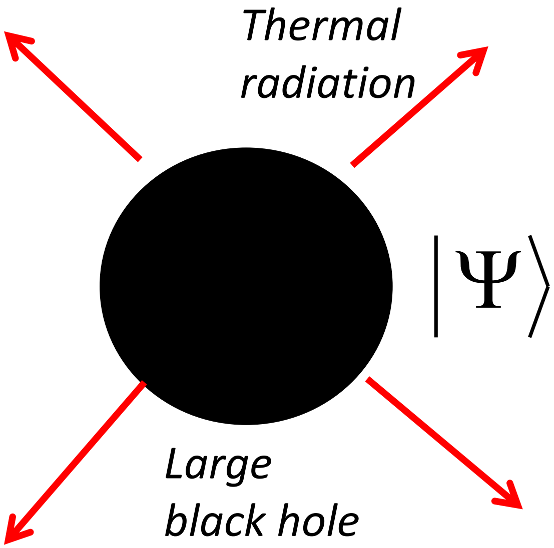
The Fall of the Black Hole Firewall: Natural Non-Maximal Entanglement for the Page Curve

Based on

M. Hotta and A. Sugita,
arXiv:1505.05870 to appear in PTEP.

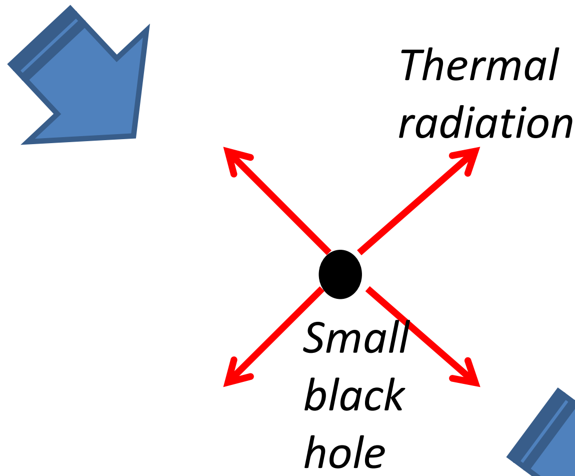
M. Hotta, R. Schützhold and W. G. Unruh,
Phys. Rev. D 91, 124060 (2015)

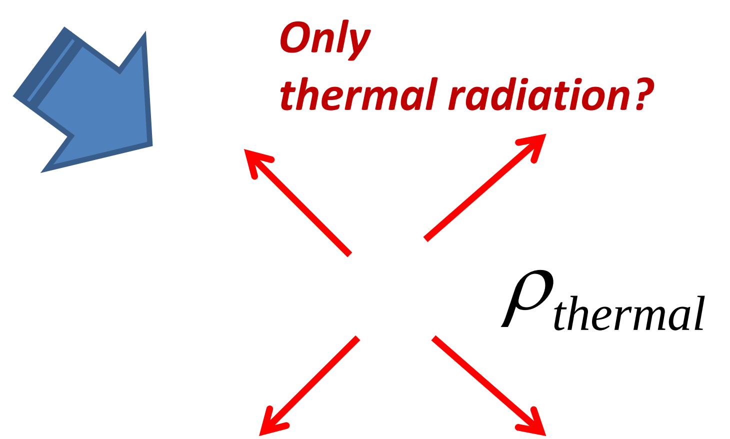
The Information Loss Problem



Hawking (1976)

$$\hat{U}|\Psi\rangle\langle\Psi|\hat{U}^{\dagger} \neq \rho_{thermal}$$

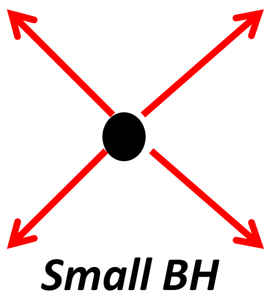




Unitarity breaking?

Information is lost!?

Why is the information loss problem so serious?



Too small energy to leak the huge amount of information.

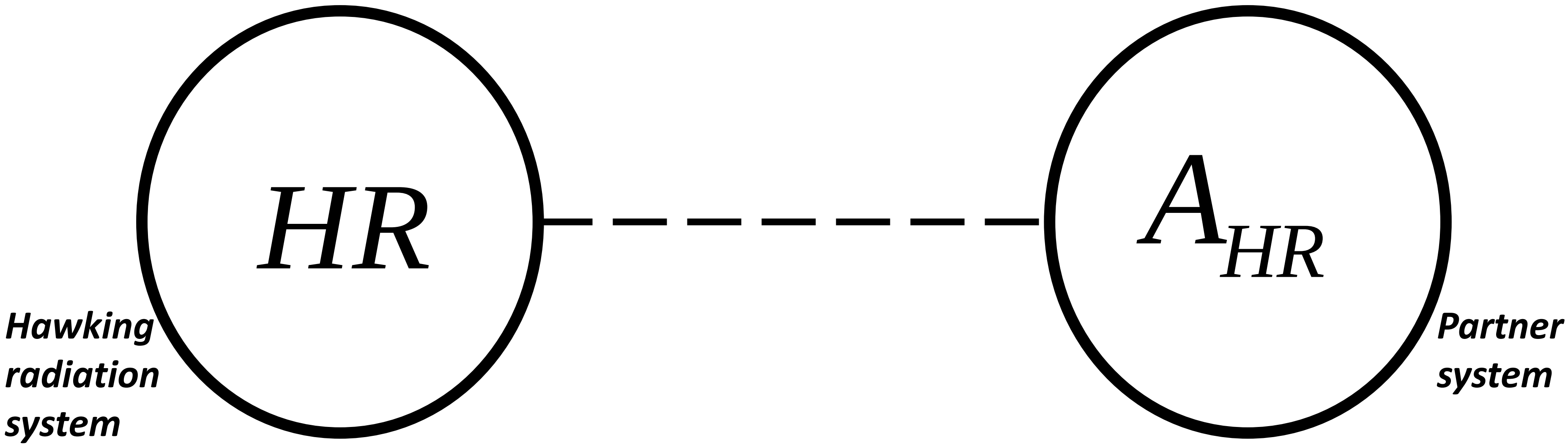
(Aharonov, et al 1987; Preskill 1992.)

If the horizon prevents enormous amount of information from leaking until the last burst of BH, only very small amount of BH energy remains, which is not expected to excite carriers of the information and spread it out over the outer space.

Purification Problem of Hawking Radiation:
from a modern viewpoint of information loss

$$\rho_{HR} = \sum_n p_n |n\rangle_{HR} \langle n|_{HR}$$

Mixed state



$$|\Psi\rangle_{HRA_{HR}} = \sum_n \sqrt{p_n} |n\rangle_{HR} |u_n\rangle_{A_{HR}}$$

Composite system in a pure state

What is the final purification partner of the Hawking radiation?

(1) Nothing, Information Loss

(2) Exotic Remnant (Aharonov, Banks, Giddings,…)

(3) Baby Universe (Dyson,..)

(4) Radiation Itself (Page,…)

- *Black Hole Complementarity ('t Hooft, Susskind, …)*
- *Fuzzi ball, **Firewall** (Mathur, Braunstein, AMPS, …)*

(5) Zero-Point Fluctuation Flow

(Wilczek, Hotta-Schützhold-Unruh, Hawking (2015))

Gravitational zero-point fluctuation with BMS charges

~MASSAGE (1)~

Canonical typicality for non-vanishing Hamiltonians yields non-maximal entanglement among black holes and the Hawking radiation,
which makes spacetimes smooth without breaking monogamy.
Thus, no reason to have BH firewalls.

~MASSAGE (2)~

Typical states must be Gibbs states for smaller quantum systems with very high precision. If we have stable Gibbs states for old Schwarzschild BH's (and small AdS BH's), the heat capacity must be positive. Actually, it is negative. Thus the states of BH evaporation are not typical!

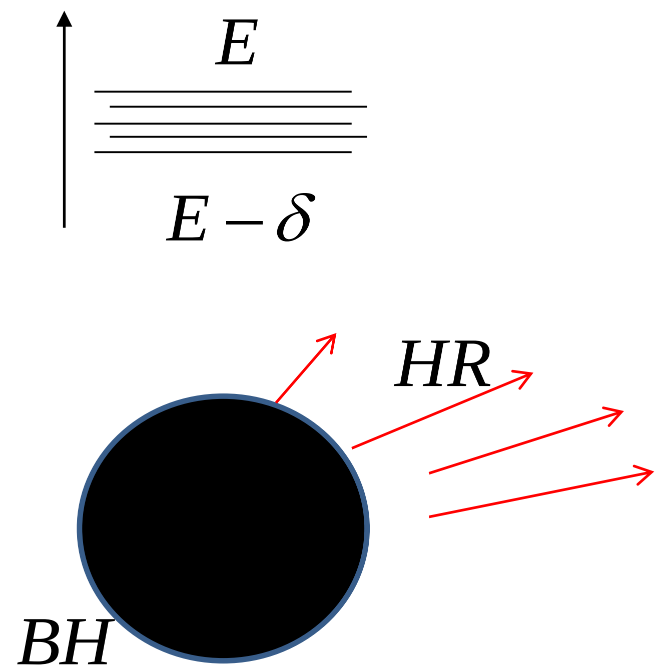
Inevitable Modification of the Page Curve

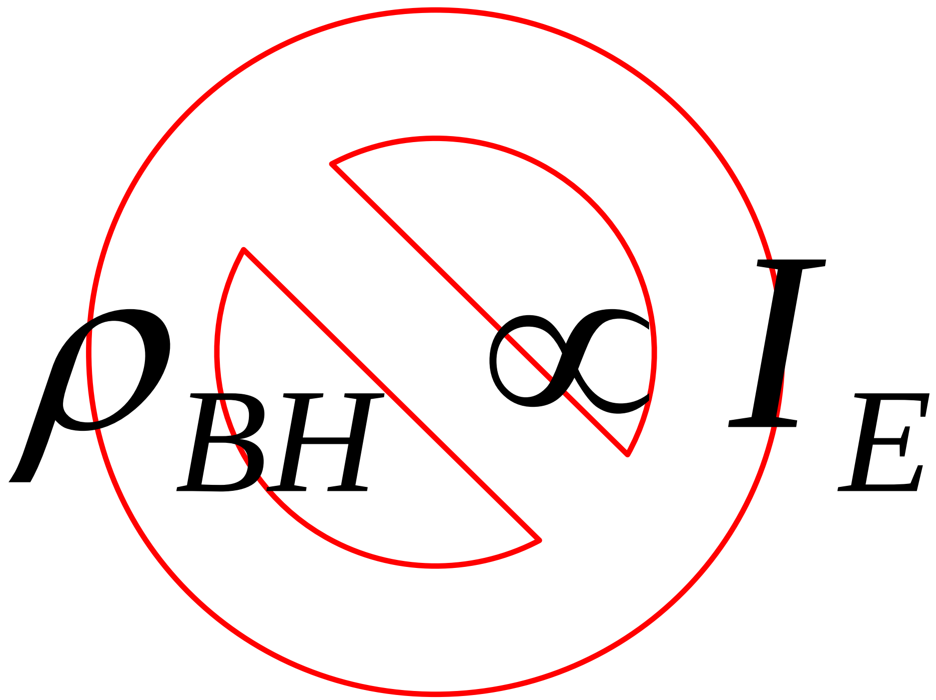
~MASSAGE (3)~

Microcanonical states** $\rho \propto I_E$ are **far from typical** for finite-temperature old BH's, even though canonical states are typical and the large entropy O(V) is merely different from the microcanonical entropy by O(lnV). Entropy difference between a typical state and the canonical state must be **exponentially small!

$$\left|S_{typical} - S_{thermal}\right| \leq O\left(\exp(-\gamma V)\right) \sim 0$$

You cannot use the microcanonical state in the typicality argument for evaporating black holes from a viewpoint of entanglement.





Plan of this talk

*I. Lubkin-Lloyd-Pagels-Page Theorem,
Page Curve Hypothesis
and BH Firewall Conjecture*

*II. Canonical Typicality
for Non-Vanishing Hamiltonians
Yields No Firewalls*

III. Summary

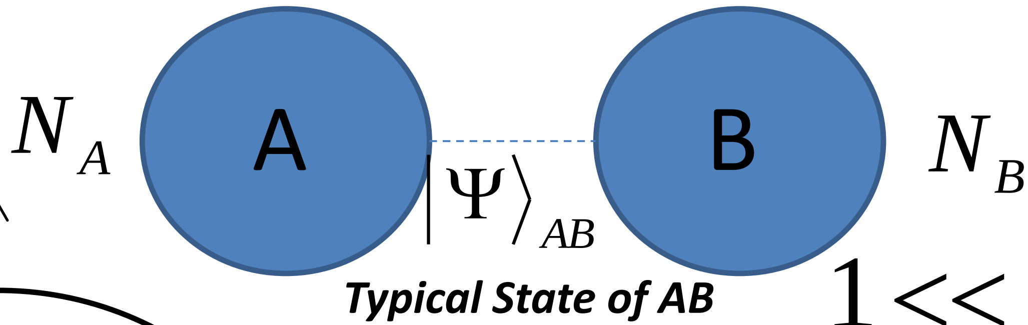
I. Lubkin-Lloyd-Pagels-Page Theorem, Page Curve Hypothesis and BH Firewall Conjecture

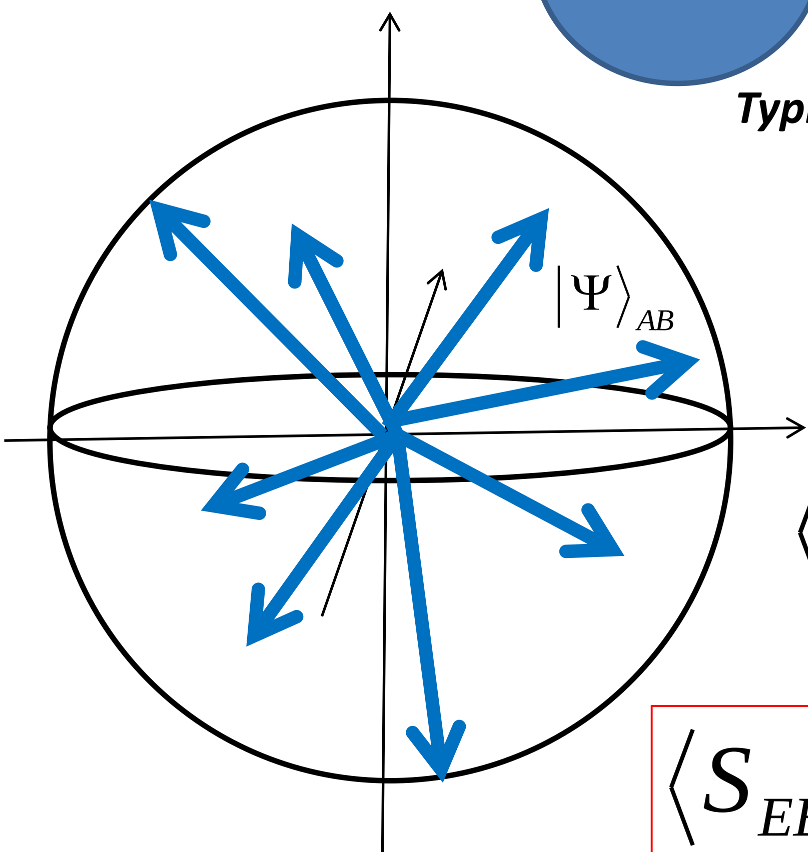
The BH firewall conjecture is based on the Page curve hypothesis,

and the hypothesis was inspired by the Lubkin-Lloyd-Pagels-Page (LLPP) theorem.

Lubkin-Lloyd-Pagels-Page Theorem:

Typical states of A and B are almost maximally entangled when the systems are large.





$$1 << N_A << N_B$$

$$\rho_A = Tr_B\left[|\Psi\rangle_{AB}\langle\Psi_{AB}|\right]$$

$$\langle S_{EE}\rangle = -\langle Tr_A[\rho_A \ln \rho_A]\rangle$$

$$\langle S_{EE}\rangle \approx \ln N_A \quad \Rightarrow \quad \rho_A \approx \frac{1}{N_A} I_A$$

Maximal Entanglement between A and B

$$\rho_A = \frac{1}{N_A} I_A \Rightarrow \left|Max\right\rangle_{AB} = \frac{1}{\sqrt{N_A}} \sum_{n=1}^{N_A} \left|u_n\right\rangle_A \left|\tilde{v}_n\right\rangle_B$$

Orthogonal unit vectors

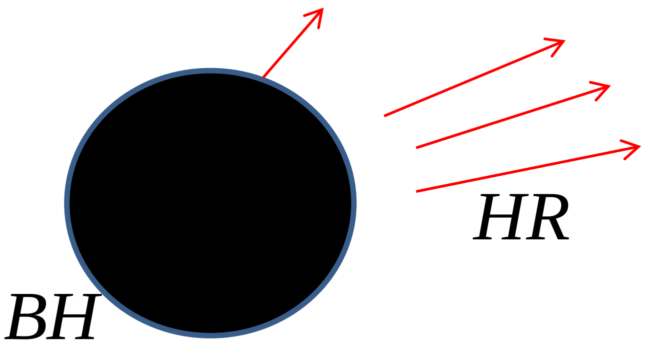
$$N_B \geq N_A$$

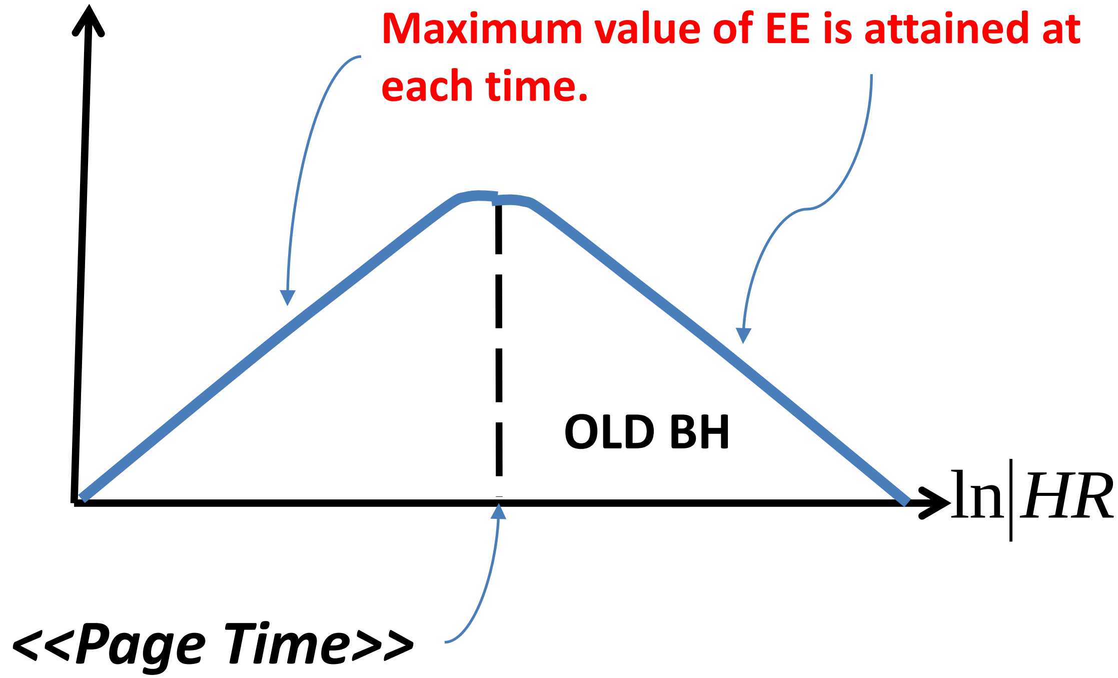
Let us assume that Hilbert-space dimensions of black holes and Hawking radiation become finite due to quantum gravity effect.

Page's Strategy for Finding States of BH Evaporation:

Nobody knows exact quantum gravity dynamics.
So let's gamble that the state scrambled by quantum gravity is one of TYPICAL pure states of the finite-dimensional composite system! That may not be so bad!

Simplified Page Curve





$$|BH| = \dim H_{BH},$$

$$|HR| = \dim H_{HR}$$

- $\ln|BH| \approx \ln|HR|$
- $M_{page} \approx 0.7 M_{bh}$

$$1 << |BH| << |HR| \Rightarrow \langle S_{EE} \rangle \approx \ln|BH| = \frac{A_{BH}}{4G}$$

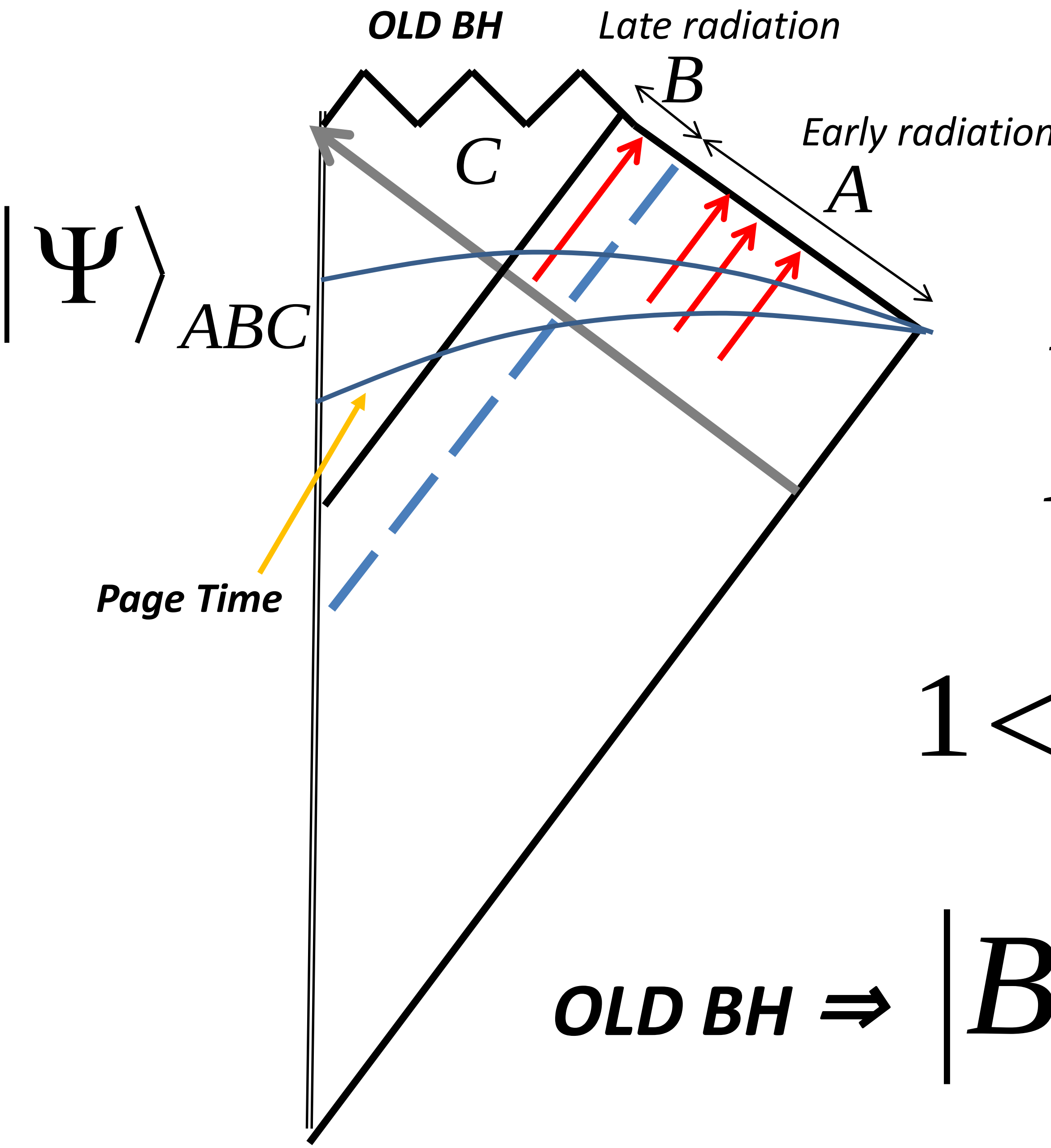
Page Curve Hypothesis for BH Evaporation:

Proposition I:

When the dimension of the BH Hilbert space is much larger or less than that of Hawking radiation, BH and HR in a typical pure state of quantum gravity share almost maximal entanglement. In other words, quantum states of the smaller system is almost proportional to the unit matrix.

Proposition II:

$$S_{EE} = S_{thermal}$$ *of the smaller system.*



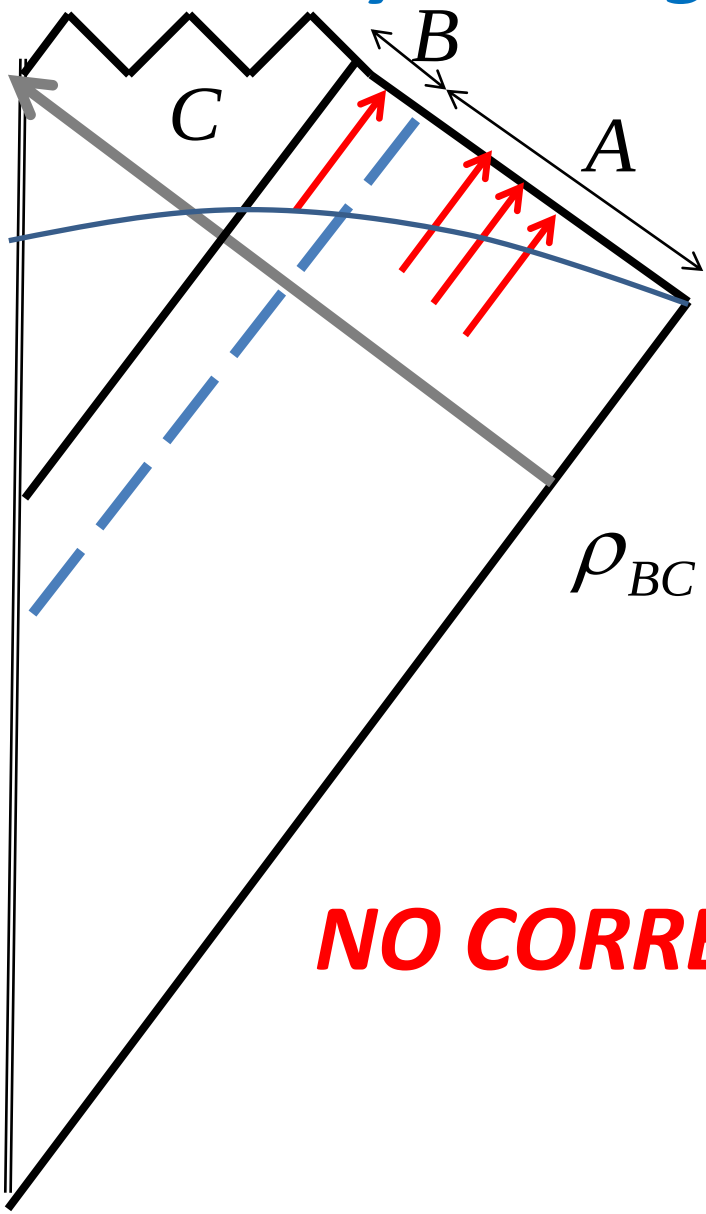
$$HR = A \cup B$$

$$BH = C$$

$$1 << |A|, |B|, |C|$$

OLD BH ⇒ $|B||C| << |A|$

Proposition I means that A and BC are almost maximally entangled with each other.



$$\rho_{BC} = \frac{1}{|BC|} I_{BC} = \left(\frac{1}{|B|} I_B \right) \otimes \left(\frac{1}{|C|} I_C \right)$$

NO CORRELATION BETWEEN B AND C!

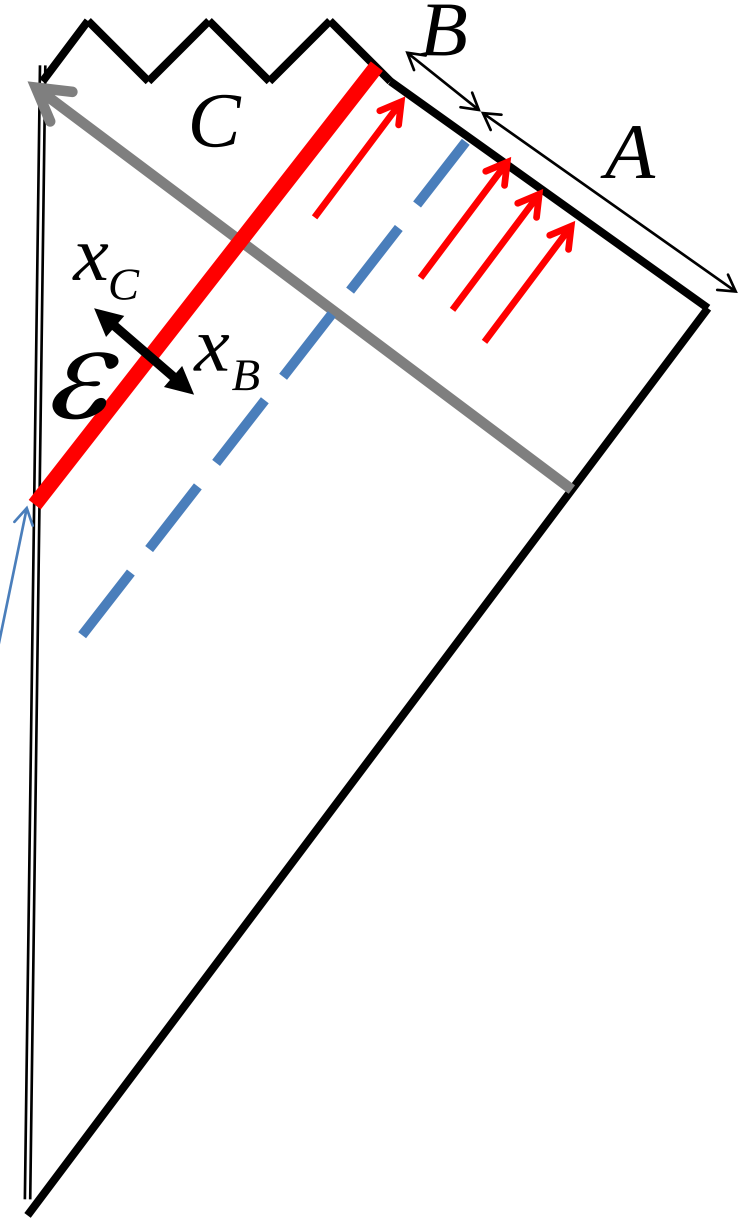
Harrow-Hayden

$$\rho_{BC} = \left(\frac{1}{|B|} I_B \right) \otimes \left(\frac{1}{|C|} I_C \right)$$

$$Tr\left[\left(\frac{\varphi(x_B) - \varphi(x_C)}{\varepsilon} \right)^2 \rho_{BC} \right] = O\left(\frac{1}{\varepsilon^2} \right)$$

$$\varepsilon \to 0$$

$$Tr\left[\left(\partial \varphi(x) \right)^2 \rho_{BC} \right] = \infty$$



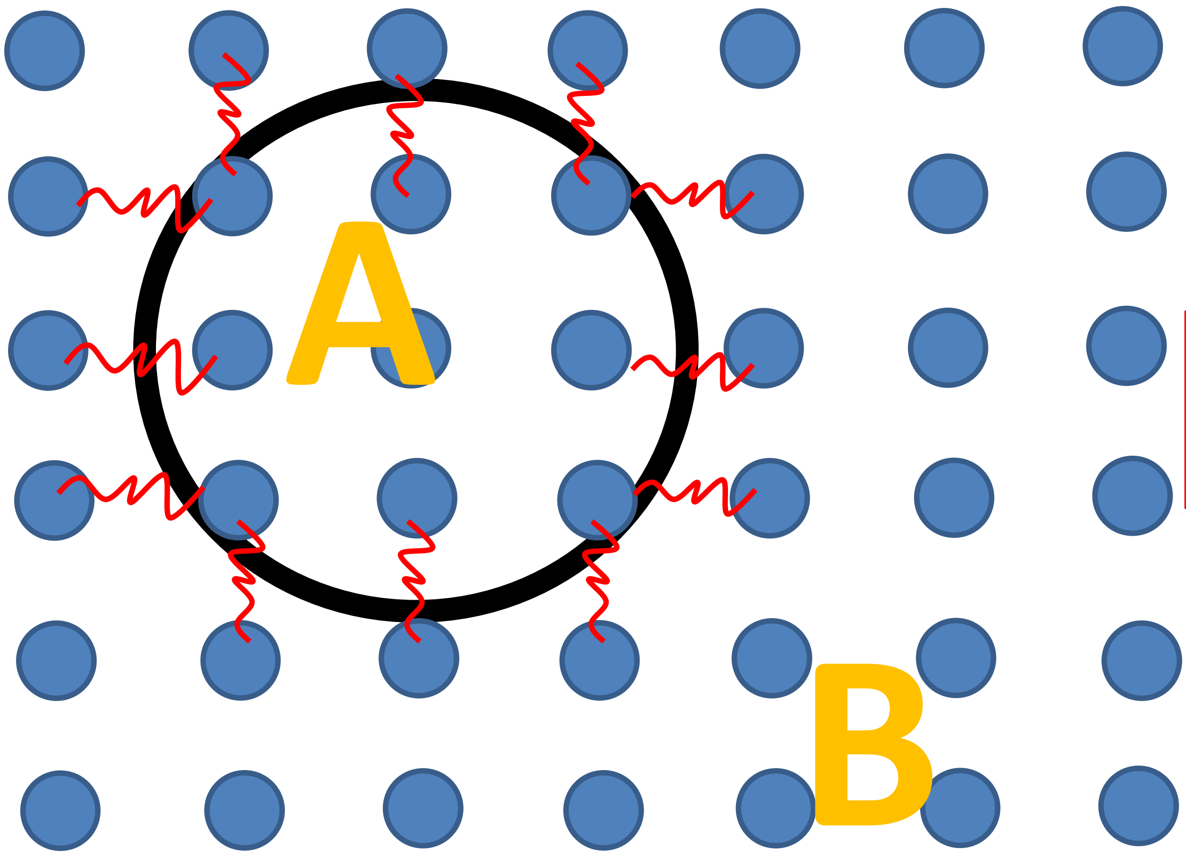
FIREWALL!

II. Canonical Typicality for Non-Vanishing Hamiltonian, and No Firewalls

Problem for Proposition I of Page Curve Hypothesis:

The area law of entanglement entropy is broken in a sense of ordinary many body physics, though outside-horizon energy density in BH evaporation is much less than the Planck scale.

$$S_{EE} \propto |\partial A| = |\partial B|$$ ← *standard area law of entanglement entropy*



$$|\Psi\rangle_{AB} \approx |0\rangle_{AB}$$

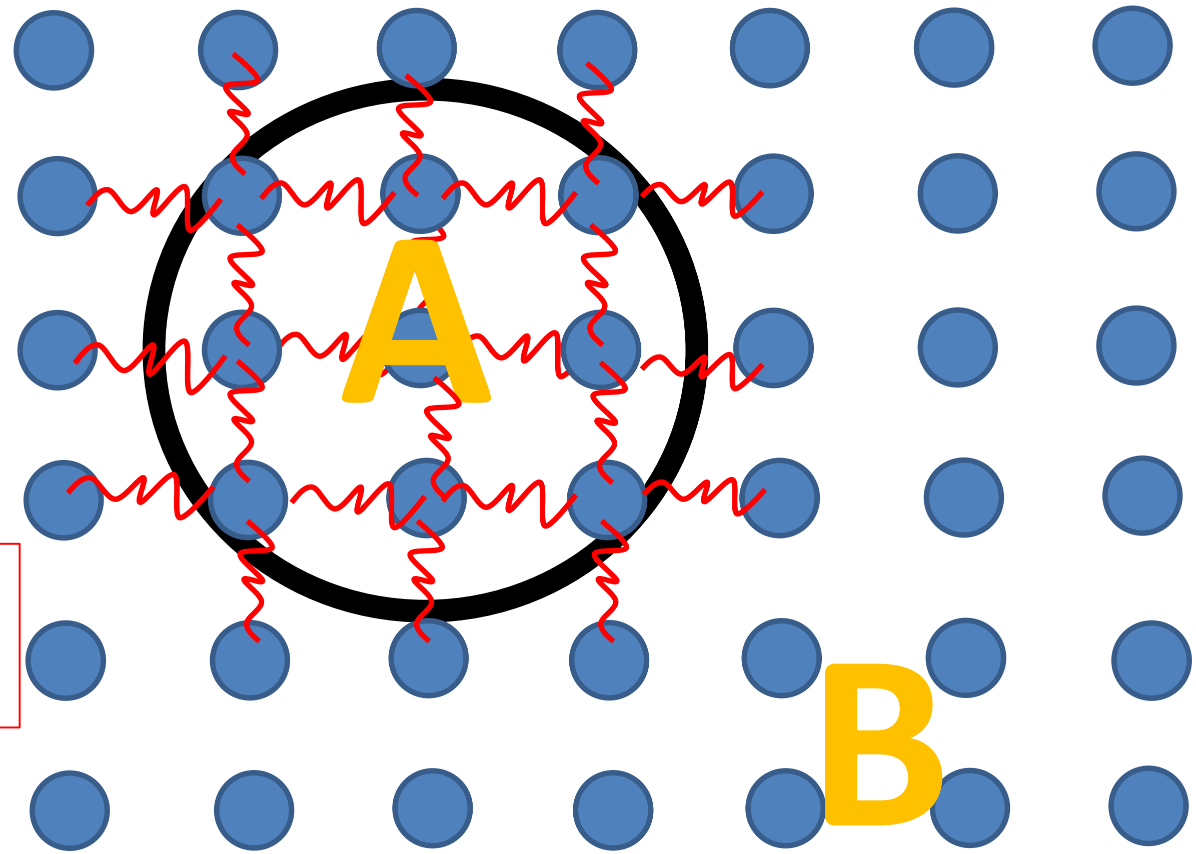
for low excited states

LLPP Typicality ⇒ $$\left|\Psi\right\rangle_{AB} = \frac{1}{\sqrt{|A|}} \sum_{n=1}^{|A|} \left|u_n\right\rangle_A \left|\tilde{v}_n\right\rangle_B$$

Qubit network model

$$|A| = 2^{V_A}$$

$$S_{EE} = \ln|A| \propto V_A$$



Not area law,

but volume law for highly excited states!

This is because zero Hamiltonian (complete degeneracy) is assumed in the LLPP theorem.
This is also an implicit premise of the Page curve hypothesis.

$$H_{AB} = 0.$$

In BH physics,
we have to treat canonical typicality with non-vanishing H in a precise manner.
Then non-maximal entanglement emerges and makes near-horizon regions smooth.
Thus no firewalls appear.

M. Hotta and A. Sugita, arXiv:1505.05870

Microcanonical Energy Shell

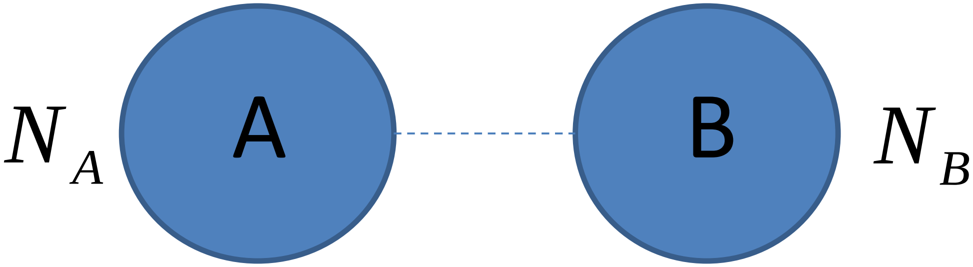
(not a tensor product of the sub-Hilbert spaces)

$$H_{AB}\left|E_j\right\rangle = E_j\left|E_j\right\rangle$$

$$\Delta(E) = \left\{ j \mid E_j \in \left[E-\delta, E\right] \right\}$$

Microcanonical Energy Shell:

$$V_{ES}(E) = \left\{ \sum_{j\in\Delta(E)} c_j \left|E_j\right\rangle \right\}$$



$$\left|\Psi\right\rangle_{AB} \in V_{ES}(E)$$

$$\rho_A = Tr_B\left[\left|\Psi\right\rangle_{AB}\left\langle\Psi\right|_{AB}\right]$$

$$= \frac{1}{N_A}\left[I_A + \sum_{n=1}^{{N_A}^2-1}\left\langle T_n\right\rangle T_n\right]$$

Bloch Representation of higher-dim quantum states

$$T^{\dagger}{}_n = T_n,\ Tr\left[T_n\right] = 0,\ Tr\left[T_n T_{n'}\right] = N_A \delta_{nn'}$$

$$\boxed{\left\langle T_n\right\rangle = Tr\left[T_n \rho_A\right]}$$

Evaluate $\langle T_n \rangle$ ***for*** $|\Psi\rangle_{AB} = \sum_{j \in \Delta(E)} c_j |E_j\rangle$.

$$D = \dim V_{ES}(E) = \dim \left\{ \sum_{j \in \Delta(E)} c_j |E_j\rangle \right\}$$

$$N_A << N_B$$

Volume of B

Hilbert space dimension of B

$$D \propto \exp(\gamma V_B(N_B)) >> 1$$

for ordinary systems.

Uniform Ensemble on Mircrocanonical Energy Shell:

$$p(c) \propto \delta\left(\sum_{j \in \Delta(E)} \left| c_j \right|^2 - 1 \right) \qquad \int p(c) d^D p = 1$$

$$\overline{f} = \int f(c) p(c) d^D p$$

$$\overline{c_j c_{j'}^{\ *}} = \frac{1}{D} \delta_{jj'},$$

$$\overline{c_j c_k c_{j'}^{\ *} c_{k'}^{\ *}} = \frac{1}{D(D+1)} \left(\delta_{jj'} \delta_{kk'} + \delta_{jk'} \delta_{j'k} \right)$$

$$\overline{\left(\langle T_n\rangle - \overline{\langle T_n\rangle}\right)^2} \le \frac{\left\| T_n{}^2 \right\|}{D+1}$$

Max eigenvalue

$$\overline{Tr_A\left(\rho_A - \overline{\rho_A}\right)^2} \le \left(\frac{1}{N_A}\sum_{n=1}^{N_A{}^2-1}\left\| T_n{}^2 \right\|\right)\frac{1}{D+1}$$

N_B **independent!**

$$D \propto \exp(\gamma V_B) = O\left(\exp\left(10^{23}\right)\right) >> 1$$

$$\left\| \rho_A - \overline{\rho_A} \right\| \le O(\exp(-\gamma V_B))$$

Hotta-Sugita (2015) as a response to a BH firewall debate with Daniel Harlow

Private Communication with D. Harlow about "Jerusalem Lectures on Black Holes and Quantum Information", [arXiv:1409.1231](https://arxiv.org/abs/1409.1231) .

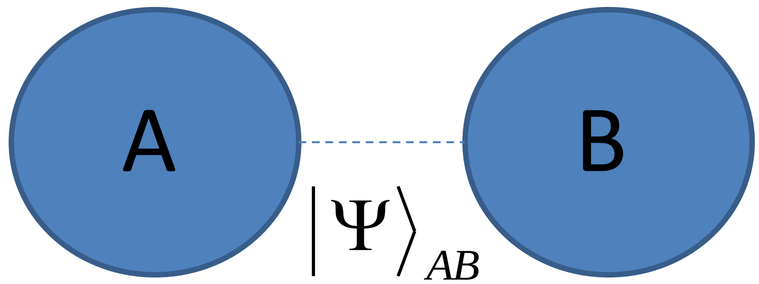
Harlow argued a canonical typicality in a weak interaction limit.

$$H = H_A + H_B + \underline{\cdots}$$

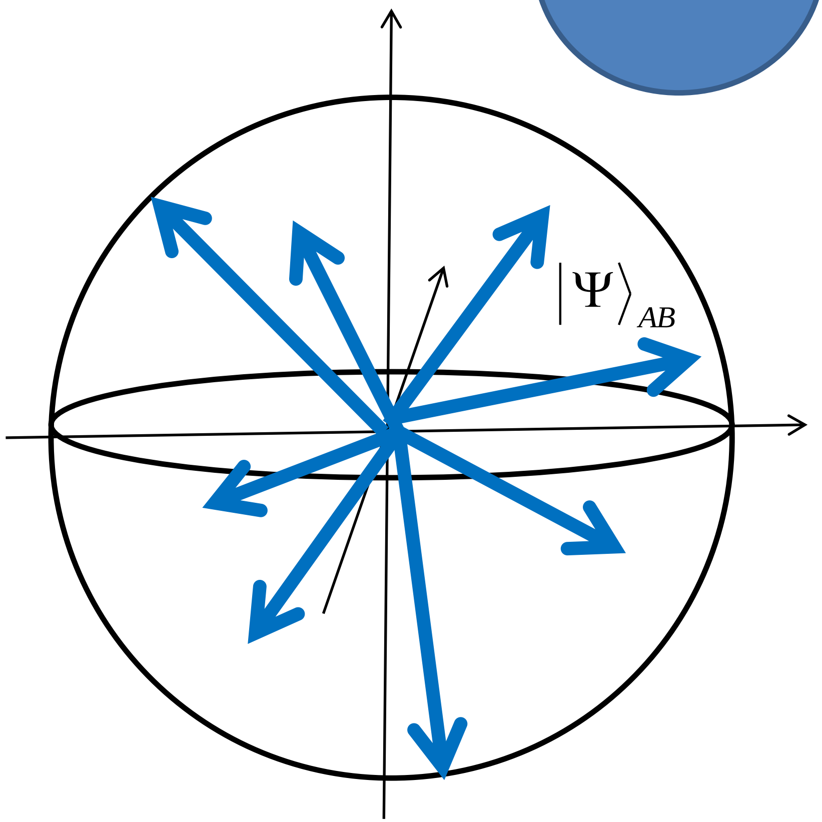
Negligibly small

$$\langle H \rangle = E_A + E_B = const.$$

$$E_A + E_B = const.$$



$$N_B >> N_A$$



$$\rho_A = Tr_B\left[|\Psi\rangle_{AB}\langle\Psi_{AB}|\right]$$

$$\langle S_{AB}\rangle = -\langle Tr_A[\rho_A \ln \rho_A]\rangle$$

Without any proof, Harlow argued these only in the weak interaction limit.

$$\rho_A \approx \frac{1}{Z_A}\exp(-\beta H_A)$$

$$\langle S_{AB}\rangle \approx S_{thermal,A}(\beta)$$

Harlow pointed out a possibility that BH firewalls may exist even after canonical typicality with non-zero Hamiltonian.

$$HR = A \cup B, \qquad BH = C$$

$$1 << |A|, |B|, |C| \quad |B||C| << |A|$$

$$\rho_{BC} \propto \exp\left(-\beta\left(H_B + H_C\right)\right)$$

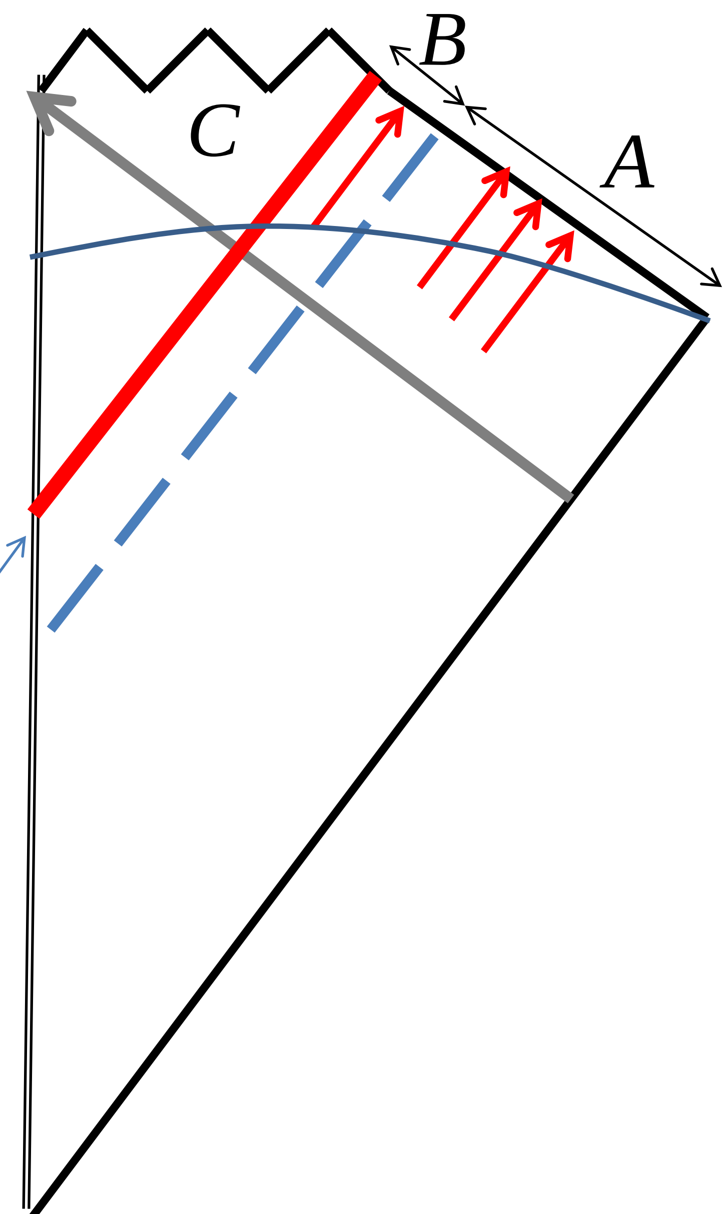
$$= \exp\left(-\beta H_B\right) \otimes \exp\left(-\beta H_C\right)$$

No Correlation, just like $I_B \otimes I_C$

$$Tr\left[\left(\partial\varphi(x)\right)^2 \rho_{BC}\right] = \infty ?$$

FIREWALL?

Harlow, arXiv:1409.1231



However, the worry is useless.
We can prove nonexistence of firewalls for general systems by using the general theory of canonical typicality.

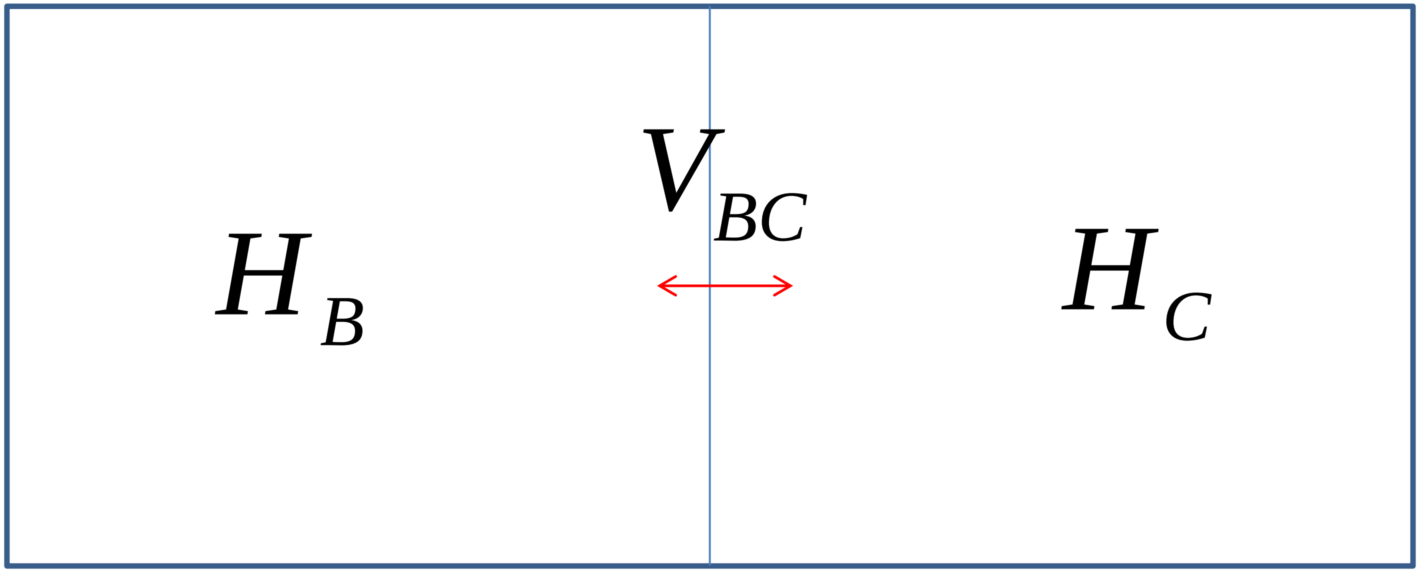
M. Hotta and A. Sugita, arXiv:1505.05870 .

Irrespective of the strength of the interaction between B and C,

$$\rho_{BC} \propto \exp\left(-\beta\left(H_B + H_C + \underline{V_{BC}}\right)\right)$$

$$|B||C| << |A|$$

Actually, a correlation exists between B and C for small interactions.



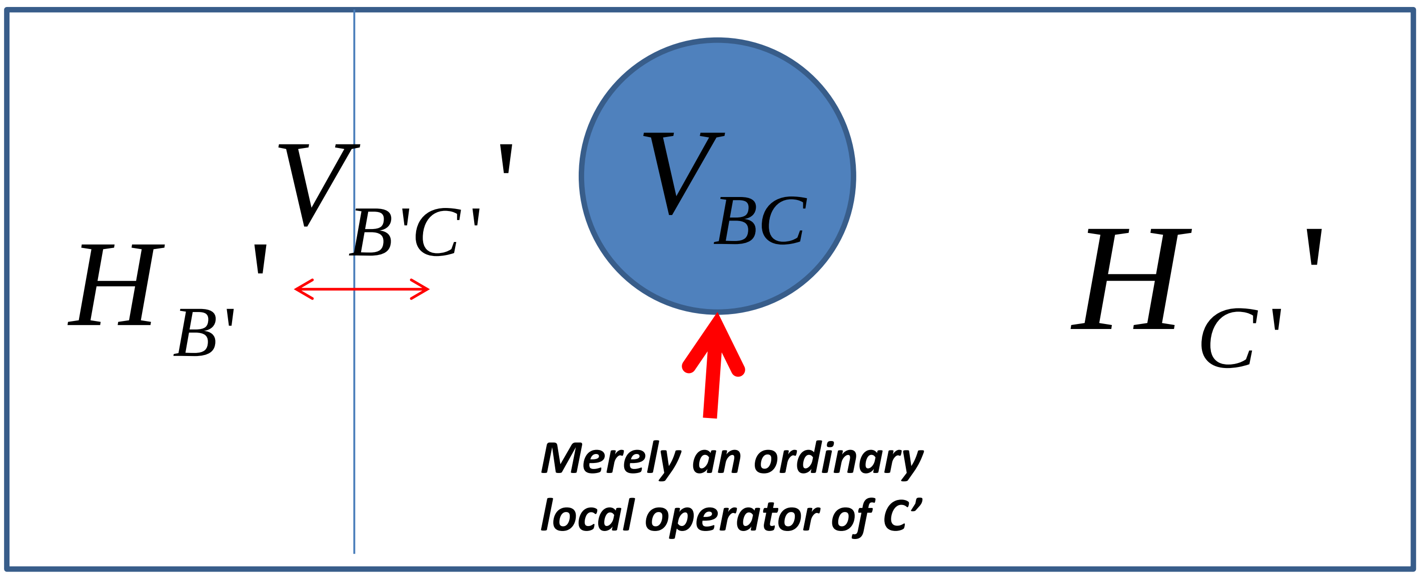
$$\rho_{BC} \propto \exp\left(-\beta\left(H_B + H_C + V_{BC}\right)\right)$$

Harlow's worry: $$\lim_{V_{BC} \to 0} \left| Tr\left[\rho_{BC} V_{BC}\right] \right| = \infty !?$$

Border shift does not change physics at all.

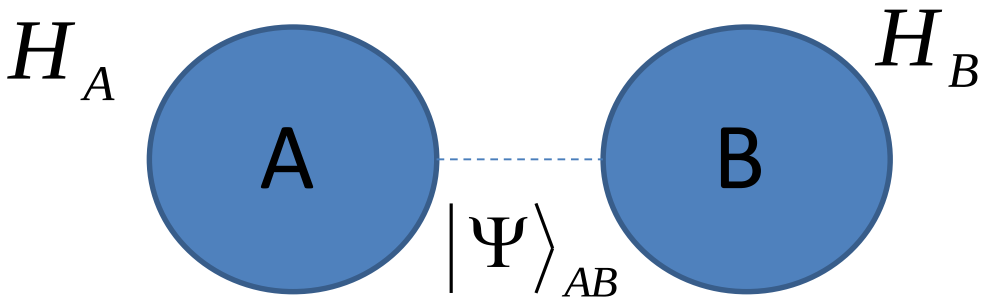
$$H_B + H_C + V_{BC} = H_{B'}' + H_{C'}' + V_{B'C'}'$$

$$\rho_{BC} = \rho_{B'C'}' \propto \exp\left(-\beta\left(H_{B'}' + H_{C'}' + V_{B'C'}'\right)\right)$$



$$\left|Tr\left[\rho_{BC} V_{BC}\right]\right| = \left|Tr\left[\rho_{B'C'}' V_{BC}\right]\right| < \infty$$ **No firewall!**

***Remark*: for ordinary weakly interacting quantum systems, entanglement entropy is upper bounded by thermal entropy, as long as stable Gibbs states exist.**



$$E_A + E_B = E$$

Arbitrary state: $\rho_A = Tr_B\left[|\Psi\rangle_{AB}\langle\Psi_{AB}|\right]$

Gibbs state: $\overline{\rho}_A = \exp(-\beta(E)H_A)/Z_A(\beta(E))$

$$S_{EE} = -Tr\left[\rho_A \ln \rho_A\right] \leq -Tr\left[\overline{\rho}_A \ln \overline{\rho}_A\right] = S_{thermal}$$

Conventional "proof":

$$I = -Tr_A[\rho_A \ln \rho_A] - \lambda_1 (Tr_A[\rho_A H_A] - E_A) - \lambda_2 (Tr_A[\rho_A] - 1)$$

$$\delta I = 0$$

$$\bar{\rho}_A = \exp(-\beta H_A) / Z_A(\beta)$$

$$-Tr[\rho_A \ln \rho_A] \le -Tr[\bar{\rho}_A \ln \bar{\rho}_A]$$

***If a stable Gibbs state exists,* it attains the maximum of the von Neumann entropy with average energy fixed.**

Unfortunately, the typicality argument cannot be applied to Schwarzschild BH evaporation!

Actually, from our result,
the typical state must be a Gibbs state,
but…

No stable Gibbs state for Schwarzschild BH

due to negative heat capacity! ***(Hawking –Page, 1983)***

$$\langle E\rangle = M_{BH} = \frac{1}{8\pi GT} \Rightarrow \frac{d\langle E\rangle}{dT} = -\frac{1}{8\pi GT^2} < 0$$

If there exists a stable Gibbs state, heat capacity must be positive.

$$Z_{BH}(\beta) = Tr\left[\exp\left(-\beta H_{BH}\right)\right]$$

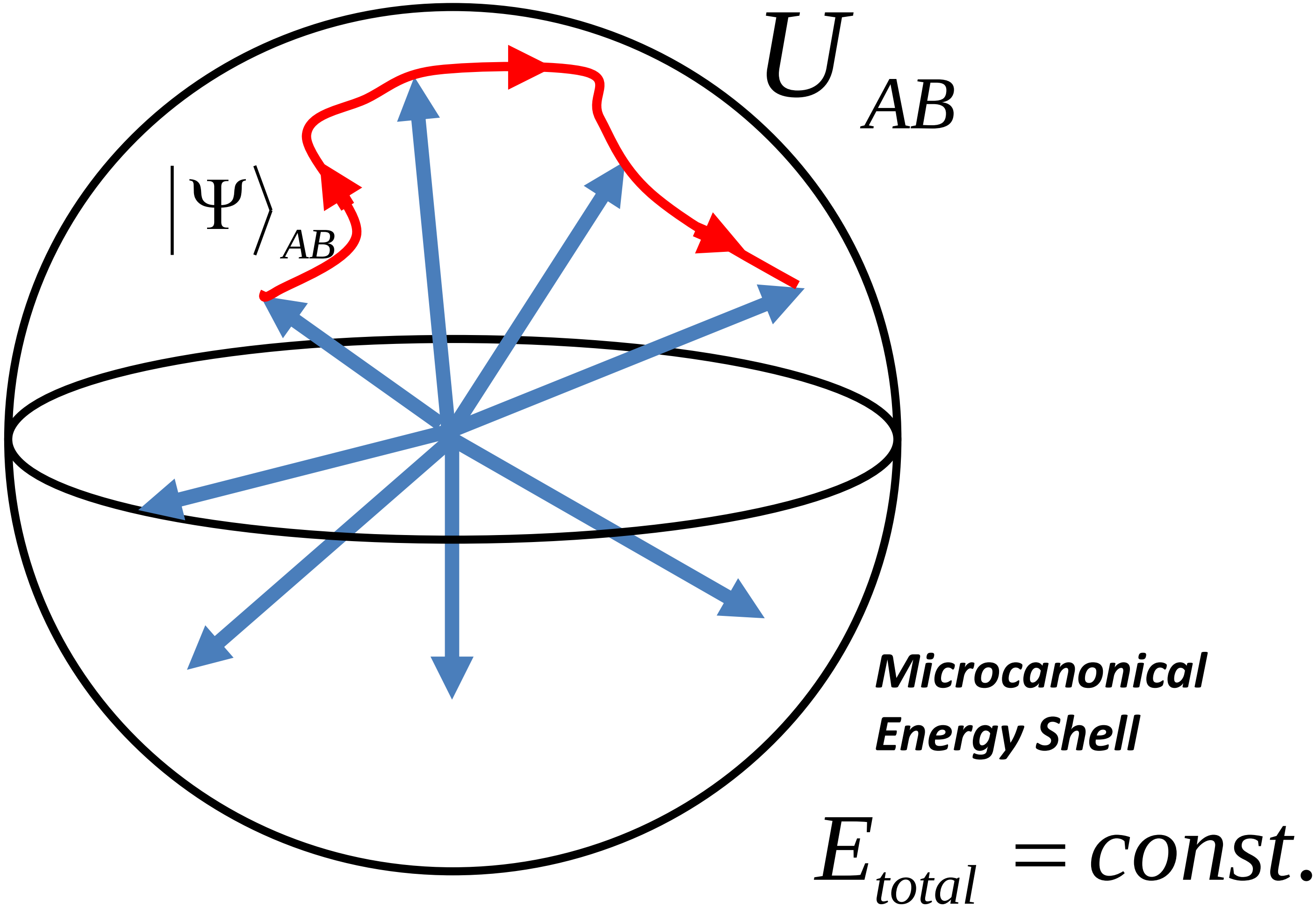
$$\Downarrow$$

$$\frac{d\langle E\rangle}{dT} = \frac{\left\langle \left(E-\langle E\rangle\right)^2\right\rangle}{T^2} > 0$$

Thus, a system of a black hole and Hawking radiation is not in typical states, at least in the sense of the Page curve hypothesis, during BH evaporation. Because we have no stable Gibbs state," thermal entropy" of Schwarzschild BH ($A/(4G)$) is not needed to be a upper bound of entanglement entropy.

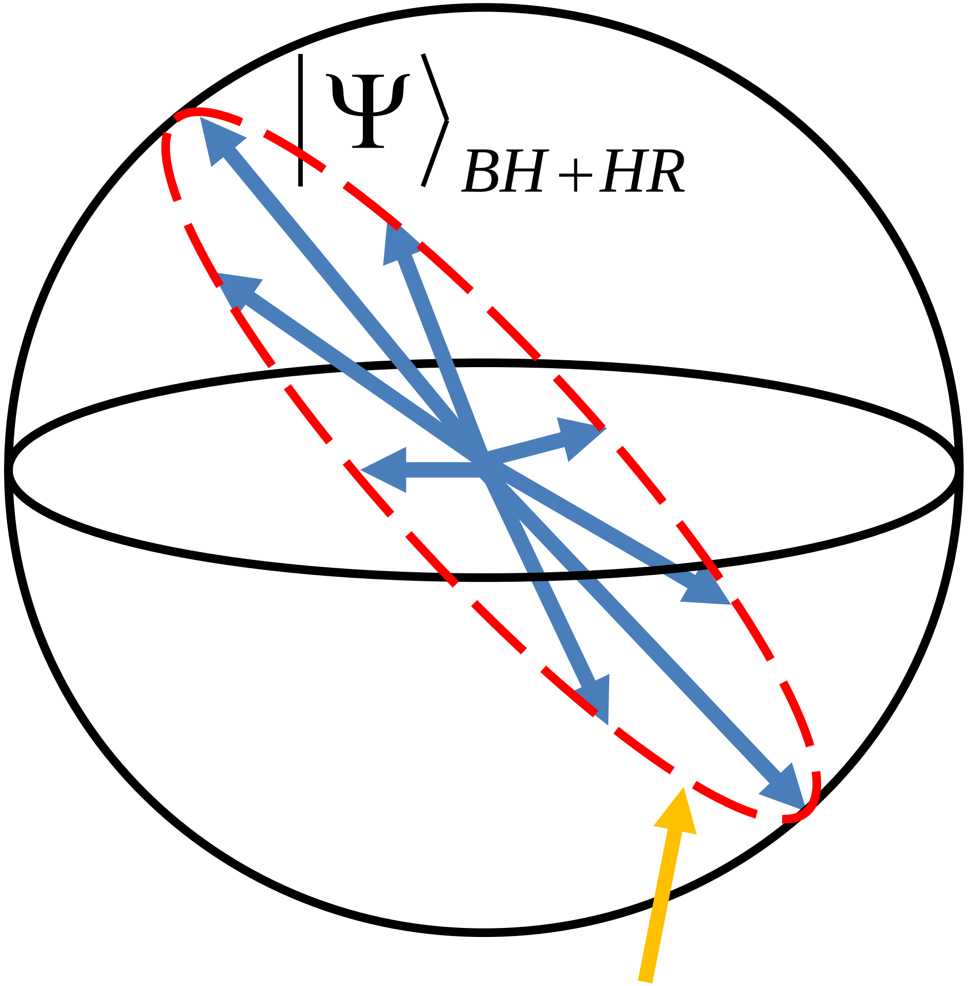
$$S_{EE} \le S_{thermal} \approx A/(4G)$$

In ordinary quantum systems,



$|\Psi\rangle_{AB}$ ***is a typical state with almost certainty after a relaxation time.***

The state of BH evaporation can be non-typical until the last burst.



Sub-Hilbert space of non-typical states

$$U_{BH} \otimes I_{HR}$$

Fast scrambling of BH does not contribute to entanglement between BH and HR.

$$U^{(emission)}$$

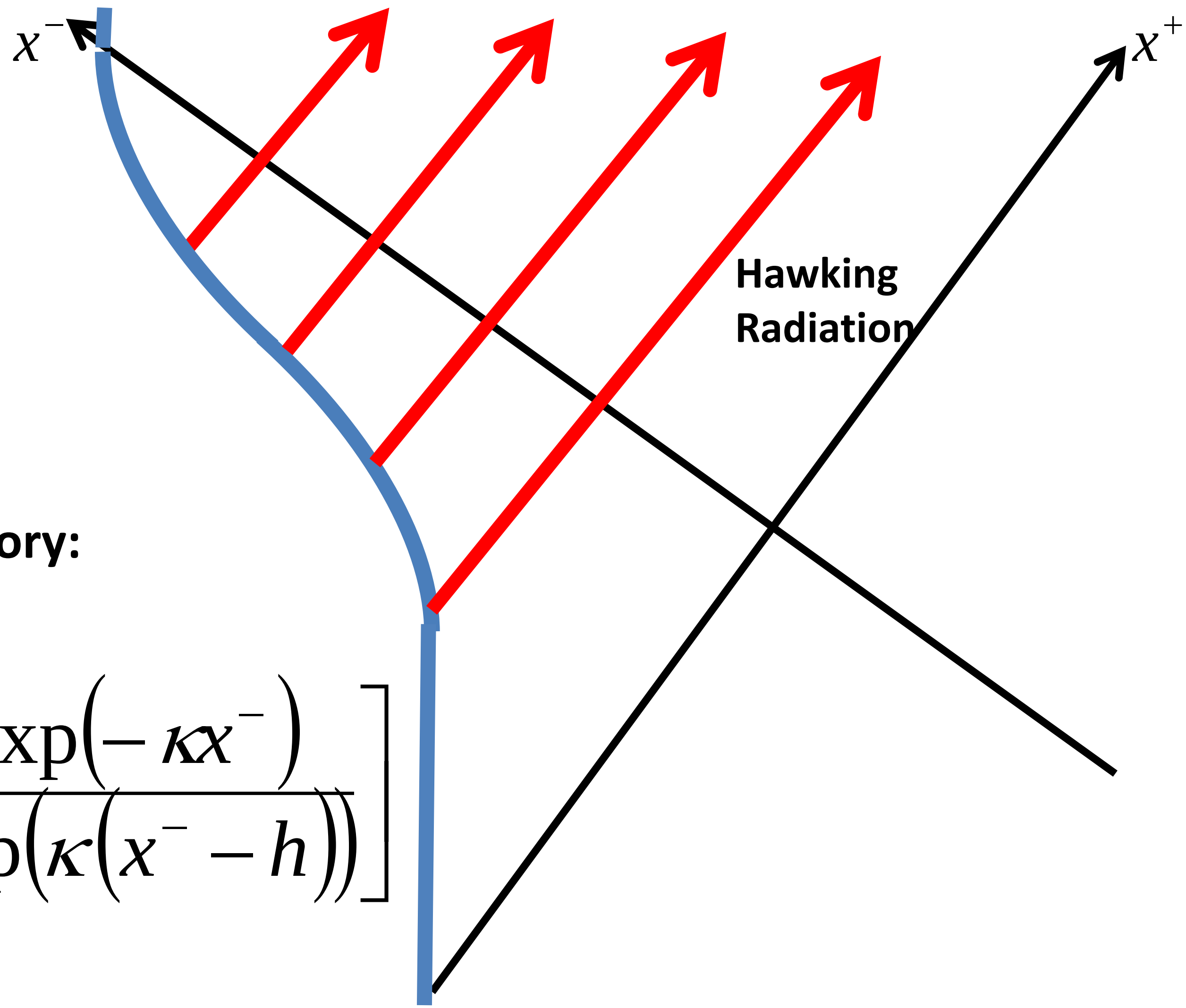
Non-chaotic HR emission generated by smooth space time curvature outside horizon

If so, how is the Page curve modified?

***The moving mirror model* is totally unitary. So we are able to learn how the information can be retrieved.**

The model is a tool to explore the Page curve hypothesis and its modification by using various mirror trajectories.

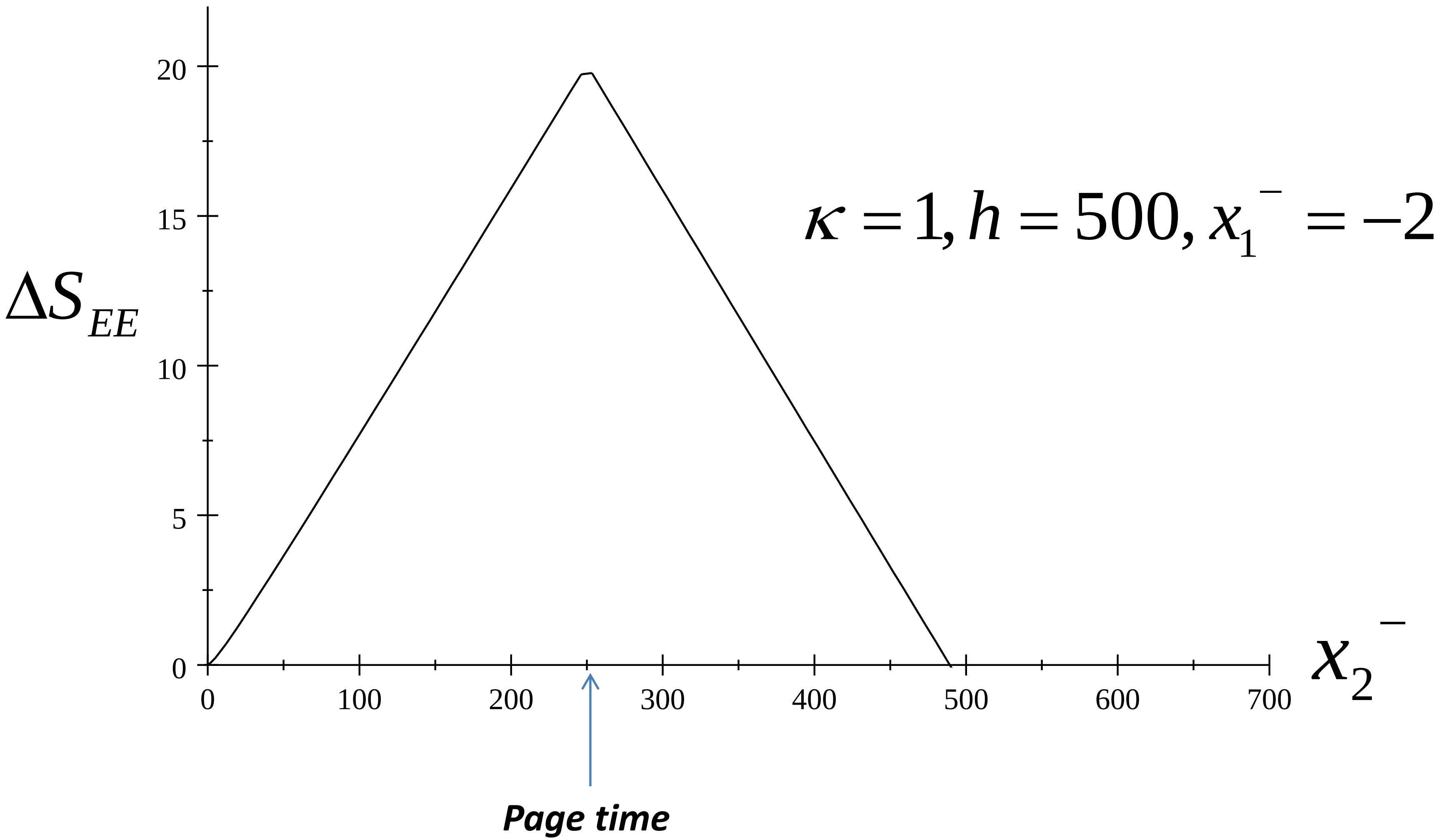
Page Curve in Moving Mirror Model



Mirror Trajectory:

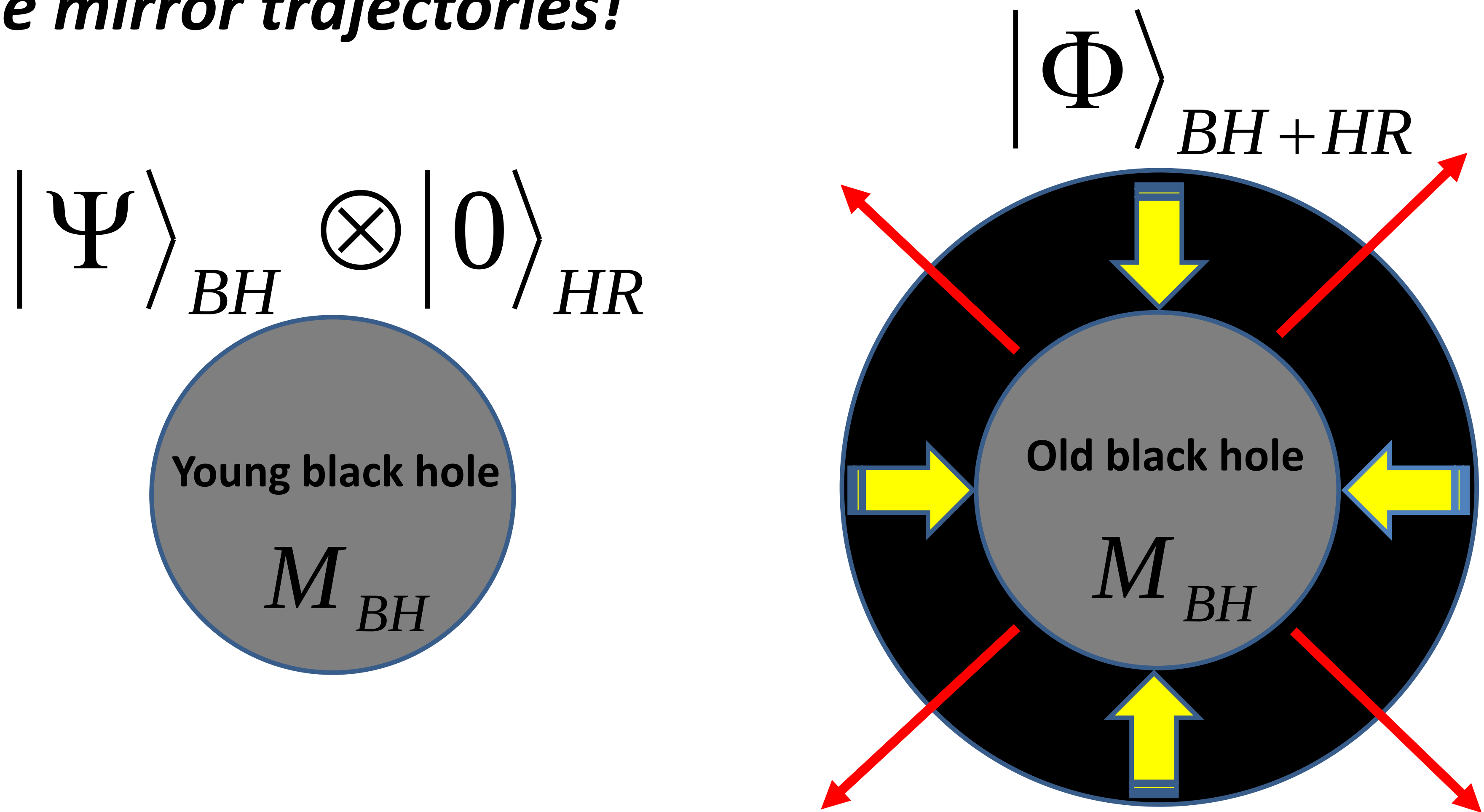
$$x^+ = -\ln\left[\frac{1+\exp(-\kappa x^-)}{1+\exp(\kappa(x^- - h))}\right]$$

Page Curve in Moving Mirror Model



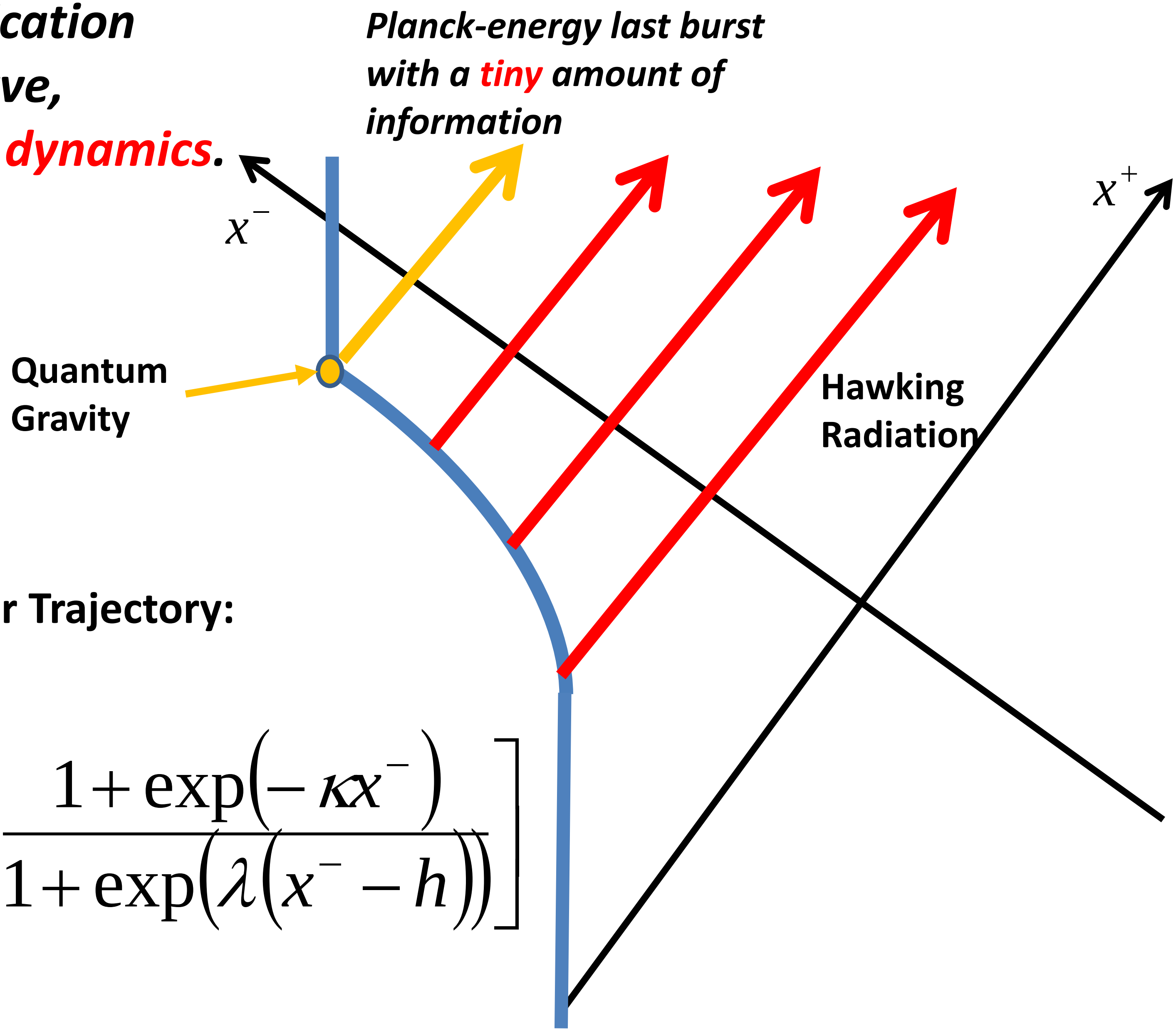
Thanks to Daniel Harlow

In order to reproduce the Page curve, very strange time evolution induced by nonlocality is required for the mirror trajectories!



Quite different time schedules of information leakage for black holes with the same mass.

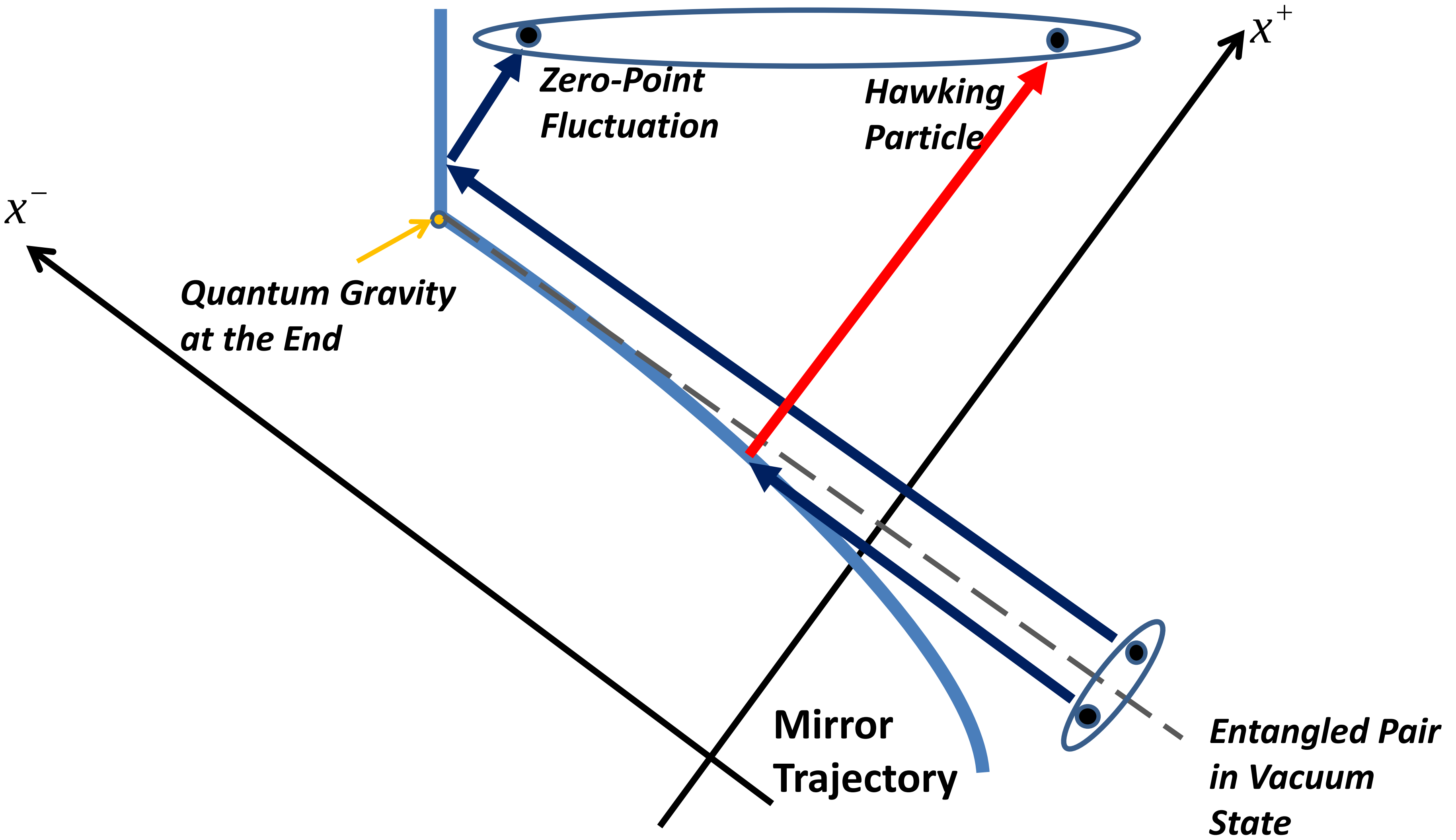
Possible modification of the Page curve, assuming local dynamics.



Mirror Trajectory:

$$x^+ = -\ln\left[\frac{1+\exp\left(-\kappa x^-\right)}{1+\exp\left(\lambda\left(x^- - h\right)\right)}\right]$$

Information Retrieval without Energy at the End



The entangled partner of the Hawking particle is zero-point fluctuation with zero energy. *(Wilczek, Hotta-Schützhold-Unruh)*

Entangled Partner

Particle A

Particle B

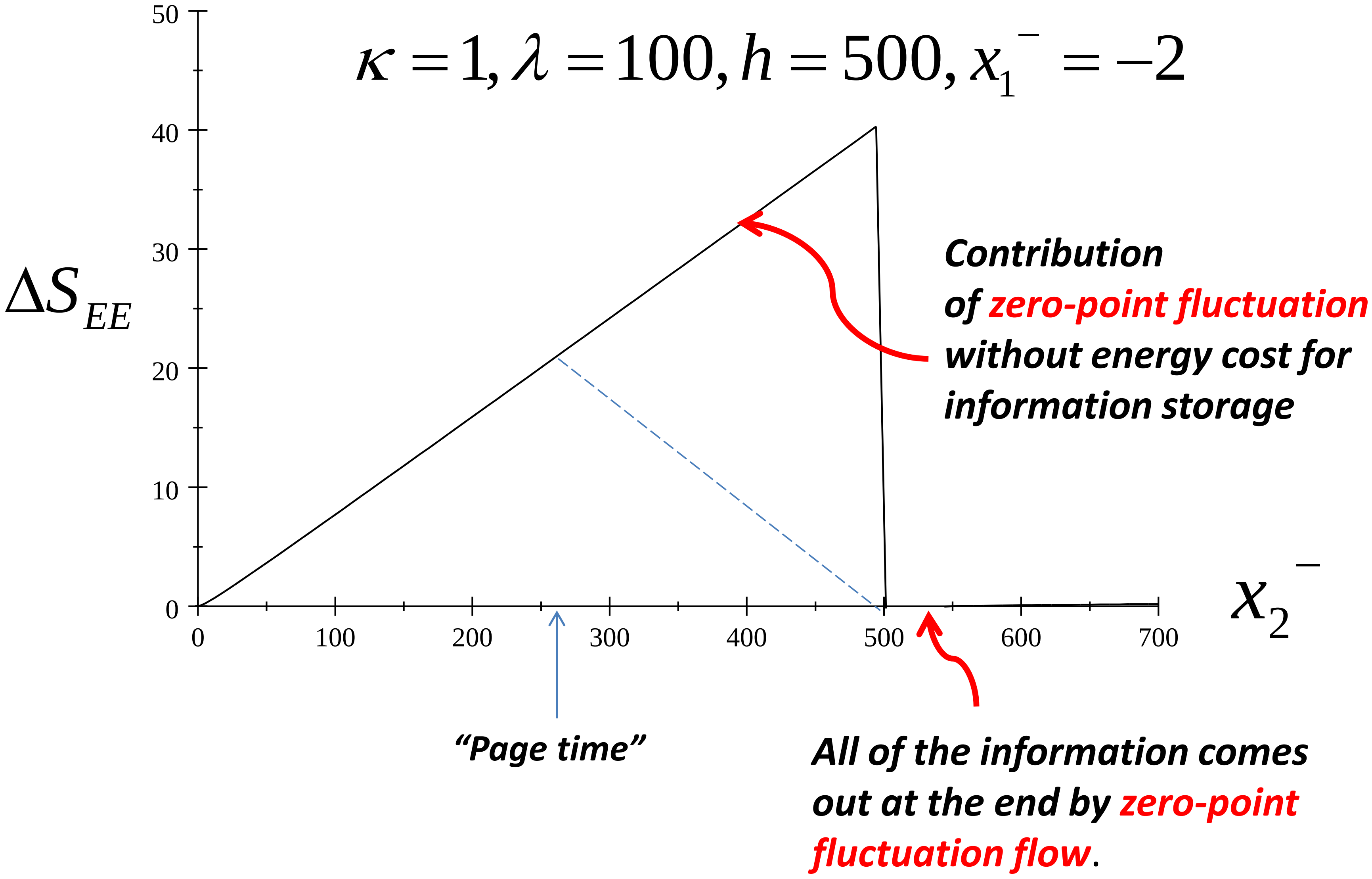
Entanglement

Hawking Particle

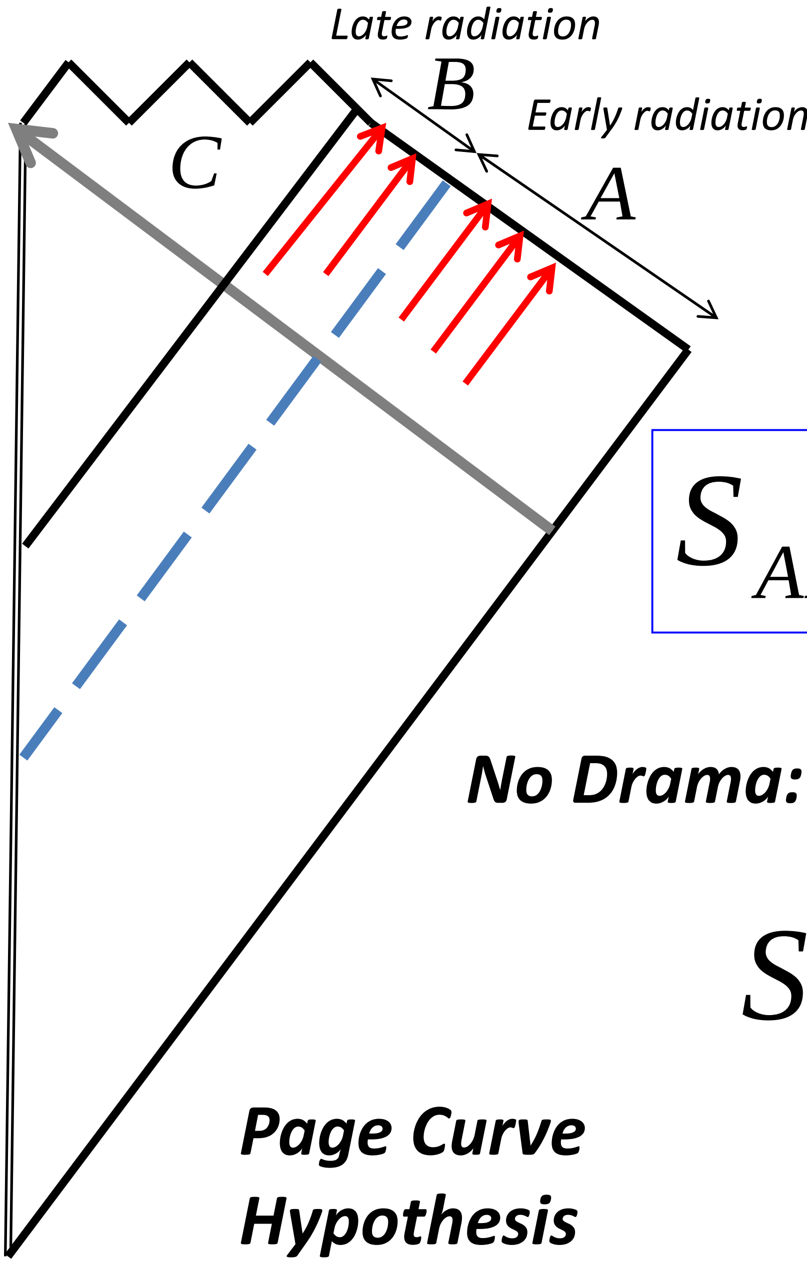
Zero-Point Fluctuation Flow with Zero Energy

(Wilczek, Hotta-Schützhold-Unruh, Hawking)

Modified Page Curve in Moving Mirror Model



Strong Subadditivity "Paradox"



Strong subadditivy:

$$S_{AB} \geq S_B + S_{ABC} - S_{BC}$$

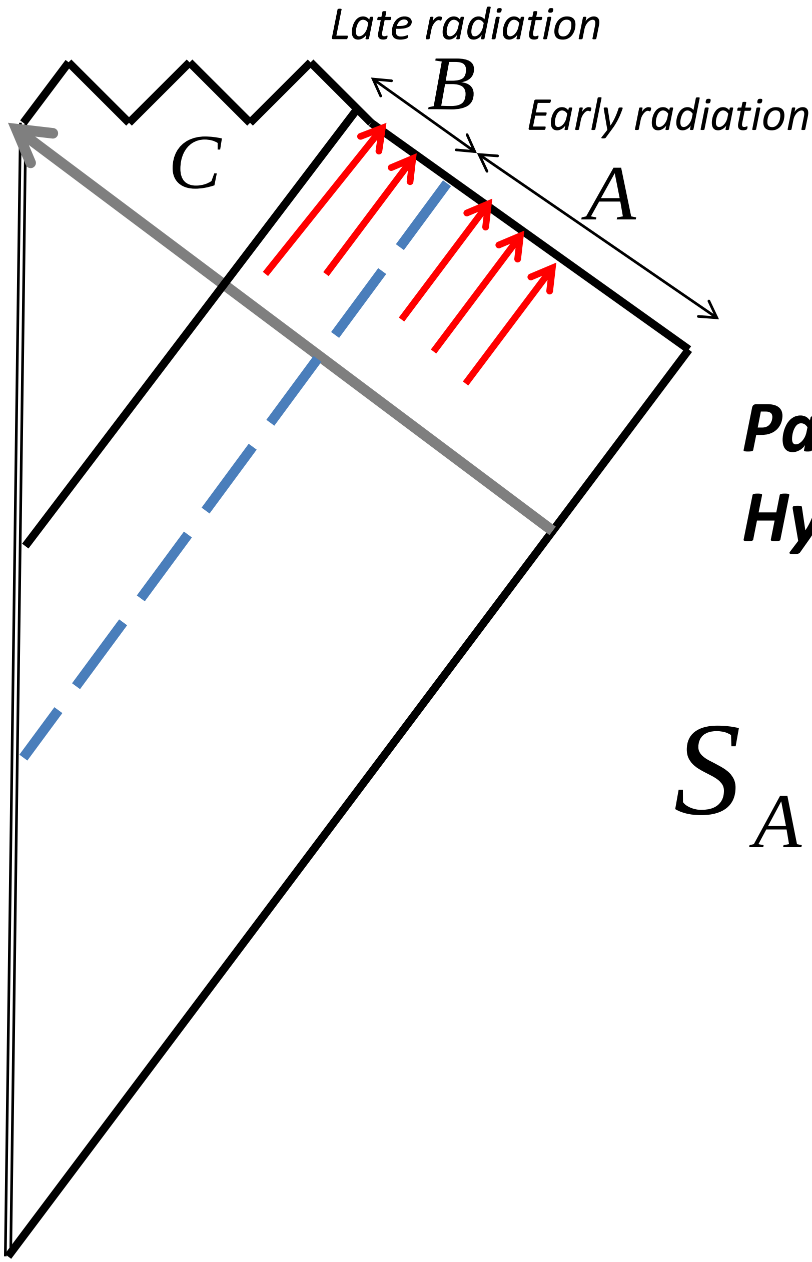
No Drama: $S_{BC} = 0,\, S_{ABC} = S_A$

$$S_{AB} \geq S_B + S_A$$

Page Curve Hypothesis

$$S_A > S_{AB}$$

Strong Subadditivity "Paradox"

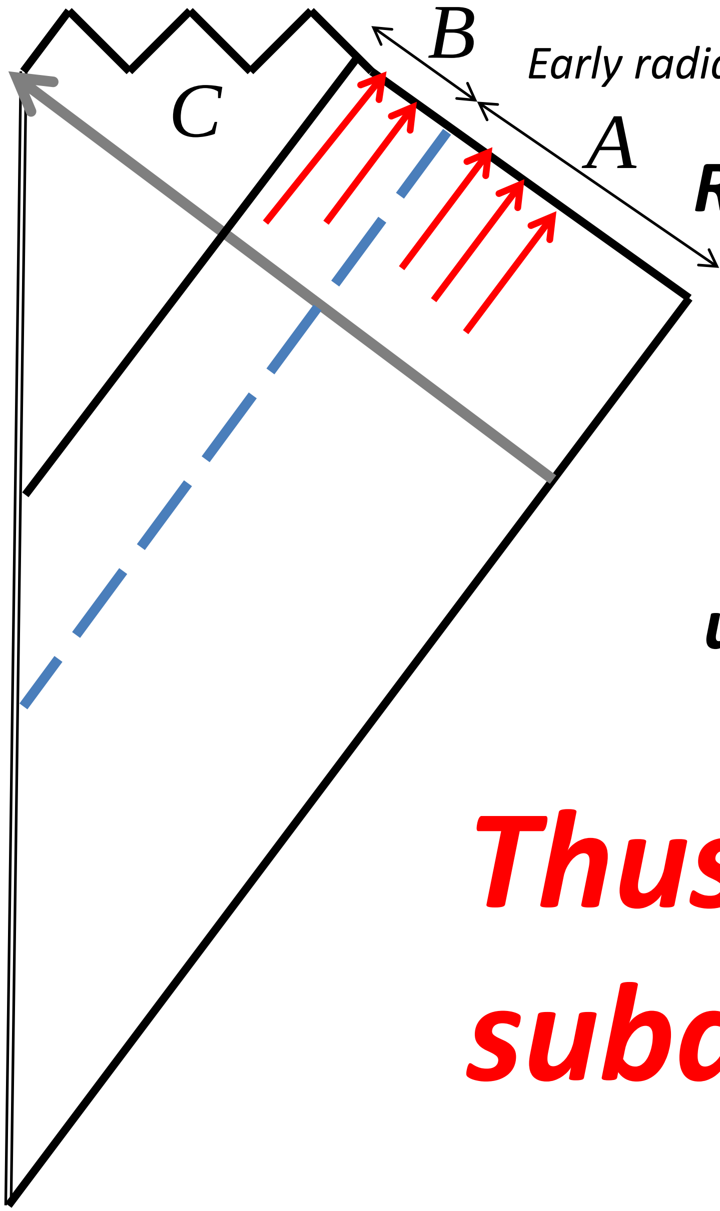


Page Curve Hypothesis $S_A > S_{AB}$

$$S_A > S_{AB} \geq S_B + S_A$$

$$0 > S_B$$

Strong Subadditivity "Paradox"



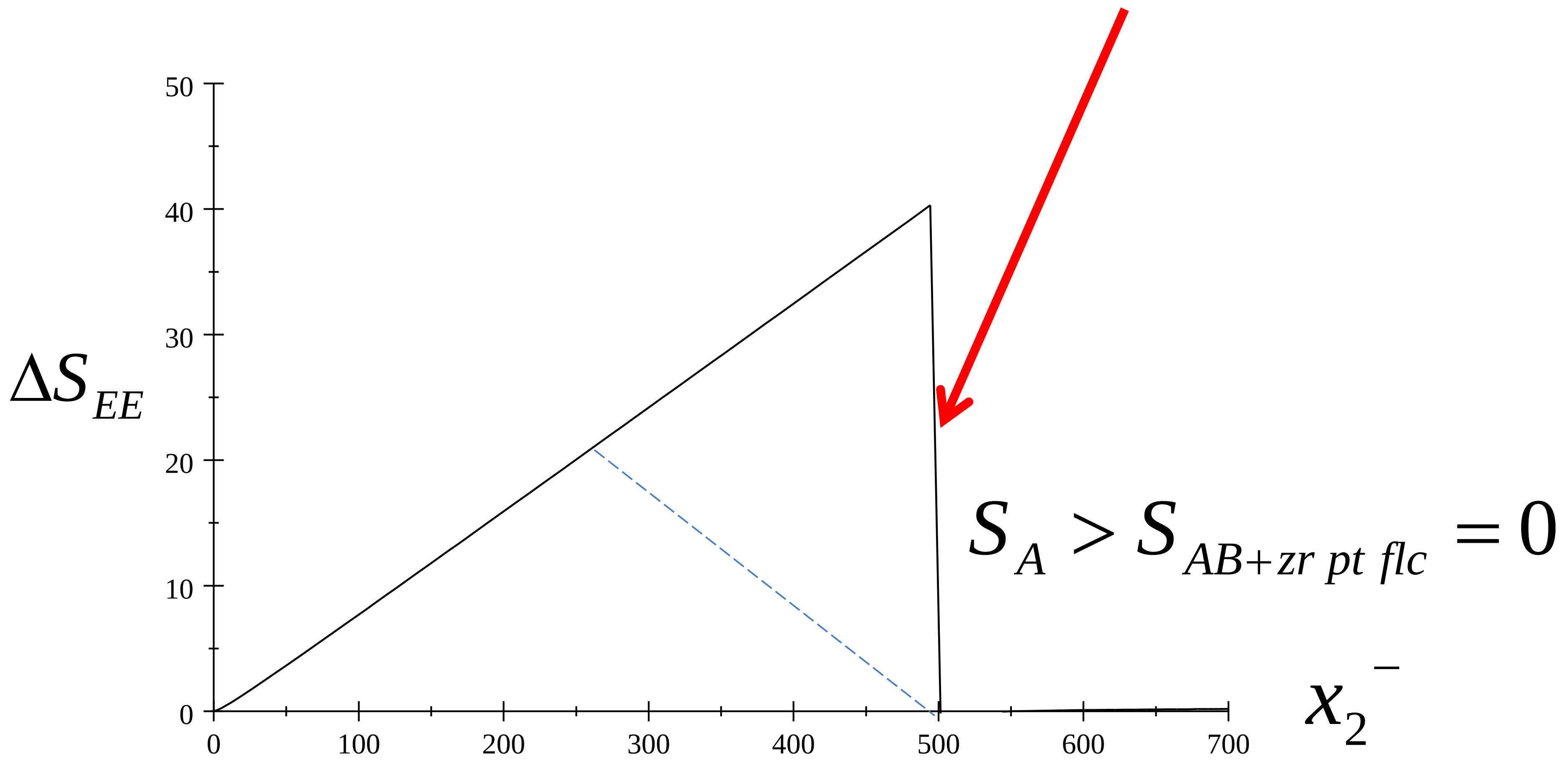
Remnant& Zero-Point Fluctuation Flow

$$S_A < S_{AB}$$

until the last burst.

Thus, no strong subadditivity paradox!

We don't care the no drama condition breaks at the last burst,
because the horizon is affected by quantum gravity.



The last burst

Summary

○ *Adopting canonical typicality for nondegenerate systems with nonvanishing Hamiltonians, the entanglement becomes non-maximal, and BH firewalls do not emerge.*

○ *Typical states must be Gibbs states for smaller quantum systems. If we have stable Gibbs states for old Schwarzschild BH's (and small AdS BH's), the heat capacity must be positive. Because it is actually negative, the states of BH evaporation are not typical.*

⇒ *Inevitable Modification of the Page Curve*

Note: for a large AdS BH and Hawking radiation in a thermal equilibrium, the entanglement entropy equals the thermal entropy of the smaller system.