

$^{12}\text{C}+^{12}\text{C}$ fusion astrophysical S-factor from a full-microscopic nuclear model

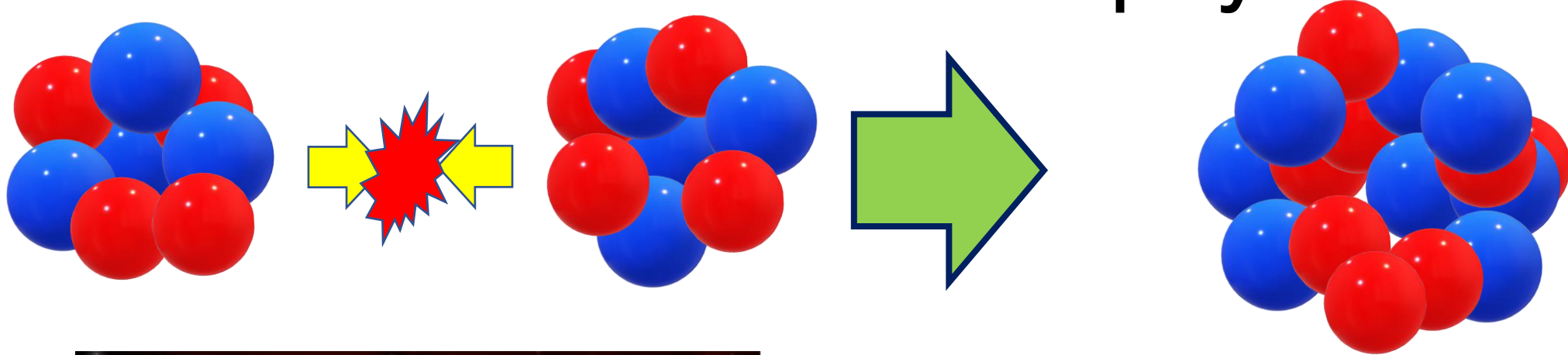
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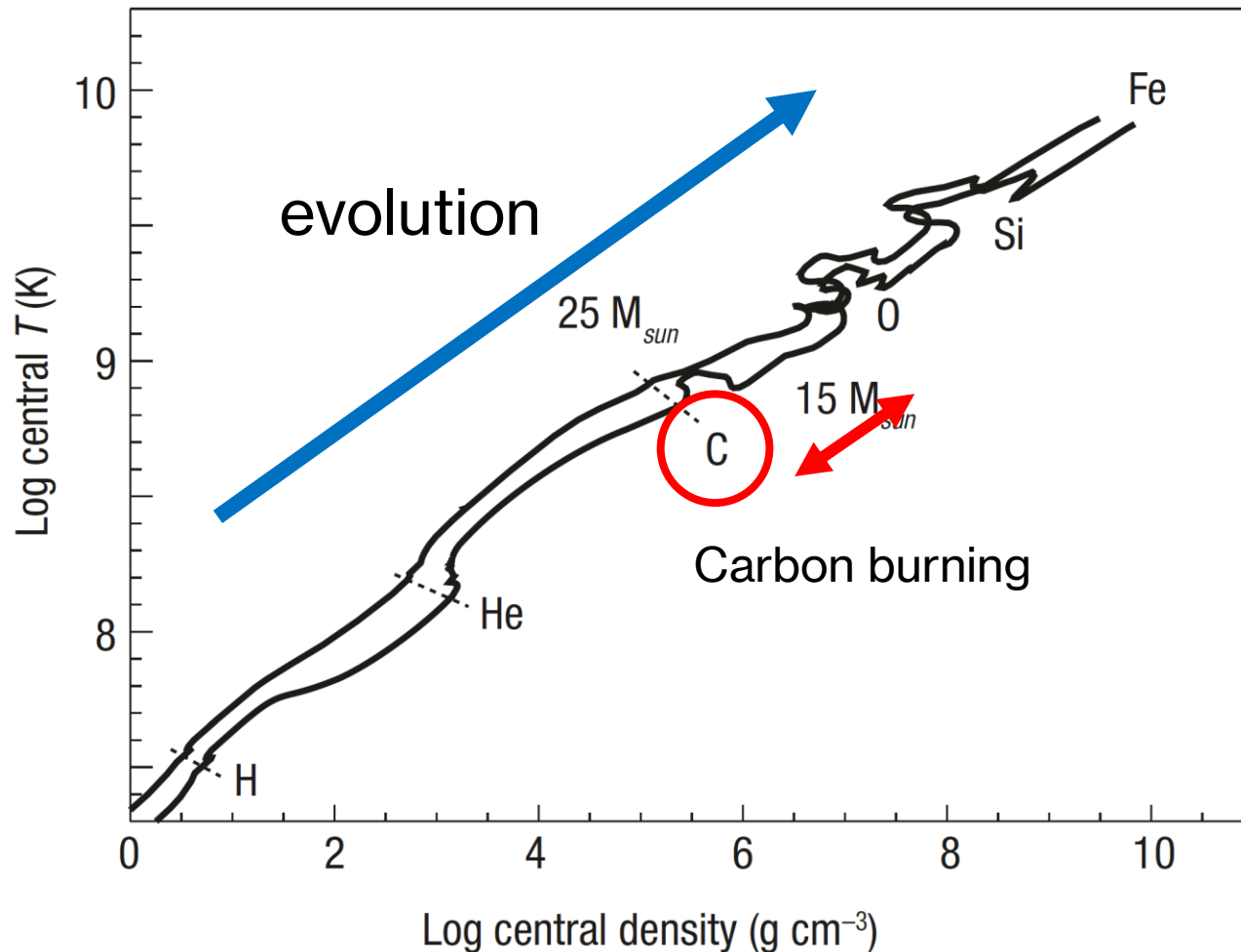
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Nuclear fusion and astrophysics



- Nuclear fusion is energy source of various astrophysical phenomena.
- $^{12}\text{C}+^{12}\text{C}$ fusion is essential reaction.
 - Evolution of massive stars
 - Supernovae (type Ia)
 - X-ray superbursts

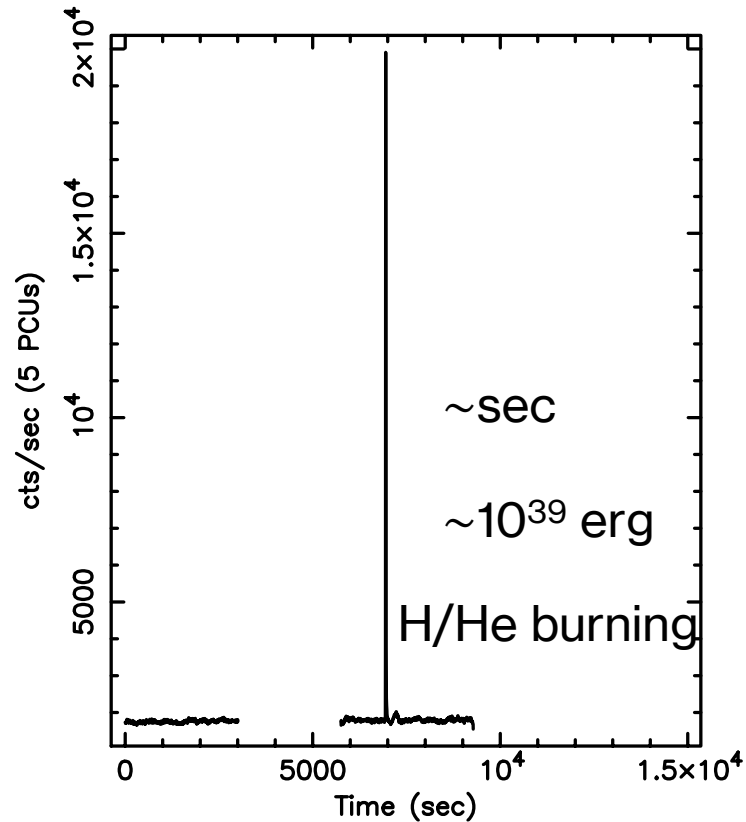
$^{12}\text{C}+^{12}\text{C}$ fusion reaction in massive stars



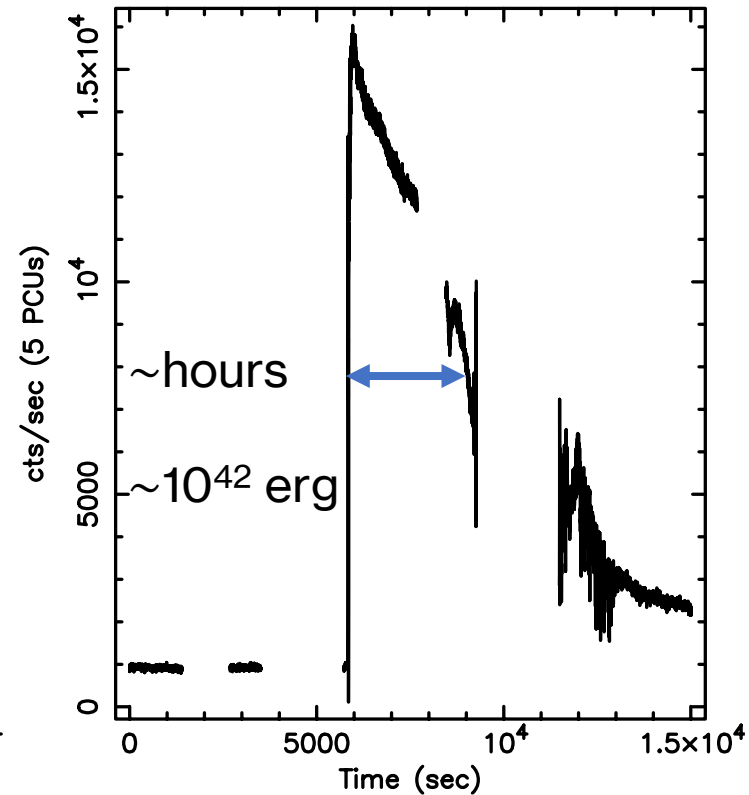
- Massive stars evolve through the carbon-burning process after the helium-burning.
- The primary reaction of the carbon-burning process.

X-ray superburst

X-ray burst



X-ray superburst

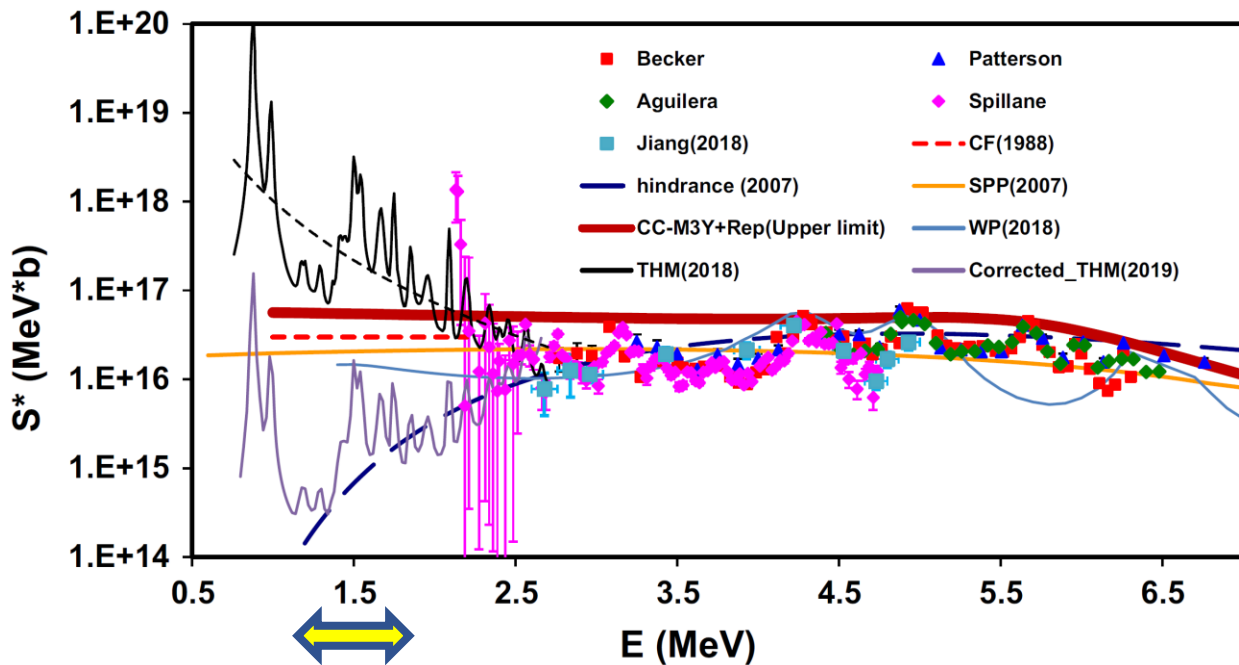


- X-ray superbursts are explosive flashes in space.
- $^{12}\text{C} + ^{12}\text{C}$ fusion reaction of the accreting matter is considered the trigger.

[E. Kuulkers, NPB**132**, 466 (2004)]

Uncertainties of $^{12}\text{C} + ^{12}\text{C}$ fusion reaction cross section

$$S^*(E) = E\sigma(E) \exp(2\pi\eta + 0.46 \text{ MeV}^{-1} E)$$



Gamow window
of X-ray superbursts

[R. L. Cooper+, ApJ **702**, 660 (2009)]

[Original fig. : C. Beck et al, EPJA56, 87 (2020)]

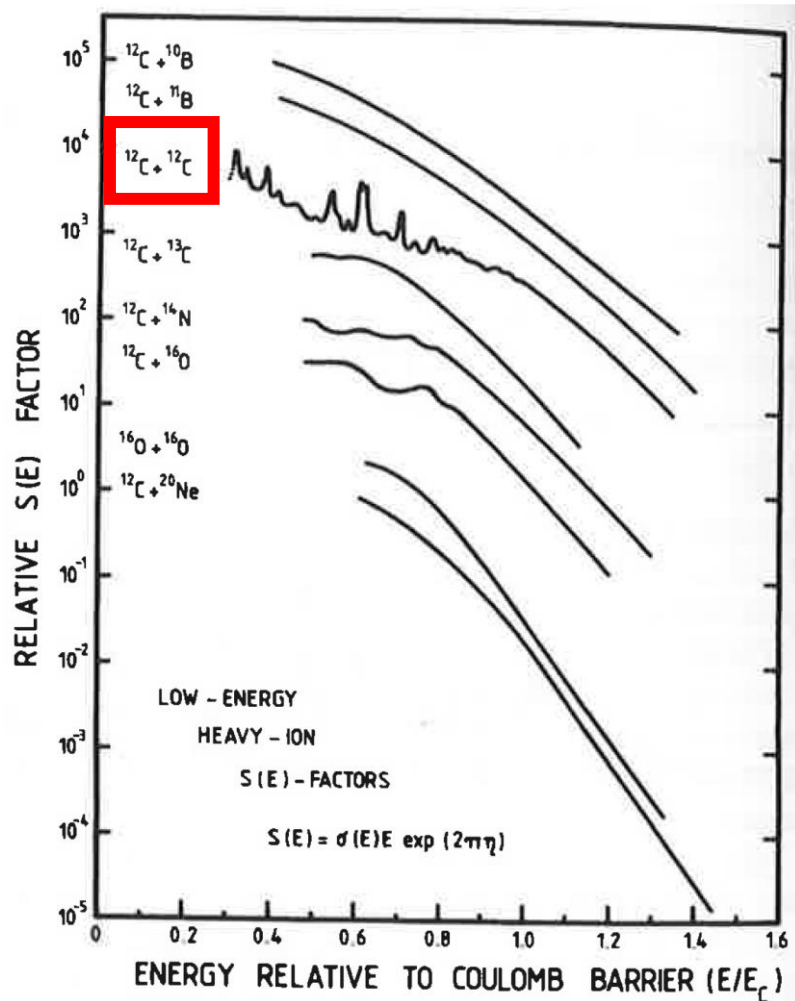
• Experiments

- **No direct experiments** in $E < 2 \text{ MeV}$.
- The indirect THM experiment (Tumino et al.) obtains a large S-factor, but the analysis is under discussion.

• Theories (just extrapolations)

- Constant: CF88.
- Strong suppression in the low-energy region: Hindrance model

Resonance-state dominance in $^{12}\text{C} + ^{12}\text{C}$ fusion reaction



➤ S-factors of $^{12}\text{C} + ^{12}\text{C}$ fusion reaction has a significant peak structure.

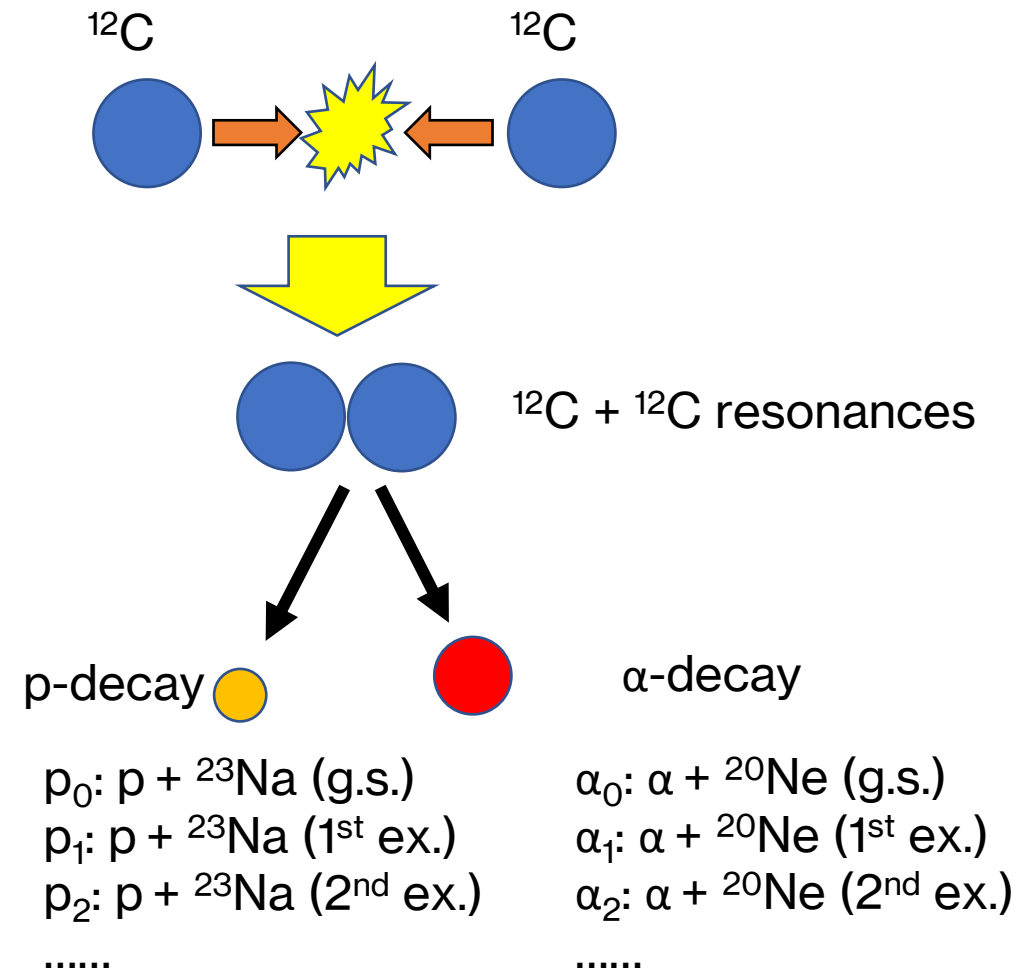
➤ **Narrow-resonance contributions** are essential.

→ Theoretical estimation of $^{12}\text{C} + ^{12}\text{C}$ fusion rate with resonance effects

Theoretical difficulties of low-energy $^{12}\text{C}+^{12}\text{C}$ fusion reaction

- Rearrangement of many nucleons
 - Entrance: $^{12}\text{C} + ^{12}\text{C}$
 - Exit: $\alpha + ^{20}\text{Ne}$, $p + ^{23}\text{Na}$
- Channel coupling effects are essential.
- Unknown macroscopic coupling potentials

→ **Microscopic framework**

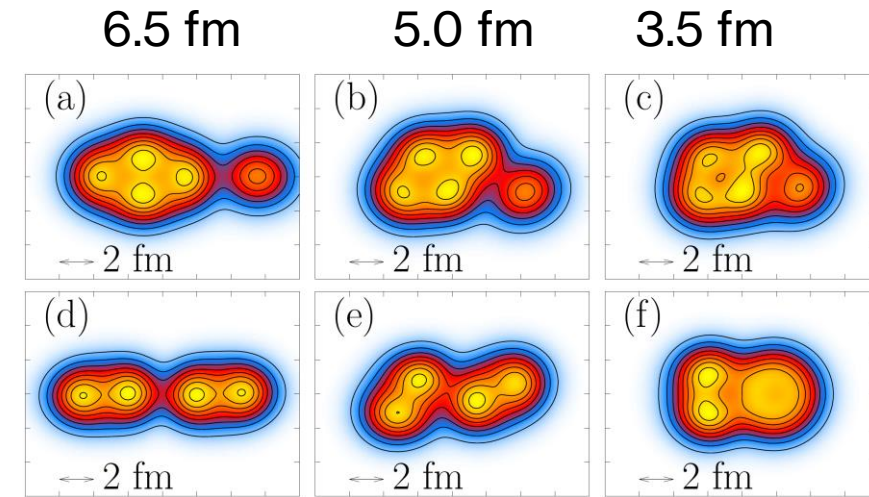


Framework: Antisymmetrized molecular dynamics (AMD)

- Slater determinant of deformed Gaussian wave packets

$$|\Phi\rangle = \mathcal{A}|\varphi_1, \varphi_2, \dots, \varphi_A\rangle \quad \alpha + {}^{20}\text{Ne}$$

$$\varphi_i(\mathbf{r}) = \exp\left[-\frac{1}{2}(\mathbf{r} - \mathbf{Z}_i) \cdot \mathbf{M}(\mathbf{r} - \mathbf{Z}_i)\right] \otimes \sigma_i \otimes \tau_i$$



- Coupling of all entrance and exit channels is treated by a linear combination of basis wave functions.

$$|\Psi\rangle = c_1 \text{ } {}^{12}\text{C}+{}^{12}\text{C} + c_2 \text{ } \alpha+{}^{20}\text{Ne} + c_3 \text{ } p+{}^{23}\text{Na} + c_4 \text{ } n+{}^{23}\text{Mg} + c_5 \text{ } \text{deformed}$$

- Fusion and decay dynamics are also treated by a linear combination of various inter-nuclear distance wave functions.

$$|\text{red circle, yellow circle}\rangle = d_1 |\text{red circle overlapping yellow circle}\rangle + d_2 |\text{red circle, yellow circle}\rangle + d_3 |\text{red circle, yellow circle}\rangle + \dots$$

- Diagonalization of Hamiltonian with the Gogny D1S density functional after parity and angular-momentum projection.

Fusion cross sections and Decay widths

resonance

- The Breit-Wigner formula

$$\sigma(E) \propto \frac{\pi}{k^2} \frac{\Gamma_{ent} \Gamma_{exit}}{(E - E_R)^2 + \Gamma^2/4}$$

- The R-matrix theory

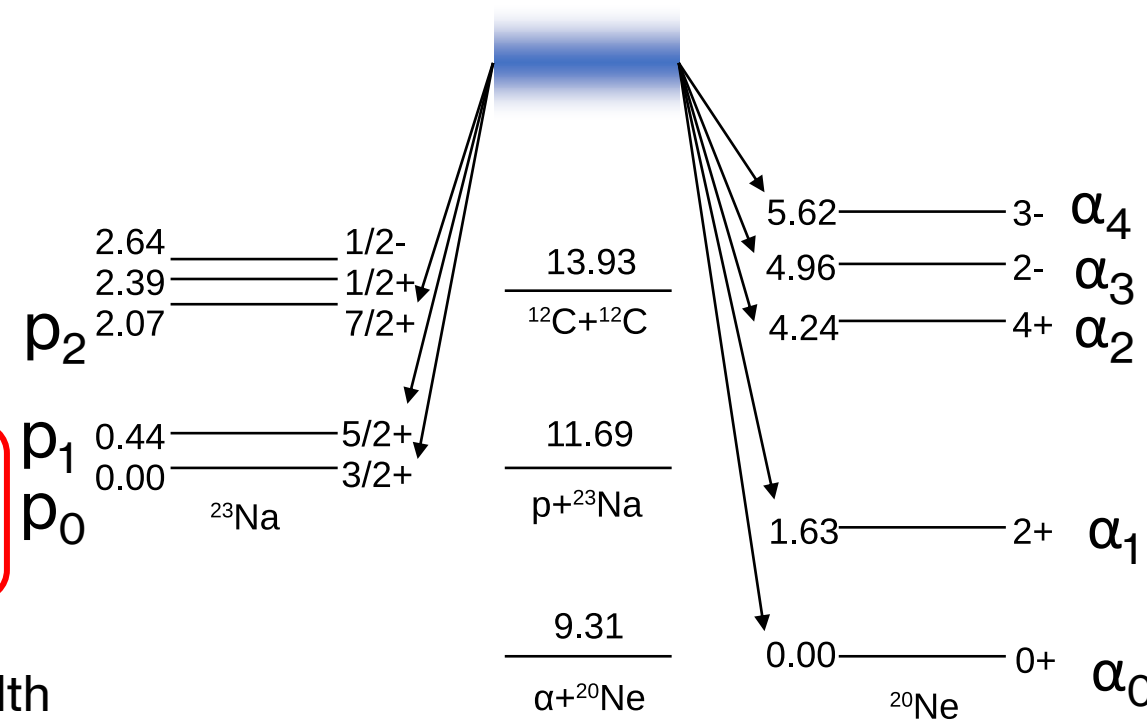
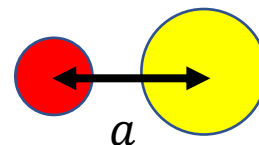
$$\Gamma_{C_1+C_2} = \frac{2ka}{F_l(ka)^2 + G_l(ka)^2} \frac{3\hbar^2}{2\mu a^2} \theta_{C_1+C_2}^2$$

$$Q + E = \frac{\hbar^2 k^2}{2\mu}$$

a : channel radius

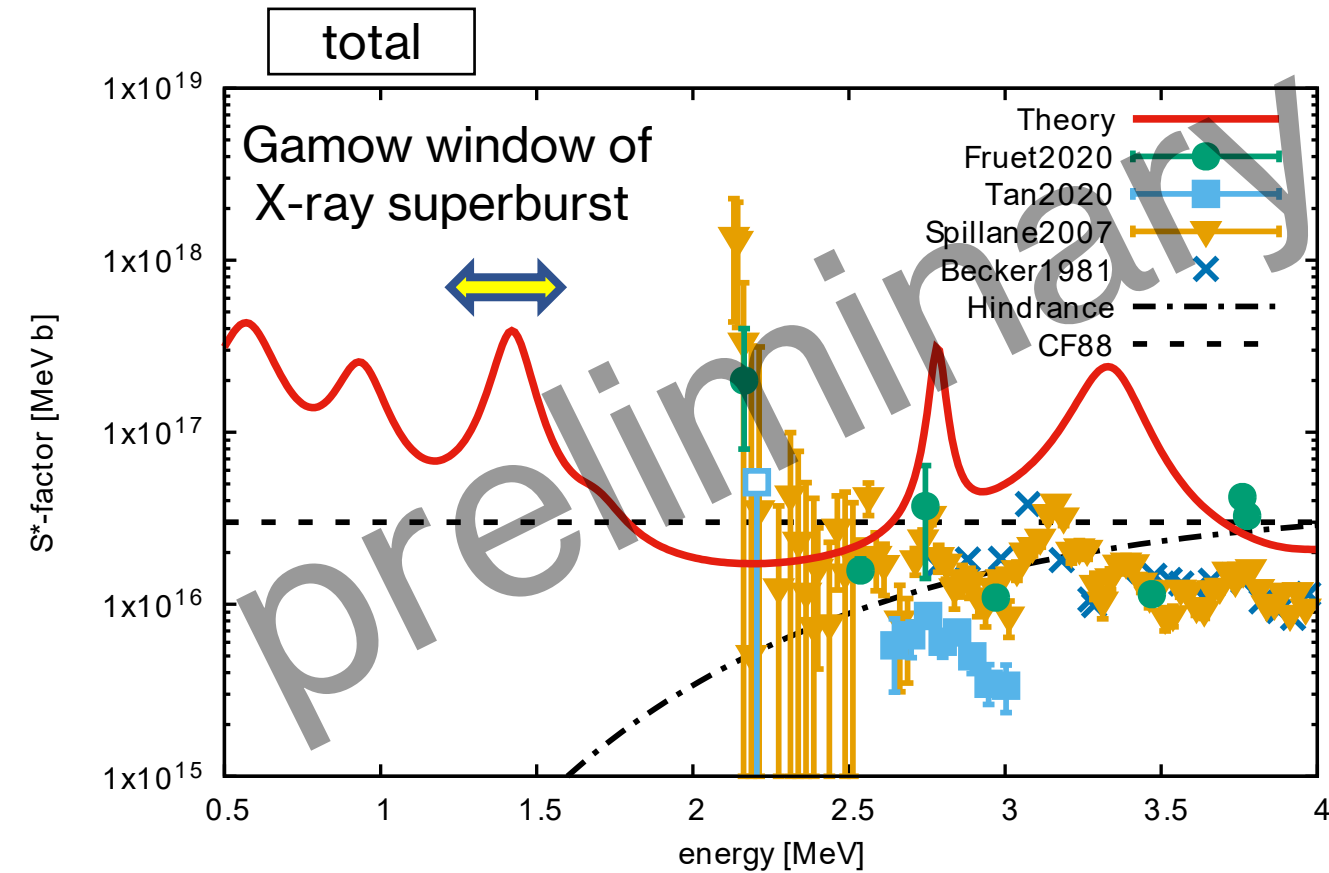
(Dimensionless) Reduced width amplitude at channel radius a
Probability of existence of clusters

Laplace expansion method
[Chiba+, PTEP (2017)]



$$\Gamma = \Gamma_{\alpha_0} + \Gamma_{\alpha_1} + \dots + \Gamma_{\alpha_4} + \Gamma_{p_0} + \Gamma_{p_1} + \Gamma_{p_2}$$

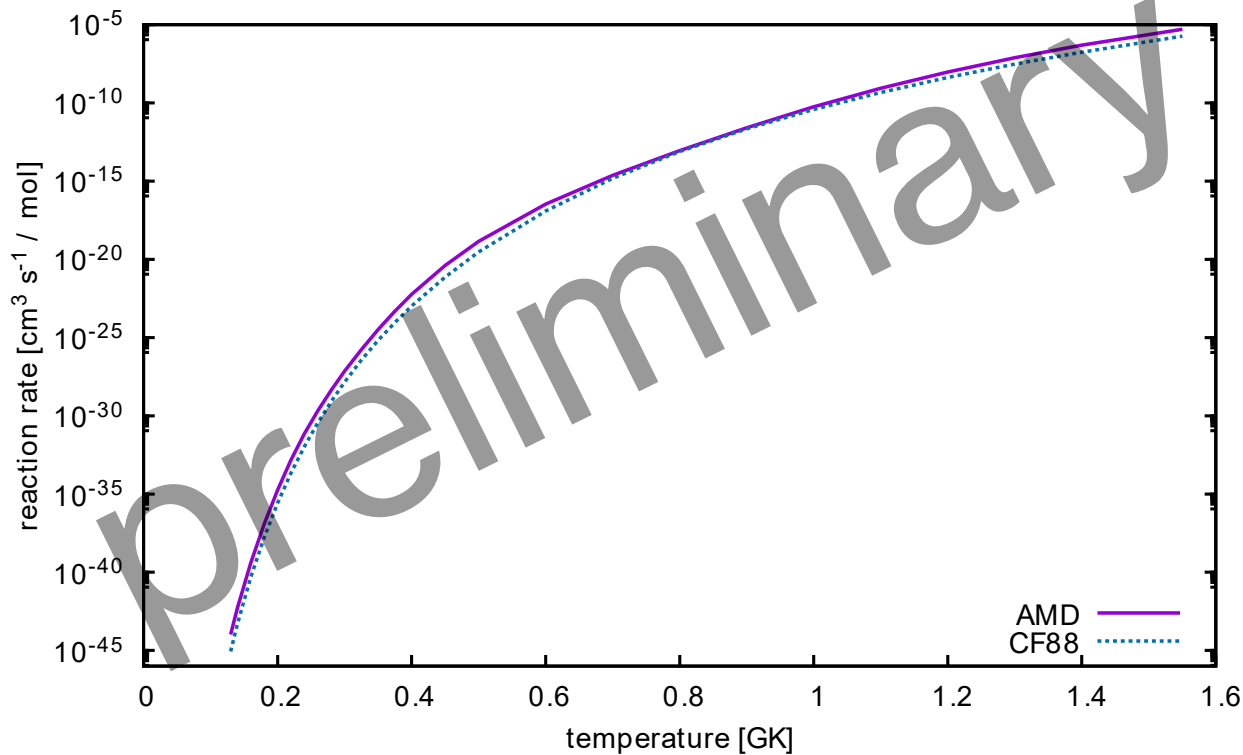
Astrophysical S factor



- Peak structure due to resonances.
- **Much larger S factors than the hindrance extrapolation.**
- A resonance is in the Gamow window of X-ray superbursts.

$^{12}\text{C}+^{12}\text{C}$ fusion reaction rate

$$\langle \sigma \rho \rangle \propto \frac{1}{(k_B T)^{3/2}} \int_0^\infty S(E) \exp\left(-\frac{E}{k_B T} - 2\pi\eta\right) dE$$

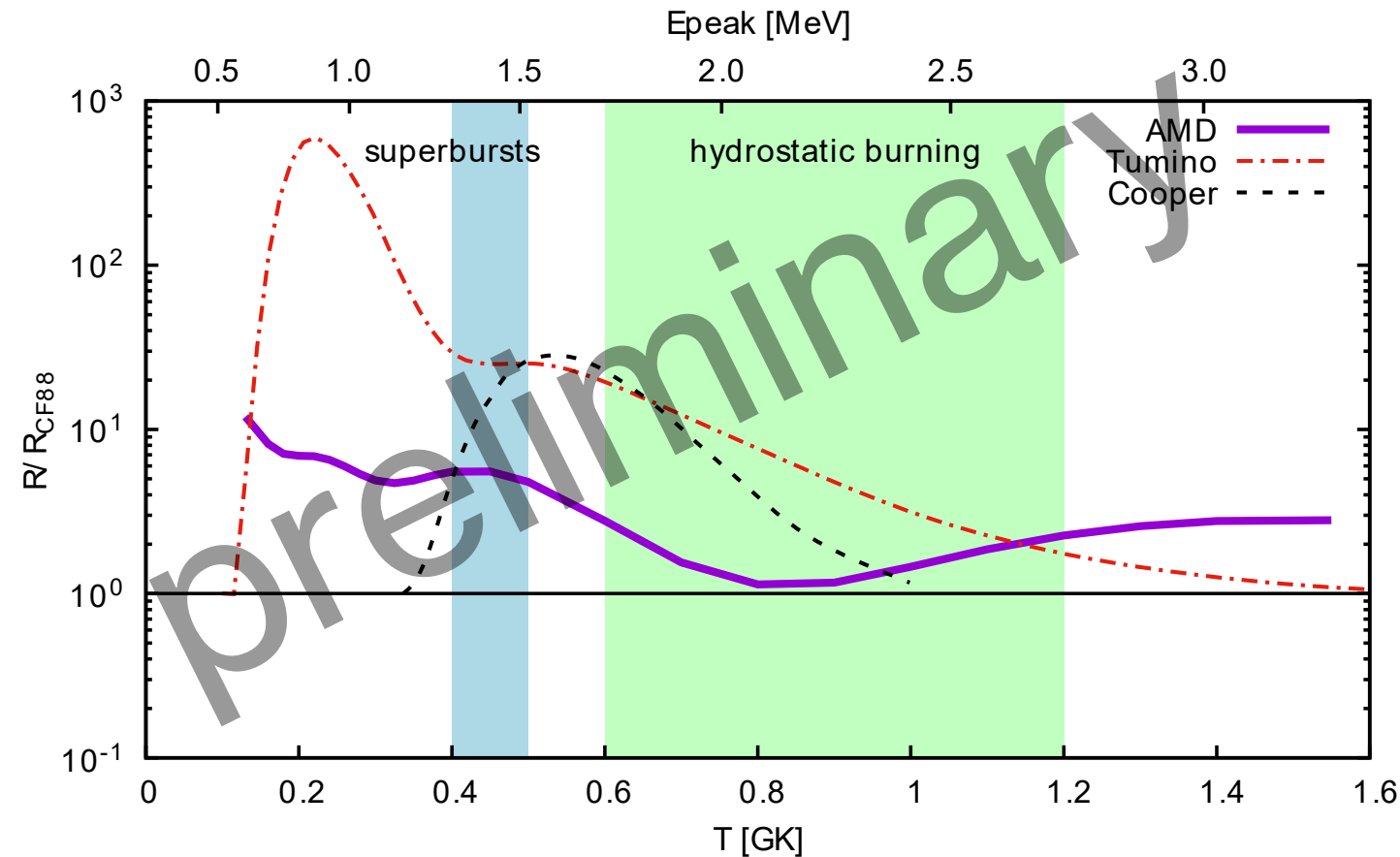


- Reasonable $^{12}\text{C}+^{12}\text{C}$ fusion reaction rate is obtained.

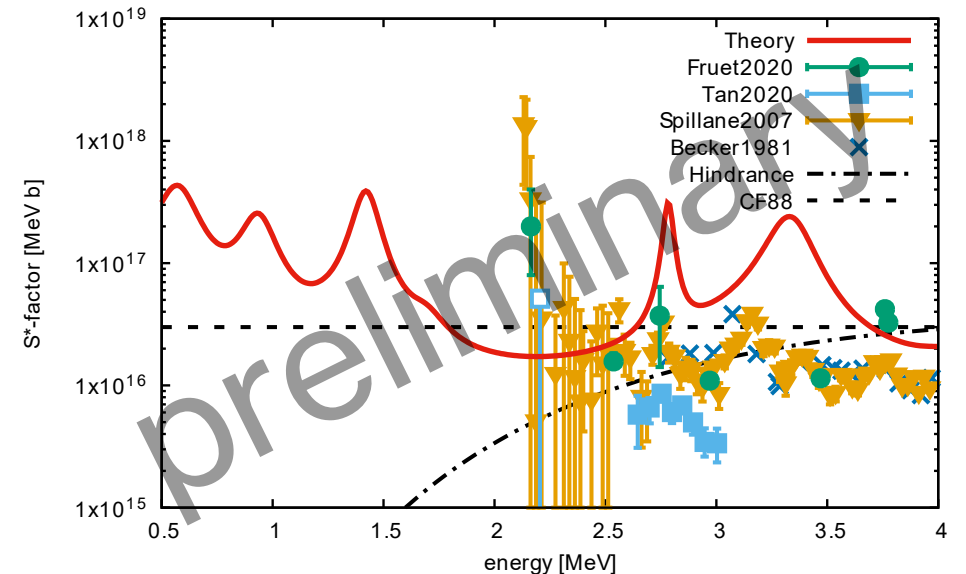
$^{12}\text{C}+^{12}\text{C}$ fusion reaction rate (detail)

$$\langle \sigma v \rangle \propto \frac{1}{(k_B T)^{3/2}} \int_0^\infty S(E) \exp\left(-\frac{E}{k_B T} - 2\pi\eta\right) dE$$

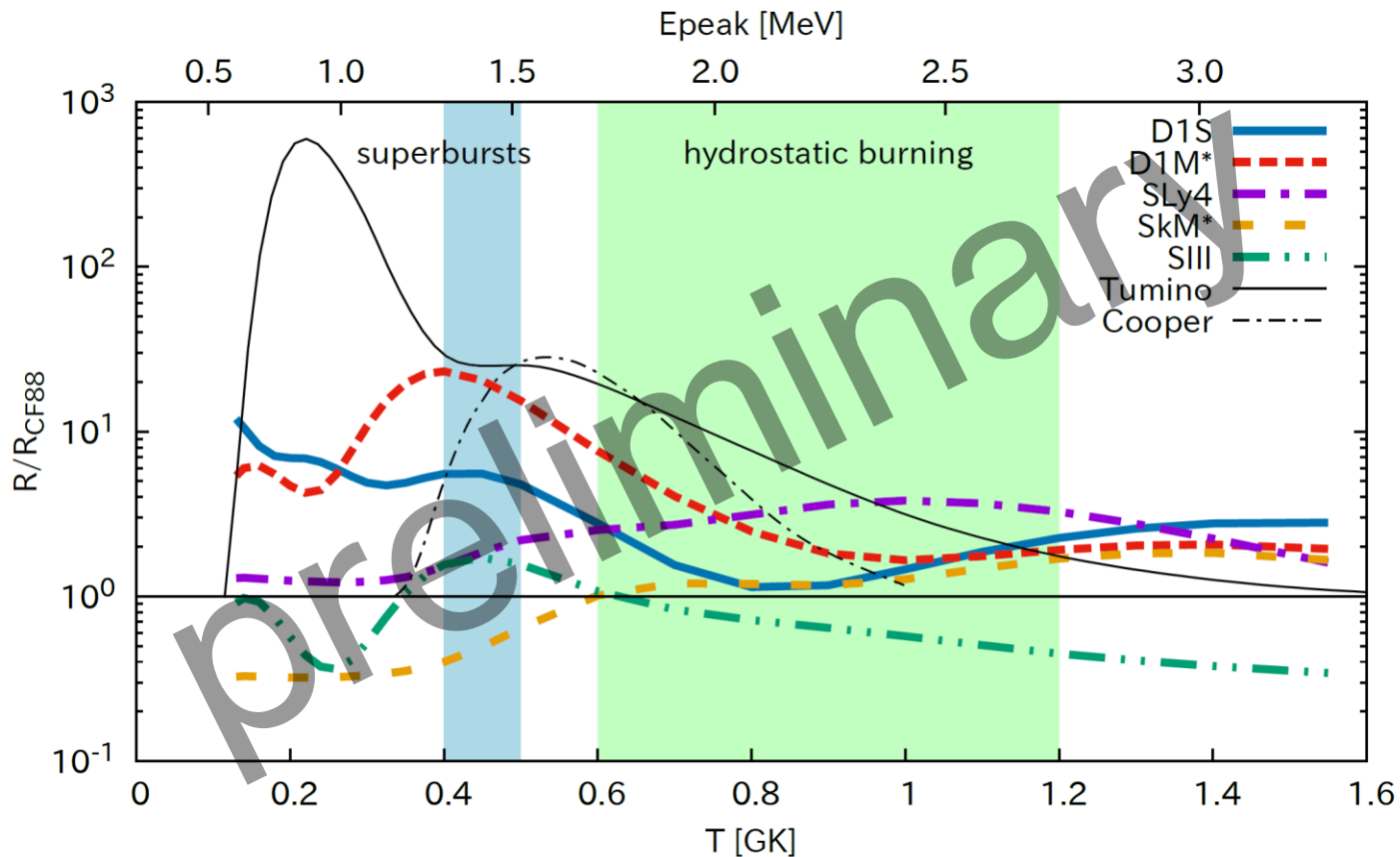
Gamow peak: $\Delta E = 4 \sqrt{\frac{E_p k_B T}{3}}$



- The same order as CF88.
- Each resonance is sensitive to the rate at low temperature due to a narrow Gamow peak.



Density functional dependence for $^{12}\text{C}+^{12}\text{C}$ fusion reaction rate



- Gogny (finite range)
 - D1S
 - D1M*
- Skyrme (zero range)
 - SLy4
 - SkM*
 - SIII
- In $T < 0.6$ GK, Gogny functionals obtain larger rates.
- Longer range of attractive ^{12}C - ^{12}C inter-nuclear potential?

Summary

- The $^{12}\text{C}+^{12}\text{C}$ fusion S-factor is calculated microscopically.
 - Channel coupling of all exit and entrance channels is treated using a nucleon-nucleon interaction.
 - All exit channels under the $^{12}\text{C}+^{12}\text{C}$ threshold are treated.
 - No hindrance effects due to contributions of low-energy resonances.
- $^{12}\text{C}+^{12}\text{C}$ fusion reaction rate is estimated in the same order as CF88 with the Gogny D1S.
- Analysis of density functional dependence for the fusion reaction rate is ongoing.