# *Microscopic Studies Of Alpha Clustering Using Configuration Interaction Approach*

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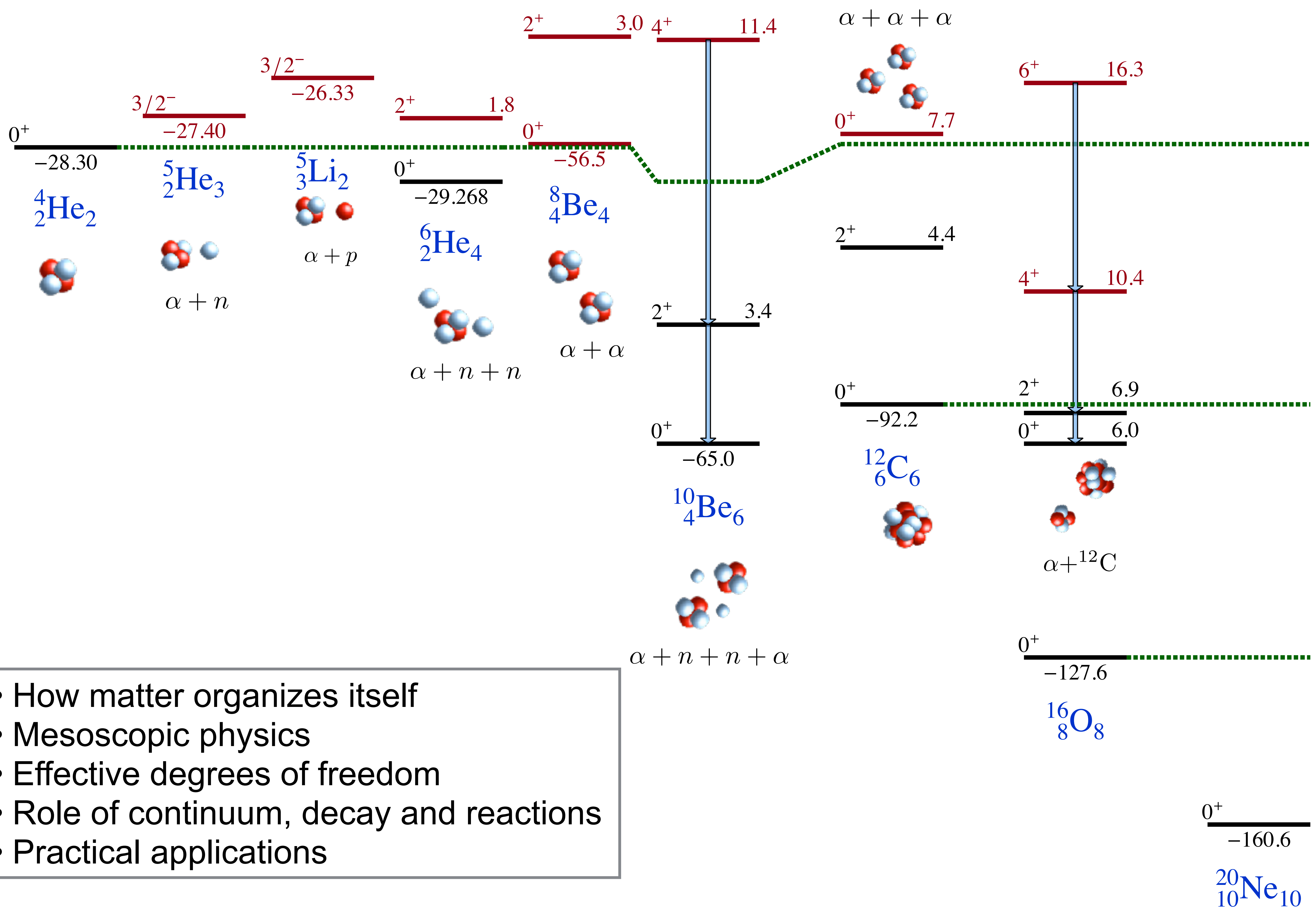
# Clustering in light nuclei




- How matter organizes itself
- Mesoscopic physics
- Effective degrees of freedom
- Role of continuum, decay and reactions
- Practical applications

# Configuration interaction approach (shell model)

## A powerful tool in studies of nuclear many-body problems

- Well-established many-body technique
- Excellent predictive power
- New computational techniques broaden applicability to nearly all nuclei
- Extensions to continuum and reaction physics.
- **Clustering**

# The Nuclear Shell Model

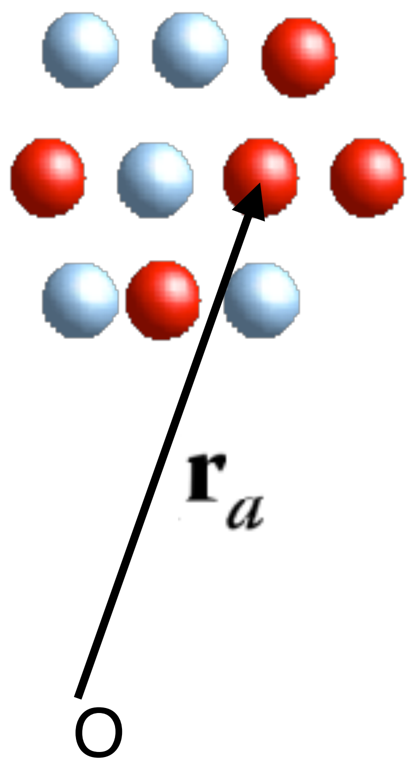
The Hamiltonian

$$H = \sum_a \frac{\mathbf{p}_a^2}{2M} + \frac{1}{2}\sum_{a\neq b} U_{ab}$$

Translational invariance tells us

$$H = \frac{\mathbf{P}^2}{2MA} + H_{\text{int}} \quad \mathbf{P} = \sum_a \mathbf{p}_a$$

$$H_{\text{int}} = \frac{1}{2A}\sum_{a,b}\frac{(\mathbf{p}_a - \mathbf{p}_b)^2}{2M} + \frac{1}{2}\sum_{a\neq b} U_{ab}$$




$$\Psi = e^{i\mathbf{PR}}\,\Psi'$$

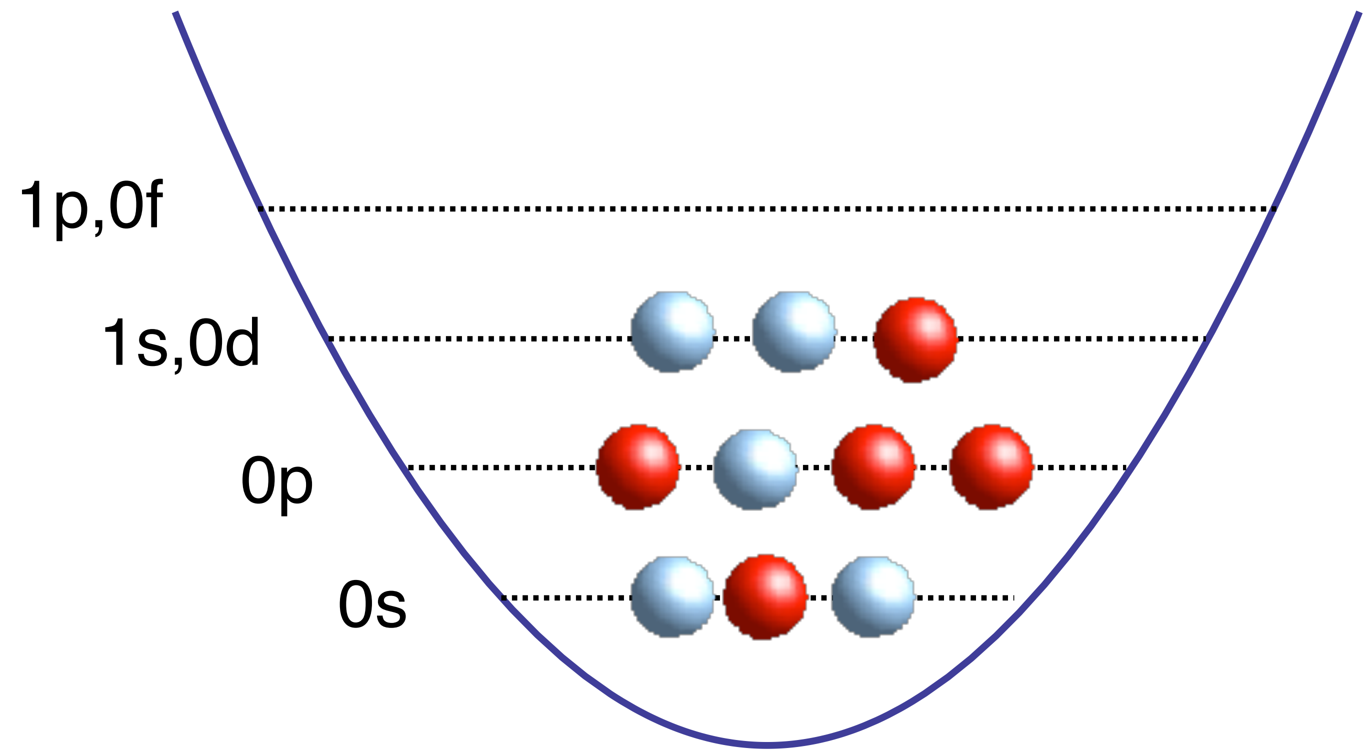
# The Nuclear Shell Model

$$H = \frac{\mathbf{P}^2}{2MA} + H_{\text{int}} \qquad H(\omega_0) = H + \frac{AM\omega_0^2}{2}\mathbf{R}^2 = H_{\text{cm}}(\omega_0) + H_{\text{int}}$$

$$H_{\text{cm}}(\omega_0) = \frac{\mathbf{P}^2}{2MA} + \frac{AM\omega_0^2}{2}\mathbf{R}^2 \qquad \Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

$$E(\omega_0) = \hbar\omega_0(N_{\text{cm}} + 3/2) + E'$$

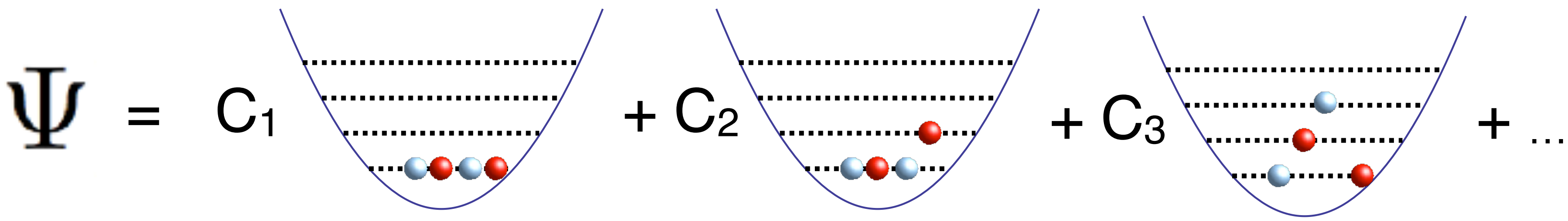
$$N_{\text{cm}} = 2n + \ell$$



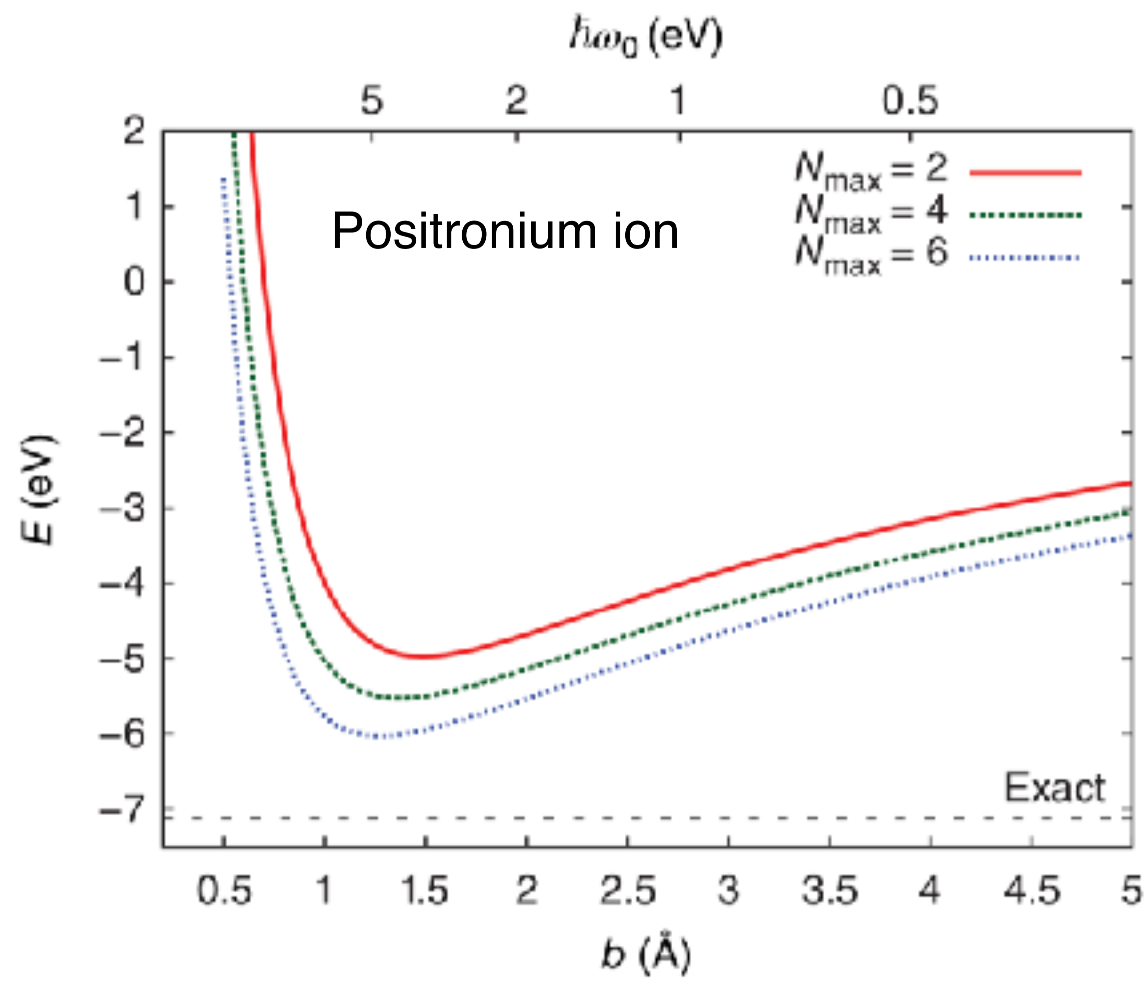
# Configuration interactions (variational principle)

**Configurations (Slater determinants or more)**

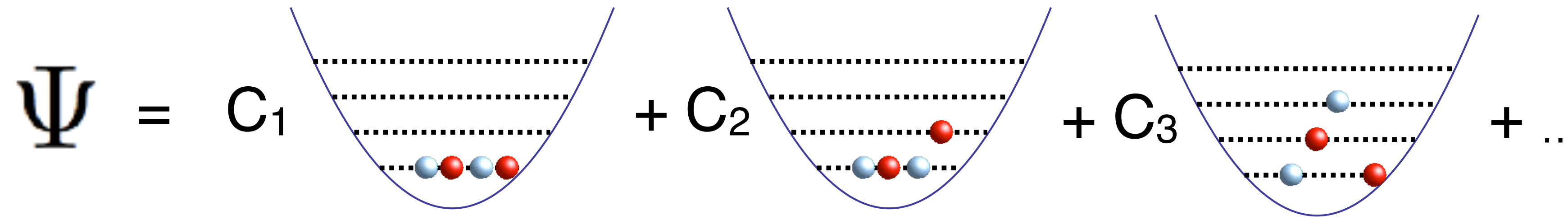
$$|\Psi(\mathbf{r_a},\mathbf{r_b})\rangle = \hat{\Psi}|0\rangle = \sum_{i,j} C_{ij} a_i^\dagger a_j^\dagger |0\rangle \qquad \langle \mathbf{r_a},\mathbf{r_b}| a_i^\dagger a_j^\dagger |0\rangle = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_i(\mathbf{r_a}) & \phi_i(\mathbf{r_b}) \\ \phi_j(\mathbf{r_a}) & \phi_j(\mathbf{r_b}) \end{vmatrix}$$



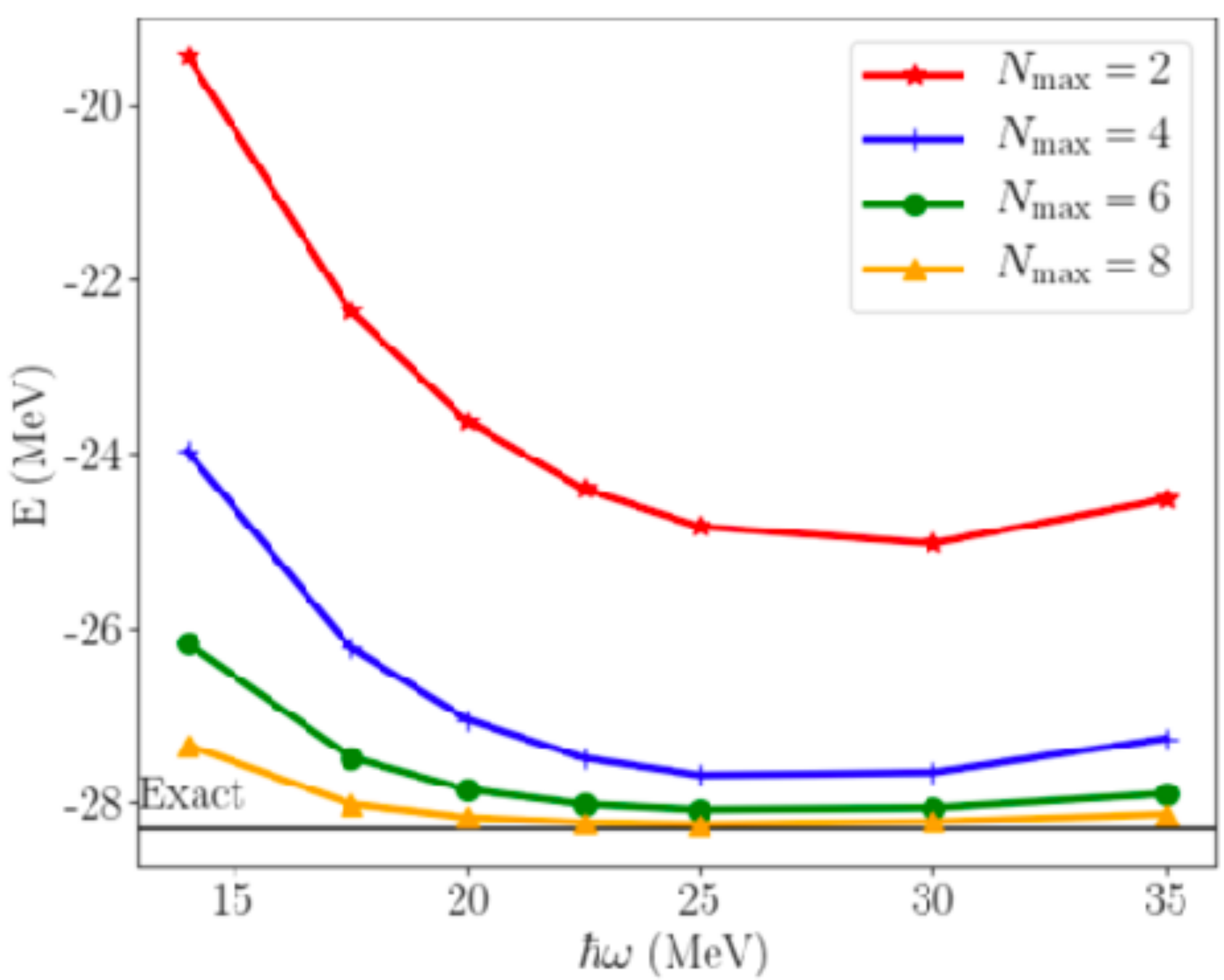

Second quantization
Full antisymmetry
Fast numeric strategies
Selection of basis and truncation

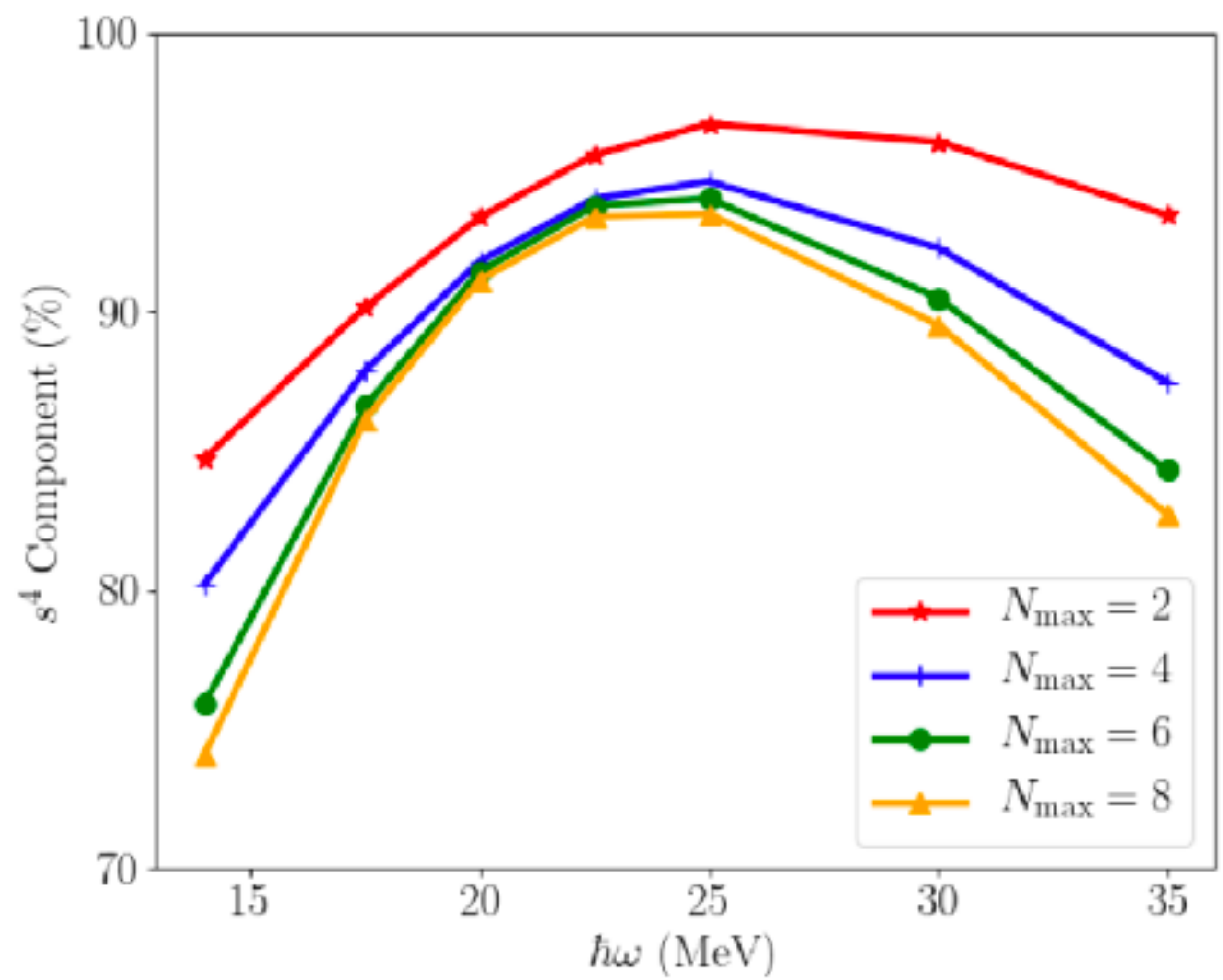


# Configuration interactions (variational principle)




**Alpha particle in no-core shell model**




JISP-16 interaction

Code at http://www.volya.net [cosmo]

# Translational invariance and Center of Mass (CM)

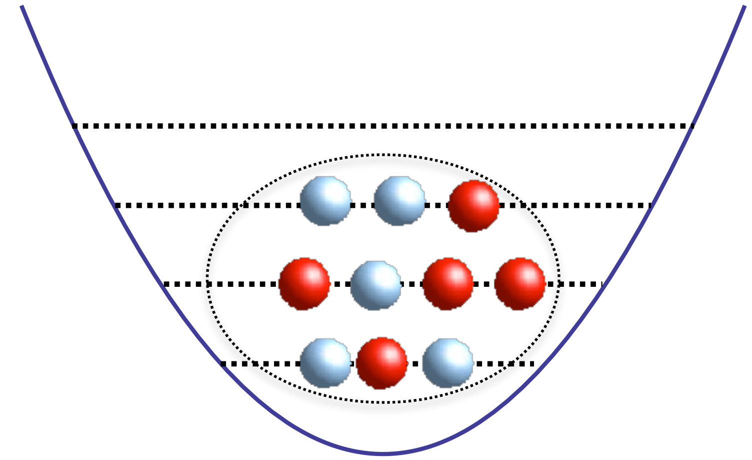
**Shell model, Glockner-Lawson procedure**

$$\Psi_D = \phi_{000}(\mathbf{R}_D)\,\Psi'_D$$

SM state

Center-of-mass vibration

Intrinsic state



**Controlling CM with operator** $\mathbf{R}$

Control only CM quanta

$$D_\mu = \sqrt{\frac{4\pi}{3}} R_\mu \qquad R_\mu = \sqrt{\frac{\hbar}{2Am\omega}}(\mathcal{B}^\dagger_\mu + \mathcal{B}_\mu)$$

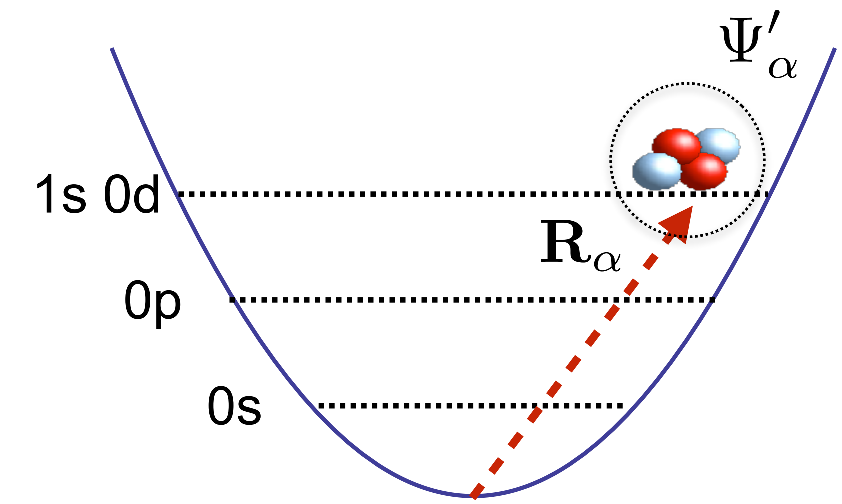
# Center-of-Mass boosts

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

$\mathcal{B}^\dagger$ and $\mathcal{B}$ CM quanta creation and annihilation (vectors)

$$\Psi_{n+1\ell m} \propto \mathcal{B}^\dagger \cdot \mathcal{B}^\dagger\,\Psi_{n\ell m}$$

$\mathcal{B}^\dagger \times \mathcal{B}$ CM angular momentum operator



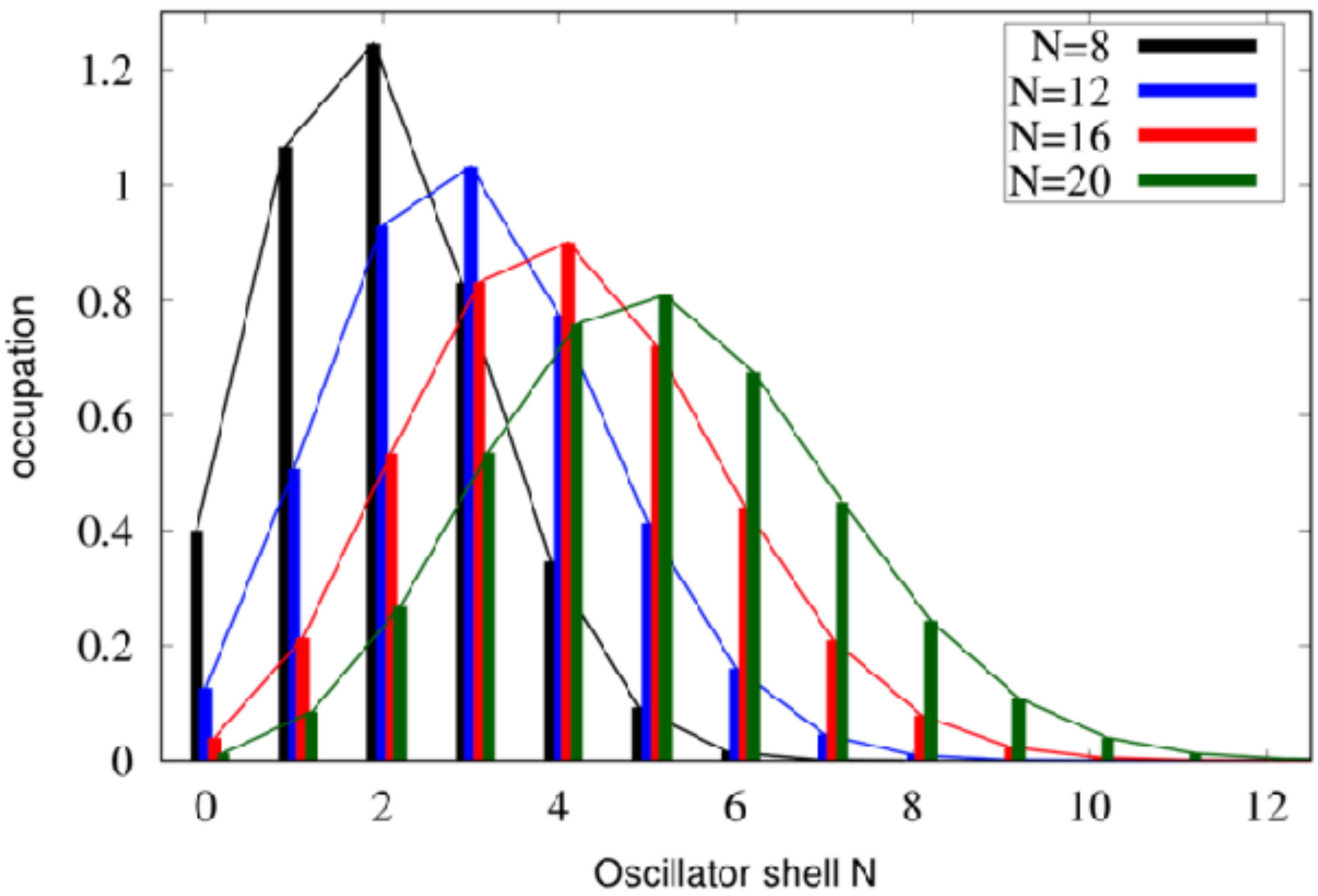

$$N = 2n + \ell$$

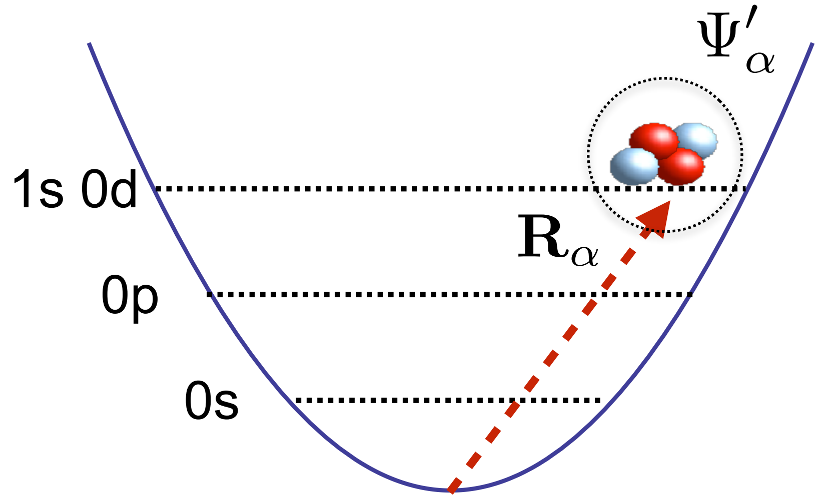
Select configuration content of NCSM wave functions for $^4$He with hΩ = 20 MeV boosted by 8 quanta (L = 0).

| Configuration | $N_{\max} = 0$ | $N_{\max} = 4$ |
|---|---|---|
| $(sd)^4$ | 0.038 | 0.035 |
| $(p)(sd)^2(pf)$ | 0.308 | 0.282 |
| $(p)^2(pf)^2$ | 0.103 | 0.094 |
| $(p)^2(sd)(sdg)$ | 0.154 | 0.141 |
| $(p)(sd)(sdg)(pfh)$ | 0.000 | 0.005 |
| $(p)(sd)(pf)(sdg)$ | 0.000 | 0.009 |


K Kravvaris and A. Volya, Journal of Phys, Conf. Proc. 863, 012016 (2017)

# CM-boosted configuration from shell model perspective

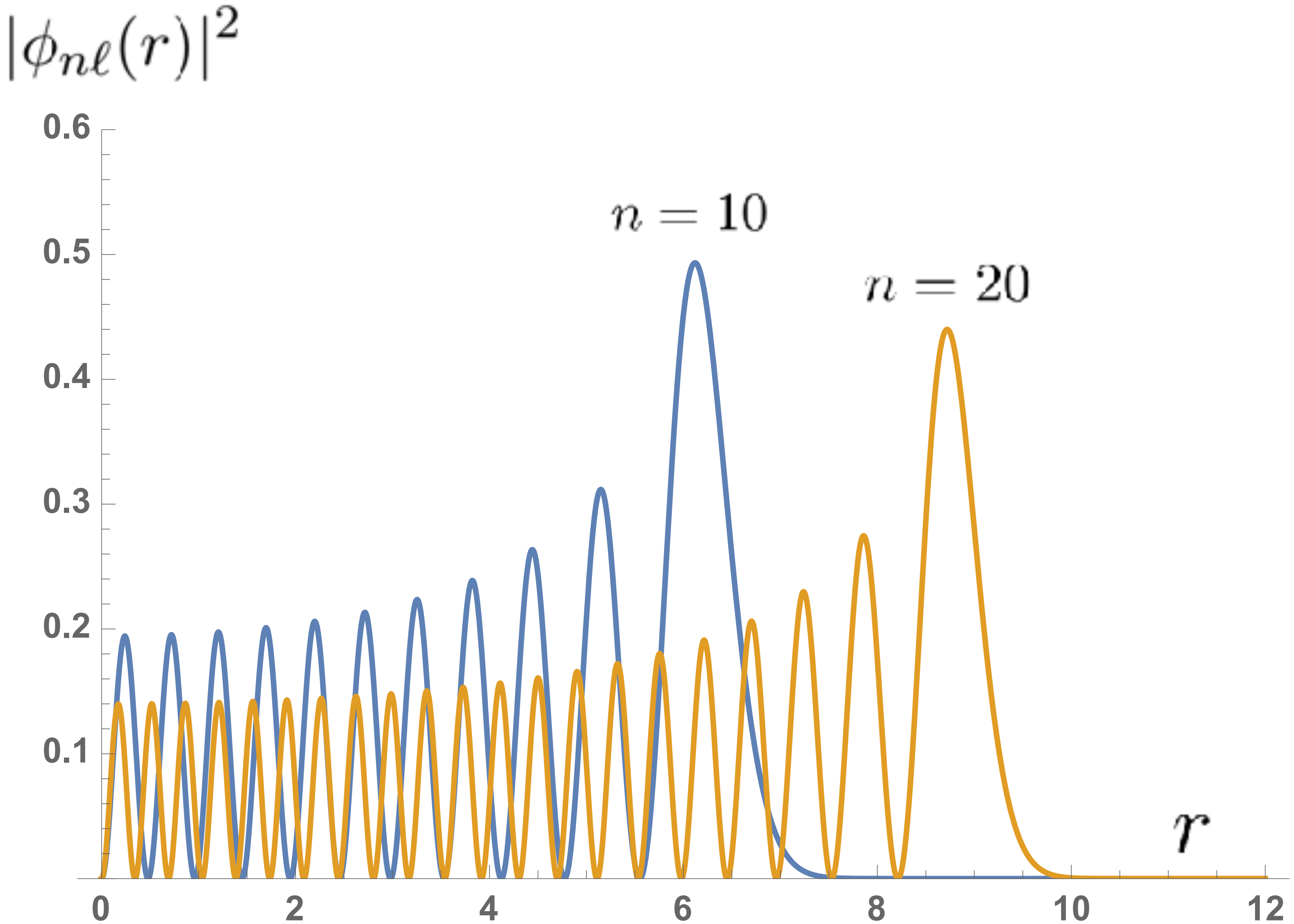


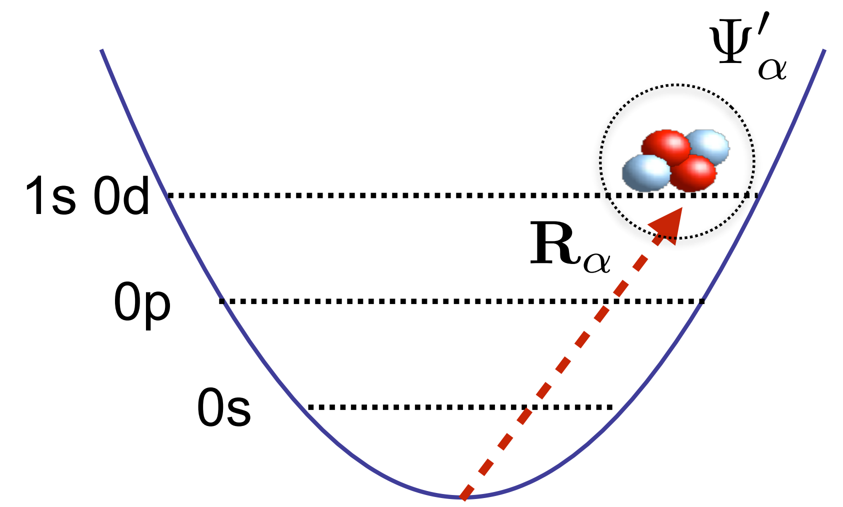




K Kravvaris and A. Volya, Journal of Phys, Conf. Proc. 863, 012016 (2017)

# CM-boosted configuration from shell model perspective

$$\langle \mathbf{R}^2 \rangle = \frac{\hbar}{M\omega_0}(N_{\rm cm} + 3/2) \qquad |\phi_{n\ell}(r)|^2 \sim \frac{1}{\sqrt{\langle \mathbf{R}^2 \rangle - r^2}}$$



# Configuration Interaction

**State, equivalent to operator (polymorphism)**

$$|\Psi\rangle \equiv \hat{\Psi}^\dagger|0\rangle = \sum_{\{1,2,3,\ldots A\}} \langle 1,2\ldots A|\Psi\rangle\, \hat{a}_1^\dagger \hat{a}_2^\dagger \ldots \hat{a}_A^\dagger|0\rangle$$

$$|\Psi_\alpha\rangle = \Psi_\alpha^\dagger|\rangle = \sum_{\{m\}} X_m^\alpha a_{m_1}^\dagger a_{m_2}^\dagger a_{m_3}^\dagger a_{m_4}^\dagger|\rangle$$

$$|\Psi_{\rm D}\rangle = \Psi_{\rm D}^\dagger|\rangle = \sum_{\{m\}} X_m^{\rm D} a_{m_1}^\dagger a_{m_2}^\dagger \ldots a_{m_{A_{\rm D}}}^\dagger|\rangle$$

Anti-symmetrized channel wave function components are generated by acting with state creation operator and forward ordering.

$$|\Psi_{\rm C}\rangle = \Psi_\alpha^\dagger \Psi_{\rm D}^\dagger|\rangle$$

Code at http://www.volya.net [cosmo]

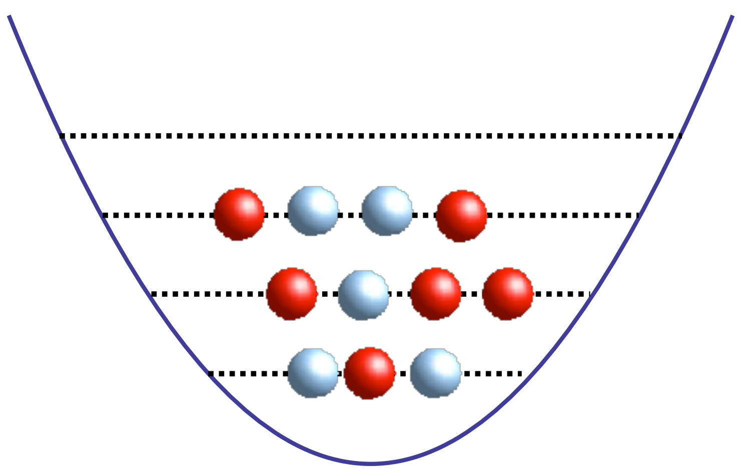
# Configuration interaction approach and clustering

**Traditional shell model configuration m-scheme**

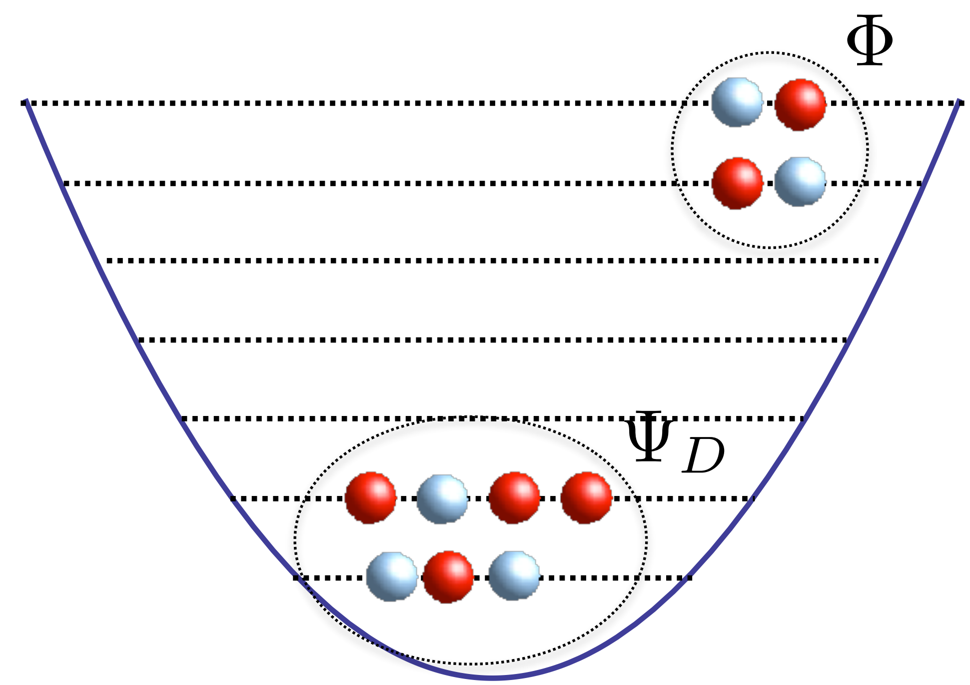
**Cluster configuration**

$$|\mathrm{channel}\rangle \sim |\Phi\Psi_D\rangle \equiv \Phi^\dagger\Psi_D^\dagger|0\rangle$$

$$|\Psi\rangle = \Psi^\dagger|0\rangle \sim a_1^\dagger a_2^\dagger \ldots a_A^\dagger|0\rangle$$

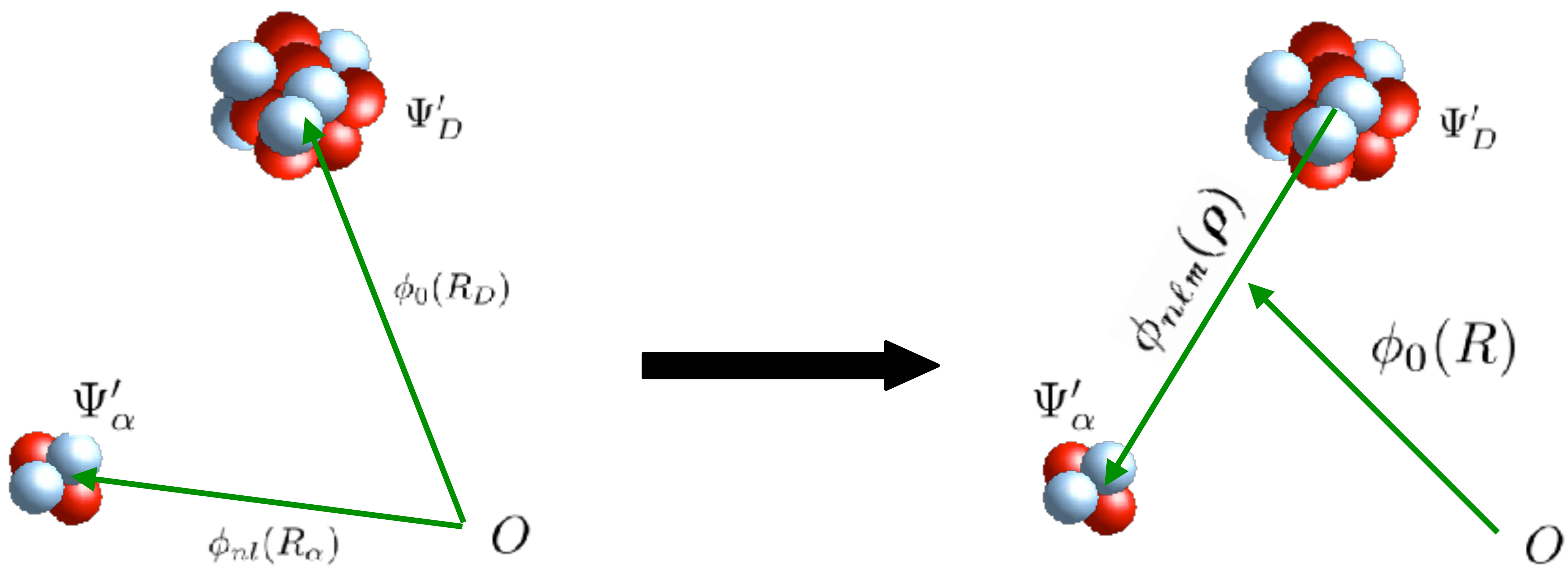


**+**




$$|\Psi\rangle \quad + \quad \Phi^\dagger|\Psi_D\rangle$$

# Recoil Recoupling

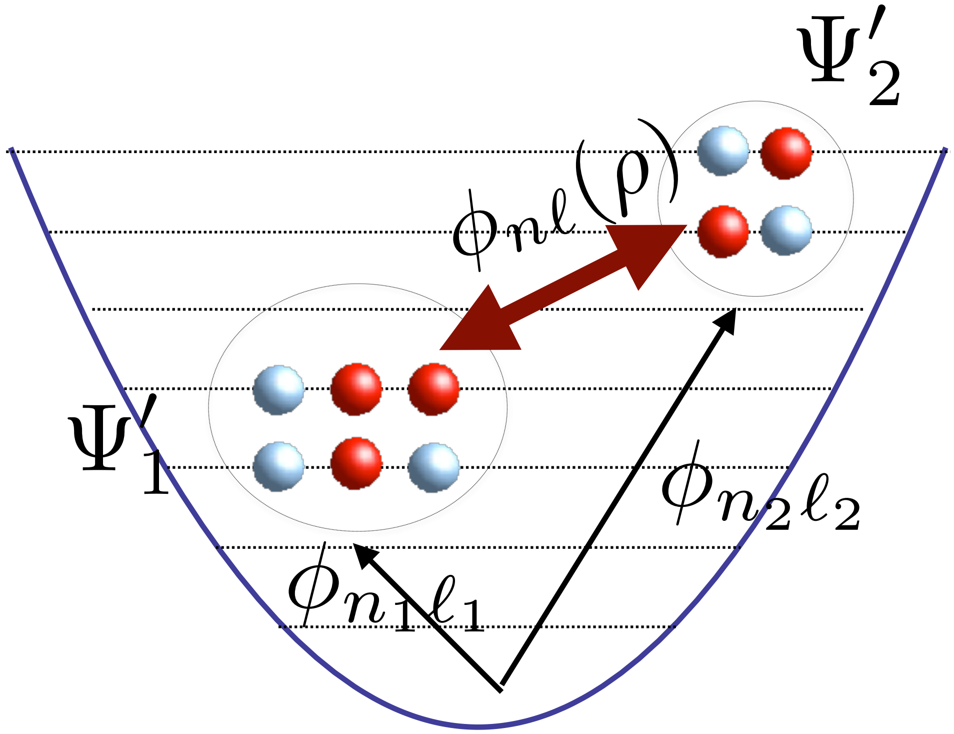



- Recoupling is done with Talmi-Moshinsky brackets

$$\Phi_{n\ell m} = \mathcal{A}\left\{\phi_{000}(\mathbf{R})\phi_{n\ell m}(\boldsymbol{\rho})\Psi'^{(1)}\Psi'^{(2)}\right\}$$

# Clustering reaction basis channel

**(basis states for clustering)**




$$\Psi = \phi_{000}(\mathbf{R})\Psi'$$

Boost

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

CM-Recouple

$$\Phi_{n\ell m} = \mathcal{A}\left\{\phi_{000}(\mathbf{R})\phi_{n\ell m}(\boldsymbol{\rho})\Psi'^{(1)}\Psi'^{(2)}\right\}$$

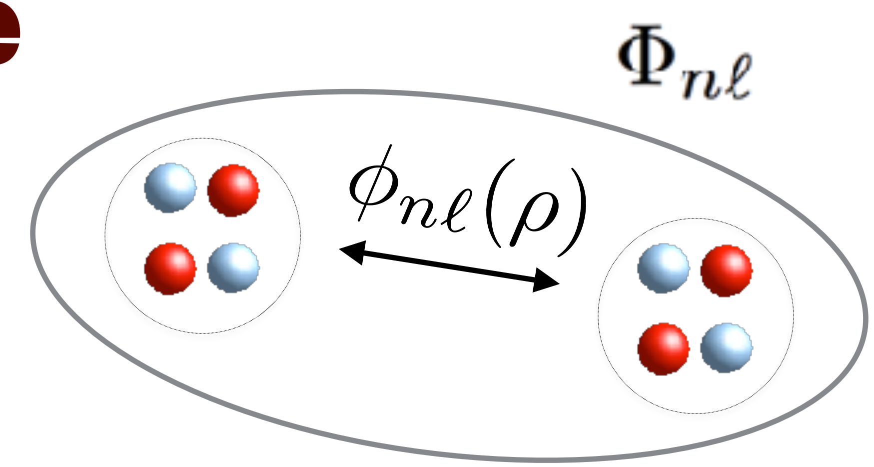
$$\Phi^{\dagger}_{n\ell} = \sum_{\substack{n_1\ell_1\\ n_2\ell_2}} \mathcal{M}^{n\ell 00;\ell}_{n_1\ell_1 n_2\ell_2}\left[\Psi^{\dagger}_{n_1\ell_1 m_1} \times \Psi^{\dagger}_{n_2\ell_2 m_2}\right]_{\ell}$$

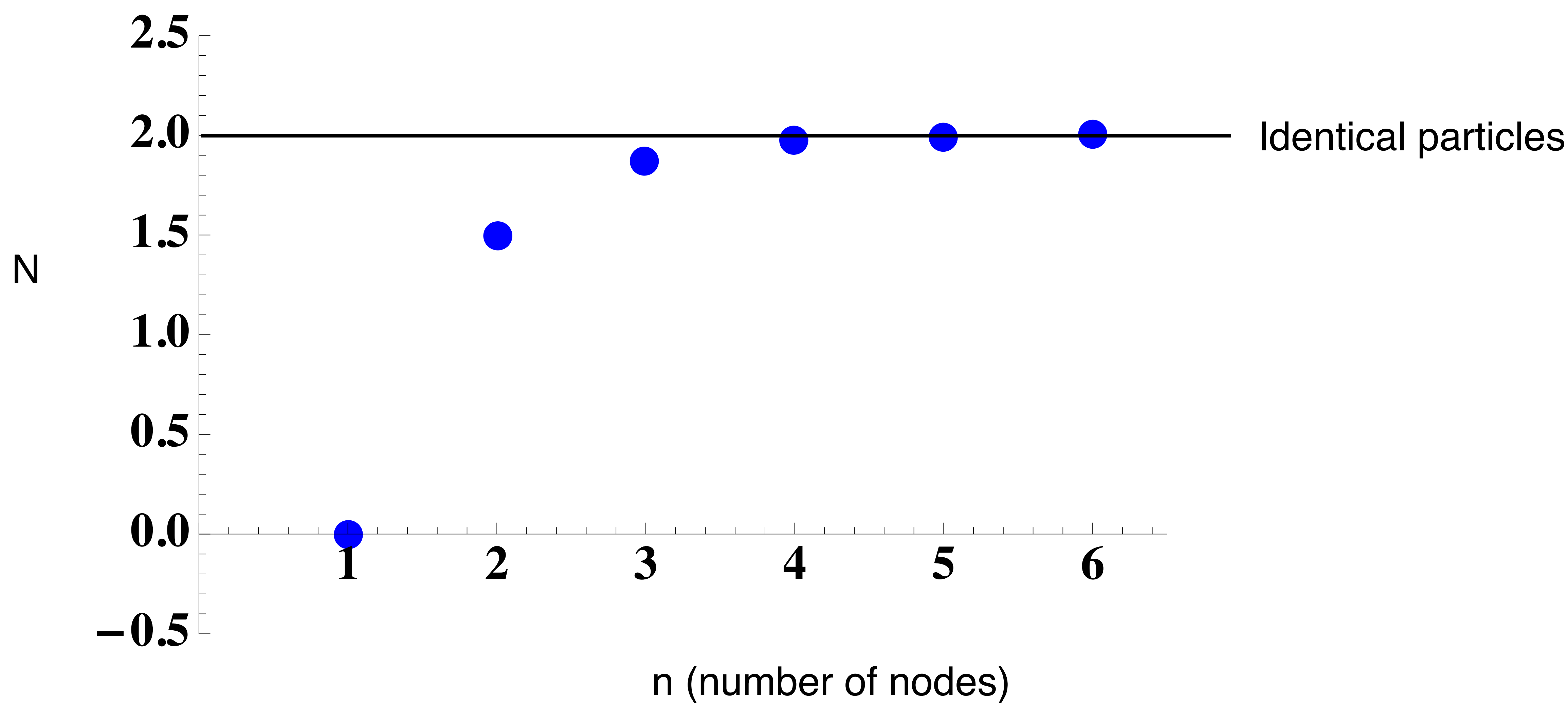
# Resonating group method $^8$Be

$$\mathcal{F}_\ell(\rho) = \sum_n \chi_n \Phi_{n\ell}$$

$$\sum_n \mathcal{H}^{(\ell)}_{nn'} \chi_{n'} = E \sum_n \mathcal{N}^{(\ell)}_{nn'} \chi_{n'}$$

$$\mathcal{H}^{(\ell)}_{nn'} = \langle \Phi_{n\ell} | H | \Phi_{n'\ell} \rangle \qquad \mathcal{N}^{(\ell)}_{nn'} = \langle \Phi_{n\ell} | \Phi_{n'\ell} \rangle$$



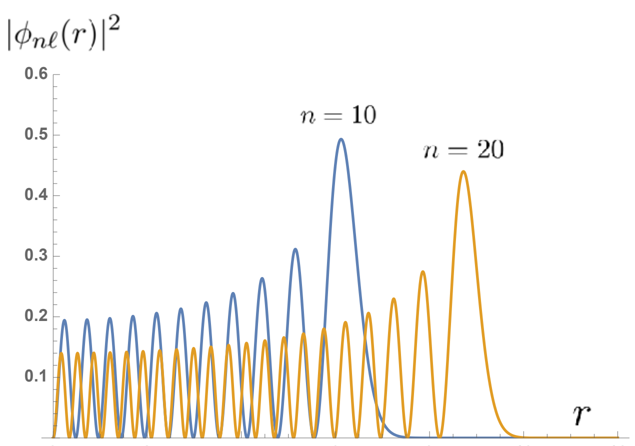



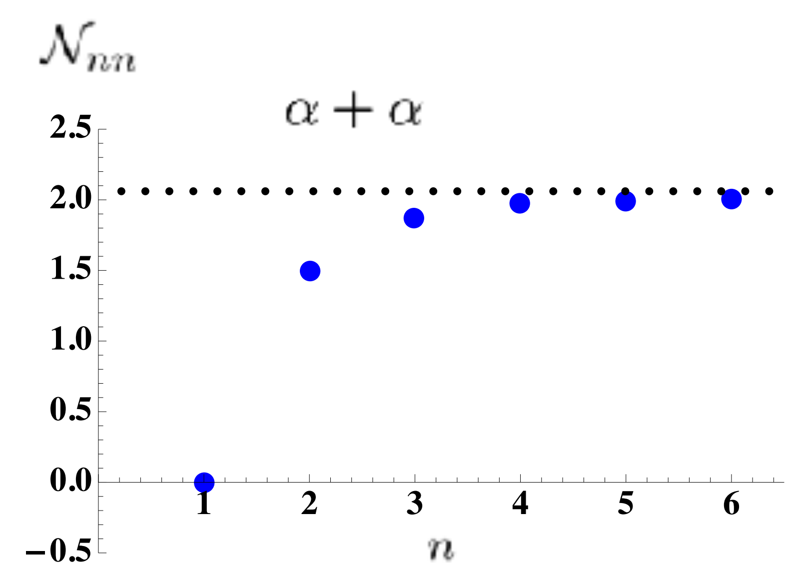
# Resonating group method and reactions

$$\sum_n \mathcal{H}^{(\ell)}_{nn'} \chi_{n'} = E \sum_n \mathcal{N}^{(\ell)}_{nn'} \chi_{n'}$$

$$\begin{pmatrix} \mathcal{H}_{00} & \dots & \mathcal{H}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{H}_{n0} & \dots & \mathcal{H}_{nn} & T_{nn+1} & 0 & \vdots \\ 0 & 0 & T_{n+1n} & T_{n+1n+1} & T_{n+1n+2} & 0 \\ 0 & \dots & 0 & T_{n+2n+1} & T_{n+2n+2} & \ddots \\ 0 & \dots & \dots & 0 & \ddots & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix} = E \begin{pmatrix} \mathcal{N}_{00} & \dots & \mathcal{N}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{N}_{n0} & \dots & \mathcal{N}_{nn} & 0 & 0 & \vdots \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & \dots & \dots & 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix}$$

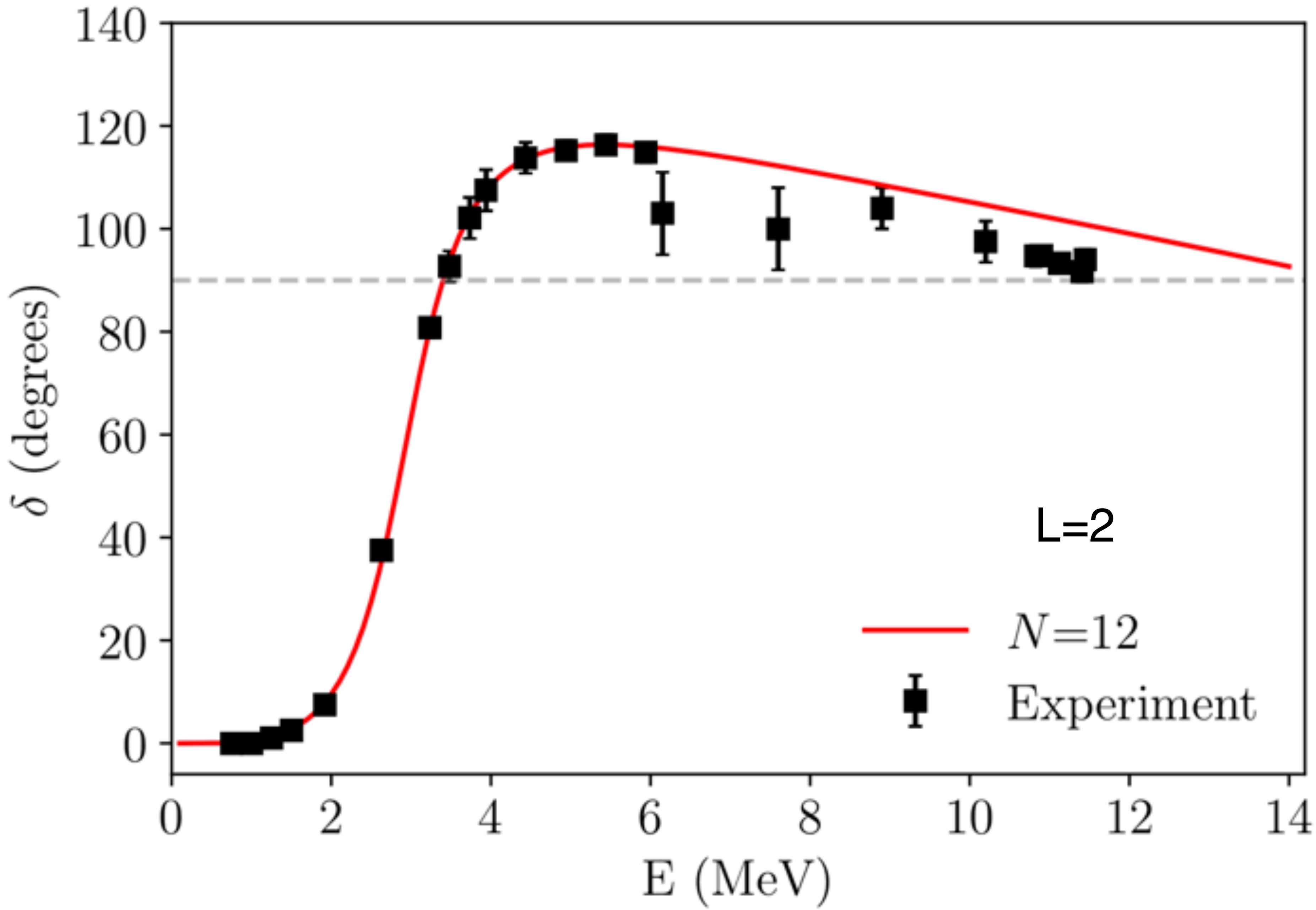
Asymptotic solution with phase shift



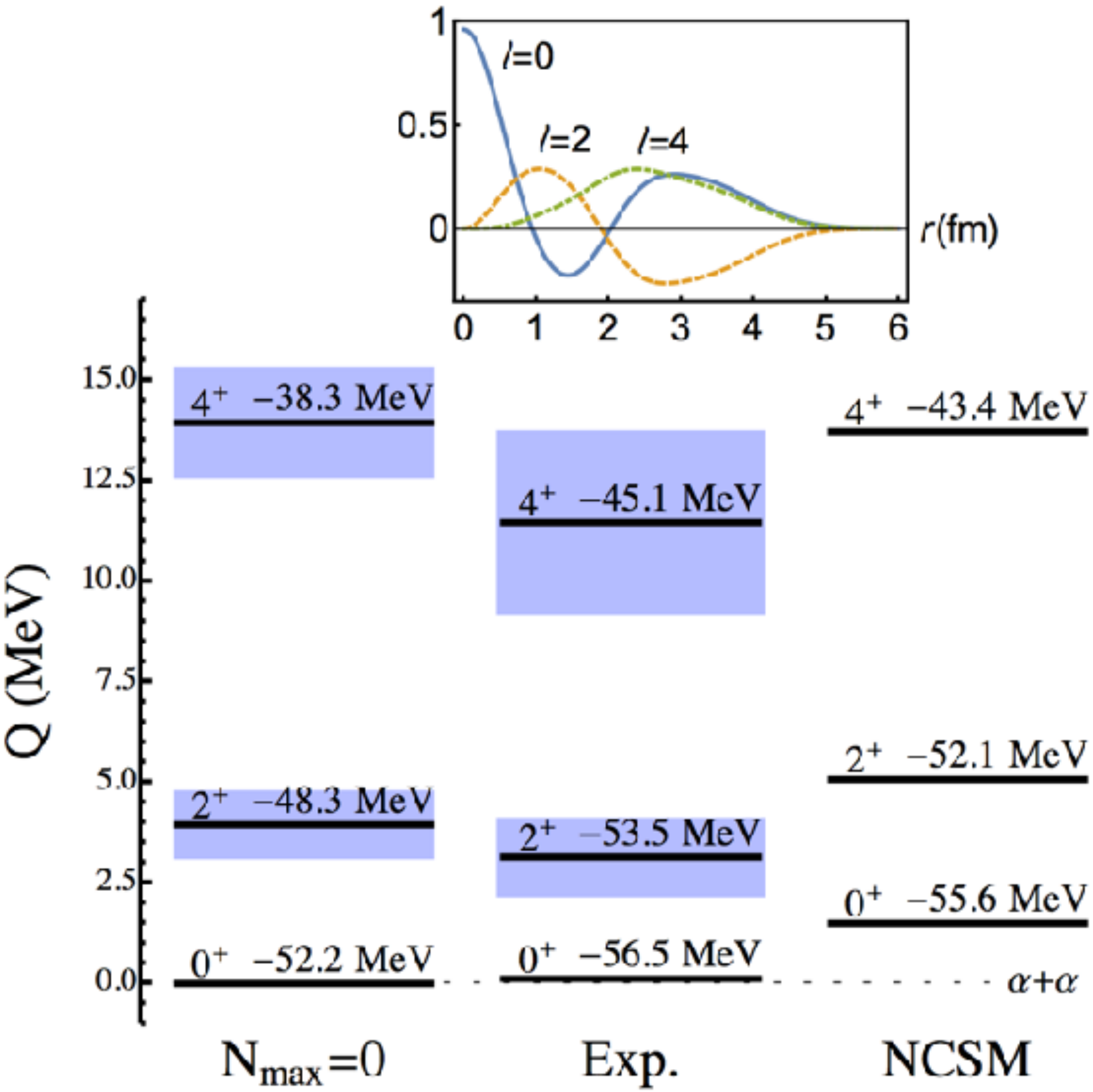

J-matrix (or HORSE) method: J. M. Bang, Annals of Physics **280**, 299 (2000)
Experimental data: Phys. Rev. 168, 1114 (1968); Nucl. Phys. **A287**, 317 (1977)

# alpha+alpha scattering phase shifts



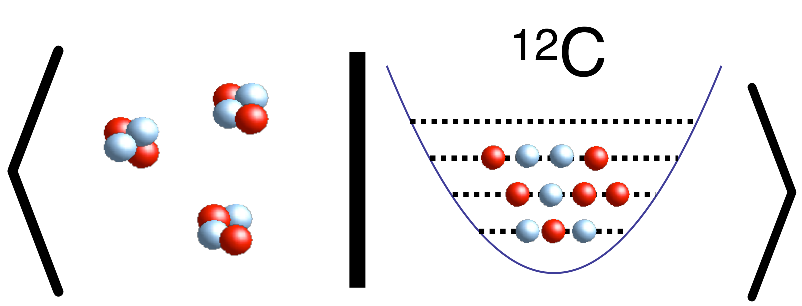

Experimental data from S. A. Afzal, A. A. Z. Ahmad, and S. Ali, Rev. Mod. Phys. 41, 247 (1969).

# Resonating group method $^8$Be results




| | | Theory | Exp. |
|---|---|---|---|
| **l=0** | ev | 8.7 | 5.6 |
| **l=2** | MeV | 1.3 | 1.5 |
| **l=4** | MeV | 2.1 | 3.5 |

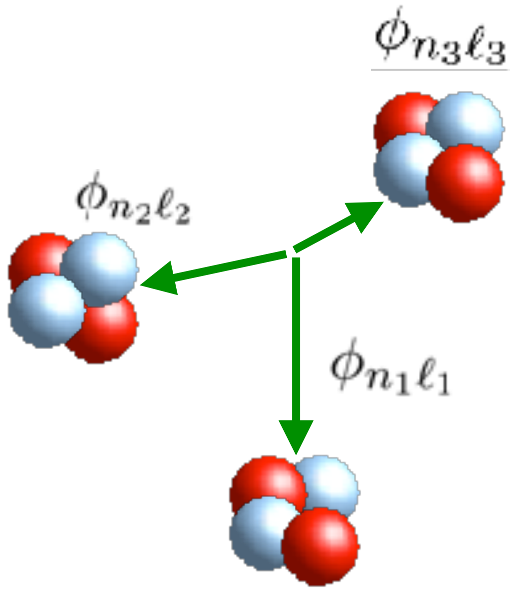
# Spectroscopic amplitudes

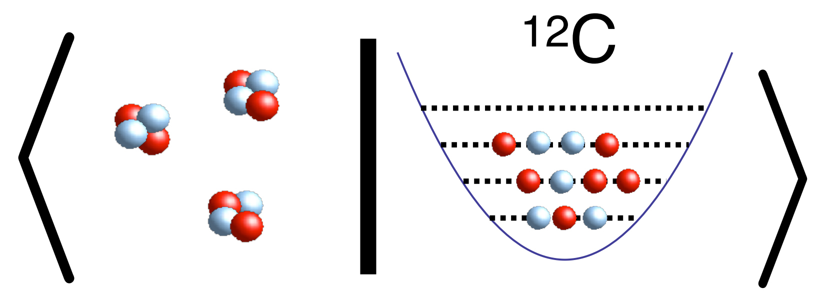



| parent | $J^\pi$ | channel | $\lvert\langle\Psi\lvert\mathcal{F}_\ell\rangle\rvert$ |
|---|---|---|---|
| $^8$Be[4] | $0^+$ | $\alpha[0]+\alpha[0]$ | 0.905 |
| $^8$Be[4] | $2^+$ | $\alpha[0]+\alpha[0]$ | 0.898 |
| $^8$Be[4] | $4^+$ | $\alpha[0]+\alpha[0]$ | 0.874 |
| $^8$Be[4] | $0^+$ | $\alpha[2]+\alpha[2]$ | 0.961 |
| $^8$Be[4] | $2^+$ | $\alpha[2]+\alpha[2]$ | 0.957 |
| $^8$Be[4] | $4^+$ | $\alpha[2]+\alpha[2]$ | 0.943 |
| $^{10}$Be[4] | $0^+$ | $^6$He[0] + $\alpha[0]$ | 0.844 |
| $^{10}$Be[4] | $0^+$ | $^6$He[4] + $\alpha[0]$ | 0.820 |
| $^{10}$Be[4] | $2^+$ | $^6$He[0] + $\alpha[0]$ | 0.834 |
| $^{10}$Be[4] | $2^+$ | $^6$He[4] + $\alpha[0]$ | 0.796 |
| $^{12}$C[4] | $0_1^+$ | $\alpha[0]+\alpha[0]+\alpha[0]$ | 0.841 |
| $^{12}$C[4] | $0_2^+$ | $\alpha[0]+\alpha[0]+\alpha[0]$ | 0.229 |

Spectroscopic amplitudes.

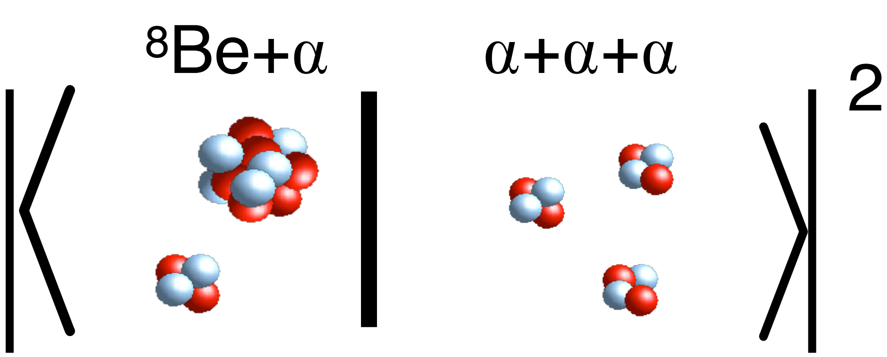
# Ttriple-alpha RGM



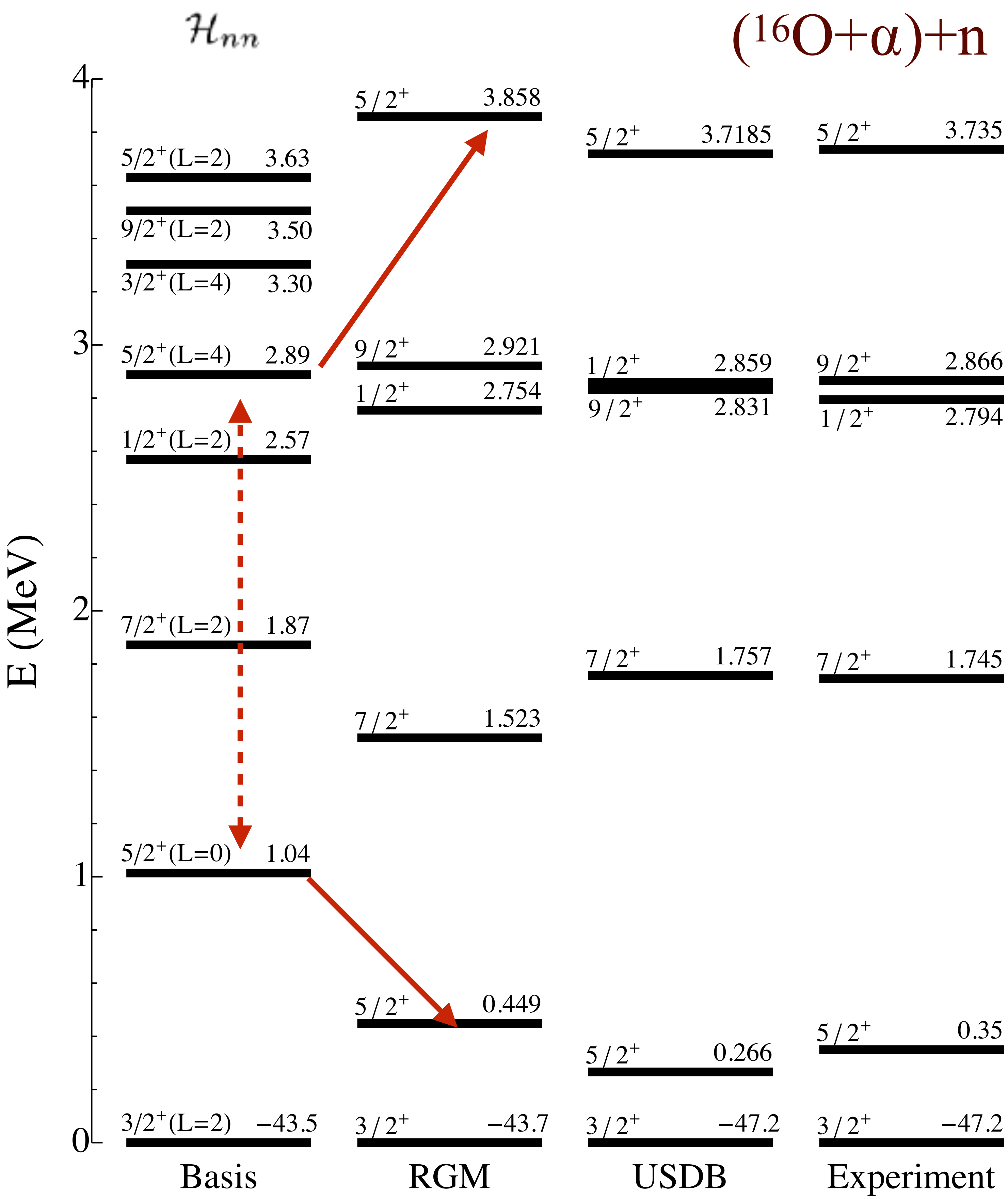

| parent | channel | overlap |
|---|---|---|
| $^{12}\mathrm{C}[4](0_1^+)$ | $\alpha[0]+\alpha[0]+\alpha[0]$ | 0.841 |
| $^{12}\mathrm{C}[4](0_2^+)$ | $\alpha[0]+\alpha[0]+\alpha[0]$ | 0.229 |

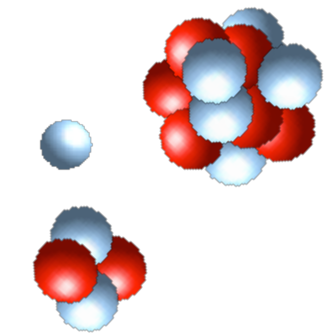
$N_{max}$(rel)=12




= 0.89

# Molecular orbits $^{21}$Ne



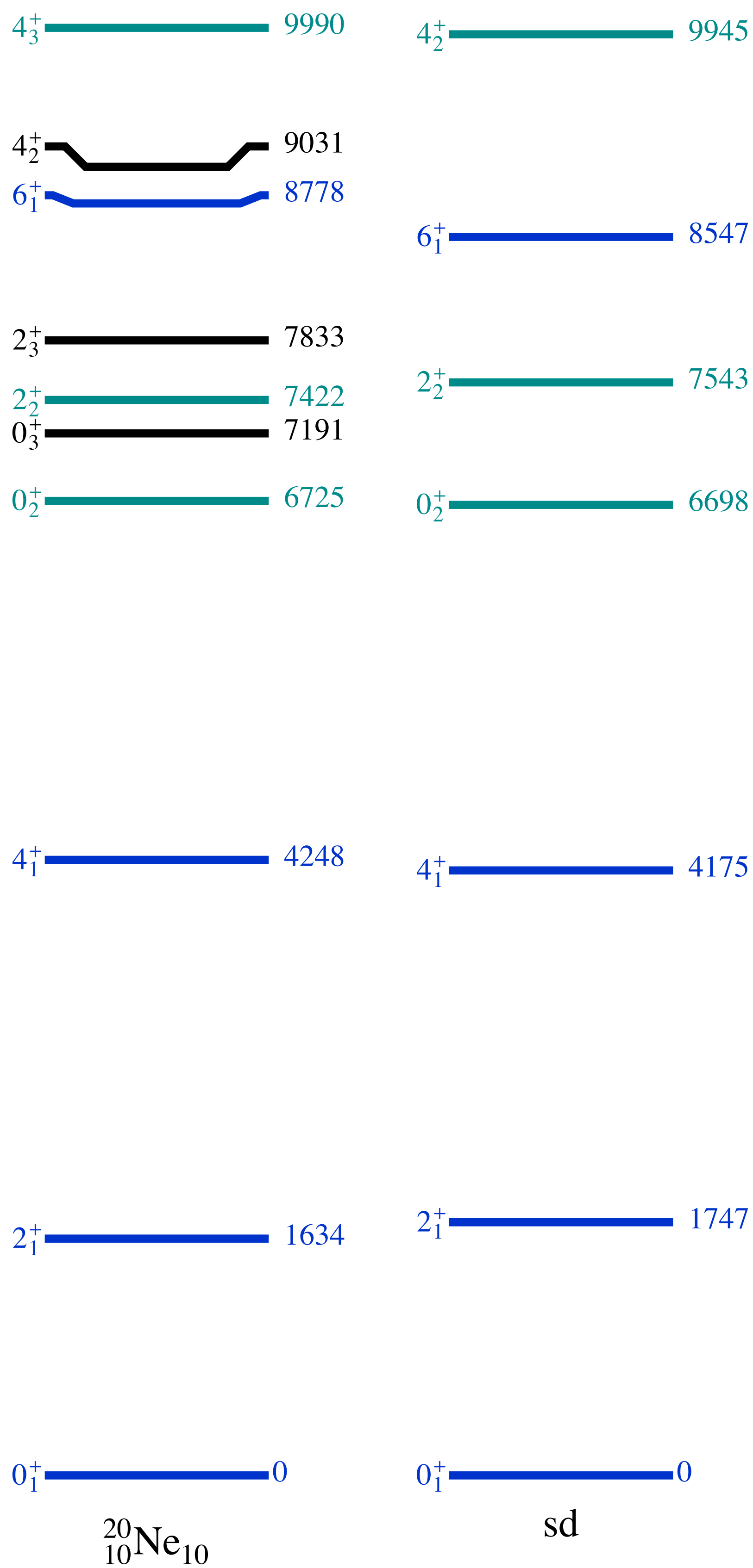
# Weak-Coupling Behavior

| $J^\pi$ | $\mathcal{S}^{(new)}$ | | | $\mathcal{S}^{(exp)}$ | | |
|---|---|---|---|---|---|---|
| | $\ell = 0$ | $\ell = 2$ | $\ell = 4$ | $\ell = 0$ | $\ell = 2$ | $\ell = 4$ |
| 3/2+ | | 1.0 | 0.18 | | $1.0 \pm 0.05$ | $0.42 \pm 0.04$ |
| 5/2+ | 0.78 | 0.02 | 0.44 | $1.04 \pm 0.41$ | ... | $0.32 \pm 0.18$ |
| 7/2+ | | 0.9 | 0.14 | | $0.91 \pm 0.08$ | $0.23 \pm 0.04$ |
| 9/2+,1/2+ | | 0.81 | 0.33 | | $0.9 \pm 0.05$ | $0.29 \pm 0.03$ |

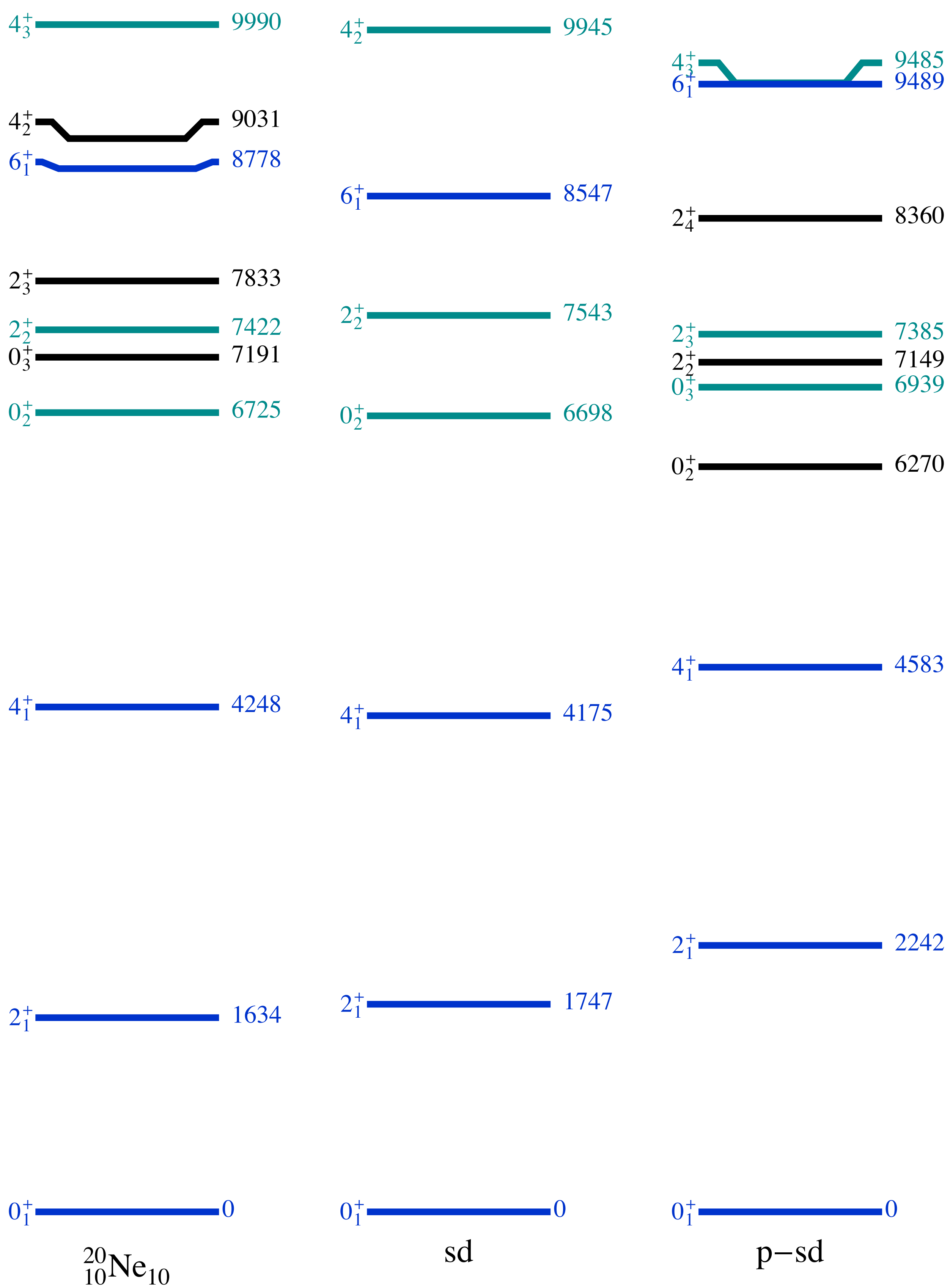

N. Anantaraman, J. P. Draayer, H. E. Gove, J. T̄oke, and H. T. Fortune. Alpha-particle stripping to $^{21}$Ne. Phys.Rev. C18, 815 (1978); Phys.Lett. 74B, 199 (1978)

A. K. Nurmukhanbetova, V. Z. Goldberg, D. K. Nauruzbayev, M. S. Golovkov, A. Volya, Phys. Rev. C 100 (2019) 062802.

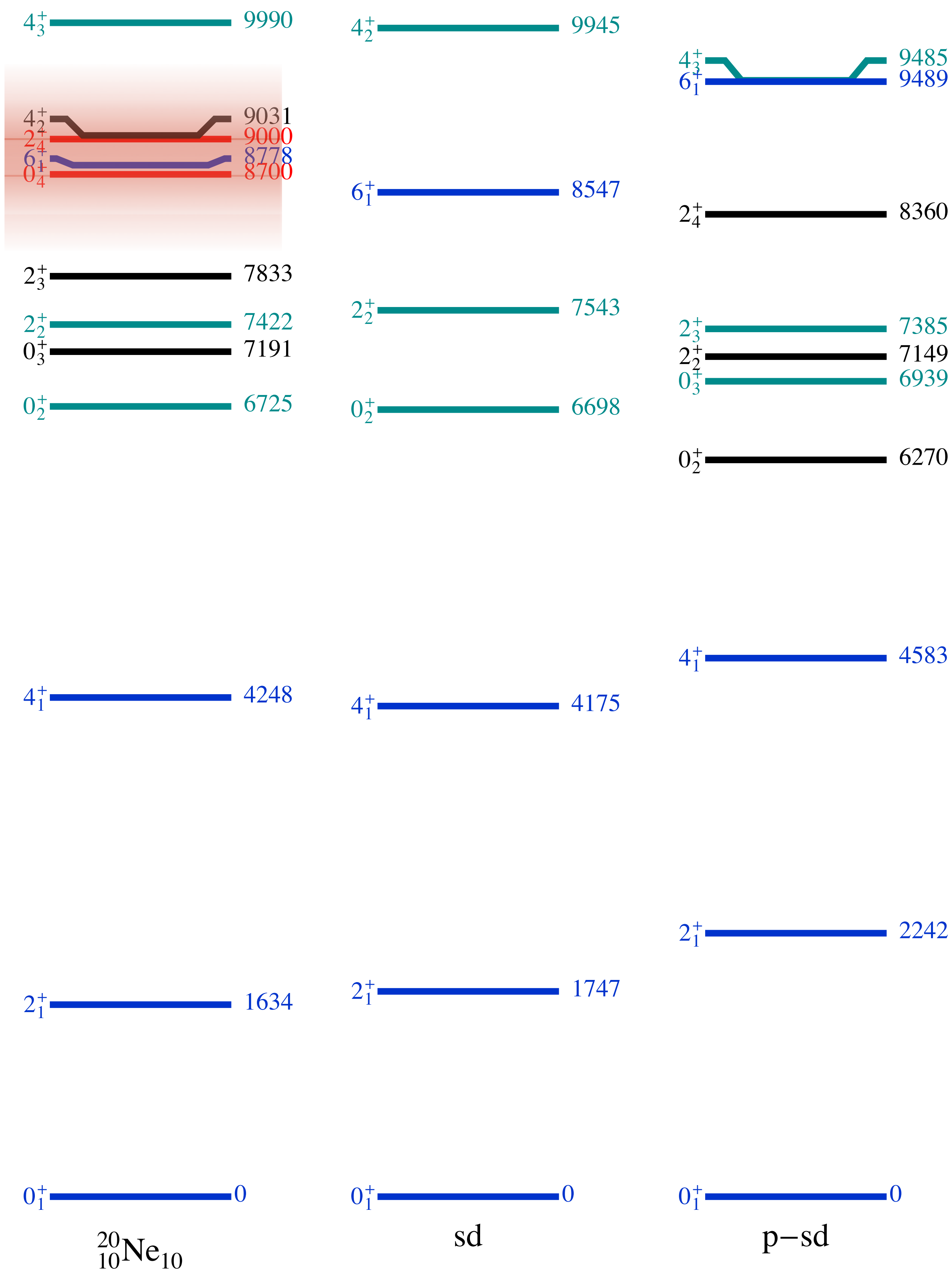
# Clustering in $^{20}Ne$



# Clustering in $^{20}$Ne



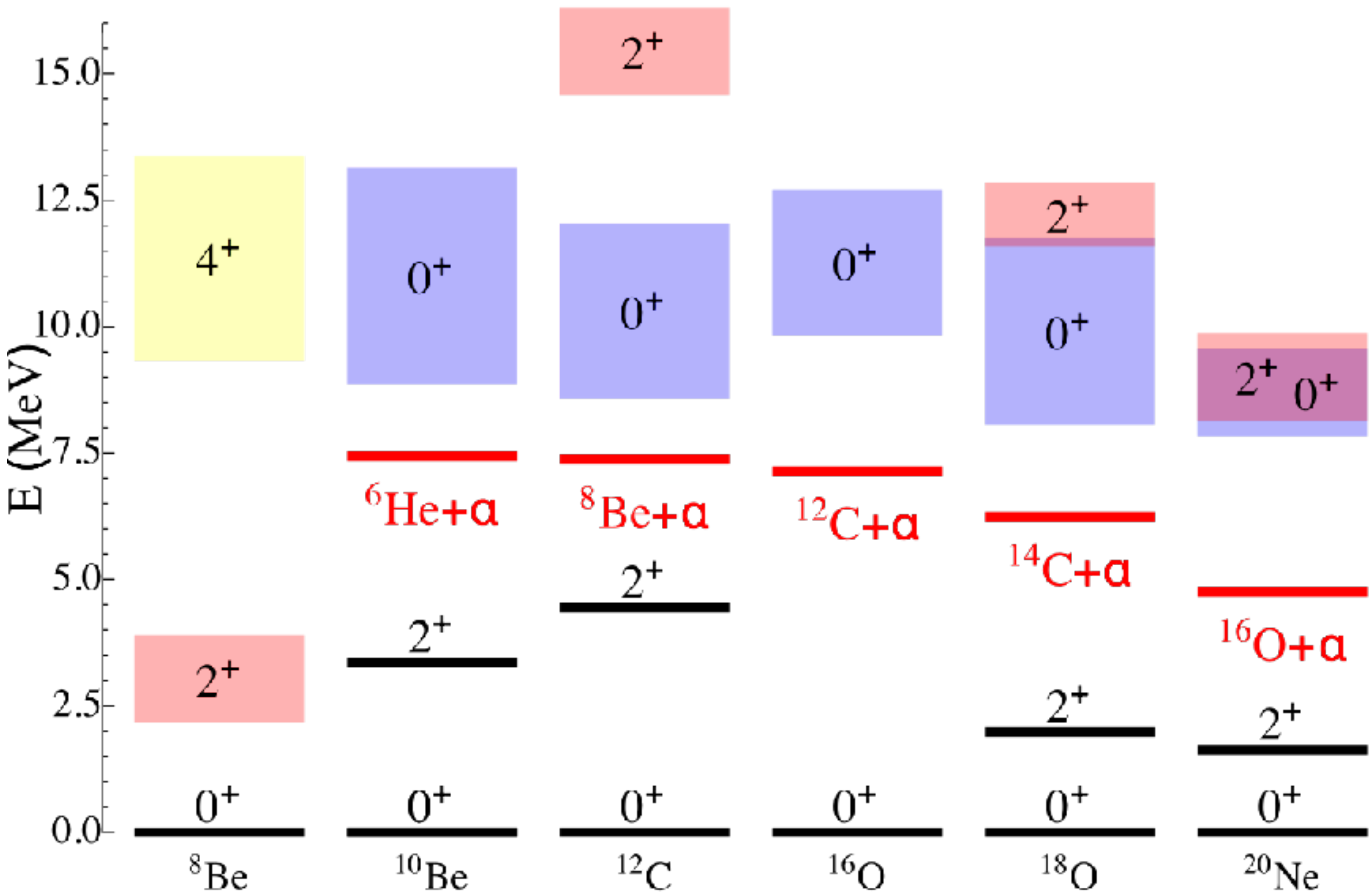
# Clustering in $^{20}$Ne



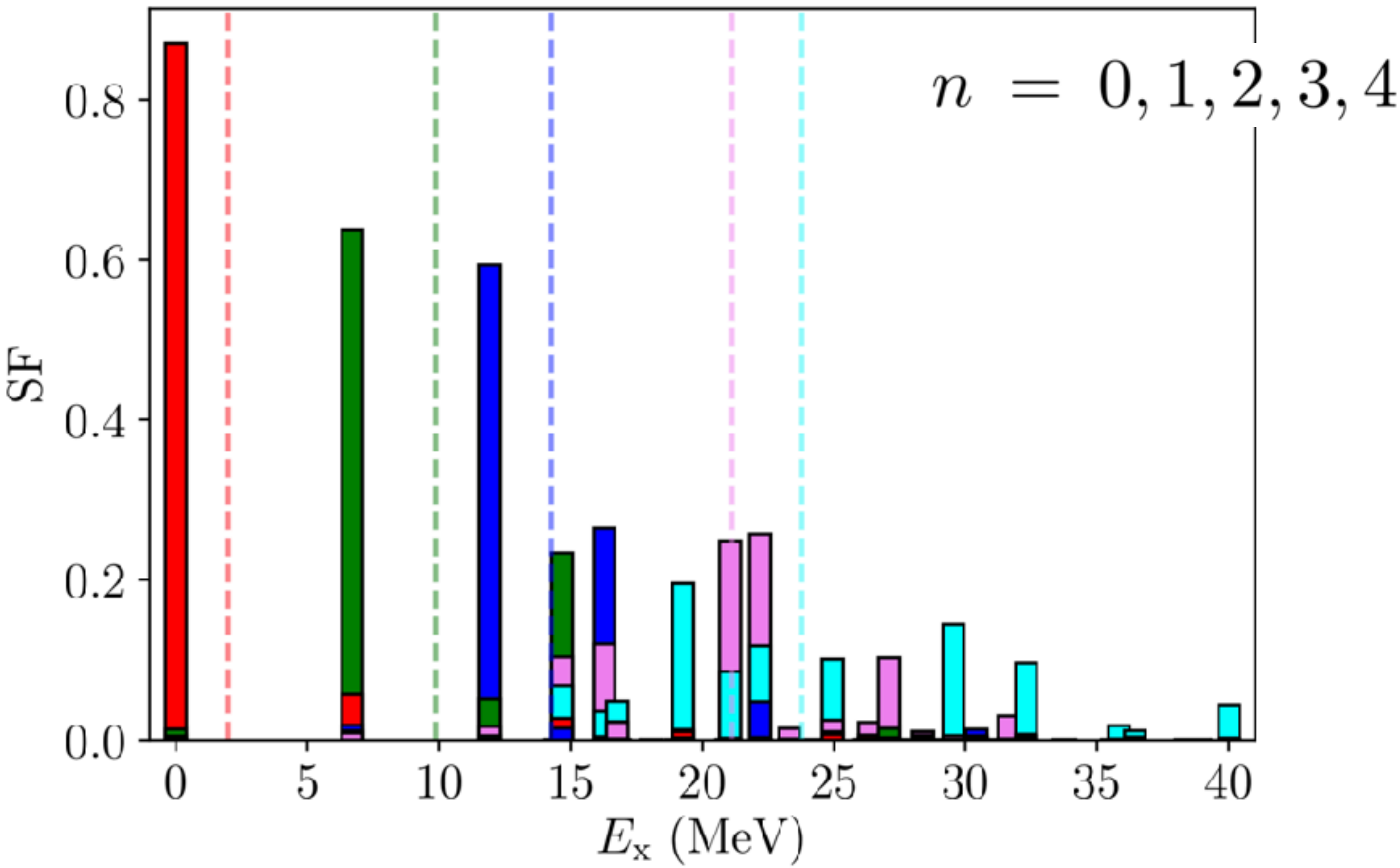

| J | E MeV | Γ width | SF ex | SF th. |
|---|---|---|---|---|
| 0+ | 0 | 0 | | 0.73 |
| 2+ | 1.63 | 0 | | 0.67 |
| 4+ | 4.25 | 0 | | 0.62 |
| 0+ | 6.73 | 19 | 0.47 | 0.46 |
| 0+ | 7.19 | 3.4 | 0.02 | 0.10 |
| 2+ | 7.42 | 15 | 0.19 | 0.12 |
| 2+ | 7.83 | 2 | 0.01 | 0.09 |
| 0+ | 8.7 | 800 | 0.3 | |
| 6+ | 8.78 | 0.11 | 0.5 | 0.51 |
| 2+ | 9.00 | 800 | 0.86 | |

D.K. Nauruzbayev et al., Phys. Rev. C **96**, 014322 (2017)

# Clustering and continuum

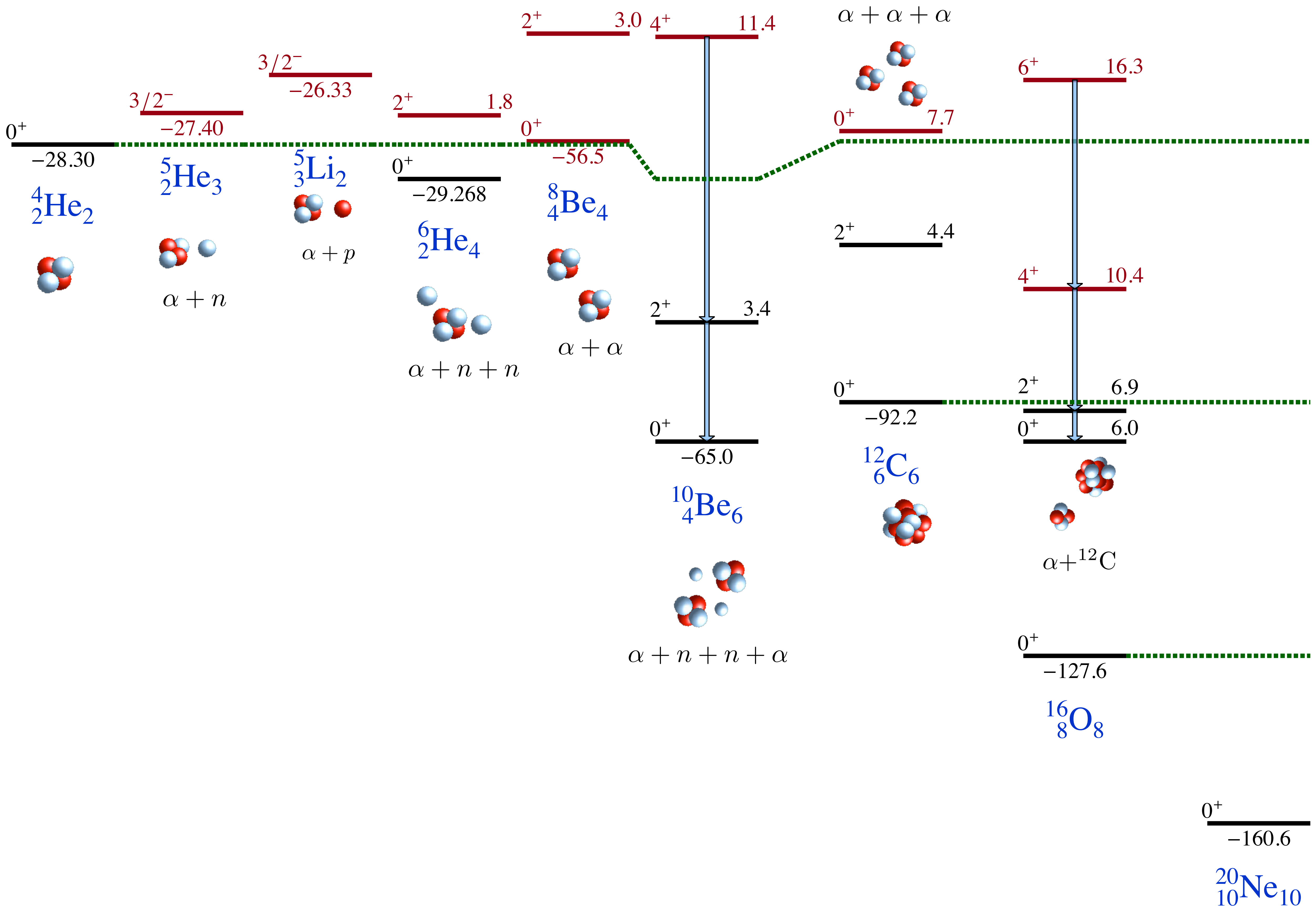


# Searching for clustering strength

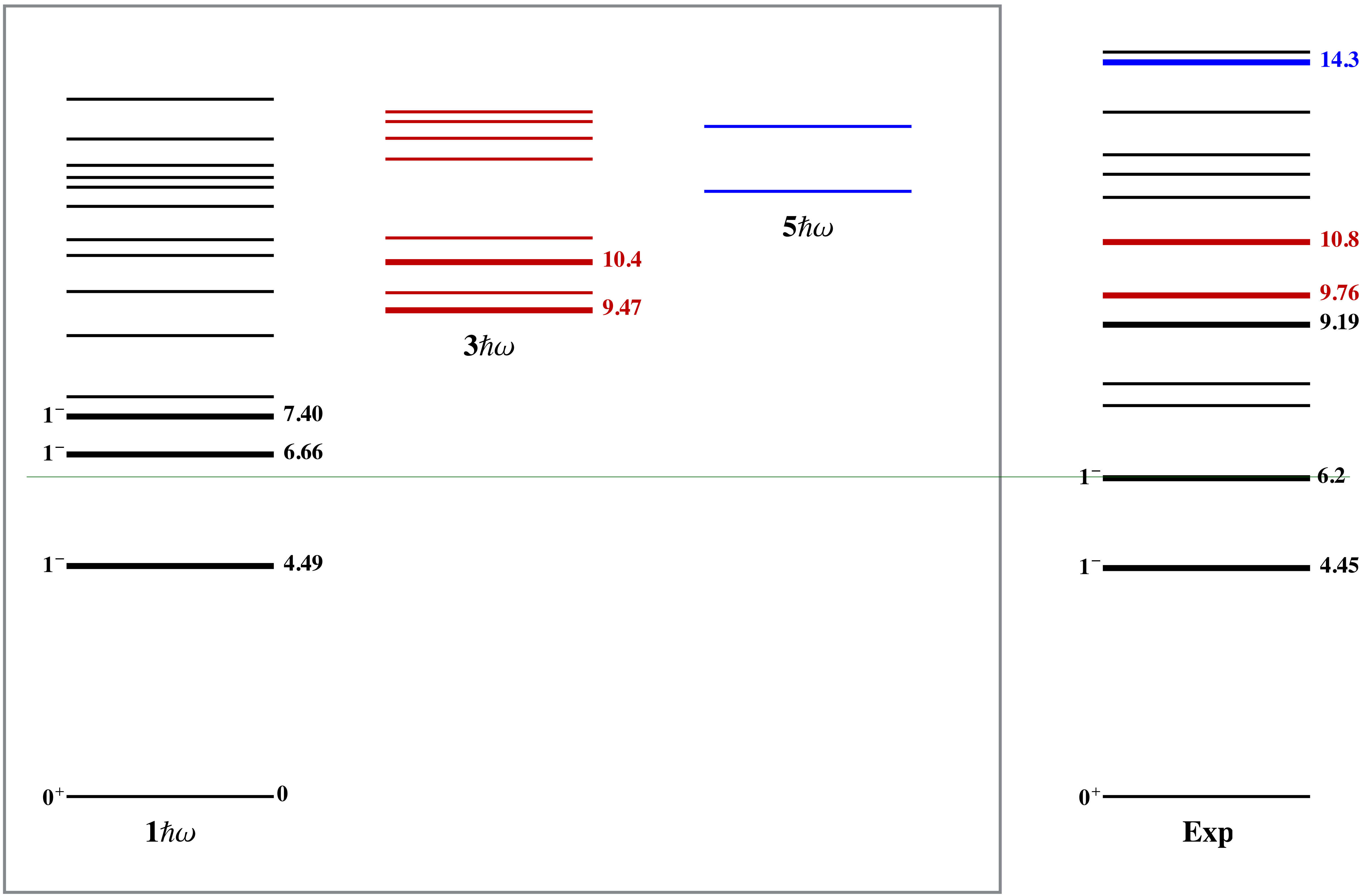



Distribution of dynamic spectroscopic factors for $^{20}$Ne → $^{16}$O(g.s.) + α. The dashed lines correspond to the RGM energies for each decay channel.

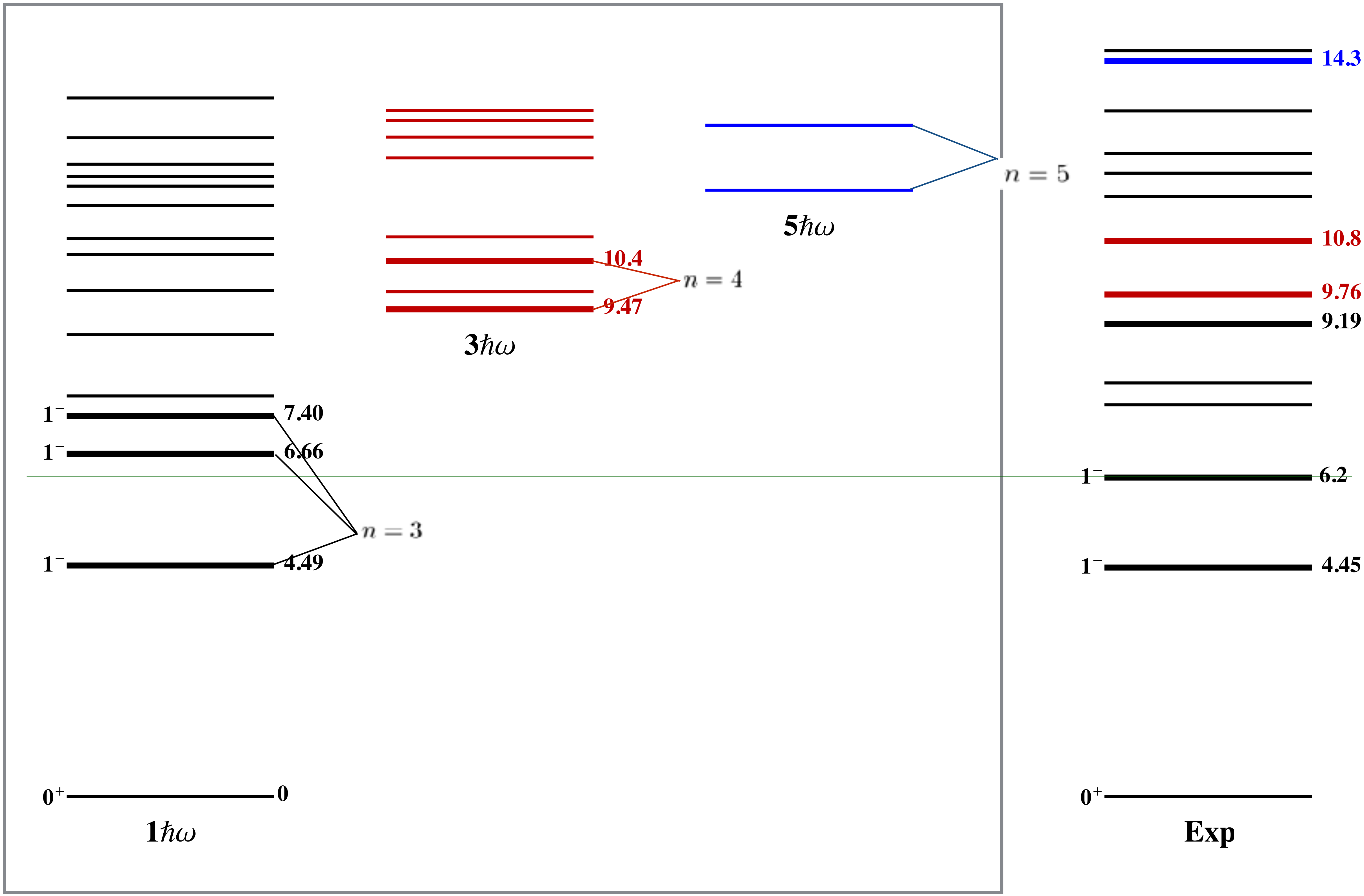
# Clustering in light nuclei



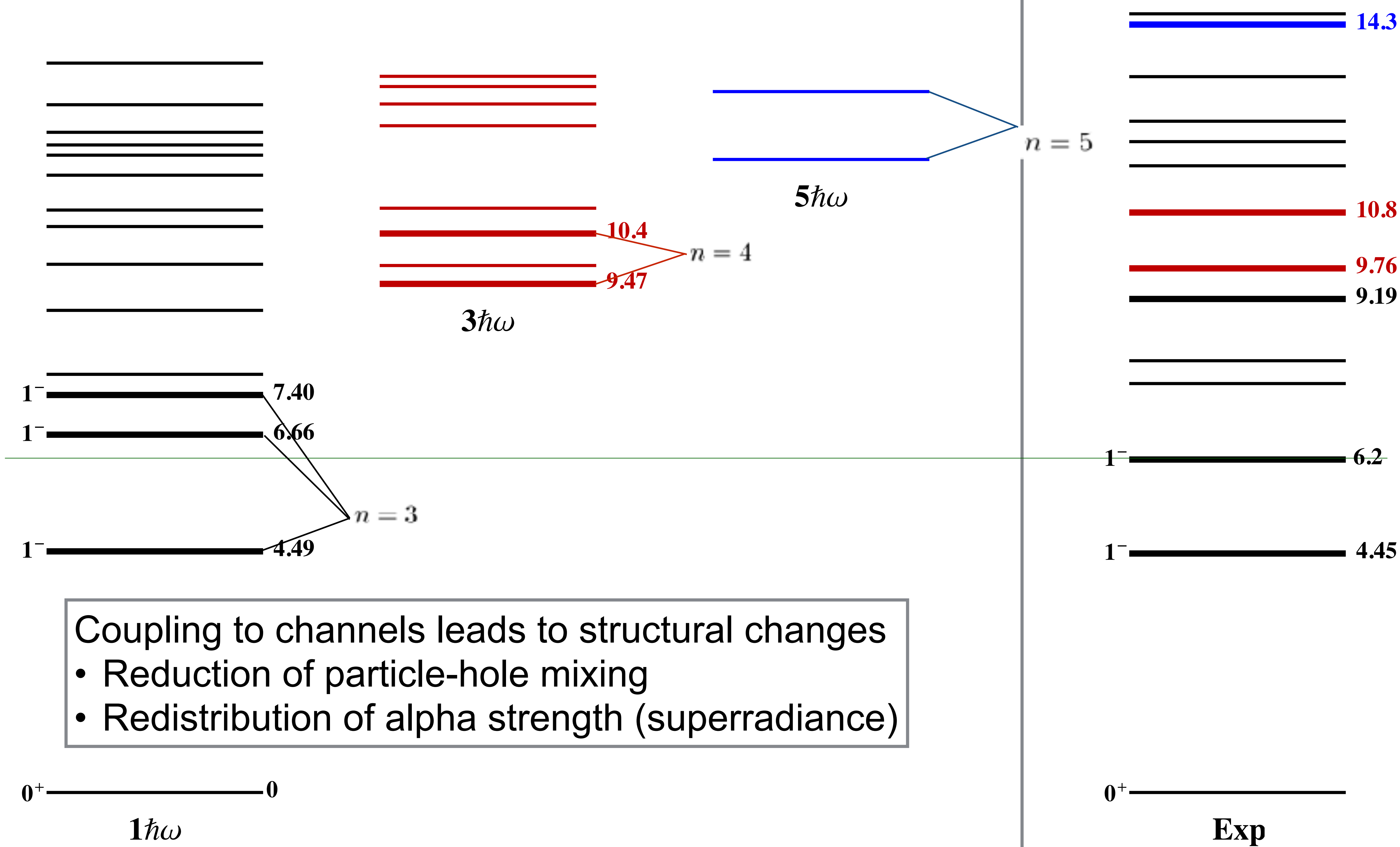
# Channel coupling in $^{18}O$
# l=1 channel



# Channel coupling in ¹⁸O
# l=1 channel



# Channel coupling in $^{18}$O
# l=1 channel




Coupling to channels leads to structural changes
- Reduction of particle-hole mixing
- Redistribution of alpha strength (superradiance)

**Thanks to all collaborators:**

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**Resources:** https://www.volya.net/ (see research, clustering)

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