Codimension-2 Solutions in Five-Dimensional Supergravity

with Masaki Shigemori [arXiv:1505.05169]

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Low Codimension Branes

- People thought they are not common
- They are peculiar, but it makes them more interesting!

For example,

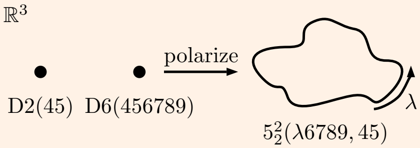
- Codimension-2: D7-brane
  → Spacetime is no longer asymptotically flat.
- Codimension-1: D8-brane
  → Spacetime terminates at finite distance from the brane.

$$\text{Codimension} = \dim(\text{spacetime}) - \dim(\text{object})$$

Low Codimension Branes

However,

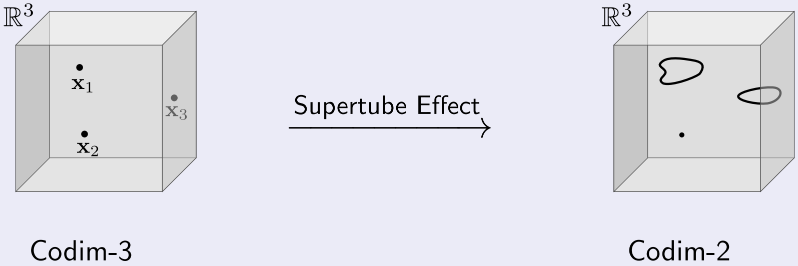
- By supertube effect,



- More common than previously thought
- Relevant to black hole physics
  - Fuzzball conjecture
  - Microstate geometry program

Purpose of This Work

Demonstrate the existence of general multi-dipole solutions



Setup

[Bates–Denef '03, Gutowski–Reall '04, Bena–Warner '04, Gauntlett–Gutowski '04]

- M-theory on $T^6 = T^2_{45} \times T^2_{67} \times T^2_{89}$
- $D = 5, \mathcal{N} = 1$ ungauged SUGRA with two vector multiplets
- Action

$$S = \frac{1}{16\pi G_5} \int \Big(- R * 1 + Q_{IJ} * F^I \wedge F^J + Q_{IJ} * \mathrm{d}X^I \wedge \mathrm{d}X^J - \frac{1}{6} C_{IJK} F^I \wedge F^J \wedge A^K \Big) ,$$

$$C_{IJK} = |\epsilon_{IJK}| , \qquad Q_{IJ} = \frac{1}{2} \mathrm{diag} \left((X^1)^{-2}, (X^2)^{-2}, (X^3)^{-2} \right)$$

The 4D/5D Solution

[Bates–Denef '03, Gutowski–Reall '04, Bena–Warner '04, Gauntlett–Gutowski '04]

- BPS solutions
  - Require SUSY
  - Assume $\mathrm{U}(1)$ symmetry $\leftarrow$ M-theory direction
- 8 harmonic functions $H = \{V, K^{I=1,2,3}, L_I, M\}$ on $\mathbb{R}^3$

$$\nabla^2 H(\mathbf{x}) = 0$$

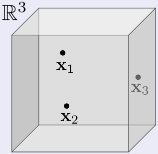
- Integrability condition

$$0 = V\nabla^2 M - M\nabla^2 V + \frac{1}{2}\left(K^I \nabla^2 L_I - L_I \nabla^2 K^I\right)$$

The 4D/5D Solution

[Bates–Denef '03, Gutowski–Reall '04, Bena–Warner '04, Gauntlett–Gutowski '04]

- Multi-center solutions with codimension-3 sources



$$H(\mathbf{x}) = h + \sum_{p=1}^{N} \frac{\Gamma_p}{|\mathbf{x} - \mathbf{x}_p|}\,, \quad \Gamma_p : \text{ charge}$$

- Brane interpretation in type IIA theory

$$V \leftrightarrow \mathrm{D6}(456789)\,, \quad \begin{array}{c} K^1 \leftrightarrow \mathrm{D4}(6789) \\ K^2 \leftrightarrow \mathrm{D4}(4589) \\ K^3 \leftrightarrow \mathrm{D4}(4567) \end{array}\,, \quad \begin{array}{c} L_1 \leftrightarrow \mathrm{D2}(45) \\ L_2 \leftrightarrow \mathrm{D2}(67) \\ L_3 \leftrightarrow \mathrm{D2}(89) \end{array}\,, \quad M \leftrightarrow \mathrm{D0}$$

- Integrability condition: force between branes has to be in balance

10D Type IIA Uplift

- Metric and other fields are given by

$$ds^2_{10,\text{str}} = -\frac{1}{\sqrt{V(Z - V\mu^2)}}(dt+\omega)^2 + \sqrt{V(Z - V\mu^2)}\,dx^i dx^i$$
$$+ \sqrt{\frac{Z - V\mu^2}{V}}\left(Z_1^{-1}dx^2_{45} + Z_2^{-1}dx^2_{67} + Z_3^{-1}dx^2_{89}\right) ,$$

$$e^{2\Phi} = \frac{(Z - V\mu^2)^{3/2}}{V^{3/2}Z} , \qquad B_2 = \left(V^{-1}K^I - Z_I^{-1}\mu\right) J_I ,$$

$$C_1 = A - \frac{V\mu}{Z - V\mu^2}(dt+\omega) ,$$

$$C_3 = \left[(V^{-1}K^I - Z_I^{-1}\mu)A + \xi^I - Z_I^{-1}(dt+\omega)\right] \wedge J_I ,$$

$$J_1 = \mathrm{d}x^4 \wedge \mathrm{d}x^5 , \quad J_2 = \mathrm{d}x^6 \wedge \mathrm{d}x^7 , \quad J_3 = \mathrm{d}x^8 \wedge \mathrm{d}x^9$$

Torus Moduli

Complexified Kähler Moduli

- τ^1 for the 2-torus T^2_{45} is

$$\tau^1 = B_{45} + i\,\mathrm{vol}(T^2_{45}) = \left(\frac{K^1}{V} - \frac{\mu}{Z_1}\right) + i\frac{\sqrt{V(Z - V\mu^2)}}{Z_1 V}$$

  where $R_4 = R_5 = l_s$
- τ^1 transforms under $\mathrm{SL}(2,\mathbb{Z})_{45}$ duality group
- τ^2 and τ^3 are defined in the same manner

Example

4-Charge Black Hole

- Supersymmetric black hole in 4D

$$N^0 \quad \mathrm{D6}(456789) \qquad \begin{array}{ll} N_1 & \mathrm{D2}(45) \\ N_2 & \mathrm{D2}(67) \\ N_3 & \mathrm{D2}(89) \end{array}$$

- Harmonic function

$$V = \frac{N^0}{r}\,, \quad K^I = 0\,, \quad L_I = 1 + \frac{N_I}{r}\,, \quad M = 0$$

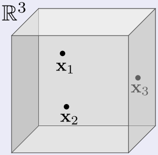
- Single-center
- Macroscopic entropy

$$S \sim \sqrt{N^0 N_1 N_2 N_3}$$

3- and 4-Charge Black Holes

Microstate Geometry Program

- By tuning parameters, we find smooth solutions without singularities $\rightarrow$ Bubbling solutions [Bena–Warner '05, Berglund–Gimon–Levi '05]
- The hope was that smooth 4D/5D solutions with codim-3 sources can reproduce 3- and 4-charge BHs entropy



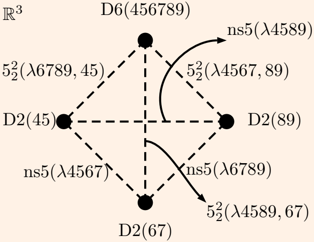
$$H(\mathbf{x}) = h + \sum_{p=1}^{N} \frac{\Gamma_p}{|\mathbf{x} - \mathbf{x}_p|}$$

- *Not enough to reproduce BH entropy!* [de Boer–El-Showk–Messamah–Van den Bleeken '09, Bena–Bobev–Giusto–Ruef–Warner '10]
- There have been many attempts to resolve this problem

3- and 4-Charge Black Holes

The 4D/5D Solutions with Codim-3 Sources

- There should be more general configurations because of the supertube effect [de Boer–Shigemori '10, '12]



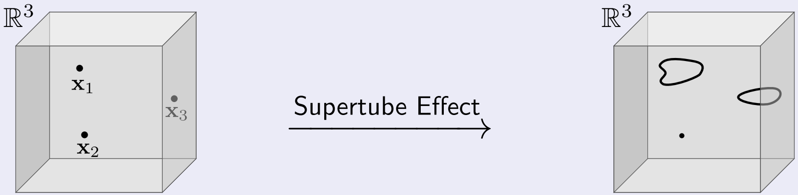
$$\mathrm{D6}(456789) \quad \begin{matrix} \mathrm{D2}(45) \\ \mathrm{D2}(67) \\ \mathrm{D2}(89) \end{matrix} \quad \rightarrow \quad \begin{matrix} 5^2_2(\lambda 6789, 45) & \mathrm{ns5}(\lambda 4567) \\ 5^2_2(\lambda 4589, 67) & \mathrm{ns5}(\lambda 6789) \\ 5^2_2(\lambda 4567, 89) & \mathrm{ns5}(\lambda 4589) \end{matrix} \quad \rightarrow \quad \cdots$$

- λ is a parameter of an arbitrary curve which describes a profile of supertube in $\mathbb{R}^3$

3- and 4-Charge Black Holes

In this work,

We restrict our attention to the simplest case, i.e. codimension-2 sources.



Codim-3 Codim-2

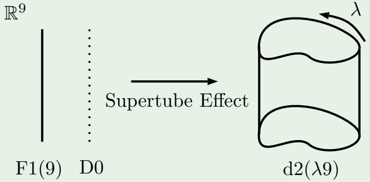
Supertube Transition

The Supertube Transition is

- a spontaneous polarization phenomenon
- branes puff up into a new dipole charges

F1-D0 System [Mateos–Townsend '01]

- Analyzed by using DBI action of D2-brane

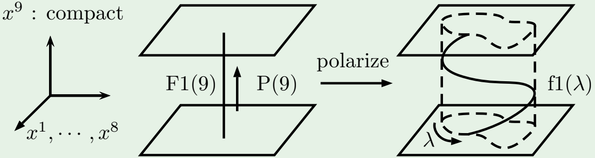


- F1 and D0 charges are dissolved in d2
- Supergravity solution is known [Emparan–Mateos–Townsend '01]

Supertube Transition

F1-P System

- $\mathrm{F1}(9) + \mathrm{P}(9) \to \mathrm{f1}(\lambda) + \mathrm{p}(\lambda)$



- To carry momentum, F1 should wiggle in transverse directions
- Arbitrary curves in transverse directions
- Supergravity solution is known [Lunin–Mathur '01]

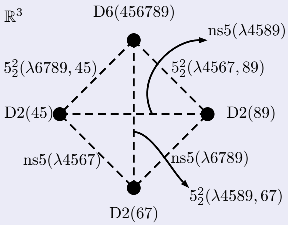
Supertube Transition

Supertubes in different frames

$$
\begin{array}{lllllll}
\mathrm{F1}(9) & + & \mathrm{D0} & \to & \mathrm{d2}(\lambda 9) & + & \mathrm{p}(\lambda) \\
\mathrm{F1}(9) & + & \mathrm{P}(9) & \to & \mathrm{f1}(\lambda) & + & \mathrm{p}(\lambda)
\end{array}
$$

- Typical supertubes in 4-charge black hole

$$
\begin{array}{lllllll}
\mathrm{D2}(67) & + & \mathrm{D2}(45) & \to & \mathrm{ns5}(\lambda 4567) & + & \mathrm{p}(\lambda) \\
\mathrm{D2}(89) & + & \mathrm{D6}(456789) & \to & 5^2_2(\lambda 4567, 89) & + & \mathrm{p}(\lambda)
\end{array}
$$



$\mathrm{D2}(67) + \mathrm{D2}(45) \to \mathrm{ns5}(\lambda 4567)$

- Harmonic functions

$$V = 1\,, \qquad K^1 = 0\,, \qquad K^2 = 0\,, \qquad K^3 = 0\,,$$
$$L_1 = 1 + \frac{Q_2}{r}\,, \quad L_2 = 1 + \frac{Q_1}{r}\,, \quad L_3 = 1\,, \qquad M = 0$$

- Integrability condition is satisfied trivially
- But this is not true bound state of this system

$\mathrm{D2}(67) + \mathrm{D2}(45) \to \mathrm{ns5}(\lambda 4567)$

- Harmonic functions

$$
\begin{aligned}
V = 1\,, \qquad K^1 = 0\,, \qquad K^2 = 0\,, \qquad K^3 = \gamma\,, \\
L_1 = f_2\,, \qquad L_2 = f_1\,, \qquad L_3 = 1\,, \qquad M = -\frac{\gamma}{2}
\end{aligned}
$$

where

$$
f_1 = 1 + \frac{Q_1}{L}\int_0^L \frac{\mathrm{d}\lambda}{|\mathbf{x} - \mathbf{F}(\lambda)|}\,, \quad f_2 = 1 + \frac{Q_1}{L}\int_0^L \frac{|\dot{\mathbf{F}}(\lambda)|^2 \mathrm{d}\lambda}{|\mathbf{x} - \mathbf{F}(\lambda)|}
$$

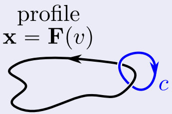
and

$$
\mathrm{d}\gamma = *_3 \mathrm{d}\alpha\,, \quad \alpha_i = \frac{Q_1}{L}\int_0^L \frac{\dot{F}_i(\lambda)\,\mathrm{d}\lambda}{|\mathbf{x} - \mathbf{F}(\lambda)|}\,.
$$

$\mathrm{D2}(67) + \mathrm{D2}(45) \to \mathrm{ns5}(\lambda 4567)$

Monodromy

- γ has monodromy



$$\int_c \mathrm{d}\gamma = \int_c *_3 \mathrm{d}\alpha = 1$$

- Complexified Kähler moduli are

$$\tau^1 = i\sqrt{f_1/f_2}\,, \quad \tau^2 = i\sqrt{f_2/f_1}\,, \quad \tau^3 = \gamma + i\sqrt{f_1 f_2}$$

- τ^3 has monodromy

$$\tau^3 \to \tau^3 + 1$$

$\mathrm{D2}(67) + \mathrm{D2}(45) \to \mathrm{ns5}(\lambda 4567)$

Metric

- 10D metric only depends on

$$Z_I = L_I + \frac{1}{2} C_{IJK} V^{-1} K^J K^K$$
$$\mu = M + \frac{1}{2} V^{-1} K^I L_I + \frac{1}{6} C_{IJK} V^{-2} K^I K^J K^K$$

- SIngle-valued

$$Z_1 = f_2 \,, \quad Z_2 = f_1 \,, \quad Z_3 = 1 \,, \quad \mu = 0$$

- Metric

$$ds^2_{10} = -(f_1 f_2)^{-1/2}(\mathrm{d}t - \alpha)^2 + (f_1 f_2)^{1/2} \mathrm{d}x^i \mathrm{d}x^i + (f_1/f_2)^{1/2} dx^2_{45} + (f_2/f_1)^{1/2} dx^2_{67} + (f_1 f_2)^{1/2} dx^2_{89} \,,$$
$$e^{2\Phi} = (f_1 f_2)^{1/2} \,, \qquad B_2 = \gamma \, \mathrm{d}x^8 \wedge \mathrm{d}x^9 \,, \quad \cdots$$

$\mathrm{D2}(89) + \mathrm{D6}(456789) \to 5^2_2(\lambda 4567, 89)$

- Harmonic functions

$$\begin{aligned} V = f_2\,, \qquad & K^1 = \gamma\,, \quad K^2 = \gamma\,, \quad K^3 = 0\,, \\ & L_1 = 1\,, \quad L_2 = 1\,, \quad L_3 = f_1\,, \qquad M = 0\,. \end{aligned}$$

- Metric is multi-valued → Non-geometric

$$\begin{aligned} ds^2_{10} &= -(f_1 f_2)^{-1/2}(\mathrm{d}t - \alpha)^2 + (f_1 f_2)^{1/2}\mathrm{d}x^i\mathrm{d}x^i \quad \left(F = 1 + \frac{\gamma^2}{f_1 f_5}\right) \\ &\quad + (f_1/f_2)^{1/2}\left(dx^2_{4567} + f_1^{-1}F^{-1}dx^2_{89}\right)\,, \\ e^{2\Phi} &= f_1^{1/2} f_2^{-3/2} F^{-1}\,, \qquad B_2 = -\frac{\gamma}{f_1 f_2 F}\,\mathrm{d}x^8 \wedge \mathrm{d}x^9\,, \quad \cdots \end{aligned}$$

- Monodromy

$$\tau'^3 \to \tau'^3 + 1\,, \quad \text{where } \tau'^3 = -\frac{1}{\tau^3}$$

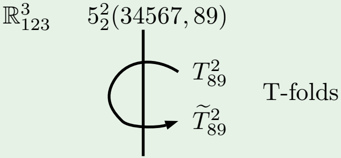
Exotic Brane

5^2_2-brane

- Obtained from NS5-brane

$$\mathrm{NS5}(34567) \xrightarrow{\mathrm{T}_8} \mathrm{KKM}(34567;8) \xrightarrow{\mathrm{T}_9} 5^2_2(34567;89)$$

- Non-geometric



- From supertube effect

$$\mathrm{D2}(89) + \mathrm{D6}(456789) \to 5^2_2(\lambda 4567;89) + \mathrm{p}(\lambda)$$

- DFT/EFT can deal with exotic branes more systematically

Summaries so far

We have seen

- 4D/5D solutions are framework of black hole research
- supertube effect is important in black hole physics
- what supertube effect is
- some examples of supertube in the 4D/5D solutions

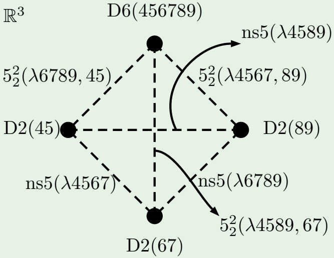
What we want to do

Constructing more general multi-dipole solutions

General Configurations

4-Charge Black Hole

- In general,



- There can be further supertube transitions

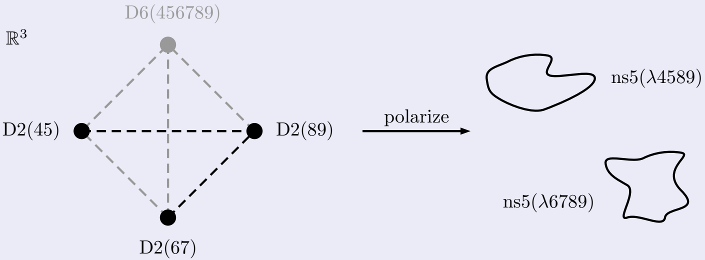
$$\text{D6}(456789) \quad \begin{array}{c} \text{D2}(45) \\ \text{D2}(67) \\ \text{D2}(89) \end{array} \rightarrow \begin{array}{cc} 5^2_2(\lambda 6789, 45) & \text{ns5}(\lambda 4567) \\ 5^2_2(\lambda 4589, 67) & \text{ns5}(\lambda 6789) \\ 5^2_2(\lambda 4567, 89) & \text{ns5}(\lambda 4589) \end{array} \rightarrow \quad \cdots$$

General Configurations

Simplest possible configuration

- 2-dipole solution

$$\begin{aligned} \mathrm{D2}(45) &+ \mathrm{D2}(89) \rightarrow \mathrm{ns5}(\lambda 4589) + \mathrm{p}(\lambda) \\ \mathrm{D2}(67) &+ \mathrm{D2}(89) \rightarrow \mathrm{ns5}(\lambda 6789) + \mathrm{p}(\lambda) \end{aligned}$$



Constructing 2-Dipole Solutions

Naive Attempt

- Superpose 1-dipole solutions

$$\mathrm{D2}(45) \quad + \quad \mathrm{D2}(89) \quad \to \quad \mathrm{ns5}(\lambda 4589) \quad + \quad \mathrm{p}(\lambda)$$

- Harmonic functions are

$$V = 1\,, \qquad K^1 = 0\,, \qquad K^2 = \gamma\,, \qquad K^3 = 0\,,$$
$$L_1 = f_1\,, \qquad L_2 = 0\,, \qquad L_3 = f_2\,, \qquad M = -\frac{\gamma}{2}$$

Constructing 2-Dipole Solutions

Naive Attempt

- Superpose 1-dipole solutions

$$\begin{array}{ccccccc} \mathrm{D2}(45) & + & \mathrm{D2}(89) & \to & \mathrm{ns5}(\lambda 4589) & + & \mathrm{p}(\lambda) \\ \mathrm{D2}(67) & + & \mathrm{D2}(89) & \to & \mathrm{ns5}(\lambda 6789) & + & \mathrm{p}(\lambda) \end{array}$$

- Harmonic functions are

$$V = 1\,, \quad K^1 = \gamma'\,, \quad K^2 = \gamma\,, \quad K^3 = 0\,,$$

$$L_1 = f_1\,, \quad L_2 = f_1'\,, \quad L_3 = f_2 + f_2'\,, \quad M = -\frac{\gamma}{2} - \frac{\gamma'}{2}$$

- This does not work. Integrability condition is not satisfied

Constructing 2-Dipole Solutions

Superthread [Niehoff–Vasilakis–Warner '12]

- Solutions in 6D SUGRA
- D1 and D5-branes with traveling waves on them

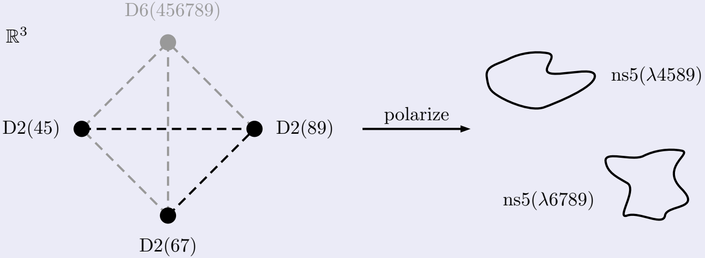
$$\begin{array}{lcccccc} \mathrm{D1(5)} & + & \mathrm{P(5)} & \to & \mathrm{d1}(\lambda) & + & \mathrm{p}(\lambda) \\ \mathrm{D5(56789)} & + & \mathrm{P(5)} & \to & \mathrm{d5}(\lambda 6789) & + & \mathrm{p}(\lambda) \end{array}$$

- Supertubes interact with each other $\to$ it solves the problem before

2-Dipole Solutions [MP–Shigemori '15]

$\mathrm{D2}(45) + \mathrm{D2}(67) + \mathrm{D2}(89) \to \mathrm{ns5}(\lambda 4589) + \mathrm{ns5}(\lambda 6789)$

- Smear and dualize superthread
- After some messy calculations, we get 2-dipole solutions



- This can be described in 4D/5D solutions
- Multi-valued harmonic functions on $\mathbb{R}^3$

2-Dipole Solutions [MP–Shigemori '15]

$\mathrm{D2}(45) + \mathrm{D2}(67) + \mathrm{D2}(89) \to \mathrm{ns5}(\lambda 4589) + \mathrm{ns5}(\lambda 6789)$

- Harmonic functions

$$V = 1\,, \qquad K^1 = \gamma_2\,, \quad K^2 = \gamma_1\,, \quad K^3 = 0\,,$$

$$L_I = 1 + \sum_p Q_{pI} \int_p \frac{1}{R_p} = Z_I\,, \qquad I = 1, 2\,,$$

$$L_3 = 1 + \sum_p \int_p \frac{\rho_p}{R_p} - K^1 K^2$$

$$+ \sum_{p,q} Q_{pq} \iint_{p,q} \left[\frac{\dot{\mathbf{F}}_p \cdot \dot{\mathbf{F}}_q}{2R_p R_q} - \frac{\dot{F}_{pi}\dot{F}_{qj}(R_{pi}R_{qj} - R_{pj}R_{qi})}{F_{pq}R_p R_q (F_{pq} + R_p + R_q)} \right],$$

$$M = \frac{1}{2} \sum_{p,q} Q_{pq} \iint_{p,q} \frac{\epsilon_{ijk}\dot{F}_{pqi}R_{pj}R_{qk}}{F_{pq}R_p R_q (F_{pq} + R_p + R_q)} - \frac{1}{2}(K^1 L_1 + K^2 L_2)$$

2-Dipole Solutions [MP–Shigemori '15]

$D2(45) + D2(67) + D2(89) \rightarrow ns5(\lambda 4589) + ns5(\lambda 6789)$

- $\gamma_{1,2}$ is multi-valued as before
- But, Z_3 and μ are single-valued $\rightarrow$ metric is single-valued

$$Z_3 = L_3 + K^1 K^2$$
$$\mu = M + \frac{1}{2}(K^1 L_1 + K^2 L_2)$$

- Condition for no closed timelike curves
- Proper monodromies for NS5-branes

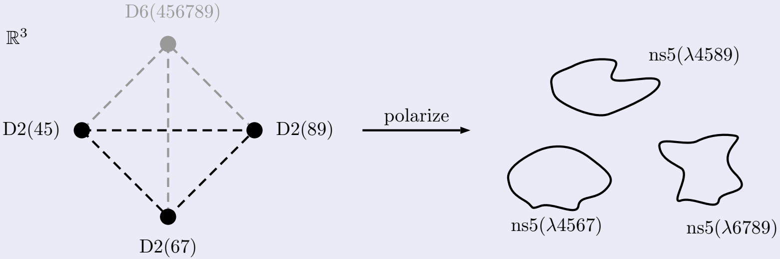
$$\tau^1 \rightarrow \tau^1 + 1\,, \quad \tau^2 \rightarrow \tau^2 + 1$$

- Unbound state $\rightarrow$ unable to use as microstate of black hole

3-Dipole Solutions [MP–Shigemori '15]

$\mathrm{D2}(45) + \mathrm{D2}(67) + \mathrm{D2}(89)$
$\rightarrow \mathrm{ns5}(\lambda 4567) + \mathrm{ns5}(\lambda 4589) + \mathrm{ns5}(\lambda 6789)$

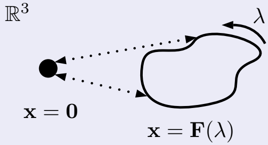
- We can extend the solutions including 3-dipole sources.



- Although it is not explicit as 2-dipole solutions, it satisfies all conditions for 3-dipole solutions.
- This system is also not bound state

Mixed Configurations [MP–Shigemori '15]

- Consider codim-3 and codim-2 sources simultaneously



with harmonic functions

$$V = n_0 + \frac{n}{r}\,,$$

$$K^1 = k_0^1 + \frac{k^1}{r}\,, \qquad K^2 = k_0^2 + \frac{k^2}{r}\,, \qquad K^3 = k_0^3 + \gamma + \frac{k^3}{r}\,,$$

$$L_1 = l_1^0 + f_2 + \frac{l_1}{r}\,, \qquad L_2 = l_2^0 + f_1 + \frac{l_2}{r}\,, \qquad L_3 = l_3^0 + \frac{l_3}{r}\,,$$

$$M = m_0 - \frac{\gamma}{2} + \frac{m}{r}\,.$$

Mixed Configurations [MP–Shigemori '15]

- Integrability condition reads

$$0 = n_0 m - m_0 n + \frac{1}{2}(k_0^I l_I - l_I^0 k^I) - \frac{1}{2}\frac{Q}{L}\int_0^L \mathrm{d}\lambda\, \frac{k^1|\dot{\mathbf{F}}(\lambda)|^2 + k^2}{|\mathbf{F}(\lambda)|}\,,$$

$$0 = n + l_3\,,$$

$$0 = k_0^2 + \frac{k^2}{|\mathbf{F}(\lambda)|} + |\dot{\mathbf{F}}(\lambda)|^2\left(k_0^1 + \frac{k^1}{|\mathbf{F}(\lambda)|}\right) \quad \text{for each value of } \lambda.$$

- First two equations are easily satisfied
- Third one gives an force balance condition at each point λ
  $\rightarrow$ Bound state!

Conclusions

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- 4D/5D solution: a paradigm for BH research
- Only codim-3 solutions are studied so far
- Codim-2 solutions are also important

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- 4D/5D solution: a paradigm for BH research
- Only codim-3 solutions are studied so far
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Future Directions

- Find systematic way to construct general multi-dipole solutions
- Applications to
  - Split attractor flow, Wall crossing, Quiver QM, etc.
- More low codimension
  - Superstratum: arbitrary surface parametrized by two variables [Bena–Giusto–Russo–Shigemori–Warner '15]