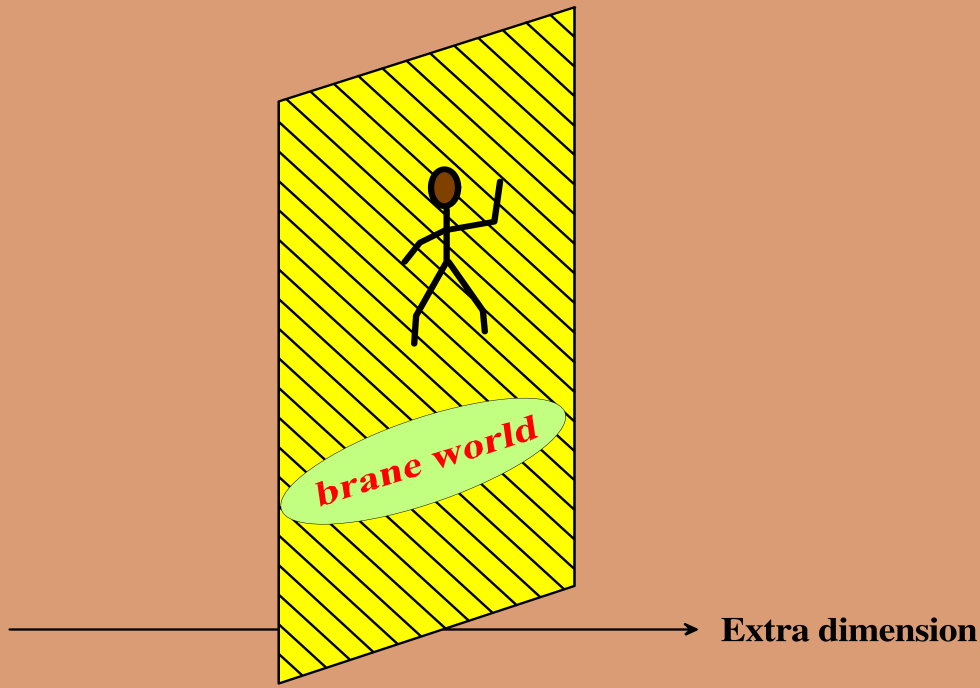
JGRG12, 2002/11/25-28

Brane-World Cosmology



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§1. Historical Notes

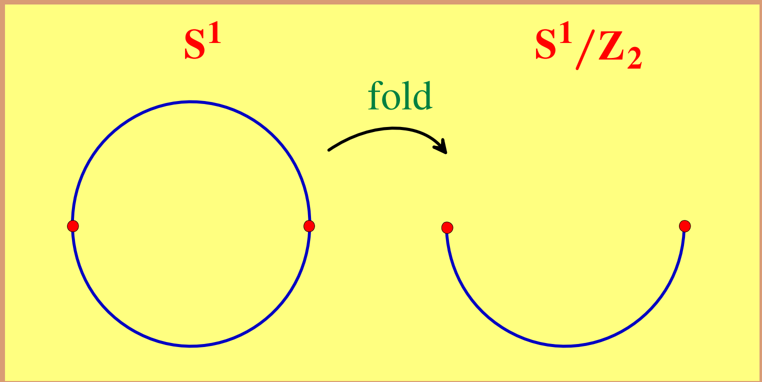
Progress in Particle Physics

Supergravity ⇒ String theory ⇒ ‘M’ theory

- Hořava & Witten (1996)

11D Supergravity with 1D compactified as $\mathbf{S^1/Z_2}$ gives a desirable string theory on the 10D boundary spacetime.

(Standard matter fields live on the boundary.)



⇓ Compactify extra 6D

$$\partial(11D) = 10D \quad \Rightarrow \quad 6D \otimes 4D$$

Putting aside the extra 6D, we have

$$\partial(5D) = 4D \quad \cdots \quad \text{brane world!}$$

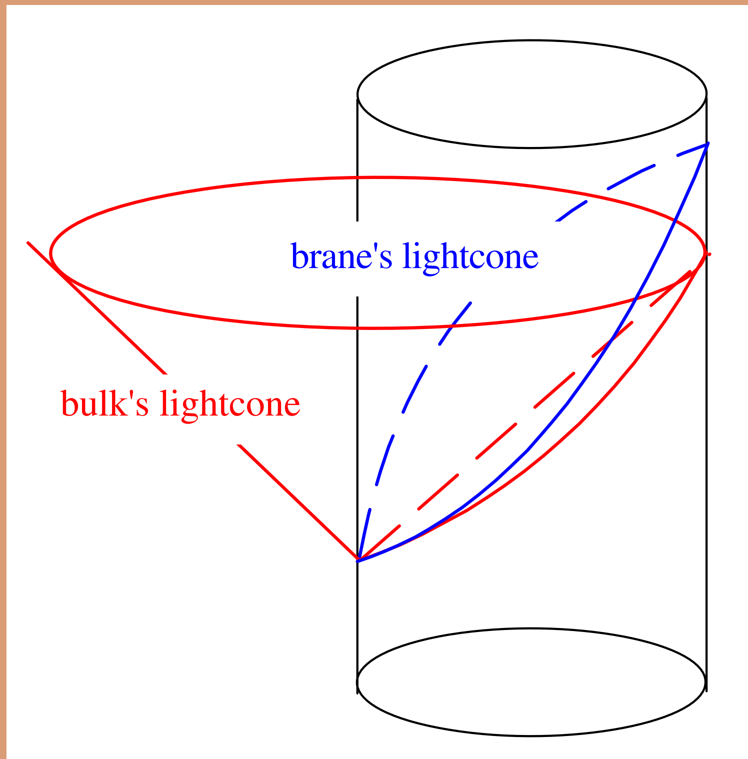
- Self-gravitating brane $\approx$ domain wall

  $\star$ $(n-1)$-brane = singular (time-like) hypersurface embedded in $(n+1)$-dim spacetime

  $\star$ brane tension (σ) = vacuum energy $(\sigma > 0?$ or $\sigma < 0?)$
  vacuum energy $\neq$ cosmological constant on the brane

  $\star$ causality on the brane $\neq$ causality in the bulk
  H. Ishihara, PRL86 (2001)

A new picture of the universe!



§2. Randall-Sundrum (RS) Brane World

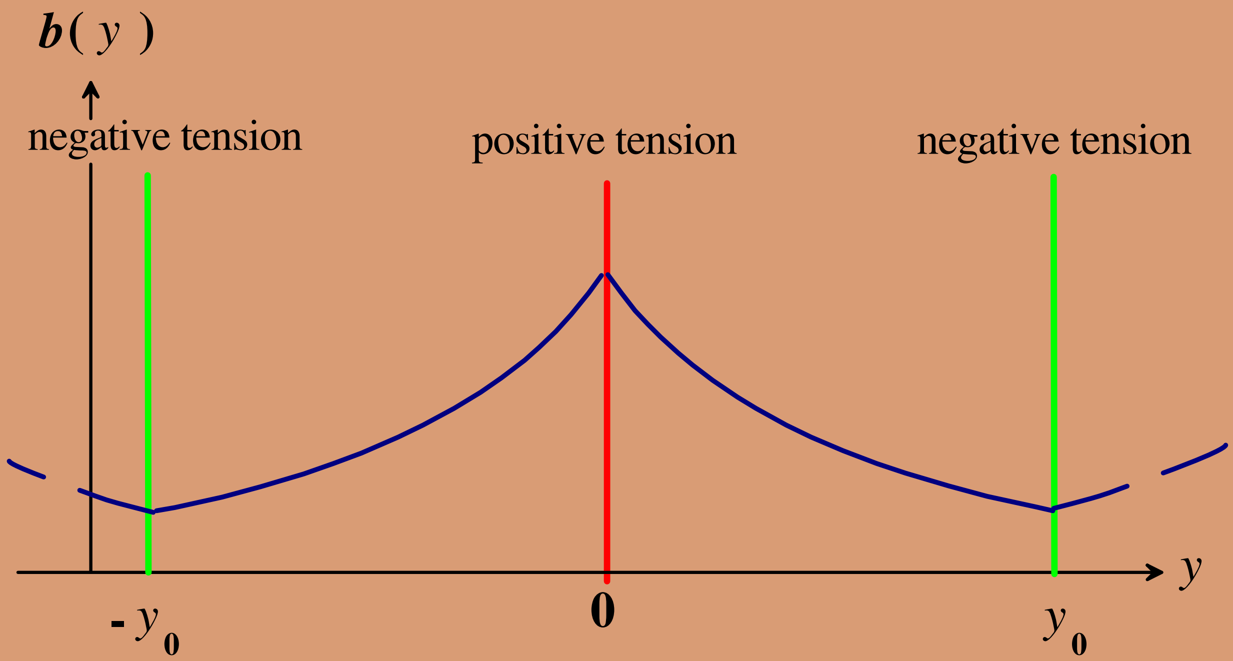
PRL 83, 3370 (1999) [RS1]; 83, 4690 (1999) [RS2]

- 5D-AdS bounded by 2 branes: $R^4 \times (S^1/Z_2)$

$$ds^2 = dy^2 + e^{-2|y|/\ell}\eta_{\mu\nu}dx^\mu dx^\nu \quad (-r_c \le y \le r_c)$$

$$\ell^2 = -\frac{6}{\Lambda_5}, \qquad \text{brane tension: } \sigma_\pm = \pm\frac{3}{4\pi G_5 \ell}$$

($b(y) = e^{-|y|/\ell}$ is called the warp factor)



- Possible solution to the Mass hierarchy problem in particle physics if we live on the negative tension brane [RS1: 2-brane model].

⋆ r_c is arbitrary ⇒ Existence of Radion mode

Brans-Dicke type gravity on the branes

unless $^{\exists}$stabilization mechanism.

$\omega_{BD} < 0$ on the negative tension brane (Garriga & Tanaka, '99)

- If we live on the positive tension brane, the negative tension brane may be absent ($r_c \to \infty$ in RS1) [RS2: 1-brane model].

⋯ 5th dimension can be non-compact

⋆ Gravity confined within $\ell \sim |\Lambda_5|^{-1/2}$ from the brane

⋆ No "radion" modes (no relative motion)

Einstein gravity is recovered on scales $\gg \ell$

$$\Phi_{\rm Newton} = -\frac{G_5 M}{r^2} \underset{r \gg \ell}{\Rightarrow} -\frac{G_5 M}{\ell\, r} = -\frac{G_4 M}{r}$$

The positive tension brane seems cosmologically favored.

§3. Brane Cosmology in AdS$_5$-Schwarzschild Bulk

Kraus ('99); Ida ('00); ⋯

- 5D AdS-Schwarzschild in Static Chart:

$$ds^2 = -A(R)dT^2 + \frac{dR^2}{A(R)} + R^2 d\Omega_K^2$$

$$A(R) = K + \frac{R^2}{\ell^2} - \frac{\alpha^2}{R^2} \quad (K = \pm 1, 0, \quad \alpha^2 = 2G_5 M)$$

⋆ For $\alpha^2 = 0$ $(K = 1)$,

$$ds^2 = dr^2 + (H\ell)^2 \sinh^2(r/\ell)[-dt^2 + \cosh^2(Ht)d\Omega_{(3)}^2]$$

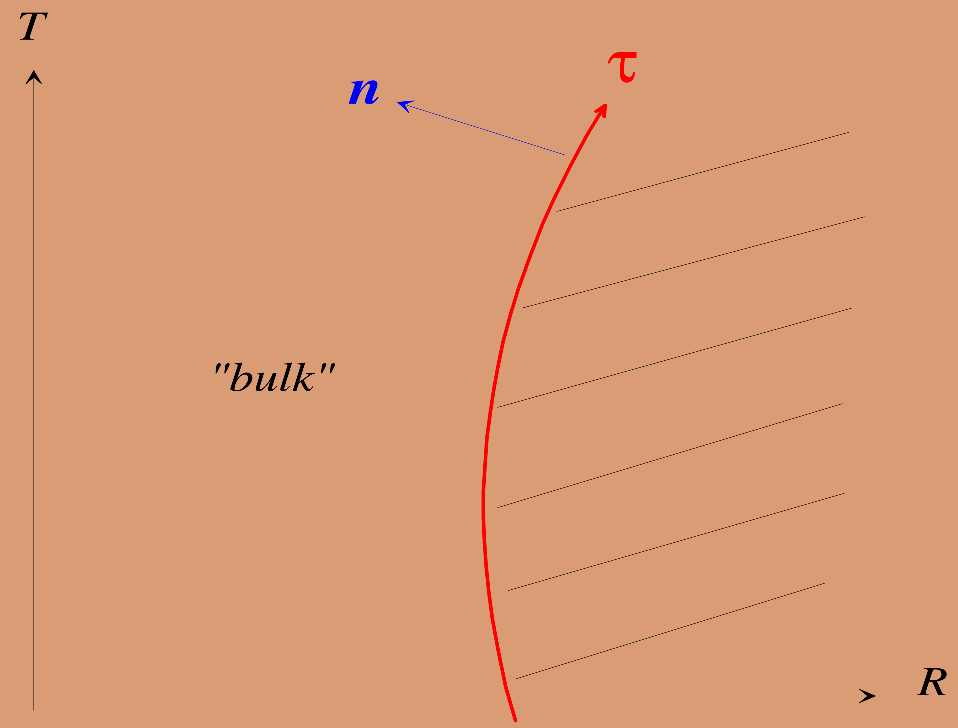
$$\begin{pmatrix} R = \ell \sinh(r/\ell)\cosh(Ht) \\ T = \ell \arctan\left(\tanh(r/\ell)\sinh(Ht)\right) \end{pmatrix} \quad H \text{ is arbitrary.}$$

Any r =const. timelike hypersurface is 4D de Sitter space.

· Pure de Sitter brane at $r = r_0$ (with Z_2 symmetry): $(T_{\mu\nu} = -\sigma\, g_{\mu\nu})$

$$\sigma = \frac{3}{4\pi G_5 \ell}\coth(r_0/\ell) \equiv \frac{3}{4\pi G_5 \ell_\sigma}, \qquad H^2\ell^2 = \frac{1}{\sinh^2(r_0/\ell)} = \frac{\ell^2}{\ell_\sigma^2} - 1$$

⋆ Deviates from de Sitter if $T_{\mu\nu}$ is non-trivial:



brane trajectory :

$$\begin{cases} R = R(\tau) \\ T = T(\tau) \end{cases}$$

$$ds^2|_{\text{brane}} = \left(-A(R)\dot{T}^2 + \frac{\dot{R}^2}{A(R)} \right) d\tau^2 + R^2(\tau) d\Omega_K^2$$

· Choose τ to be Proper time on the Brane:

$$-A(R)\dot{T}^2 + \frac{\dot{R}^2}{A(R)} = -1$$

$$\Rightarrow \quad \boxed{A^2(R)\dot{T}^2 = \dot{R}^2 + A(R)}$$

· Junction condition under Z_2-symmetry:

$$[K_{\mu\nu}]^{+}_{-} = 2K_{\mu\nu}(+0) = -8\pi G_5[T_{\mu\nu} - (1/3)Tg_{\mu\nu}]$$

$$K_{\mu\nu} = \frac{1}{2}\mathcal{L}_{\boldsymbol{n}} q_{\mu\nu}\,; \quad n_a = (\dot{R}, -\dot{T}, 0, 0, 0)$$

$q_{\mu\nu} \cdots$ induced metric on the brane

$$T^{\mu}{}_{\nu} = \mathrm{diag}(-\rho, p, p, p) - \sigma_c\, \delta^{\mu}{}_{\nu}\,; \quad \sigma_c = \frac{3}{4\pi G_5 \ell}$$

$$\Rightarrow \quad \boxed{\frac{A(R)}{R}\dot{T} = \frac{4\pi G_5}{3}(\rho + \sigma_c) = \frac{4\pi G_5}{3}\rho + \frac{1}{\ell}}$$

$G_4 = G_5/\ell$ $\cdots$ 4D Newton const.

$$\Downarrow \quad A(R) = K + R^2\ell^2 - \frac{\alpha^2}{R^2}$$

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{8\pi G_4}{3}\rho + \ell^2\left(\frac{4\pi G_4}{3}\rho\right)^2 + \frac{\alpha^2}{R^4}$$

- Friedmann equation on the brane:

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{8\pi G_4}{3}\rho + \ell^2\left(\frac{4\pi G_4}{3}\rho\right)^2 + \frac{\alpha^2}{R^4}$$

⋆ presence of $\propto \rho^2$ and α^2/R^4 terms.

· ρ^2-term dominates in the early universe: $H \propto \rho$.

For $\rho \propto R^{-4}$, $K = \alpha^2 = 0$; $\quad R \propto \left(t + \frac{2t^2}{\ell}\right)^{1/4}$

· reduces to standard Friedmann equation for $\ell^2 G_4 \rho \ll 1$.

$(\ell^2 G_4 \rho \ll 1 \;\Leftrightarrow\; \rho \ll \sigma_c)$

BBN constraint: $\sigma_c \gtrsim (100\,\mathrm{MeV})^4$ $\;(\Leftrightarrow \ell \lesssim 10^6\,\mathrm{cm})$

· α^2/R^4-term: "dark radiation" $\quad \alpha^2 = 2G_5 M \sim$ BH mass

$$\frac{\alpha^2}{R^4} = -\frac{E_{tt}}{3}\,; \quad E_{\mu\nu} = C^{(5)}_{\mu a \nu b}\, n^a n^b \quad \text{(5D Weyl contribution)}$$

Hawking radiation from BH? $\cdots$ AdS/CFT

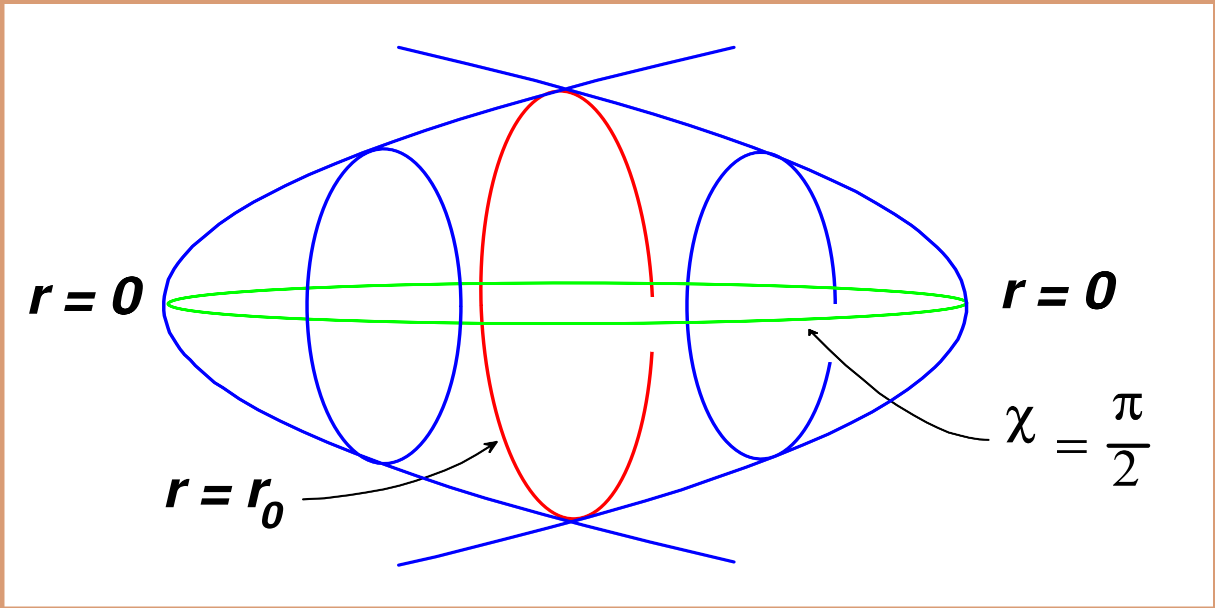
(T. Tanaka, gr-qc/0203082)

§4. Quantum Brane Cosmology

- Euclidean AdS: H^5 ($O(5,1)$-symmetric)

$$ds^2 = dr^2 + \ell^2 \sinh^2(r/\ell)(d\chi^2 + \sin^2\chi \, d\Omega_{(3)}^2)$$

⋆ Brane at $r = r_0$ (with Z_2-symmetry) = de Sitter-brane instanton



topology $\sim S^5$

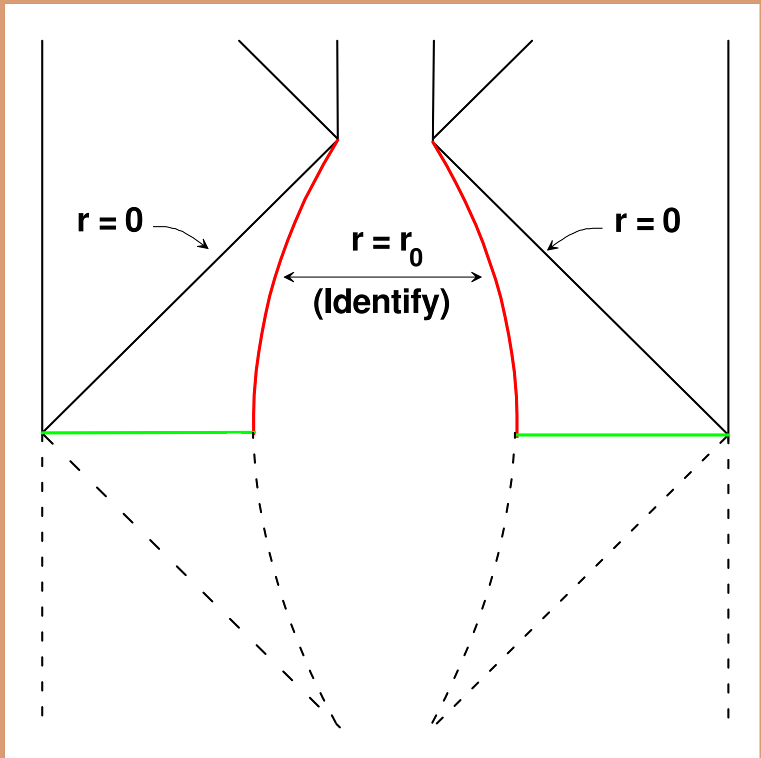
$\chi = \pi/2$: totally geodesic 4-surface

- Creation of inflating brane-world

Garriga & MS ('00); Koyama & Soda ('00)

Analytic continuation: $\chi \to iHt + \pi/2$

$$ds^2 = dr^2 + (H\ell)^2 \sinh^2(r/\ell)(-dt^2 + H^{-2}\cosh^2 Ht\, d\Omega_{(3)}^2) \quad (r \leq r_0)$$



Spatially Compact 5D Universe

(RS flat brane is recovered in the limit $r_0 \to \infty$)

§5. 4D Graviton and Kaluza-Klein Excitations

- Gravitational perturbation of de Sitter brane universe:

$$\begin{aligned} ds^2 &= dr^2 + (H\ell)^2 \sinh^2(r/\ell) ds^2_{\mathrm{dS}_4} + h_{ab}dx^a dx^b \\ &= b^2(\eta)\left(d\eta^2 + H^2 ds^2_{\mathrm{dS}_4}\right) + h_{ab}dx^a dx^b \end{aligned}$$

· $dr = b(r) d\eta$ (conformal radial coordinate):

$$b(r) = \ell \sinh(r/\ell) \quad \Rightarrow \quad b(\eta) = \frac{\ell}{\sinh(|\eta| + \eta_0)}$$

$$\left(\sinh \eta_0 = \frac{1}{\sinh(r_0/\ell)}\,; \qquad -\infty < \eta < \infty\right)$$

· (generalized) Randall-Sundrum gauge:

$$h_{55} = h_{5\mu} = h^\mu{}_\mu = D_\mu h^{\mu\nu} = 0\,;$$

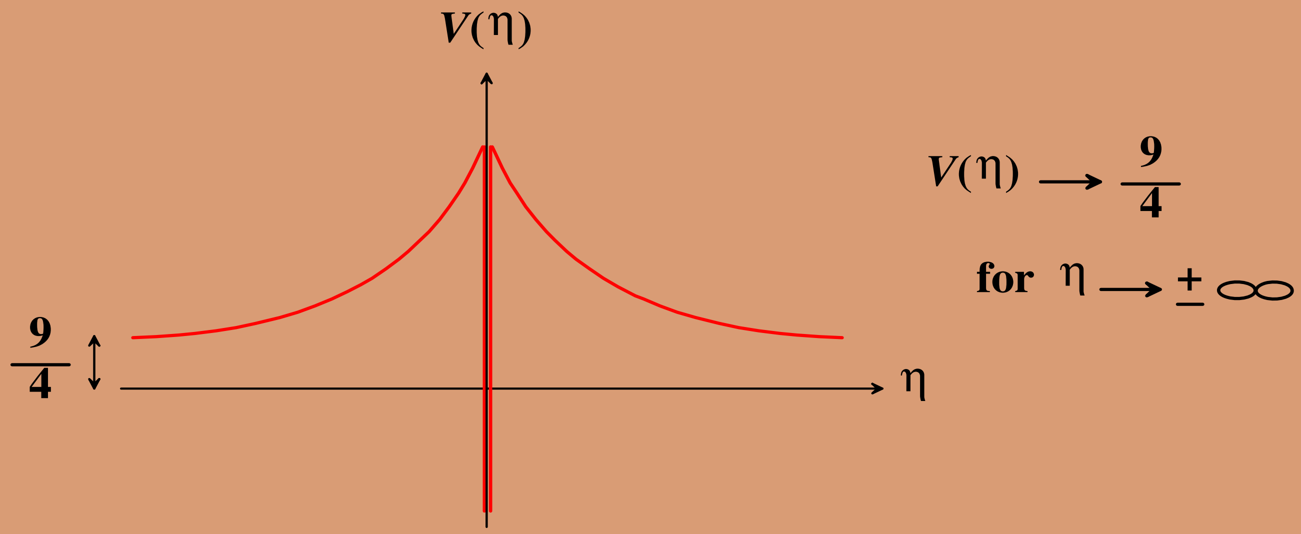
D_μ : 4D covariant derivative

- Perturbation equations:

$$h_{\mu\nu} = b^{1/2}\,\varphi(\eta) H_{\mu\nu}(x) \quad \Rightarrow \quad \begin{cases} -\varphi'' + \dfrac{(b^{3/2})''}{b^{3/2}}\varphi = \dfrac{m^2}{H^2}\varphi \\ (-\overset{(4)}{\Box} + 2H^2 + m^2) H_{\mu\nu} = 0 \end{cases}$$

⋆ "Volcano" potential for φ:

$$\boxed{V(\eta) \equiv \frac{(b^{3/2})''}{b^{3/2}} = \frac{15}{4\sinh^2(|\eta| + \eta_0)} + \frac{9}{4} - 3\coth\eta_0\delta(\eta)}$$

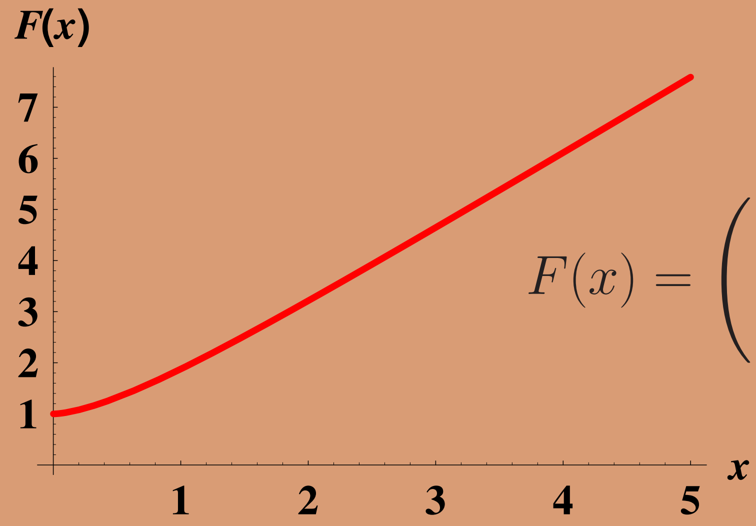


⋆ 4D graviton (zero mode $m = 0$): $\varphi \propto b^{3/2} \;\Rightarrow\; h_{\mu\nu} \propto b^2$

⋆ KK excitations ($m > 0$): $V \to (9/4) \;\Rightarrow\; m > (3/2)H$

· Zero mode is confined just like the case of the RS flat brane.
When quantized, however, the normalization (amplitude) is non-trivial:

$$\frac{P_{gw}(k)k^3}{G_4} \sim \left(\frac{H}{2\pi}\right)^2 F(H\ell) \qquad \text{(Langlois, Maartens \& Wands ('00))}$$



$$F(x) = \left(\sqrt{1+x^2} - x^2 \ln\left[\frac{1+\sqrt{1+x^2}}{x}\right]\right)^{-1}$$

– large enhancement at $H\ell \gg 1$: $F \sim \frac{3}{2}H\ell$.

· Mass gap in KK spectrum: $\Delta m = (3/2)H$

– The same is true for any bulk scalar field with $m \lesssim H$.

– No 'zero-mode' (bound state mode) for $m \gg H$.

§6. Brane-world Inflation Driven by a Bulk Scalar Field

Himemoto & MS ('00), Himemoto, Sago & MS ('01), $\cdots$

$\star$ Randall-Sundrum's "default" parameters:

$$\text{brane tension:} \quad \sigma_c = \frac{3}{4\pi G_5 \ell}\,; \quad \ell = \left|\frac{6}{\Lambda_5}\right|^{1/2}.$$

If $|\sigma| > \sigma_c$, then inflation occurs on the brane:

$$H^2 = \frac{1}{\ell_\sigma^2} - \frac{1}{\ell^2} = \frac{1}{\ell_\sigma^2} - \frac{|\Lambda_5|}{6}\,; \quad \sigma \equiv \frac{3}{4\pi G_5 \ell_\sigma}$$

If $|\sigma| = \sigma_c$ but $|\Lambda_{5,eff}| < |\Lambda_5|$, inflation also occurs on the brane:

$$H^2 = \frac{|\Lambda_5 - \Lambda_{5,eff}|}{6}$$

$$\Downarrow$$

Brane-world inflation can be driven solely by bulk (gravitational) fields.

cf. Hawking, Hertog & Reall ('01) , Nojiri, Odintsov & Osetrin ('01), $\cdots$

- Toy Model

$$L_5 = \frac{1}{16\pi G_5} R - \frac{1}{2} g^{ab} \partial_a \phi \partial_b \phi - U(\phi)$$

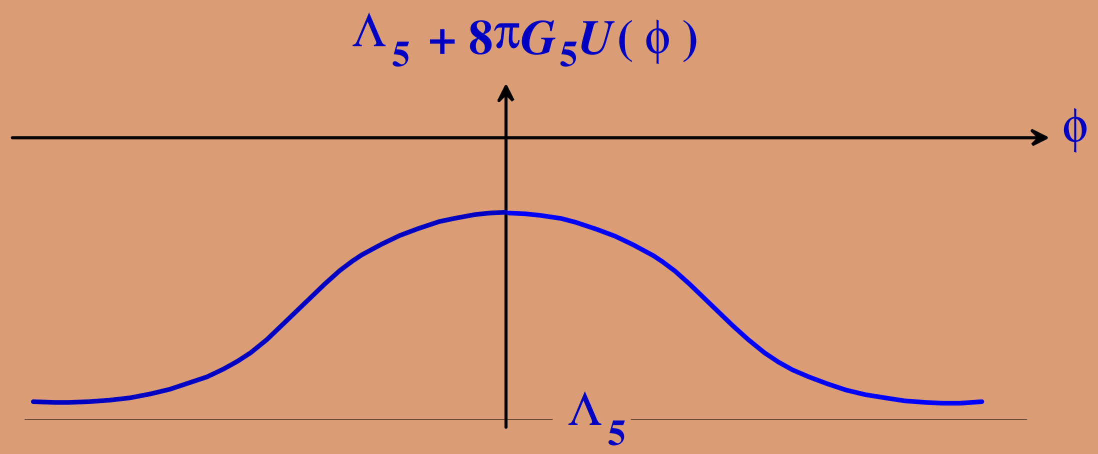
$\sim$ a conformally transformed scalar-tensor gravity

If ϕ varies very slowly,

$$|\Lambda_{5,eff}| = |\Lambda_5 + 8\pi G_5 U(\phi)| \; < |\Lambda_5|,$$

$$H^2 = \frac{4\pi G_5 U(\phi)}{3} = \frac{8\pi G_4}{3} U_4 \,; \quad G_5 = G_4 \ell, \quad U_4 = \frac{\ell}{2} U(\phi).$$

(ℓ is arbitrary here.)



- Friedmann equation in the presence of a bulk scalar:

$$\left(\frac{\dot a}{a}\right)^2 + \frac{K}{a^2} \equiv H^2 + \frac{K}{a^2} = \frac{8\pi G_4}{3}\tilde\rho_\phi + \frac{1}{3}E^t{}_t\,;$$

$$\tilde\rho_\phi = \ell\left(\frac{1}{4}\dot\phi^2 + \frac{1}{2}U(\phi)\right)\,, \quad E^t{}_t = \overset{(5)}{C}{}^t{}_{rtr}\,.$$

From Bianchi Ids. on the brane and ϕ equation in the bulk,

$$E^t{}_t = -\frac{4\pi G_5}{a^4}\int^t a^4\dot\phi(\partial_r^2\phi + \frac{\dot a}{a}\dot\phi)\,dt$$

$$= 2\pi G_5\dot\phi^2 + \frac{C}{a^4}\,, \qquad \text{if} \quad \ddot\phi + 3H\dot\phi + \frac{1}{2}\partial_\phi U = 0 \text{ on the brane.}$$

$$\Rightarrow \quad \rho_{\text{eff}} = \tilde\rho_\phi + \frac{E^t{}_t}{8\pi G_4} = \ell\left(\frac{1}{2}\dot\phi^2 + \frac{1}{2}U(\phi)\right) + \frac{C}{a^4}\,.$$

⋆ 5D scalar ϕ behaves like a 4D scalar $\Phi = \sqrt{\ell}\,\phi$ with

$$U_{\text{eff}}(\Phi) = \frac{\ell}{2}U(\Phi/\sqrt{\ell})$$

When and in what situations will this be true?

- "Zero mode" and KK modes $\left(U = \frac{1}{2}M^2\phi^2\right)$

⋆ For de Sitter brane at $r = r_0$,

$$\Phi(r, x^\mu) = u_0(r)\phi_0(x^\mu) + \int_{3/2}^{\infty} d\lambda\, u_\lambda(r)\phi_\lambda(x^\mu)$$

ϕ_0 : "zero mode" (bound state mode)

ϕ_λ : Kaluza-Klein modes $M_\lambda^2 = \lambda^2 H^2 \quad (\lambda > 3/2)$

Effective 4d mass of zero mode when $M^2 \lesssim H^2$:

$$M_0^2 = \begin{cases} M^2/2 & \text{for } H^2\ell^2 \ll 1 \\ 3M^2/5 & \text{for } H^2\ell^2 \gg 1 \end{cases}$$

⋆ No bound state when $M^2 > H^2$.

(But there is a quasi-normal mode with $M_0 = M/\sqrt{2} - i\Gamma$)

· Zero-mode dominance ⇔ consistent with the effective potential picture.

· KK modes are important when $H\ell \gg 1$

(⇔ gravity zero-mode is non-trivial when $H\ell \gg 1$).

Quantum fluctuations of KK modes need be evaluated.

- Case of a scalar coupled to brane tension   Langlois and MS, in prep.

$$S=\int d^5x\sqrt{-g}\left(\frac{1}{16\pi G_5}R-\frac{1}{2}(\nabla\phi)^2-V_5(\phi)\right)-\int d^4x\sqrt{-q}\,\sigma(\phi)$$

$$8\pi G_5 V_5(0)=\Lambda_5=-6\,\ell^2$$

⋆ Friedmann equation on the brane:

$$H^2=\frac{8\pi G_5}{3}\left(\frac{1}{4}\dot{\phi}^2+\frac{1}{2}V_5+\frac{8\pi G_5}{12}\sigma^2-\frac{1}{16}\left(\frac{\partial\sigma}{\partial\phi}\right)^2\right)+\frac{E^0{}_0}{3}$$

If $\quad {}^{\exists}V_{\rm eff}\quad$ s.t. $\quad \ddot{\Phi}+3H\dot{\Phi}+V'_{\rm eff}=-J\,,\quad \Phi\equiv\sqrt{\ell}\,\phi$

$$\Rightarrow\quad\begin{cases} H^2=\dfrac{8\pi G_4}{3}\left[\dfrac{1}{2}\dot{\Phi}^2+V_{\rm eff}(\Phi)+\rho_{\mathcal{E}}\right]\,, \\ \dot{\rho}_{\mathcal{E}}+4H\rho_{\mathcal{E}}=J\dot{\Phi}\,;\quad V_{\rm eff}=\dfrac{1}{2}V_5+\dfrac{8\pi G_5}{12}\sigma^2-\dfrac{1}{16}\left(\dfrac{\partial\sigma}{\partial\phi}\right)^2\,, \end{cases}$$

where $G_4=G_5/\ell$ and $E^0{}_0=8\pi G_4\left(\rho_{\mathcal{E}}+\frac{1}{4}\dot{\Phi}^2\right)$.

Does this hold whenever $H^2\ell^2\ll1$? AdS/CFT?

§7. Large-scale Cosmological Perturbations on the Brane

- General formalism ⇒ Kodama et al., Langlois, Mukohyama, $\cdots$.
- Essentially a 5-dimensional, PDE problem.
- However, some simplifications on super-horizon scales.
  Langlois, Maartens, MS & Wands ('01)

- Basic equations (in AdS$_5$ bulk background; no bulk scalar)

$$G_{\mu\nu} + \Lambda_4\, q_{\mu\nu} = 8\pi G_4\, T_{\mu\nu} + (8\pi G_5)^2\, \Pi_{\mu\nu} - E_{\mu\nu}$$
$$\equiv 8\pi G_4\, T^{\rm tot}_{\mu\nu} \quad (\Pi_{\mu\nu} \sim \text{ quadratic in } T_{\mu\nu})$$

"$-E_{\mu\nu}$" : **Weyl fluid** (or "dark radiation")

For $T_{\mu\nu} = \rho\, u_\mu u_\nu + P\, h_{\mu\nu} + \pi_{\mu\nu}$ ($\pi_{\mu\nu}$: anisotropic stress $= O(\epsilon)$)

$$u^\mu D^\nu E_{\mu\nu} = O(\epsilon^2)$$

Weyl fluid is decoupled on superhorizon scales.

standard 4-d theory is applicable ($E^0{}_0 = 8\pi G_4\, \rho_{\mathcal{E}}$, etc.)

- Large angle CMB anisotropy

$$\left(\frac{\delta T}{T}\right)_{sw}(\vec{\gamma},\eta_0) = (\zeta_{\rm r}+\Theta)\,(\eta_{\rm dec},\vec{x}(\eta_{\rm dec})) + \int_{\eta_{\rm dec}}^{\eta_0} d\eta\,\partial_\eta\Theta(\eta,\vec{x}(\eta)).$$

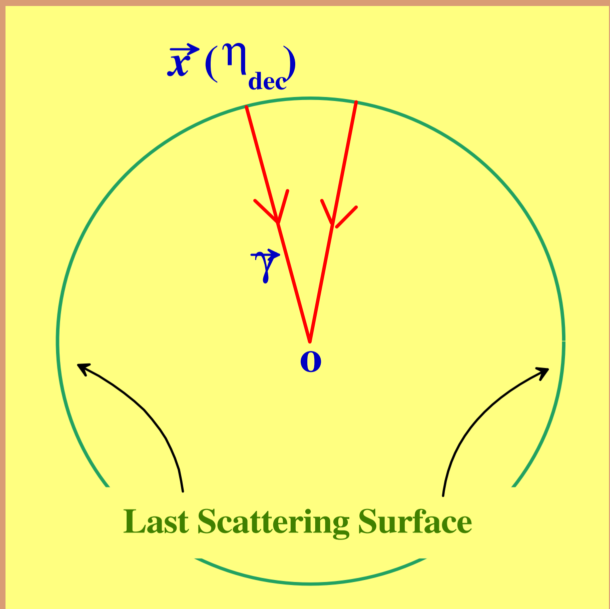
(Sachs-Wolfe) (Integrated Sachs-Wolfe)

$\zeta_{\rm r} \sim$ curvature perturbation on $\rho_{\rm photon} =$ const. surfaces

$\Theta = \Psi - \Phi$

$\Psi \sim$ Newton potential

$\Phi \sim$ curvature perturbation in Newton gauge



For a dust-dominated universe at decoupling,

$$\text{SW:} \quad \zeta_{\rm r} + \Theta = -\frac{1}{5}\zeta_* - \frac{2}{5}S_{\rm dr} - \frac{8}{3}\frac{\rho_{\rm r}}{\rho_{\rm d}}S_{\mathcal{E}} - 8\pi G_4 a^2\,\delta\pi_{\rm tot} + \frac{16\pi G_4}{a^{5/2}}\int_0^a \delta\pi_{\rm tot}\,a^{7/2}da$$

$$\text{ISW:} \quad \partial_\eta\Theta = -\partial_\eta\left[\frac{8}{3}\frac{\rho_{\rm r}}{\rho_{\rm d}}S_{\mathcal{E}} + 8\pi G_4 a^2\,\delta\pi_{\rm tot} - \frac{16\pi G_4}{a^{5/2}}\int_0^a \delta\pi_{\rm tot}\,a^{7/2}da\right]$$

where ζ_* is the adiabatic curvature perturbation,

$$S_{\mathcal{E}} := \frac{\delta\rho_{\mathcal{E}}}{4\rho_{\rm r}} - \frac{\rho_{\mathcal{E}}\,\delta\rho}{3\rho_{\rm r}(\rho+P)} \quad \sim \text{ Weyl entropy perturbation}$$

$$S_{\rm dr} := \frac{\delta\rho_{\rm d}}{\rho_{\rm d}} - \frac{3}{4}\frac{\delta\rho_{\rm r}}{\rho_{\rm r}} \quad \sim \text{ standard entropy perturbation}$$

$$\delta\pi_{\rm tot} = \left(1 - \frac{\rho+3P}{2\sigma}\right)\delta\pi + \delta\pi_{\mathcal{E}} \quad \sim \text{ anisotropic stress}$$

$\delta\pi_{\mathcal{E}}$: traceless part of Weyl fluid $(E^i{}_j - \frac{1}{3}\delta^i_j E^k{}_k)$

$\delta\pi_{\mathcal{E}}$ cannot be determined within this 4-d approach

§8. Summary

Brane-world gives a new picture of the universe

Can we find cosmological evidence?

- **Quantum brane cosmology**

⋆ Spatially compact 5D Universe created from nothing

· Well-posed initial value problem

⋆ 4D Universe created in de Sitter (inflationary) phase

· Non-trivial quantum fluctuations if $H\ell \gg 1$

· Effects of KK modes need to be investigated.

⋆ Inflation without inflaton on the brane

· Inflation as a result of 5D gravitational dynamics

⋆ Mass gap ($\Delta m = (3/2)H$) in the KK spectrum

· Isolation of the zero mode ⇔ Stability of the brane world

• Evolution of a brane universe

⋆ Presence of ρ^2 term in Friedmann equation

· Modified evolution when $\ell^2 G_4 \rho \gtrsim 1$

⋆ Weyl fluid term (dark radiation) in Friedmann equation

· Effect of 5D bulk gravity

⋆ Large scale perturbation can be solved without 5D equations.

· 5D effect is encoded in CMB through Weyl anisotropy.

• Some issues on brane-world cosmology

⋆ Search for a natural brane-world inflation model

related to cosmological constant problem ?

⋆ Quantitative analysis of cosmological perturbations

5D dynamics of "$E_{\mu\nu}$" (poster by Minamitsuji)

⋆ Analysis of the two-brane cosmological model (radion dynamics)

"born-again braneworld" (talk by Kanno)

Need a good approximation method,
e.g., Kanno-Soda's (Shiromizu-Koyama's) approach