One-Armed Spiral Instability in Differentially Rotating Stars

Motoyuki Saijo (Kyoto University)

MS, Baumgarte, Shapiro, Astrophysical J 595 (2003) in press
(astro-ph/0302436)

CONTENTS

1. Introduction

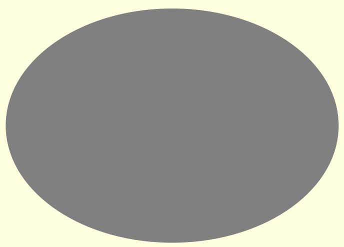
Rotating Configuration

$\beta \equiv T/W$

T: Rotational Kinetic Energy
W: Gravitational Binding Energy

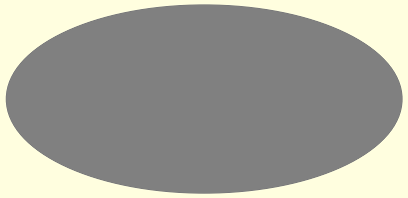
Maclaulin Spheroid

Uniformly rotating incompressible stars

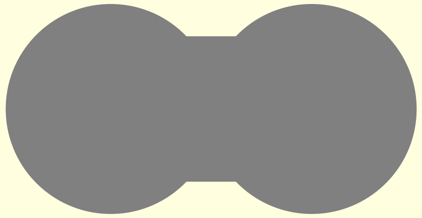


Jacobi Ellipsoid

Ellipsoid with unequal axis, incompressible stars



Bar-like Configurations



Instability ➡ Candidate of GW sources

Secular Instability

$\beta_{\rm sec} \simeq 0.14 \qquad \tau_{\rm sec} \sim \tau_{\rm vis}$ or $\tau_{\rm GW}$

Mass-shedding Limit

Dynamical Instability

$\beta_{\rm dyn} \simeq 0.27 \qquad \tau_{\rm dyn} \sim (G\bar{\rho})^{-1/2} \ll \tau_{\rm sec}$

Differential rotation release the limit

$$m R \Omega_{\rm eq}^2 \lesssim \frac{Mm}{R^2}$$

Centrifugal force   Gravity

β

Effects which enhance the onset of dynamical instability

- **High degree of differential rotation**

  Tohline, Hachisu (1990); Pickett, Durisen, Davis (1996); Shibata, Karino, Eriguchi (2002)

  $\Omega_c/\Omega_{eq} \gtrsim 10$ $\quad$ Ω_c : Central angular velocity

  $T/|W| \lesssim 0.20$ $\quad$ Ω_{eq} : Equatorial surface angular velocity

- **Strong relativistic gravitation**

  Shibata, Baumgarte, Shapiro (2000); MS, Shibata, Baumgarte, Shapiro (2001)

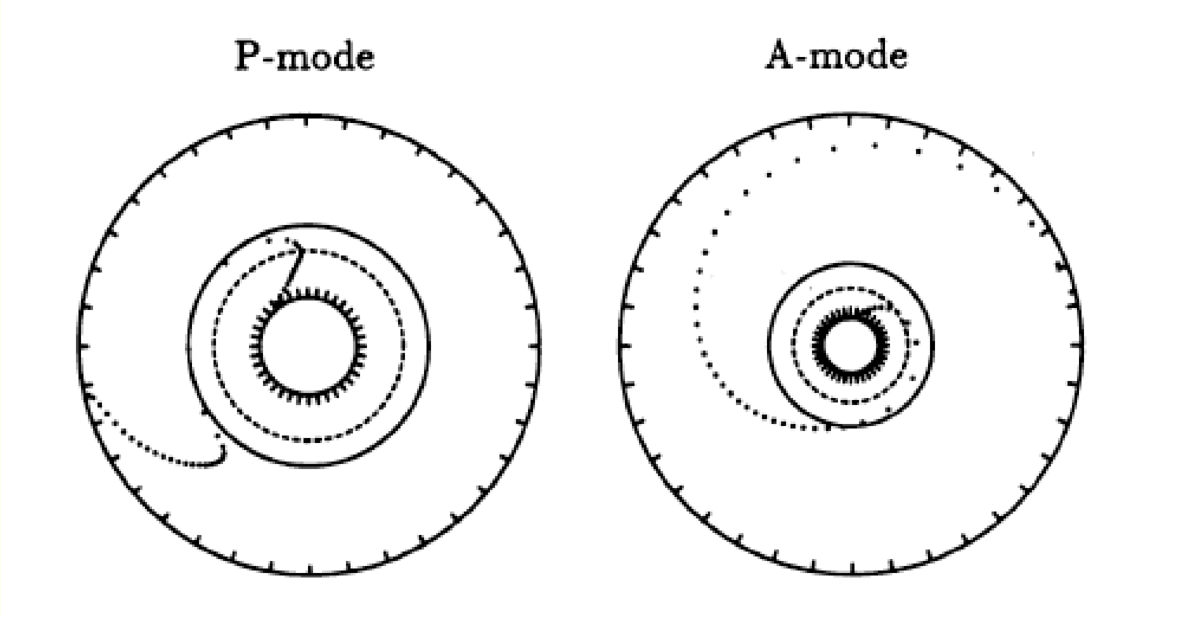
  $$\Omega_c/\Omega_{eq} \sim 3$$

  $$T/|W| \sim 0.24 - 0.26$$

m=1 instability in protostar system

- **Woodward, Tohline, Hachisu (1994)**

  Nonlinear stability analysis of accretion system
  (Point mass + thick self-gravitating disk)



As increasing T/W, disk first becomes m=1 unstable resembles the "eccentric instability" and further increasing T/W resembles the Papaloziou-Pringle instability

- Picket, Durisen, Davis (1996)

  Discovered the m=1 instability in n=3/2 polytropic star with strong concentration of the angular momentum in the envelope region of the star

Dynamical One-Armed Spiral Instability (m=1)

Centrella et al. (2001)

・Discovered the m=1 instability in toroidal stars
・Requires soft equation of state (n=3.33) and high degree of differential rotation ($\Omega_{\rm c}/\Omega_{\rm eq} = 26$)

➡ Dense torus causes the excitation of m=1 instability

Purpose

・Identify necessary conditions for triggering the m=1 dynamical instability in stellar system.
・Discover the nature of the m=1 dynamical instability by comparing with that of the m=2 bar mode instability.
・Find an effect of the m=1 dynamical instability on gravitational waves.

2. Newtonian Hydrodynamics

Features of our Newtonian hydrodynamics code

- Newtonian hydrodynamics with artificial viscosity
- Equatorial plane symmetry
- Adiabatic evolution

Continuity Equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v^i)}{\partial x^i} = 0$$

Equation of State

$$P = (\Gamma - 1)\rho\varepsilon$$

Energy Equation

$$\frac{\partial e}{\partial t} + \frac{\partial (e v^j)}{\partial x^j} = -\frac{1}{\Gamma}(\rho\epsilon)^{-1+1/\Gamma} P_{\text{vis}} \frac{\partial v^i}{\partial x^i}$$

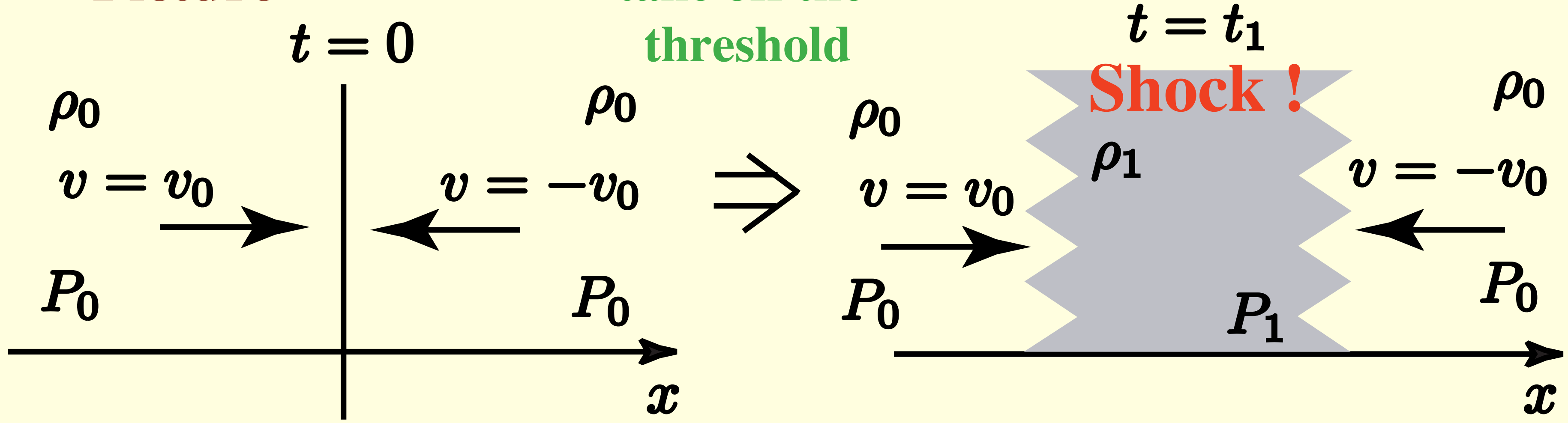
Euler Equation

$$\frac{\partial (\rho v_i)}{\partial t} + \frac{\partial (\rho v_i v^j)}{\partial x^j} = -\frac{\partial}{\partial x^i}(P + P_{\text{vis}}) - \rho \frac{\partial \Phi}{\partial x^i}$$

1D Newtonian Wall Shock Problem

Check the validity to treat shock (e.g. Hawley, Smarr, Wilson 1984)

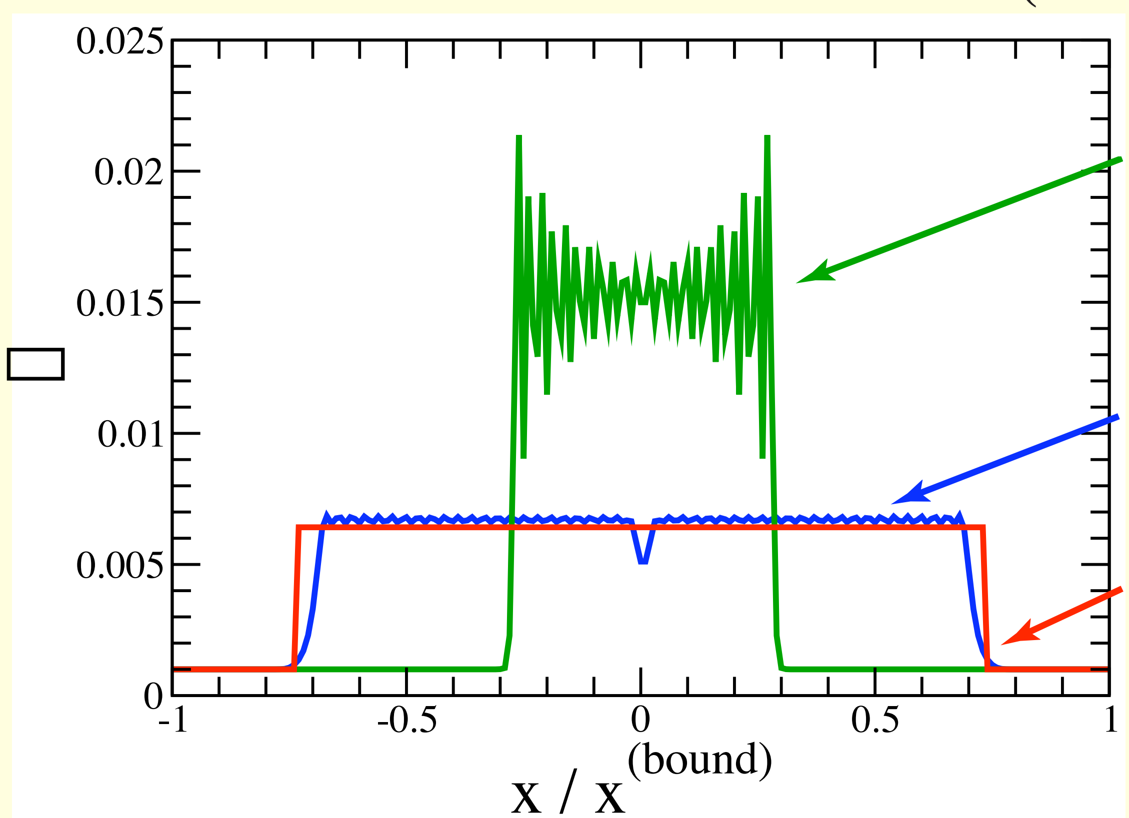
Picture



2 phases of the fluid collide at supersonic speed

Self similar solution
(Analytic solution exists!)

Comparison $t = 1.0$



Numerical solution without artificial viscosity

Numerical solution with artificial viscosity

Analytical solution

$$\rho_0 = 1.0 \times 10^{-3}$$
$$v_0 = 2 v_{\mathrm{sound}}$$
$$n = 3.33$$

Can reproduce the analytic solution with the numerical one about 5% of the maximum density.

3. Code Tests

1. Bar formation Test

Confirm the ability of our code to identify bar formation

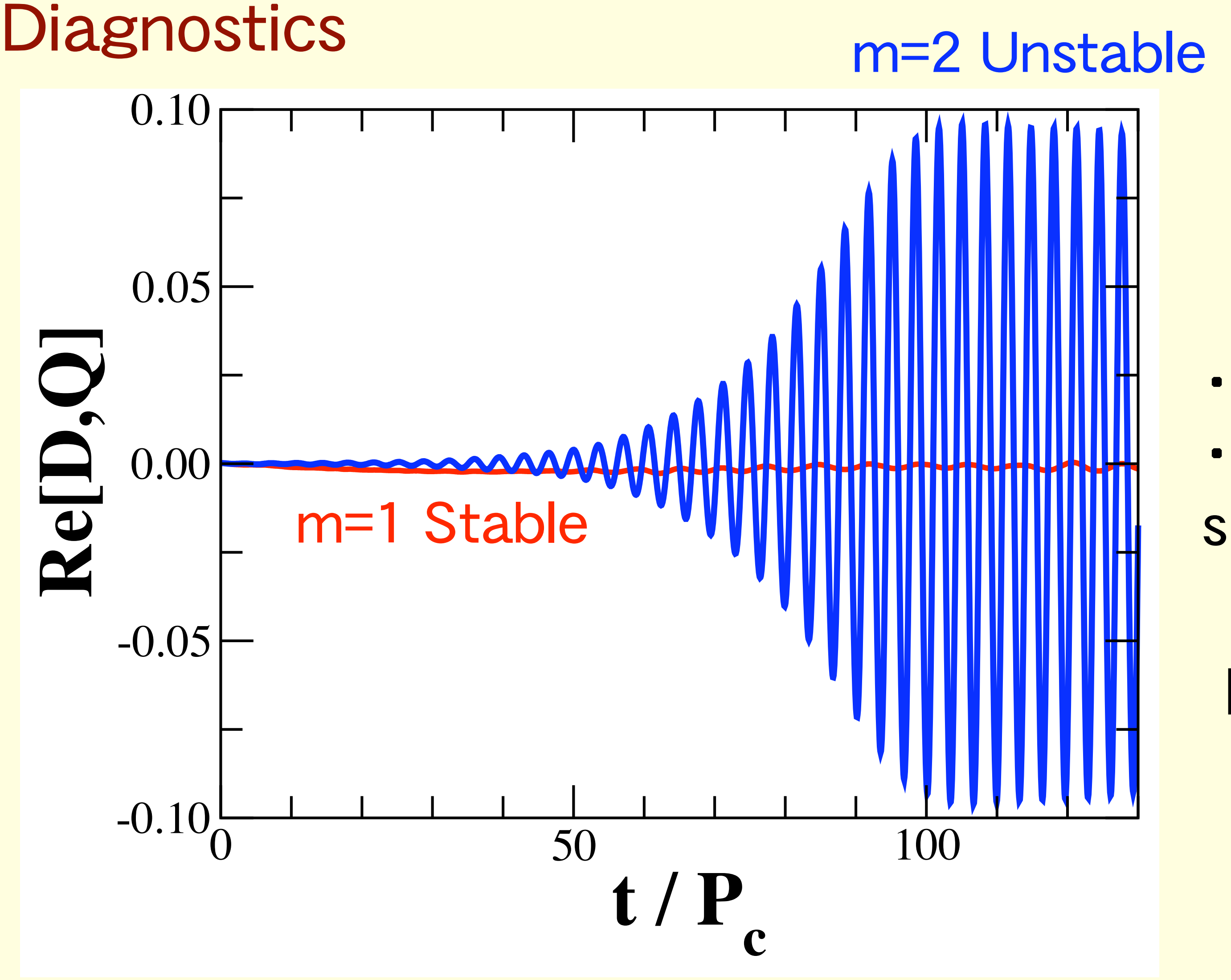
(Shibata, Karino, Eriguchi 2002)

Initial Condition

n	Ω_c/Ω_{eq}	T/W
1	26.0	0.119

To probe the stability, we put small m=1 and m=2 perturbation in the initial density from equilibrium.

Diagnostics



density weighted avarage

$$D = \left\langle e^{im\varphi} \right\rangle_{m=1}$$

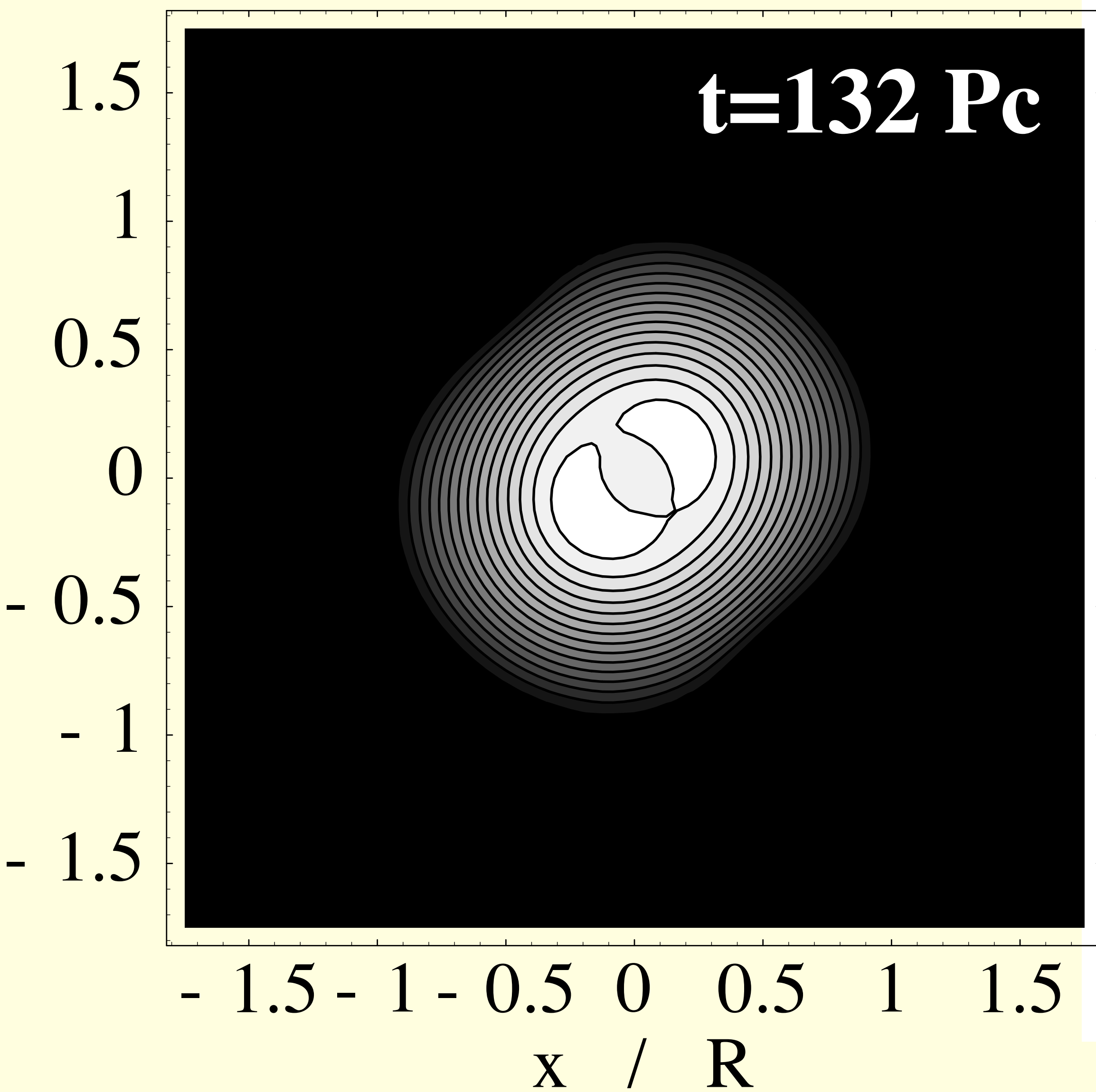
$$Q = \left\langle e^{im\varphi} \right\rangle_{m=2}$$

- Clear evidence for bar structure
- Bar persists without decay for over one surface rotation period.

➡ The amplitude survive without decay until gravitational radiation reaction forces destroy the bar

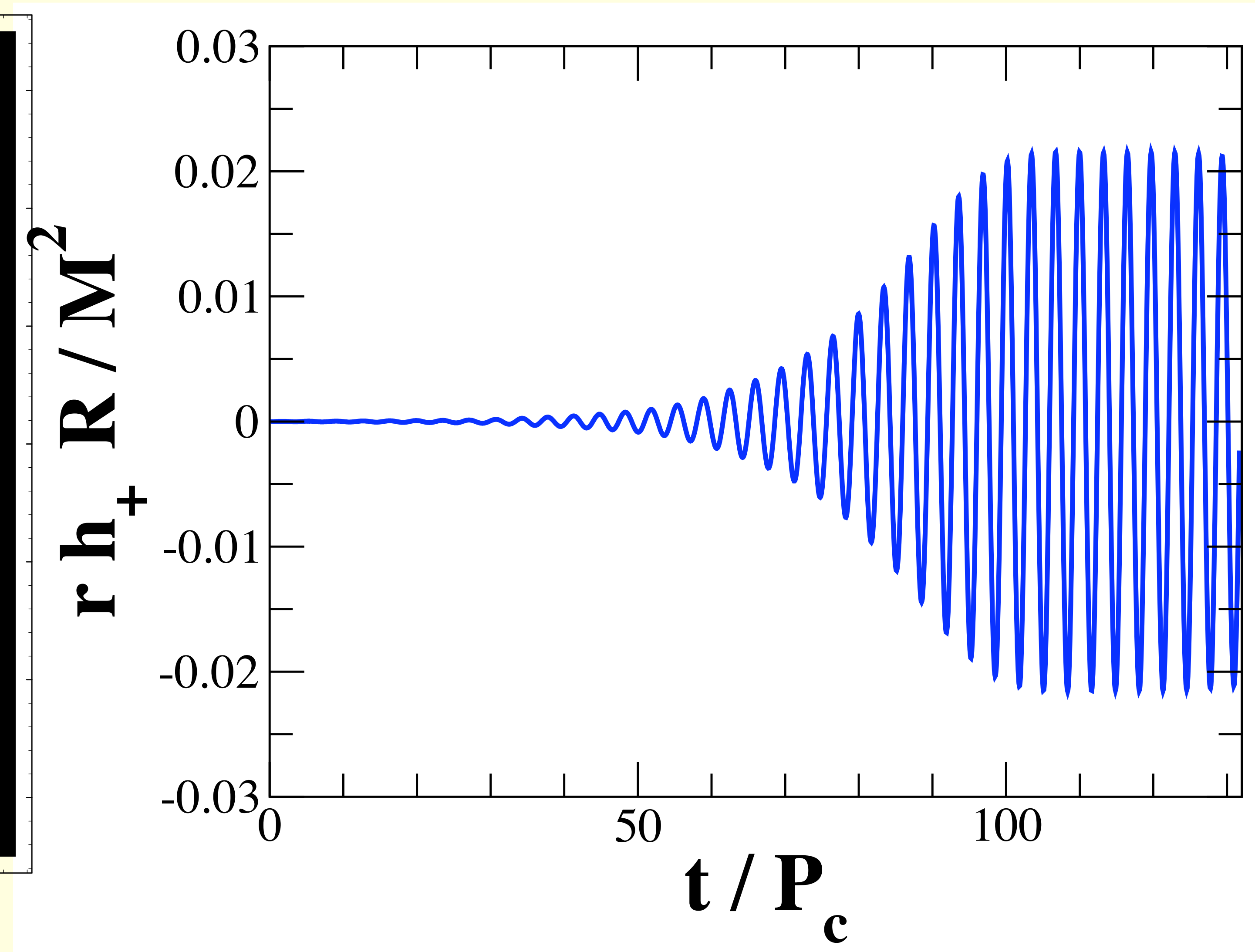
$$\sim (R/M)^{5/2} t_{\rm dyn} \gg t_{\rm dyn}$$

Final density snapshots in the equatorial plane



Clear evidence for bar structure

Gravitational Waveform using quadrupole formula



Observed from z-axes

Amplitude persists for at least over one surface rotation period

m=1 Dynamical Instability

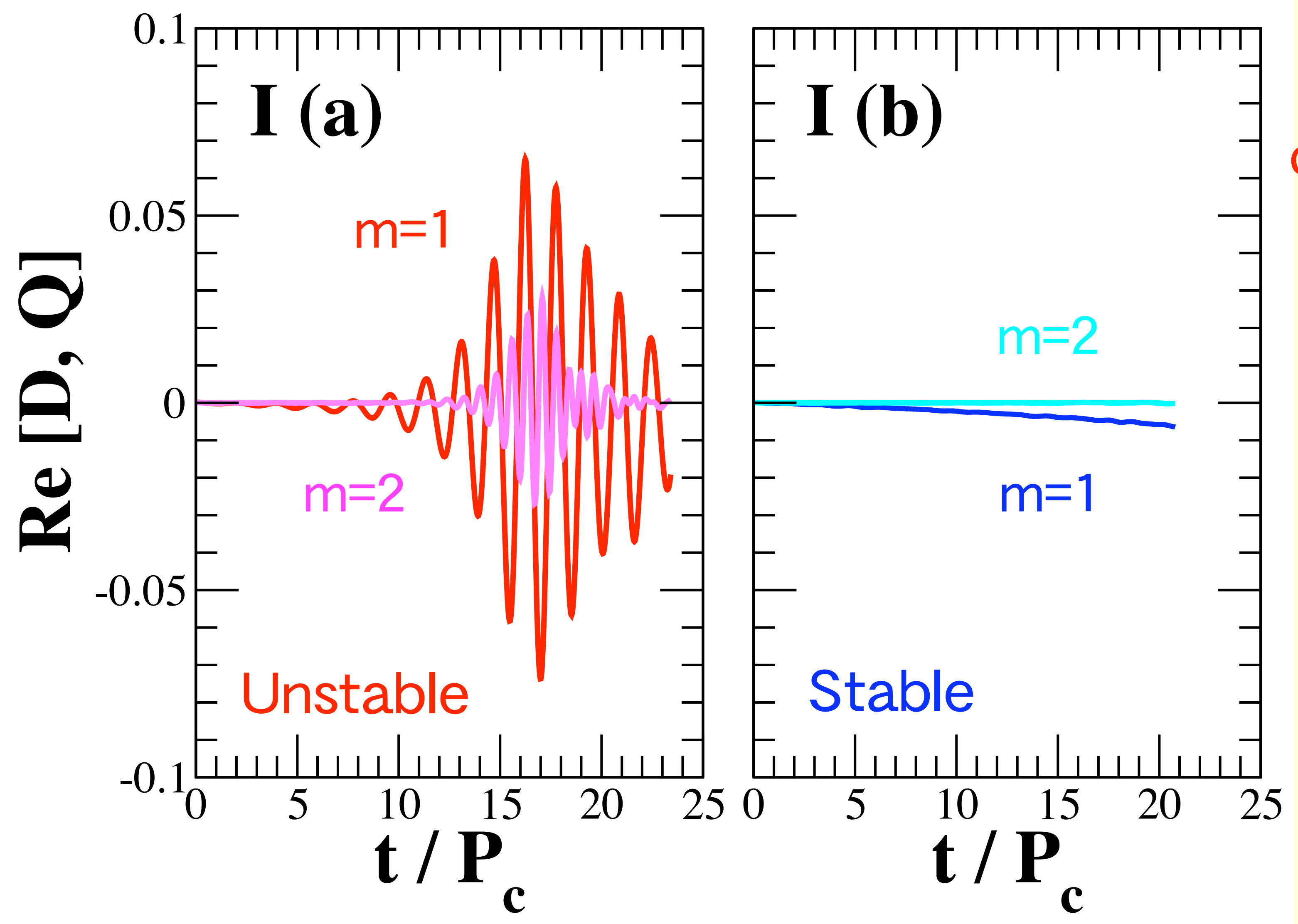
Confirm the ability of our code to identify the m=1 instability
(Centrella et al. 2001)

Initial Conditions (n=3.33, Ω_c/Ω_{eq}=26)

Model	T/W	structure
(a)	0.144	toroidal
(b)	0.090	spheroidal

To probe stability, we slightly put the m=1 and m=2 perturbation in the initial density from equilibrium.

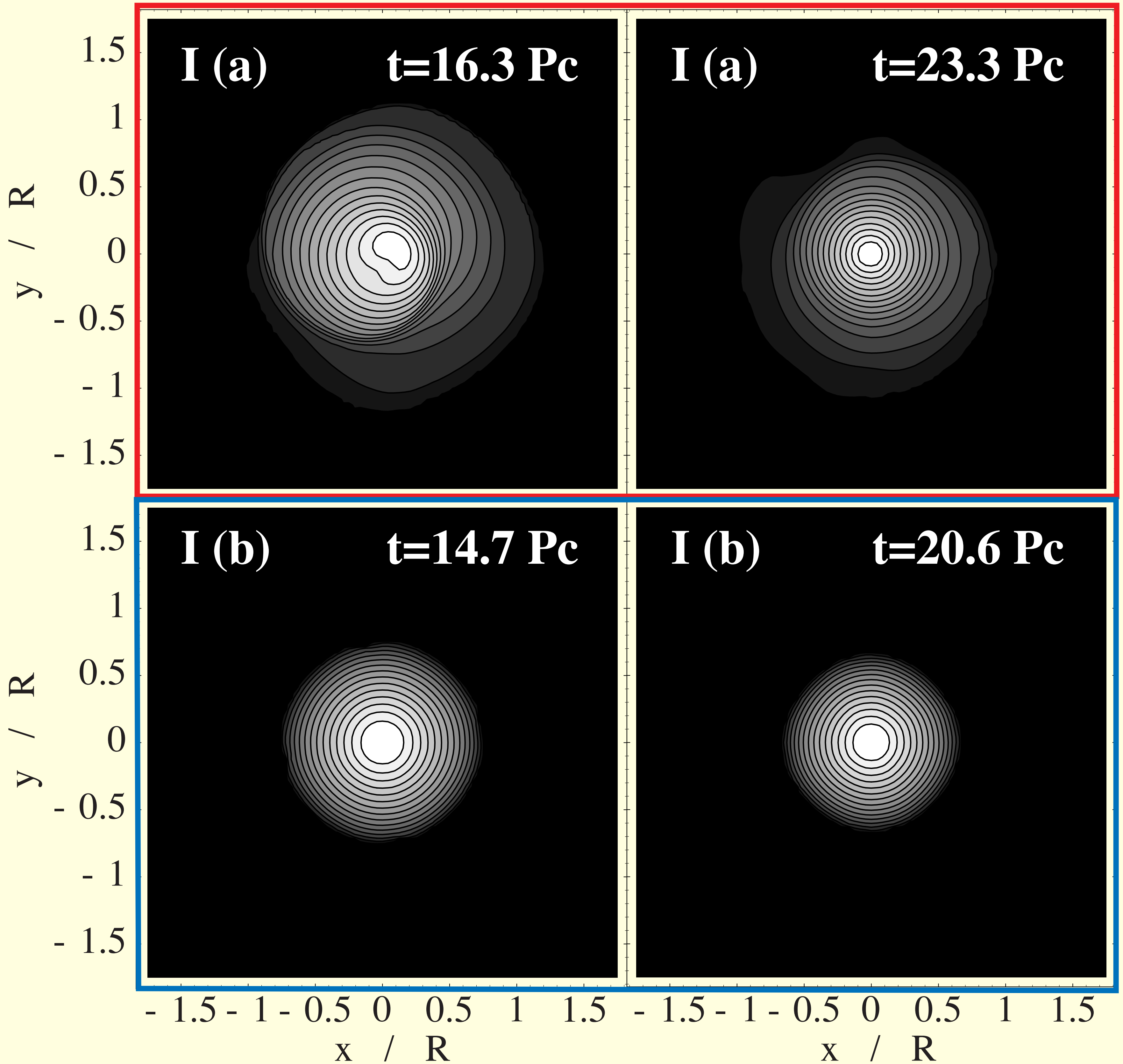
m=1 and m=2 diagnostics



・D and Q grows up exponentially during the evolution
・Small growth of an m=2 mode

D and Q remains oscillation around zero
(absence of exponential growth)

Density snapshots in the equatorial plane



- Clear single spiral arm at the intermediate stage
- Finally the spiral arm is destroyed

Instability rearranges the matter in the star, and, as a consequence, it eliminates the toroidal structure.

Remains equilibrium

Toroidal star triggers m=1 instability

4. One-Armed Spiral Instability

Centrally condensed protostellar disk systems are known to experience the m=1 instability

(Picket, Dursen, Davis 1996)

➡ This suggests that in differentially rotating stars

1. Softness of equation of state
2. High degree of differential rotation

might trigger the same m=1 instability

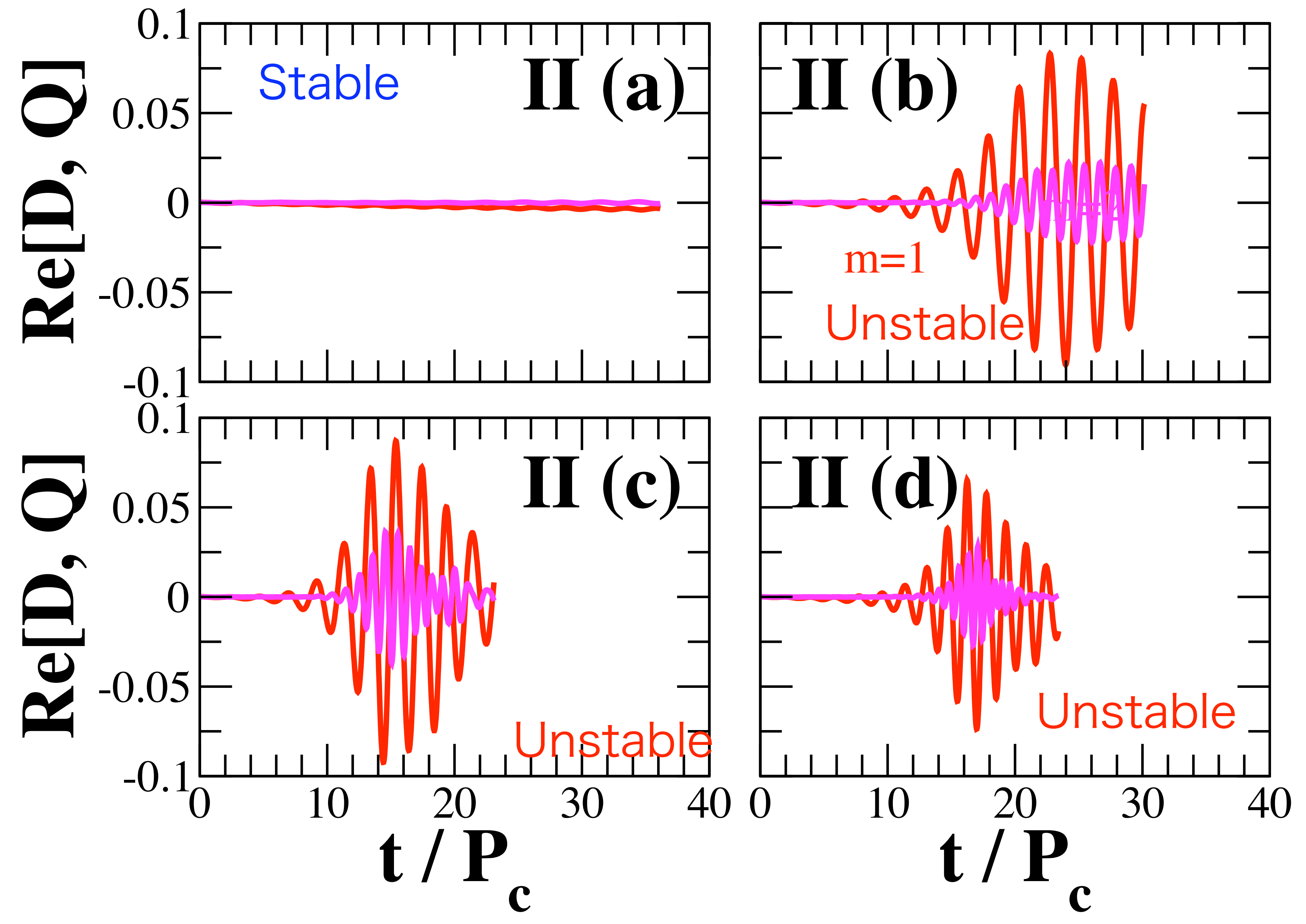
Dependence of m=1 instability on the polytropic index

Initial Conditions (toroidal stars)

Model	n	Ω_c/Ω_{eq}	T/W
(a)	1	26.0	0.145
(b)	2	26.0	0.145
(c)	3	26.0	0.147
(d)	3.33	26.0	0.144

To probe stability, we put a small m=1 and m=2 perturbation in the initial density from equilibrium.

Diagnostics



- D and Q grows up exponentially during the evolution
- Small growth of an m=2 mode

D and Q remains oscillation around zero (absence of exponential growth)

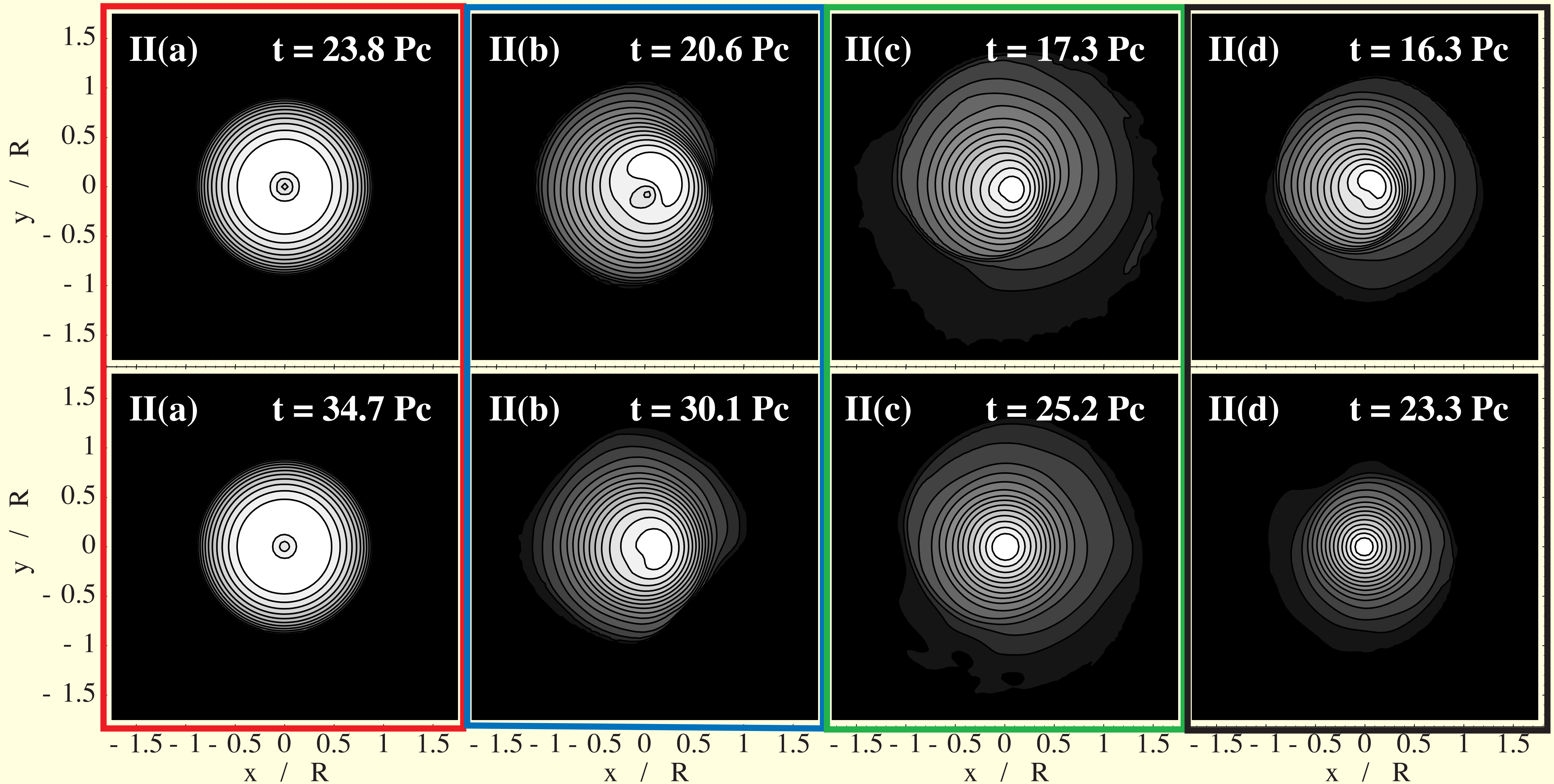
Pattern period of the diagnostics

Model	m=1[Pc]	m=2[Pc]	Pattern[Pc]
(b)	2.5	1.2	2.4
(c)	2.0	1.0	2.0
(d)	1.6	0.7	1.4

m=2 mode is regarded as a higher mode of m=1 due to the same pattern period.

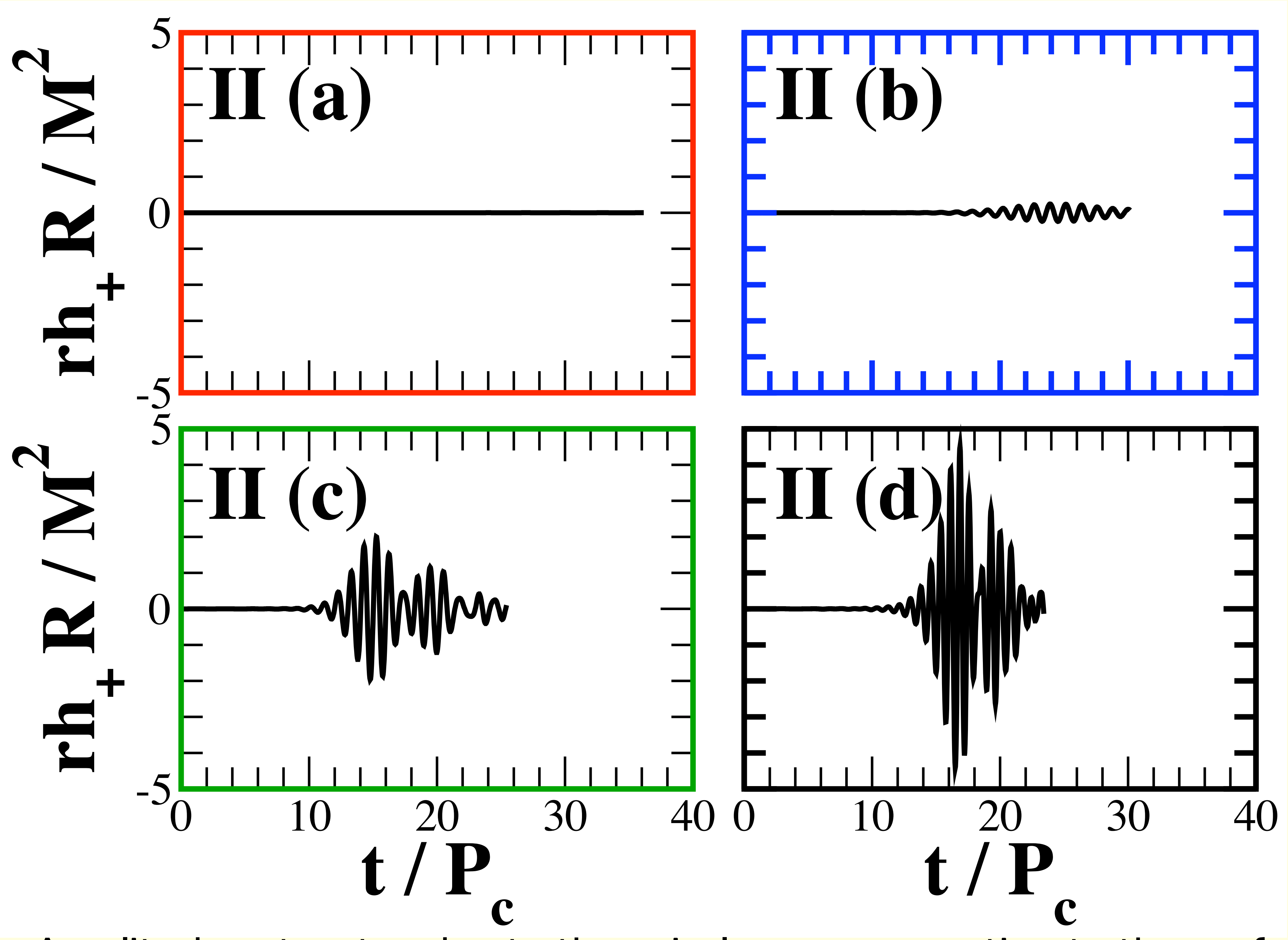
Density snapshots in the equatorial plane

Instability rearranges the matter in the star,
and as a consequence, it finally eliminates the toroidal structure



Stable Unstable Unstable Unstable

Gravitational Waves



・Amplitude saturates due to the spiral arm propagating to the surface

・Oscillation period is related to the central rotation period, which suggest that the instability is generated around the density maximum at t=0.

(N.B. We cannot determine the compaction of the star in Newtonian gravity.)

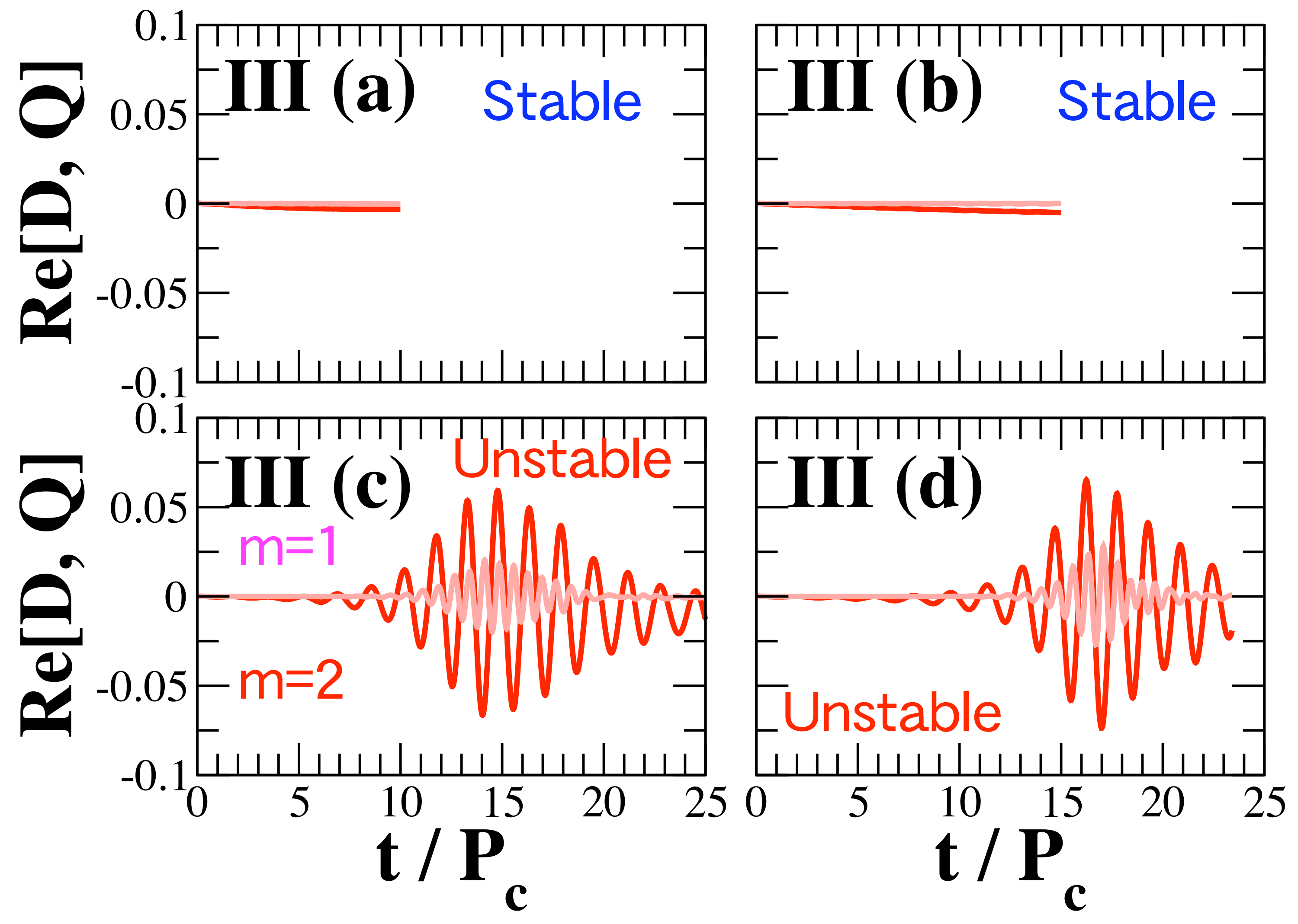
Dependence of m=1 instability on the degree of differential rotation

Initial Conditions

Model	n	Ω_c/Ω_{eq}	T/W
(a)	1	3.60	0.150
(b)	2	6.95	0.150
(c)	3	17.0	0.147
(d)	3.33	26.0	0.144

To probe stability, we put a small m=1 and m=2 perturbation in the initial density from equilibrium.

Diagnostics



・D and Q grows up exponentially during the evolution

・Small growth of an m=2 mode

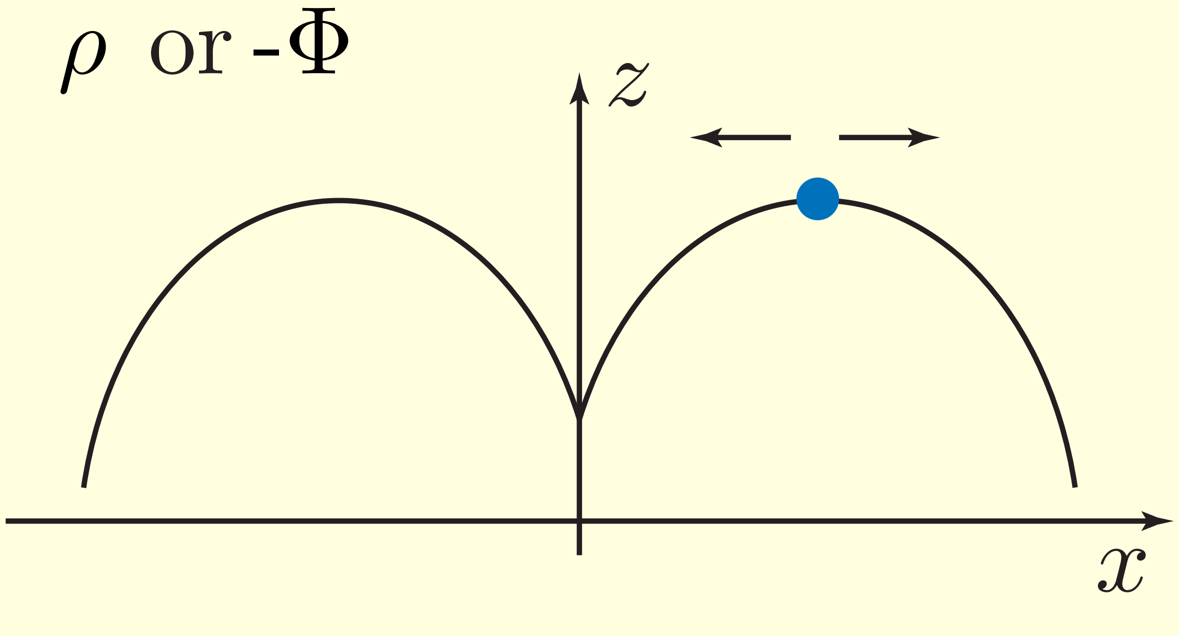
D and Q remains oscillation around zero (absence of exponential growth)

Pattern Period

Model	m=1[Pc]	m=2[Pc]	Pattern[Pc]
(c)	1.6	0.7	1.4
(d)	1.6	0.7	1.4

m=2 mode is regarded as a higher mode of m=1 due to the same pattern period.

Interpretation of m=1 instability



Model	tep / tsd	Stability
II (a)	3.25	Stable
II (b)	0.37	Unstable
II (c)	0.037	Unstable
II (d)	0.0052	Unstable
III (a)	233	Stable
III (b)	1.87	Stable
III (c)	0.037	Unstable
III (d)	0.0052	Unstable

typical timescale

- sound crossing time

$L = 2\pi\varpi_{\max}$

typical length scale for nonaxisymmetric instability

$$t_{\rm sound} = \frac{2\pi\varpi_{\max}}{v_{\rm sound}|_{\varpi=\varpi_{\max}}}$$

$$v^2_{\rm sound} = \frac{dP}{d\rho}$$ depends on EOS

- epicyclic period (ring instability)

$$t_{\rm epicyclic} = \frac{2\pi}{\kappa_{\rm epicyclic}|_{\varpi=\varpi_{\max}}}$$

$$\kappa^2_{\rm epicyclic} = 2\frac{\Omega}{\varpi}\frac{d}{d\varpi}(\varpi^2\Omega)$$ depends on the degree of differential rotation

m=1 instability is excited in the star when tep < tsd.

The mechanism of generating m=1 instability could be the same to that of eccentric instability.

5. Conclusions

We investigate the dynamical instability of the one-armed spiral m=1 mode in differential rotating stars by means of hydrodynamical simulations in Newtonian gravitation.

- Both soft equation of state and high degree of differential rotation are necessary for the m=1 instability to be triggered.
- m=1 instability rearranges the matter of the star, and, as a consequence, it eliminates the toroidal structure.
- A quasi-periodic gravitational wave persists for several rotation periods, decaying as the spiral arm instability propagates outward to the surface.