A holographic description of superstrata

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(with I. Bena, E. Moscato, R. Russo, M. Shigemori, N. Warner)

Outline

- A quick review of the D1-D5-P system:
  - the Strominger-Vafa counting
- The dual CFT side:
  - microstates of the D1-D5 CFT
- Supergravity construction of D1-D5-P microstates:
  - superstrata
- Holography:
  - deriving geometry from the CFT

The D1-D5-P system

- The simplest BPS black hole with a finite-area horizon is

$$\boxed{\text{D1-D5-P on } \mathbb{R}^{4,1} \times S^1 \times T^4}$$

- At small gravitational coupling ($g_s \to 0$) the bound state of D-branes is described by a CFT
- Microstates of the CFT can be counted

$$\log(\#\text{microstates}) = 2\pi\sqrt{n_1 n_5 n_p} = S_{BH}$$

(Note: index equals degeneracy at leading order in n_i)

What happens to the microstates at finite gravitational coupling ($g_s N \gg 1$)?

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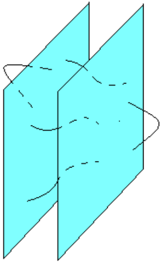
Microstate geometries

- For large $g_s N$, D-branes backreact on spacetime
- For particular microstates (coherent states), the backreaction is well described by supergravity

$g_s N \sim 0$ $\qquad\qquad$ $g_s N \gg 1$



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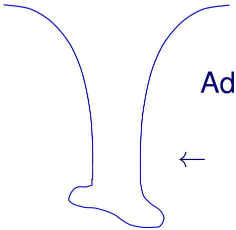

$\mathbb{R}^{4,1} \times S^1$

$AdS_3 \times S^3$

$\leftarrow \quad r \sim R_{\rm Hor}$

no horizon!

D-brane microstate $\qquad$ microstate geometry

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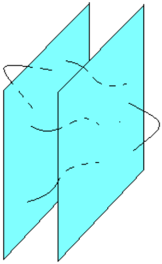

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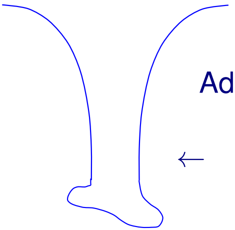

$\mathbb{R}^{4,1} \times S^1$

$\mathrm{AdS}_3 \times S^3 \rightarrow$ CFT

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no horizon!

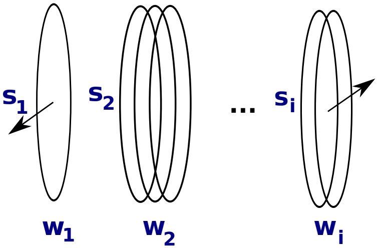
D-brane microstate $\qquad$ microstate geometry

The D1-D5 CFT

- At a special point in moduli space, the low energy limit of the D1-D5 system is described by the

$$(T^4)^N/S_N \text{ orbifold with (4,4) susy}$$

  where $N = n_1 n_5$
- States carrying D1-D5 charges are RR ground states
- Fermion zero-modes $\psi_0^{\alpha\dot{A}}$, $\tilde{\psi}_0^{\dot{\alpha}\dot{A}}$ carry spin under $SU(2)_\alpha \times SU(2)_{\dot{\alpha}} \sim SO(4)$



$\sum_i N_i w_i = N$

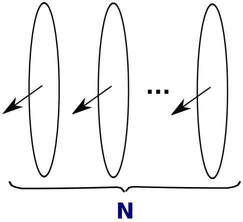
N_i # strands of winding w_i and "spin" s_i

$$\Leftrightarrow \quad \prod_i (|s_i\rangle_{w_i})^{N_i}$$

Examples I

- The simplest D1-D5 state is the maximally rotating one

$$|+,+\rangle_1^N \quad \leftrightarrow$$



- Spectral flow maps this state into the SL(2,$\mathbb{C}$) invariant vacuum:

$$|+,+\rangle_1^N \xrightarrow{s.f.} |0\rangle_{NS}$$

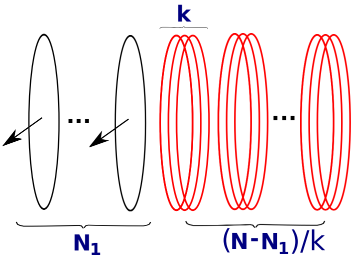
- The dual geometry is (in appropriate coordinates)

$$\mathrm{AdS}_3 \times S^3$$

Examples II

- A more general state is

$$|+,+\rangle_1^{N_1} |0,0\rangle_k^{(N-N_1)/k} \quad \leftrightarrow$$



- The dual geometry is a deformation of $AdS_3 \times S^3 \times T^4$
- The deformation is controlled by one scalar warp factor

$$Z_4 = b\, R \left(\frac{a}{\sqrt{r^2+a^2}} \right)^k \frac{\sin^k \theta}{r^2 + a^2 \cos^2 \theta} \cos k\phi$$

where $\quad |a|^2 \propto N_1 \,, \quad |b|^2 \propto (N - N_1)$

Coherent states and supergravity

- The state $|+,+\rangle_1^{N_1}|0,0\rangle_k^{(N-N_1)/k}$ is an eigenstate of R-charge but the geometry (Z_4) depends on ϕ
- The state dual to the geometry is actually the “coherent state”

$$\sum_{N_1} a^{N_1}\, b^{N-N_1}\, |+,+\rangle_1^{N_1}\, |0,0\rangle_k^{(N-N_1)/k}$$

(Kanitscheider, Skenderis, Taylor)

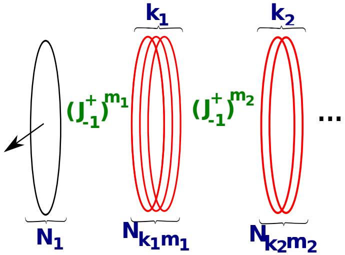
- The sum over N_1 is peaked on $N_1 \approx \overline{N}_1 \propto |a|^2$
- When $\overline{N}_1, (N-\overline{N}_1)/k \gg 1$ the state is well described by supergravity
- The supergravity parameters $a\,, b\,, \ldots$ determine the average numbers $\overline{N}_i$ of strands of each type

Adding momentum

- Susy: momentum is carried by left-moving excitations on the CFT
- For example, one can act with modes of the global chiral algebra

$$L_{-1} \xrightarrow{s.f.} L_{-1} - J^3_{-1} \quad , \quad J^+_0 \xrightarrow{s.f.} J^+_{-1}$$

- We concentrate here on J^+_{-1} ($L_{-1} - J^3_{-1}$ is work in progress)



Note:

$$J^+_{-1}|+,+\rangle_1 = 0$$

$$(J^+_{-1})^m|0,0\rangle_k = 0 \text{ for } m > k$$

How to construct the dual geometries?

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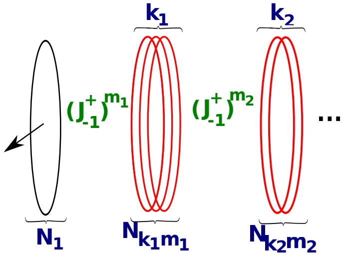

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Linearized perturbation

- If $N_{k_i m_i} \ll 1$ the states are described by a linearized perturbation around $AdS_3 \times S^3$, encoded in Z_4
- The perturbation can be derived by acting on the D1-D5 geometry $|+,+\rangle_1^{N_1} |0,0\rangle_k^{N-N_1}$ with the diffeomorphism dual to J^+_{-1} (Mathur, Saxena, Srivastava; Shigemori)

$$Z_4 = \sum_{k,m} b_{k,m} Z_4^{(k,m)} \, , \; Z_4^{(k,m)} = \Delta_{k,m}(r,\theta) \cos\Big(\frac{\sqrt{2}v}{R} + (k-m)\phi - m\psi\Big)$$

where $|b_{k,m}|^2 \propto N_{k,m}$

(note the dependence on $\phi\,,\psi$ and $v = t + y$)

- If $N_{k_i m_i} \gg 1$ non-linear terms in $b_{k,m}$ become important
- The non-linear geometry is not a descendant of a D1-D5 geometry

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General susy ansatz

- The most general geometry preserving the same supercharges as the D1-D5-P black hole and T^4-invariant is

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}}(dv+\beta)\Big(du+\omega+\frac{\mathcal{F}}{2}(dv+\beta)\Big)+\sqrt{\mathcal{P}}ds_4^2\,,\ \mathcal{P}=Z_1Z_2-Z_4^2$$

where $v=\frac{t+y}{\sqrt{2}}$, $u=\frac{t-y}{\sqrt{2}}$

- It is encoded by

0) ds_4^2 (4D euclidean metric), β (1-form in 4D)

1) Z_1 , Z_2 , Z_4 (0-forms)

2) ω (1-form in 4D), $\mathcal{F}$ (0-form)

- Susy implies that u is an isometry. Everything depends on v , x^i

Almost linear structure

0) The sugra equations for ds_4^2 , β are non-linear (they define an “almost hyperkahler” structure)
1) Assuming ds_4^2 , β, the equations for Z_1 , Z_2 , Z_4 are linear and homogeneous
2) The equations for ω , $\mathcal{F}$ are linear and inhomogeneous: the sources are quadratic in Z_i’s

Strategy: given ds_4^2 , β, first solve 1) and then solve 2)

Non-linear completion

- Given the linear structure, one can assume that
  - ds_4^2 , β and Z_2 do not receive corrections in $b_{k,m}$
  - Z_4 remains linear in $b_{k,m}$
- Given Z_1 , Z_2 , Z_4 one can solve the sugra eqs. for ω , $\mathcal{F}$
- Regularity:
  - ω is singular unless one includes in Z_1 terms quadratic in $b_{k,m}$
- Result:
  - for any $\{b_{k,m}\}$ there is a unique regular geometry
  - $\{b_{k,m}\}$ $\leftrightarrow$ Fourier modes of an arbitrary function of two variables $\Rightarrow$ supestrata

D1-D5-P microstates depend on functions of at least two variables

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D1-D5-P microstates depend on functions of at least two variables

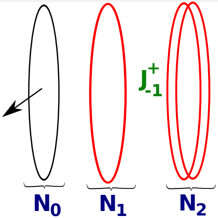
Holographic 1-point functions

- Can we test the connection between geometries and states?
- Terms of order r^{-2-d} in the asymptotic expansion of the geometry are related to vevs of dimension d operators in the microstate (Kanitscheider, Skenderis, Taylor)
- The vevs of chiral primary operators (and their descendants) in 1/4 and 1/8 BPS states are protected
- Examples: operators of dimension 1
  - O: $O|++\rangle_k = |00\rangle_k$ $\Rightarrow$ $Z_4 \sim \frac{\langle O \rangle Y^1}{r^3}$
  - Σ_2: $\Sigma_2 (|++\rangle_{k_1} \otimes |++\rangle_{k_2}) = |++\rangle_{k_1+k_2}$ $\Rightarrow$ $Z_1 \sim \frac{\langle \Sigma_2 \rangle Y^1}{r^3}$

(Y^1 : S^3 scalar spherical harmonic of order 1)

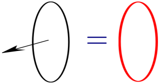
A D1-D5-P example

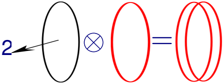
- Consider the state: $|s\rangle = \sum_{N_i}$



$b_1^2 \propto \overline{N}_1$

$b_2^2 \propto \overline{N}_2$

O  $\Rightarrow \quad \langle s|O|s\rangle \propto b_1 \quad \leftrightarrow \quad Z_4 \propto b_1$

Σ_2  $\Rightarrow \quad \langle s|\Sigma_2|s\rangle \propto e^{iv}\, b_1 b_2 \quad \leftrightarrow \quad Z_1 \propto e^{iv}\, b_1 b_2$

- Gravity and CFT match (including numerical coefficients)
- The CFT implies the regularity of spacetime

Entanglement entropy

- Consider the EE of one interval of length l in the state $|s\rangle$: $S_l^{(s)}$
- CFT: in the limit of $l \to 0$, $S_l^{(s)}$ is encoded by the vevs $\langle s|\mathcal{O}_K|s\rangle$

$$S_l^{(s)} = -\frac{\partial S_n^{(s)}}{\partial n}|_{n=1} \,,\; S_n^{(s)} = \left(\frac{l}{R}\right)^{-4\Delta_n}\left[1 + \sum_K \left(\frac{l}{R}\right)^{\Delta_K+\bar{\Delta}_K} \mathcal{C}_K \,\langle s|\mathcal{O}_K|s\rangle\right]$$

- Gravity: $S_l^{(s)}$ is given by the area of a minimal co-dimension 2 surface in the 6D geometry (Ryu, Takayanagi; Hubeny, Rangamani)

$$S_l^{(s)} = \frac{\mathrm{area}(\gamma_l)}{4G_N}$$

- $S_l^{(s)}$ is not protected, but if one includes only chiral primary $\mathcal{O}_K$ $\Rightarrow$ gravity and CFT match!

Summary

- We have constructed a family of regular and horizonless D1-D5-P geometries
- We have identified their CFT dual states
- We have checked the gravity-CFT map by computing 1-point functions and entanglement entropy

Outlook

- The states we have constructed are all the ones that reduce (in the limit $N_i \ll 1$) to linear perturbations around $AdS_3 \times S^3$ $\Rightarrow$ "graviton gas"
- These states are insufficient to produce an entropy which scales like $(n_1 n_5 n_p)^{1/2}$ (fractional modes are missing)
- Which CFT states (outside the "graviton gas") admit a description in supergravity?
- How well can one resolve typical states in supergravity? (need to know the vevs of operators of high enough dimension)
- What can one say about non-BPS microstates?