YITP Workshop on Microstructures of Black Holes

November 26, 2015

On “Exotic” Brane Dynamics, Gravity and Gauge Theories

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Contents

- A motivation
- Aspects of exotic branes
- GLSM
- Brane configurations
- Questions (which I would like to ask you)

A motivation

Compactify 11-dim. supergravity (SUGRA) on a torus $\boldsymbol{T^d}$

$\rightarrow$ $(\mathbf{11} - \boldsymbol{d})$-dim maximal SUGRA has global symmetry $\boldsymbol{G_0}$

$\boldsymbol{G_0 = E_{d(d)}}$: U-duality symmetry

Deformation of maximal SUGRA by...

1. introducing mass parameter Romans' mass
2. gauging subgroup of $\boldsymbol{G_0}$ embedding tensors

their String/M-theory origin ?

embedding tensors and M-theory: de Wit, Nicolai, Samtleben arXiv:0801.1294, etc.

1.	Romans' mass	→	D8-brane
2.	embedding tensors	→	domain walls

String/M-theory origins of many domain walls are not so common, but

referred to as

Exotic Branes

from standard branes via string dualities

in lower dimensions

Exotic branes should be ingredients of black hole microstructures,

as discussed in this workshop.

Exotic brane is ...

1. brane of codim $\boldsymbol{T \leq 2}$

   $\boldsymbol{T = 2}$: defect brane (vortex), $\boldsymbol{T = 1}$: domain wall (kink)

2. mass/tension $\propto \boldsymbol{1/g_{\mathrm{s}}^{n}}$ (often with $\boldsymbol{n \geq 2}$)
3. monodromy on background spacetime geometry by string dualities

ex)

$\boldsymbol{D}$	domain walls	defect branes
10	D8	D7, 7_3
8	D6, $6_3^{(1,1)}$	D5, NS5, KK5, 5_2^2, 5_3^2

Nomenclature $b_n^{(d,c)}$:

$$\text{mass} = \frac{R_1 R_2 \cdots R_b (R_{b+1} \cdots R_{b+c})^2 (R_{b+c+1} \cdots R_{b+c+d})^3}{g_s^n \, \ell_s^{b+2c+3d+1}}$$

String dualities

$$\mathrm{T}_y \quad : \quad R_y \to \frac{\ell_s^2}{R_y}, \qquad g_s \to \frac{\ell_s}{R_y} g_s$$

$$\mathrm{S} \quad : \quad g_s \to \frac{1}{g_s}, \qquad \ell_s \to g_s^{1/2} \ell_s$$

$$\begin{tikzcd} \text{D5(12345)} \arrow[r, "\mathrm{S}"] & \text{NS5(12345)} \arrow[r, "\mathrm{T}_9"] & \text{KK5(12345,9)} \arrow[r, "\mathrm{T}_8"] & 5_2^2\text{(12345,89)} \arrow[r, "\mathrm{S}"] & 5_3^2\text{(12345,89)} \\ 5_1 & 5_2 & 5_2^1 & & \end{tikzcd}$$

SUGRA solutions corresponding to exotic branes are not well-defined :

1. asymptotically non-flat logarithmic ($T = 2$), linear ($T = 1$)
2. multi-valued monodromy by string dualities

SUGRA solution of exotic 5^2_2-brane (string frame)

$$\begin{aligned} ds^2 &= dx^2_{012345} + H\{(d\varrho)^2 + \varrho^2(d\vartheta)^2\} + \frac{H}{K}\{(dx^8)^2 + (dx^9)^2\} \\ B_{89} &= -\frac{\vartheta}{K}, \qquad e^{2\phi} = \frac{H}{K} \\ H &= 1 + \log\frac{\Lambda}{\varrho}, \qquad K = H^2 + \vartheta^2 \end{aligned}$$

Brane charge is not given in a conventional way.

defect branes:	Bergshoeff, Ortín, Riccioni arXiv:1109.4484
various setup:	de Boer, Shigemori arXiv:1209.6056
Alice strings:	Okada, Sakatani arXiv:1411.1043

GLSM

Exotic branes are subject to string dualities.

It is important to study them in string theory beyond SUGRA.

2D Gauged Linear Sigma Model (GLSM)

Witten hep-th/9301042; Hori, Vafa hep-th/0002222

We studied 2D $\mathcal{N} = (4, 4)$ GLSM

TK, Sasaki arXiv:1304.4061

whose IR effective theory is string sigma model on exotic 5^2_2-brane geometry.

This is a generalization of Tong's GLSM for ALF space.

ALE : Douglas, Moore hep-th/9603167; Johnson, Myers hep-th/9610140

ALF : Tong hep-th/0204186; Harvey, Jensen hep-th/0507204; Okuyama hep-th/0508097

GLSM for five-branes

- 2D $\mathcal{N} = (4,4)$ gauge theory
  with charged/adjoint hypermultiplets

- Higgs branch moduli space
  = background geometry of five-branes

- Those for NS5-branes/KK5-branes (codim 3)
  are well known

- Those for exotic five-branes (codim $\leq$ 2) are not so

... skip details

GLSM

T-duality is realized by Roček-Verlinde (Hori-Vafa) transformation,

and its generalization :

$$-\frac{1}{g^2}\int \mathrm{d}^4\theta\, |\Theta|^2 - \Big(\int \mathrm{d}^2\widetilde{\theta}\, \Theta\Sigma + \text{(h.c)} \Big) \ \to\ +g^2 \int \mathrm{d}^4\theta \left(\Gamma + \overline{\Gamma} + V \right)^2$$

$$+\frac{1}{g^2}\int \mathrm{d}^4\theta\, |\Psi|^2 - \Big(\int \mathrm{d}^2\theta\, \Psi\Phi + \text{(h.c)} \Big) \ \to\ -g^2 \int \mathrm{d}^4\theta \left\{ \left(\Xi + \overline{\Xi} - (C + \overline{C}) \right)^2 - (\Psi - \overline{\Psi})(C - \overline{C}) \right\}$$

$$(\Phi = \overline{D}_+ \overline{D}_- C)$$

The second transformation is performed without shift symmetry $\Psi \to \Psi + \mathrm{i}\alpha$

co-existence of the scalar parts of Im Ξ and Im Ψ
implies the doubled space by T-duality

Duality transformation with F-term : TK, Yata arXiv:1406.0087

Brane configurations

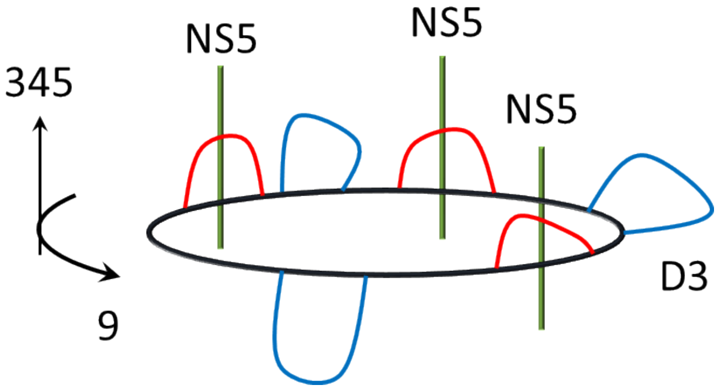
GLSM is also powerful when we embed the system into string theory.

Brane configurations

Tong's 2D GLSM is realized by intersection of $\boldsymbol{k}$ NS5 and a D3 wrapped on circles :

type IIB	0	1	②	③	④	⑤	6	7	8	⑨
k NS5	—	—	—	—	—	—				
D3	—	—	—							—

gauge Coulomb Higgs



gauged flavor $U(1)_F$ as fluctuation on NS5-branes

→ type IIB setup (6D $\mathcal{N} = (1,1)$ vector) ~~type IIA setup (6D $\mathcal{N} = (2,0)$ tensor)~~

2345-directions are compactified

Brane configurations

Once the GLSM is embedded into brane configurations,

we can apply string dualities to it and investigate various aspects of gauge theories :

type IIB	0	1	②	③	④	⑤	6	7	8	⑨
k NS5	—	—	—	—	—	—				
D3	—	—	—							—
	gauge		Coulomb				Higgs			

standard

$$\xleftrightarrow{\mathrm{T}_{89}}$$

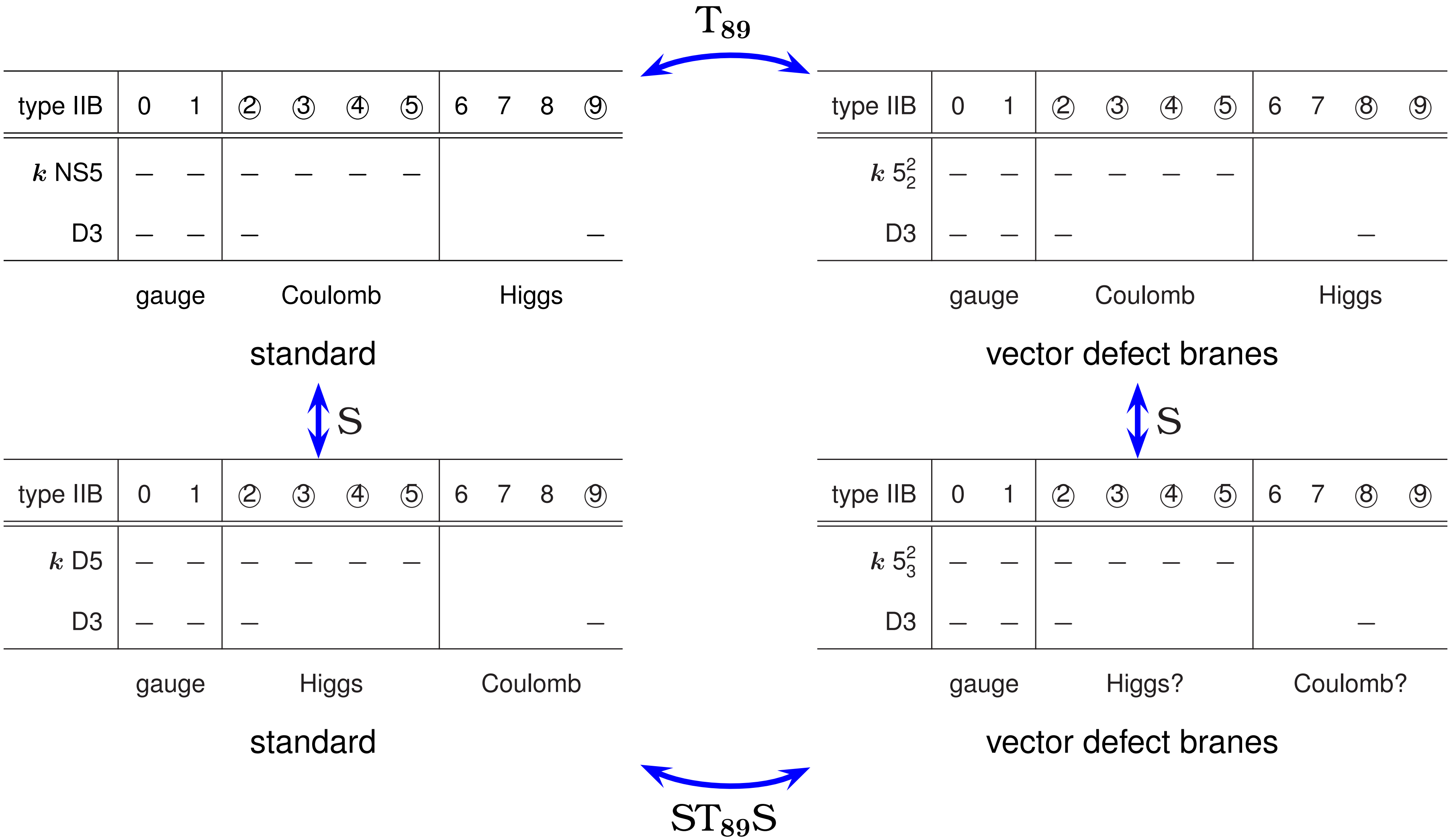
type IIB	0	1	②	③	④	⑤	6	7	⑧	⑨
k 5^2_2	—	—	—	—	—	—				
D3	—	—	—						—	
	gauge		Coulomb				Higgs			

vector defect branes

Brane configurations

Once the GLSM is embedded into brane configurations,

we can apply string dualities to it and investigate various aspects of gauge theories :



T_{89}

type IIB	0	1	②	③	④	⑤	6	7	8	⑨
k NS5	—	—	—	—	—	—				
D3	—	—	—							—

gauge Coulomb Higgs

standard

type IIB	0	1	②	③	④	⑤	6	7	⑧	⑨
k 5^2_2	—	—	—	—	—	—				
D3	—	—	—							—

gauge Coulomb Higgs

vector defect branes

S

S

type IIB	0	1	②	③	④	⑤	6	7	8	⑨
k D5	—	—	—	—	—	—				
D3	—	—	—							—

gauge Higgs Coulomb

standard

type IIB	0	1	②	③	④	⑤	6	7	⑧	⑨
k 5^2_3	—	—	—	—	—	—				
D3	—	—	—							—

gauge Higgs? Coulomb?

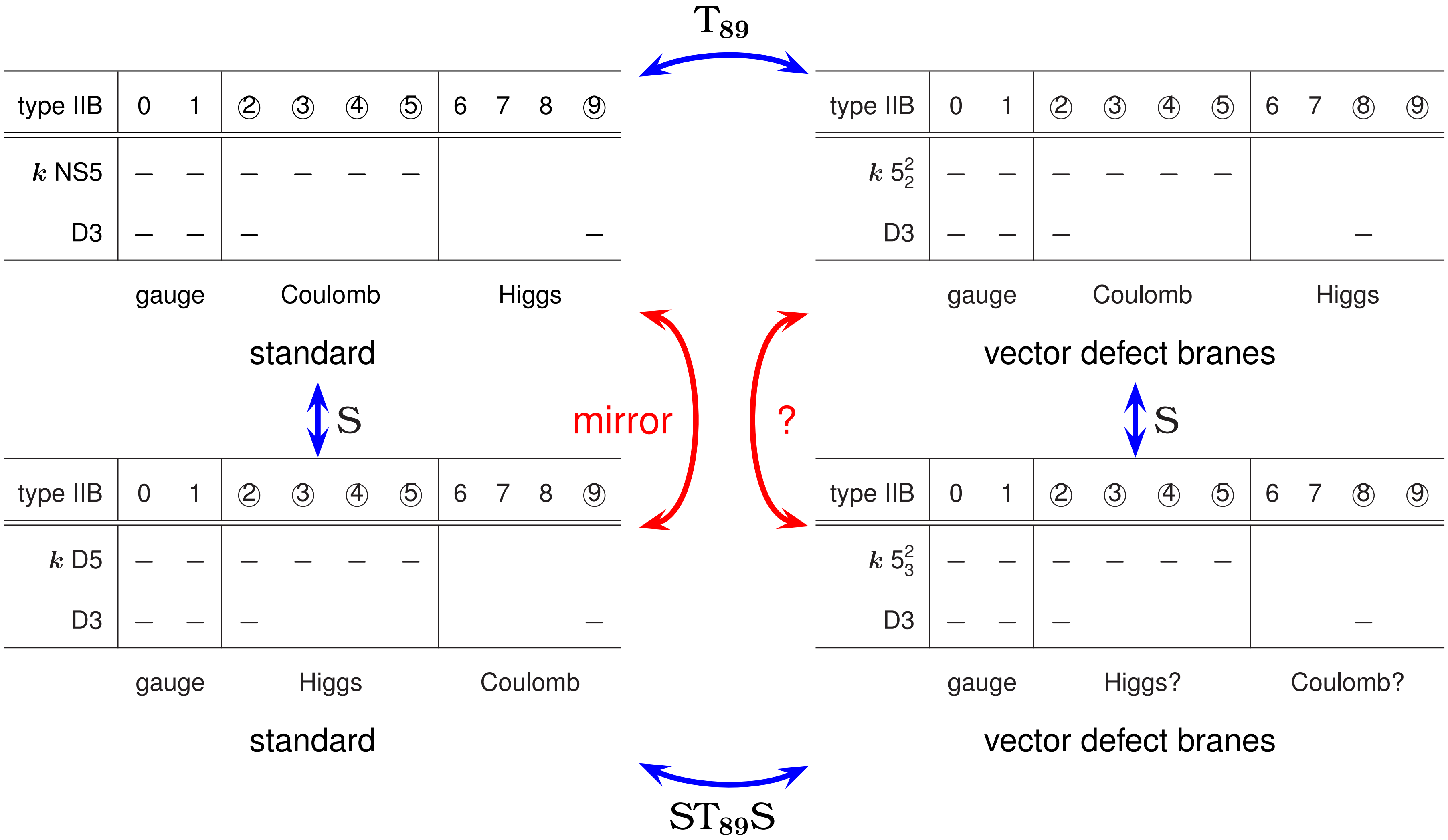
vector defect branes

$ST_{89}S$

Brane configurations

Once the GLSM is embedded into brane configurations,

we can apply string dualities to it and investigate various aspects of gauge theories :



T_{89}

type IIB	0	1	②	③	④	⑤	6	7	8	⑨
k NS5	—	—	—	—	—	—				
D3	—	—	—							—
	gauge		Coulomb				Higgs			

standard

type IIB	0	1	②	③	④	⑤	6	7	⑧	⑨
k 5^2_2	—	—	—	—	—	—				
D3	—	—	—						—	
	gauge		Coulomb				Higgs			

vector defect branes

S

mirror

?

S

type IIB	0	1	②	③	④	⑤	6	7	8	⑨
k D5	—	—	—	—	—	—				
D3	—	—	—							—
	gauge		Higgs				Coulomb			

standard

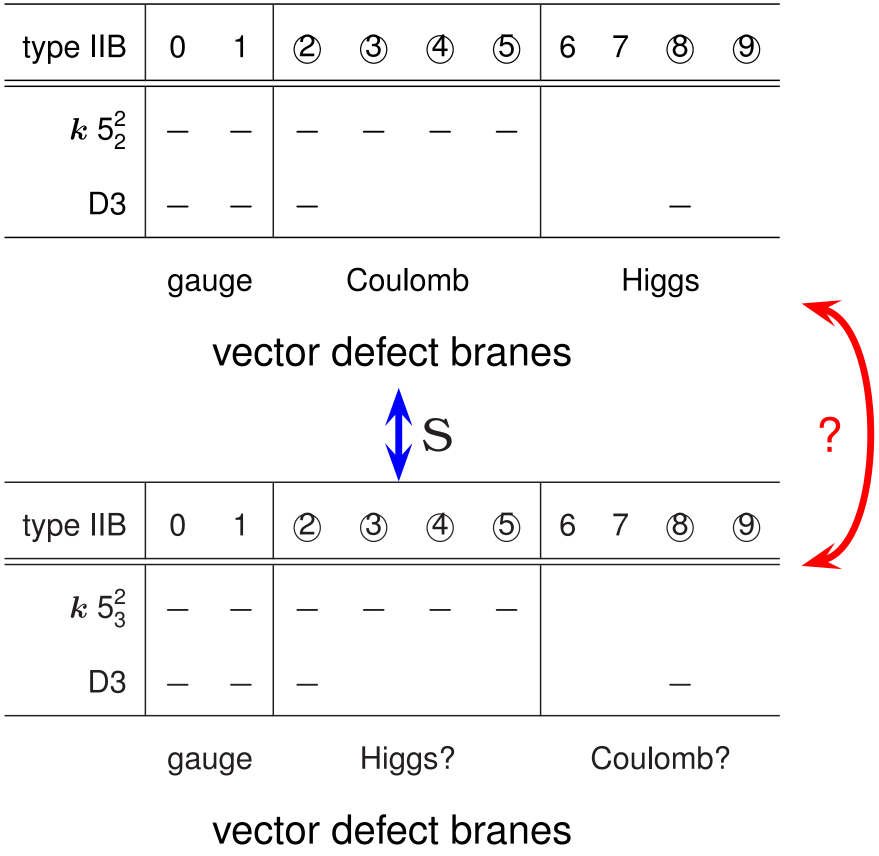
type IIB	0	1	②	③	④	⑤	6	7	⑧	⑨
k 5^2_3	—	—	—	—	—	—				
D3	—	—	—						—	
	gauge		Higgs?				Coulomb?			

vector defect branes

$\mathrm{ST}_{89}\mathrm{S}$

Once the GLSM is embedded into brane configurations,

we can apply string dualities to it and investigate various aspects of gauge theories :



type IIB	0	1	②	③	④	⑤	6	7	⑧	⑨
$k\ 5^2_2$	—	—	—	—	—	—				
D3	—	—	—						—	
	gauge		Coulomb				Higgs			

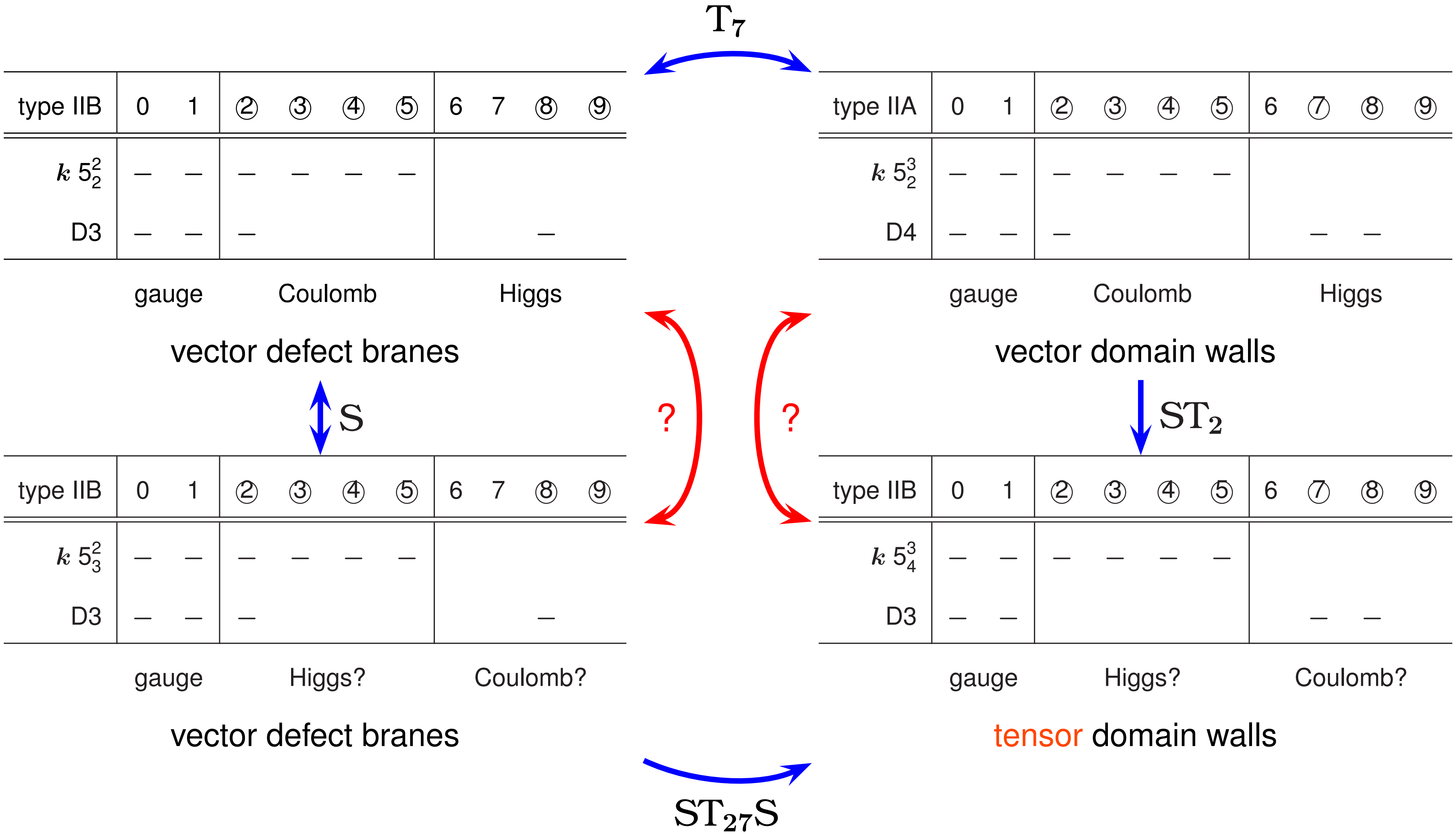
vector defect branes

type IIB	0	1	②	③	④	⑤	6	7	⑧	⑨
$k\ 5^2_3$	—	—	—	—	—	—				
D3	—	—	—						—	
	gauge		Higgs?				Coulomb?			

vector defect branes

Once the GLSM is embedded into brane configurations,

we can apply string dualities to it and investigate various aspects of gauge theories :



T_7

type IIB	0	1	②	③	④	⑤	6	7	⑧	⑨
k 5^2_2	—	—	—	—	—	—				
D3	—	—	—						—	

gauge Coulomb Higgs

vector defect branes

type IIA	0	1	②	③	④	⑤	6	⑦	⑧	⑨
k 5^3_2	—	—	—	—	—	—				
D4	—	—	—					—	—	

gauge Coulomb Higgs

vector domain walls

S

?

?

ST_2

type IIB	0	1	②	③	④	⑤	6	7	⑧	⑨
k 5^2_3	—	—	—	—	—	—				
D3	—	—	—						—	

gauge Higgs? Coulomb?

vector defect branes

type IIB	0	1	②	③	④	⑤	6	⑦	⑧	⑨
k 5^3_4	—	—	—	—	—	—				
D3	—	—						—	—	

gauge Higgs? Coulomb?

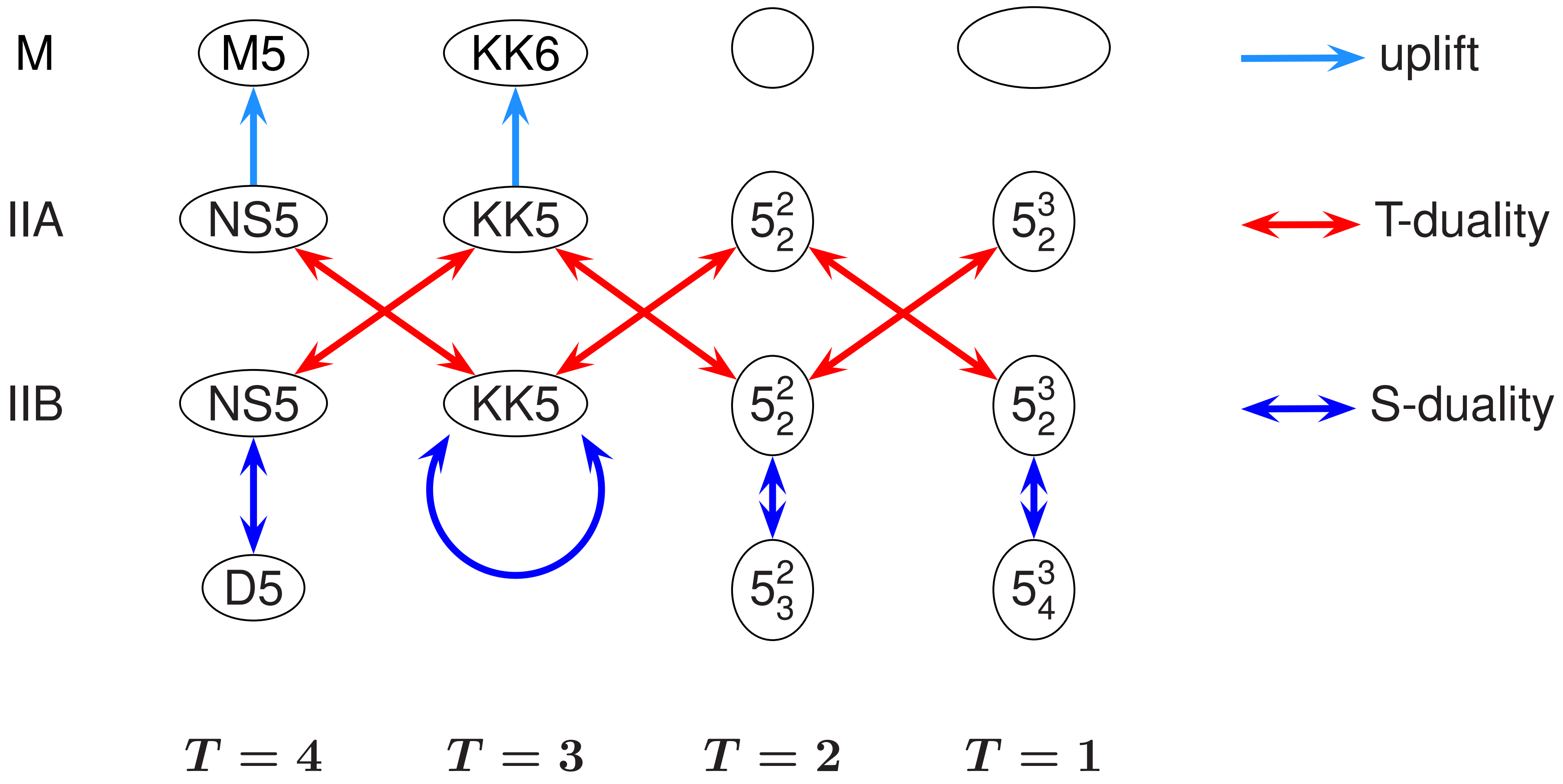
tensor domain walls

$\mathrm{ST}_{27}\mathrm{S}$

Questions which I would like to ask you are ...

1. Uplift exotic branes to M-theory
2. Exotic brane ending on exotic brane
3. Hanany-Witten effect across exotic branes

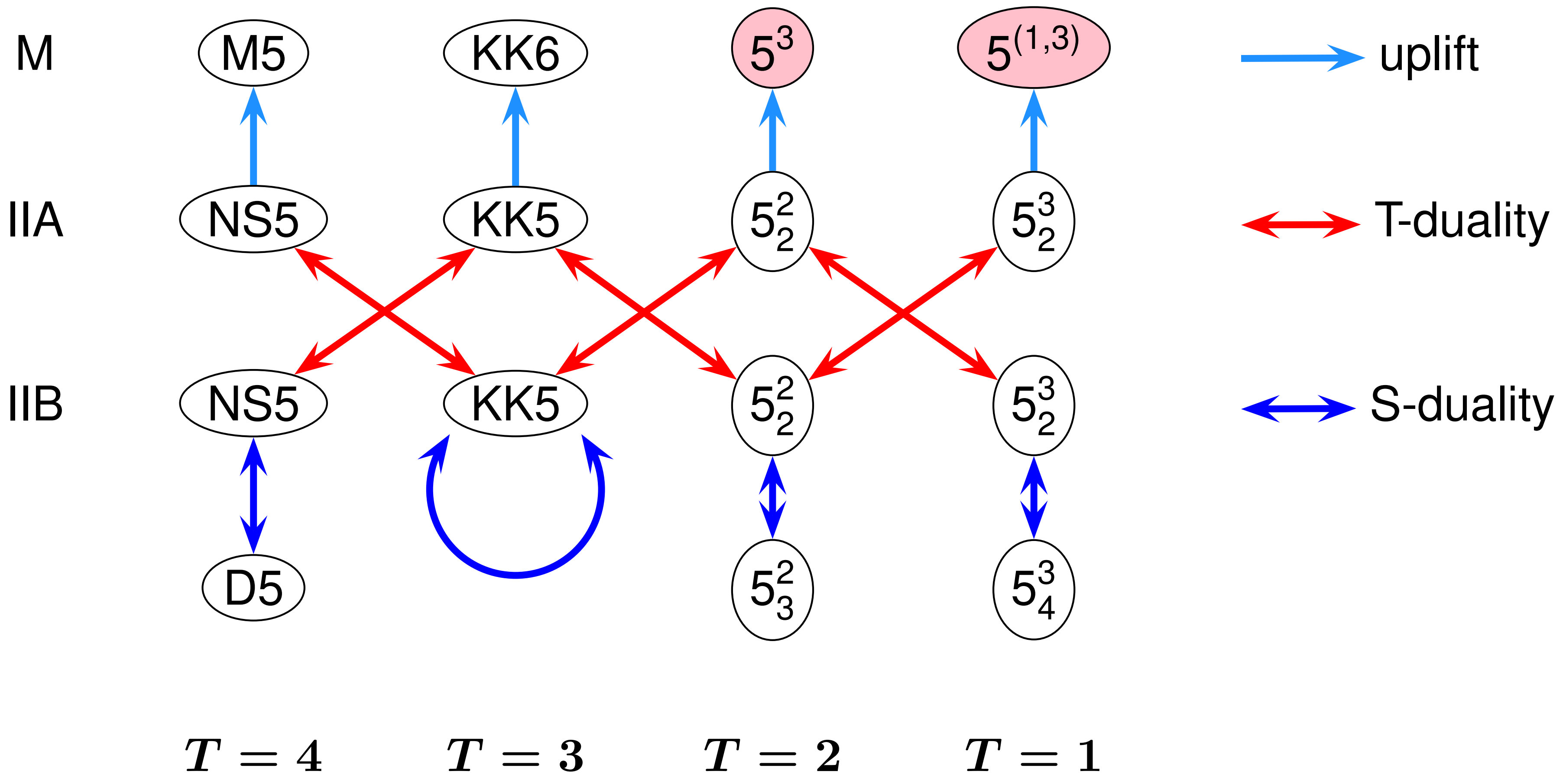
Questions: M-branes



How can we use exotic M-branes (and other exotic branes)?

Worldvolume action : TK, Sasaki, Yata arXiv:1404.5442
TK, Sasaki to appear

Questions: M-branes



M M5 KK6 5^3 $5^{(1,3)}$

IIA NS5 KK5 5^2_2 5^3_2

IIB NS5 KK5 5^2_2 5^3_2

D5 5^2_3 5^3_4

$T = 4$ $T = 3$ $T = 2$ $T = 1$

uplift

T-duality

S-duality

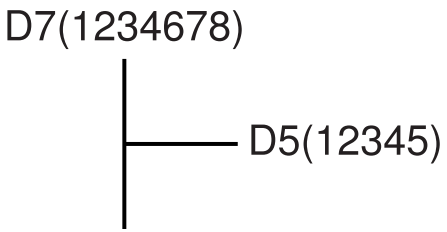
How can we use exotic M-branes (and other exotic branes)?

Worldvolume action : TK, Sasaki, Yata arXiv:1404.5442
TK, Sasaki to appear

Questions: exotic brane ending on exotic brane

We start from D5-D7 system such as

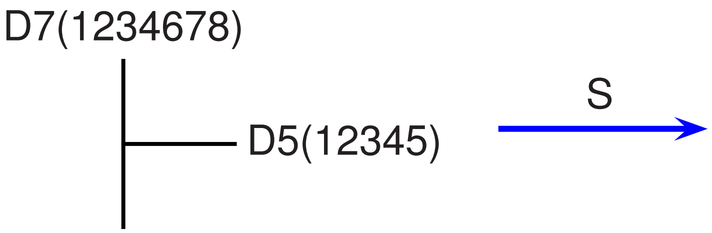
	0	1	2	3	4	5	6	7	8	9
D7	—	—	—	—	—		—	—	—	
D5	—	—	—	—	—	—				



Questions: exotic brane ending on exotic brane

We start from D5-D7 system such as

	0	1	2	3	4	5	6	7	8	9
D7	—	—	—	—	—		—	—	—	
D5	—	—	—	—	—	—				


D7(1234678)

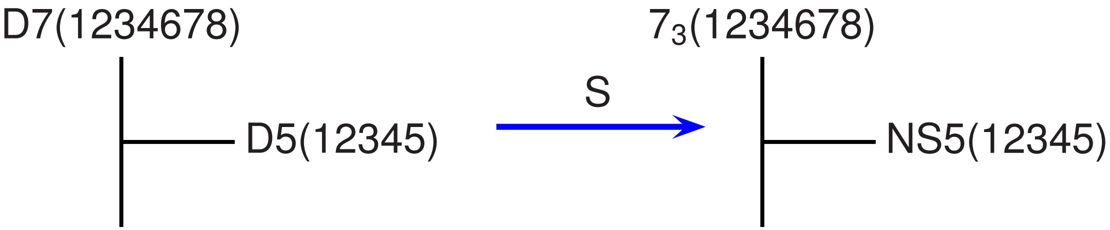
D5(12345)

S

Questions: exotic brane ending on exotic brane

We start from D5-D7 system such as

	0	1	2	3	4	5	6	7	8	9
D7	—	—	—	—	—		—	—	—	
D5	—	—	—	—	—	—				


D7(1234678)

D5(12345)

$\xrightarrow{S}$

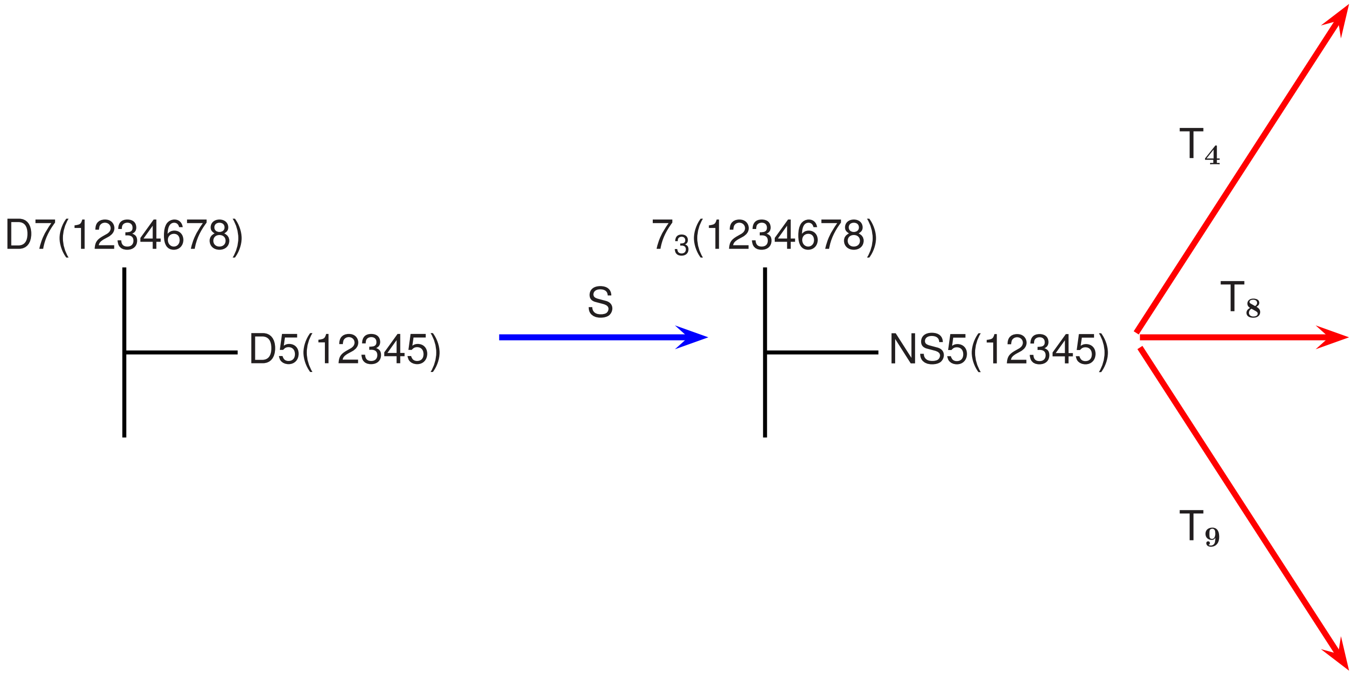
7_3(1234678)

NS5(12345)

Questions: exotic brane ending on exotic brane

We start from D5-D7 system such as

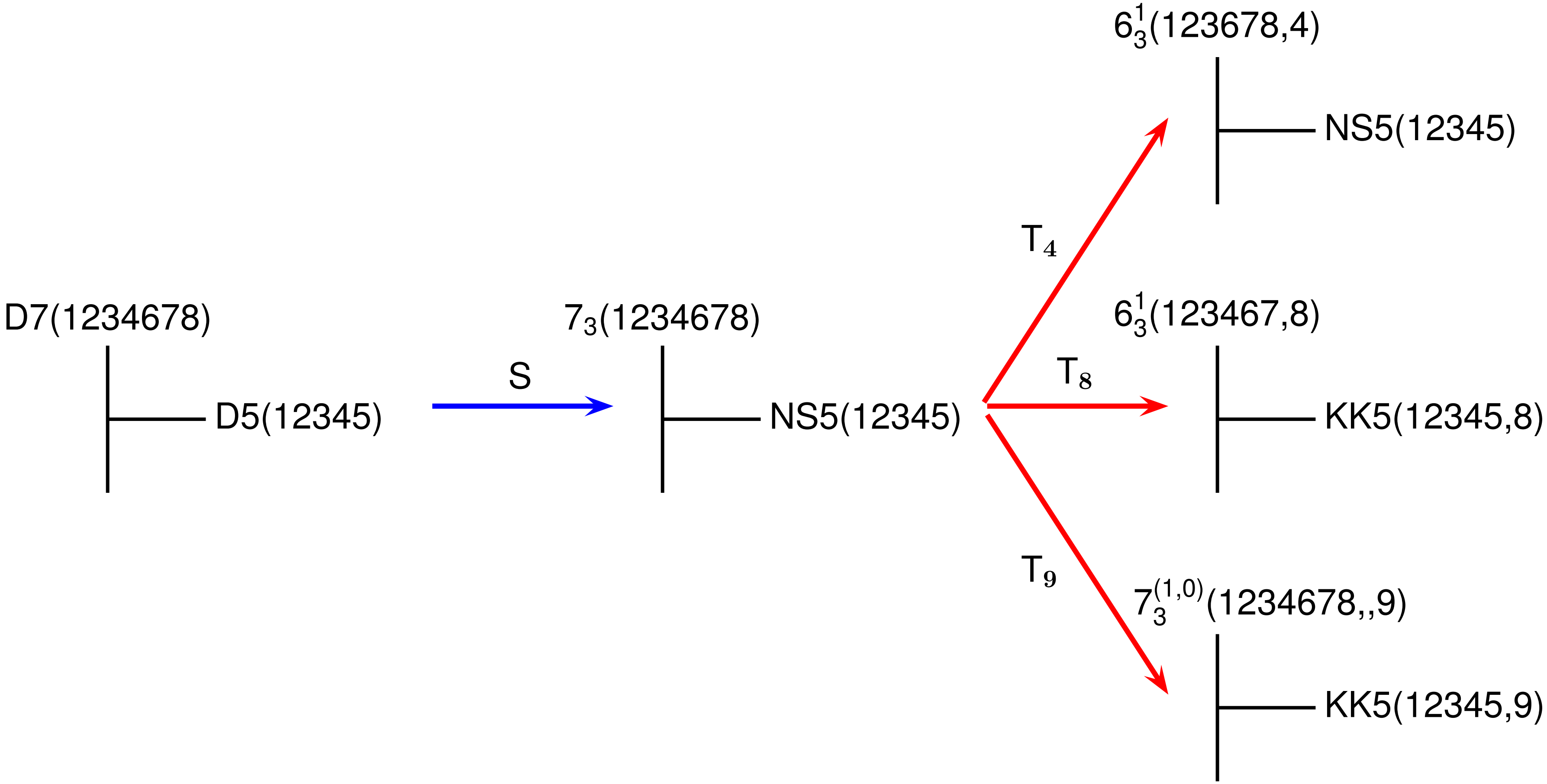
	0	1	2	3	4	5	6	7	8	9
D7	—	—	—	—	—		—	—	—	
D5	—	—	—	—	—	—				



Questions: exotic brane ending on exotic brane

We start from D5-D7 system such as

	0	1	2	3	4	5	6	7	8	9
D7	—	—	—	—	—		—	—	—	
D5	—	—	—	—	—	—				



D7(1234678)

D5(12345)

S

7_3(1234678)

NS5(12345)

T_4

6^1_3(123678,4)

NS5(12345)

T_8

6^1_3(123467,8)

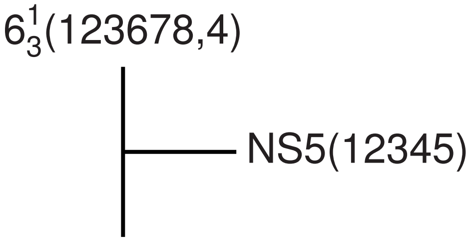
KK5(12345,8)

T_9

$7^{(1,0)}_3$(1234678,,9)

KK5(12345,9)

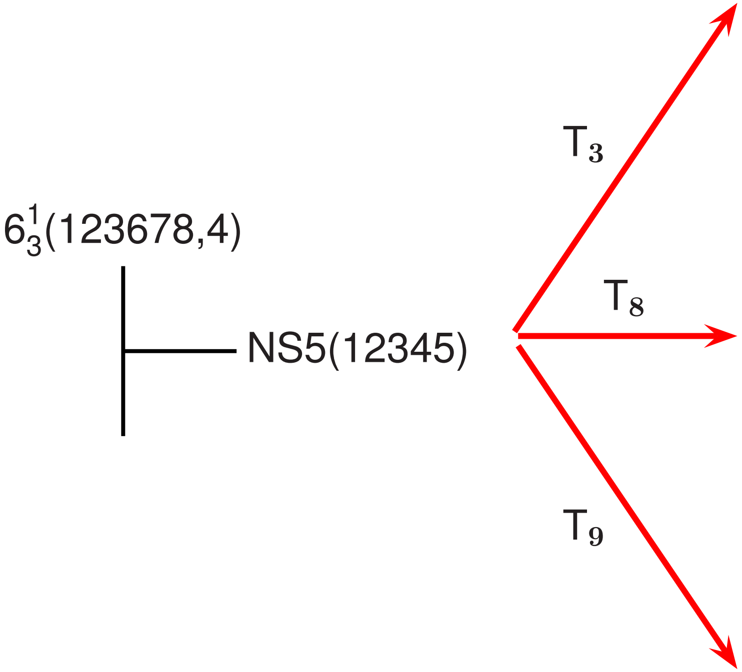
Questions: exotic brane ending on exotic brane



6^1_3(123678,4)

NS5(12345)

Questions: exotic brane ending on exotic brane



6^1_3(123678,4)

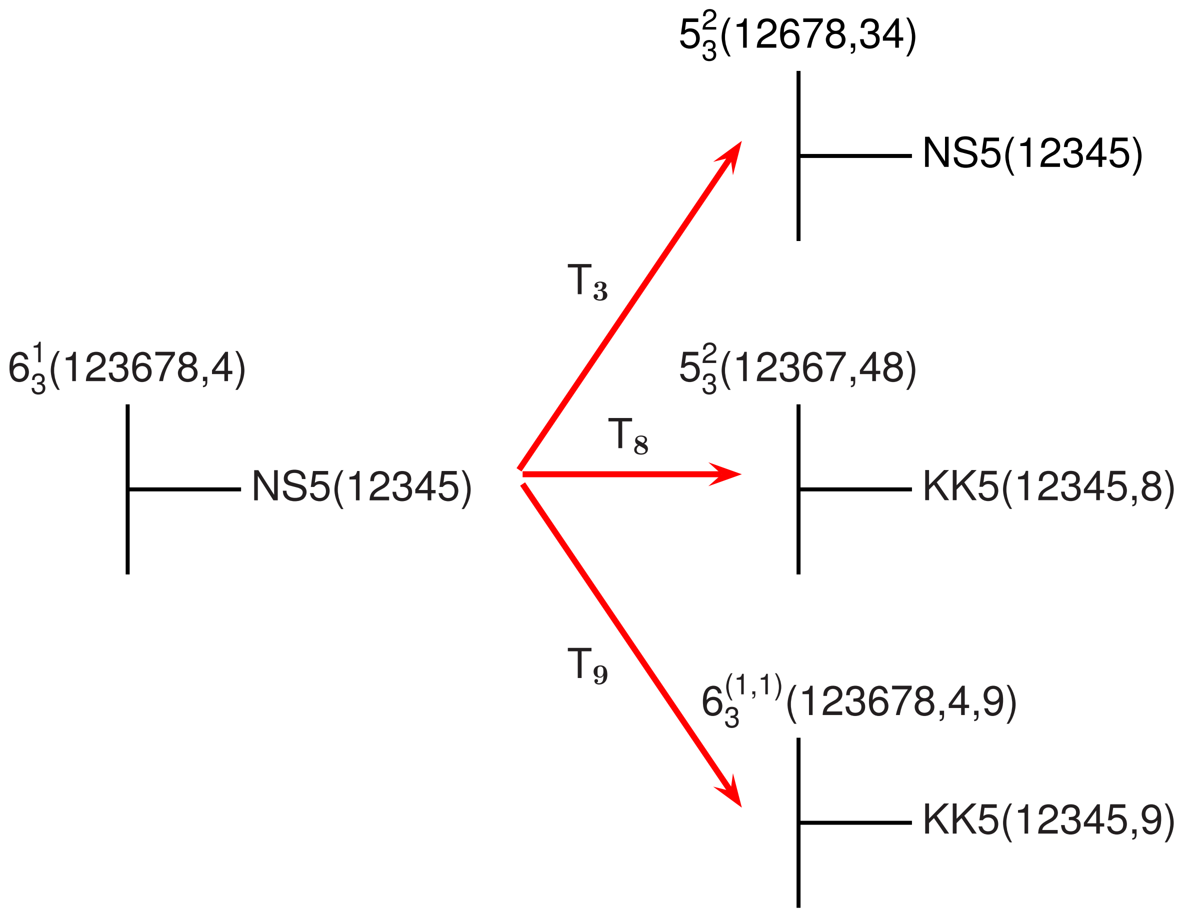
NS5(12345)

T_3

T_8

T_9

Questions: exotic brane ending on exotic brane



6^1_3(123678,4)

NS5(12345)

T_3

T_8

T_9

5^2_3(12678,34)

NS5(12345)

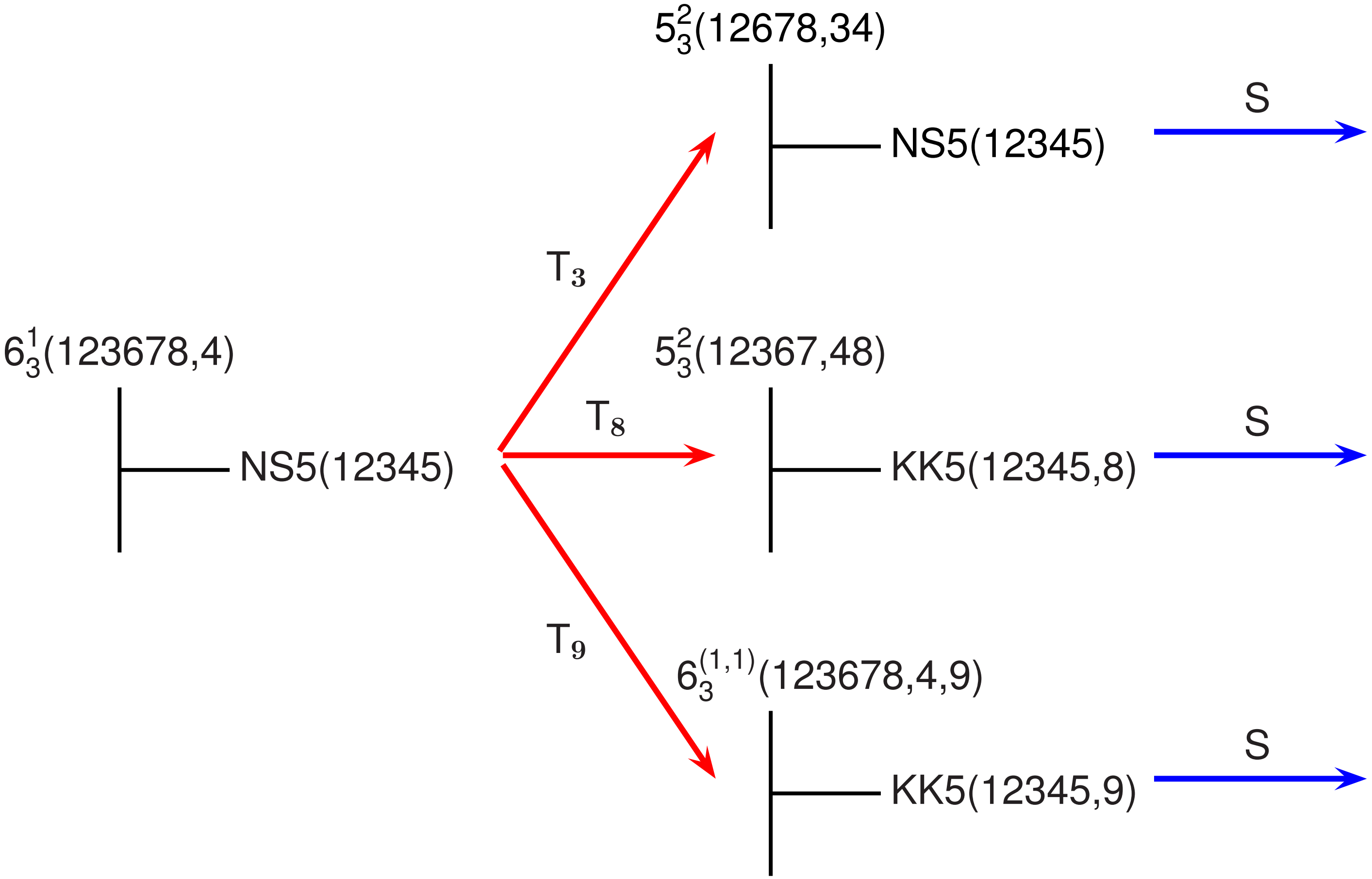
5^2_3(12367,48)

KK5(12345,8)

$6^{(1,1)}_3$(123678,4,9)

KK5(12345,9)

Questions: exotic brane ending on exotic brane



6^1_3(123678,4)

NS5(12345)

T_3

T_8

T_9

5^2_3(12678,34)

NS5(12345)

5^2_3(12367,48)

KK5(12345,8)

$6^{(1,1)}_3$(123678,4,9)

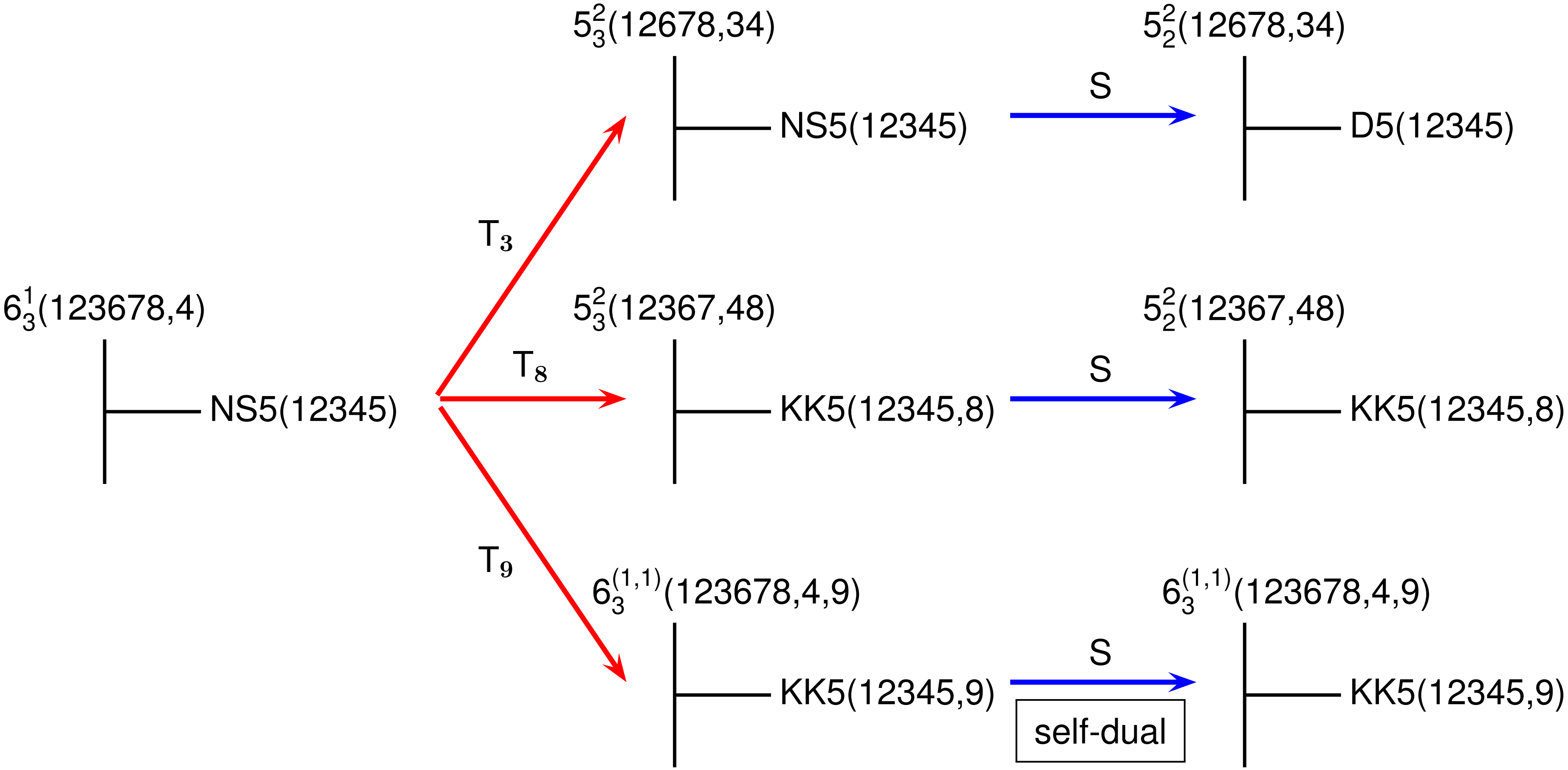
KK5(12345,9)

S

S

S

Questions: exotic brane ending on exotic brane



5^2_3(12678,34) — NS5(12345) $\xrightarrow{S}$ 5^2_2(12678,34) — D5(12345)

6^1_3(123678,4) — NS5(12345)

T_3

T_8

T_9

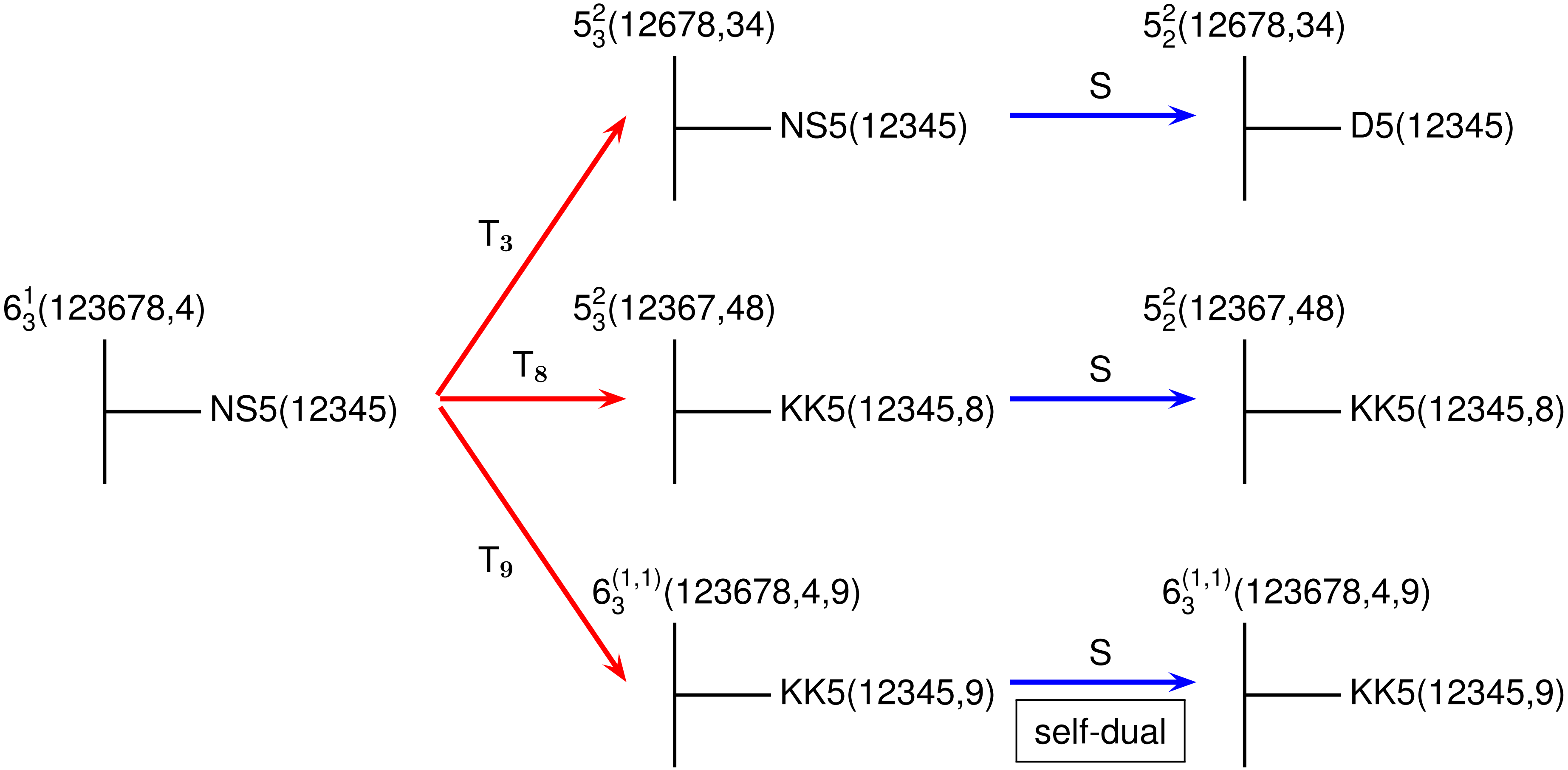
5^2_3(12367,48) — KK5(12345,8) $\xrightarrow{S}$ 5^2_2(12367,48) — KK5(12345,8)

$6^{(1,1)}_3$(123678,4,9) — KK5(12345,9) $\xrightarrow{S}$ $6^{(1,1)}_3$(123678,4,9) — KK5(12345,9)

self-dual

Which modes are localized at intersection?

Questions: exotic brane ending on exotic brane



5^2_3(12678,34) — NS5(12345) $\xrightarrow{S}$ 5^2_2(12678,34) — D5(12345)

6^1_3(123678,4) — NS5(12345) $\xrightarrow{T_3}$ 5^2_3(12678,34) — NS5(12345)

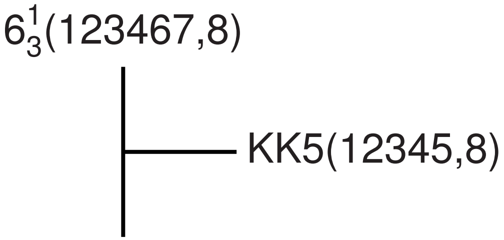
6^1_3(123678,4) — NS5(12345) $\xrightarrow{T_8}$ 5^2_3(12367,48) — KK5(12345,8) $\xrightarrow{S}$ 5^2_2(12367,48) — KK5(12345,8)

6^1_3(123678,4) — NS5(12345) $\xrightarrow{T_9}$ $6^{(1,1)}_3$(123678,4,9) — KK5(12345,9) $\xrightarrow{S}$ $6^{(1,1)}_3$(123678,4,9) — KK5(12345,9)

self-dual

common 0123-directions: 4D exotic brane world scenario?

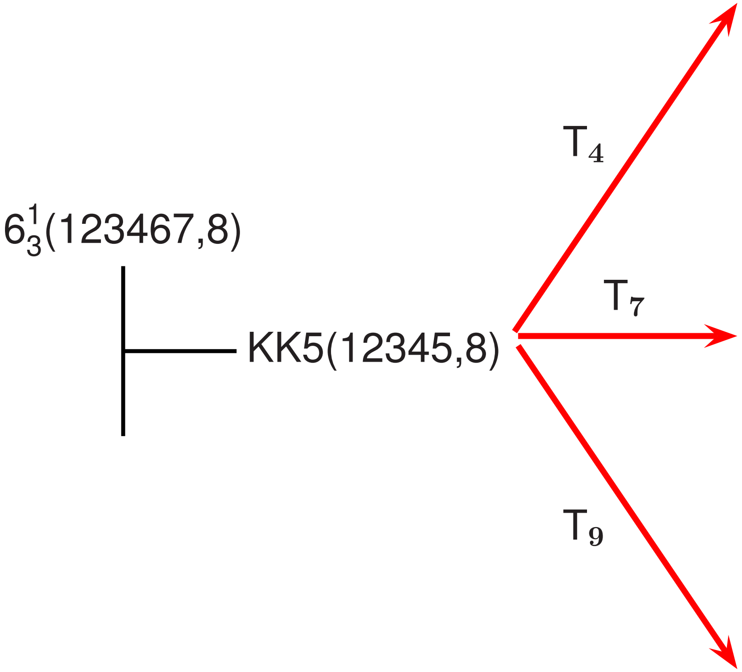
Questions: exotic brane ending on exotic brane



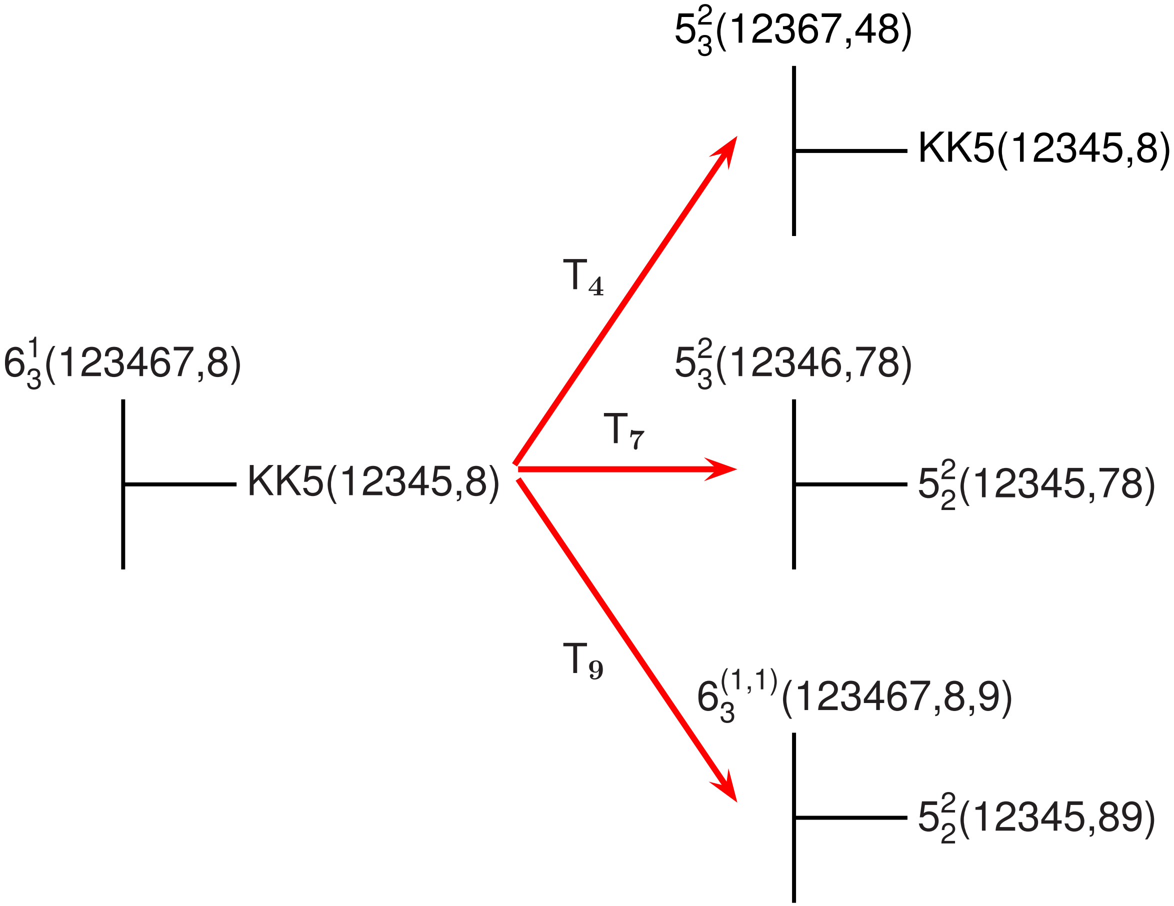
6^1_3(123467,8)

KK5(12345,8)

Questions: exotic brane ending on exotic brane



Questions: exotic brane ending on exotic brane



5^2_3(12367,48)

KK5(12345,8)

T_4

6^1_3(123467,8)

KK5(12345,8)

5^2_3(12346,78)

T_7

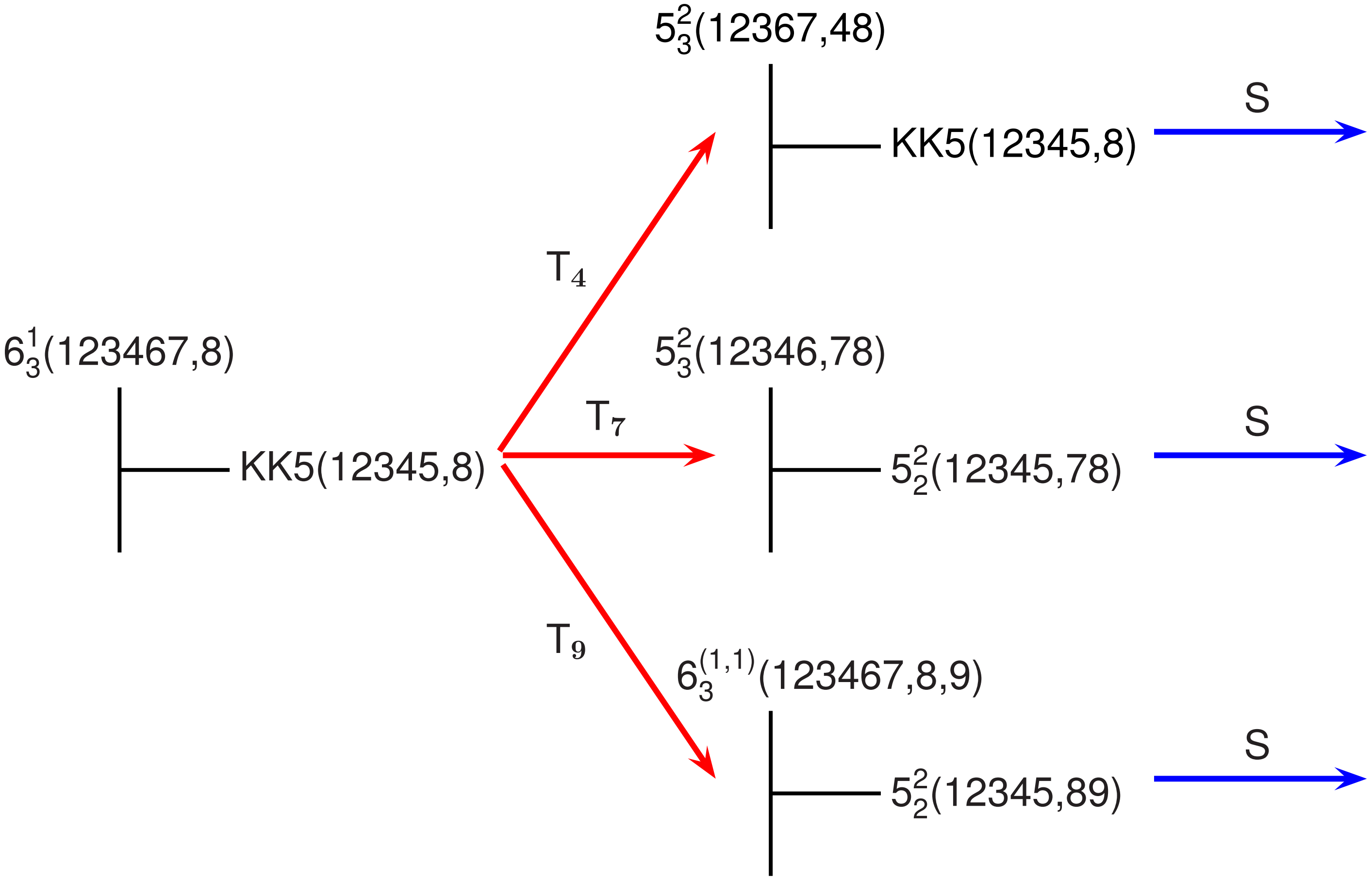
5^2_2(12345,78)

T_9

$6^{(1,1)}_3$(123467,8,9)

5^2_2(12345,89)

Questions: exotic brane ending on exotic brane



6_3^1(123467,8)

KK5(12345,8)

T_4

T_7

T_9

5_3^2(12367,48)

KK5(12345,8)

S

5_3^2(12346,78)

5_2^2(12345,78)

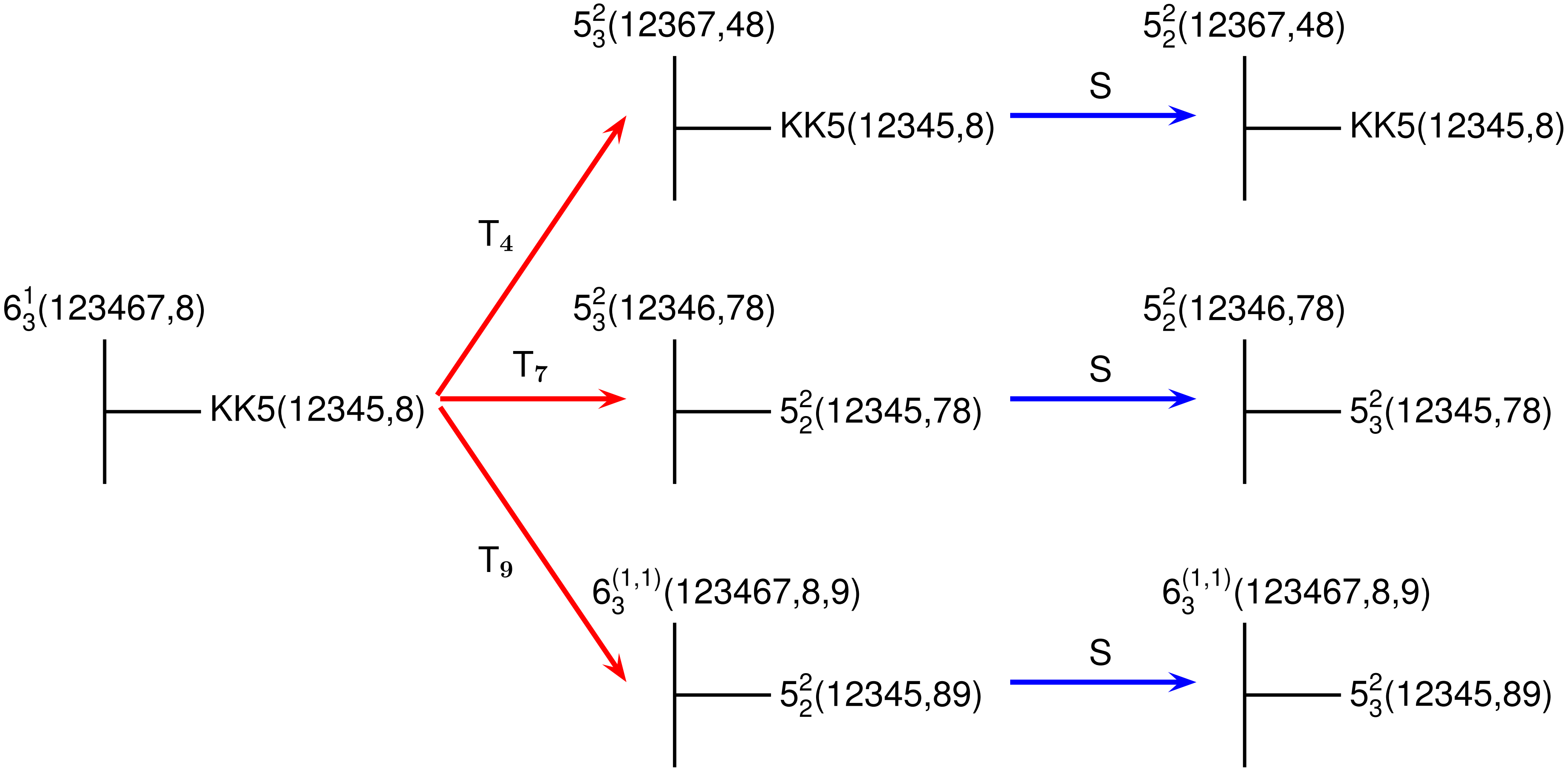
S

$6_3^{(1,1)}$(123467,8,9)

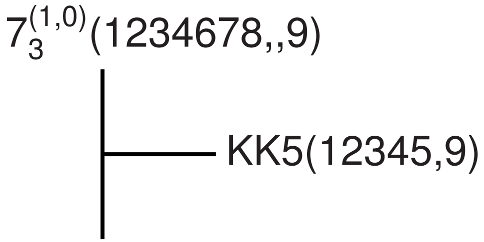
5_2^2(12345,89)

S

Questions: exotic brane ending on exotic brane

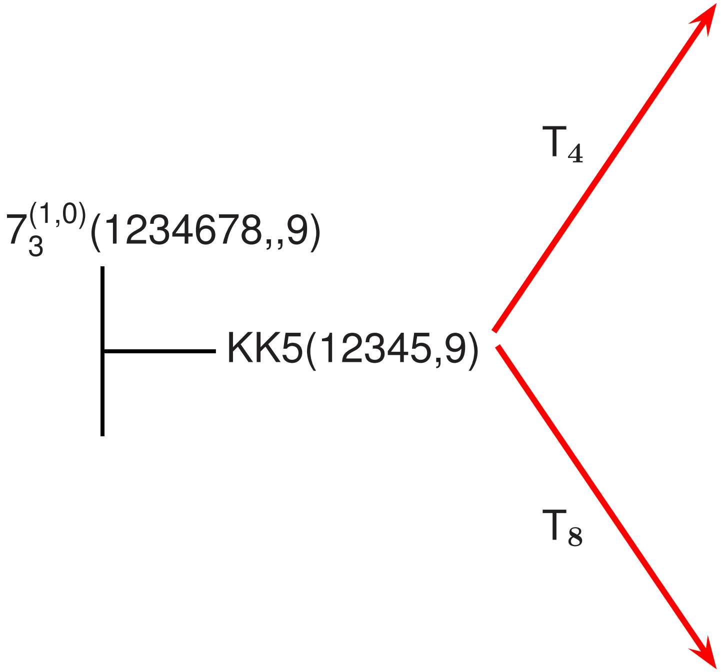


Questions: exotic brane ending on exotic brane



$7_3^{(1,0)}(1234678,,9)$

KK5(12345,9)



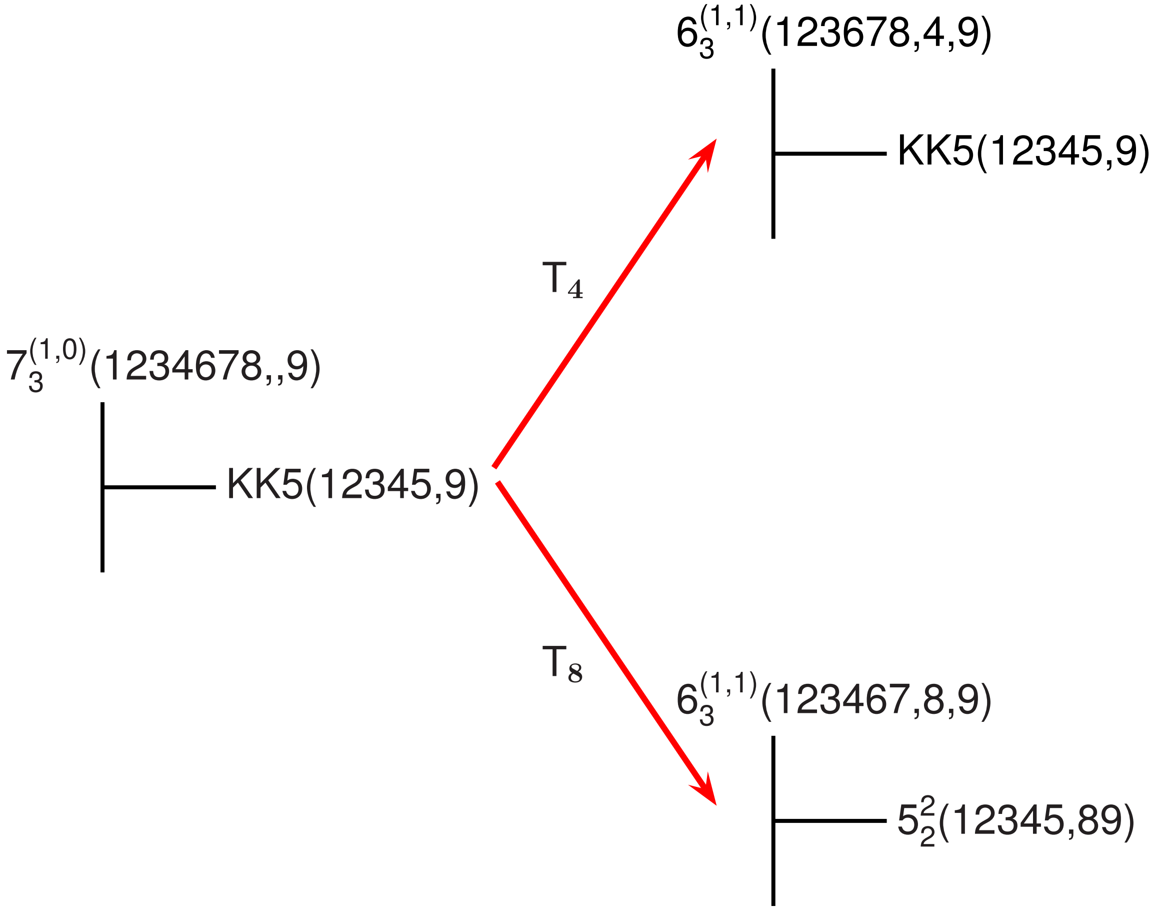
$7_3^{(1,0)}(1234678,,9)$

KK5(12345,9)

T_4

T_8

Questions: exotic brane ending on exotic brane



$6_3^{(1,1)}$(123678,4,9)

KK5(12345,9)

T_4

$7_3^{(1,0)}$(1234678,,9)

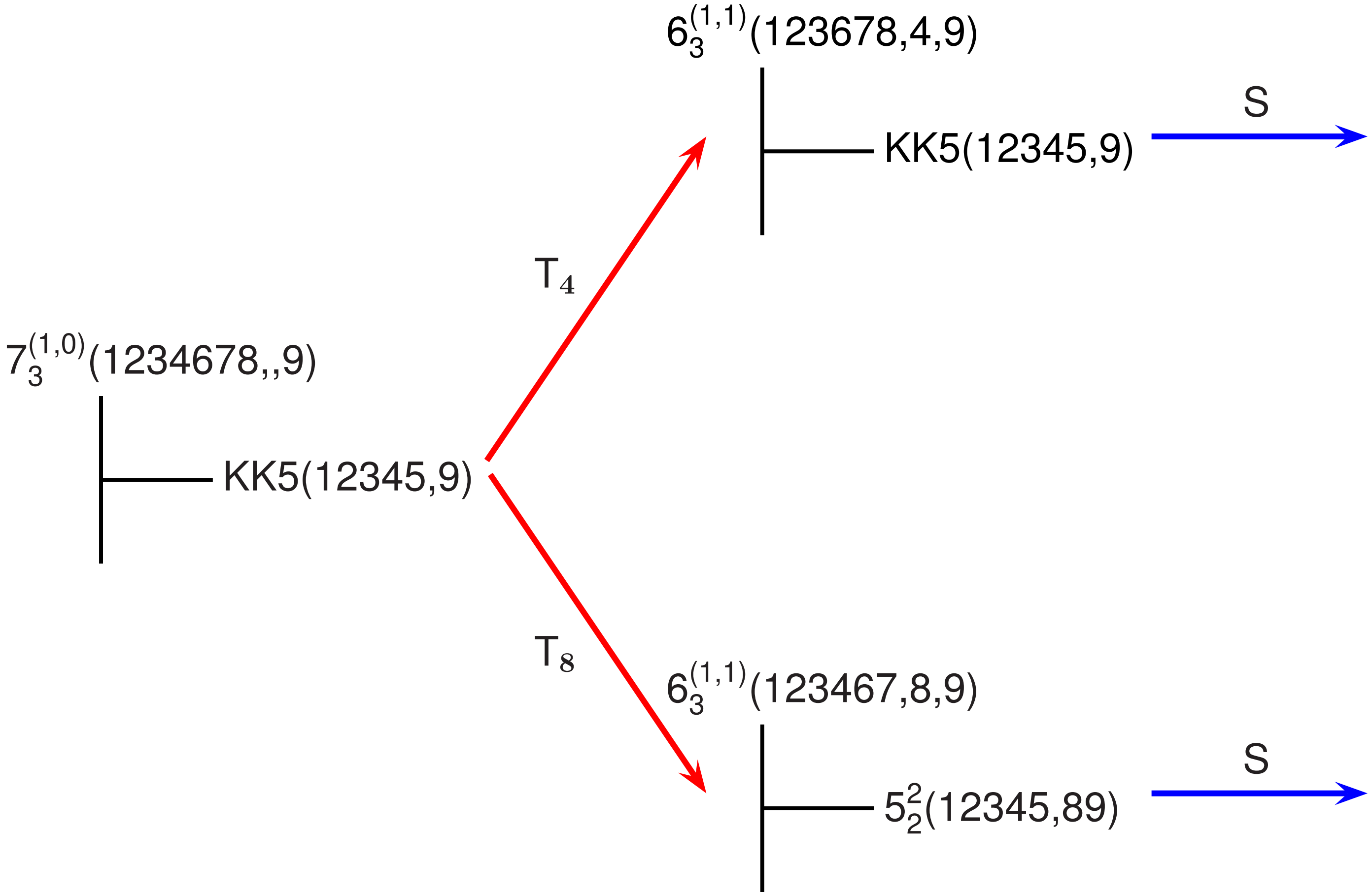
KK5(12345,9)

T_8

$6_3^{(1,1)}$(123467,8,9)

5_2^2(12345,89)

Questions: exotic brane ending on exotic brane



$7_3^{(1,0)}$(1234678,,9)

KK5(12345,9)

T_4

$6_3^{(1,1)}$(123678,4,9)

KK5(12345,9)

S

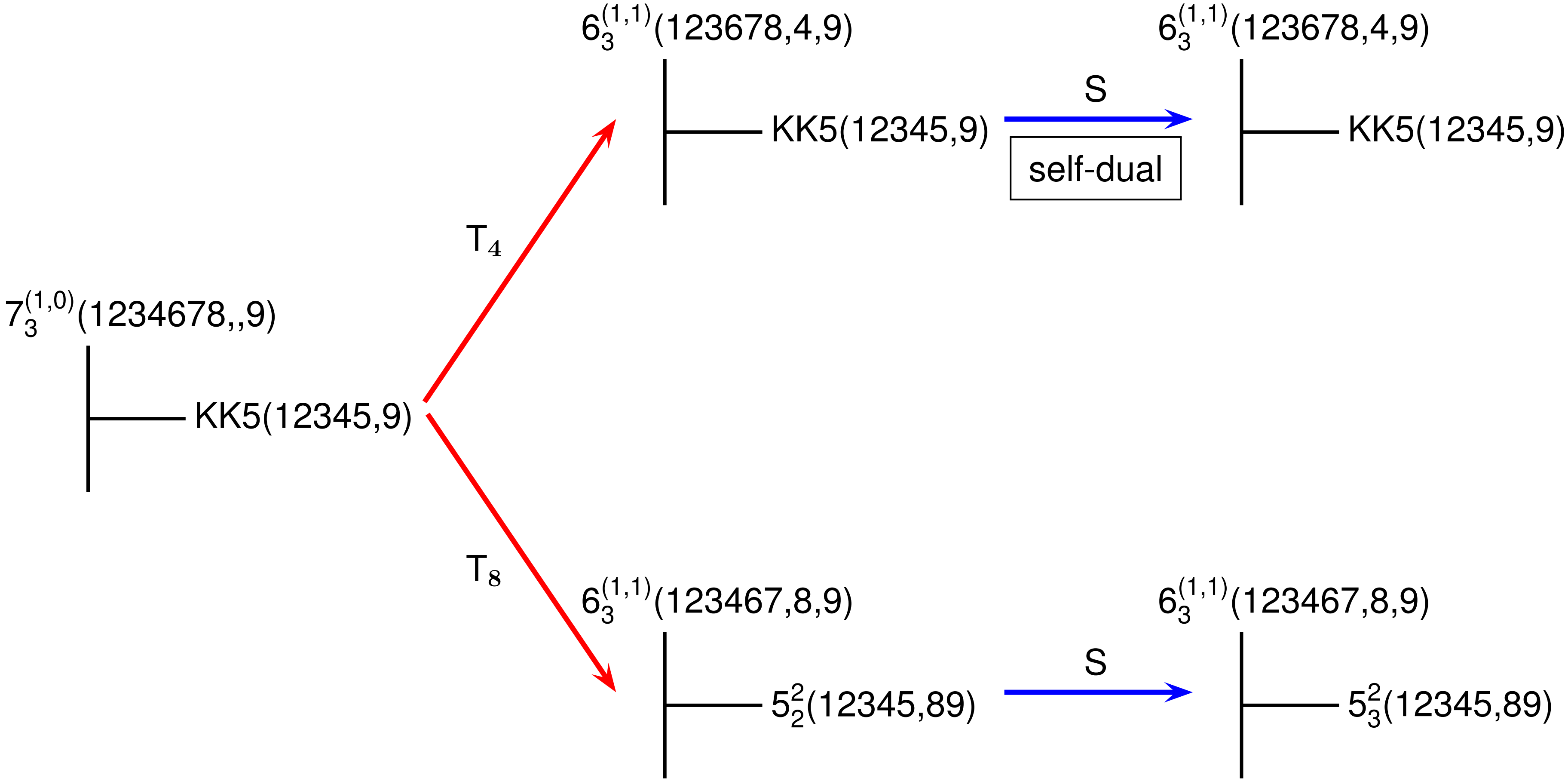
T_8

$6_3^{(1,1)}$(123467,8,9)

5_2^2(12345,89)

S

Questions: exotic brane ending on exotic brane



$7_3^{(1,0)}$(1234678,,9)

KK5(12345,9)

T_4

$6_3^{(1,1)}$(123678,4,9)

KK5(12345,9)

S

self-dual

$6_3^{(1,1)}$(123678,4,9)

KK5(12345,9)

T_8

$6_3^{(1,1)}$(123467,8,9)

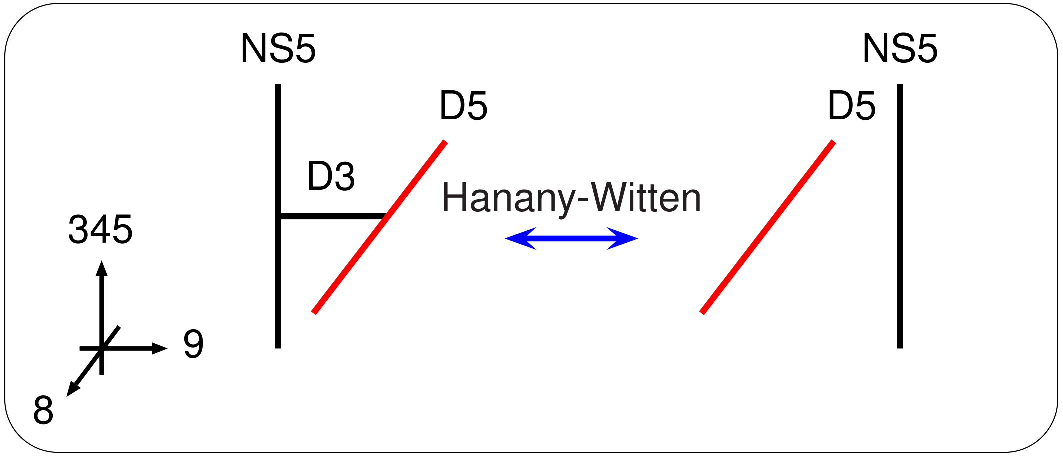
5_2^2(12345,89)

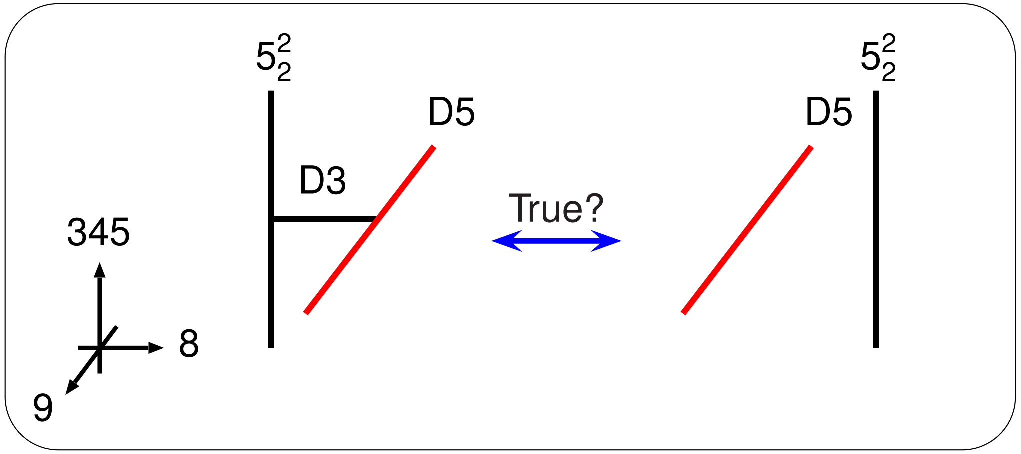
S

$6_3^{(1,1)}$(123467,8,9)

5_3^2(12345,89)

Questions: Hanany-Witten effect





Linking number is defined as a sum of brane charges :

$$\boldsymbol{L_B \ = \ \frac{1}{2}(r - l) + L - R}$$

However, can we define meaningful Dp-brane charges in the 5^2_2 background?

branch cut as D7 background?

instructive works : de Boer, Shigemori arXiv:1209.6056
Okada, Sakatani arXiv:1411.1043

Summary

Summary

✓ Exotic brane configurations will give us new examples of mirror symmetry between two gauge theories.

✓ Exotic brane configurations will tell us nobel features of gravity and gauge theories if we understand the followings :

1. exotic M-branes
2. exotic brane ending on exotic brane
3. Hanany-Witten effect in the presence of exotic brane

Orientifold planes are also reconsidered in exotic backgrounds.

✓ Exotic branes should be ingredients of BH microstructures (we hope).

Thanks

Appendix

Defect branes in diverse dimensions

	fundamental	Dirichlet	solitonic	S_D-dual of (Dirichlet)	S_D-dual of (fundamental)
D	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
IIB		D7 $[C_8]$		7_3 $[E_8 = \mathsf{S}_{10}(C_8)]$	
9		D6 $[C_7]$		6^1_3 $[E_{8,1} = \mathsf{S}_9(C_7)]$	
8		D5 $[C_6]$	NS5 $[D_6]$ KK5 $[D_{7,1}]$ 5^2_2 $[D_{8,2}]$	5^2_3 $[E_{8,2} = \mathsf{S}_8(C_7)]$	
7		D4 $[C_5]$		4^3_3 $[E_{8,3} = \mathsf{S}_7(C_5)]$	
6		D3 $[C_4]$		3^4_3 $[E_{8,4} = \mathsf{S}_6(C_4)]$	
5		D2 $[C_3]$		2^5_3 $[E_{8,5} = \mathsf{S}_5(C_3)]$	
4	F1 $[B_2]$	D1 $[C_2]$		1^6_3 $[E_{8,6} = \mathsf{S}_4(C_2)]$	1^6_4 $[F_{8,6} = \mathsf{S}_4(B_2)]$
3	P	D0 $[C_1]$		0^7_3 $[E_{8,7} = \mathsf{S}_3(C_1)]$	$0^{(6,1)}_4$ $[F_{8,7,1}]$

E.A. Bergshoeff et al arXiv:1009.4657, 1102.0934, 1108.5067, etc.

Domain walls in diverse dimensions

	fundamental	Dirichlet	solitonic		
D	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
IIA		D8 $[C_9]$			
9		D7 $[C_8]$		$7_3^{(1,0)}$ $[\boldsymbol{E}_{9,1,1}]$	
8		D6 $[C_7]$		$6_3^{(1,1)}$ $[\boldsymbol{E}_{9,2,1}]$	
7		D5 $[C_6]$	NS5 $[D_6]$ KK5 $[D_{7,1}]$ 5_2^2 $[D_{8,2}]$	$5_3^{(1,2)}$ $[\boldsymbol{E}_{9,3,1}]$	5_4^3 $[\boldsymbol{F}_{9,3}]$
6		D4 $[C_5]$		$4_3^{(1,3)}$ $[\boldsymbol{E}_{9,4,1}]$	$4_4^{(1,3)}$ $[\boldsymbol{F}_{9,4,1}]$
5		D3 $[C_4]$		$3_3^{(1,4)}$ $[\boldsymbol{E}_{9,5,1}]$	$3_4^{(2,3)}$ $[\boldsymbol{F}_{9,5,2}]$
4		D2 $[C_3]$		$2_3^{(1,5)}$ $[\boldsymbol{E}_{9,6,1}]$	$2_4^{(3,3)}$ $[\boldsymbol{F}_{9,6,3}]$
3	F1 $[B_2]$	D1 $[C_2]$		$1_3^{(1,6)}$ $[\boldsymbol{E}_{9,7,1}]$	$1_4^{(4,3)}$ $[\boldsymbol{F}_{9,7,4}]$

E.A. Bergshoeff et al arXiv:1009.4657, 1102.0934, 1108.5067, etc.

$$
\begin{aligned}
\mathscr{L}_{\text{NS5}} \ = \ & \sum_{a=1}^{k} \int d^4\theta \, \frac{1}{e_a^2} \Big\{ - |\Sigma_a|^2 + |\Phi_a|^2 \Big\} \\
& + \sum_a \int d^4\theta \, \Big\{ |Q_a|^2 \, e^{V_a} + |\widetilde{Q}_a|^2 \, e^{-V_a} \Big\} + \sum_a \Big[\int d^2\theta \, \widetilde{Q}_a \Phi_a Q_a + (\text{h.c.}) \Big] \\
& + \int d^4\theta \, \frac{1}{g^2} \Big\{ - |\Theta|^2 + |\Psi|^2 \Big\} \\
& + \sum_a \Big[\int d^2\theta \, (s_a - \Psi) \Phi_a + \int d^2\widetilde{\theta} \, (t_a - \Theta) \Sigma_a + (\text{h.c.}) \Big]
\end{aligned}
$$

(Σ_a, Φ_a) : $\mathcal{N} = (4,4)$ vector multiplets

$(Q_a, \widetilde{Q}_a)$: $\mathcal{N} = (4,4)$ charged hypermultiplets

(Ψ, Θ) : $\mathcal{N} = (4,4)$ adjoint hypermultiplet

(s_a, t_a) : FI parameters, positions of NS5-branes

GLSM for five-branes

$$
\begin{aligned}
\mathscr{L}_{\text{KK5}} \;=\; & \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \frac{1}{e_a^2} \Big\{ - |\Sigma_a|^2 + |\Phi_a|^2 \Big\} \\
& + \sum_a \int \mathrm{d}^4\theta \, \Big\{ |Q_a|^2 \, \mathrm{e}^{V_a} + |\widetilde{Q}_a|^2 \, \mathrm{e}^{-V_a} \Big\} + \sum_a \Big[\int \mathrm{d}^2\theta \, \widetilde{Q}_a \Phi_a Q_a + \text{(h.c.)} \Big] \\
& + \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{g^2} |\Psi|^2 + \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + \sum_a V_a \Big)^2 \Big\} \\
& + \sum_a \Big[\int \mathrm{d}^2\theta \, (s_a - \Psi) \, \Phi_a + \int \mathrm{d}^2\widetilde{\theta} \, t_a \Sigma_a + \text{(h.c.)} \Big]
\end{aligned}
$$

$$
\frac{1}{g^2}(\Theta + \Theta) \;\doteq\; -(\Gamma + \overline{\Gamma} + \sum_a V_a)
$$

GLSM for five-branes

$$
\begin{aligned}
\mathscr{L}_{5^2_2} \;=\; & \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \frac{1}{e_a^2} \Big\{ -|\Sigma_a|^2 + |\Phi_a|^2 \Big\} \\
& + \sum_a \int \mathrm{d}^4\theta \, \Big\{ |Q_a|^2 \, \mathrm{e}^{V_a} + |\widetilde{Q}_a|^2 \, \mathrm{e}^{-V_a} \Big\} + \sum_a \Big[\int \mathrm{d}^2\theta \, \widetilde{Q}_a \Phi_a Q_a + (\text{h.c.}) \Big] \\
& + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big\{ \Big(\Gamma + \overline{\Gamma} + \sum_a V_a \Big)^2 - \Big(\Xi + \overline{\Xi} - \sum_a (C_a + \overline{C}_a) \Big)^2 \Big\} \\
& + \int \mathrm{d}^4\theta \, \Big\{ -(\Psi - \overline{\Psi}) \sum_a (C_a - \overline{C}_a) \Big\} \\
& + \sum_a \Big[\int \mathrm{d}^2\theta \, s_a \Phi_a + \int \mathrm{d}^2\widetilde{\theta} \, t_a \Sigma_a + (\text{h.c.}) \Big]
\end{aligned}
$$

$$
\begin{aligned}
\Phi_a \;&=\; \overline{D}_+ \overline{D}_- C_a \\
\frac{1}{g^2}(\Psi + \overline{\Psi}) \;&\doteq\; -\Big(\Xi + \overline{\Xi} - \sum_a (C_a + \overline{C}_a) \Big)
\end{aligned}
$$