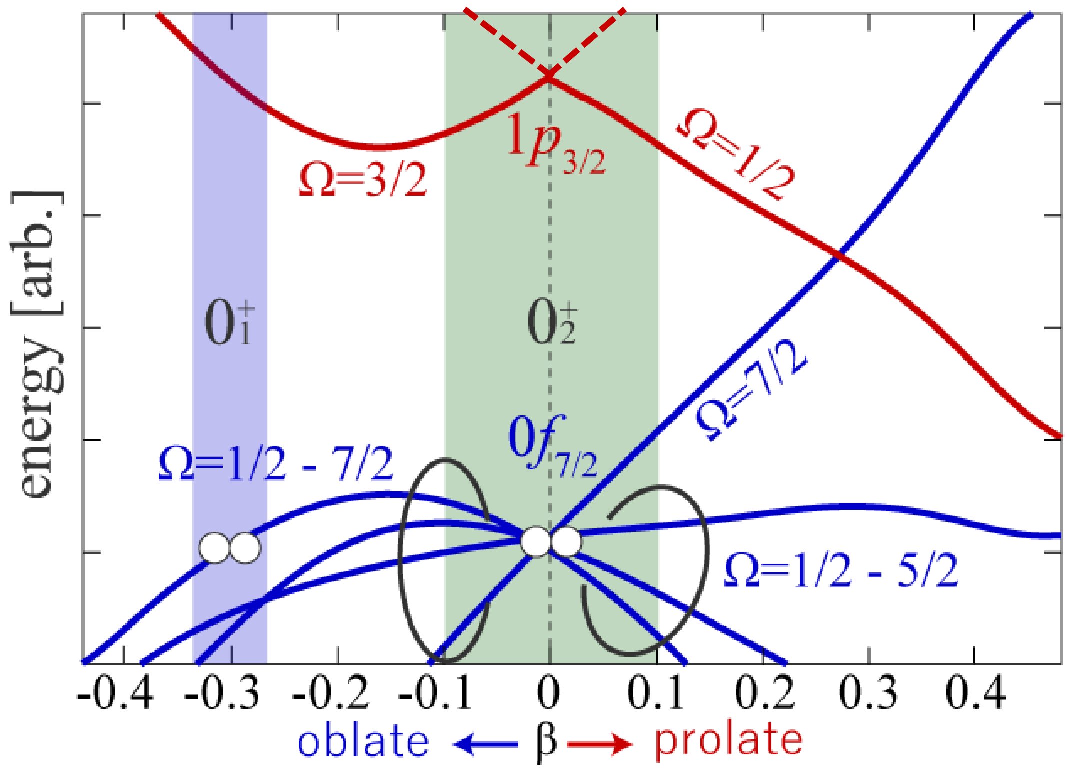

Oblate and spherical states are nonorthogonal (although they are not identical)

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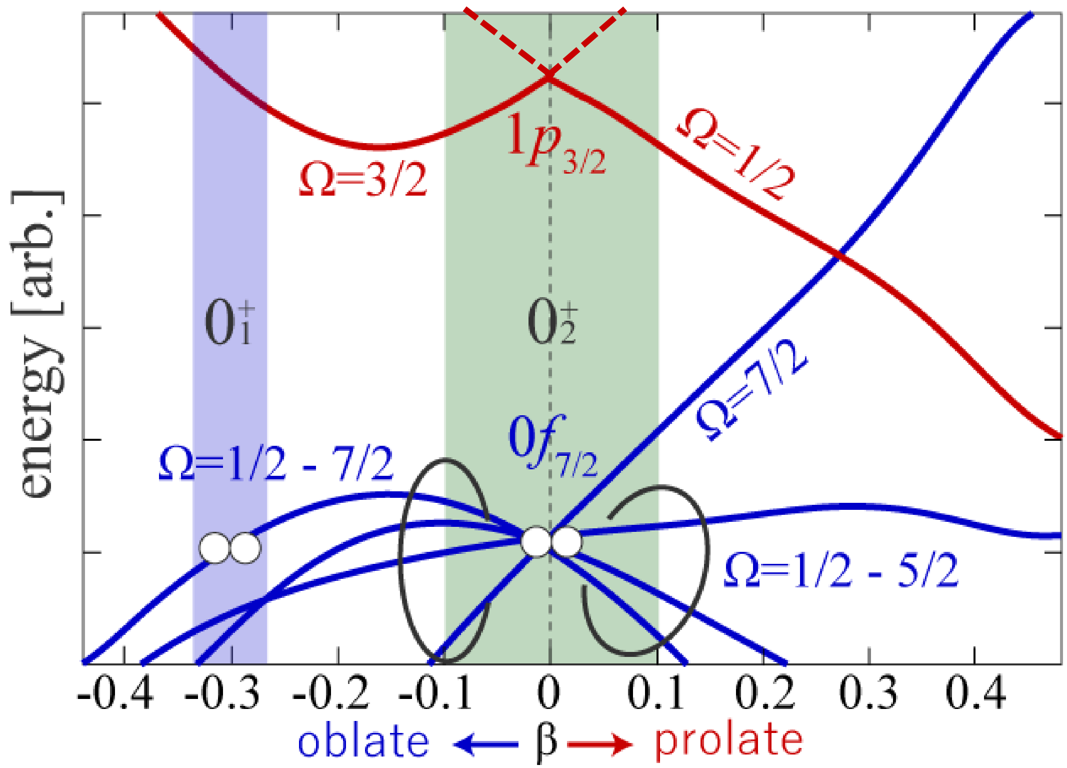


Oblate and spherical states are nonorthogonal (although they are not identical)

⇒ **The matrix element has a finite value and increases when the overlap of two configurations is large!**

$$\langle 0_2^+ | \mathcal{M} | 0_1^+ \rangle = \langle \mathrm{spherical} | \mathcal{M} | \mathrm{oblate} \rangle$$

Comparison of rigid rotors (^{40}Mg v.s. ^{42}Si)

Short Summary

^{40}Mg

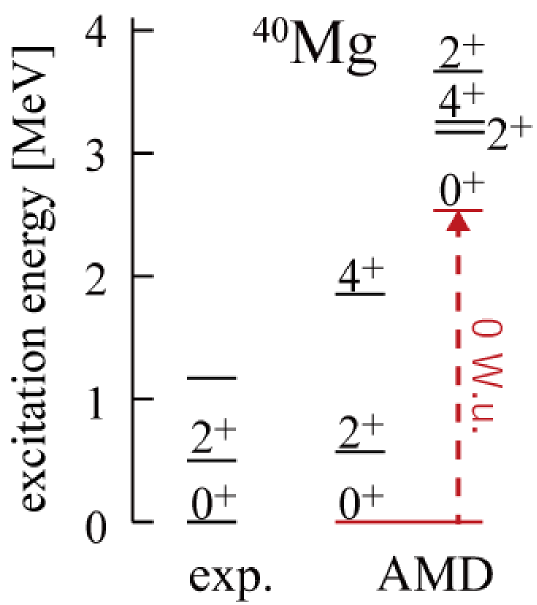
0_1^+ state : prolate

$|\Omega = \pm 1/2\rangle^2 | \pm 3/2\rangle^2 | \pm 5/2\rangle^2 | \pm 1/2\rangle^2$

0_2^+ state : oblate

$|\Omega = \pm 1/2\rangle^2 | \pm 3/2\rangle^2 | \pm 5/2\rangle^2 | \pm 7/2\rangle^2$

Transition is **hindered!**



^{42}Si

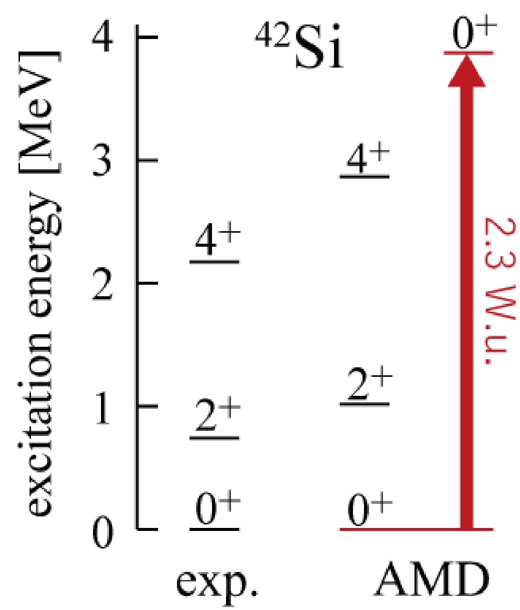
0_1^+ state : oblate

$|\Omega = \pm 1/2\rangle^2 | \pm 3/2\rangle^2 | \pm 5/2\rangle^2 | \pm 7/2\rangle^2$

0_2^+ state : spherical

$|\Omega = \pm 1/2\rangle^2 | \pm 3/2\rangle^2 | \pm 5/2\rangle^2 | \pm 7/2\rangle^2$

Transition is **allowed!**

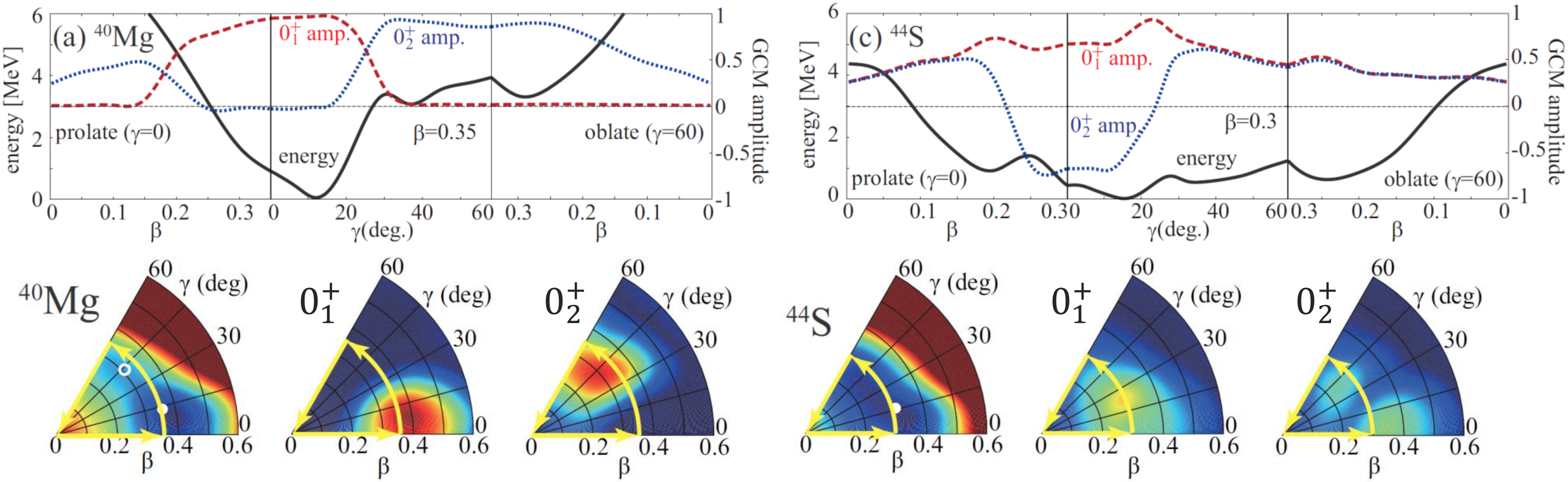


Comparison of rigid rotor and fluctuating γ deformation

Comparison of rigid rotor and fluctuating γ deformation

^{40}Mg v.s. ^{44}S

The energy and the collective amplitude (not squared)
as functions of β and γ along the fan-shaped path on the β-γ plane



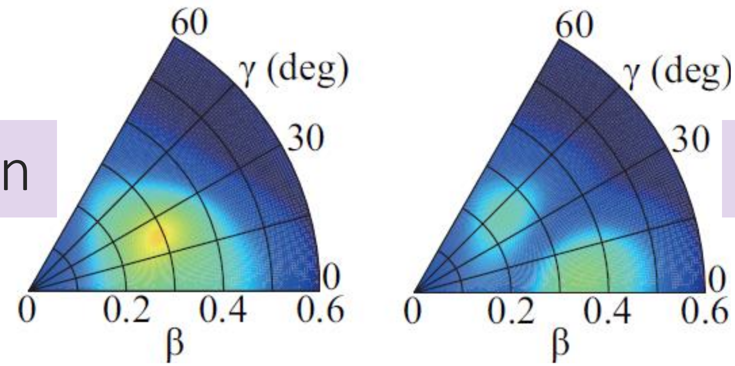
0_1^+ and 0_2^+ states have
rigid shapes with small fluctuation

- The energy changes only 1 MeV as a function of γ, when β is fixed to 0.3
- 0_1^+ and 0_2^+ states show similar large fluctuations, while 0_2^+ state exhibits a node near γ=30 due to the orthogonality to 0_1^+ state

Comparison of rigid rotor and fluctuating γ deformation

In the case of ^{44}S (strong transition, $B(\mathrm{IS0}) = 0.4$ W. u.)

0_1^+ state : fluctuation



0_2^+ state : fluctuation

$$|0_1^+\rangle = a|A\rangle + b|B\rangle, \quad |0_2^+\rangle = b|A\rangle - a|B\rangle.$$

Assume that there is a mixture of prolately- and oblately-deformed shapes with equal amplitude

$$|A\rangle = |\mathrm{prolate}\rangle, |B\rangle = |\mathrm{oblate}\rangle, a = b = 1/\sqrt{2}$$

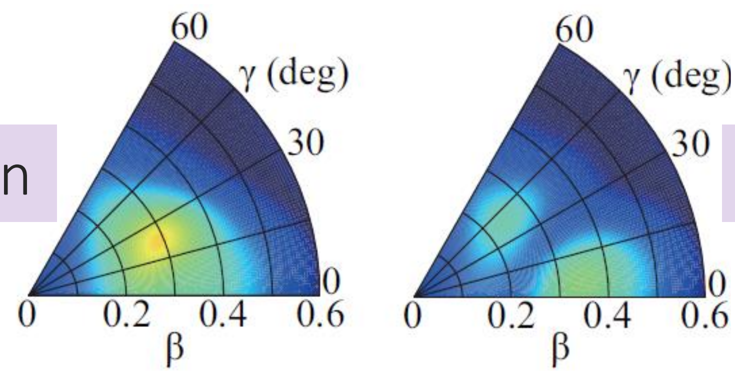
The matrix element is proportional to the size difference between the prolately- and oblately-deformed shapes

$$\langle 0_2^+|\mathcal{M}|0_1^+\rangle = \frac{1}{2}(\langle \mathrm{prolate}|\mathcal{M}|\mathrm{prolate}\rangle - \langle \mathrm{oblate}|\mathcal{M}|\mathrm{oblate}\rangle)$$

Comparison of rigid rotor and fluctuating γ deformation

In the case of ^{44}S (strong transition, $B(\mathrm{IS0}) = 0.4\ \mathrm{W.u.}$)

0_1^+ state : fluctuation



0_2^+ state : fluctuation

$$\langle 0_2^+ | \mathcal{M} | 0_1^+ \rangle = \frac{1}{2}(\langle \mathrm{prolate} | \mathcal{M} | \mathrm{prolate} \rangle - \langle \mathrm{oblate} | \mathcal{M} | \mathrm{oblate} \rangle)$$

Apply the AMD w.f. with (β,γ) = (0.31, 16) and (0,23, 49) as $|\mathbf{prolate}\rangle$ and $|\mathbf{oblate}\rangle$

⇒ The above equation yields $B(\mathrm{IS0}) = \mathbf{0.4\ W.u.}$, which corresponds to the GCM calc.

The mixture of the prolate and oblate shapes with different radii makes the monopole transitions stronger!

(even though the transition between two shapes is forbidden)

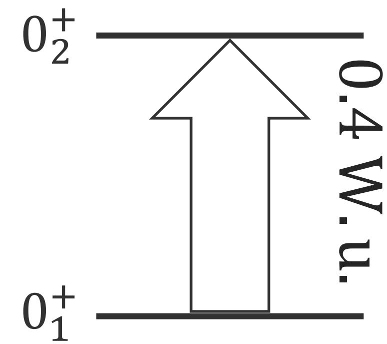
Summary

N=28 isotones show **different aspects of shape coexistence**, which can be **probed by the monopole transition strengths!**

^{44}S

$0_1^+, 0_2^+$ states : fluctuation

The mixture of the prolate and oblate shapes with different radii makes the monopole transitions stronger

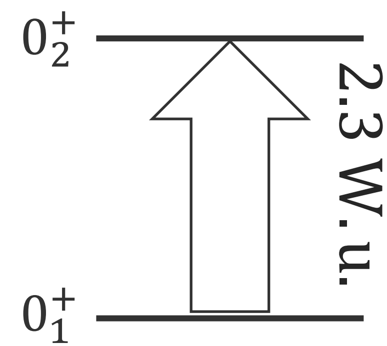


^{42}Si

0_1^+ state : oblate

0_2^+ state : spherical

Monopole transition is allowed because the neutron orbits have the same quantum numbers



^{40}Mg

0_1^+ state : prolate

0_2^+ state : oblate

Monopole transition is hindered because the neutron orbits have different quantum numbers