A holographic construction of CFT excited states

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Microstructure of Black Holes
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The microstructure of black holes

- A central question in black hole physics is what are the microstates that account for entropy of the black hole.
- With AdS/CFT, it became clear that we are counting are states of the dual CFT.
- BUT what are the microstates in the geometric regime?

AdS/CFT and Fuzzballs

The AdS/CFT argument [KS, Taylor (2007)]:

- For every state of the CFT, there should exist a regular Asymptotically AdS solution that captures the vevs of gauge invariant operator in that state.
- Consider now one of the supersymmetric states that accounts for the entropy of, say, the Strominger-Vafa black hole. Associated with this state there must exist an Asymptotically AdS solution.
- Since this is a pure state the corresponding geometry must by horizon-free. Solutions with horizons have entropy and thus cannot be associated with pure states; horizons are associated with thermal states.

AdS/CFT and Fuzzballs

- Thus there should exist $\exp(S)$ regular horizon-free solutions, one for each state counted to account for the black hole entropy. This is precisely the fuzzball proposal.
- This argument however does not imply that the solutions would be well-described by supergravity. Most of the states will be associated with stringy solutions.

➢ How many solutions do we expect within SUGRA?

Holographic anatomy

- In previous works, one typically started from a supergravity solution and extracted the holographic data (sources and/or vevs)
- One may then use this data to check whether educated guesses for the dual state are consistent with the holographic data.
- This was applied successfully in a number of non-trivial examples (Coulomb branch solution [KS, Taylor (2006)], LLM solutions [KS, Taylor (2007)], fuzzball solutions [Kanitscheider, KS, Taylor (2007)].

Holographic reconstruction

- In this talk, we will go back to basics and ask the converse:

  Given a state can we explicitly work out the bulk solution?

References

- KS, Ariana Christodoulou, Holographic construction of CFT excited states, to appear

Earlier relevant work:

- S. de Haro, S. Solodukhin, KS, Holographic reconstruction of space-time and renormalization in AdS/CFT Commun.Math.Phys. 217 (2001) 595-622 . hep-th/0002230.
- KS, B. van Rees, Real-time gauge/gravity duality, Phys.Rev.Lett. 101 (2008) 081601, 0805.0150; Real-time gauge/gravity duality: prescription, renormalization and examples, JHEP 0905 (2009) 085, 0812.2909.

Outline

Outline

QFT

- To define a theory we typically give the Lagrangian.
- Depending on context we may be interested in:
- vacuum correlators

$$\langle 0|O_1(x_1)\cdots O(x_n)|0\rangle$$

- correlators in a non-trivial states,

$$\langle \alpha|O_1(x_1)\cdots O(x_n)|\alpha\rangle$$

For example, $|\alpha\rangle$ may be a state that spontaneously breaks some of the symmetries, a thermal state, a non-equilibrium state, etc.

- The correlators may be time-ordered, Wightman functions, advanced, retarded ...

QFT

- Given a theory, a state and a set of operators there is a unique answer for their correlators.
- Any dual formulation should have the same properties.
- How does it work in holography?

Holography

- Within the gravity approximation:
- The dual QFT is represented by:
  An (asymptotically) AdS solution of $(d+1)$-dimensional gravity coupled to matter.
- Operators are dual to bulk fields and the state is encoded in subleading terms in bulk solutions.

Euclidean vs Lorentzian signature

- If we are interested in time-independent states (vacuum state, thermal state) we may Wick rotate to Euclidean signature.
- In Lorenzian signature there are different types of correlators (time-ordered, Wightman, retarded, etc.).
- In Euclidean QFT there is a unique type of correlators.
- So to simplify matters let's initially focus on Euclidean signature.
- We will return to full fledged Lorentzian discussion afterwards.

Outline

Bulk scalar field

- Bulk scalar fields Φ of mass $m^2 = \Delta(\Delta - d)$ are dual to operators O_Δ of dimension Δ.
- The solution of the bulk field equation has an asymptotic expansion of the form [de Haro, Solodukhin, KS (2000)]:

$$\Phi(x,r) = r^{d-\Delta}\phi_{(0)} + \cdots + \log r^2 \psi_{(2\Delta-d)} + r^{\Delta}\phi_{(2\Delta-d)} + \cdots$$

- $\phi_{(0)}$ is the source for the dual operator and all blue terms are locally determined by it.
- $\phi_{(2\Delta-d)}$ is related with the expectation value of O,

$$\langle O_\Delta \rangle \sim \phi_{(2\Delta-d)}$$

QFT vs gravity

- We argued earlier that the Lagrangian and the state uniquely specify the correlators.
- The counterpart of this statement on the gravity side is that $(\phi_{(0)}, \phi_{(2\Delta-d)})$ uniquely specify a bulk solution.
- One way to see this is to use a radial Hamiltonian formalism, where the radial direction plays the role of time.
- The renormalised radial canonical momentum is

$$\pi_\Delta = \langle O_\Delta \rangle \sim \phi_{(2\Delta-d)}$$

- By a standard Hamiltonian argument, specifying the conjugate pair $(\phi_{(0)}, \pi_\Delta)$ uniquely picks a solution. [Papadimitriou, KS (2004)]

Holographic reconstruction I

Given QFT data $(\phi_{(0)}, \langle O_\Delta \rangle)$ there is a unique solution $\Phi(x, r)$ of the bulk field equations.

QFT vs gravity

- In QFT the vacuum structure is a dynamical question: in general, one cannot tune the value of $\langle O_\Delta \rangle$.
- The counterpart of this statement in gravity is that regularity in the interior selects $\langle O_\Delta \rangle$.
- A generic pair $(\phi_{(0)}, \pi_\Delta)$ leads to a singular solution.

Outline

New issues

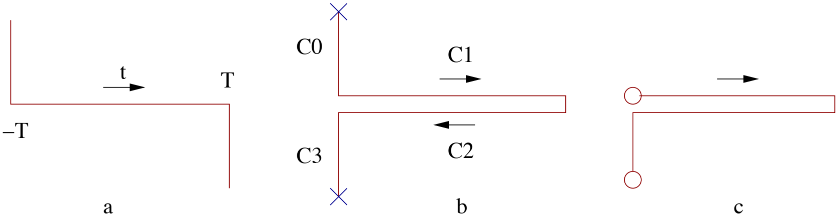
- On the gravity side:
  - In Lorentzian signature the bulk equations of motion do not have a unique solution given boundary conditions. One needs in addition initial conditions.
  - There are non-trivial normalisable modes that vanish at the boundary.
- On the QFT side:
  - Given a Lagrangian and operators there is more than one type of correlator one may wish to compute.
  - One may wish to compute time-ordered, anti-time ordered, Wightman products, etc. on non-trivial states.

QFT

On the QFT side one can summarise the new data needed by

- providing a contour in the complex time plane
- specifying where operators are inserted in this contour.

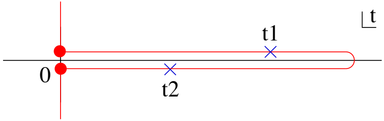
Examples



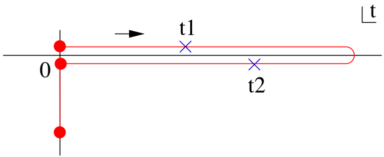
- a Vacuum-to-vacuum contour: computes correlators $\langle 0|T(\cdots)|0\rangle$
- b In-in contour: computes correlators $\langle \alpha|\cdots|\alpha\rangle$
- c Thermal contour: computes thermal correlators

Examples

- Wightman in-in 2-point function, $\langle 0|O(x)O(y)|0\rangle$



- Thermal Wightman 2-point function.



Real-time gauge/gravity duality [van Rees, KS (2008)]

The holographic prescription is to use "piece-wise" holography:

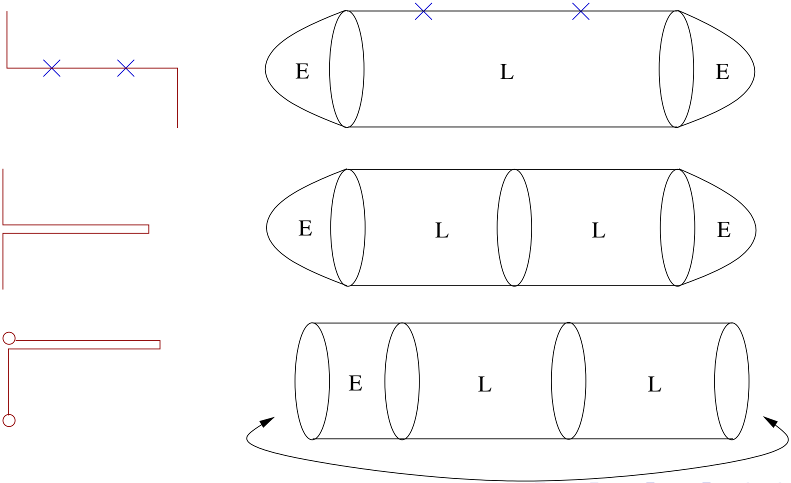
- Real segments are associated with Lorentzian solutions,
- Imaginary segments are associated with Euclidean solutions,
- Solutions are matched at the corners.

Examples

a

b

c



Comments

- In the extended space-time associated to a given contour, boundary conditions and regularity in the interior uniquely fix the bulk solution.
- The 'Euclidean caps' can also be thought of as Hartle-Hawking wave functions.

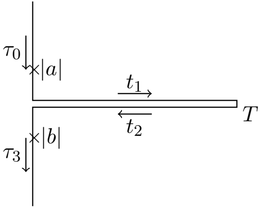
Holographic reconstruction II: Lorentzian case

Given QFT data $(\phi_{(0)}, \langle O_\Delta \rangle)$ and a contour in the complex time plane there is a unique solution $\Phi(x, r)$ of the bulk field equations.

Outline

Excited states

- Excited states are obtained by acting with operators on the CFT vacuum,
$$|\Delta\rangle = O_\Delta|0\rangle$$
- Using the in-in formalism one may compute correlation functions in this state:



- This means that we are considering the path integral

$$Z[\phi_{(0)}] = \int [\mathcal{D}\varphi] \exp\left(-i \int_C dt d^{d-1}x \sqrt{-g_{(0)}} \left(\mathcal{L}_{QFT} + \phi_{(0)} O_\Delta\right)\right)$$

1-point functions

- In preparation for the holographic discussion let us discuss the 1-point function,

$$\langle O_\Delta(x)\rangle = \int [\mathcal{D}\varphi] O_\Delta(x) \exp\left(-i\int_C dt d^{d-1}x \sqrt{-g_{(0)}}\left(\mathcal{L}_{QFT} + \phi_{(0)} O_\Delta\right)\right)$$

- To leading order in the sources,

$$\begin{aligned}\langle O_\Delta(x)\rangle &= \langle 0|O_\Delta(x) O_\Delta(0)|0\rangle \phi^-_{(0)} + \phi^+_{(0)}\langle 0|O^\dagger_\Delta(\infty) O_\Delta(x)|0\rangle \\ &= \langle 0|O_\Delta(x)|\Delta\rangle \phi^-_{(0)} + \phi^+_{(0)}\langle \Delta|O_\Delta(x)|0\rangle\end{aligned}$$

The 2-point functions here are Wightman functions, not time-ordered.

Remarks

- Setting $\phi^{+}_{(0)} = 0, \phi^{-}_{(0)} = 1$ and taking $x \to \infty$ computes the norm of the state.
- The orthogonality of the 2-point functions implies that

$$\langle O_{\Delta_i}(x) \rangle = 0, \qquad \Delta_i \neq \Delta$$

  to linear order in sources.
- However, to higher order in the sources,

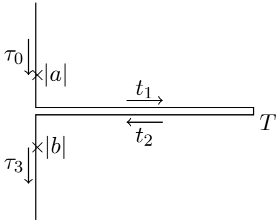
$$\langle O_{\Delta_i}(x) \rangle \neq 0, \qquad \Delta_i \neq \Delta$$

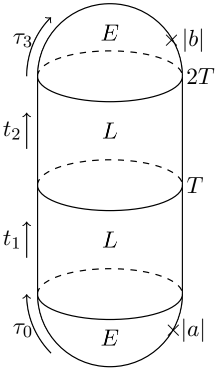
Holographic construction

- In holography each gauge invariant operator O_{Δ_i} is mapped to a corresponding bulk field Φ_i.
- If we are interested in the holographic construction of the CFT state $|\Delta\rangle$ to leading order in the sources, it suffices to consider only the corresponding field Φ to linear order.
- To higher order one needs to consider other bulk fields as well.

Holographic construction

The time contour and the corresponding bulk space-time are:





Procedure

- We need to find regular solutions of the (linear) bulk equations with the prescribed boundary conditions in each part of the extended spacetime.
- Impose the matching conditions.
- I will present the results for a scalar field in AdS_3 in global coordinates.
- We have also carried the same computation in Poincaré coordinates.

Obtaining the solution: Euclidean solutions

- In the past cap we consider a solution with a delta function source:

$$\Phi_E^- = r^{d-\Delta}\phi_{(0)}^-(\tau_0,\phi) + \dots, \qquad \phi_{(0)}^-(\tau_0,\phi) = \delta(\tau_0 - a)\delta(\phi)$$

- This leads to

$$\Phi_E^-(\tau_0,r,\phi) = \sum_{n=0}^{\infty}\sum_{k\in\mathbb{Z}}\left(\frac{i}{4\pi^2}\left[\theta(-\tau_0-|a|)\phi_{(0)}^-(\omega_{nk}^-,k)e^{-\omega_{nk}^-\tau_0+ik\phi}+\right.\right.$$
$$\left.\left.+\theta(\tau_0+|a|)\phi_{(0)}^-(\omega_{nk}^+,k)e^{-\omega_{nk}^+\tau_0+ik\phi}\right] + d_{nk}^- e^{-\omega_{nk}^-\tau_0+ik\phi}\right) g(\omega_{nk},|k|,r)$$

  where $g(\omega_{nk},|k|,r) \sim r^{\Delta}$ as $r \to 0$ and $\omega_{nk}^{\pm}$ are the frequencies of the normalizable modes.

- There is a similar expression at the future EAdS cap.

Lorentzian solution I

- The Lorentzian solution in the first Lorentzian part is a sum of normalisable modes

$$\Phi_L^1(t_1, r, \phi) = \sum_{n,k} (a_{nk} e^{-i\omega_{nk}^+ t_1 + ik\phi} + a_{nk}^\dagger e^{i\omega_{nk}^+ t_1 - ik\phi}) i g(\omega_{nk}, |k|, r).$$

- There is a similar expression for the second Lorenzian part.

Matching conditions

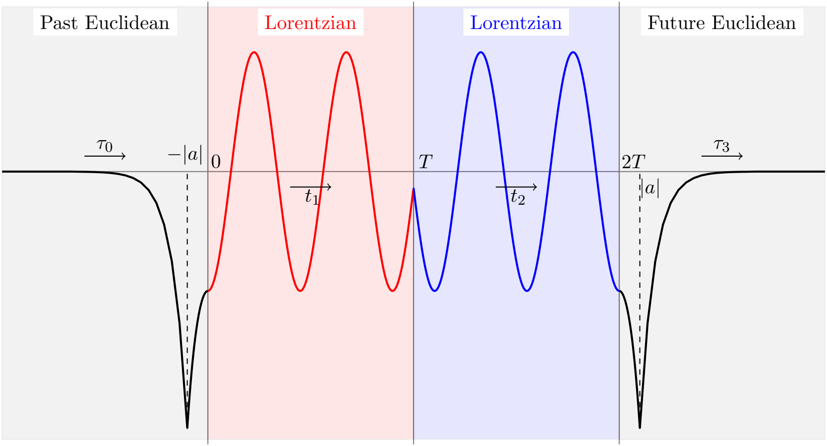
- The matching conditions are continuity of fields and momenta along the contour:

$$\begin{aligned}
\Phi_E^-\big|_{\tau_0=0} &= \Phi_L^1\big|_{t_1=0}, \quad \partial_{\tau_0}\Phi_E^-\big|_{\tau_0=0} = -i\partial_{t_1}\Phi_L^1\big|_{t_1=0} \\
\Phi_L^1\big|_{t_1=T} &= \Phi_L^2\big|_{t_2=T}, \quad \partial_{t_1}\Phi_L^1\big|_{t_1=T} = -\partial_{t_2}\Phi_L^2\big|_{t_2=T} \\
\Phi_L^2\big|_{t_2=2T} &= \Phi_E^+\big|_{\tau_3=0}, \quad \partial_{t_2}\Phi_L^2\big|_{t_2=2T} = -i\partial_{\tau_3}\Phi_E^+\big|_{\tau_3=0}.
\end{aligned}$$

- The matching conditions fix all constants in terms of the sources. In particular, they imply

$$a_{nk} \sim \phi_{(0)}^-(\omega_{nk}^+, k), \qquad a_{nk}^\dagger \sim \phi_{(0)}^+(\omega_{nk}^-, k)$$

The solution across the matching surface



Amplitude of a single mode as a function of (Euclidean and then Lorentzian) time for fixed r, ϕ.

Holographic 1-point function

- The 1-point function is extracted from the asymptotic,

$$\Phi(x, r) = \cdots + r^{\Delta}\phi_{(2\Delta-d)} + \cdots\,, \qquad \langle O_\Delta \rangle \sim \phi_{(2\Delta-d)}$$

- In our case the solution is normalizable, so

$$\langle O_\Delta \rangle = \lim_{r\to 0} r^{-\Delta}\Phi^1_L(r, t, \phi)$$
$$\sim \lim_{r\to 0}\sum_{n,k}\left(\phi^-_{(0)}(\omega^+_{nk}, k)e^{-i\omega^+_{nk}t+ik\phi} + \phi^+_{(0)}(\omega^-_{nk}, k)e^{i\omega^+_{nk}t-ik\phi}\right) ig(\omega_{nk}, |k|, r)$$

- The limit and the sums can be evaluated exactly [van Rees, KS (2008)] leading to

$$\langle O_\Delta(x)\rangle = \langle 0|O_\Delta(x)O_\Delta(0)|0\rangle\phi^-_{(0)} + \phi^+_{(0)}\langle 0|O^\dagger_\Delta(\infty)O_\Delta(x)|0\rangle$$

in exact agreement with our earlier QFT computation!

Summary

Bulk excitations (normalizable modes) are dual to excited CFT states.

The ‘extrapolation dictionary’

- An *a priori* different dictionary was suggested in the early days of AdS/CFT [Banks, Douglas, Horowitz, Martinec (1998)].
- Boundary correlation functions can be obtained as a limit of bulk correlation functions:

$$\langle O_\Delta(x_1)\cdots O_\Delta(x_n)\rangle_{QFT} = \lim_{r\to 0} r^{-n\Delta}\langle\Phi(x_1,r)\cdots\Phi(x_n,r)\rangle_{Bulk}$$

- This prescription is equivalent to the standard one for scalars in a fixed background [Harlow, Stanford (2011)]. It also agrees with the real-time prescription for non-equilibrium scalar 2-point functions [Keranen, Kleinert (2014)].

Bulk reconstruction

- The extrapolate dictionary suggests the following map between boundary and bulk operators:

$$\Phi(x,r) = \int d^d x' K(x'|r,x) O_\Delta(x')$$

  K is called the smearing function.

- Bulk correlation functions are then related to boundary correlation functions by

$$\langle \Phi(x_1,r_1)\cdots\Phi(x_n,r_n)\rangle_{Bulk} = \int d^d x_1' ... d^d x_n' K(x_1'|r_1,x_1)\ldots K(x_n'|r_n,x_n)\langle O_\Delta(x_1')\cdots O_\Delta(x_n')\rangle_{QFT}$$

[Banks, Douglas, Horowitz, Martinec (1998)], ... [Hamilton, Kabat, Lifschytz, Lowe (2006)] ...

Smearing function

- In our discussion we reconstructed the bulk solution from $\langle O_\Delta(x')\rangle$
- One can also follow similar steps to those in [Hamilton, Kabat, Lifschytz, Lowe (2006)] to show that

$$\Phi_L(x,r) = \int d^d x' K(x'|r,x)\langle O_\Delta(x')\rangle$$

In our derivation the smearing function is automatically convergent due to $i\epsilon$'s that originate from the Euclidean caps. (In *Hamilton et al* these insertions were added by hand.)

Remarks

- In our derivation the relation was between classical fields in the bulk and expectation values of boundary operators in a specific state.
- This appears different than a map between bulk and boundary operators.
- If we consider a quantized bulk field

$$\hat{\Phi}_L(t_1, r, \phi) = \sum_{n,k} (\hat{a}_{nk} e^{-i\omega^+_{nk} t_1 + ik\phi} + \hat{a}^\dagger_{nk} e^{i\omega^+_{nk} t_1 - ik\phi}) ig(\omega_{nk}, |k|, r)$$

  where now $a_{nk}, a^\dagger_{nk}$ are creation and annihilation operators, then our results would imply that $\phi^\pm_{(0)}$ are quantum operators.
- However, from the QFT perspective $\phi^\pm_{(0)}$ are classical sources that couple to quantum operators.

Beyond linear fields

This boundary-to-bulk map cannot hold in this form in complete generality.

- If the QFT contains operators with dimensions, $O_{\Delta_1}, O_{\Delta_2}, O_{\Delta_3}$, with $\Delta_3 = \Delta_1 + \Delta_2$. Then [KS, Taylor (2006)],

$$\langle O_{\Delta_3} \rangle = \pi_{\Delta_3} + c_{12} \pi_{\Delta_1} \pi_{\Delta_2}$$

  where c_{12} is a constant. This suggests

$$\Phi_3(x,r) = \int d^d x' K(x'|r,x) \langle O_{\Delta_3}(x') \rangle + \int d^d x' K_1(x'|r,x) \langle O_{\Delta_1}(x') \rangle \langle O_{\Delta_2}(x') \rangle$$

- Locality also suggest non-linear terms should be added to the map [Kabat, Lifschytz, Lowe (2011)] ...[Kabat, Lifschytz (2015)].

Outline

How many fuzzballs do we expect within SUGRA?

- We argued earlier that regular supergravity solutions are uniquely specified by the 1-point functions of the dual operators,

$$\langle \alpha | O_i | \alpha \rangle$$

- Within supergravity we only have access to fields dual to chiral primary operators.

D1-D5 system

- Ramond ground states are given by

$$|\alpha\rangle = O^R_\alpha |O\rangle$$

  where O^R_α is a 1/4 BPS operator obtained from chiral primary by spectra flow.
- Therefore, our earlier analysis shows that there is a linearized solution for every such state.
- Barring linearization instability, there should exist a regular supergravity solution for every such state.

➠ All of the entropy of D1-D5 system may be accounted for within supergravity.

D1-D5-P system

- In this case we need to add left moving excitations

$$|\alpha\rangle = O_\alpha |O\rangle$$

  The operator O_α breaks another 1/2 of SUSY relative to R ground states.

- Our earlier analysis shows now that there is no a linearized solution for any such state.

  Since CFT 2-point functions are diagonal and here we would need the 2-point function of a 1/4 BPS and 1/8 BPS operator.

D1-D5-P system

- At next order we would need the 3-point function of a R ground state operator with two 1/8 BPS operators to be non-zero.
- For this to be case the OPE of two O_α operators to contain a R ground state.
- ➠ These operators should be in correspondence with R ground states.
- ➠ This suggests that the number of supergravity fuzzball solutions for the D1-D5-P system has the same growth as the 2 charge solutions.

Outline

Conclusions

- I discussed how QFT data reconstruct classical fields in the bulk, both in Euclidean and Lorentzian signature.
- The map works in full generality.
- I explicitly worked out the dual of the CFT excited state at the linearized level.
- It is straightforward (but tedious) to work out the dual at the non-linear order. The dual solution would involve many bulk fields.
- I discuss implications for the fuzzball program.