Microstate solutions from black hole deconstruction

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Microstructures of Black Holes, YITP, 25.11.2015 based on

- J.R. and D. Van den Bleeken, *"Microstate solutions from black hole deconstruction,"* arXiv:1510.00583 [hep-th]

see also

- JR and D. Van den Bleeken, *Unlocking the Axion-Dilaton in 5D Supergravity*, JHEP **1411**, 029 (2014) [arXiv:1407.5330 [hep-th]].
- T. S. Levi, J. R., D. Van den Bleeken, W. Van Herck and B. Vercnocke, *Godel space from wrapped M2-branes*, JHEP **1001**, 082 (2010) [arXiv:0909.4081 [hep-th]].

Overview

- Black hole deconstruction proposal for microstate solutions

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- Explicit construction of backreacted solutions

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- Open issues

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- String theory has been successful in explaining the Bekenstein-Hawking entropy $S = \frac{A}{4}$ microscopically for susy black holes (Strominger, Vafa 1996)

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- Focus here on the 4D D0-D4 black hole (Maldacena, Strominger, Witten 1997). Black hole deconstruction proposal (Denef, Gaiotto, Strominger, Van Den Bleeken, Yin 2007): microstates from multicentered brane configurations.

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- Focus here on the 4D D0-D4 black hole (Maldacena, Strominger, Witten 1997). Black hole deconstruction proposal (Denef, Gaiotto, Strominger, Van Den Bleeken, Yin 2007): microstates from multicentered brane configurations.
- These are microstate solutions with singular brane sources.

D0-D4 BLACK HOLE AND DECOUPLING LIMIT

- Consider type IIA on CY. Make a a D0-D4 black hole with charges ($\Gamma \equiv (Q_{D6}, Q_{D4}, Q_{D2}, Q_{D0})$),

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- M-theory decoupling limit:

$$\frac{R_M}{l_{11}} \to \infty\,, \qquad V_\infty \equiv \frac{V_X}{l_{11}^6} \quad \text{fixed,}$$

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- Large λ: 11D supergravity in $AdS_3 \times S^2 \times CY$ attractor throat geometry (decoupling limit of black hole is BTZ $\times S^2 \times CY$)
- Small λ: worldvolume theory on M5-brane on a smooth holomorphic 4-cycle D, with $[D] = P$

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- Cardy's formula reproduces S_{BH}

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  - $\Gamma_{\bar{D}6} = (-1, \frac{P}{2}, -\frac{P^2}{8}, -\frac{P^3}{48})$: anti-D6-brane with flux $-\frac{P}{2}$
  - $\Gamma_{D0} = (0, 0, 0, q_0 + \frac{P^3}{24})$: D0-brane charge

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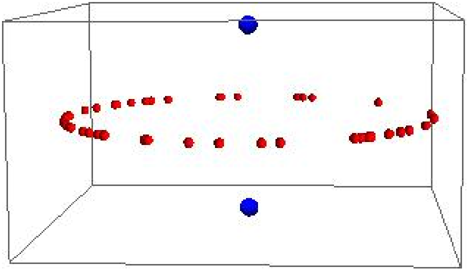
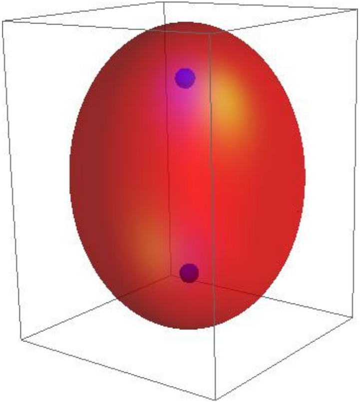
  - $\Gamma_{D0} = (0, 0, 0, q_0 + \frac{P^3}{24})$: D0-brane charge
- the D0-charge can be added in two distinct ways:

I. Single D0-brane centers



Moduli space of such solutions, but not large enough to account for entropy (De Boer, El-Showk, Messamah, Van Den Bleeken 2009).

II. Supereggs: D2-branes with D0-brane flux



BPS (Gaiotto, Simons, Strominger, Yin 2004), induced D0 charge through $\int_{D2} C_1 \wedge F$

Counting argument

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- Quantum mechanics on each D2-probe has lowest Landau level degeneracy $\sim P^3$
- Combinatorics of partitioning q_0 over the D2's gives, for $q_0 \gg P^3$, an entropy

$$S = 2\pi\sqrt{q_0 P^3}$$

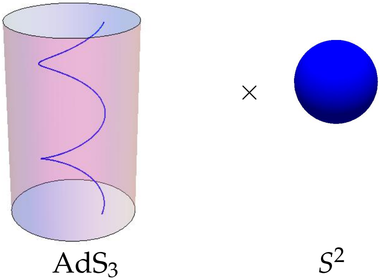
Matches with Bekenstein-Hawking entropy

M-theory decoupling limit

- Lift of D6-anti-D6 gives 2 KK monopoles with opposite charge. Is a bubbled geometry with near geometry $AdS_3 \times S^2 \times CY$

M-THEORY DECOUPLING LIMIT

- Lift of D6-anti-D6 gives 2 KK monopoles with opposite charge. Is a bubbled geometry with near geometry $AdS_3 \times S^2 \times CY$
- Lift of D2 is M2 wrapping the S^2, pointlike in AdS_3



Backreaction

M2's were treated as probes, fully backreacted solution was unknown: subject of this talk. Remarks:

- Approximation: M2-charge is smeared over CY. Can reduce to 5D on CY, or to 3D on $S^2 \times CY$

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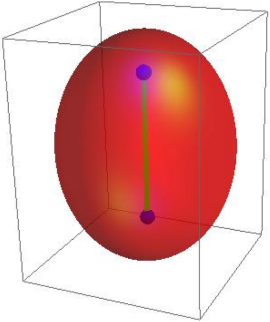
- Hoped to capture 'pure Higgs' states of D-brane system (Martinec, Niehoff 2015)
- Goals:
  - construct backreacted solution
  - check absence of pathologies such as closed timelike curves
  - check that is asymptotically AdS $\rightarrow$ holographic interpretation

Tadpole and a disclaimer

- Coupling $\int_{D6} G_6 \wedge A$ induces charge on compact D6 worldvolume which must be cancelled by adding an F1-string (Brodie, Susskind, Toumbas 2001).



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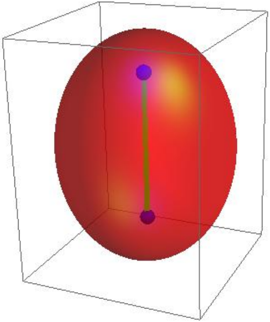

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- Have not yet succeeded in adding this F1

Reduction to 3D

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$$S = \frac{1}{16\pi G_3} \int_{\mathcal{M}} \left[d^3x \sqrt{-g} \left(R + \frac{2}{l^2} - \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{2\tau_2^2} \right) + \frac{l}{2} \mathcal{A} \wedge d\mathcal{A} \right]$$

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- τ_1: axion from dualizing 1-form A ($C_3 = A \wedge vol_{S^2}$)
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- wrapped M2 is point-particle source for τ:

$$S_{M2} = \frac{1}{16\pi G_3} \left[-2\pi p \int_{\mathcal{W}} d\xi \frac{\sqrt{-{}^*g}}{\tau_2} \right] + 2\pi p \int_{\mathcal{W}} A$$

Ansatz, BPS equations

Levi, JR, Van den Bleeken, Van Herck, Vercnocke 2011

- Ansatz for BPS solutions: fibration over 2D base

$$ds^2 = \frac{l^2}{4}\left[-\left(dt + 2\Im m(\partial\Phi) + \Lambda\right)^2 + \tau_2 e^{-2\Phi} dw d\bar{w}\right]$$
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- equation for conformal factor

$$
4\partial_w \partial_{\bar{w}} \Phi + \tau_2 e^{-2\Phi} = 0
$$

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- This is global AdS + leftmoving Wilson line

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- In dual (4,0) MSW CFT, represents (R vacuum with maximal R-charge) $\otimes$ vacuum

$$(h, \bar{h}) = (0, -c/24), \qquad j_L = c/12$$

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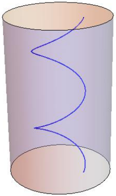

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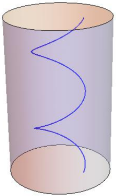

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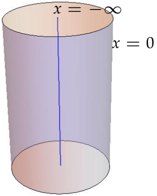
- M2 of charge $2\pi p$ adds to the vacuum

$$\Delta h = \frac{1}{2}(H_T - P_\phi) = \frac{p}{V_\infty}\frac{c}{24} \quad \leftarrow \text{puzzle!}$$

$$\Delta \bar{h} = \frac{1}{2}(H_T + P_\phi) = -\frac{c}{24} + \frac{p}{V_\infty}\frac{c}{24}\cosh 2\rho_0$$

M2 in center of AdS

First let's consider the M2 in the 'center' $x \to -\infty$ of AdS. Shift coordinate x such that AdS boundary at $x = 0$.



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V_∞: asymptotic Calabi-Yau volume

- τ has monodromy

$$\tau \to \tau + 2\pi p \qquad \text{under } \psi \to \psi + 2\pi$$

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- If we turn off M2 charge, must recover D6-anti-D6 background
- Will work perturbatively in ϵ around D6-anti-D6 solution

M2 IN CENTER OF ADS

To second order in ϵ one finds

$$
\begin{aligned}
\Phi &= \Phi_0 + \epsilon\Phi_1 + \epsilon^2\Phi_2 + \dots \\
\Phi_0 &= \ln\sinh(-x) \\
\Phi_1 &= \frac{1}{2}(-x + x\coth(x) - 1) \\
\Phi_2 &= \frac{1}{24}\Big(-6\mathrm{Li}_2\left(e^{2x}\right)\coth(x) - 3\Big(2x^2 + x^2\mathrm{csch}^2(x) \\
&\quad -4\log(-\mathrm{csch}(x)) - 1 + \log(16)\Big) \\
&\quad + \left(\pi^2 - 6(x-2)x\right)\coth(x)\Big)
\end{aligned}
$$

M2 in center of AdS

At higher order:

$$\Phi = \sum_{\epsilon=0}^{\infty} \Phi_n \epsilon^n$$

$$\Phi_n'' - \frac{2}{\sinh^2 x}\Phi_n = S_n(x)$$

$$S_n(x) = \frac{1}{\sinh^2 x}\left(x \sum_{\vec{p}\in\mathcal{P}_{n-1}} \prod_{l=1}^{n-1} \frac{(-2\Phi_l)^{p_l}}{p_l!} - \sum_{\vec{p}\in\mathcal{P}'_n} \prod_{l=1}^{n-1} \frac{(-2\Phi_l)^{p_l}}{p_l!} \right)$$

Have the iterative solution, explicit up a a single integral

$$\Phi_n(x) = (x\coth x - 1)\int_{-\infty}^{x} S_n(u)\coth u\, du + \coth x \int_x^0 S_n(u)(u\coth u - 1)du$$

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- absence of closed timelike curves:
  - Our solution has 'nice' (no CTC) behaviour near symmetry axis and boundary
  - Under these conditions, using null energy condition, can show that no CTCs can develop in the interior (JR 2011)

Holographic interpretation

Asymptotically AdS with Fefferman-Graham expansions

$$
\begin{aligned}
ds_3^2 &= l^2\left[\frac{dy^2}{4y^2} + \frac{g_{(0)}}{y} + g_{(2)++}dx_+^2 + g_{(2)--}dx_-^2 + \ln y\,\tilde{g}_{(2)--}dx_-^2 + \ldots\right. \\
\Psi &= \Psi_{(0)} + y\Psi_{(2)} + \ldots \\
\tau_1 &= \tau_{1(0)} \\
\mathcal{A} &= \mathcal{A}_{(0)+}dx_+ + \mathcal{A}_{(0)-}dx_-
\end{aligned}
$$

CFT Sources

- Source terms in CFT are

$$\begin{aligned} g_{(0)+-} &= 1 \\ \Psi_{(0)} &= -\ln V_\infty \\ \tau_{1(0)} &= p x_- \\ \mathcal{A}_{(0)-} &= \frac{\epsilon}{2} \end{aligned}$$

CFT Sources

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$$\begin{aligned} g_{(0)+-} &= 1 \\ \Psi_{(0)} &= -\ln V_\infty \\ \tau_{1(0)} &= p x_- \\ \mathcal{A}_{(0)-} &= \frac{\epsilon}{2} \end{aligned}$$

- Dual field theory is deformed by source terms for weight $(1,1)$ primary $\mathcal{O}_{\tau_1}$ and weight $(1,0)$ R-current J^3_+:

$$\delta S_{CFT} = -\int dx_+ dx_- \left(\tau_{1(0)}\mathcal{O}_{\tau_1} + \mathcal{A}_{(0)-}J^3_+\right)$$

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- Null deformation, preserves leftmoving conformal symmetry (see e.g. Caldeira Costa, Taylor 2010)
- Killing spinor analysis suggests that a leftmoving chiral $N = 2$ is preserved

VEVs

Holographic renormalization (de Haro, Solodukhin, Skenderis 2001) gives

$$\langle T_{++}\rangle = \frac{c}{6}\left(g_{(2)++} + \frac{1}{4}\mathcal{A}^2_{(0)+}\right)$$

$$\langle T_{--}\rangle = \frac{c}{6}\left(g_{(2)--} - \frac{1}{2}\tilde{g}_{(2)--} + \frac{1}{4}\mathcal{A}^2_{(0)-}\right)$$

$$\langle J^3_+\rangle = \mathcal{A}_{(0)+}$$

$$\langle \mathcal{O}_\Psi\rangle = -\frac{c}{6}\Psi_{(2)}$$

$$\langle \mathcal{O}_{\tau_1}\rangle = 0$$

VEVs

Taking Fourier modes one finds the VEVs

$$
\begin{aligned}
h &= 0 \qquad \leftarrow \text{R groundstate} \\
\bar{h} &= -\frac{c}{24}\left(1-\epsilon+\frac{\pi^2}{6}\epsilon^3-(0.819\ldots)\epsilon^4+(0.621\ldots)\epsilon^5+\mathcal{O}(\epsilon^6)\right) \\
j_L &= \frac{c}{12}\left(1-\frac{\epsilon}{2}\right) \\
\langle(\mathcal{O}_\Psi)_0\rangle &= \frac{c\epsilon}{12}\left(1-\frac{\epsilon}{2}\right) \\
\langle(\mathcal{O}_{\tau_1})_0\rangle &= 0
\end{aligned}
$$

$\bar{h}$ agrees with probe probe analysis to order ϵ

OTHER SOLUTIONS

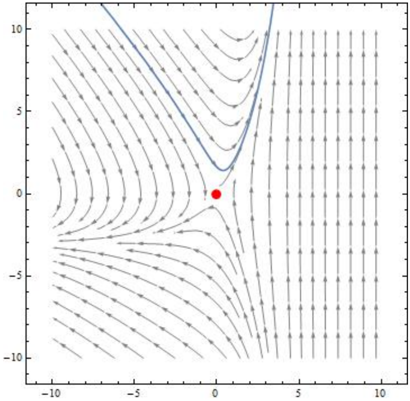
Defining

$$\begin{aligned} 1-\epsilon x &= e^{-\epsilon u} \\ X &= -\left(2\Phi + 3\epsilon u + \ln \frac{3}{2V_\infty}\right) \\ Y &= \dot{X} \end{aligned}$$

Equation for Φ becomes flow equation

$$(\dot{X}, \dot{Y}) = (Y, -\epsilon Y + 3(e^X - \epsilon^2))$$

OTHER SOLUTIONS



Other solutions

- Fixed point corresponds to exact solution

$$\Phi = \ln\left(\sqrt{\frac{2}{3\epsilon^2}}(1-\epsilon x)^{3/2}\right)$$

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- Geometry is that of 3D Gödel universe (Levi, JR, Van den Bleeken, Van Herck, Vercnocke 2011)

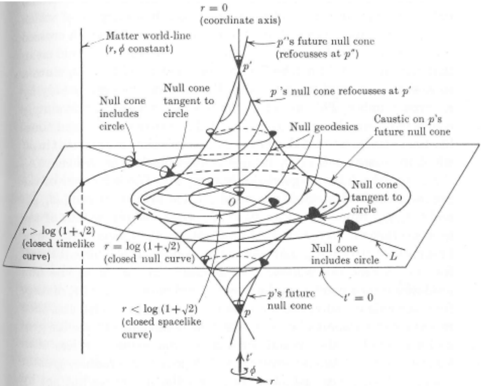
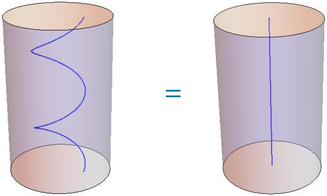


FIGURE 31. Gödel's universe with the irrelevant coordinate z suppressed. The space is rotationally symmetric about any point; the diagram represents correctly the rotational symmetry about the axis $r = 0$, and the time invariance. The light cone opens out and tips over as r increases (see line L) resulting in closed timelike curves. The diagram does not correctly represent the fact that *all* points are in fact equivalent.

M2 at constant radius

- Probe picture: obtained by acting with broken generators



$$\times g(\rho_0)$$

$$g(\rho_0) = \begin{pmatrix} \cosh \rho_0 & \sinh \rho_0 \\ \sinh \rho_0 & \cosh \rho_0 \end{pmatrix}$$

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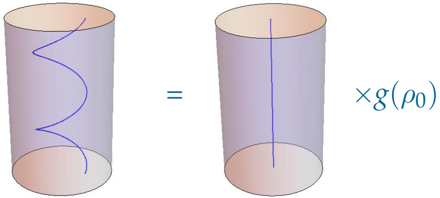

$$g(\rho_0) = \begin{pmatrix} \cosh\rho_0 & \sinh\rho_0 \\ \sinh\rho_0 & \cosh\rho_0 \end{pmatrix}$$

- Can be seen as large coordinate transformation

$$\begin{aligned}
\cosh^2\tilde{\rho} &= \cosh^2(\rho+\rho_0) - \sin^2\frac{\psi}{2}\sinh 2\rho \sinh 2\rho_0 \\
\tilde{x}_+ &= x_+ - x_- + \arg\left[(1+e^{ix_-}\coth\rho_0\tanh\rho)(1+e^{ix_-}\tanh\rho_0\tanh\rho)\right] \\
\tilde{x}_- &= \arg\left[(1+e^{ix_-}\coth\rho_0\tanh\rho)(1+e^{-ix_-}\tanh\rho_0\tanh\rho)\right]
\end{aligned}$$

M2 at constant radius

- Propose to get the backreacted solution by similar coordinate transformation on M2 in center. Near the boundary it acts as

$$\begin{aligned} \tilde{x}_+ &= x_+ + \mathcal{O}(e^{-2\rho}) \\ e^{i\tilde{x}_-} &= \frac{\cosh\rho_0 e^{ix_-} + \sinh\rho_0}{\sinh\rho_0 e^{ix_-} + \cosh\rho_0} + \mathcal{O}(e^{-2\rho}) \end{aligned}$$

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- Implements $e^{\rho_0(\tilde{L}_{-1}-\tilde{L}_1)}$
- Acting on our solution, obtain new solution with

$$h = 0$$
$$\bar{h} = -\frac{c}{24}\,[1 - \epsilon\cosh 2\rho_0$$
$$-\epsilon^2\left(2\rho_0\cosh 2\rho_0 - e^{-4\rho_0}{}_2F_1^{(0,1,0,0)}(\frac{1}{2}, 2, 1, 1 - e^{-4\rho_0})\right) + \ldots$$

5D point of view

- Lift to 5D: solution with VMs and $\tau \in$ universal hyper excited:

$$
\begin{aligned}
ds_5^2 &= \frac{l^2}{4}\left[-\left(dt + 2\Im m(\partial\Phi) + \Lambda\right)^2 + \tau_2 e^{-2\Phi} dw d\bar{w} + d\theta^2 + \sin^2\theta (d\phi - \mathcal{A})^2\right] \\
\mathcal{A} &= dt + \Lambda, \qquad d\Lambda = 0, \qquad \tau = \tau(w), \qquad l = 2\left(\frac{P^3}{6}\right)^{\frac{1}{3}} \\
F^I &= \frac{P^I}{2}\sin\theta d\theta \wedge (d\phi - \mathcal{A}), \qquad Y^I = \frac{P^I}{l}
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$$

- Can be written as t-fibration over 4D base, which contains 3 ASD 2-forms with structure

$$
\begin{aligned}
d\Phi^{\pm} \mp i\frac{d\tau_1}{2\tau_2}\wedge\Phi^{\pm} &= 0\\
d\Phi^3 &= 0 \qquad \leftarrow \text{Kahler form}
\end{aligned}
$$

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- In our solutions: base is toric Kahler with
  - 1 translational Killing vector ∂_ψ
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  - 1 translational Killing vector ∂_ψ
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- requires generalization of Bellorin, Meessen, Ortin 2007, see JR, Van den Bleeken 2014

KILLING SPINORS

- Killing spinor analysis: all our sols preserve common 'zero-mode' Killing spinors

$$G_0^{\beta\gamma} = e^{-\frac{i}{2}\beta\phi\sigma_3} e^{\frac{i\theta}{2}\gamma^{\hat{\phi}}} g_0^{\beta\gamma} \qquad \beta,\gamma = \pm 1$$

with $g_0^{\beta\gamma}$ certain constant spinors.

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- Independent of AdS_3 coordinates, in Ramond sector
- Without the M2, all 4 $G_0^{\beta\gamma}$, without the M2 only the 2 with $\beta = 1$
- Further non-common (4 resp. 2) KS depending on x^+: extra ± 1 modes preserved by maximal R-charged Ramond groundstate

Outlook

Constructed backreacted super-egg. Consistent with R ground state in leftmoving sector. M2 charge turns on deformation in rightmoving sector. Some open question:

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- Understand interpretation in MSW CFT better
- Analyze closer what happens when $\bar{L}_0 > 0$
- Missing ingredient: stretched $F1$-string
- Full BH entropy from quantizing solution space? Unlikely in 5D/3D sugra, though expect to get scaling with D0-charge right. Perhaps in 11D?

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- M2 in center is prototype W-brane solution (Martinec, Niehoff 2015) conjectured to describe pure Higgs states

EXTRA: COMMENT ON BOUNDARY STRESS TENSOR

- Despite broken right-moving conformal invariance, our solutions have, surprisingly

$$\langle T_{+-}\rangle = \partial_+ \langle T_{--}\rangle = 0$$

- For sources of the type

$$\Psi_{(0)} = \text{constant}, \qquad \partial_+ \tau_{1(0)} = \partial_+ \mathcal{A}_{(0)-} = 0$$

derive holographic Ward identities

$$\begin{aligned} \langle T_{+-}\rangle &= 0 \\ \partial_- \langle T_{++}\rangle &= 0 \\ \partial_+ \langle T_{--}\rangle &= -\frac{1}{2}\langle \mathcal{O}_{\tau_1}\rangle \tau'_{1(0)} \end{aligned}$$

- Our solutions have vanishing $\langle \mathcal{O}_{\tau_1}\rangle$ and therefore $\partial_+ \langle T_{--}\rangle = 0$