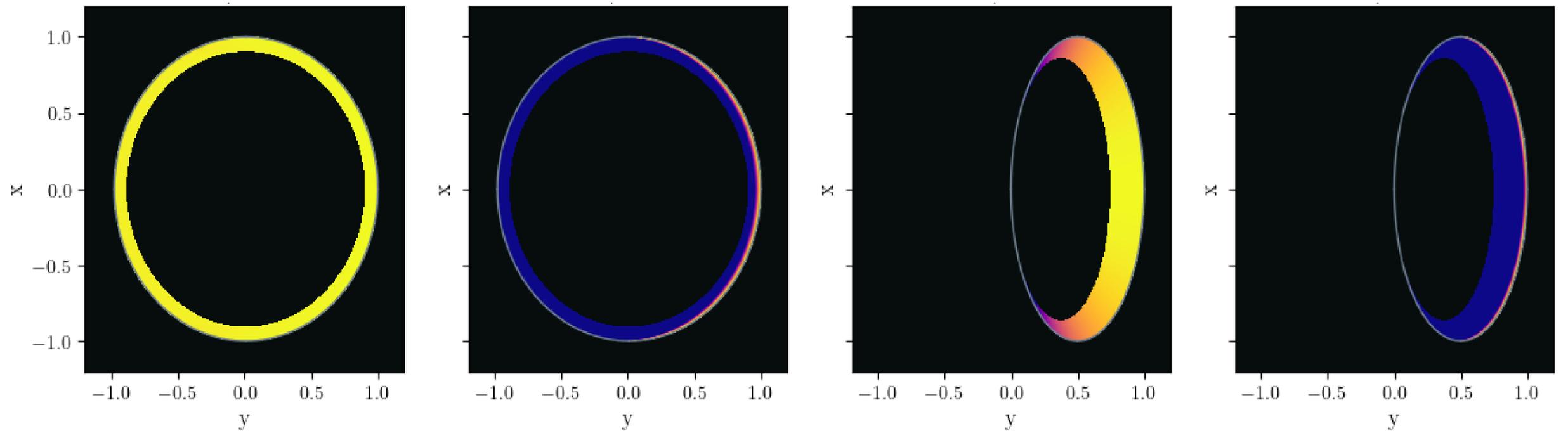


Numerical Modeling of Relativistic Effects in Synchrotron-Emitting Shocks

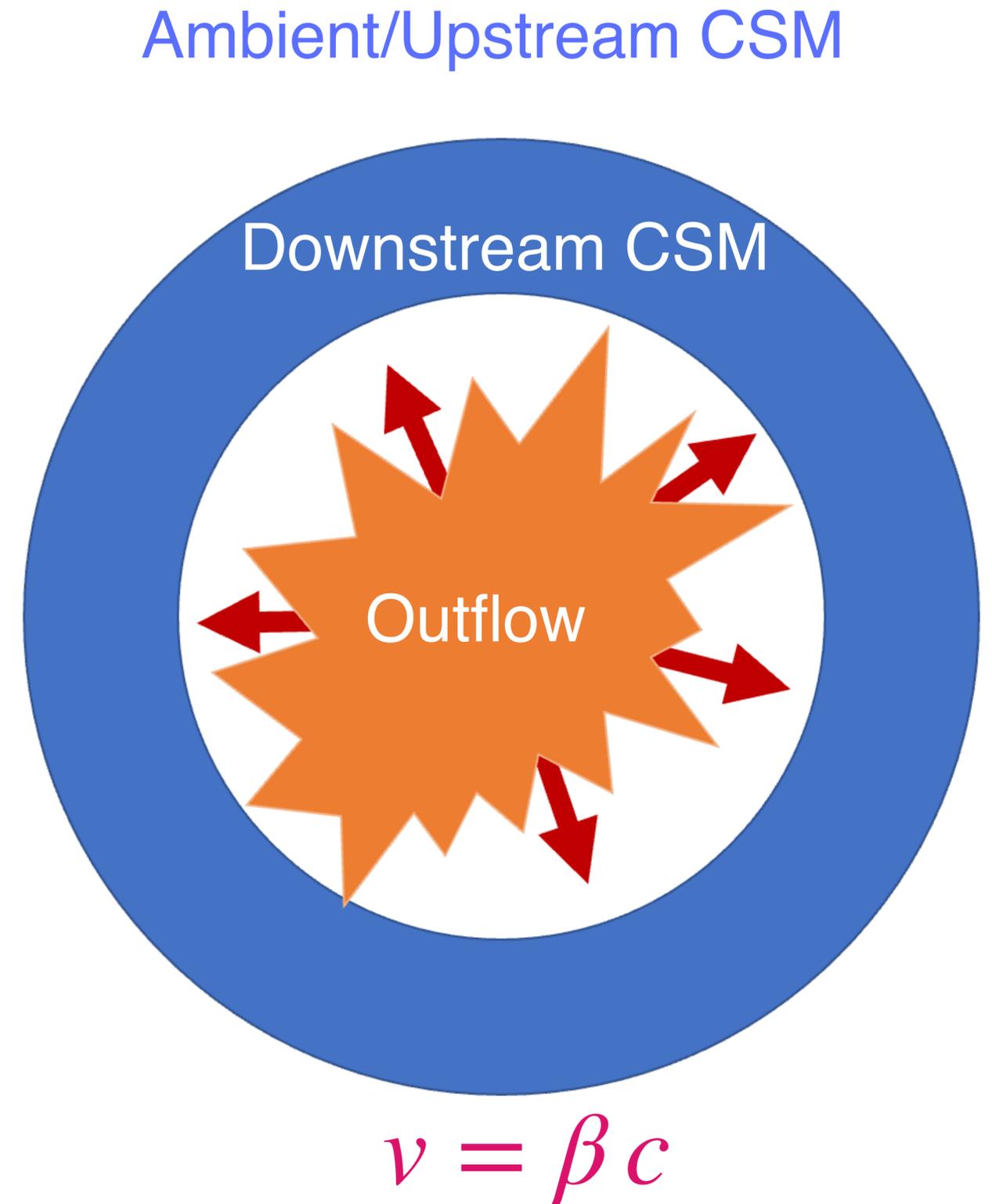
Ross Ferguson

1/30/2026



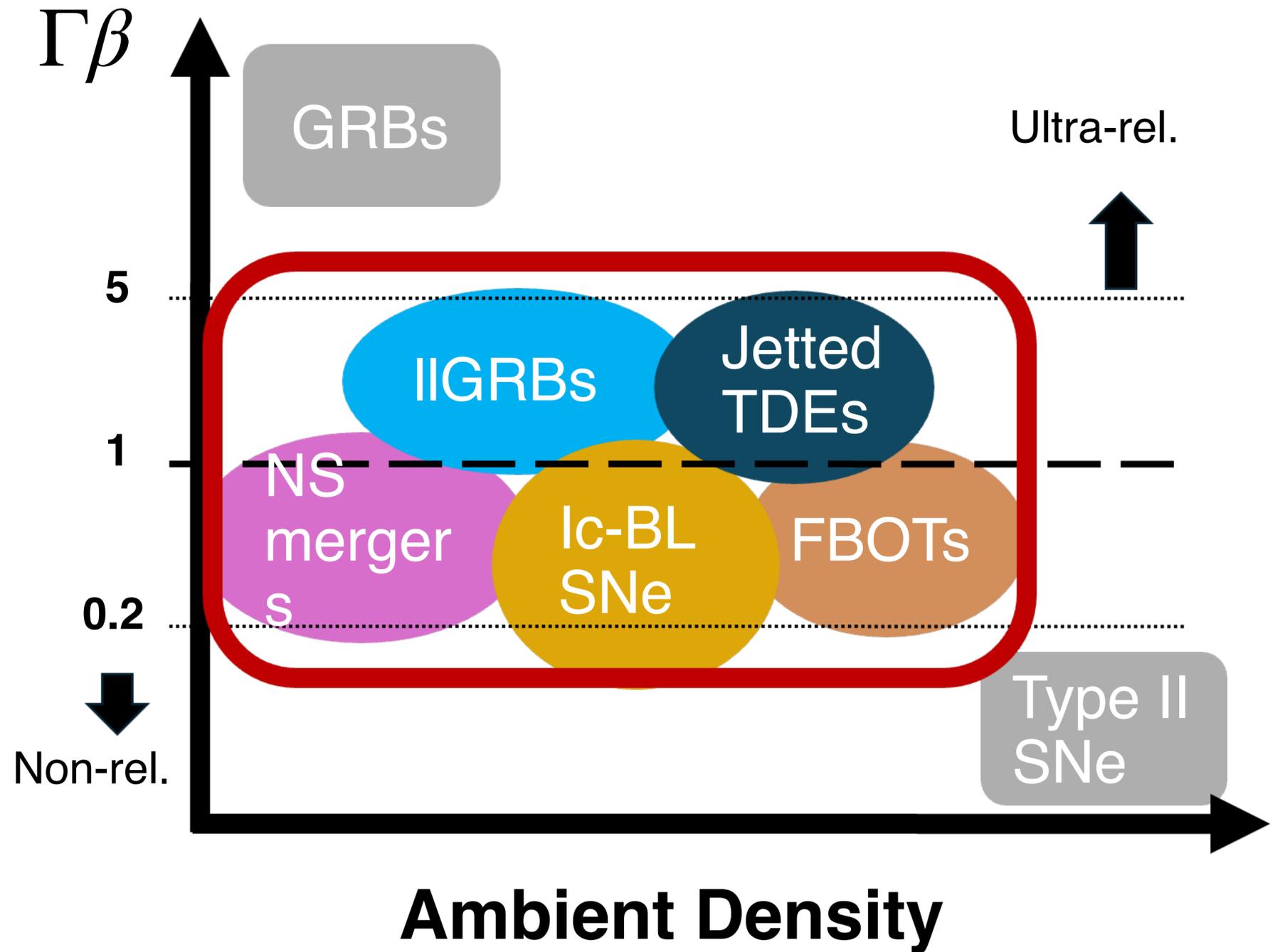
Shock Waves

- Cataclysmic events → Shocks
 - $E \sim 10^{51}$ ergs
- Shock + circumstellar medium (CSM)
 - Synchrotron radiation



Motivation

- Typical models make invalid assumptions
 - Relativistic effects relevant for $\Gamma\beta \gtrsim 1$



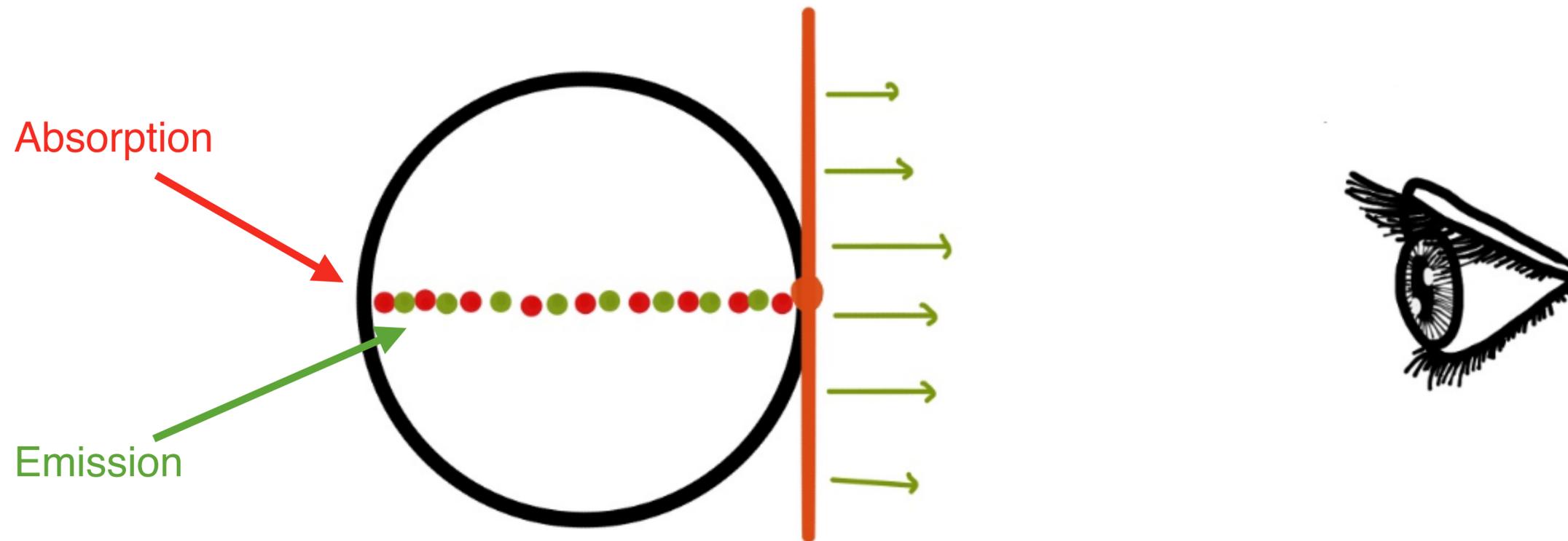
Accurate Afterglow Models?

Yes

No

Effective Line-of-Sight (LOS) Approximation

- Assume emission along a ray representative of entire shock



- Analytic models: Chevalier 1998, Sari et al. 1998
- Best for non-rel. shocks

Relativistic Effects

- Doppler effect (lab vs. post-shock fluid frames)

$$\nu' = \nu\gamma(1 - \beta\mu) \equiv \frac{\nu}{D(\gamma, \mu)}$$



$\mu = \cos \theta$

Relativistic Effects

- Doppler effect (lab vs. post-shock fluid frames)

$$\nu' = \nu\gamma(1 - \beta\mu) \equiv \frac{\nu}{D(\gamma, \mu)}$$

- Time-of-flight effects

$$t = T + \frac{r\mu}{c}$$

$$\mu = \cos \theta$$

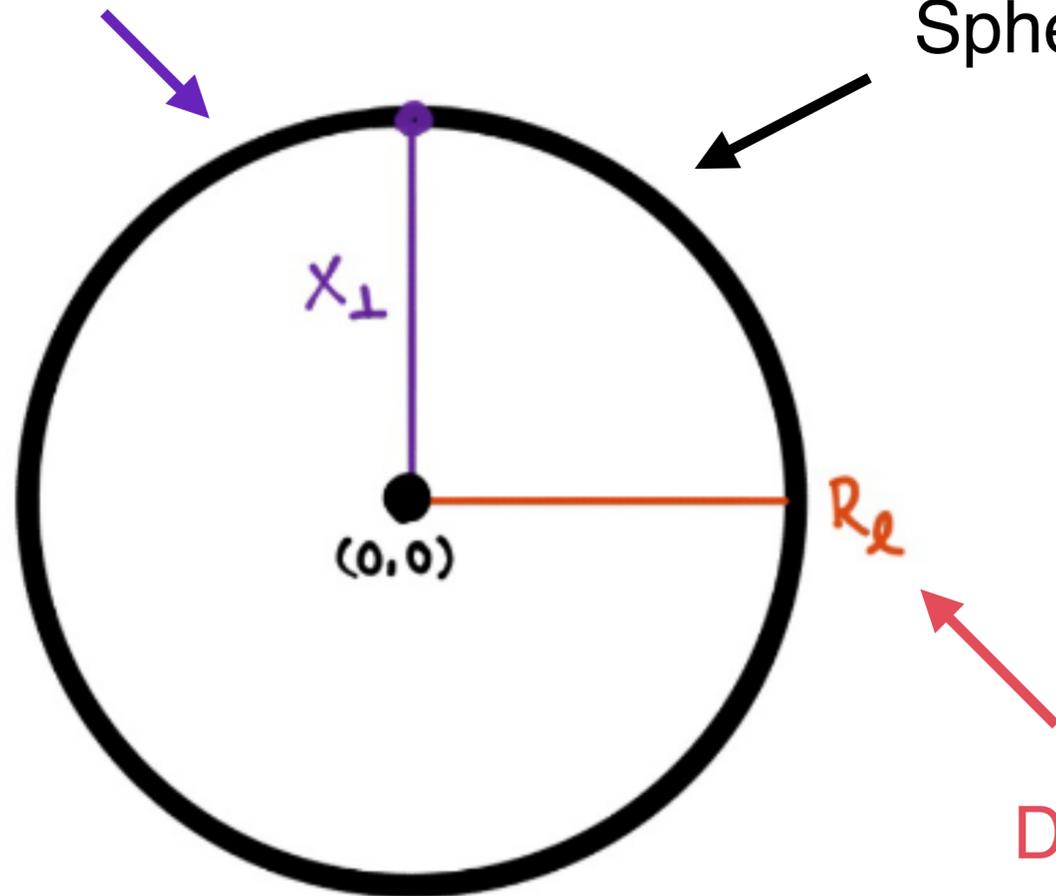
Retarded time

Observation time

Equal Arrival Time Surface (EATS)

Distance perp. to LOS ('x')

Spherical expansion



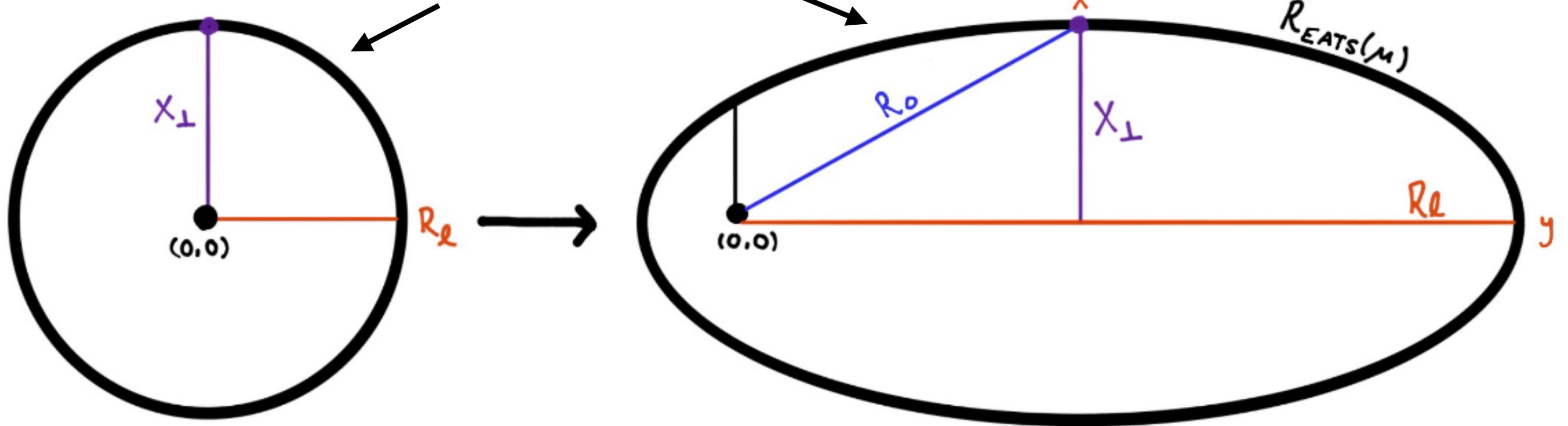
Distance parallel to LOS ('y')

$$r \ll 1$$
$$R_e \approx x_{\perp}$$

Granot et al. 1999a,b

Equal Arrival Time Surface (EATS)

Spherical expansion

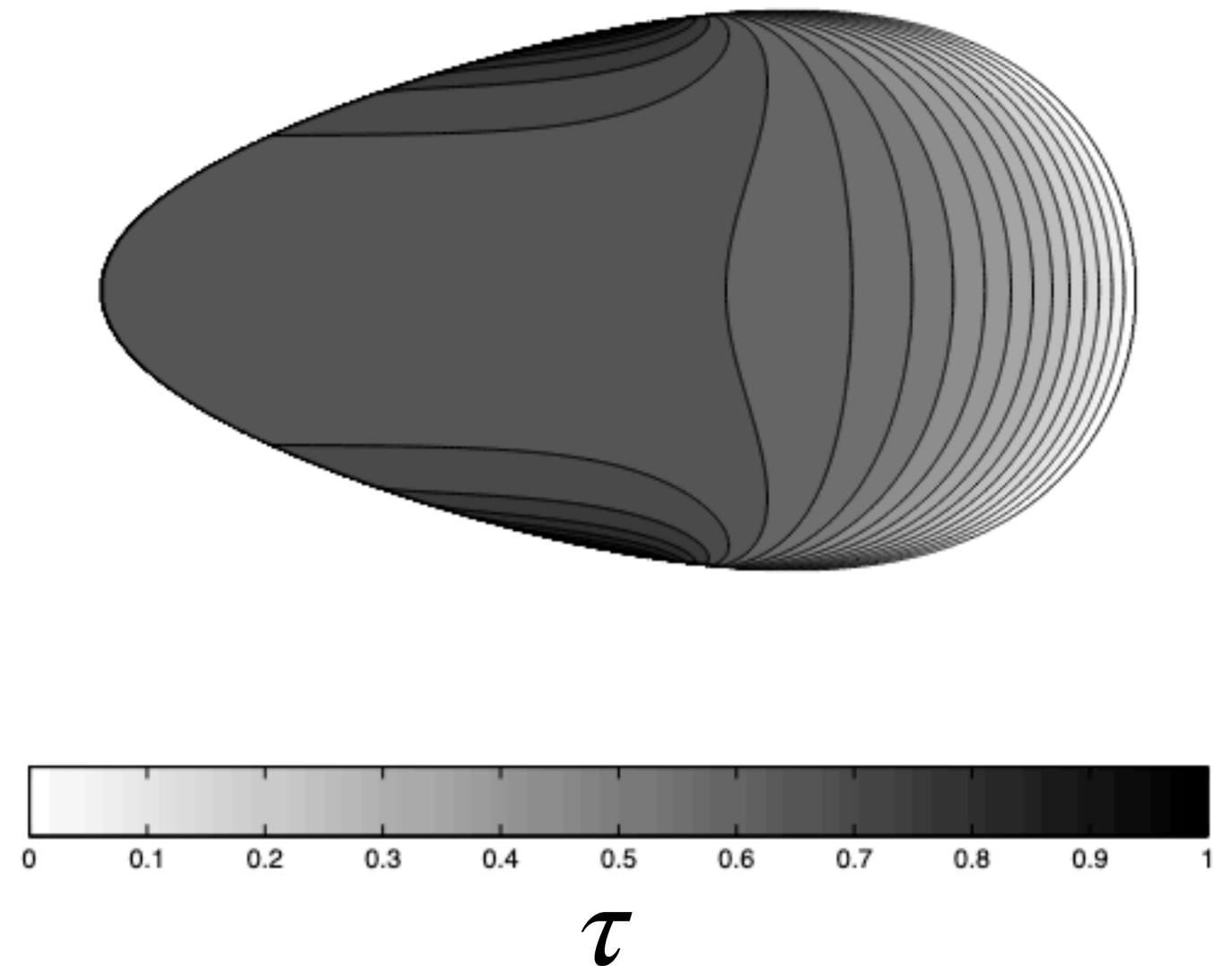


$\Gamma\beta \ll 1$
 $R_e \approx x_{\perp}$

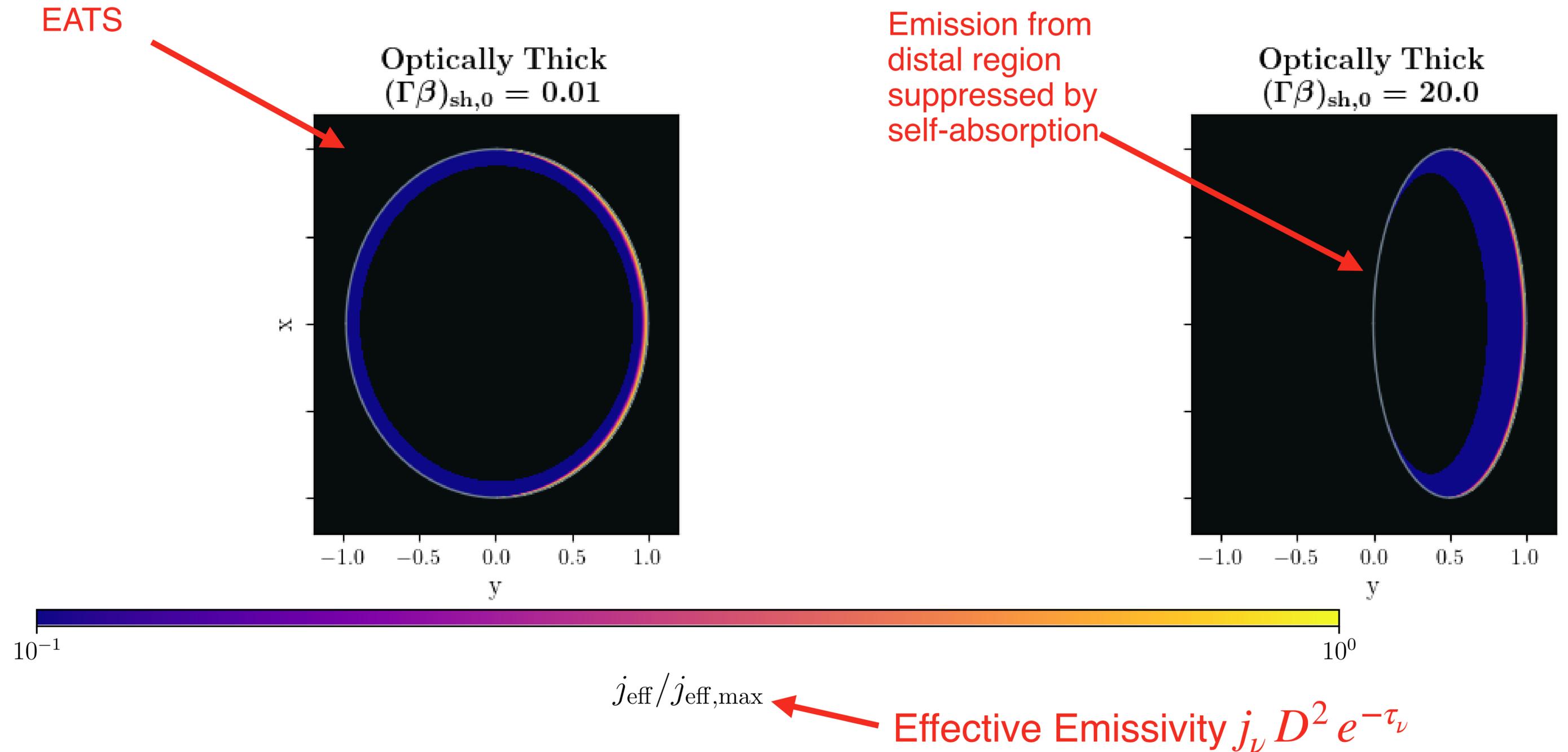
$\Gamma\beta \gg 1$
 $R_e > x_{\perp}$

Previous Work on GRB Afterglows

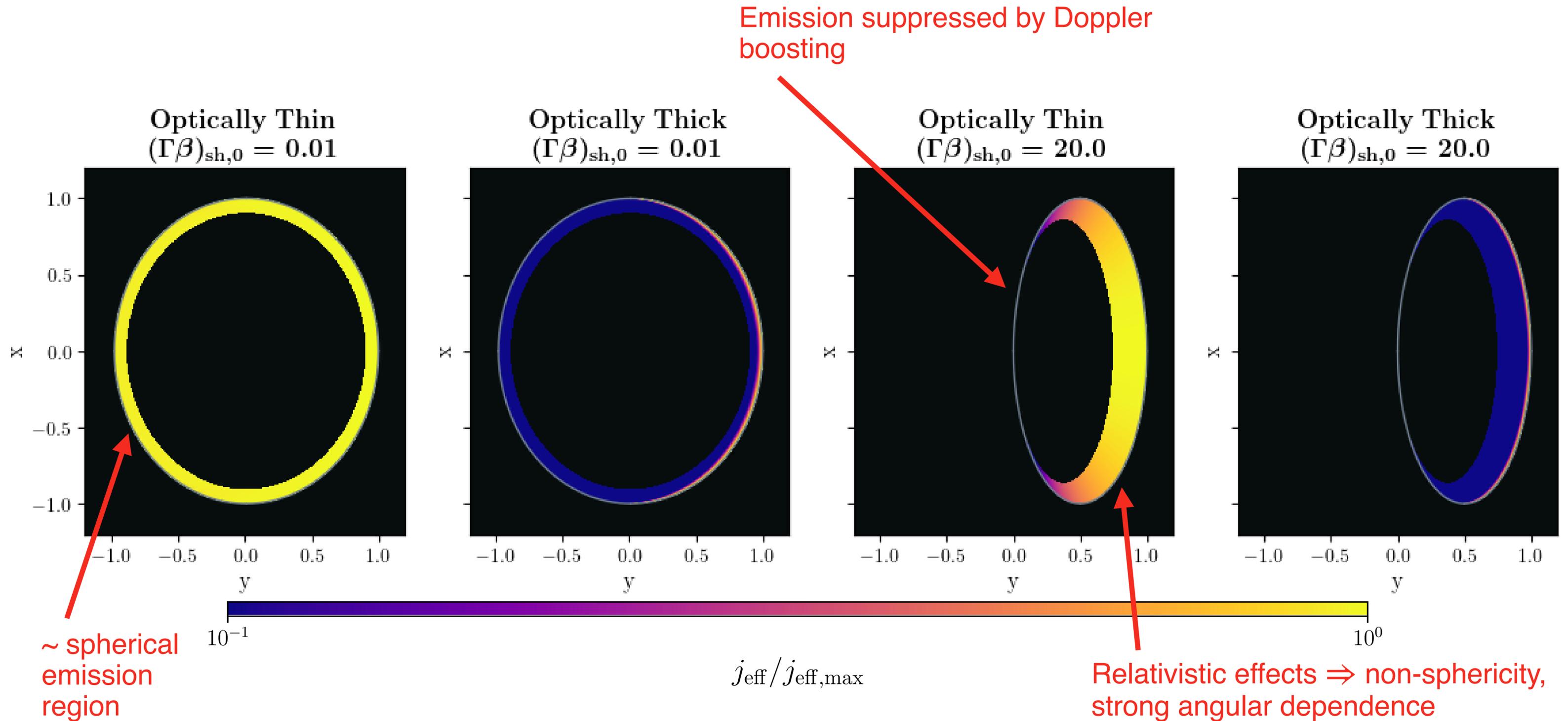
- EATS & synchrotron afterglow investigated in detail for GRBs (e.g., Granot et al. 1999a,b)
- Key differences
 - Generic hydrodynamics vs. Blandford-McKee solution
 - Applicable to arbitrary velocities (no implicit assumption that $\Gamma \gg 1$)



Emission Region Contour Plots



Emission Region Contour Plots



Radiative Transfer

Radiative Transfer

- Total flux emitted behind entire shock
- Specific intensity along ray: radiative transfer equation

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$$

Radiative Transfer

- Total flux emitted behind entire shock
- Specific intensity along ray: radiative transfer equation

$$\frac{1}{R_\ell} \frac{dI_\nu}{dy} = D^2(\gamma, \mu) j'_\nu - \frac{1}{D(\gamma, \mu)} \alpha'_\nu I_\nu$$

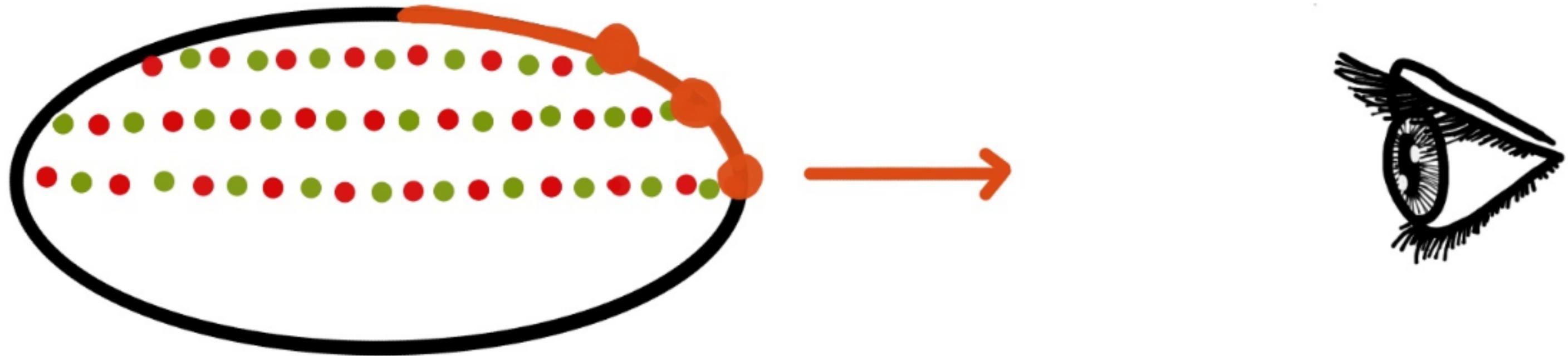
Radiative Transfer

- Total flux emitted behind entire shock
- Specific intensity along ray: radiative transfer equation

$$\frac{1}{R_\ell} \frac{dI_\nu}{dy} = D^2(\gamma, \mu) j'_\nu(t, y) - \frac{1}{D(\gamma, \mu)} \alpha'_\nu(t, y) I_\nu$$

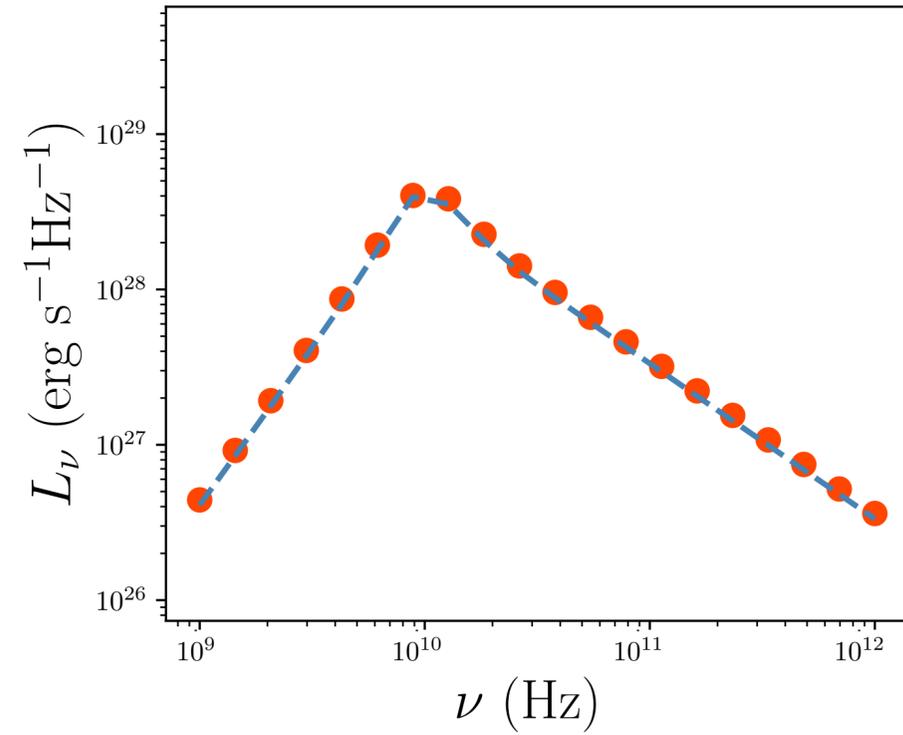
Radiative Transfer: Full-Volume Model

- Total flux: $F_\nu = 2\pi \frac{X_\perp^2}{d_L^2} \int_0^1 I_\nu(x) dx$



Results and Implications for Observed Events

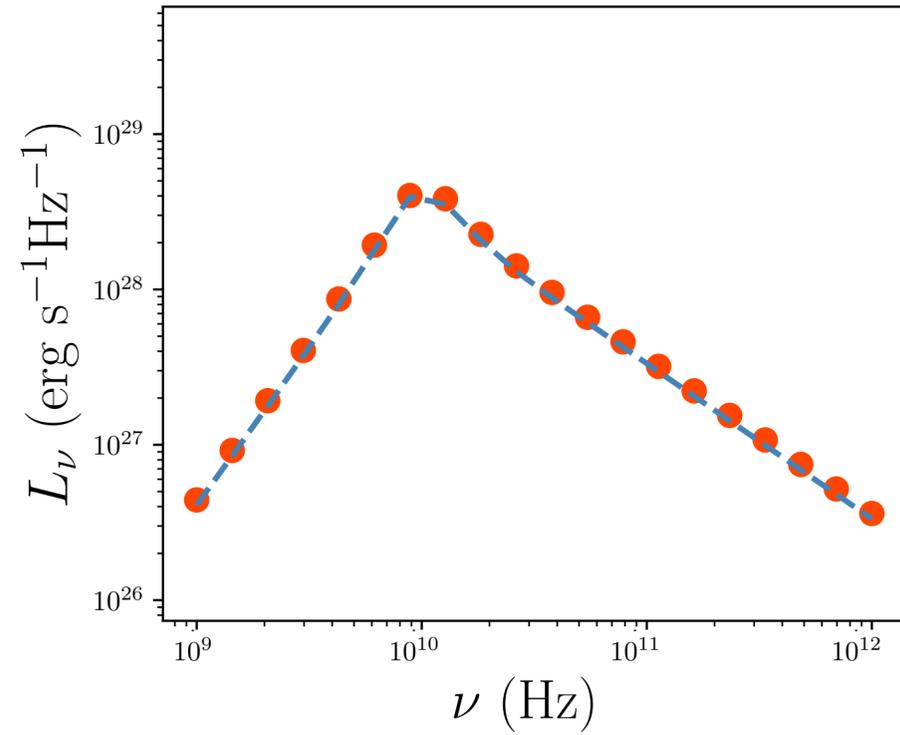
$$(\Gamma\beta)_{\text{sh},0} = 0.3$$



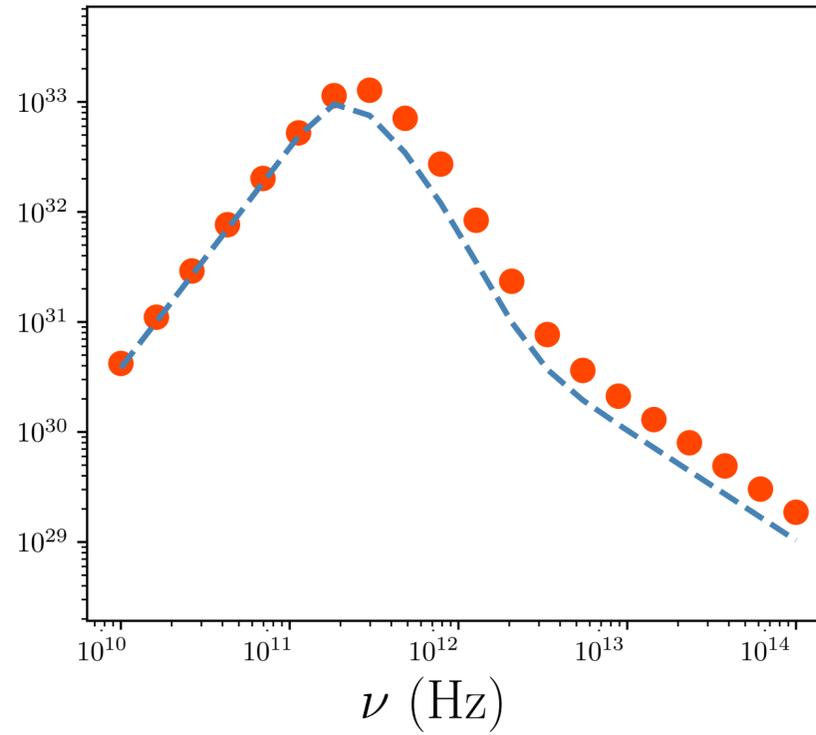
Constant
velocity

Spectral Energy Distributions (SEDs)

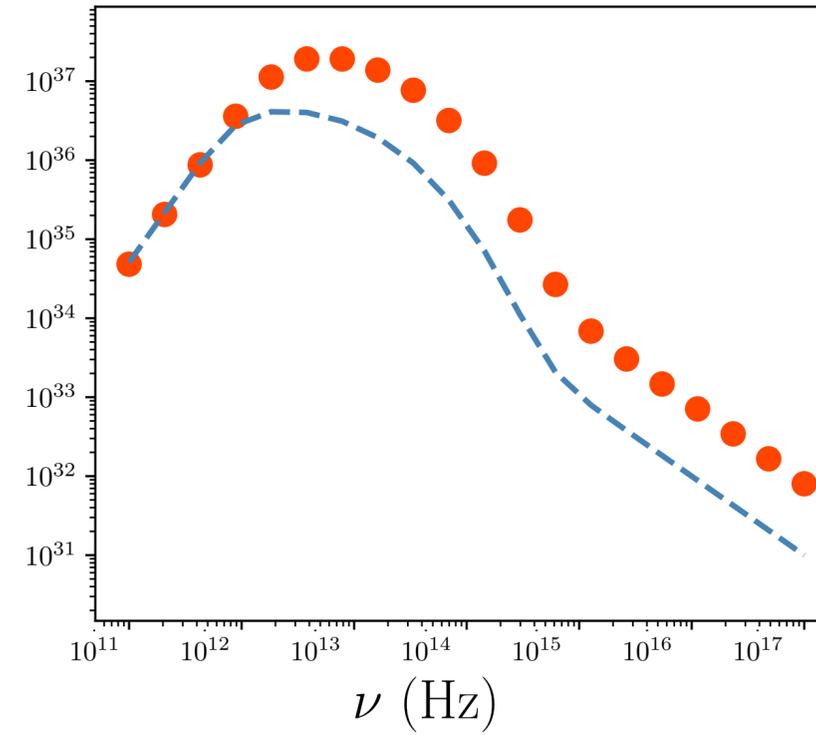
$(\Gamma\beta)_{\text{sh},0} = 0.3$



$(\Gamma\beta)_{\text{sh},0} = 0.9$



$(\Gamma\beta)_{\text{sh},0} = 2.7$

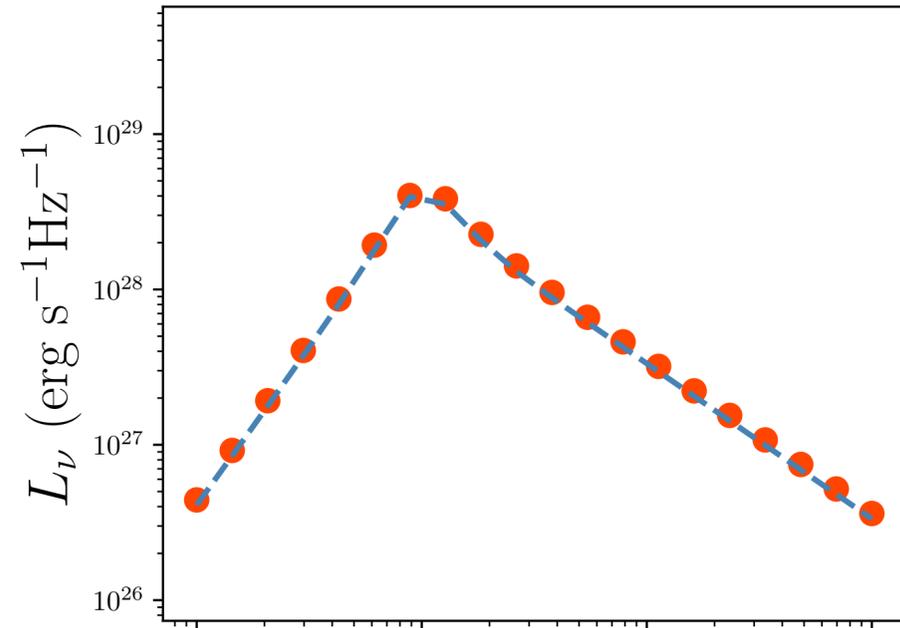


Constant
velocity

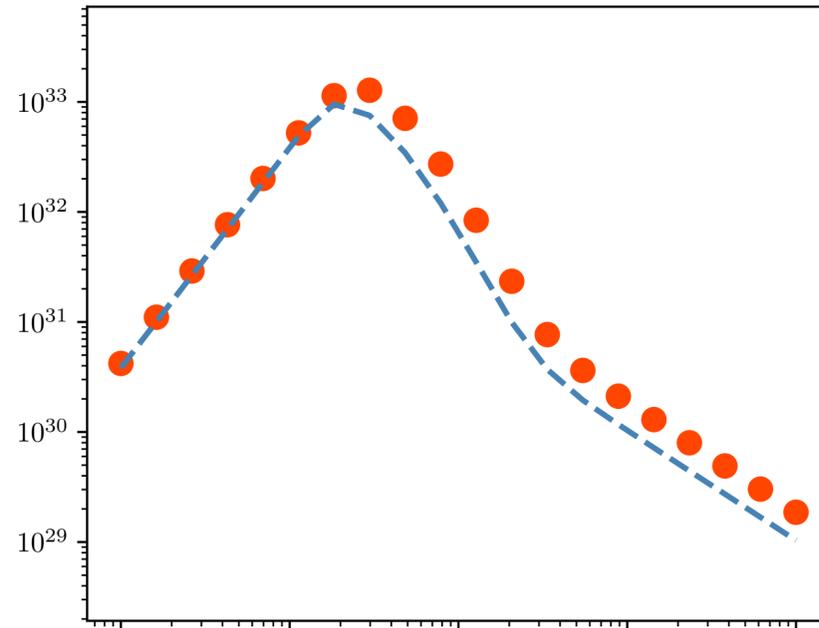


Spectral Energy Distributions (SEDs)

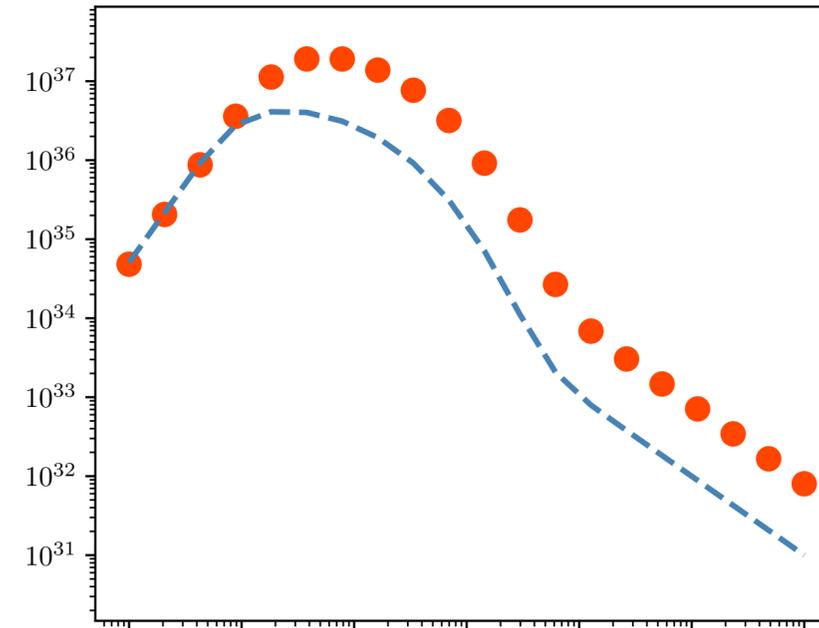
$(\Gamma\beta)_{\text{sh},0} = 0.3$



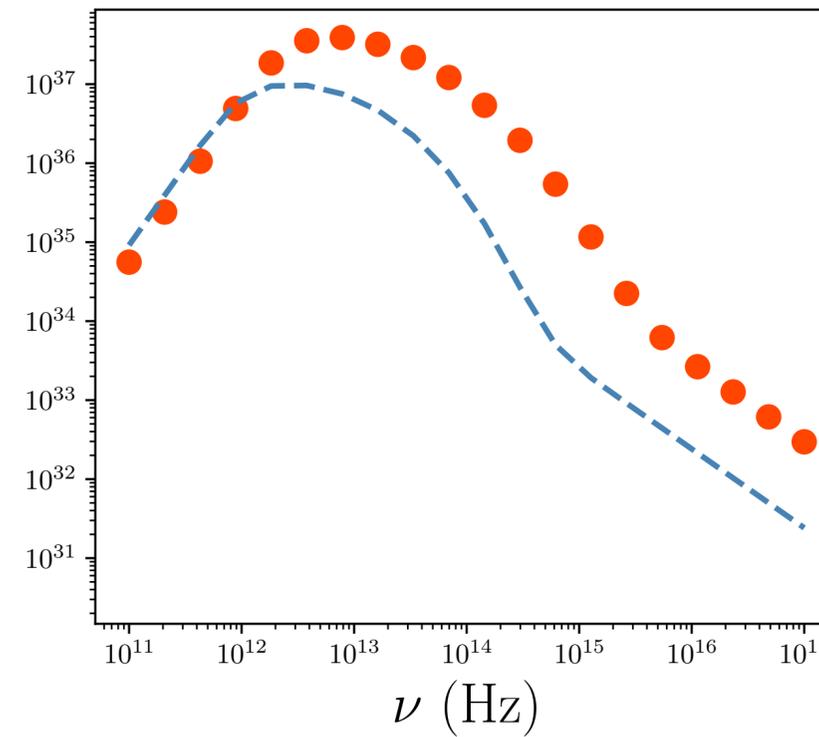
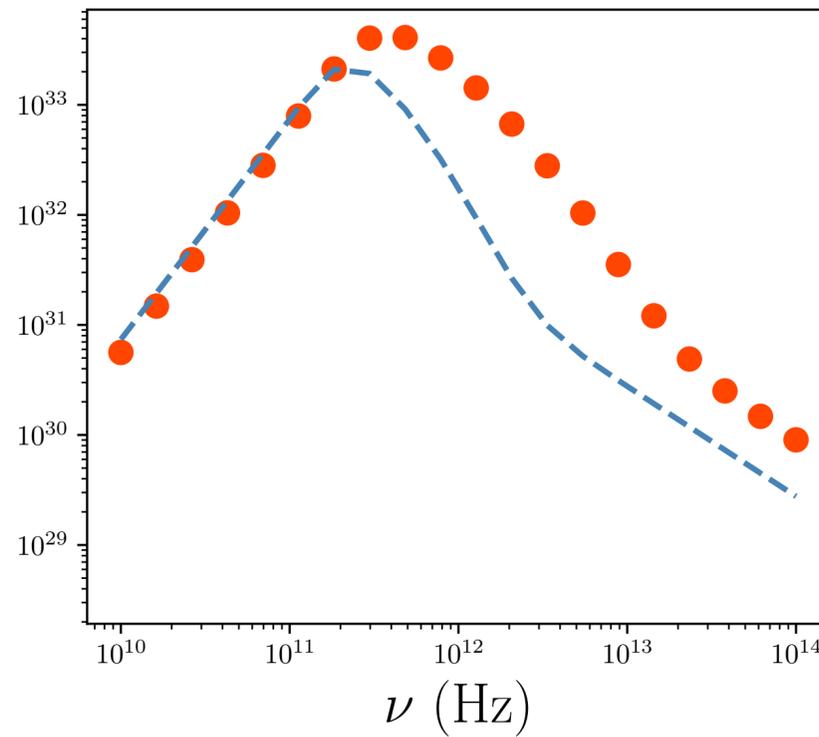
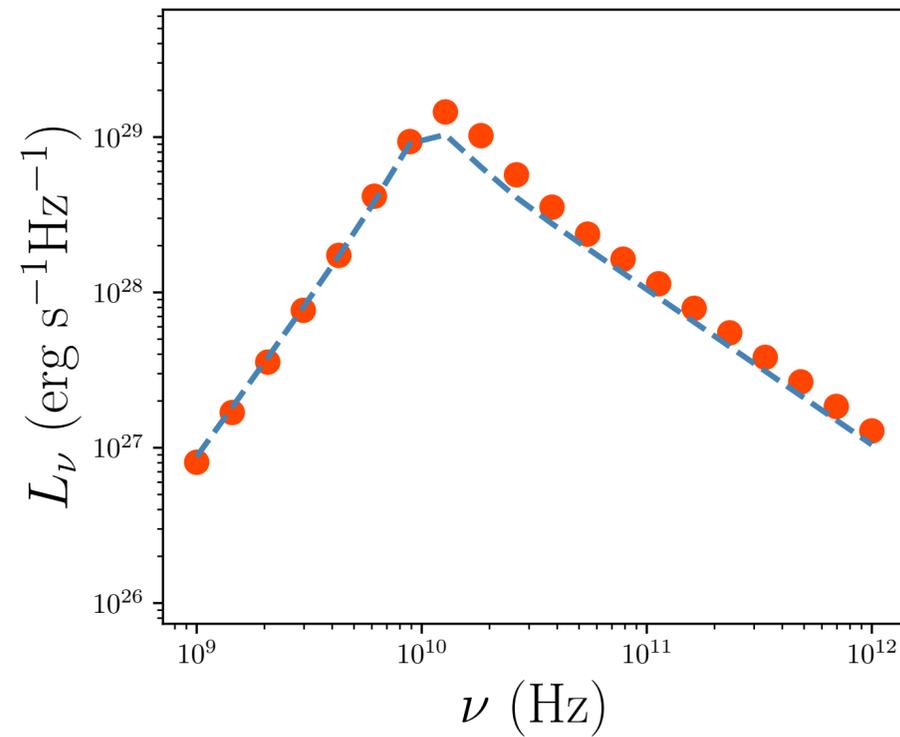
$(\Gamma\beta)_{\text{sh},0} = 0.9$



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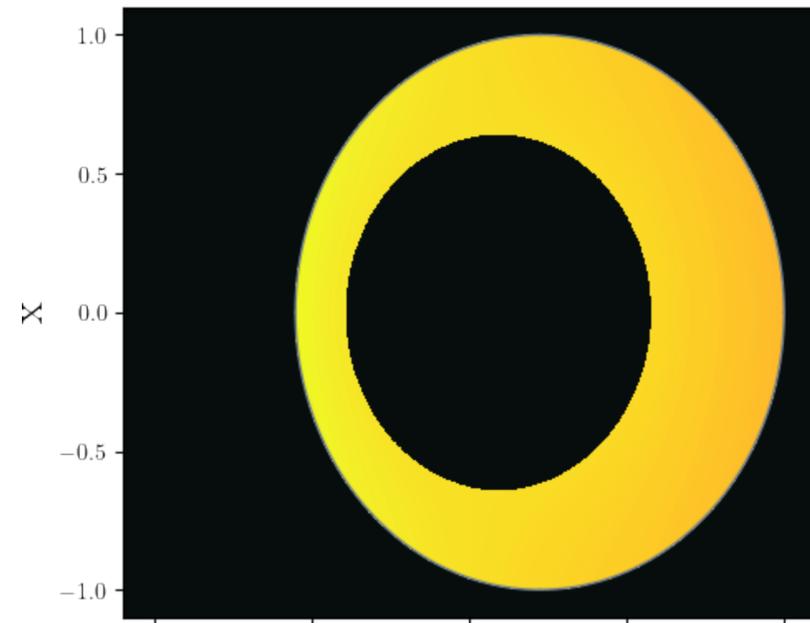
Constant
velocity



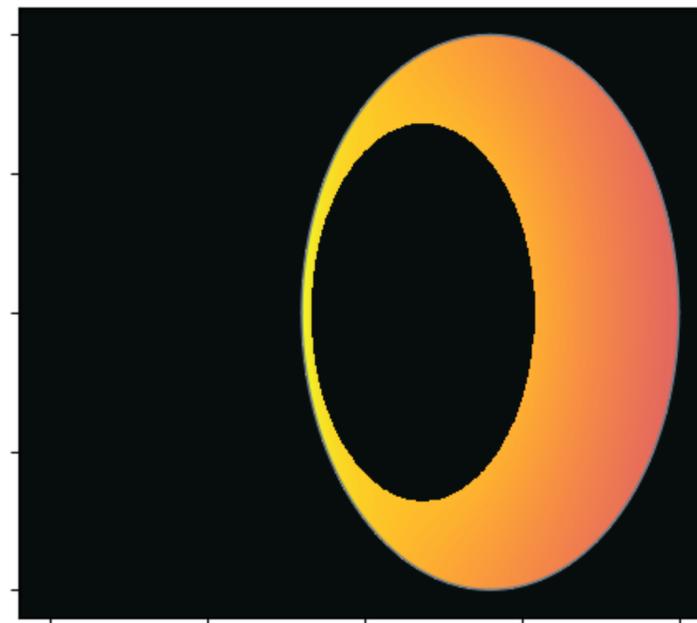
Strong
deceleration

● Full Calculation - - - LOS

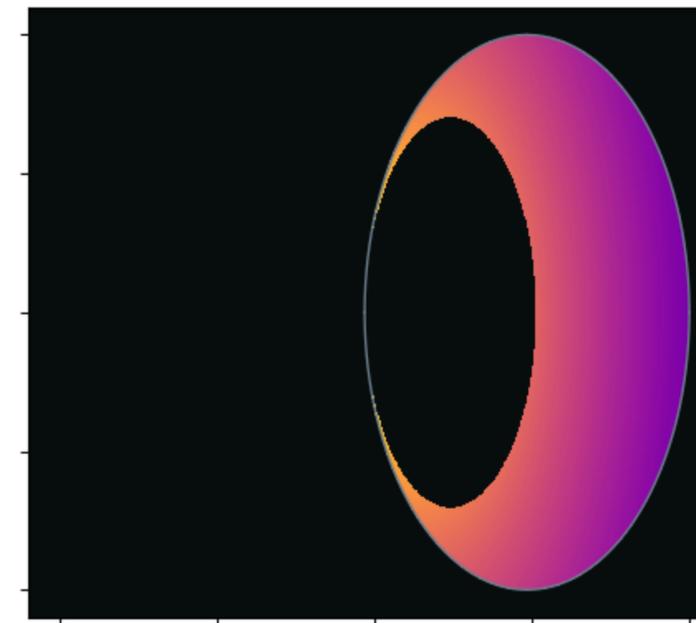
$(\Gamma\beta)_{\text{sh},0} = 0.3$



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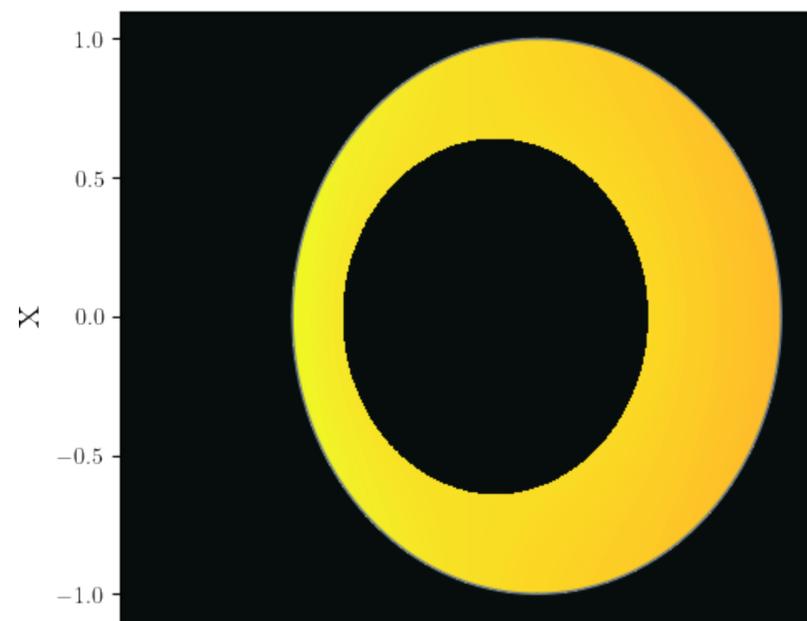
$(\Gamma\beta)_{\text{sh},0} = 2.7$



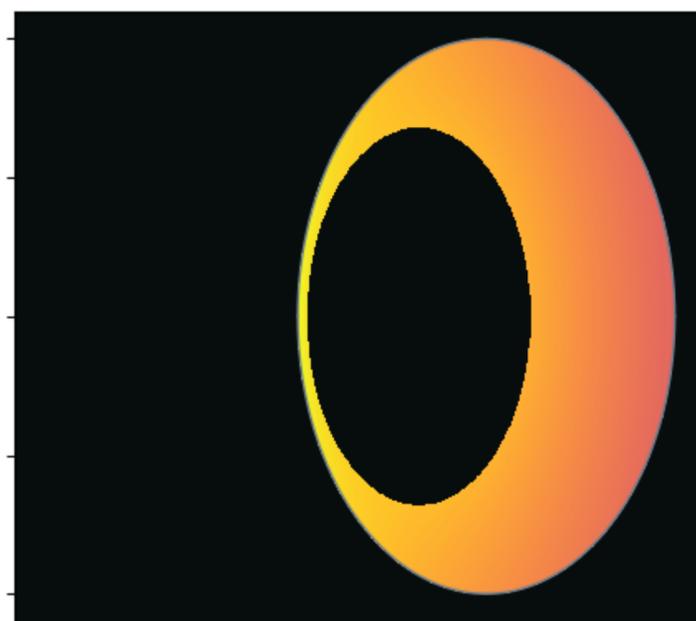
Constant
velocity



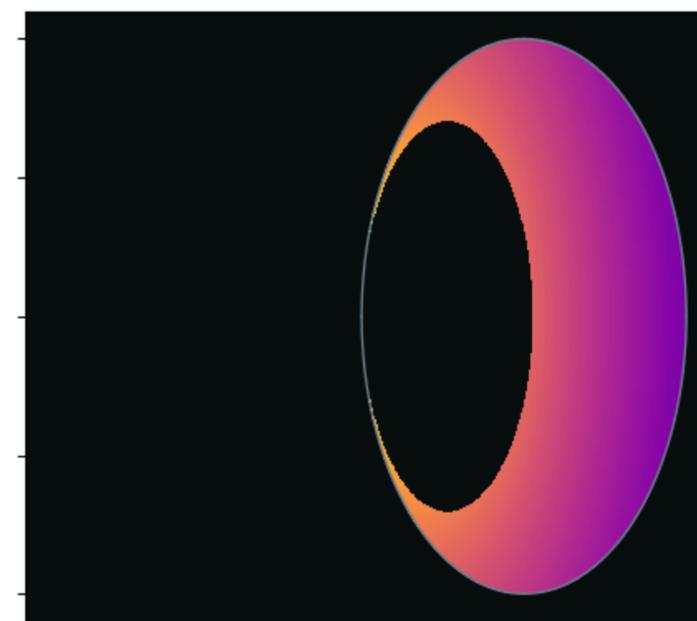
$(\Gamma\beta)_{\text{sh},0} = 0.3$



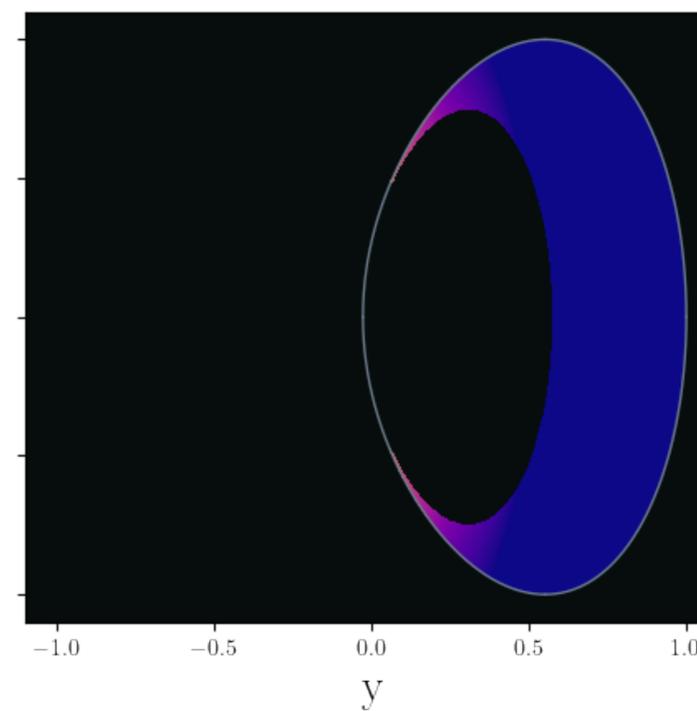
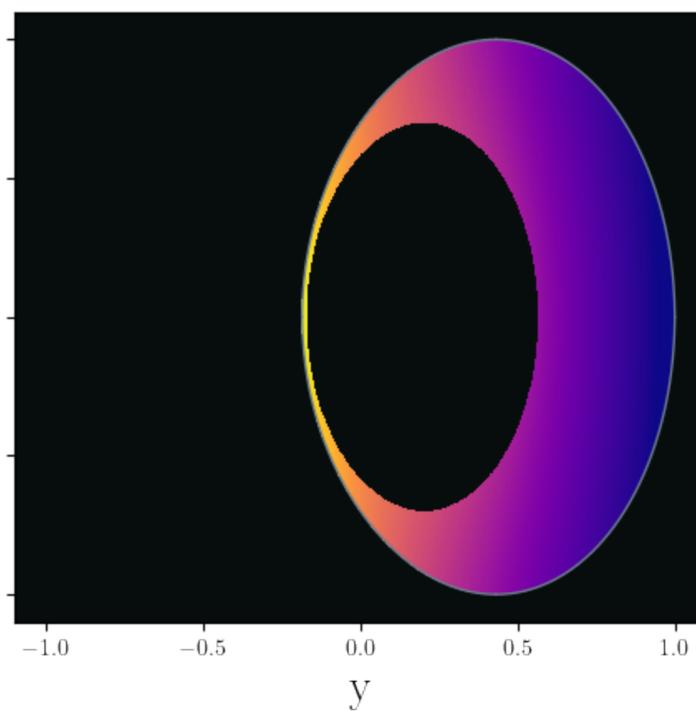
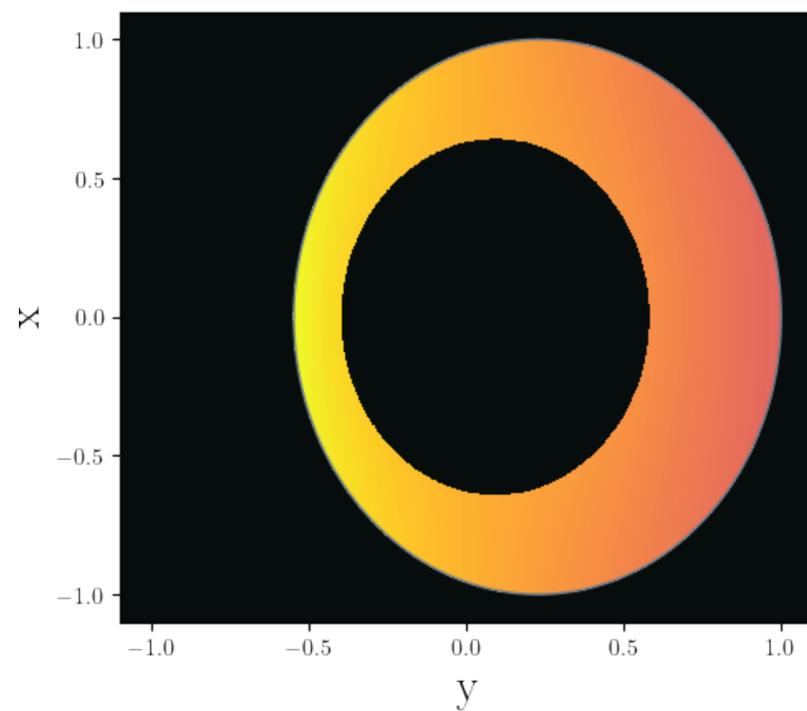
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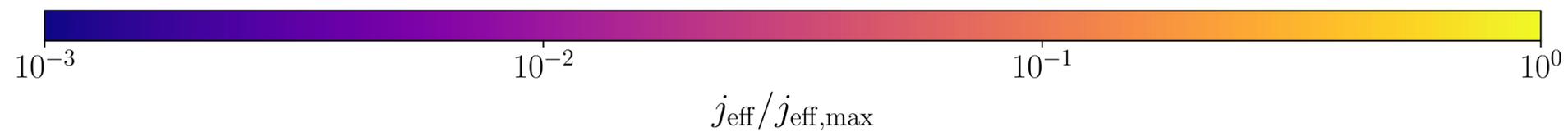
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Constant
velocity

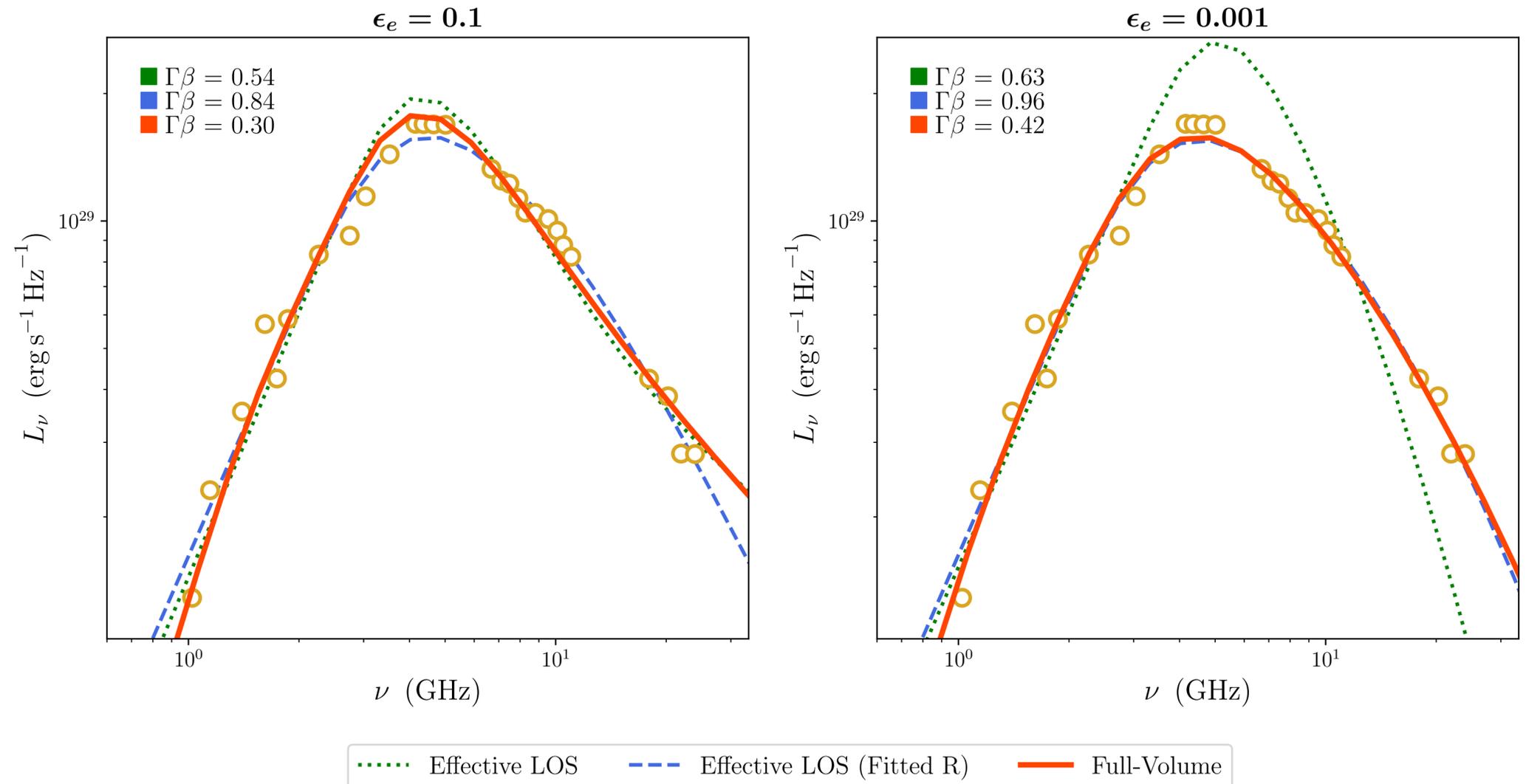


Strong
deceleration



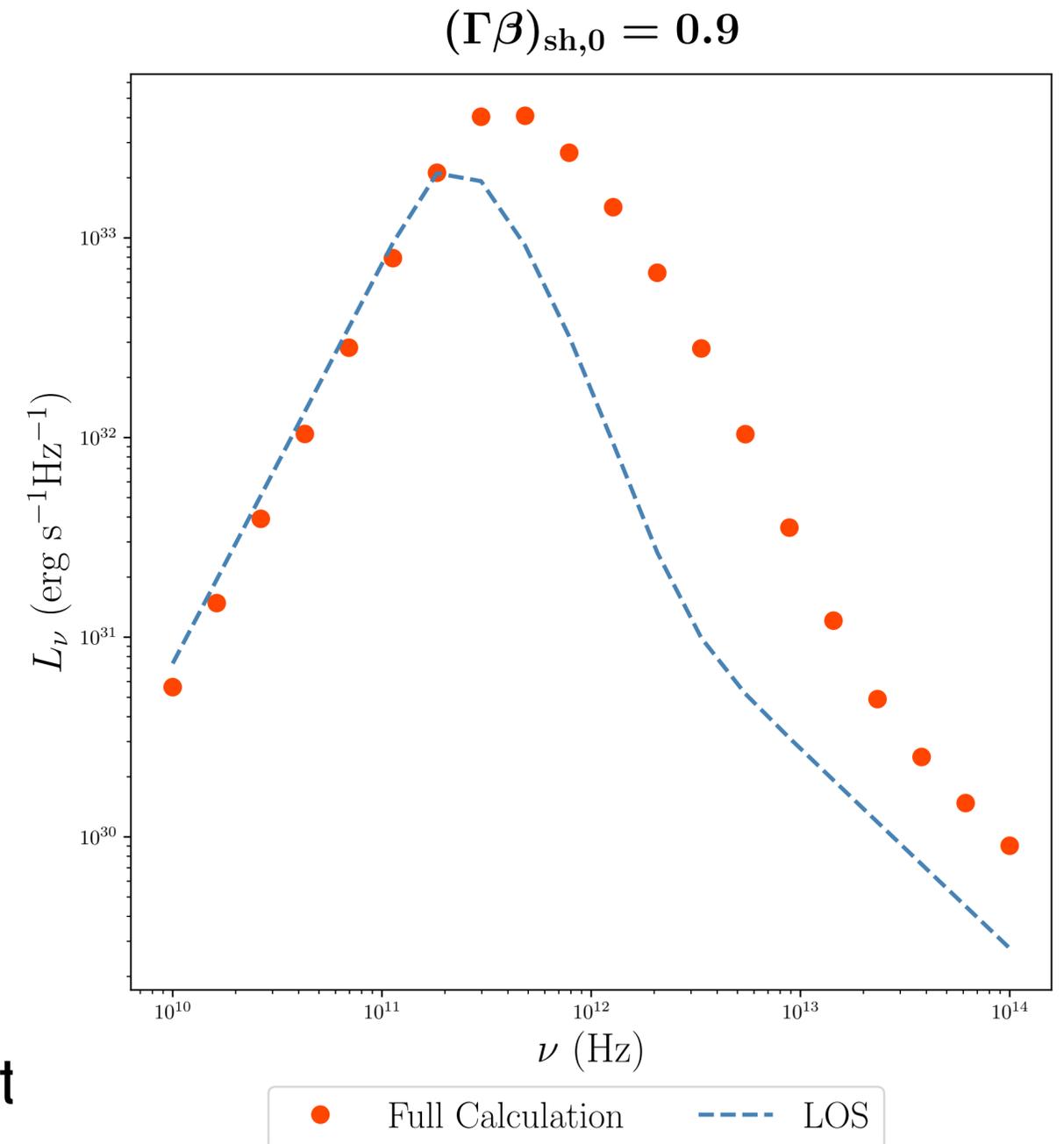
Comparison to Observation: CSS161010

- Full-volume model, LOS approximation yield similar fitted curves
- Notice discrepancy in fitted parameters



Conclusion

- Built radiative transfer code to calculate full-volume model
 - Relativistic effects
 - Applicable to arbitrary velocities and hydrodynamics
- **Detailed models especially important for $\Gamma\beta \sim 1$**
 - Discrepancy between models can be orders of magnit



Future Work

- Modeling FBOs, jetted TDEs, Ic-BL SNe, etc.
- Realistic hydrodynamics, electron cooling, inhomogeneities
- FRB synchrotron maser model (afterglow)
- Magnetar-boosted NS mergers

Thank you!