The Oates Relation: testing a structured jet origin

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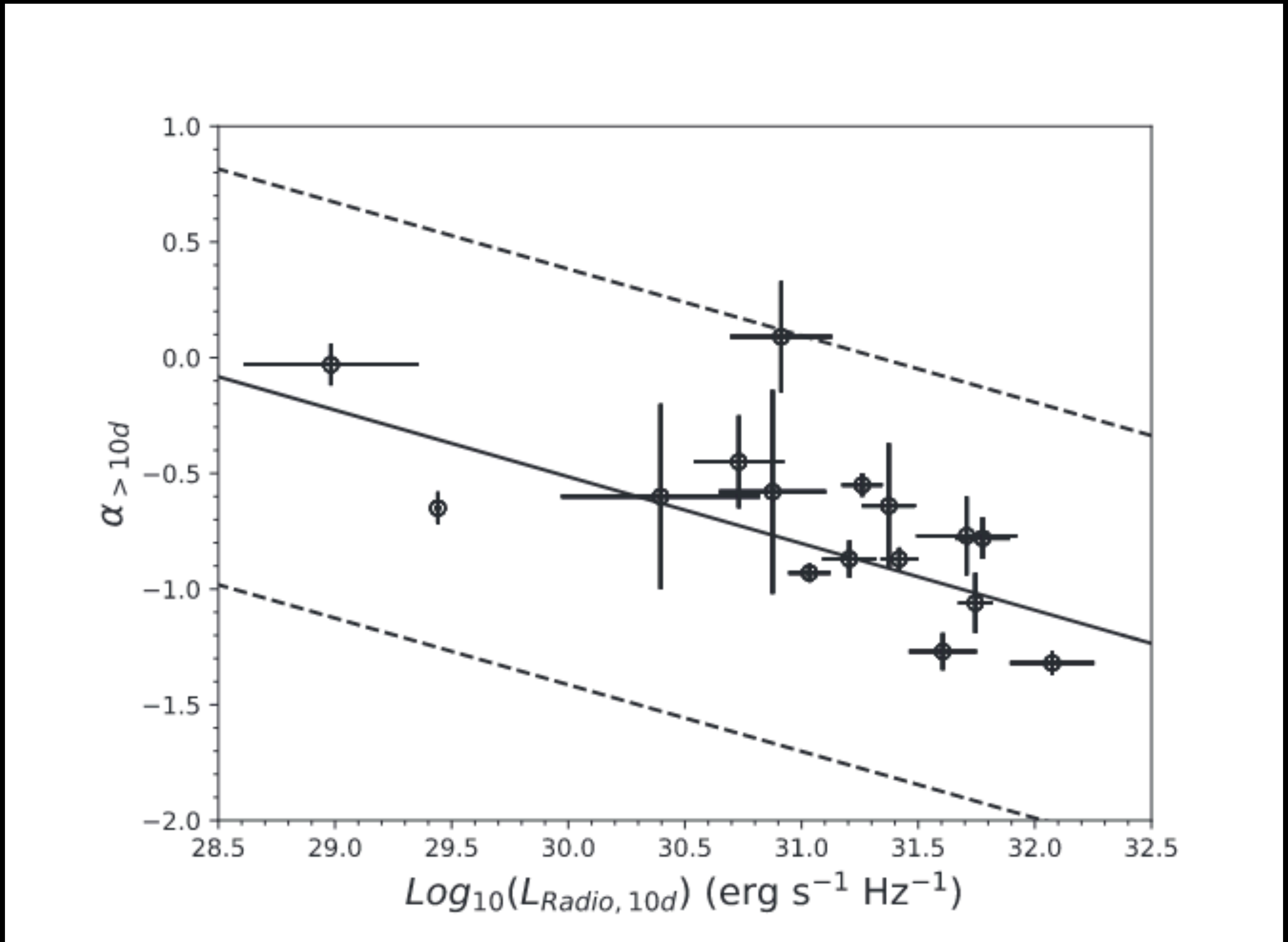
The correlation

- Empirical relation between early-time brightness and average decay rate of LGRB afterglows
- More luminous afterglows typically decay faster than lower luminosity counterparts
- Not predicted by standard external shock afterglow model— possible explanations of regulated energy release, jet structure or observer viewing angle
- The relation is observed over 15 orders of magnitude in frequency from radio to GeV energies and all consistent with one another within 1σ — suggests a common underlying physical mechanism

The correlation
Radio

- 16 LGRB 8.5GHz rest frame light curves
- Correlation of luminosity at 10 days and average rate of decay past this time:

$$\alpha_{\mathrm{radio},>10\mathrm{d}} = -0.29^{+0.19}_{-0.28} \log(L_{\mathrm{Radio},10\,\mathrm{d}}) + 8.12^{+8.86}_{-5.88}$$



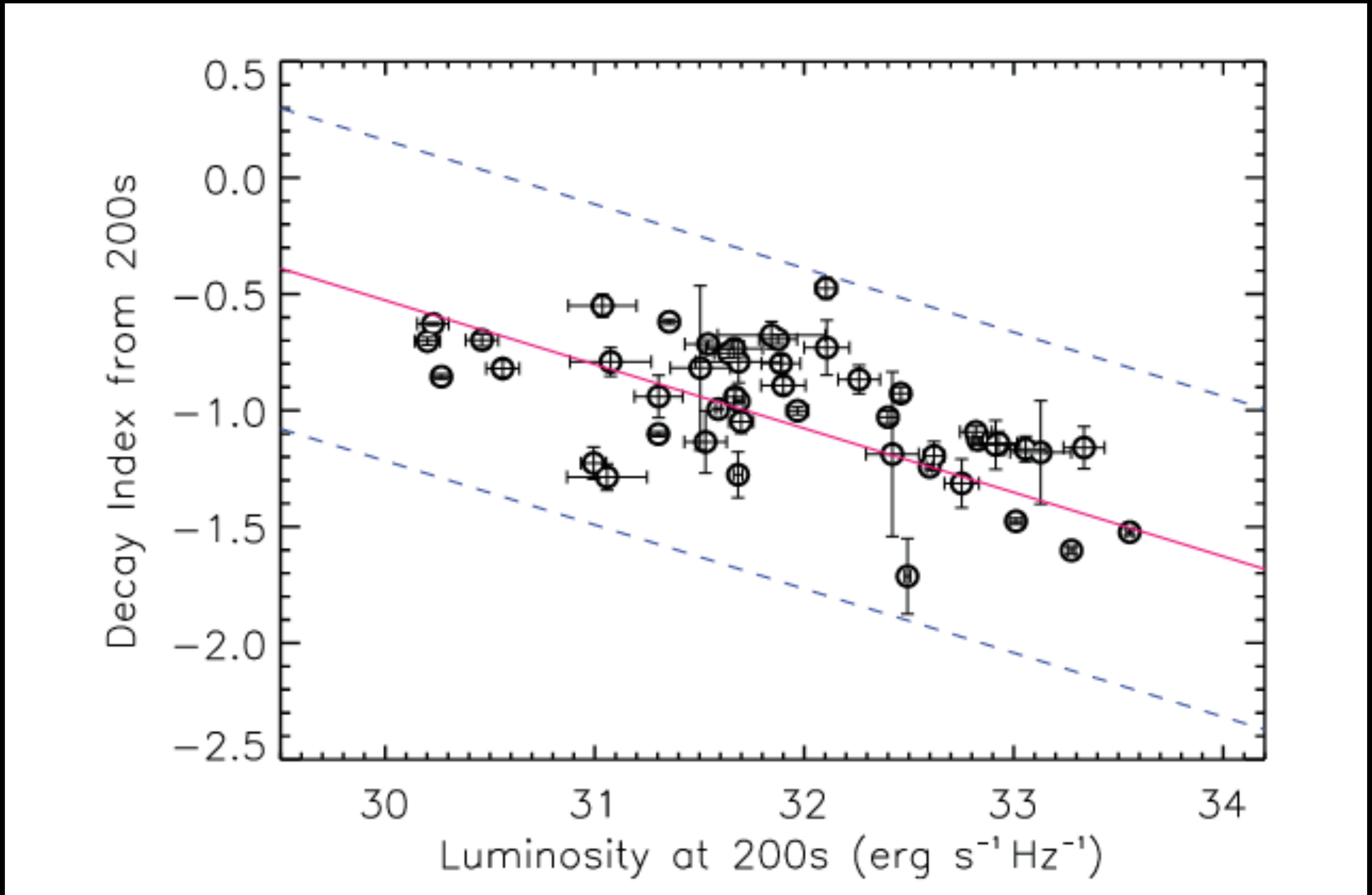
Shilling et al. 2025

Correlation

Optical/UV

- 48 *Swift*/UVOT LGRB 1600 Å rest frame light curves
- Correlation of luminosity at 200s and average rate of decay past this time:

$$\alpha_{\mathrm{O},>200\,\mathrm{s}} = -0.28^{+0.04}_{-0.04}\log(L_{\mathrm{O},>200\,\mathrm{s}}) + 7.72^{+1.31}_{-1.31}$$



Oates et al. 2012

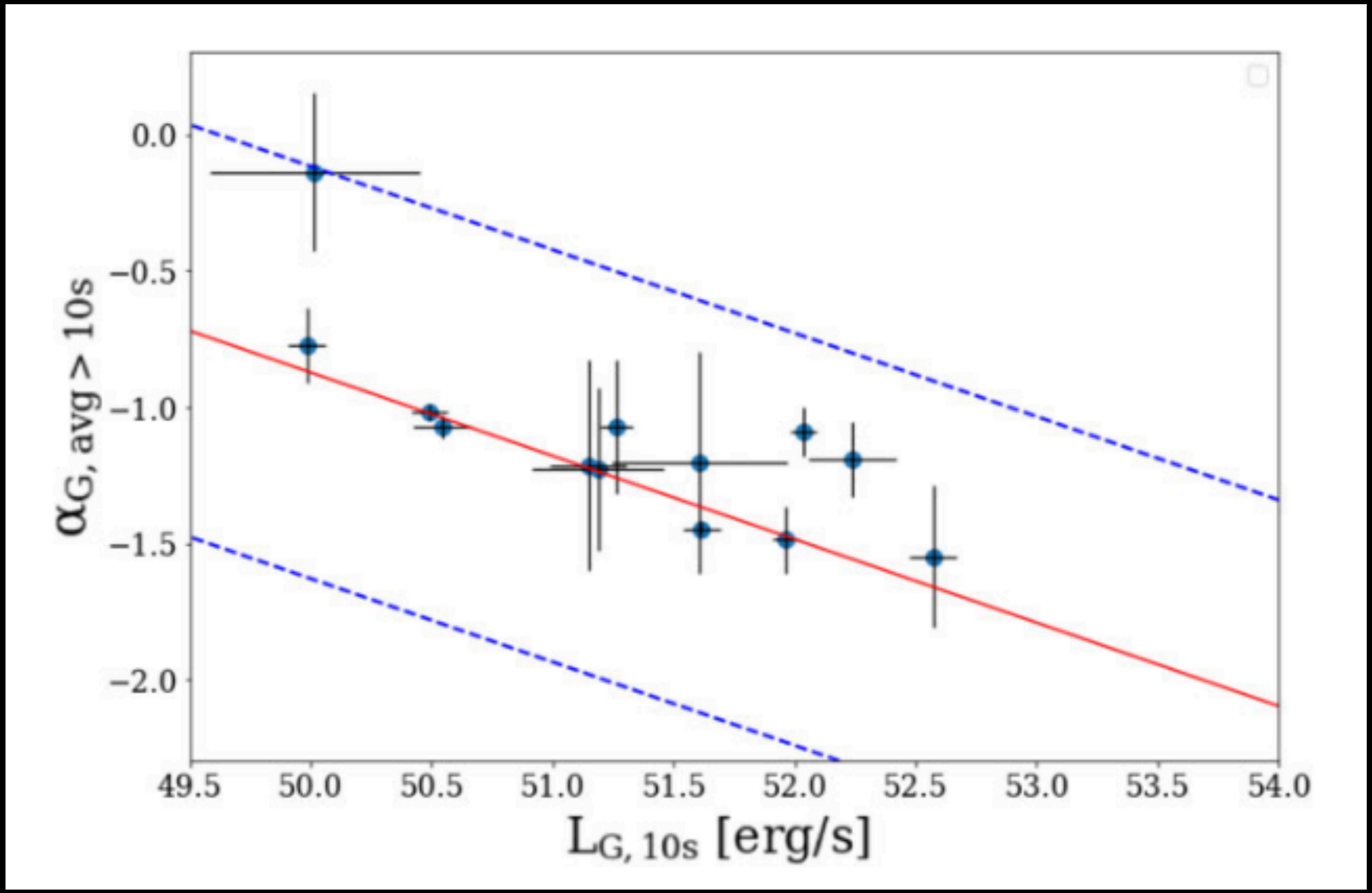
Correlation

GeV

- 13 *Fermi*/LAT 100MeV - 100 GeV rest frame light curves

- Correlation of luminosity at 10s and average rate of decay past this time:

$$\alpha_{G,>10\,s} = -0.31^{+0.12}_{-0.09}\log(L_{G,\,10\,s}) + 14.43^{+4.55}_{-5.97}$$



Hinds et al. 2023

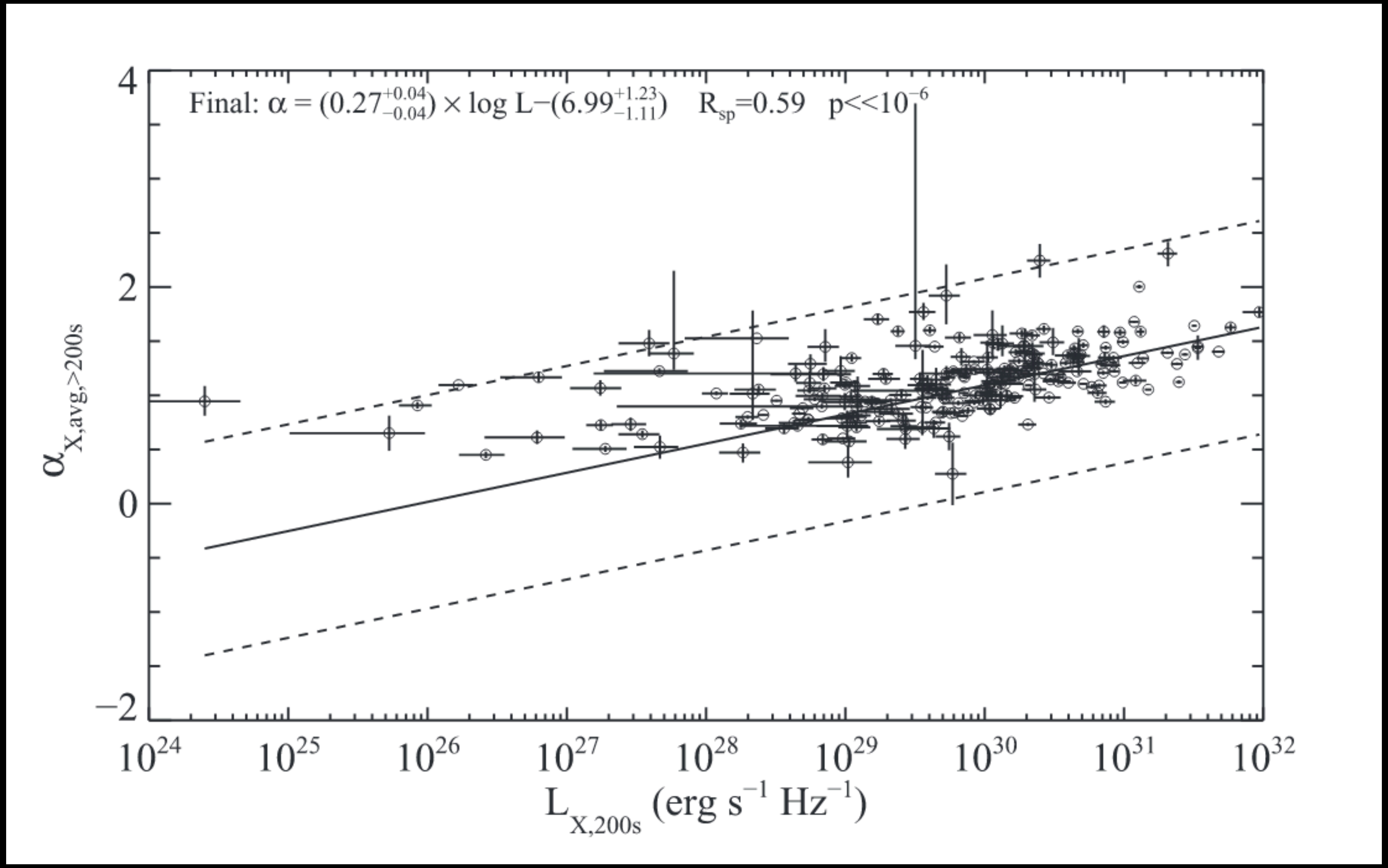
Correlation

X-ray

- 237 *Swift*/XRT 1keV rest frame light curves
- Correlation of luminosity at 200s and average rate of decay past this time:

$$\alpha_{X,>200\,s} = 0.27^{+0.04}_{-0.04} \log L_{X,\,200\,s} - 6.99^{+1.23}_{-1.11}$$

- $F(t, \nu) \propto t^{-\alpha}\nu^{-\beta}$ convention

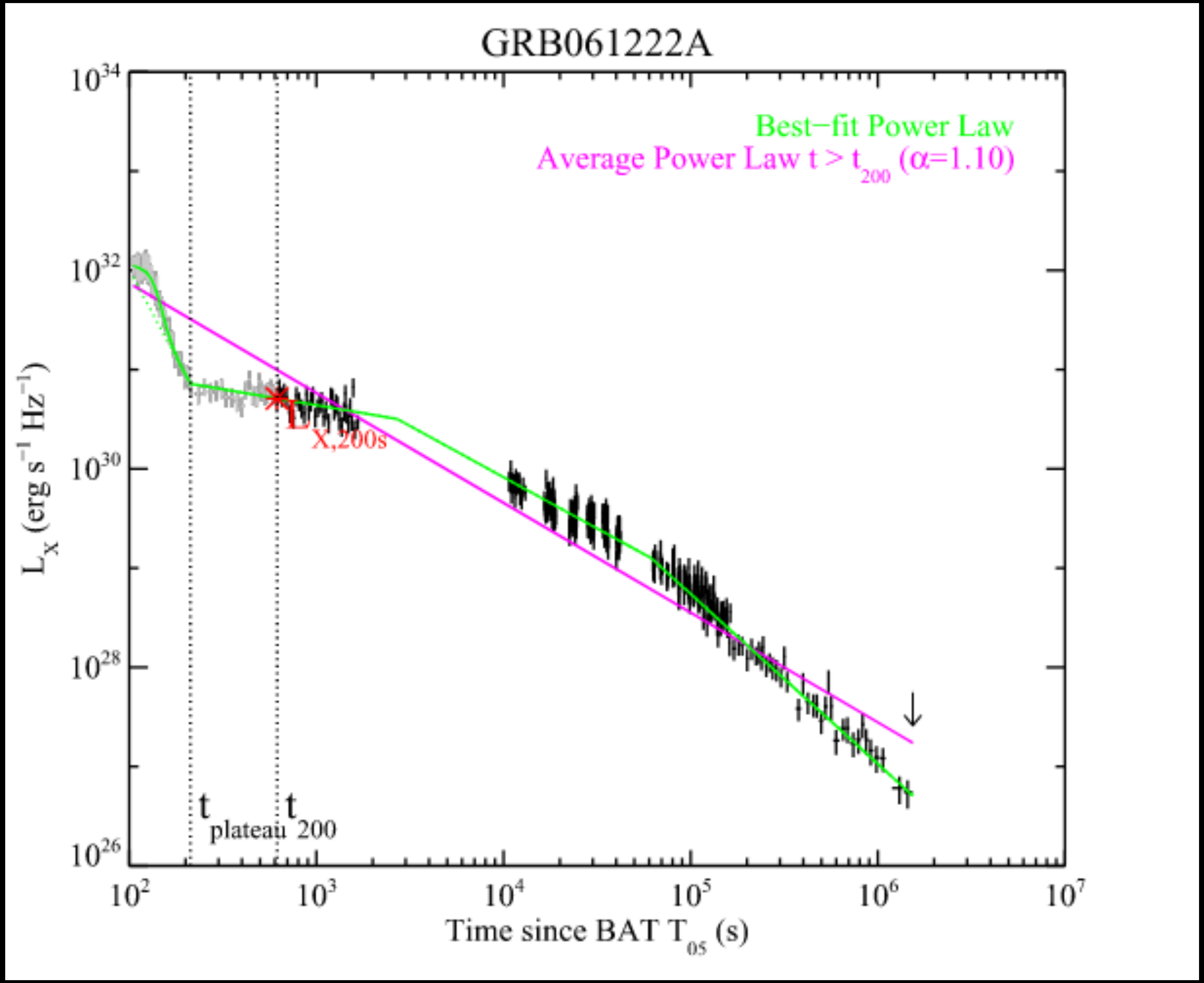


Racusin et al. 2016

Racusin et al. 2016

Light curves

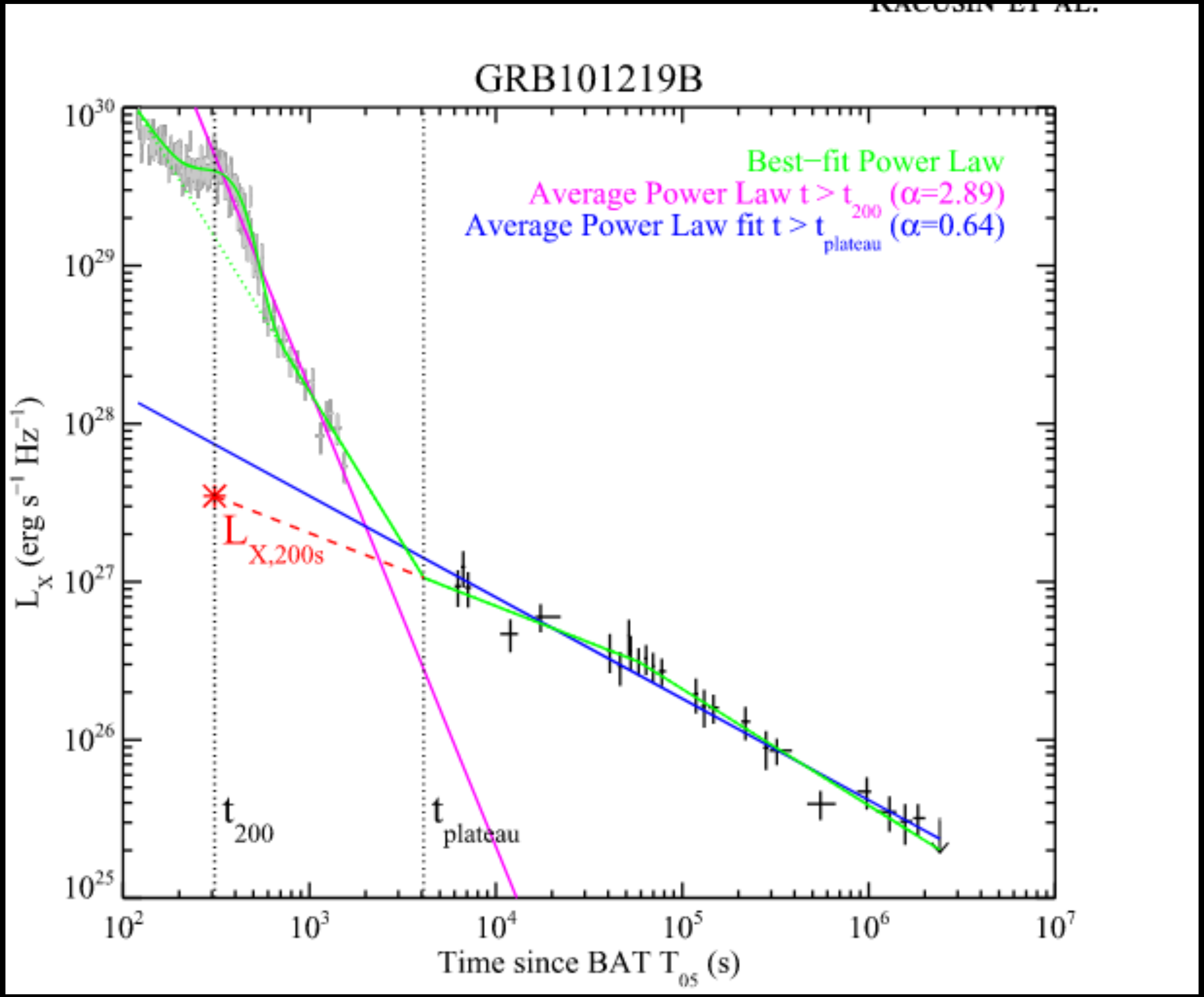
- Fit light curve with X-ray canonical segments
- Interpolate $L_{200\,\mathrm{s}}$ from the fit
- Simple average power law fit for $t > 200\,\mathrm{s}$ gives $\alpha_{\mathrm{X},>200\,\mathrm{s}}$



Racusin et al. 2016

Light curves - Steep decay contamination

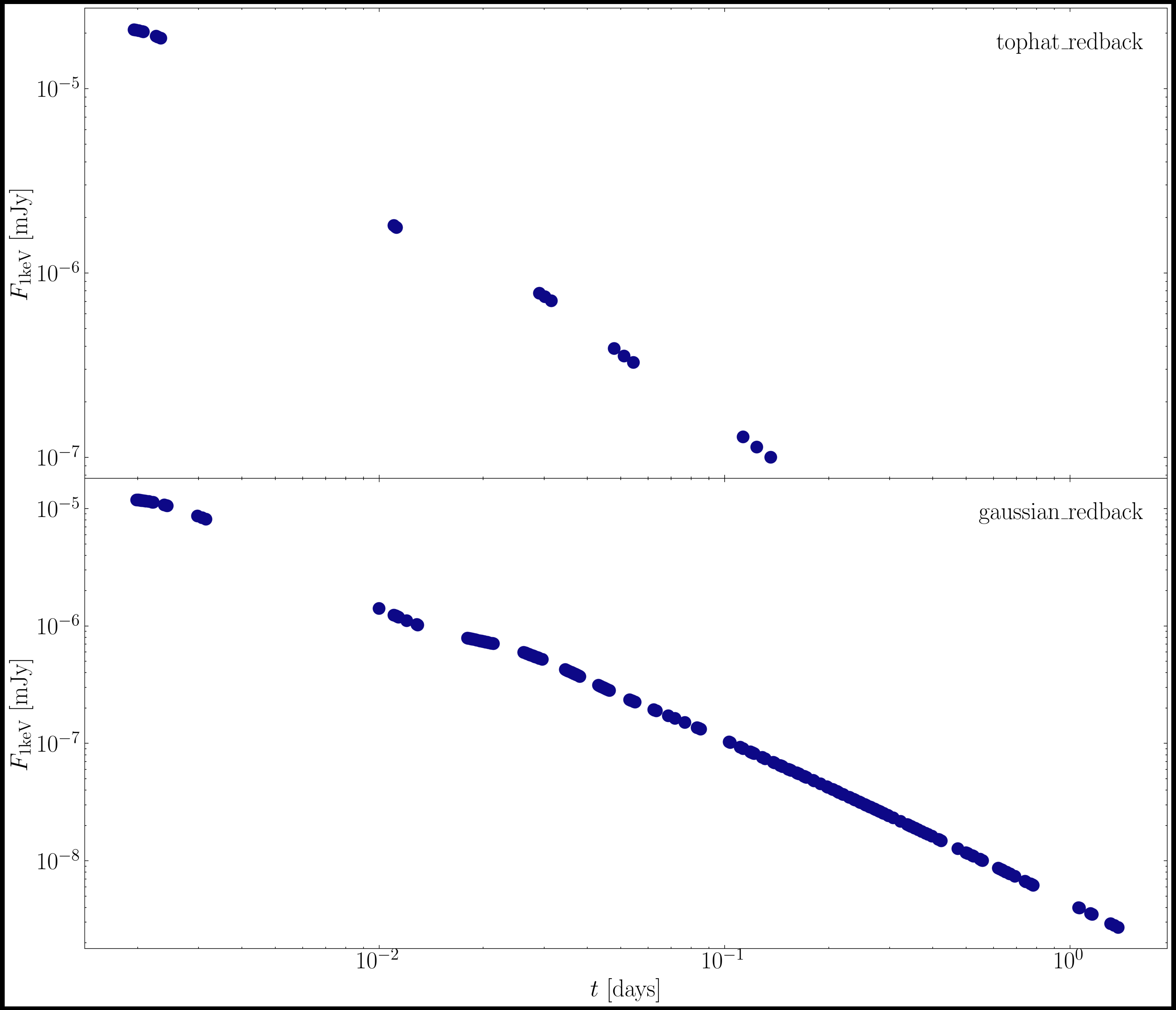
- Where 200s occurs during steep (extended prompt) decay segment
- Extrapolate $L_{200\,\mathrm{s}}$ from the segment fit following steep decline
- Ignore steep decline data for average power law fit for $\alpha_{\mathrm{X},>200\,\mathrm{s}}$



Method

Light curve generation

- Simulated 237 light curves using the native tophat and Gaussian jet structure models[1] in the REDBACK[2] python tool
- Cadence implemented from random sampling of XRT light curves used in Racusin et al. 2016
- XRT-like flux density threshold



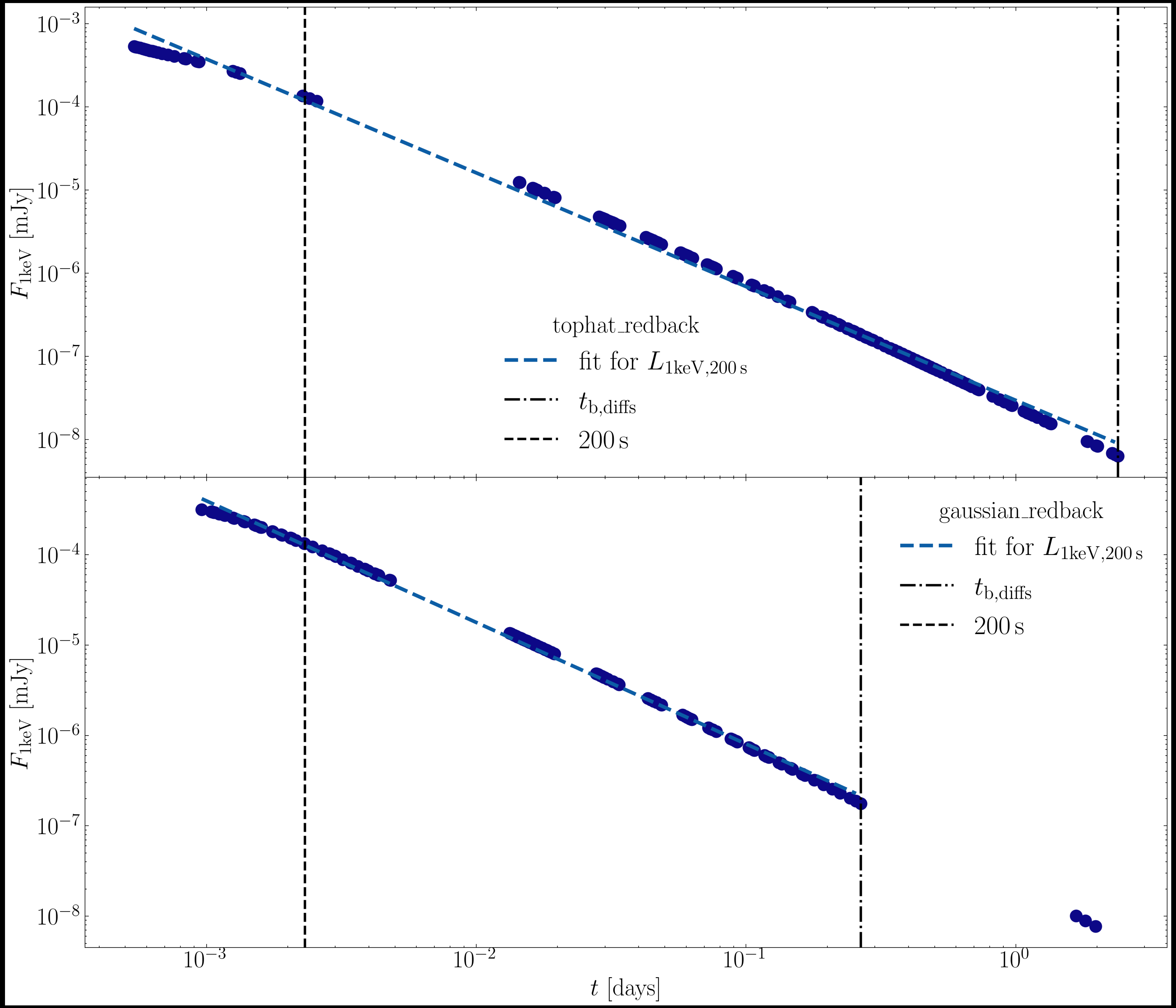
[1] Lamb, Mandel & Resmi 2018
[2] Sarin et al. 2024

Method

Luminosity

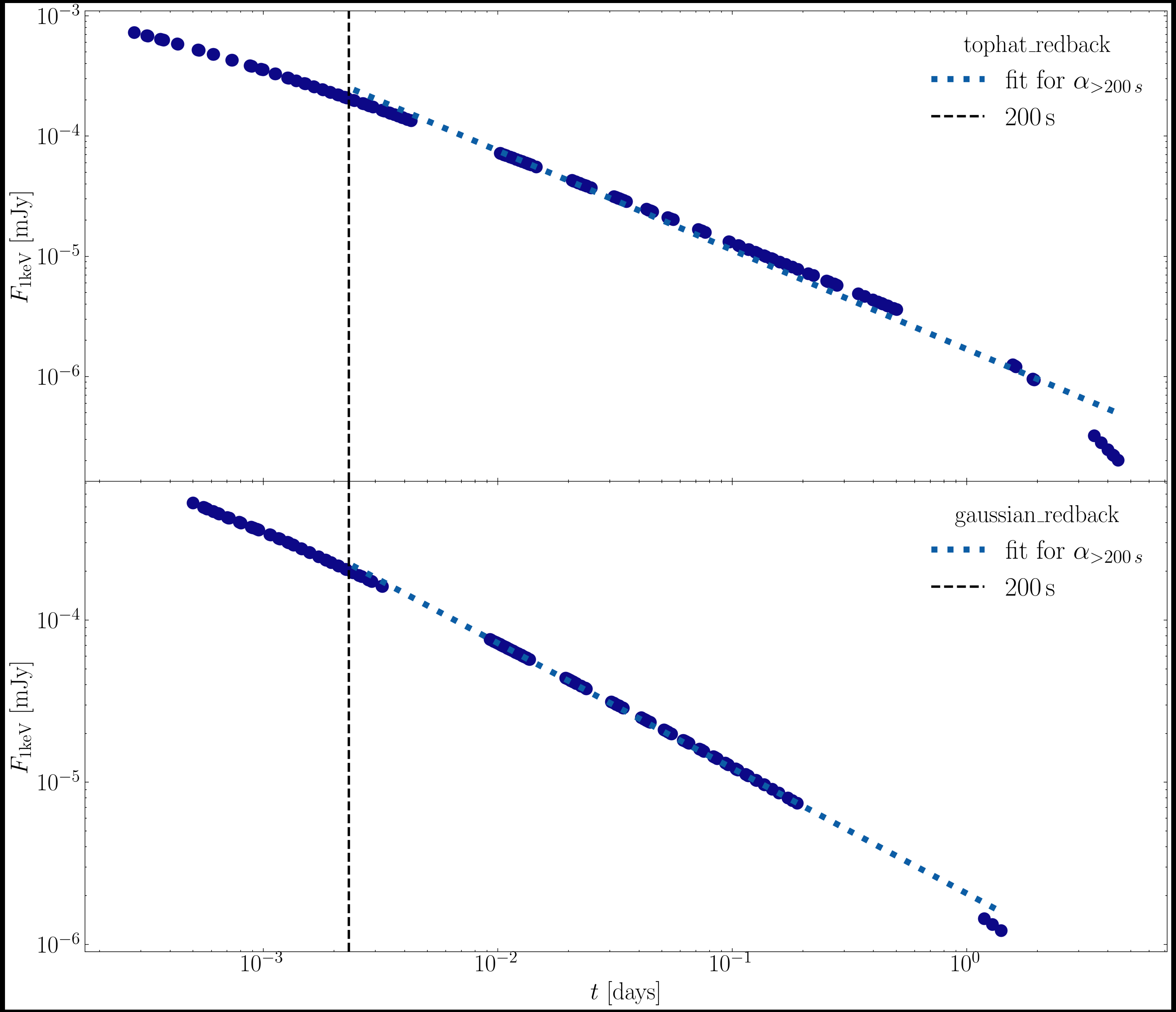
- Inferred $L_{200\,\mathrm{s}}$ from peak-to-break segment fitting and interpolation/extrapolation at rest frame 200s

- Break found with constraint $|\dot{F}_{\nu,i+1} - \dot{F}_{\nu,i}|_{\mathrm{min}} \leq -0.2$, otherwise "break" set as end of light curve



Method
Decline index

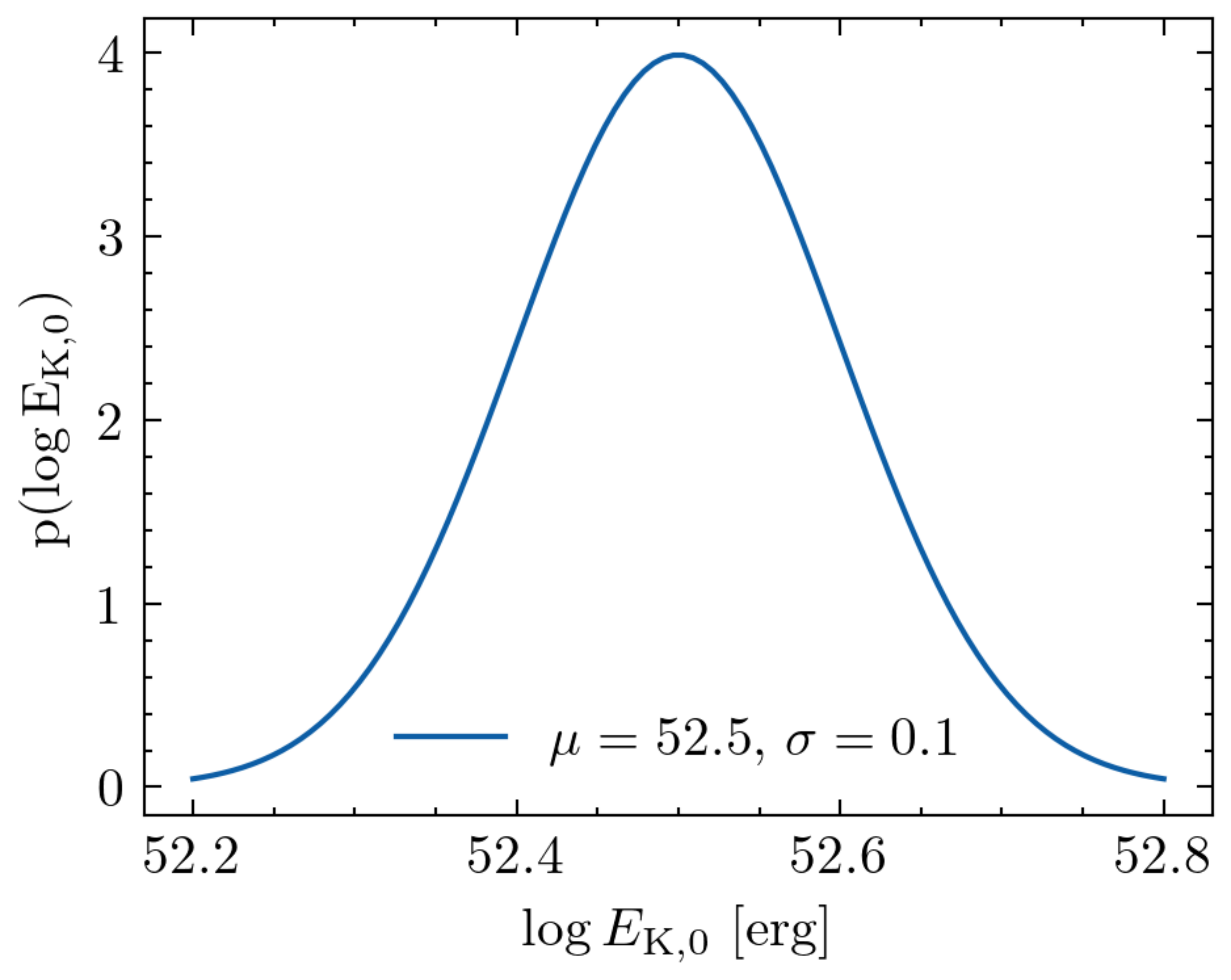
- Inferred $\alpha_{\rm avg}$ from average power law fit from $\rm t = max(200\,s,\ t_{peak})$ to the end of light curve



Method

Maximum viewing angle and jet kinetic energy spread

- Modelled the jet kinetic energy distribution as a Gaussian with $\sigma_{\mathrm{K},0}$
- Inclinations sampled from within θ_{max} as a fraction of θ_{c}
- Varied θ_{max}, $\sigma_{\mathrm{K},0} \in [0,...,2.0]$ in increments of 0.1



Method

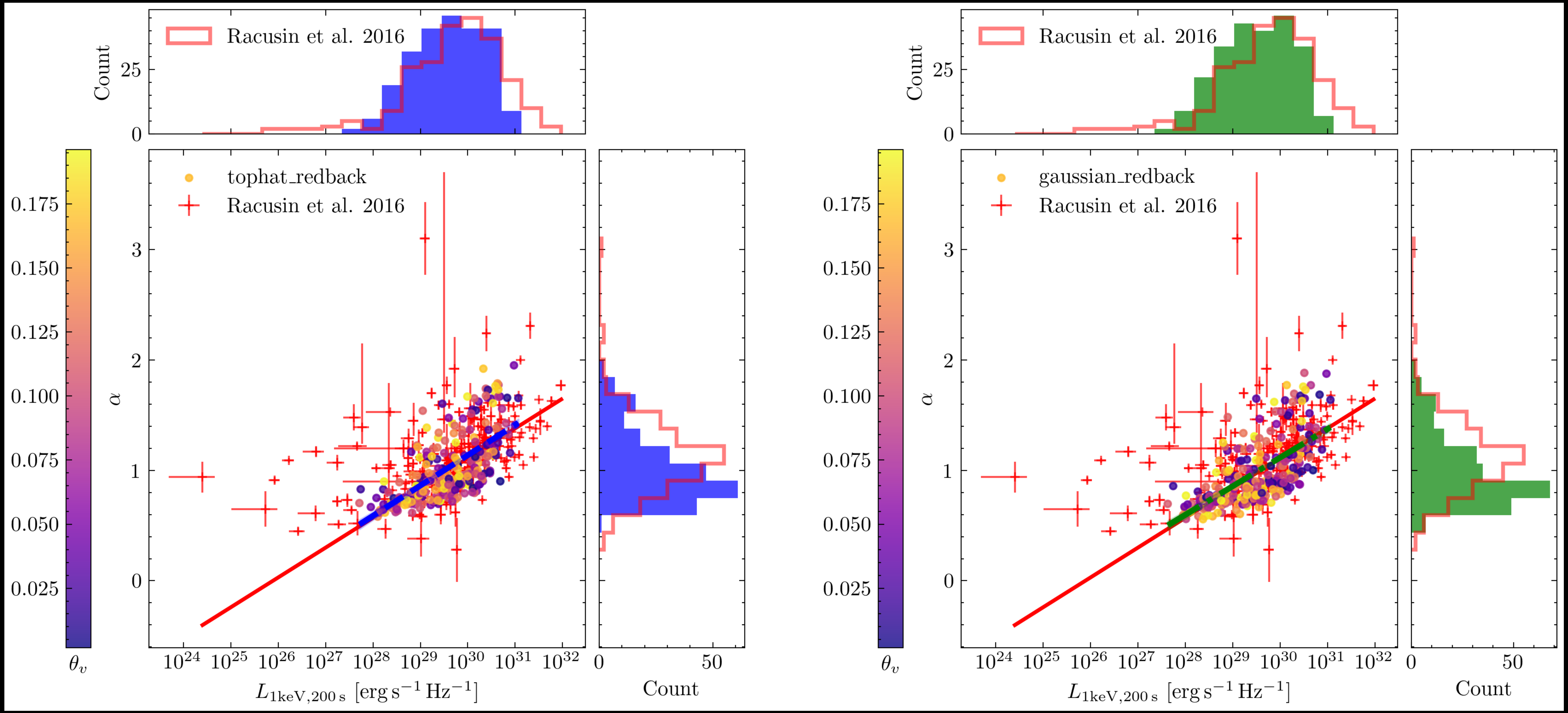
Fiducial Parameter Distributions

Parameter	Profile	Min	Central	Max
θ_i	Cosine	0	N/A	$1.0\ \theta_\mathrm{c}$
θ_c	Constant	N/A	0.2	N/A
θ_j	Constant	N/A	0.4	N/A
p	Truncated Gaussian ($\sigma_p = 0.7$)	1.3	1.9	3.0
Γ_0	Constant	N/A	500	N/A
$\log n_0$	Loguniform	-3	N/A	1
z	Empirical[1]	0	N/A	8
$\log E_{\mathrm{K},0}$	Gaussian ($\sigma_E = 0.1$)	$-\infty$	52.5	$+\infty$
k	Constant	N/A	0	N/A
$\log \epsilon_e$	Constant	N/A	-0.5	N/A
$\log \epsilon_B$	Constant	N/A	-3.0	N/A

[1] Wanderman & Piran 2010

Results

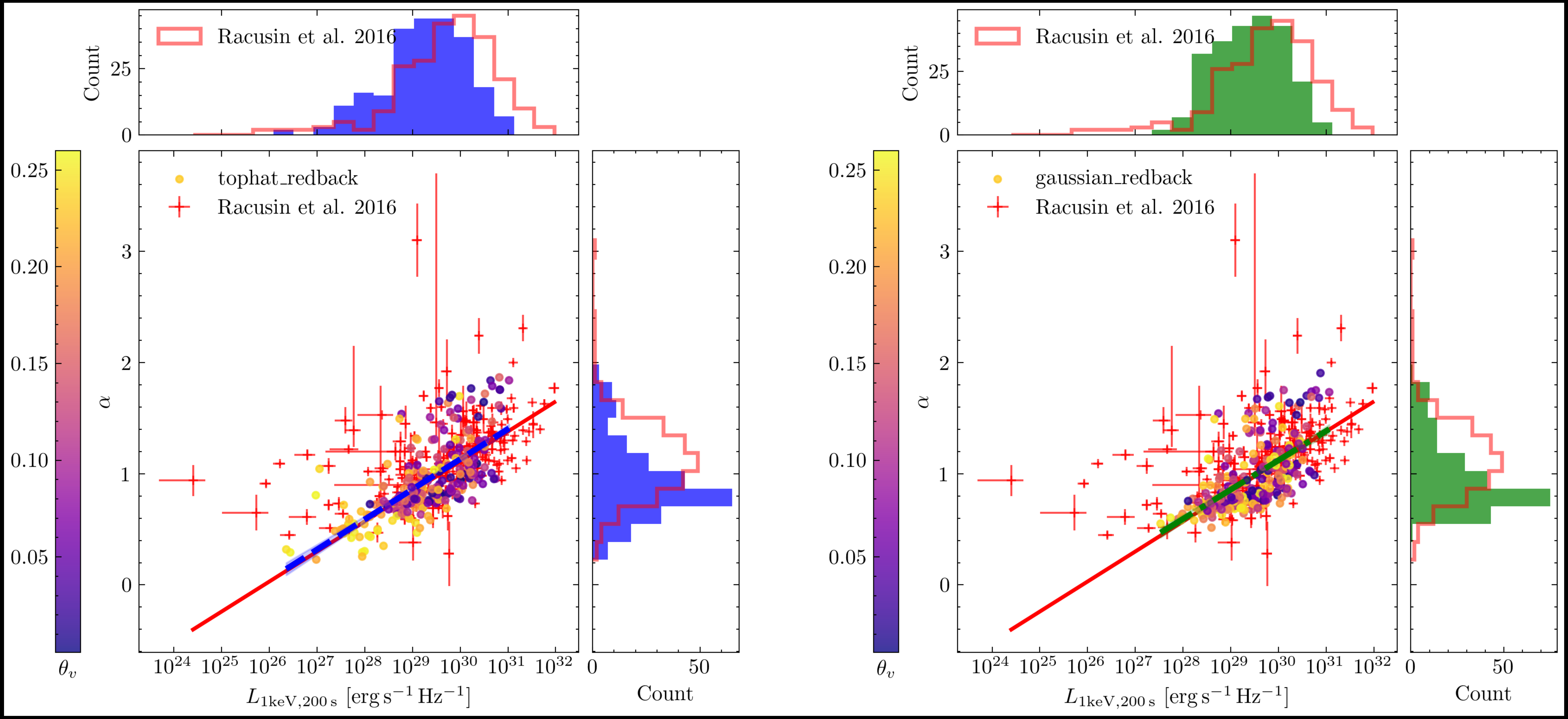
Correlation



Turnbull et al. (in prep.)

Results

Correlation - $\theta_{\mathrm{max}} = 1.3\,\theta_{\mathrm{c}}$

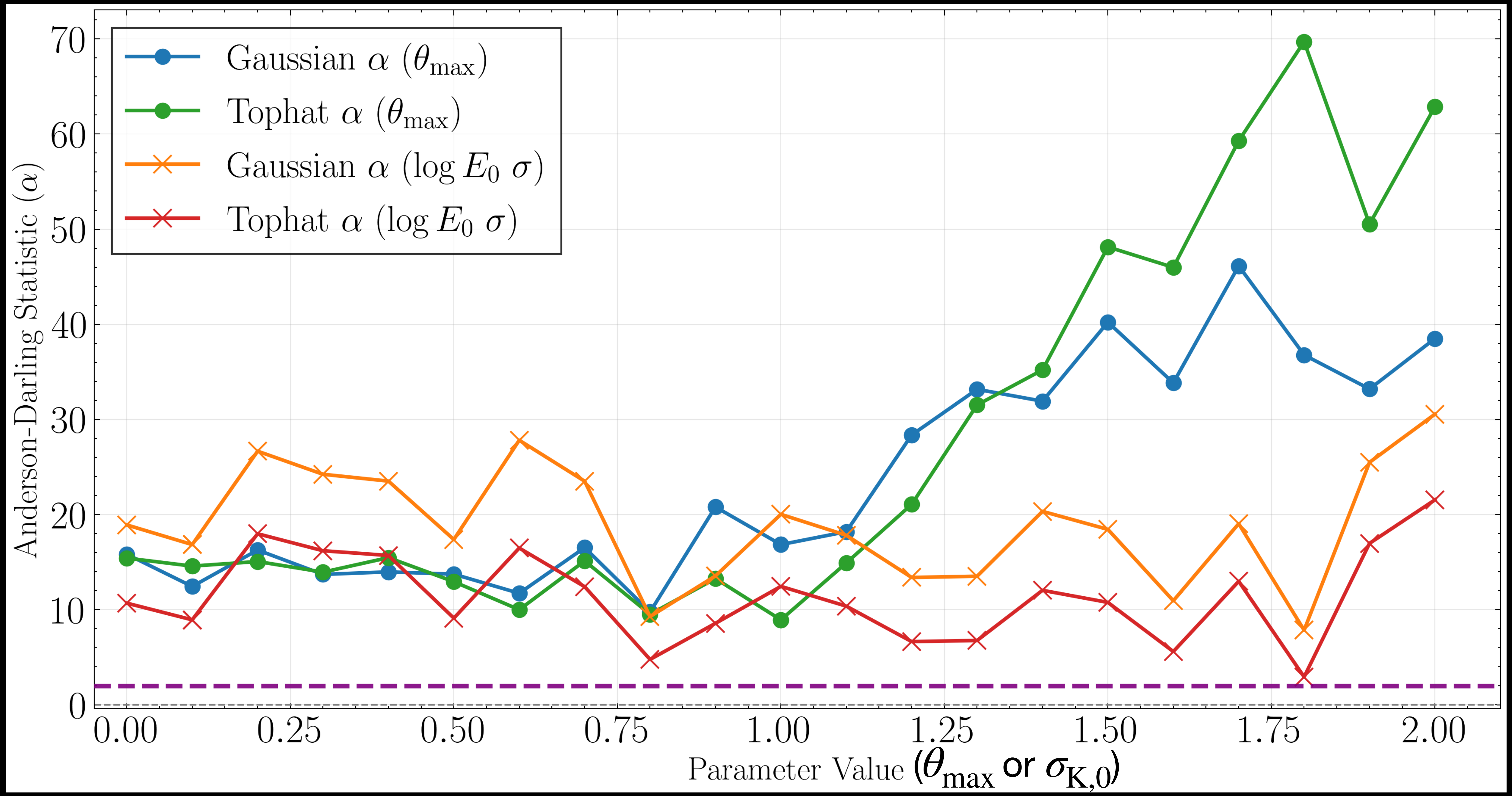


Turnbull et al. (in prep.)

Results

Two-sample Anderson-Darling statistic test

- Tests null hypothesis that both samples are drawn from the same distribution
- Tested varying θ_{max} or $\sigma_{\mathrm{K,0}}$ from 0 to 2.0 in increments of 0.1
- Divergence seen for the α_{avg} samples at $\theta_{\mathrm{max}} \geq 1.2\,\theta_{\mathrm{c}}$ for both structures
- Little evolution in α_{avg} from varying $\sigma_{\mathrm{K,0}}$ for either structure

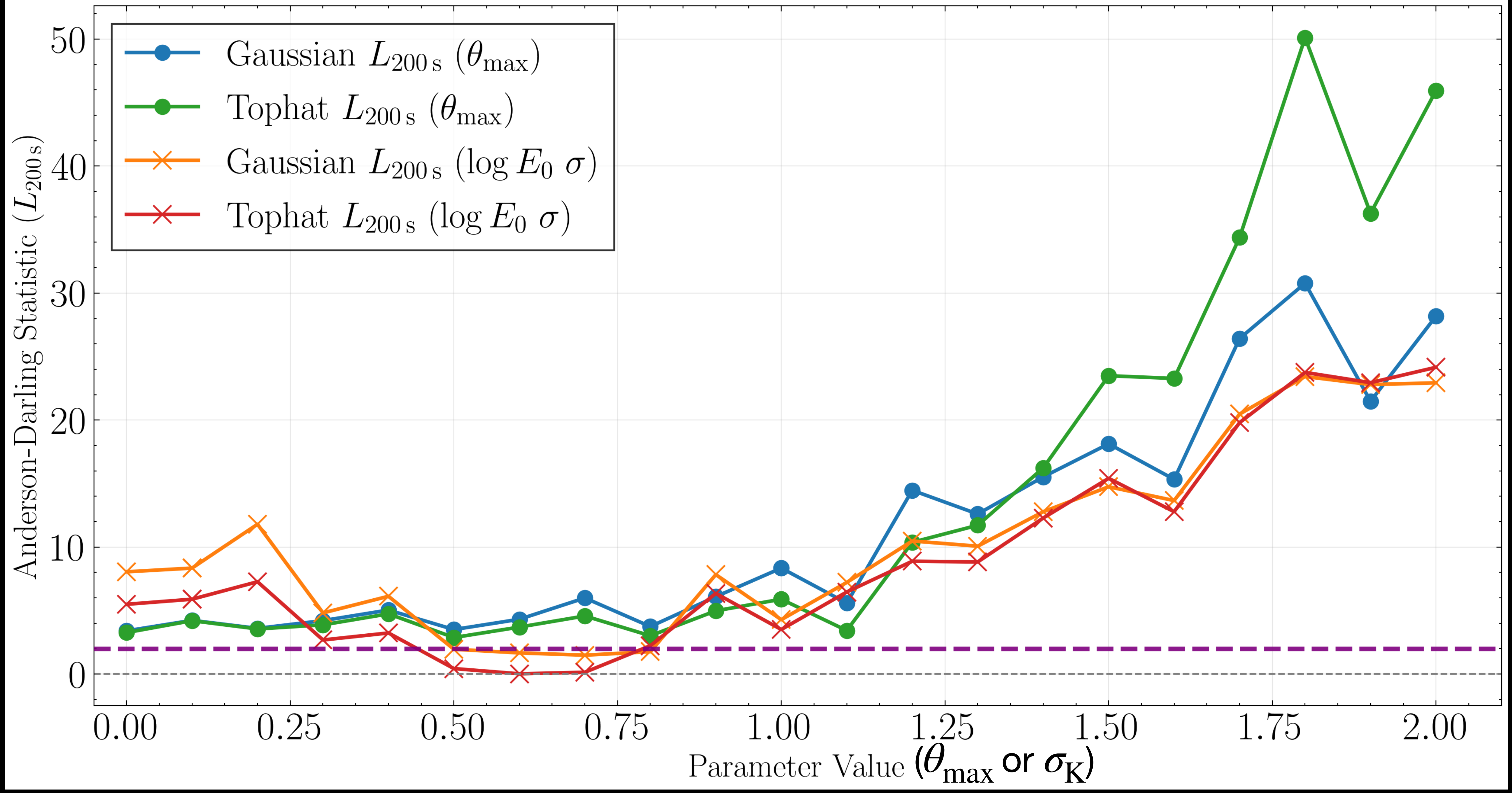


Turnbull et al. (in prep.)

Results

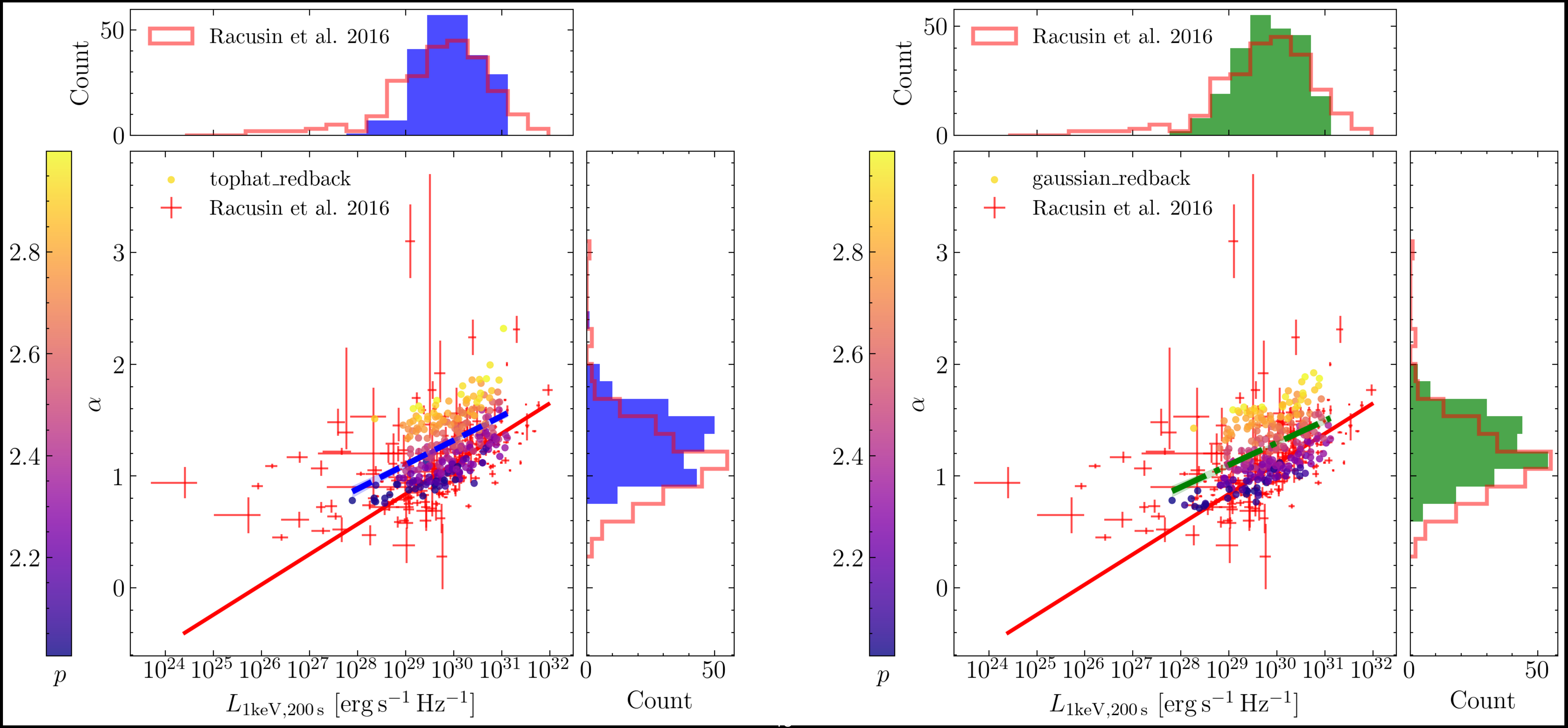
Two-sample Anderson-Darling statistic test

- Divergence at >1.3 for both parameters ($\theta_{\max}$ and $\sigma_{\mathrm{K},0}$)
- Both structures drop below 5% critical value from $\sigma_{\mathrm{K},0} = 0.5$ to $\sigma_{\mathrm{K},0} = 0.7$
- Possible support for quasi-universal jet energies for both structures



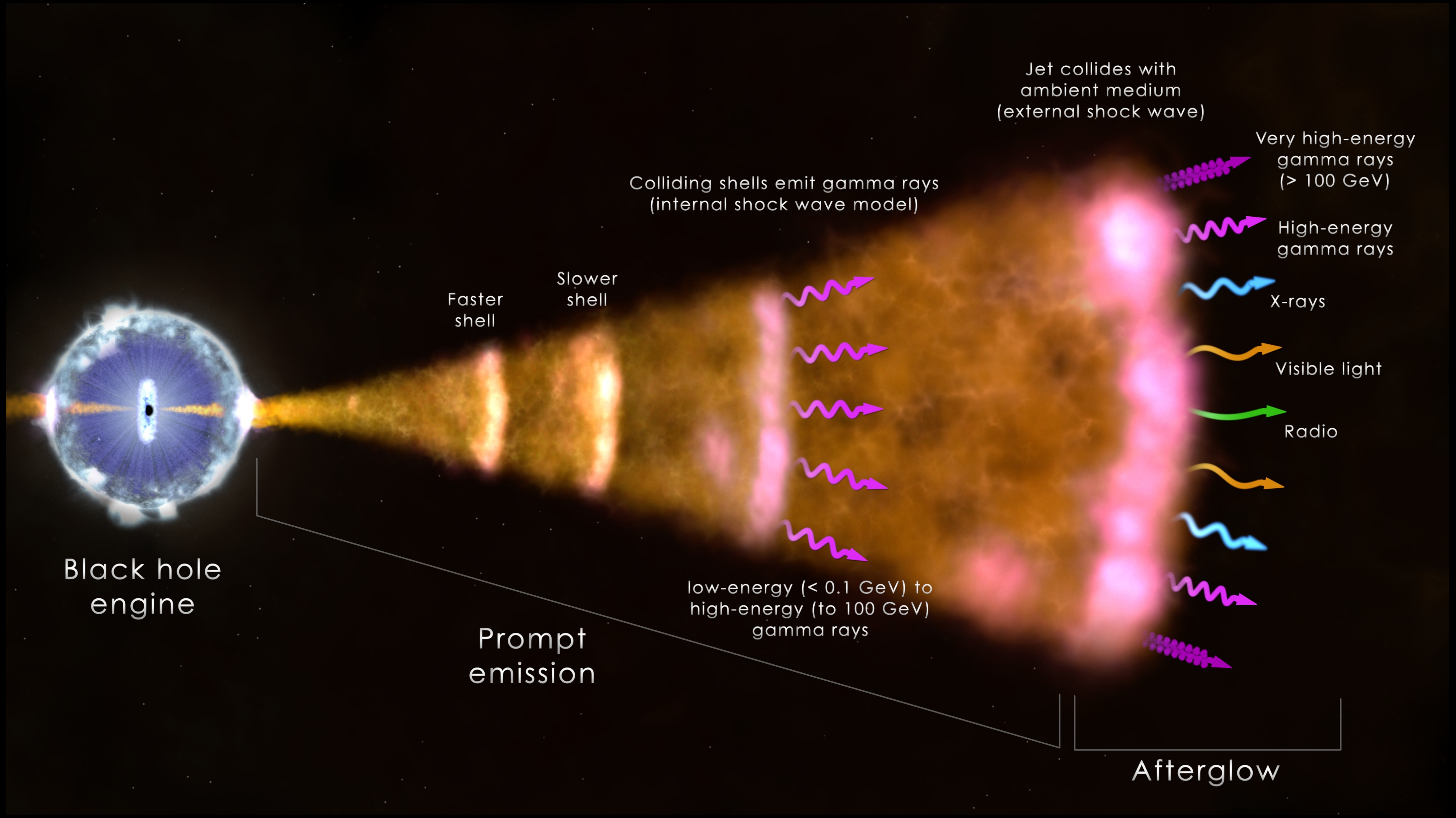
Turnbull et al. (in prep.)

$\mu_p = 2.2, p \geq 2.0$



Implications

- Luminosity constraints would allow for a cosmological probe once the underlying cause is known, seemingly viewing angle
- Next step is to compare $E_{\gamma,\mathrm{iso}}$ of Racusin et al. 2016 GRBs with their position on correlation space
- Analytical still to be done



Obligatory GRB bingo square

Conclusions

- We are able to synthetically reproduce the Oates correlation in 1keV
- We do not find the correlation is caused by jet structure
- The relation instead seems to be due to differences in the observer viewing angle including off-axis viewing angles
- Analysis of the fiducial parameter distributions suggests a quasi-universality of jet energies of $\log E_{K,0} \approx 52.5^{+0.7}_{-0.7}$
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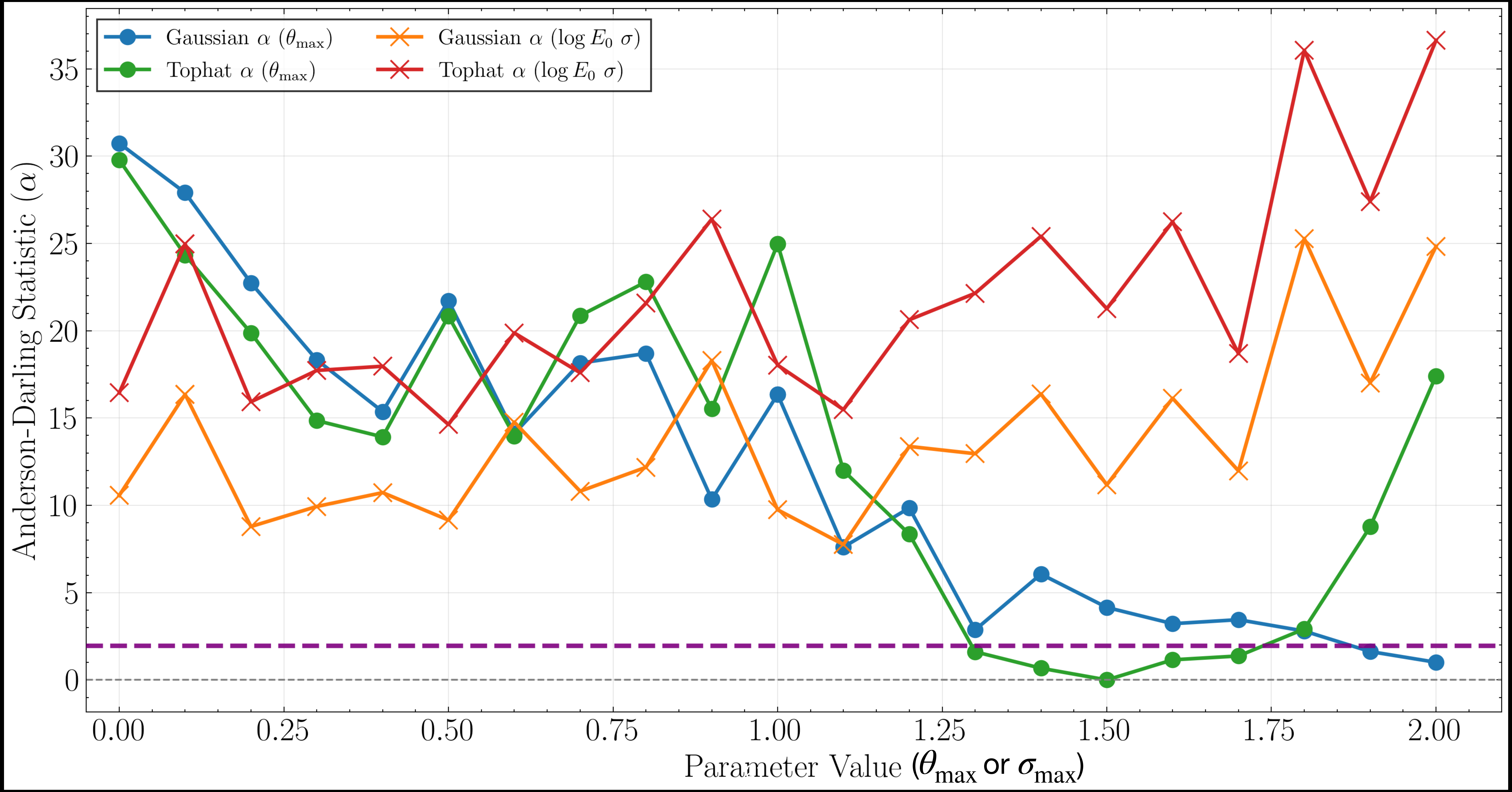
$\mu_p = 2.2,\ p \geq 2.0$

Two-sample Anderson-Darling statistic test (α)



$p \geq 2.0,\ \mu_p = 2.2$

Two-sample Anderson-Darling statistic test (L_{200s})