ASTROPHYSICAL *S* FACTOR AND RATE OF $p+{}^{7}Be \rightarrow {}^{8}B+\gamma$ DIRECT CAPTURE REACTION IN A POTENTIAL MODEL

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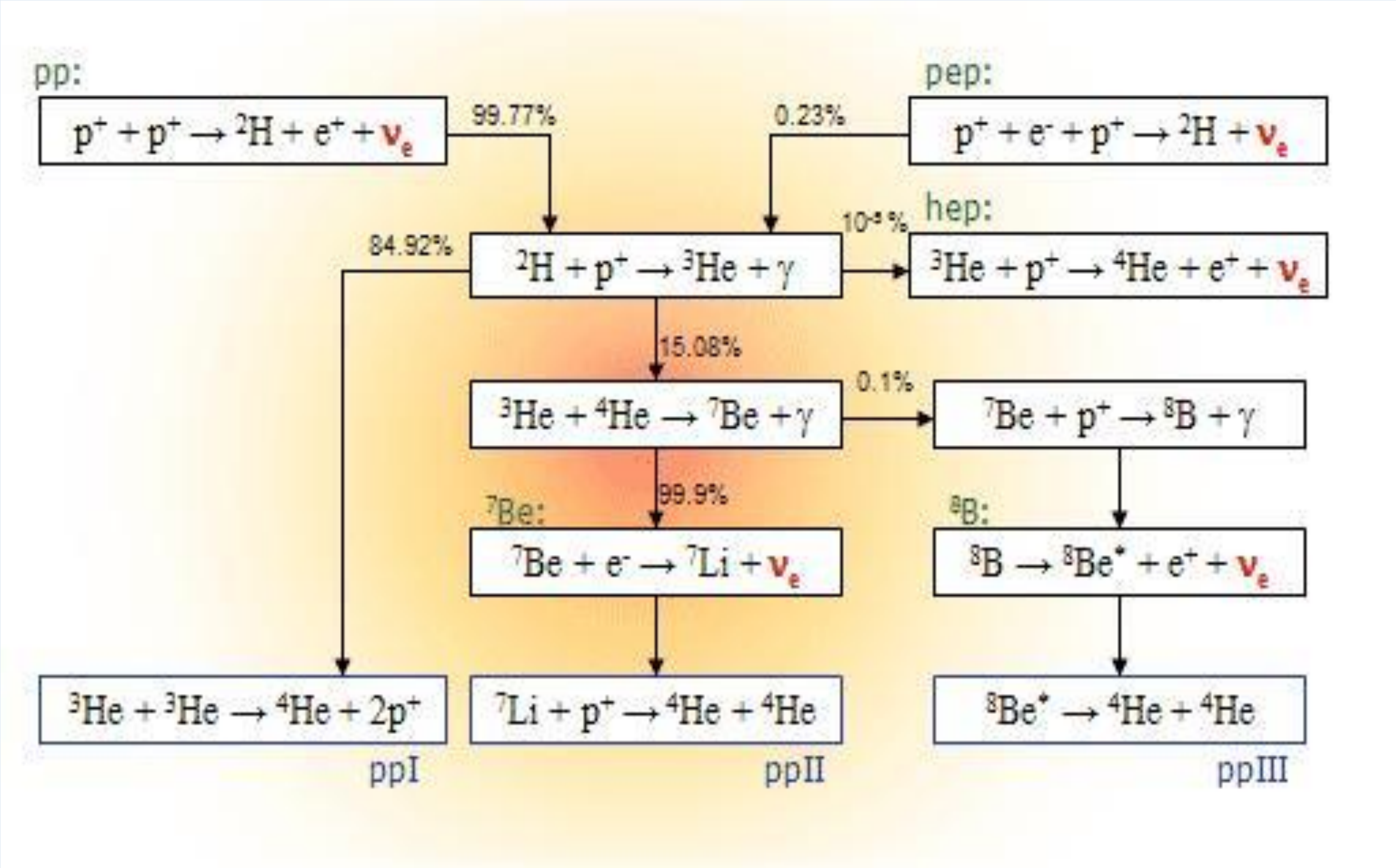
E.M. Tursunov, S.A. Turakulov, A.S. Kadyrov, L.D. Blokhintsev, Phys. Rev. C104, 045806 (2021)

YKIS2022b

pp-chain


pp:
p⁺ + p⁺ → ²H + e⁺ + νₑ
99.77%
0.23%
pep:
p⁺ + e⁻ + p⁺ → ²H + νₑ
hep:
10⁻⁵ %
84.92%
²H + p⁺ → ³He + γ
³He + p⁺ → ⁴He + e⁺ + νₑ
15.08%
0.1%
³He + ⁴He → ⁷Be + γ
⁷Be + p⁺ → ⁸B + γ
99.9%
⁷Be:
⁸B:
⁷Be + e⁻ → ⁷Li + νₑ
⁸B → ⁸Be* + e⁺ + νₑ
³He + ³He → ⁴He + 2p⁺
⁷Li + p⁺ → ⁴He + ⁴He
⁸Be* → ⁴He + ⁴He
ppI
ppII
ppIII

Motivation

Solar Fusion II (SFII) workshop estimation

E.G. Adelberger *et al., Rev. Mod. Phys.* ***83, 195 (2011).***

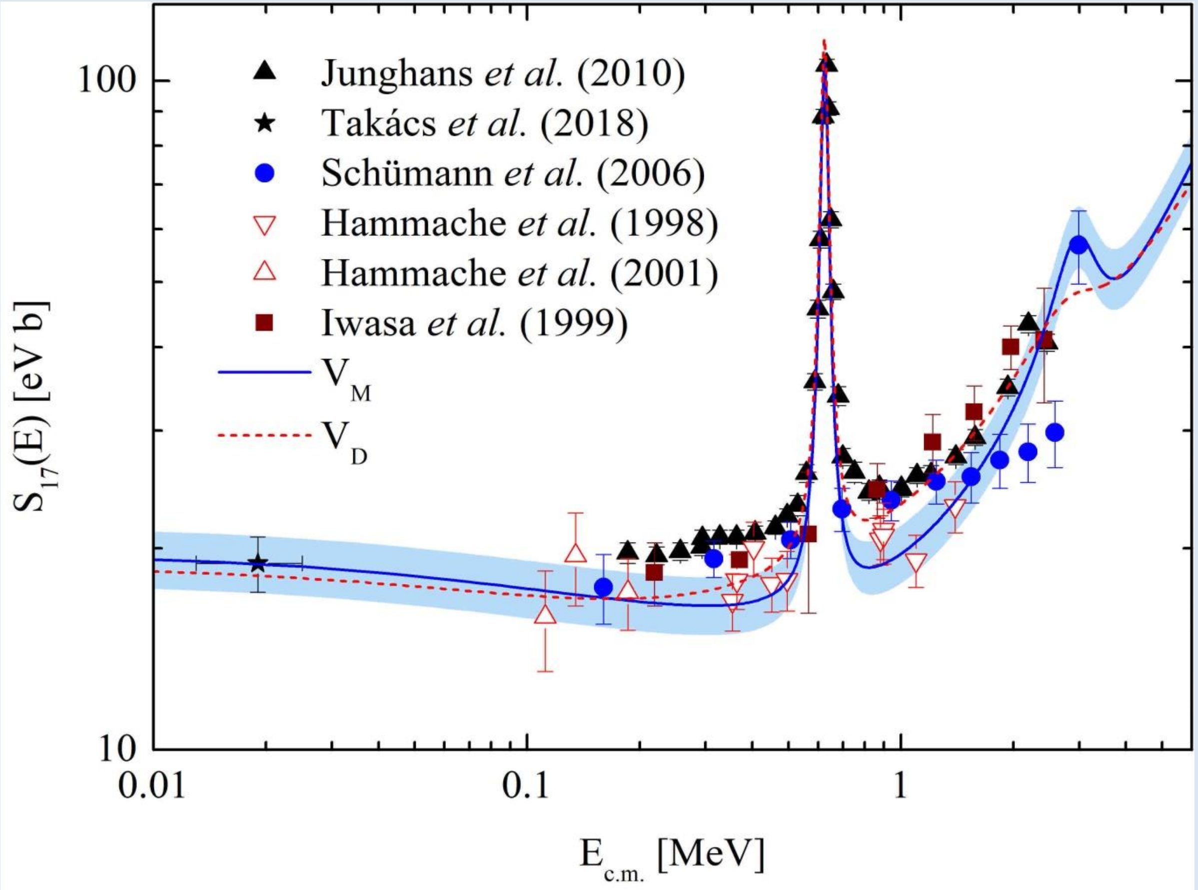
$$S_{17}(0) = (20.8 \pm 0.7_{\rm expt} \pm 1.4_{\rm theor})\ {\rm eV\ b} \qquad (1)$$

Solar neutrino flux extracted

M. P. Takács, et al. Nucl. Phys. **A970, 78 (2018).**

$$S_{17}(19^{+6}_{-5}\ {\rm keV}) = (19.0 \pm 1.8)\ {\rm eV\ b}. \qquad (2)$$

Astrophysical S-factor for the $^7Be(p, \gamma)^8B$ capture process



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Connection of the S factor with the S-wave scattering length and ANC

D. Baye, Phys. Rev. C **62, 065803 (2000).**

$$S_s(0)/C^2 \approx 35.6(1 - 0.0014a_{01})\ \text{eV b fm}$$
$$\approx 34.74\ \text{eV b fm},$$

Experimental value for the S-wave p^{7}Be-scattering length: S.N. Paneru et al. PRC99, 045807 (2019)

$$a_{01} = 17.34^{+1.11}_{-1.33}\ \text{фм}$$

$p+{}^7Be(3/2^-)$ scattering and bound state wave functins in

3S1, 3P0, 3P1, 3P2, 3D1, 3D2, 3D3, and 3F3 partial waves

$$\Psi^J_{lS} = \frac{u_E^{(lSJ)}(r)}{r}\{Y_l(\hat{r}) \otimes \chi_S(\xi)\}_{JM}, \tag{3}$$

$$\Psi^{J_f}_{l_f S'} = \frac{u^{(l_f S' J_f)}(r)}{r}\{Y_{l_f}(\hat{r}) \otimes \chi_{S'}(\xi)\}_{J_f M_f}, \tag{4}$$

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right) + V^{lSJ}(r)\right]u_E^{(lSJ)}(r) = E u_E^{(lSJ)}(r), \tag{5}$$

$$u_E^{(lSJ)}(r) \underset{r\to\infty}{\longrightarrow} \cos\,\delta_{lSJ}(E) F_l(\eta, kr) + \sin\,\delta_{lSJ}(E) G_l(\eta, kr), \tag{6}$$

Potentials and cross section

S.B. Dubovichenko et al. Nucl. Phys. A983 (2019) 175-194

$$V^{lSJ}(r) = V_0 \exp(-\alpha_0 r^2) + V_c(r), \quad (7)$$

$$V_{Coul}(\mathrm{r}) = \frac{Z_1 Z_2 e^2}{r}$$

$$\sigma(E) = \sum_{J_f \lambda \Omega} \sigma_{J_f \lambda}(\Omega), \quad (8)$$

$$\sigma_{J_f \lambda}(\Omega) = \sum_J \frac{(2J_f + 1)}{[S_1][S_2]} \frac{32\pi^2(\lambda + 1)}{\hbar\lambda([\lambda]!!)^2} k_\gamma^{2\lambda+1} C^2(S) \times \sum_{lS} \frac{1}{k_i^2 v_i} \left| \langle \Psi_{l_f S'}^{J_f} \| M_\lambda^\Omega \| \Psi_{lS}^J \rangle \right|^2, \quad (9)$$

EM transition operators

$$M^E_{\lambda\mu} = e \sum_{j=1}^{A} Z_j {r'}_j^{\lambda} Y_{\lambda\mu}(\hat{r}'_j), \tag{10}$$

$$\begin{aligned} M^M_{1\mu} &= \sqrt{\frac{3}{4\pi}} \sum_{j=1}^{A} \left[\mu_N \frac{Z_j}{A_j} \hat{l}_{j\mu} + 2\mu_j \hat{S}_{j\mu} \right] \\ &= \sqrt{\frac{3}{4\pi}} \left[\mu_N \left(\frac{A_2 Z_1}{A A_1} + \frac{A_1 Z_2}{A A_2} \right) \hat{l}_{r\mu} + 2(\mu_1 \hat{S}_{1\mu} + \mu_2 \hat{S}_{2\mu}) \right], \end{aligned} \tag{12}$$

Parameters of the **p+^{7}Be** potentials. Models V_D *and* V_M.

$^{(2S+1)}L_J$	**V_0, MeV**	**α, fm^{-2}**	**E_{FS}, MeV**
$^3S_1$	-343.0	1.0	-110.13
$^3S_1$ (V_M)	**-100.0**	**0.876**	**-2.42**
$^3P_0$	-580.0	1.0	-102.25
$^3P_1$	-709.85	0.83	-205.38
$^3P_2$	–330.414634	0.375	-96.59
$^3P_2$ (V_M)	**–300.5003**	**0.34**	**-87.86**
$^3P_2$ (V_M+)	**-272.2387**	**0.307**	**79.61**
$^3P_2$ (V_M-)	**-333.8405**	**0.379**	**95.59**
$^3D_1$	-343.0	1.0	**-**
$^3D_2$	–116.04	0.095	-20.45
$^3D_2$ (V_M)	**–193**	**0.15**	**-37.92**
$^3D_3$	-343.0	1.0	**-**
$^3F_3$	-104.555	0.055	-15.99

S.B. Dubovichenko et al. Nucl. Phys. A983 (2019) 175-194. Parameters were fitted to g.s. binding energy E_b=0.137 MeV in $^3P_2$ and resonance properties in other channels.

Original potential model V_D yields an estimate a_{01}= -0.26 fm for the S-wave p+^{7}Be scattering length.

The modified potential model V_M yields a_{01}= 17.34 fm.

Experimental value for the S-wave p^{7}Be-scattering length:

S.N. Paneru et al. PRC99, 045807 (2019)

$$a_{01} = 17.34^{+1.11}_{-1.33}\ \text{fm}$$

$$a_{02} = -3.18^{+0.55}_{-0.50}\ \text{fm}$$

Scattering amplitude in the presence of Coulomb interaction (L.D. Blokhintsev et al, PRC95, 044618 (2017))

$$f_0 = \lim_{k\to 0} \frac{1}{k \cdot C_0^2 (ctg\delta_0 - i)} = -a_0$$

$$f_l = \frac{e^{2i\delta_l} - 1}{2ik} \cdot C_l^{-2} = \frac{1}{k \cdot C_l^2 (ctg\delta_l - i)}$$

1) **Without Coulomb interaction**

$$k \cdot ctg\delta_0 |_{k\to 0} = -\frac{1}{a_0}$$

2) **With Coulomb interaction**

$$C_0^2 = 2\pi\eta / (\mathrm{e}^{2\pi\eta} - 1)$$

$$k \cdot C_0^2 \cdot ctg\delta_0 |_{k\to 0} = -\frac{1}{a_0}$$

$$a_0 = -\frac{1}{k \cdot C_0^2 \cdot ctg\delta_0} = -\frac{tg\delta_0}{k \cdot C_0^2} = -\frac{\sin\delta_0}{k \cdot C_0^2 \cdot \cos\delta_0}$$

- formula for the calculations at small values of the energy.

$$\eta = \frac{\mu \cdot Z_1 \cdot Z_2 \cdot e^2}{\hbar^2 k} = \frac{\mu \cdot Z_1 \cdot Z_2 \cdot e^2}{\hbar\sqrt{2\mu E}} = \sqrt{\frac{\mu}{2E}} \cdot \frac{Z_1 \cdot Z_2 \cdot e^2}{\hbar} = \sqrt{\frac{\frac{A_1 \cdot A_2}{A_1 + A_2} m_N c^2}{2E}} \cdot \frac{Z_1 \cdot Z_2 \cdot e^2}{\hbar c}$$

Phase shifts of the **p+^{7}Be scattering.** Models V_D *and* V_M.

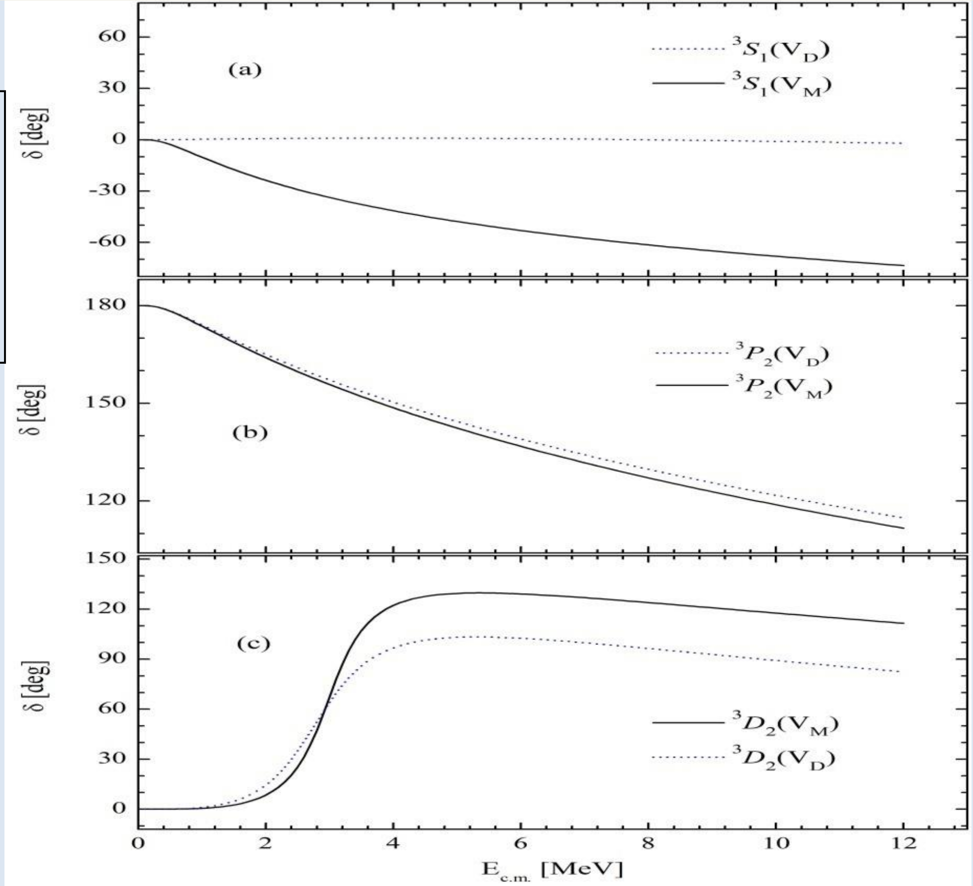
Original potential model V_D *yields an estimation* a_{01}= -0.26 fm for the S-wave p+^{7}Be scattering length

The modified potential model V_M yields a_{01}= 17.34 fm.

ANC values:

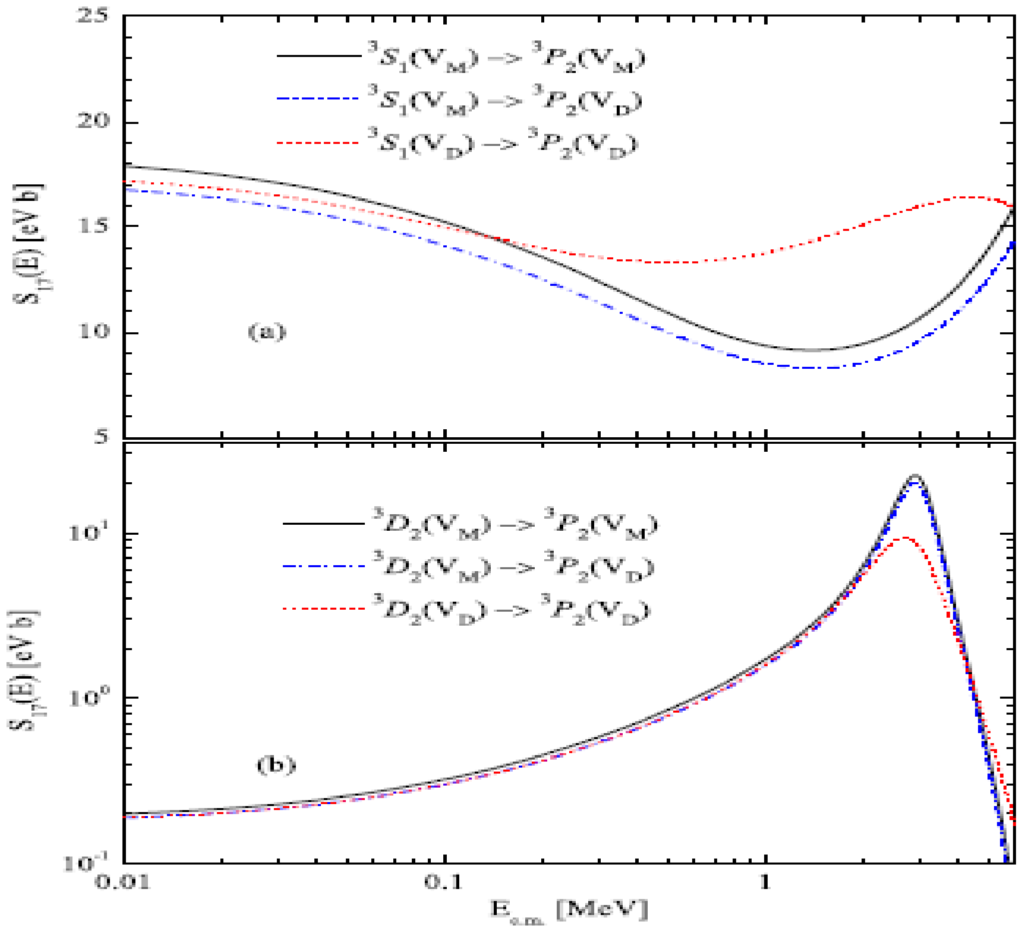
$C^2 = 0.496\ fm^{-1}$ for V_D

$C^2 = 0.538\ fm^{-1}$ for V_M



E1 partial **astrophysical** S factors for the **p+^{7}Be capture**

Comparison for the Models $\boldsymbol{V_D}$ ***and*** $\boldsymbol{V_M}$.



Selection rules p + ^{7}Be → ^{8}B + γ

$1/2^+$ $3/2^-$ 2^+

Parity:

$$\pi_i \pi_f = \begin{cases} (-1)^{\lambda}; E\lambda \\ (-1)^{\lambda+1}; M\lambda \end{cases}$$

Moments:

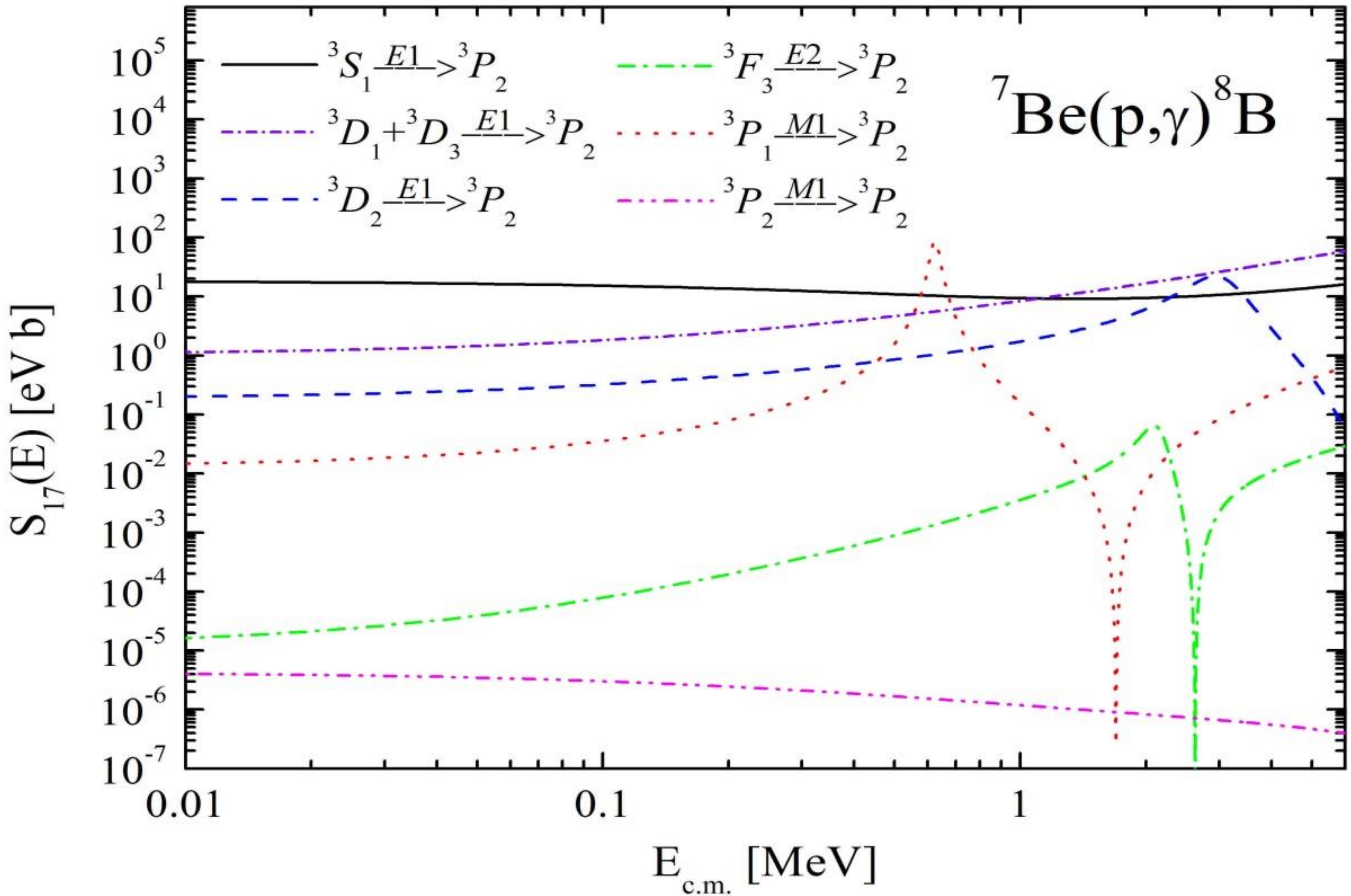
$$\left| J_i - J_f \right| \le \lambda \le J_i + J_f$$

$$E1: \quad (+)(-)(-)^{l_i} = -1 \quad \Longrightarrow \quad l_i = 0, 2$$

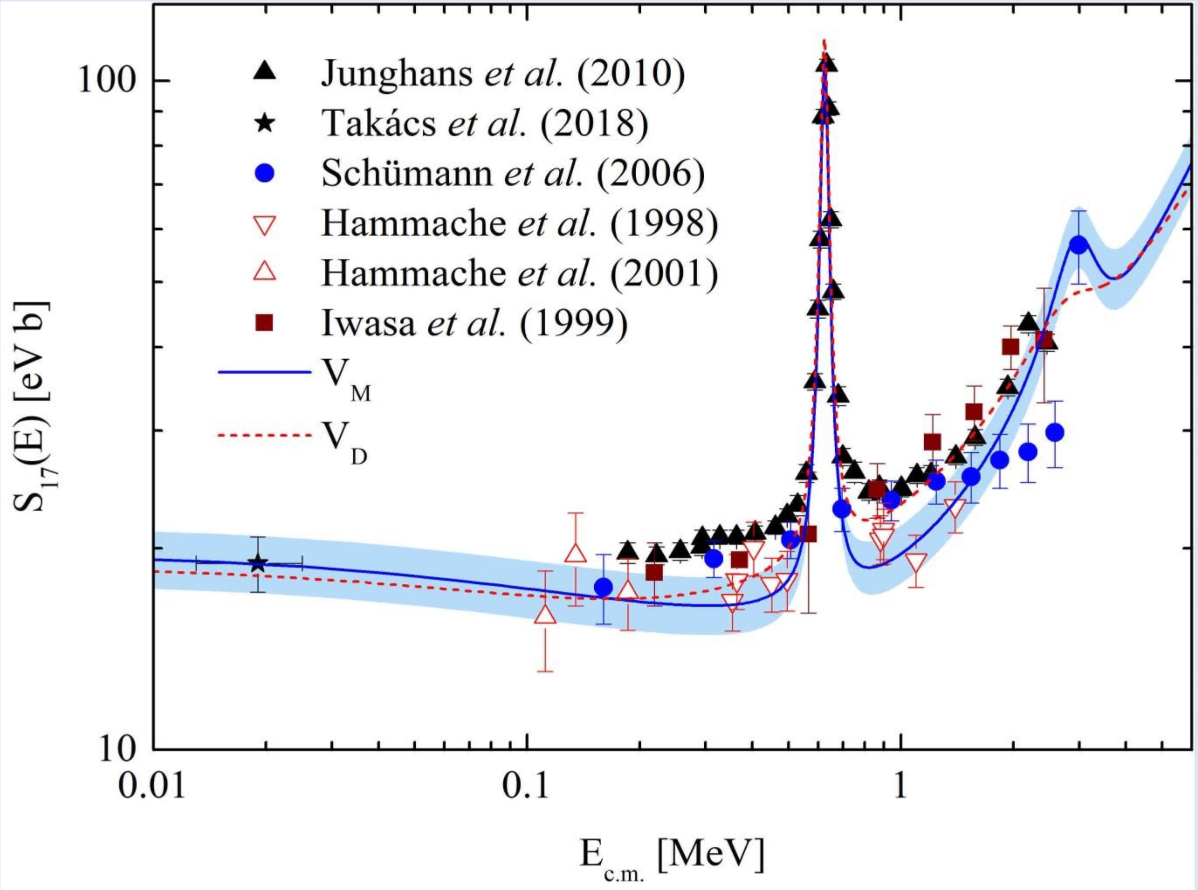
$$E2: \quad (+)(-)(-)^{l_i} = 1 \quad \Longrightarrow \quad l_i = 1, 3$$

$$M1: \quad (+)(-)(-)^{l_i} = 1 \quad \Longrightarrow \quad l_i = 1$$

Partial astrophysical S factors



Astrophysical S-factor for the $^{7}Be(p, \gamma)^{8}B$ capture process



F. Hammache *et al.*, *Phys. Rev. Lett.* ***80, 928 (1998).***
[11] F. Hammache *et al.*, *Phys. Rev. Lett.* ***86, 3985 (2001).***

A.R. Junghans et al. Phys.Rev. C68(2003) 065803

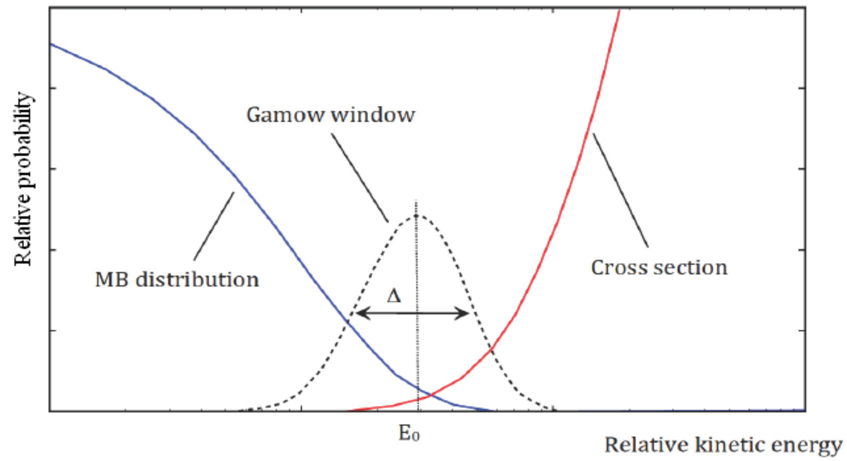
$$S_{17}(0) \approx 20.51^{+2.02}_{-1.85} \text{ eV b}$$

Reaction rates $N_{17}(\sigma v)$ [sm^3 mol^{-1} s^{-1}]

$$N_{17}(\sigma \, v) = N_A \frac{(8/\pi)^{1/2}}{\mu^{1/2}(k_B T)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-E/k_B T\right) dE \qquad (13)$$

N_A=6.0221 x 10^{23} mol^{-1}, $k_B T = T_9/11.605$ MeV and T_9 is a temperature in units of 10^9 K.

$$N_{17}(\sigma \, v) = 3.7313 \times 10^{10} A^{-1/2} T_9^{-3/2} \int_0^\infty \sigma(E) E \exp\left(-11.605 E/T_9\right) dE \qquad (14)$$



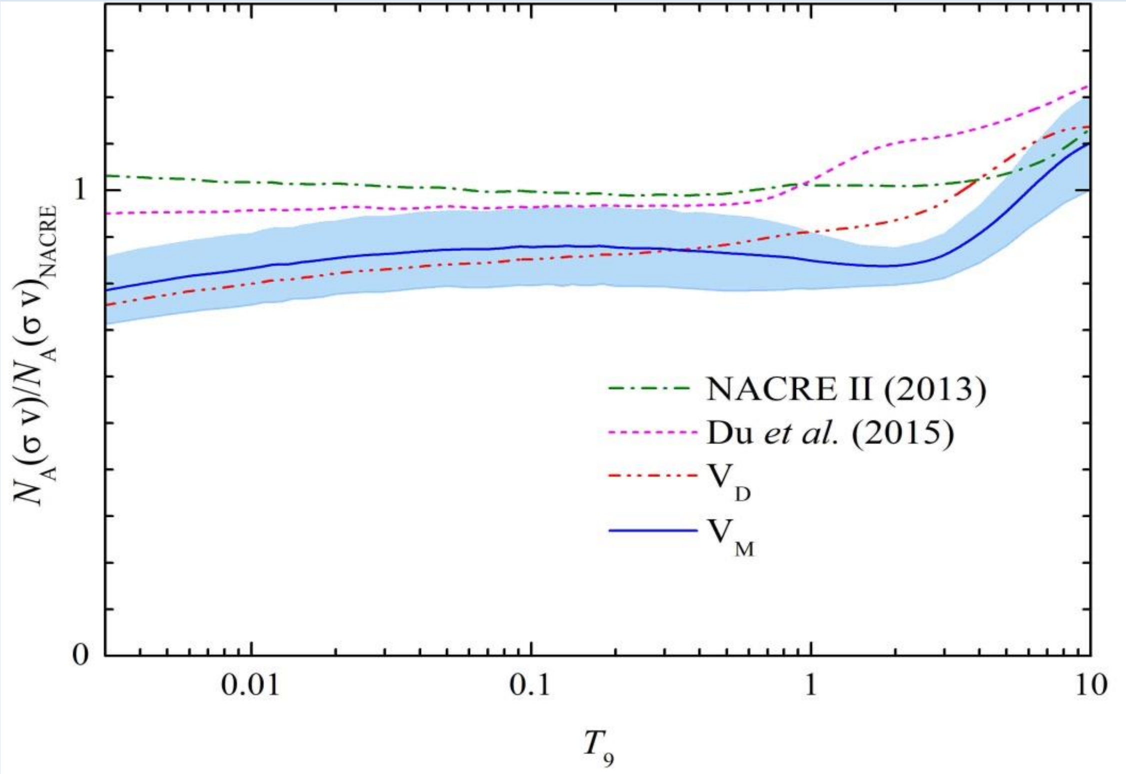
Gamow window

$$E_0 = 0.122 (Z_1^2 Z_2^2 A)^{1/3} T_9^{2/3}$$

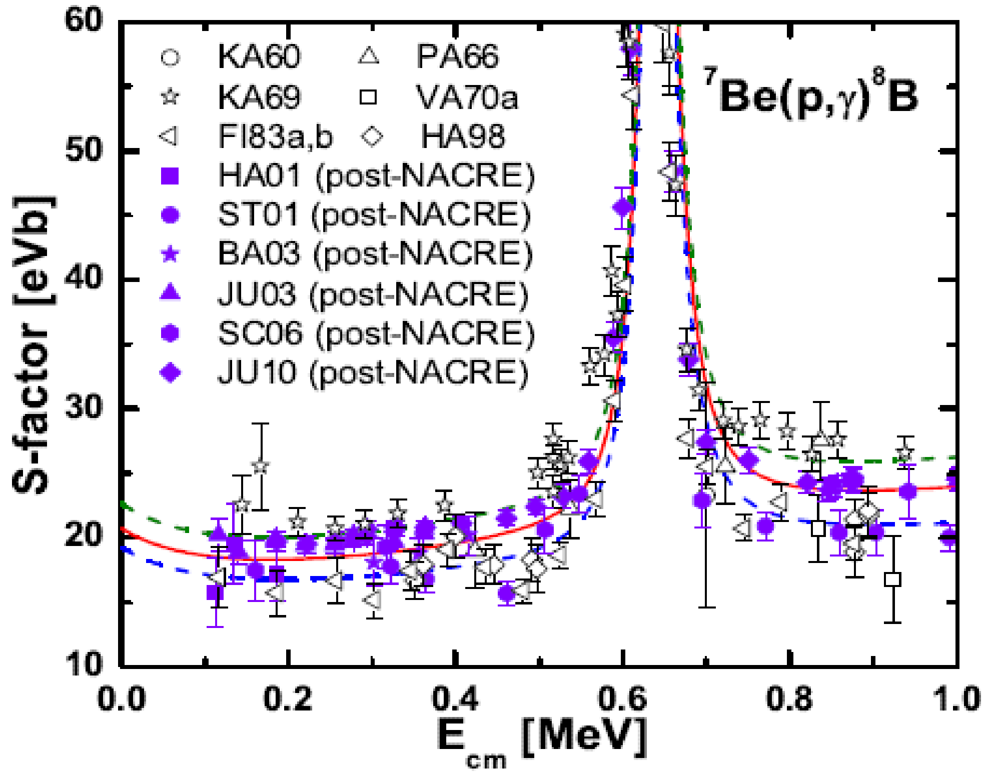
Width of the Gamow window

$$\Delta E_0 = 0.2368 (Z_1^2 Z_2^2 A)^{1/6} T_9^{5/6}$$

Results for the **Reaction rates of ^{7}Be(p, γ)^{8}B** ($10^{6} \leq T \leq 10^{10}$ K)



Y.Xu et al. NACRE II: Nuclear Physics A918 (2013)61



Next step: Three-body model. Energy calculations (MeV).

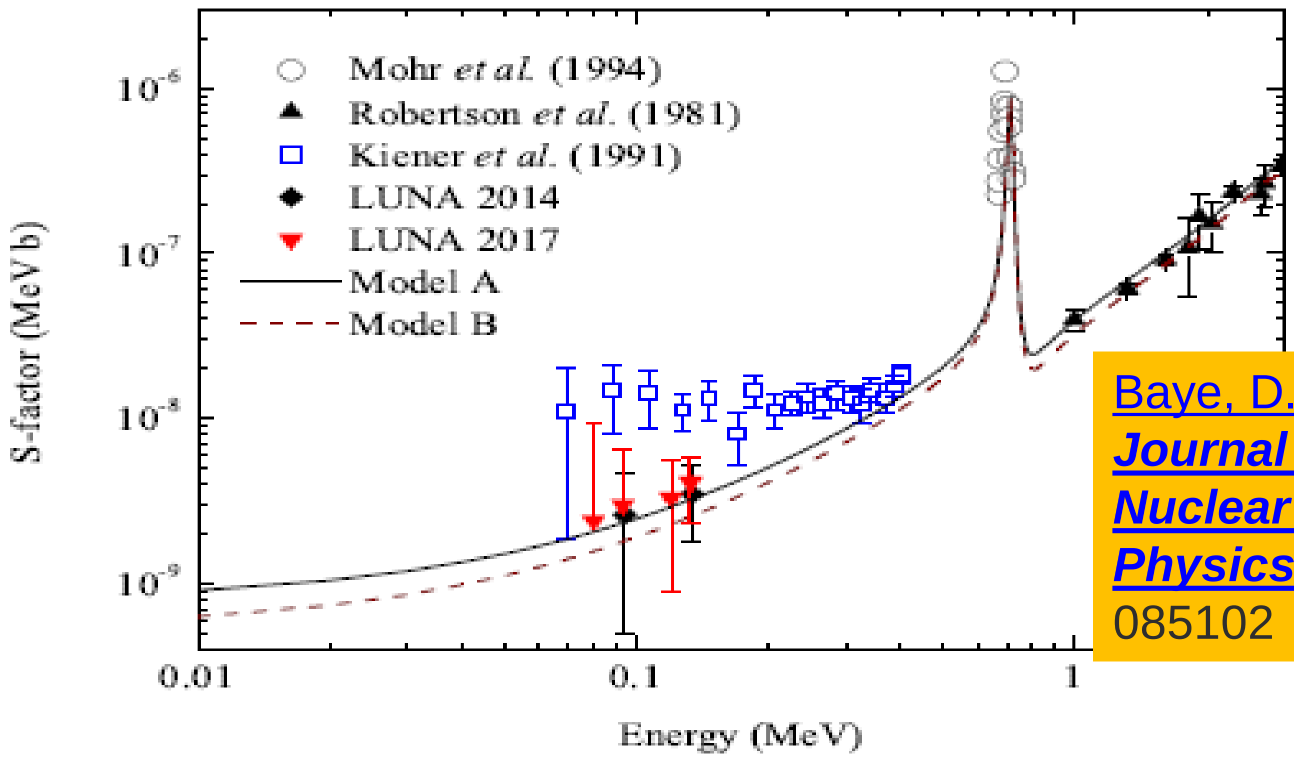
$^8B = p + \alpha + ^3He$

$^8Li = n + \alpha + ^3H$

Hyperspherical Lagrange-mesh method.
P. Descouvemont et al. Phys. Rev. C67, 044309 (2003)

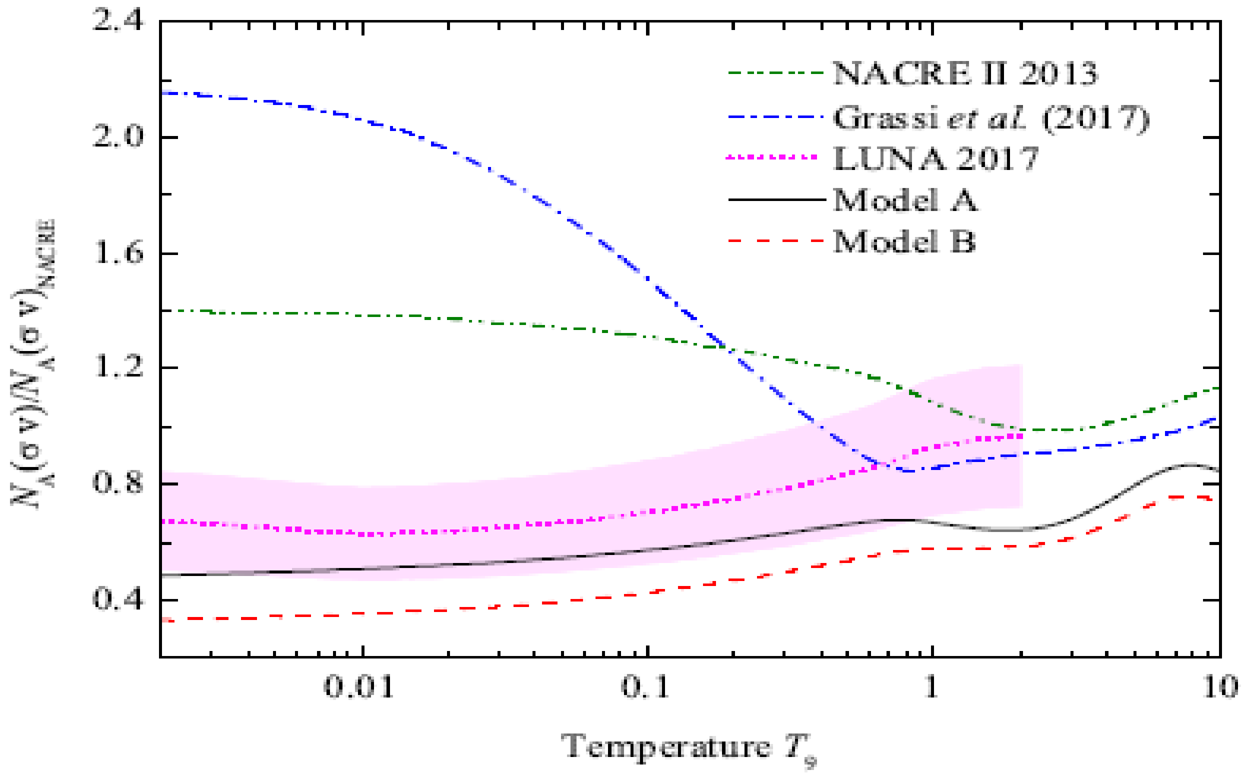
Kmax	8	12	16	18	20	22	24	26	28	30	exp
8B (2+)	-1.390	-1.622	-1.689	-1.704	-1.712	-1.718					-1.725
8Li (2+)	-4.101	-4.285	-4.328	-4.335	-4.338	-4.340					-4.501
8B (1+)	0.133	-0.204	-0.333	-0.369	-0.395	-0.416	-0.433	-0.447	-0.460	-0.472	-0.951
8Li (1+)	-2.466	-2.726	-2.802	-2.819	-2.828	-2.834	-2.837	-2.840	-2.842	-2.843	-3.520

d+α→ ^{6}Li +γ: 3-body model Astrophysical S factor



Baye, D., Tursunov, E.M. ***Journal of Physics G: Nuclear and Particle Physics***, 2018, 45(8), 085102

d+α→ ^{6}Li +γ: 3-body model
Reaction rates



E. M. Tursunov, et al. Phys. Rev. C **98**, 055803 (2018)

CONCLUSION

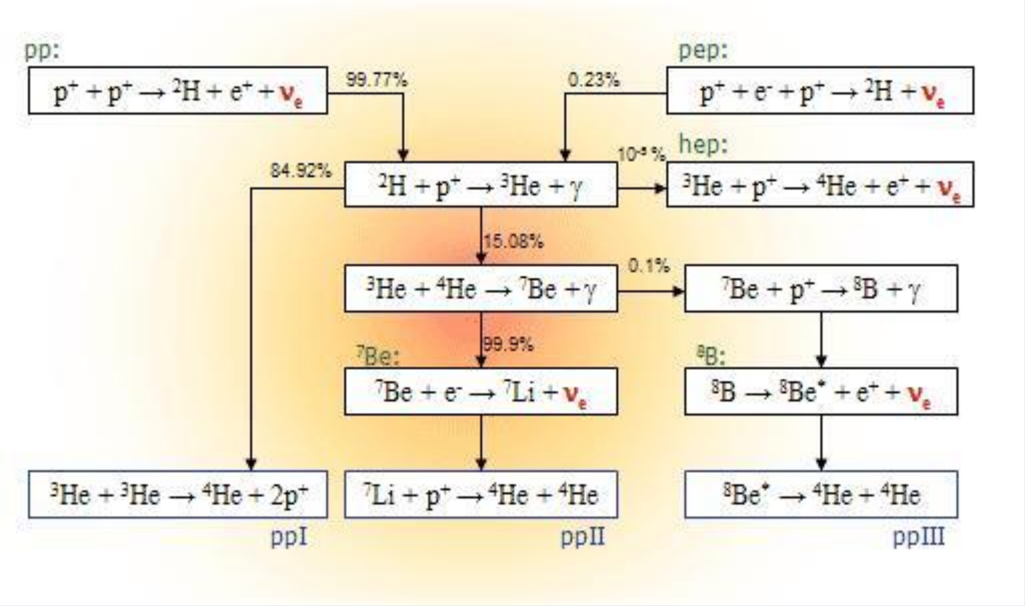
1. Astrophysical direct capture process $p+{}^{7}Be \rightarrow {}^{8}B+\gamma$ was studied in a single channel potential model
2. The modified potential is consistent with the theory of D. Baye which connects the astrophysical S factor at zero energy divided by ANC with the value of the S-wave $p+{}^{7}Be$ scattering length.
3. For the astrophysical S factor we obtained $S_{17}(0) \approx 20.51^{+2.02}_{-1.85}$ eV b

   At intermediate energies the results are very consistent with the two data sets of

   F. Hammache et al.
4. Reaction rates are lower than the estimations of the NACRE II collaboration.
5. Study of the process within the 3-body model is in progress.

THANKS!

pp- chain



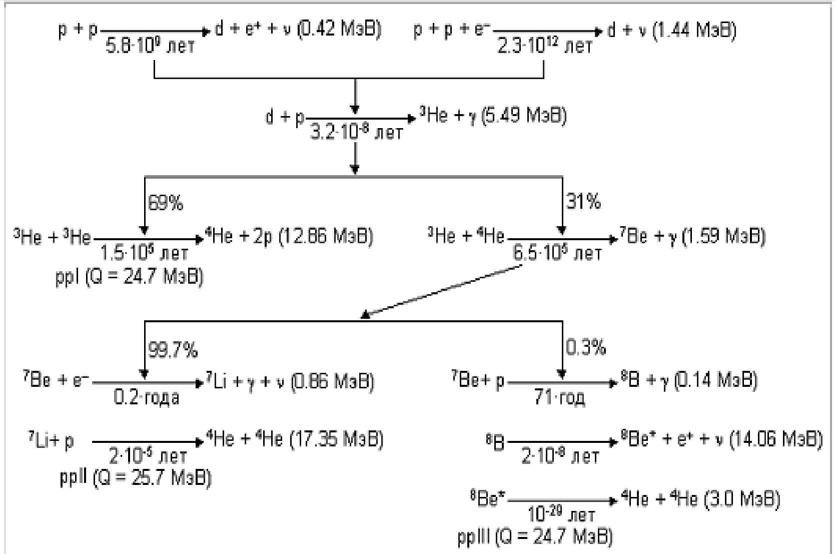


Рис. 14. Протон - протонная цепочка.