Unsolved issue in Computational Relativistic Astrophysics

Tsvi Piran
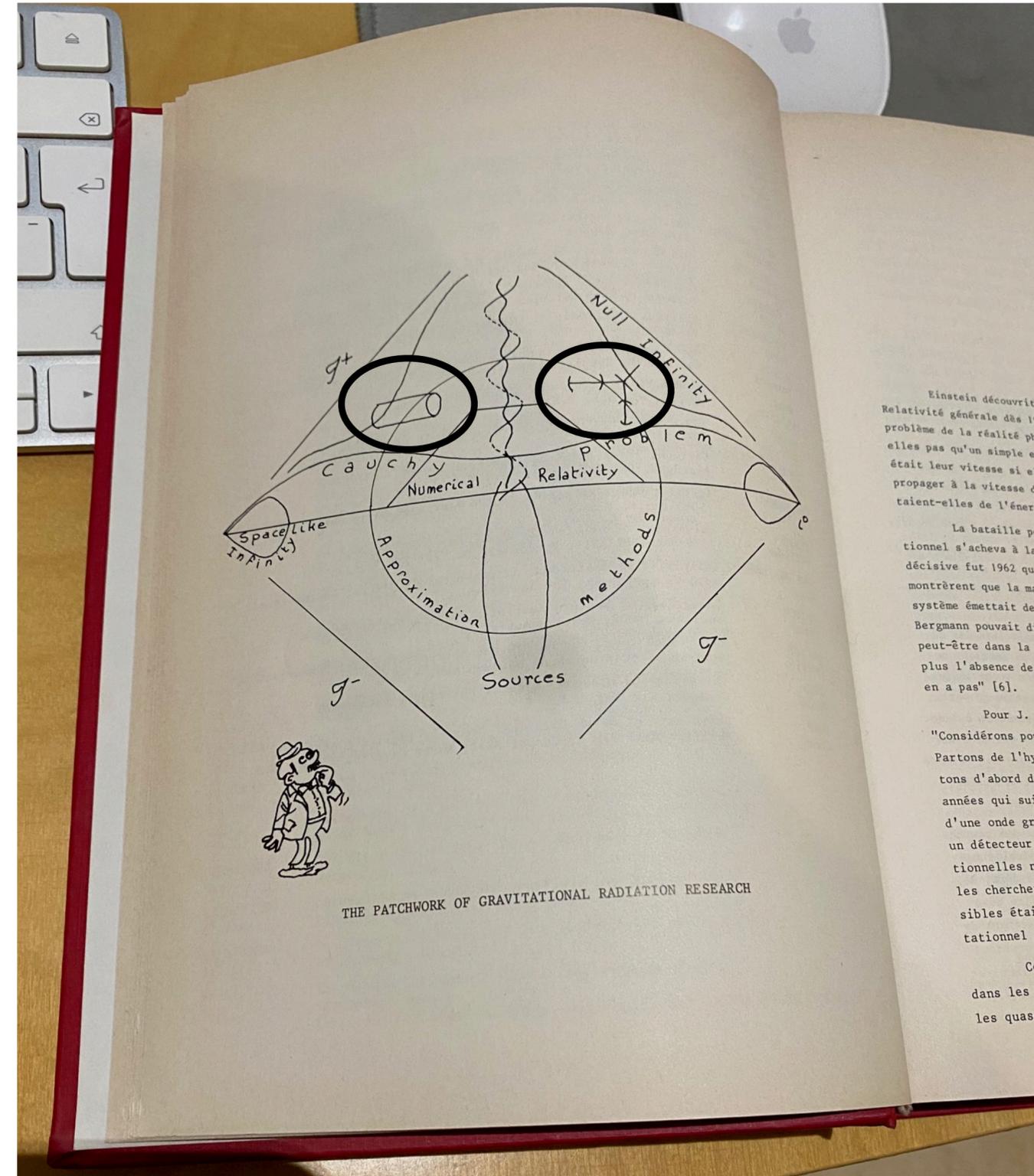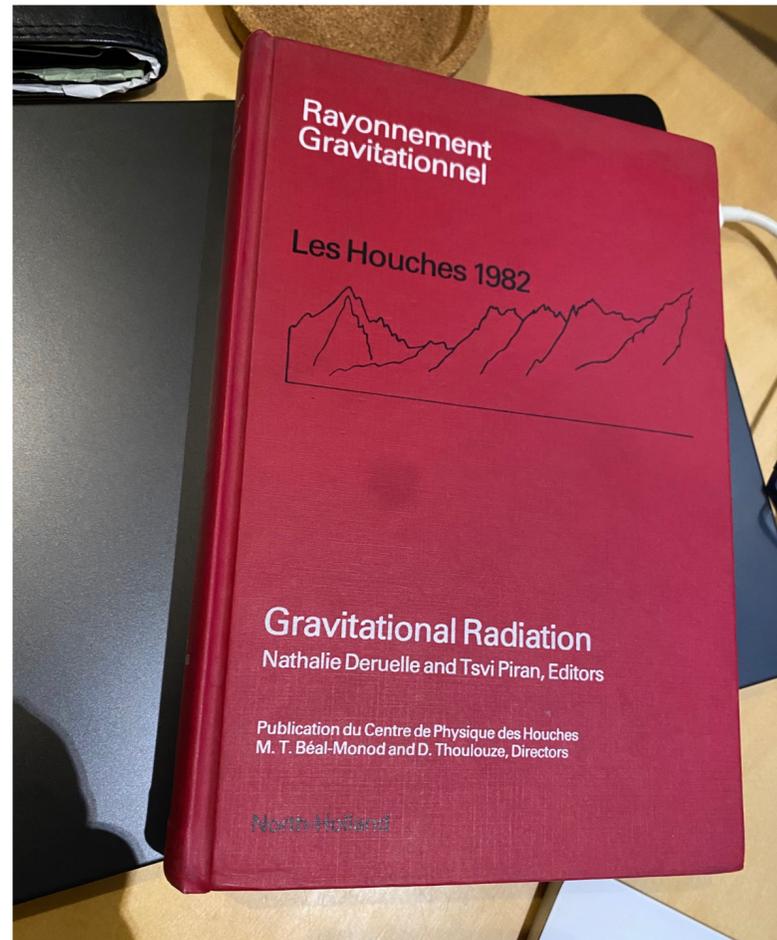
The Hebrew University, Jerusalem

Prehistory
Les Houches 1982

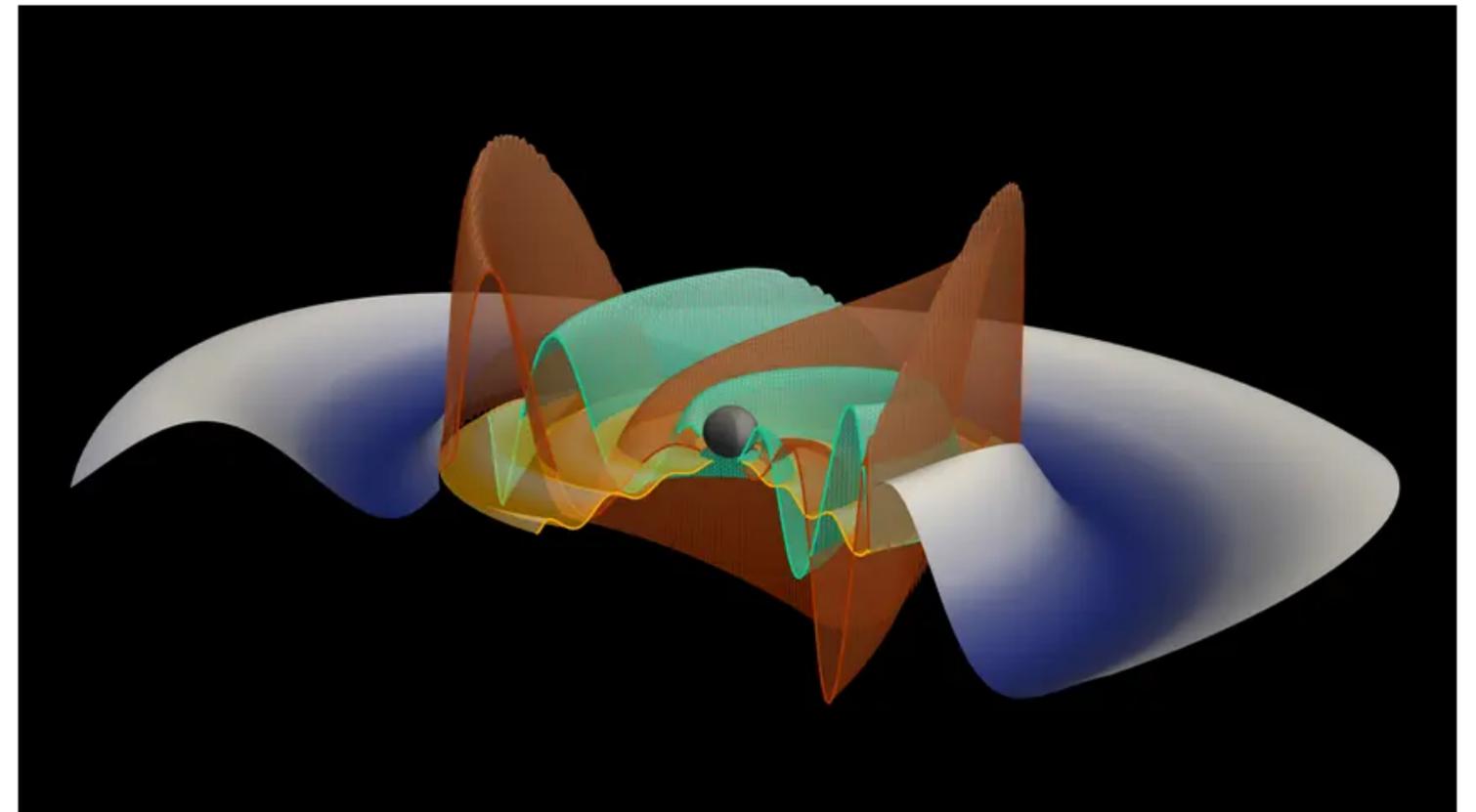- Can numerical simulationsreveal the observed gravitational waves?

- Damour - no need analytic

- Piran - yes

THE PATCHWORK OF GRAVITATIONAL RADIATION RESEARCH

Most recent success
LIGO-Virgo-Kagra observations of Kerr Normal modes

- First and second normal modes identified in the gravitational waves signal of GW250114

Visualization of a binary black hole ringdown consistent with the gravitational-wave event GW250114.The gravitational waves are separated into two modes of the ringing remnant black hole, identified in the observation: the fundamental mode (green) and its first overtone (red). It also shows a predicted third tone (yellow) that the data places limits on. Visualization performed at the Max Planck Institute for Gravitational Physics (Albert Einstein Institute), based on a numerical relativity simulation of the Simulating Extreme Spacetimes (SXS) Project.
H. Pfeiffer, A. Buonanno (Max Planck Institute for Gravitational Physics), K. Mitman (Cornell University)

Pre History
Stark and Piran 1985

- Carried out on Cray 1 = 80-150 Mflops

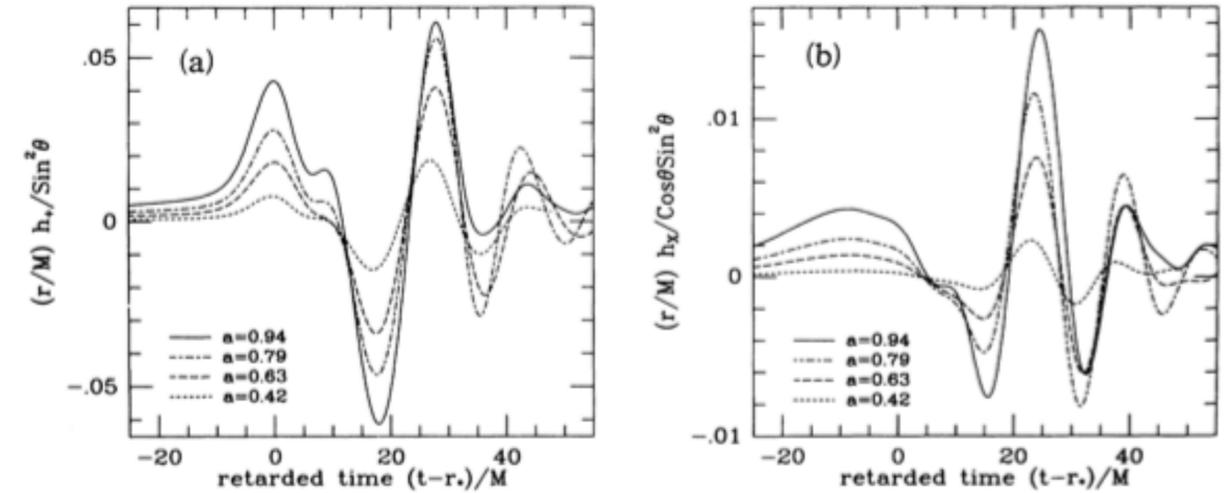- 2025 Macbook pro - 10s Tflops

FIG. 2. (a) Plus and (b) cross mode wave forms for the collapse to a black hole by stars ($f_p = 0.01$) of various angular momenta a.

4人組時代からの課題　an unsolved issue from the time of "gang of four".

1. ブラックホール(BH)形成時の放出重力波を定量的に評価したい。
   need to make quantitative estimate of GW emission at BH formation.

Prehistory

The gang of four - Takashi Nakamura, Ken-Ichi Madea, Kei-ichi Maeda, Shoken Miyama, Misao Sasaki

- M2になった頃(?)中村さんに誘われる・・・何か新しくてできることをしよう！

Nakamura-san proposed: let's work on something interesting and feasible!

⟹ 数値相対論
numerical relativity

4人組（中村・前田・観山・佐々木）
Nakamura, Maeda, Miyama, Sasaki: "gang of four"

CAR - Computer Aided Relativity

T Nakamura,   K Maeda,   S Miyama & M. Sasaki

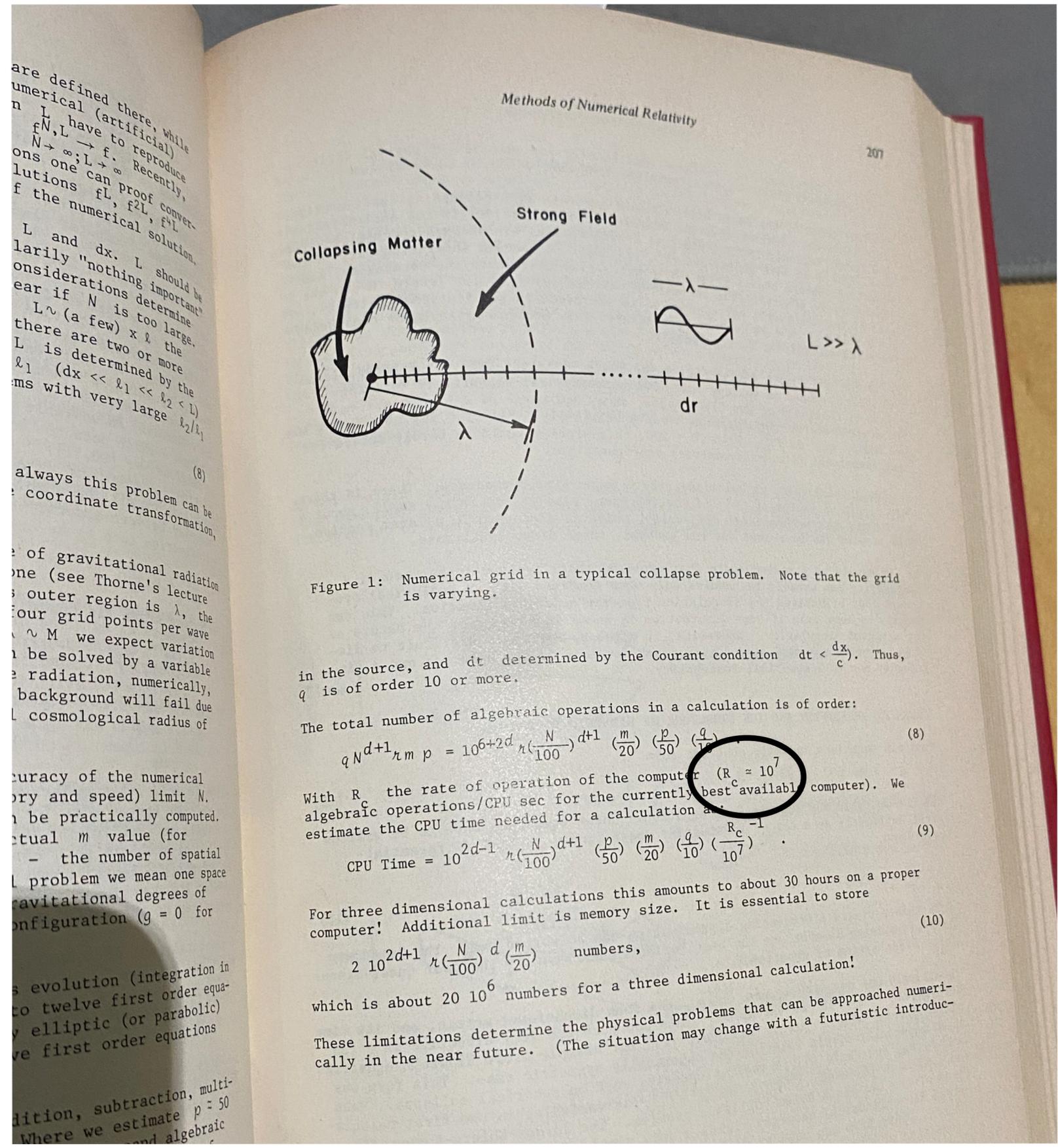Boss   semi-Boss

privates ...

S Miyama

better known (?) as
Buddhist monk

Prehistory
Les Houches 1982

- Computation time and memory

Collapsing Matter

Strong Field

$L \gg \lambda$

dr

λ

Figure 1: Numerical grid in a typical collapse problem. Note that the grid is varying.

in the source, and dt determined by the Courant condition $dt < \frac{dx}{c}$. Thus, q is of order 10 or more.

The total number of algebraic operations in a calculation is of order:

$$q N^{d+1} n m p = 10^{6+2d} n (\frac{N}{100})^{d+1} (\frac{m}{20}) (\frac{p}{50}) (\frac{q}{10}) \tag{8}$$

With R_c the rate of operation of the computer ($R_c \approx 10^7$ algebraic operations/CPU sec for the currently best available computer). We estimate the CPU time needed for a calculation as

$$\text{CPU Time} = 10^{2d-1} n (\frac{N}{100})^{d+1} (\frac{p}{50}) (\frac{m}{20}) (\frac{q}{10}) (\frac{R_c}{10^7})^{-1} . \tag{9}$$

For three dimensional calculations this amounts to about 30 hours on a proper computer! Additional limit is memory size. It is essential to store

$$2 \cdot 10^{2d+1} n (\frac{N}{100})^d (\frac{m}{20}) \quad \text{numbers,} \tag{10}$$

which is about $20 \cdot 10^6$ numbers for a three dimensional calculation!

These limitations determine the physical problems that can be approached numerically in the near future. (The situation may change with a futuristic introduc-

BNS mergers
How far are we from reproducing that from beginning to end?

- GR is not the main issue but:

  - "Clean Initial values"

  - Accuracy of GW signals phases

- Microphysics within the merger

  - EOS

  - Neutrino & Radiation transport

  - Magnetic fields

  - Short-scale turbulence; viscosity

  - Composition and nuclear physics

SWIFT NEUTRON STAR
COLLISION V. 2

ANIMATION: DANA BERRY
310-441-1735
PRODUCED BY ERICA DREZEK

BNS mergers
How far are we from reproducing that from beginning to end?

- Long-term evolution

  - Pre merger - accurate initial values, Magnetic field interaction, *tidal deformation*, residual eccentricity

  - Post merger - Accretion disk, Jets, Macronova, Afterglow

- Multiscales

  - Low side - Small scale turbulence, magnetic evolution? Dynamos?

  - High side - Orbital scale, macronova scale, Jet scale and afterglow scale
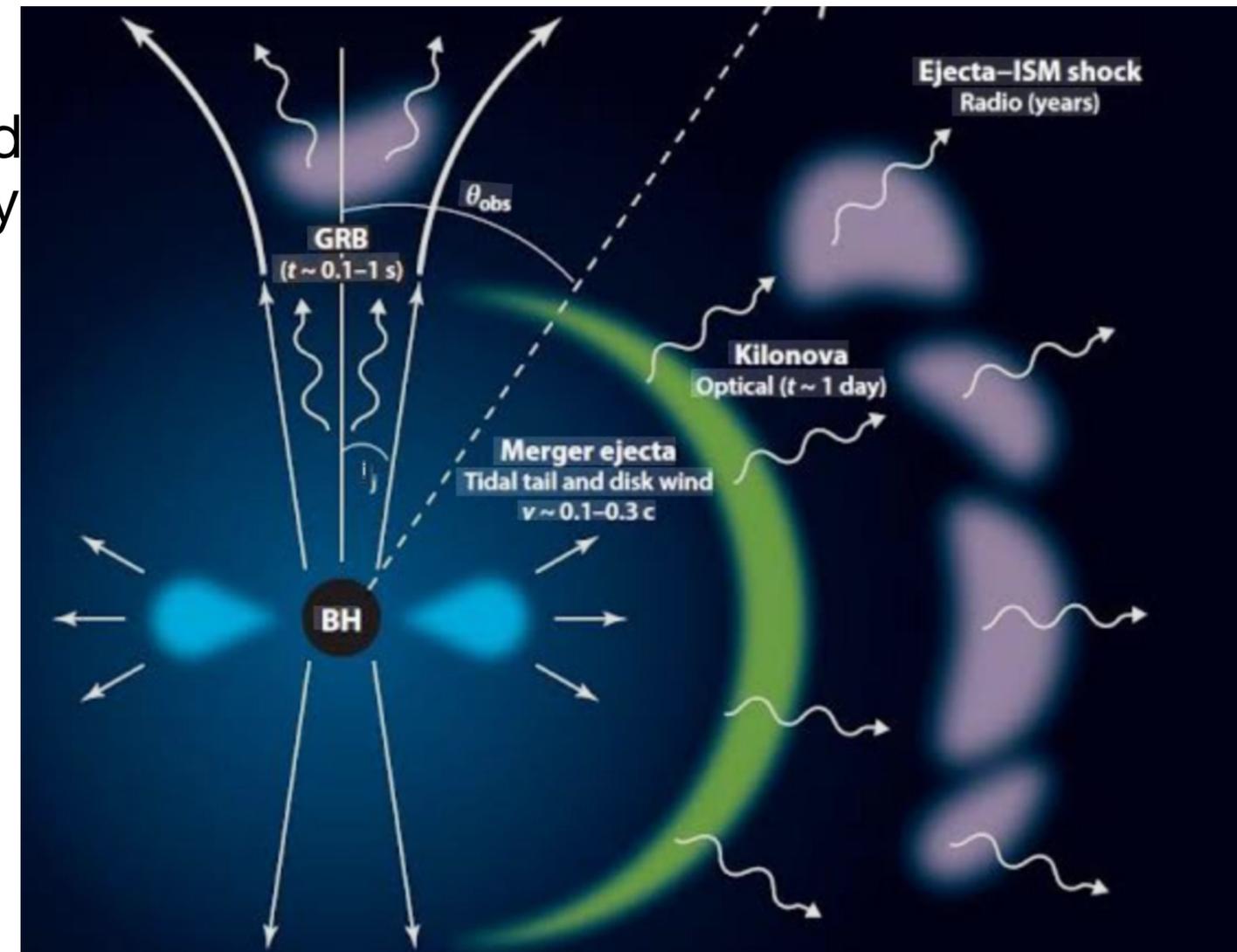
BNS mergers
How far are we from reproducing that from beginning to end?

- The EM signal

  - Atomic physics of the ejecta

  - Shock and reconnection physics

- Numerical issues:

  - Boundary conditions

  - AMR effects

  - Moving from one kind of code to another codes, requires interpolation between grids and physics.;
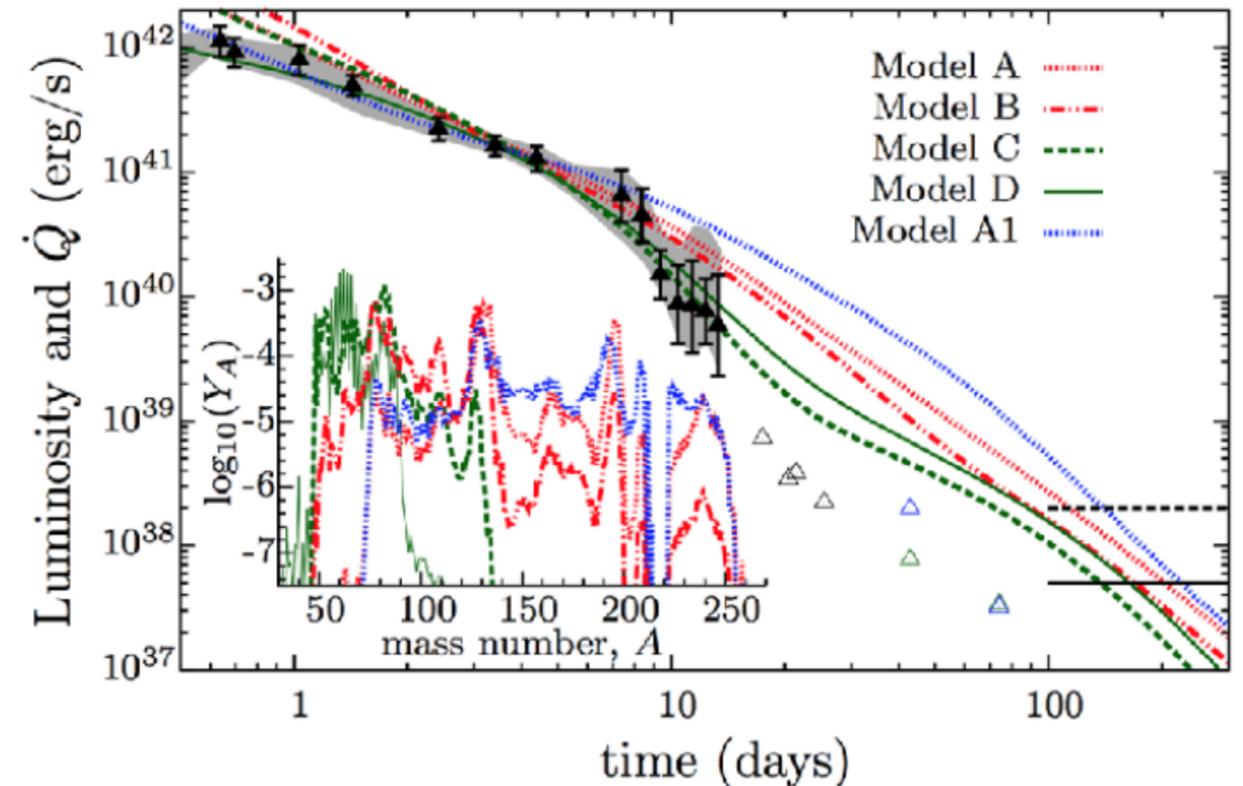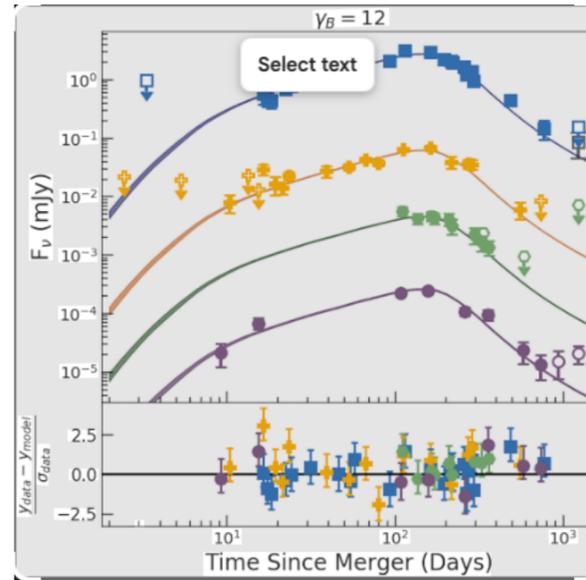
Bolometric luminosity of the kilonova AT2017gfo associated with GW170817 from Smartt et al. (2017) with uncertainties derived from the range given in the literature (Smartt et al. 2017; Waxman et al. 2018; Cowperthwaite et al. 2017; Arcavi 2018). Also shown are lower limits (empty triangles) on the latetime luminosity as inferred from the Ks band with VLT/HAWK-I (Tanvir et al. 2017) (black) and the 4.5 μm detections by the Spitzer Space Telescope from Villar et al. (2018, green) and (Kasliwal et al. 2019, blue). Colored lines show the ejecta heating rate for models with different values for the ejecta mass and average electron fraction as follows: A (Y e = 0.15; M ej = 0.04 M), B (Y e = 0.25; M ej = 0.04 M), C (Y e = 0.35; M ej = 0.055 M), D (Y e = 0.45; M ej = 0.03 M). While models A − D assume the FRDM nuclear mass model (Möller et al. 1995), Model A1 (Y e = 0.15; M ej = 0.02 M) uses the DZ31 nuclear mass model (Duflo and Zuker 1995). Their corresponding r -process abundance distributions at t = 1 days are shown in the inset. Thermalization is calculated following (Kasen and Barnes 2019) for an assumed ejecta velocity 0.1 c. The black solid (dashed) horizontal lines in the lower right corner represent the approximate observation limits of the NIR (MIR) instruments on the James Webb Space Telescope for a merger at 100 Mpc. Image reproduced with permission from Wu et al. (2019b), copyright by APS

BNS mergers

How far are we from reproducing that from beginning to end?

- The GW signal

  - Gauge effects

  - Phase errors

  - Finite radius extraction

  - Initial conditions
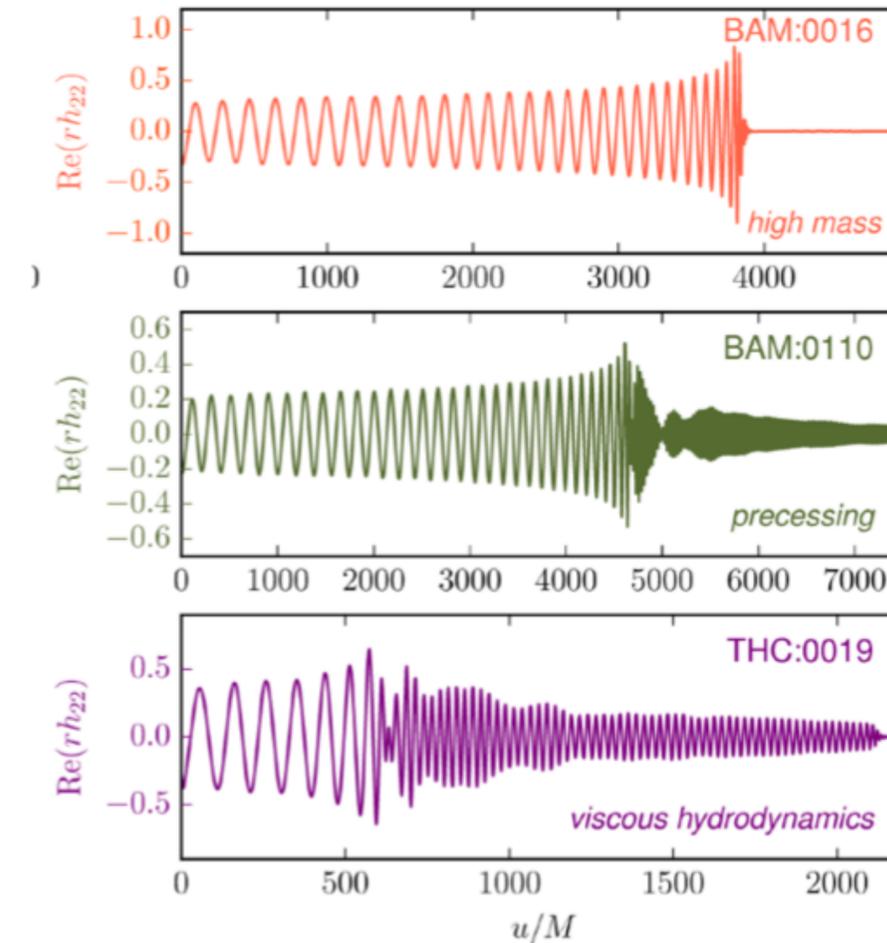
2nd release of the database (590 waveforms)
González+ 2023
2210.16366

BNS mergers
Some surprises are still possible

- Jet propagation inside Merger ejecta

  - Choked short GRB jets (Moharana)?

- Relativistic radiation-mediated shocks (Itoh)

  - Nuclear Physics within jet-cocoon propagation (A. Granot)

    - Composition change

    - r-process nucleosynthesis

    - Kilonova light curves

Kiuchi et al.,

Pais et al.,
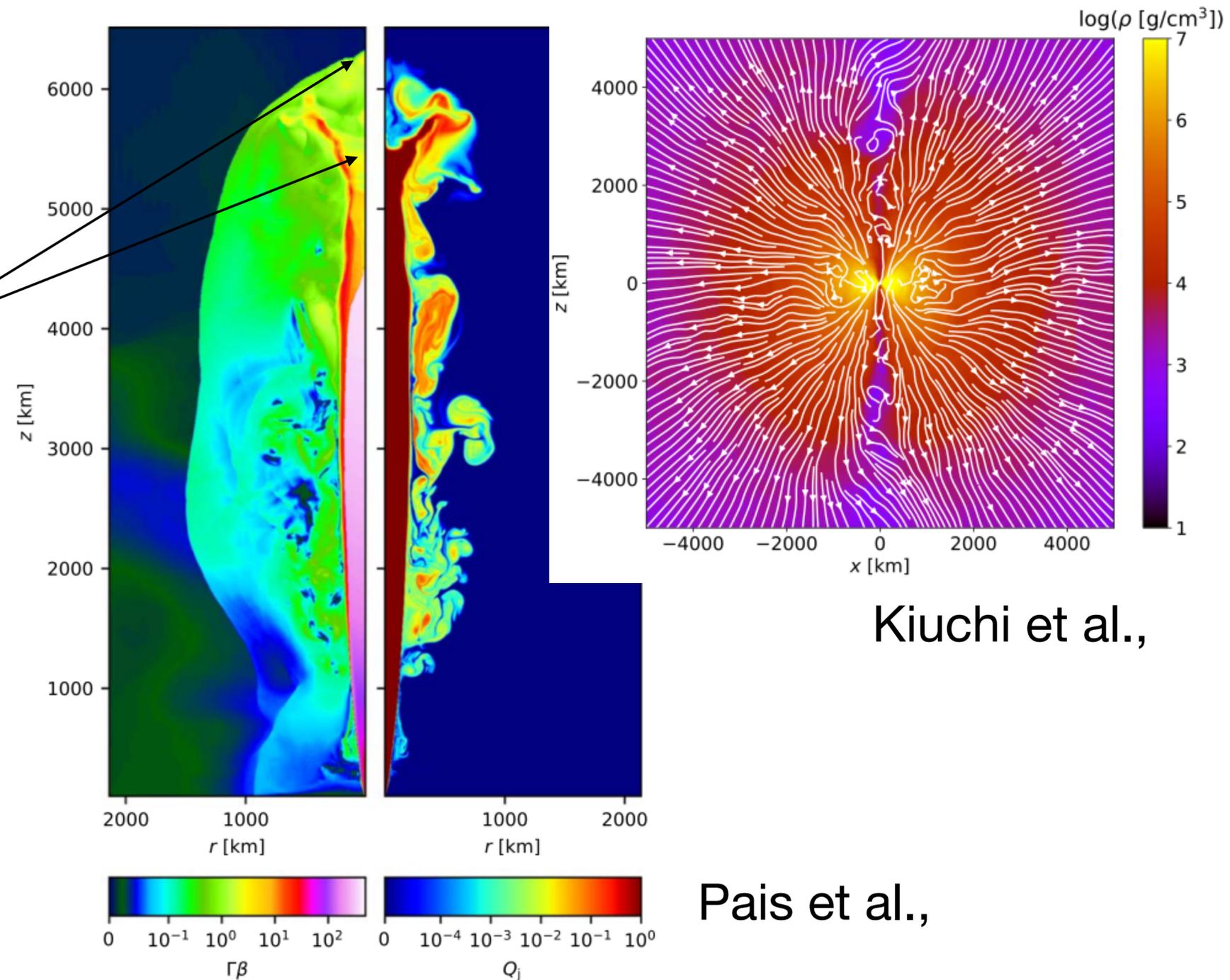
BNS mergers
The ultimate relativistic computational problem

- Almost all issues that one can think about in relativistic computations appear in the BNS merger problem.

- Need different codes for different steps

- Some unexplored regimes that may harbour surprises.

Collapsars and long GRBs
Some open issues

- The supernova problem

- Jet formation

- Jet - star interaction

- Jet CSM interaction

- Emission mechanisms
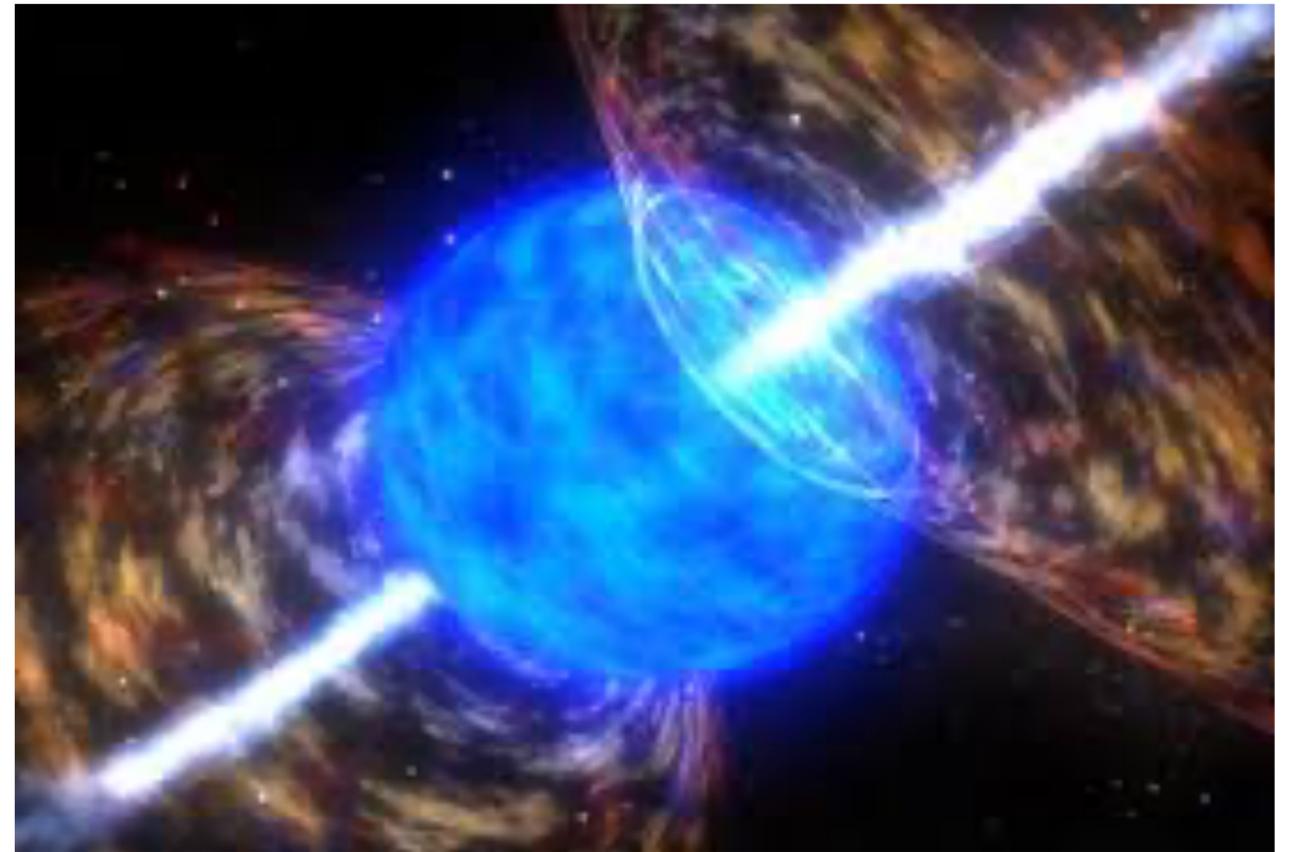
Collapsars and long GRBs
The jet-star interaction, choked jets and low-luminosity GRBs
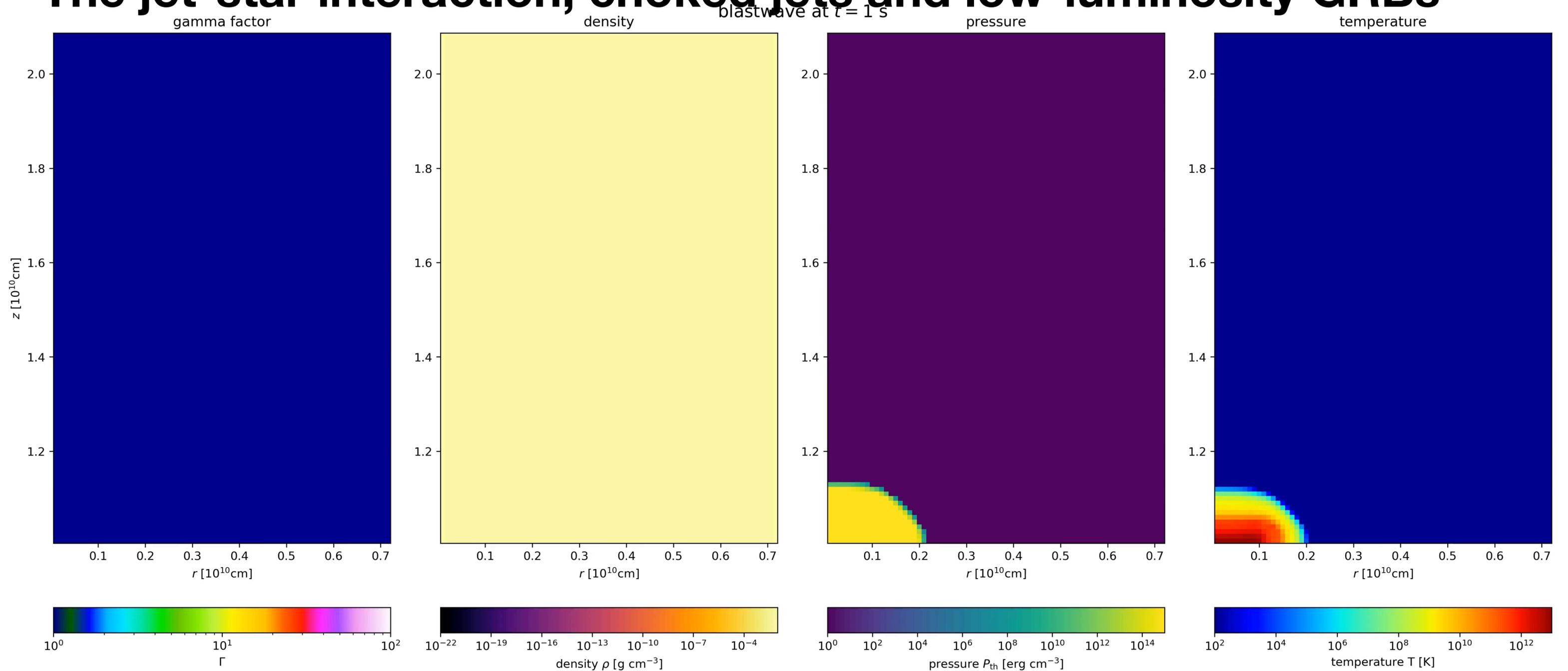
Simulation of choket jet in a stellar envelop by Matteo Pais

Low luminosity GRBs

Do we see choked jets?

Dim GRBs (low luminosity, soft and smooth bursts)

Typical energy 10^{47} erg

Typical redshift z< 0.1

Most common in nature but only few have been observed.

Produced by a collapsing star.

The core produces a jet that DOES NOT punches the envelope. The observed gamma-rays are produce by a hot cocoon when it emerges from the stellar envelope.

t = 14.7 s
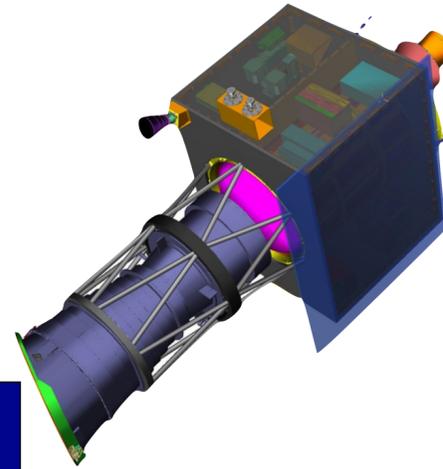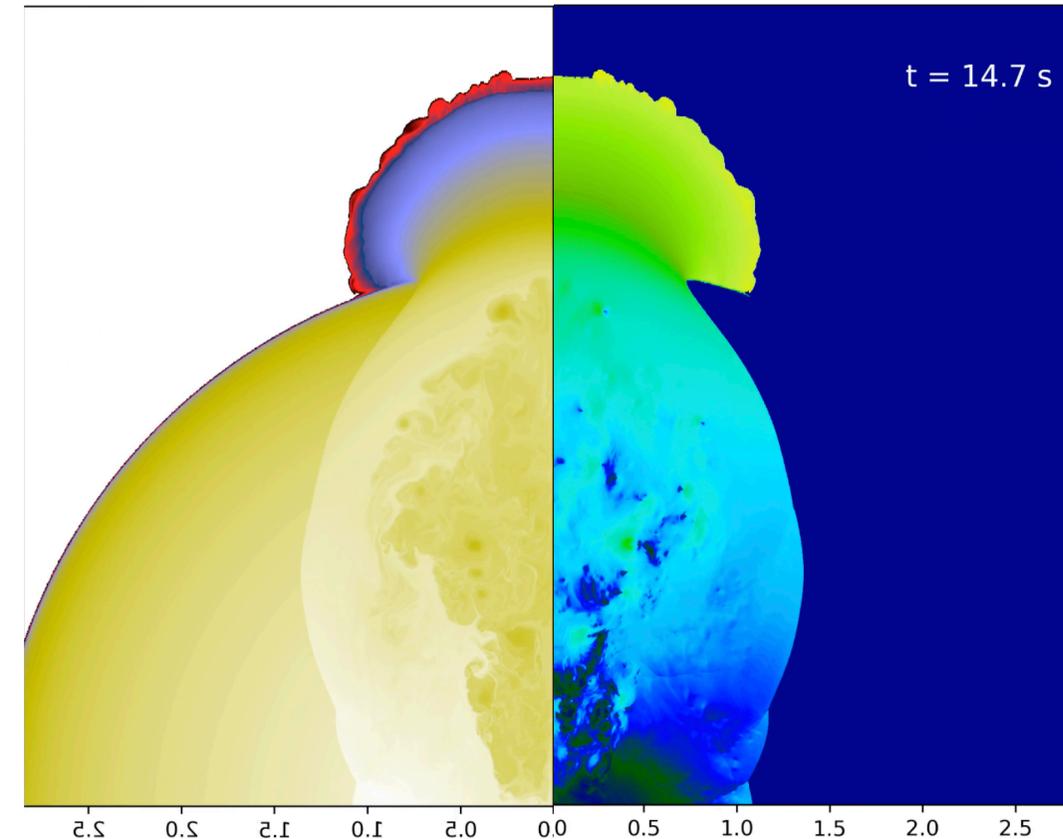
Choked jet
The missing signature
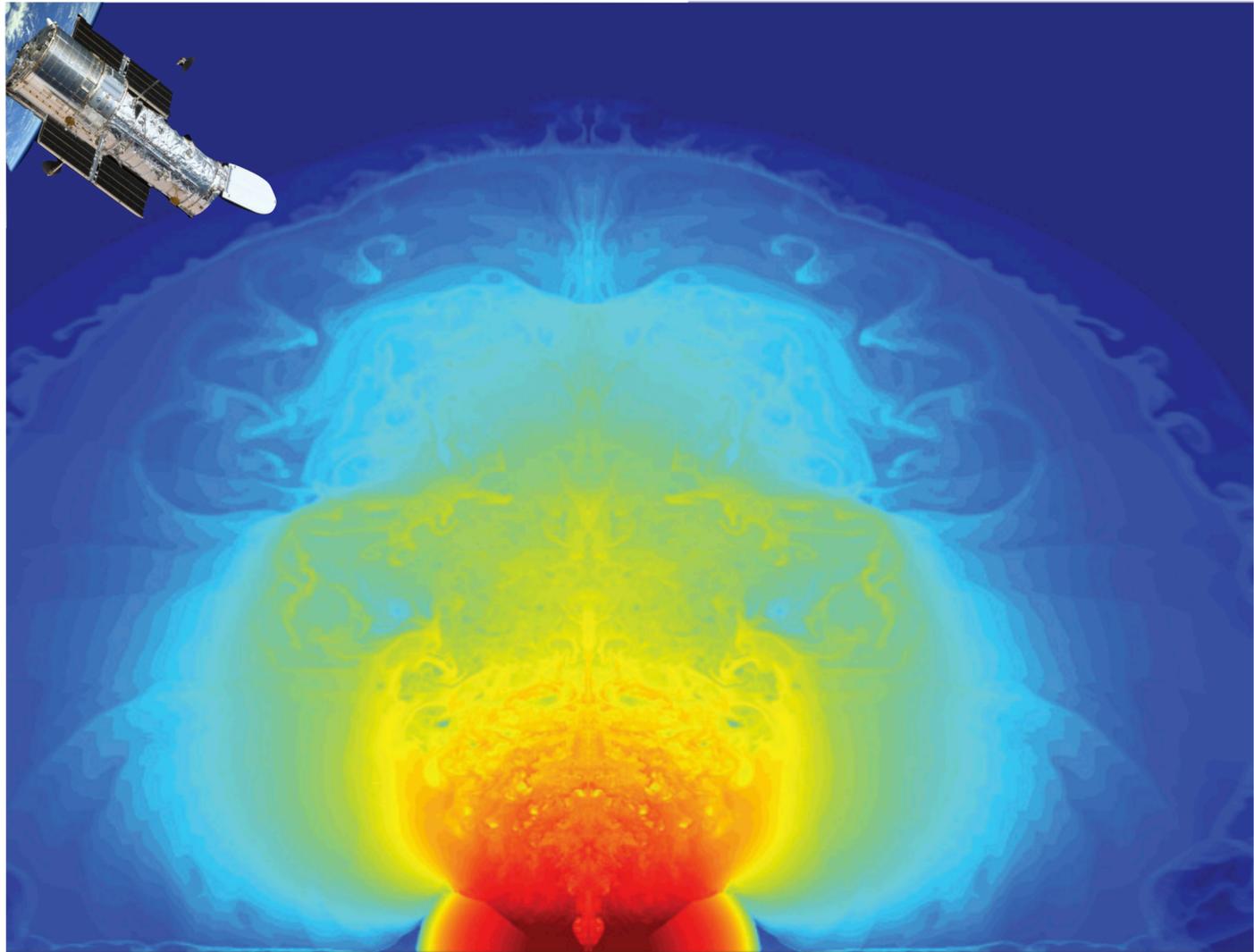
Spectrum
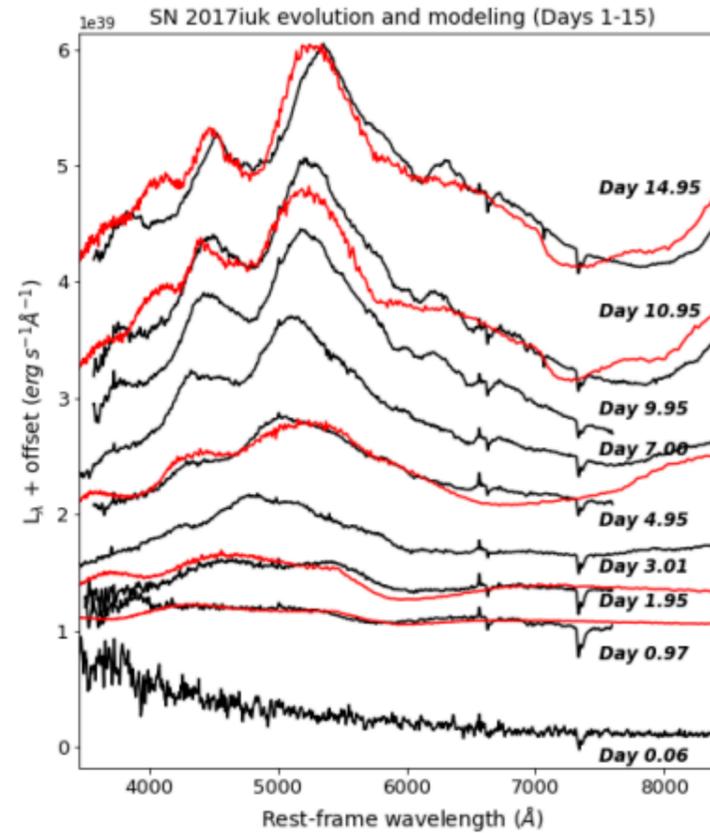
Regular SNe   **Cocoon**

Izzo et al., 2019   *TP et al., 2017, 2019*
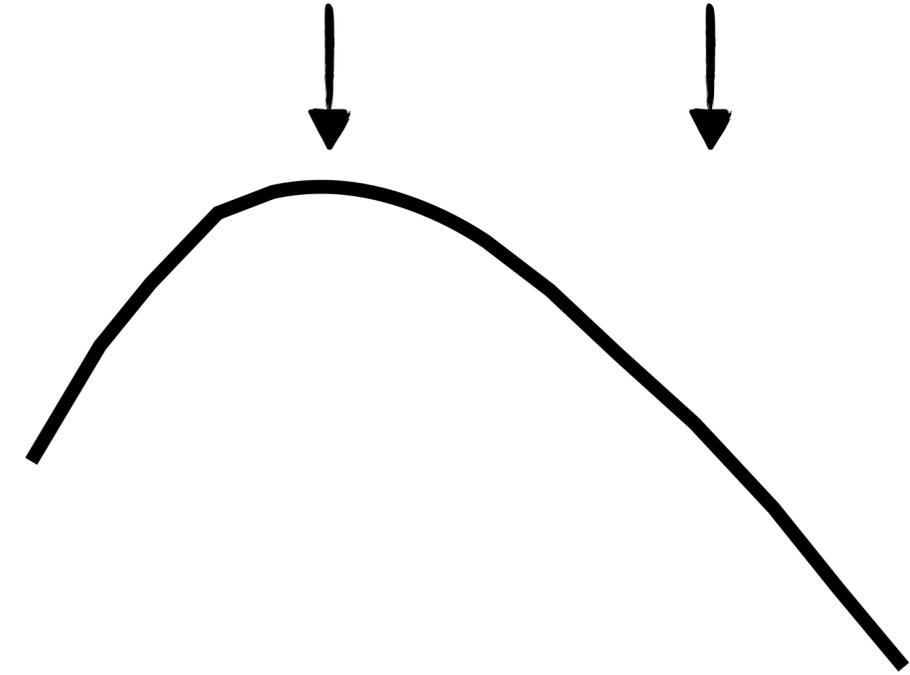
TP et al., 2017, 2019

Expansion velocity ~10^5 km/sec

Missing is a detailed simulation of the observational cocoon breakout

Collapsars and long GRBs
The LHASSO's TeV emission - who ordered that?

- The LHASSO TeV signal is the first evidence for the very early jet-CSM interaction.
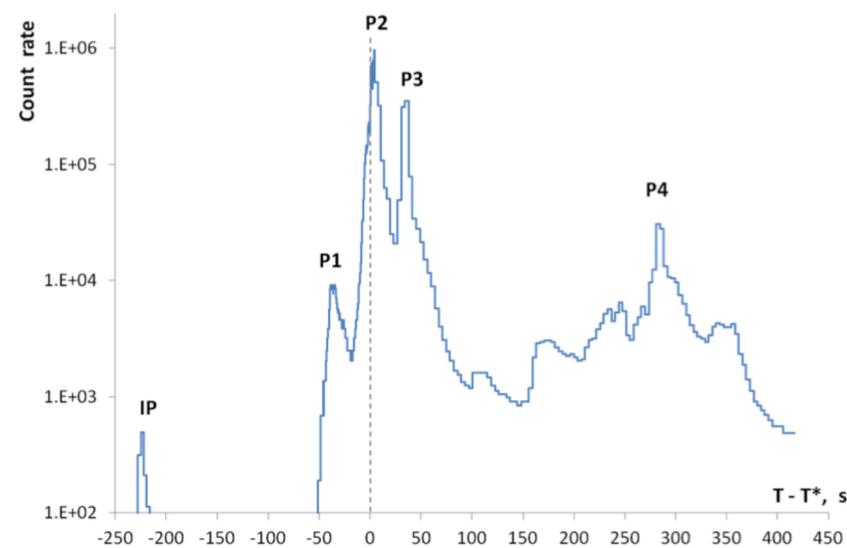
LHAASO

0.3-7 TeV afterglow

Prompt Emission
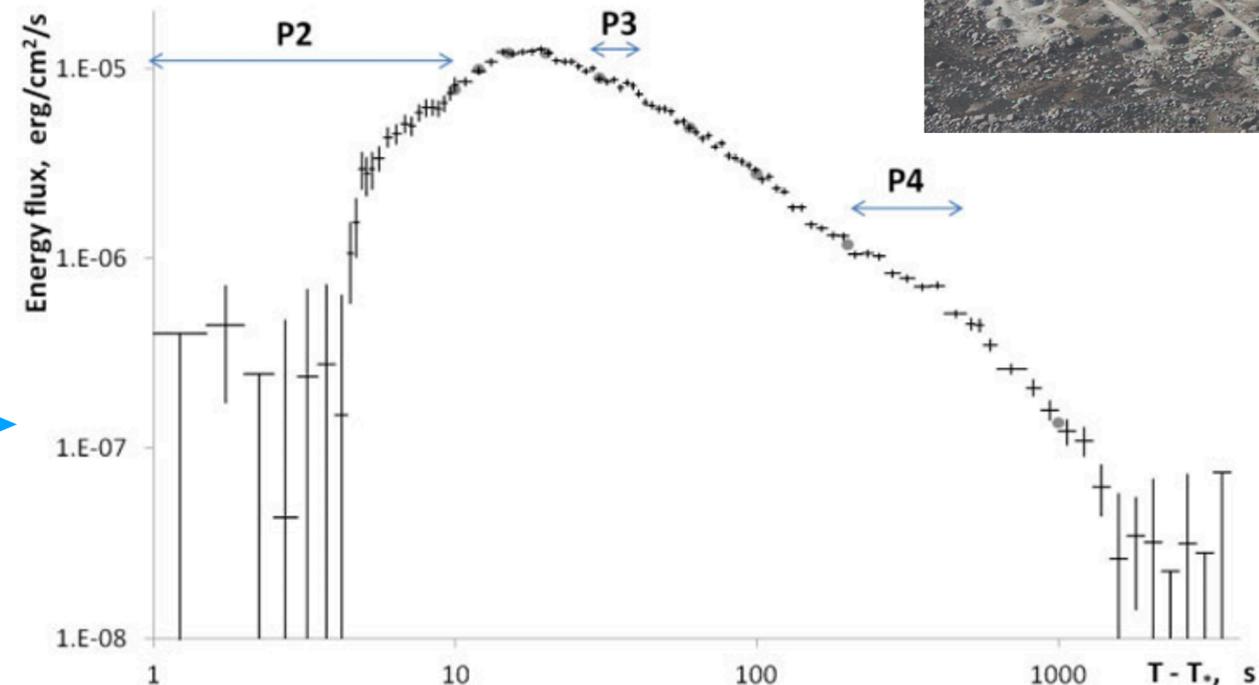
It takes time to accelerate the shock material

Derishev & Piran 2024

$$\frac{\mathrm{d}\Gamma_{\mathrm{sh}}}{\mathrm{d}R} = -\sqrt{2}\frac{\langle\nabla p\rangle}{\langle w\rangle} = \sqrt{2}\left(\frac{p_{\mathrm{in}}}{p_{\mathrm{out}}} - 1\right)\frac{1}{\Delta R'}\frac{p_{\mathrm{out}}}{\langle w\rangle}.$$

$$\frac{R}{\Gamma_{\mathrm{sh}}}\frac{\mathrm{d}\Gamma_{\mathrm{sh}}}{\mathrm{d}R} = \left(\frac{p_{\mathrm{in}}}{p_{\mathrm{out}}} - 1\right)\frac{2(3-k)\Gamma_{\mathrm{sh}}^2 M_{\mathrm{fs}}c^2}{3E_{\mathrm{fs}}}$$

If $E_{\mathrm{fs}} = E_{\mathrm{BM}}$ this gives the BM solution
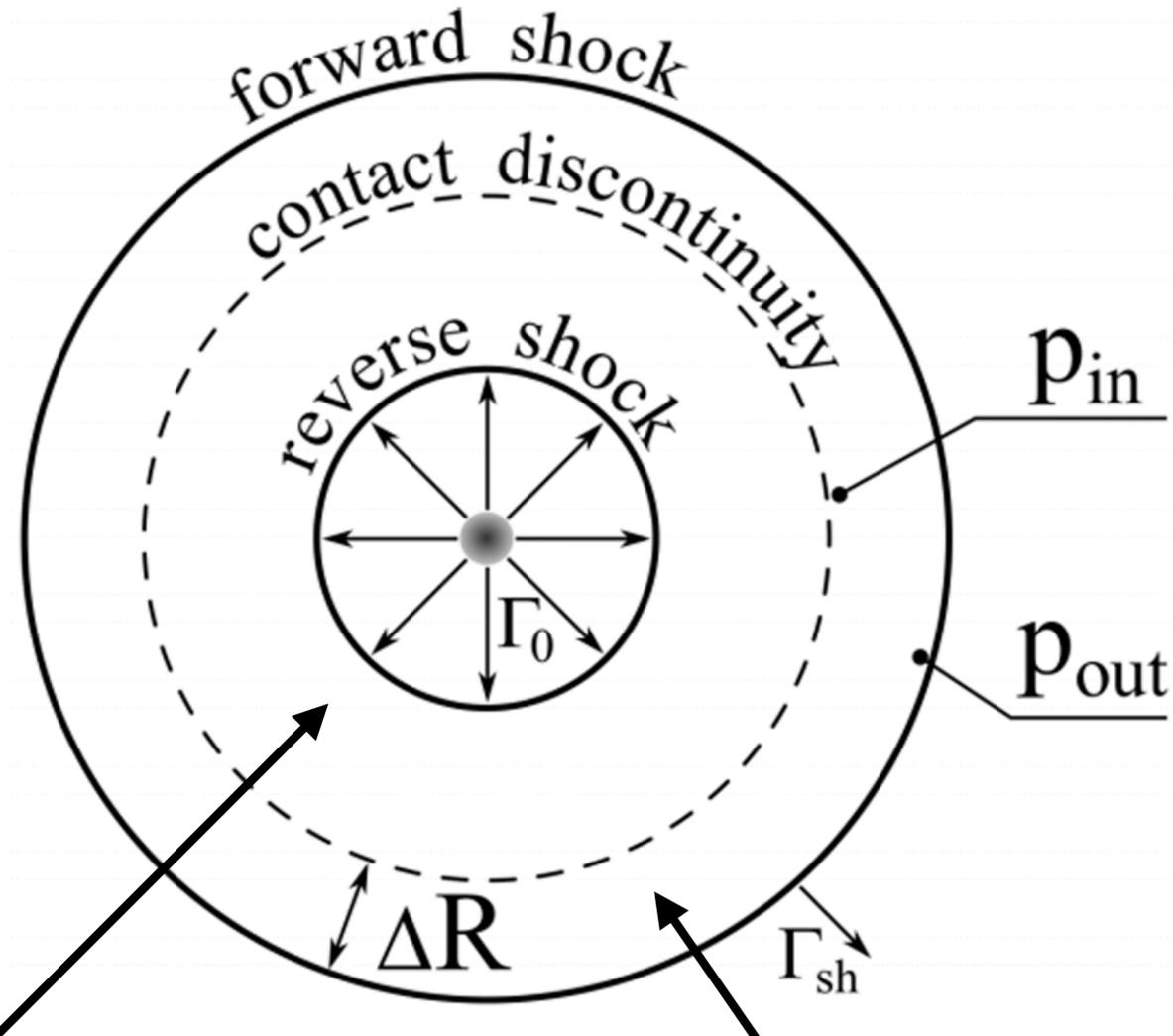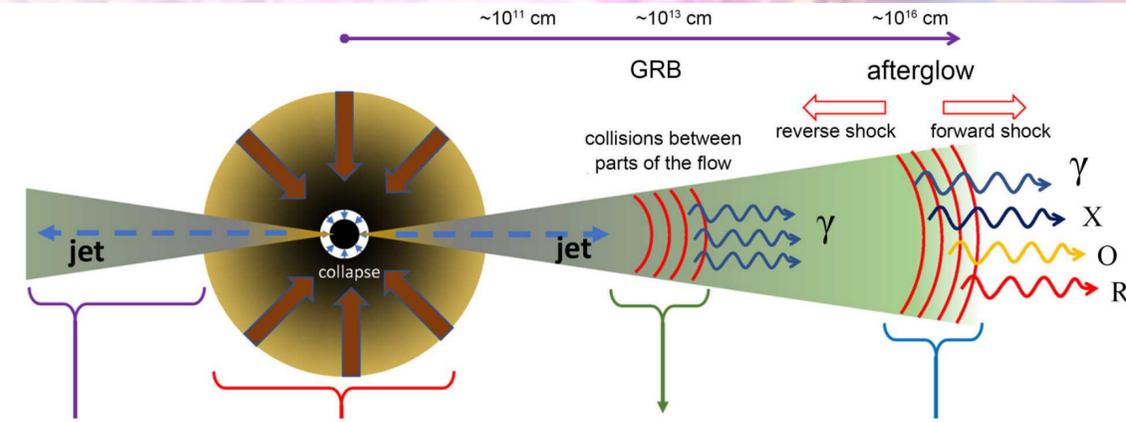
$$E_{\mathrm{BM}} = C_{\mathrm{E}}\Gamma_{\mathrm{sh}}^2 M_{\mathrm{fs}}c^2,$$

$$\Gamma_{\mathrm{sh}}M_{\mathrm{se}}c^2 + E_{\mathrm{se}}$$

$$E_{\mathrm{fs}} \equiv C_{\mathrm{E}}\Gamma_{\mathrm{sh}}^2 M_{\mathrm{fs}}c^2,$$

$$\frac{R}{\Gamma_{\mathrm{sh}}}\frac{\mathrm{d}\Gamma_{\mathrm{sh}}}{\mathrm{d}R} = C_{\mathrm{A}}\left(1 - \frac{E_{\mathrm{BM}}}{E_{\mathrm{fs}}}\right) - \frac{3-k}{2}, \quad C_{\mathrm{A}} = \frac{4(3-k)}{3\,C_{\mathrm{E}}}.$$

The *Inverse Jet Break*
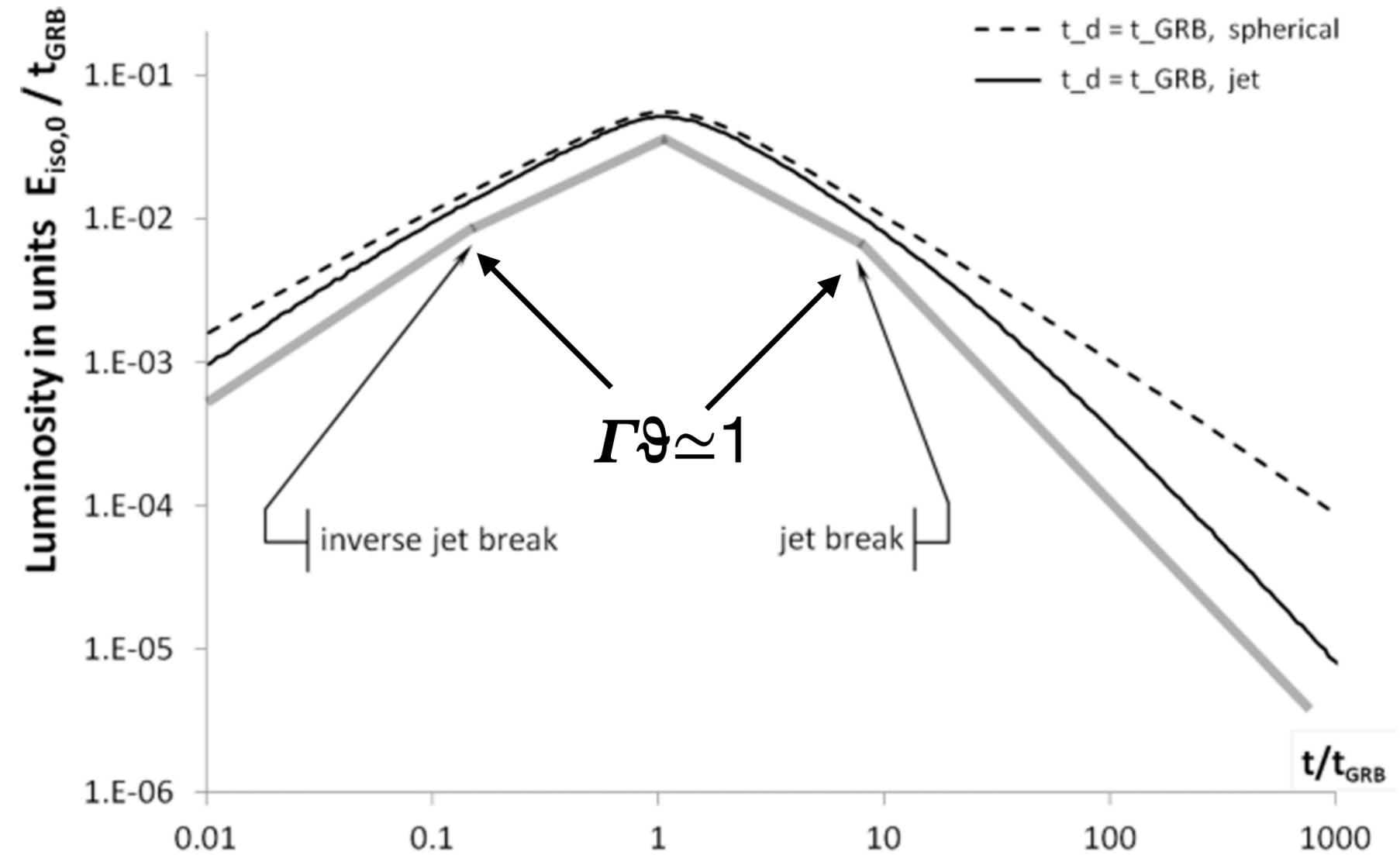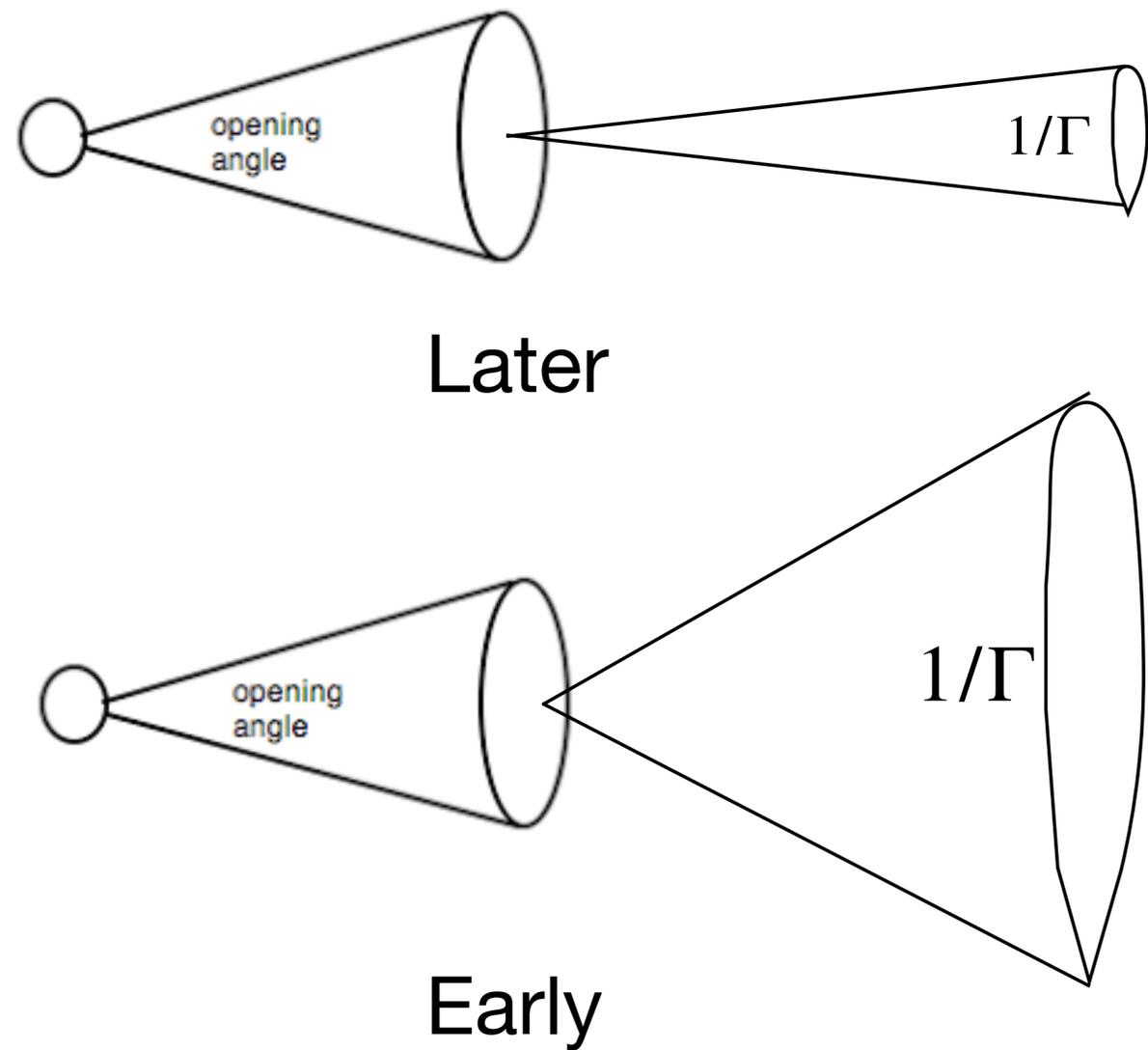
The best fit model

The numerical challange - interaction of ultrarelativistic narrow jet with CSM

$\vartheta = 0.6/\Gamma_0$

$t_d = 130 \text{sec}$

$300 < \Gamma_0 < 800$

For $\Gamma = 500$

$\vartheta = 0.0012 \simeq 0.1°$

$E_{tot} \simeq 10^{50} \text{ erg}$

$R_d = 2 \text{ ly}$

$\dot{m} = 1.2 \times 10^6 \text{ m}_\odot/\text{y}$

Derishev & Piran 2024
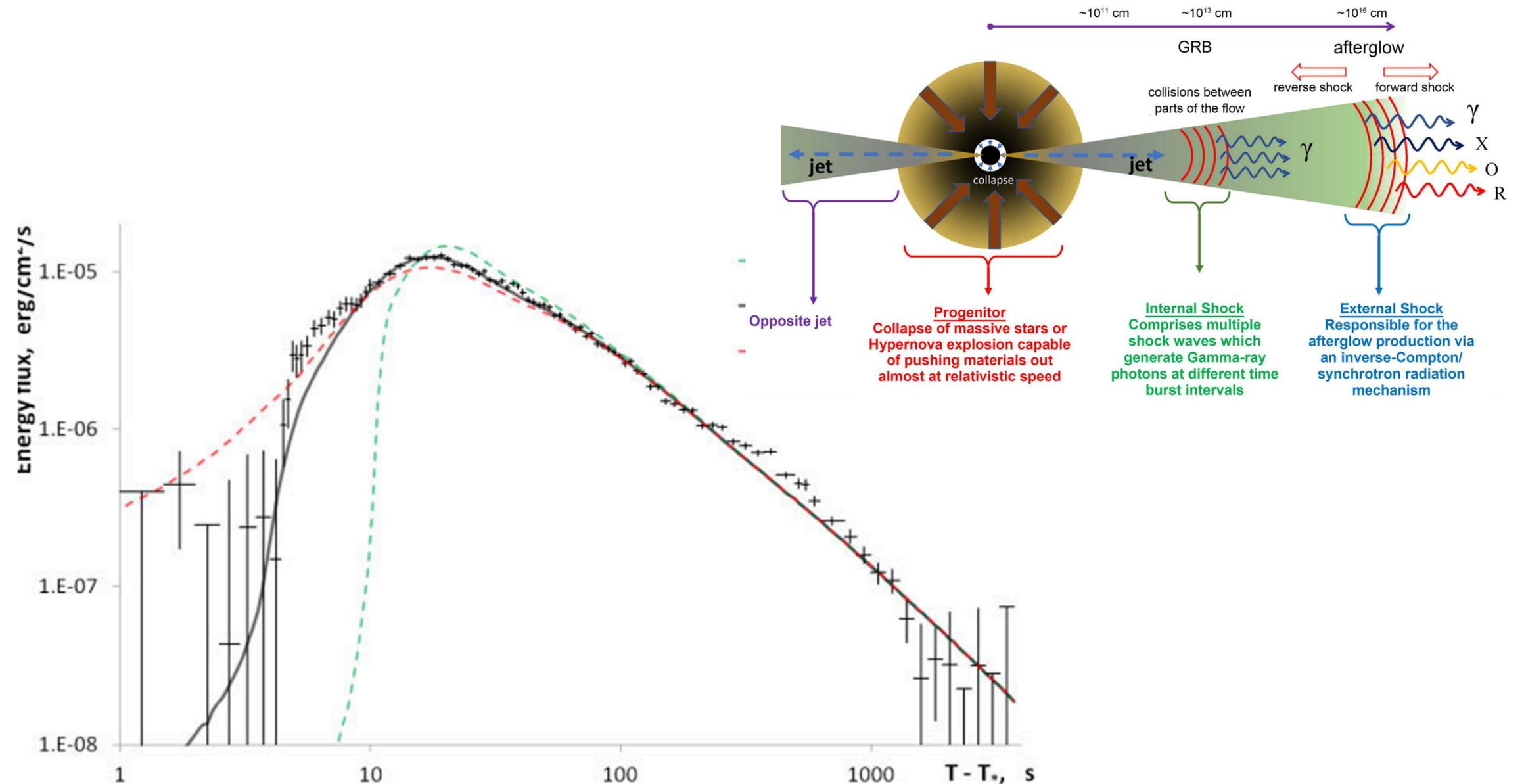
Jet-Gravitational waves

Can we detect choked jets and jet acceleration in general?

- The jet acceleration produces a memory type gravitaiontal wave signal

- Segalis and Ori 2001

- Piran 2002

- Sago, Ioka, Nakamura Yamazaki, 2004
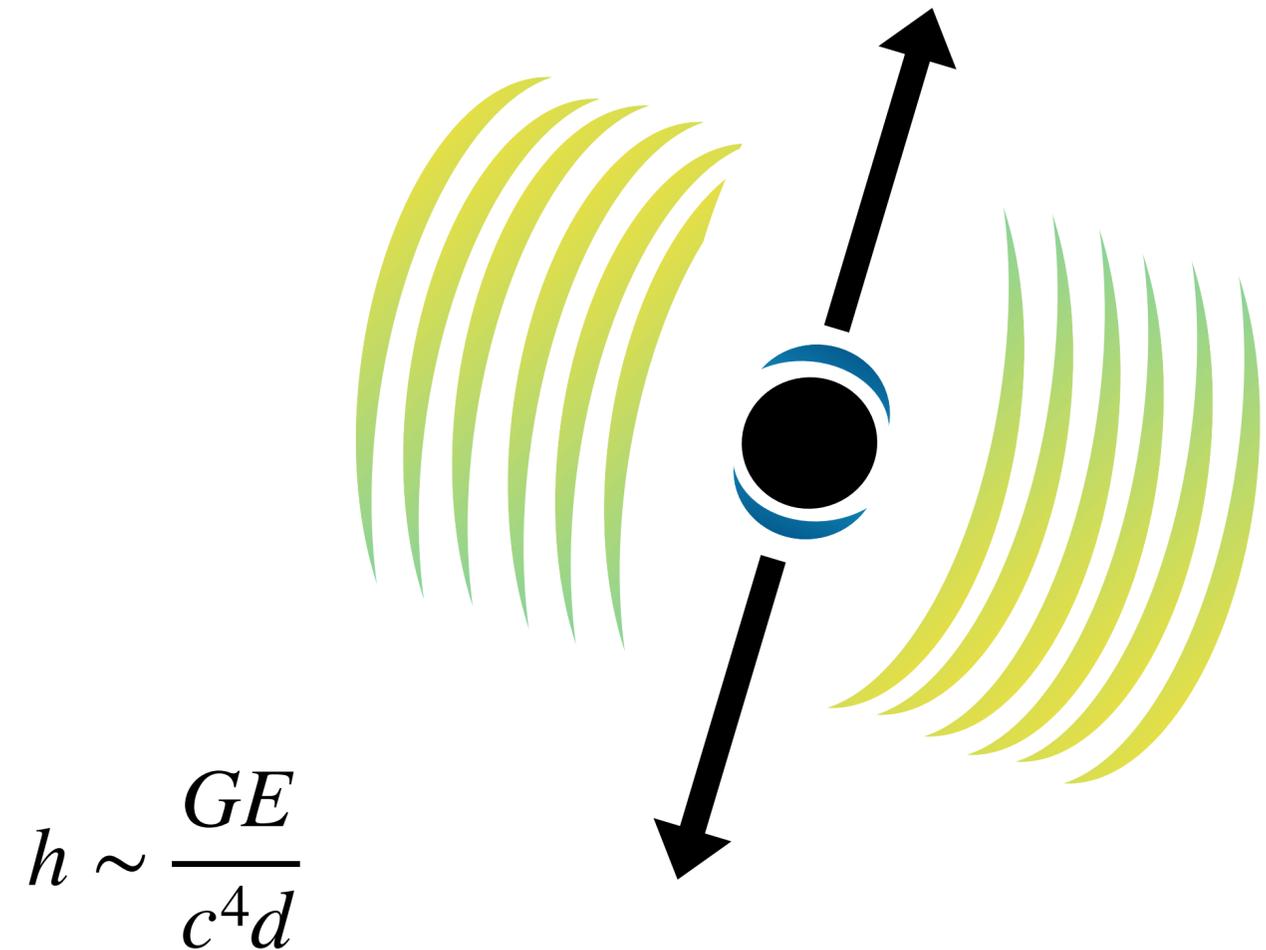
$$h \sim \frac{GE}{c^4 d}$$

Jet-Gravitational waves

Need to solve $\Box h_{\mu\nu} = 8\pi G S_{\mu\nu}$ **using retarded Green's function**

- The jet acceleration produces a memory type gravitaiontal wave signal

- Segalis and Ori 2001
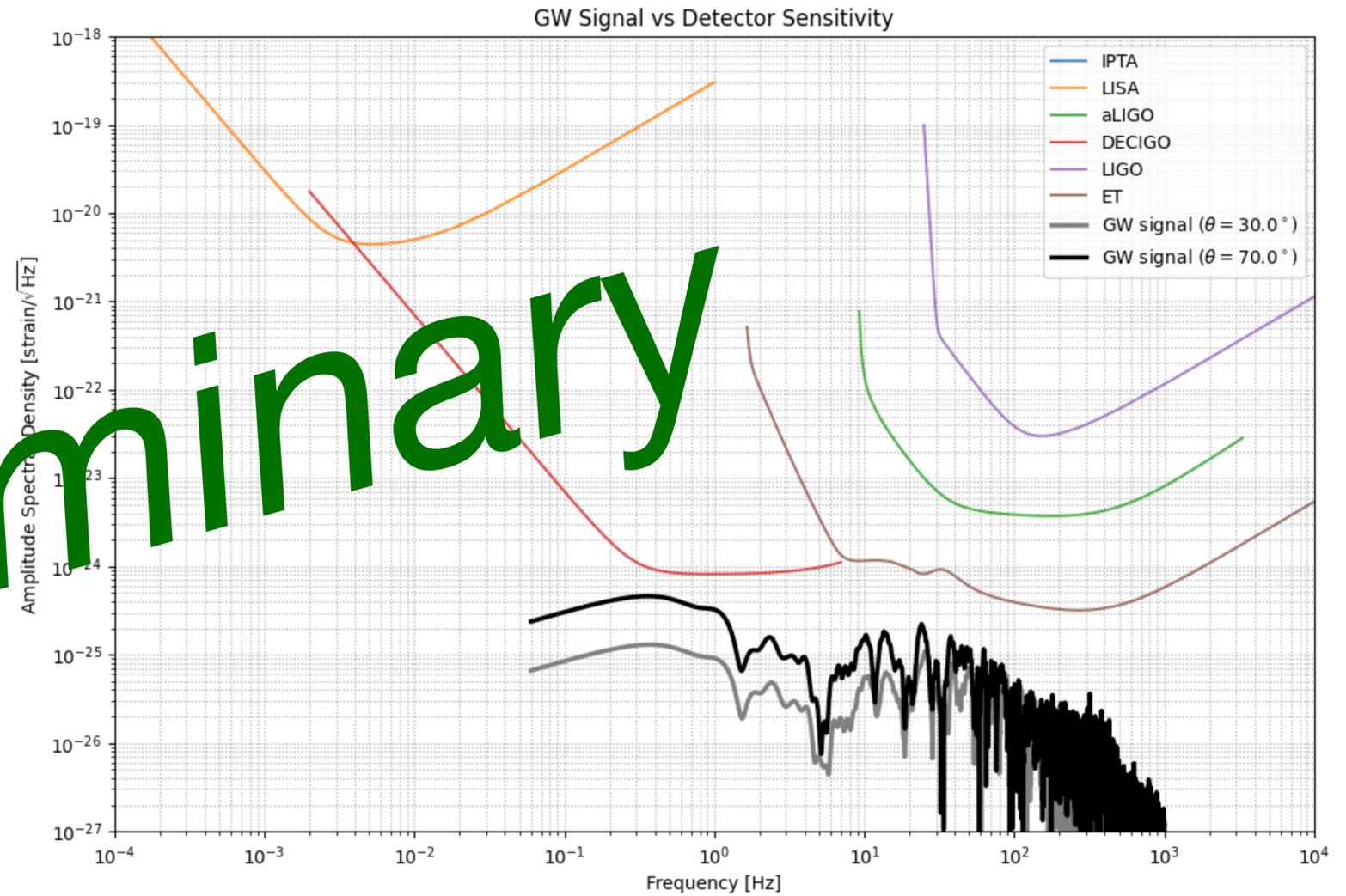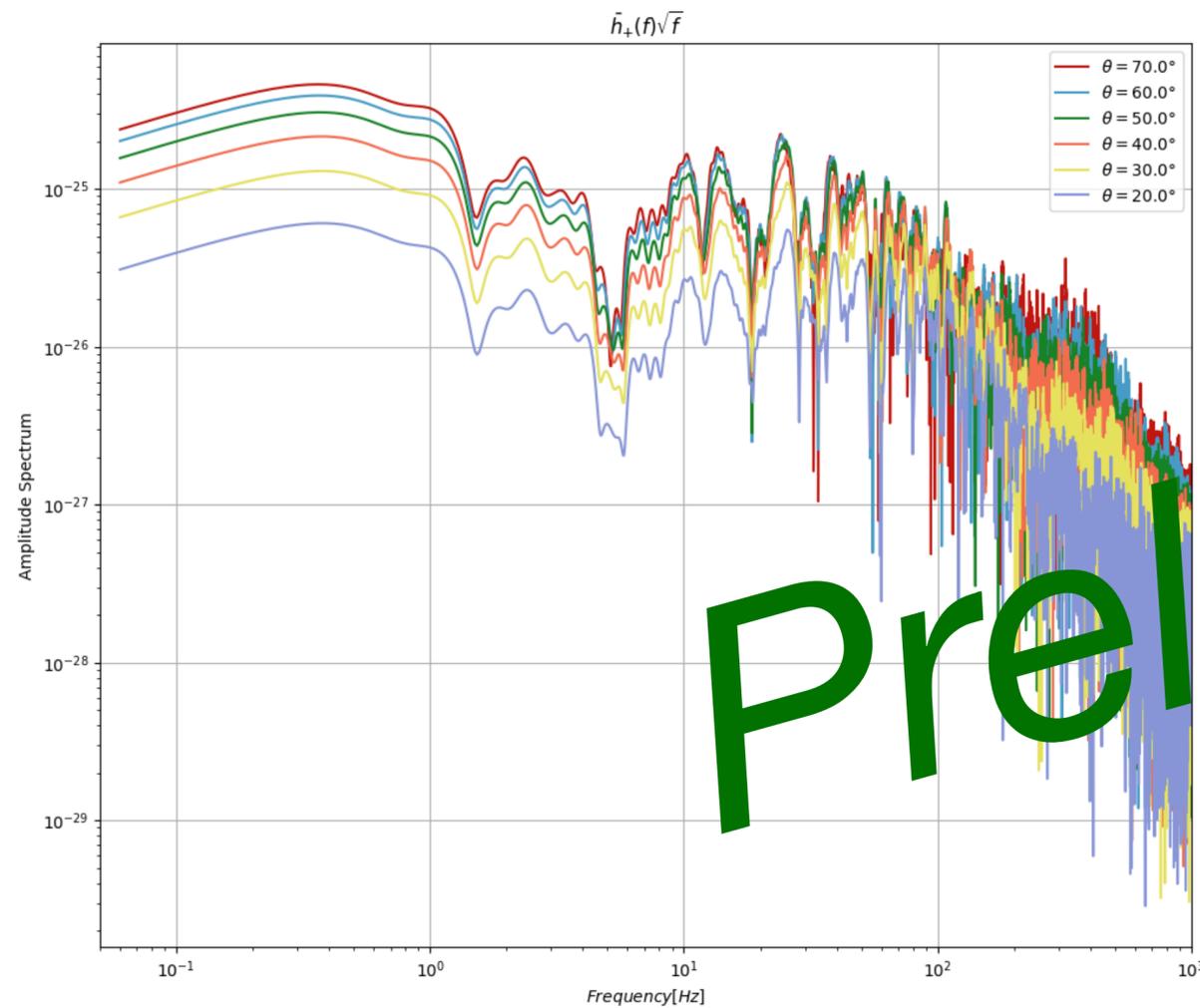
- Piran 2002

- Sago, Ioka, Nakamura Yamazaki, 2004
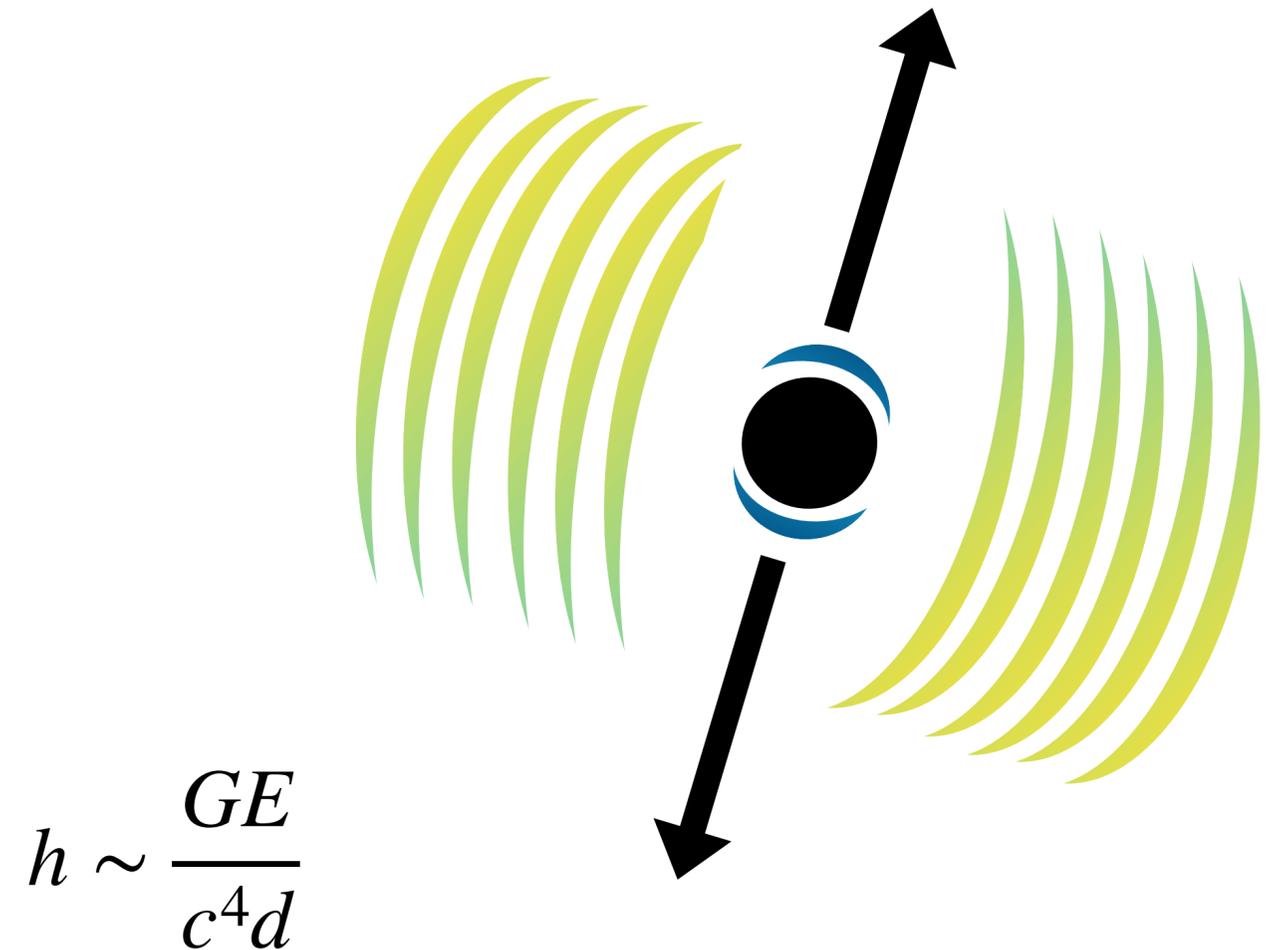
$$h \sim \frac{GE}{c^4 d}$$

Jet-Gravitational waves

A test on jet simulation (Zuriel, Ofengeim, Bromberg, Piran)
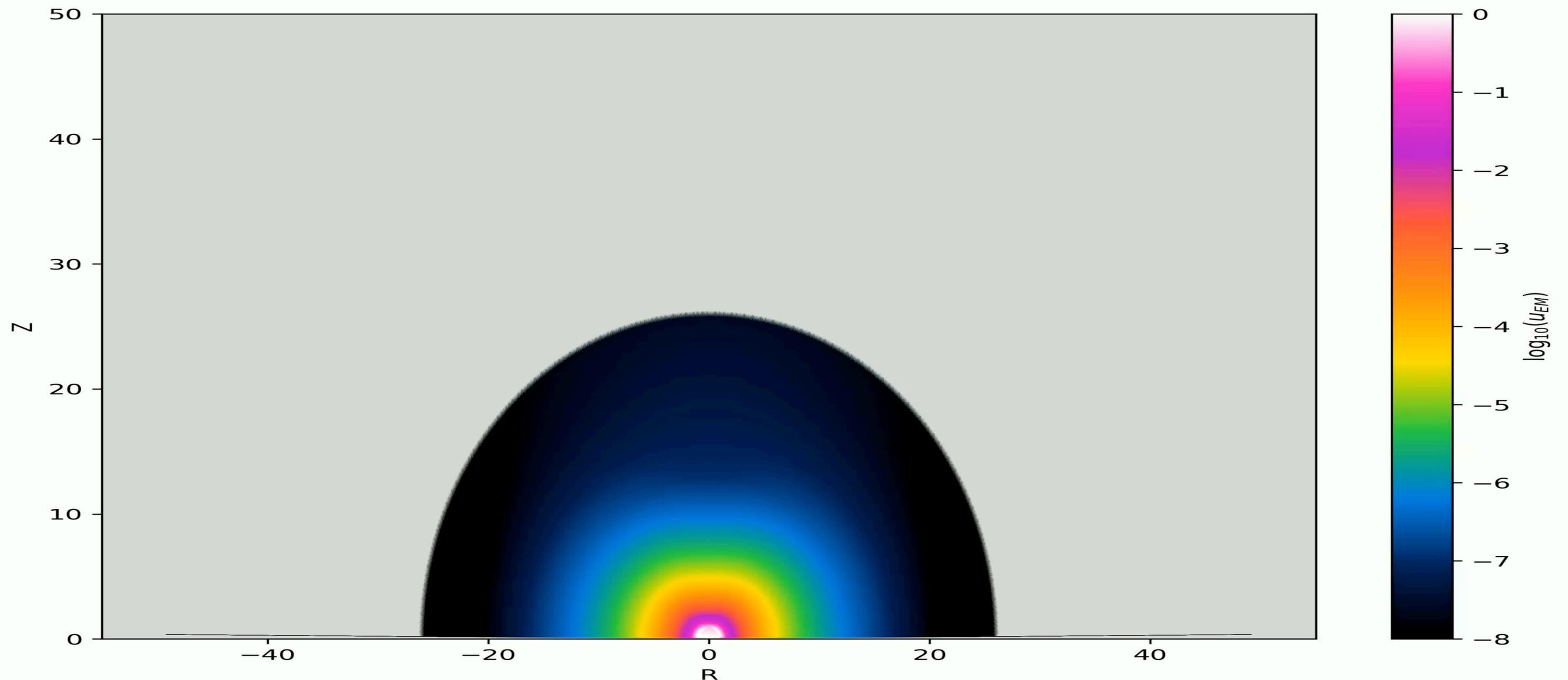
$$h \sim \frac{GE}{c^4 d}$$

Simulaiton: Omer Bromberg

Jet-Gravitational waves
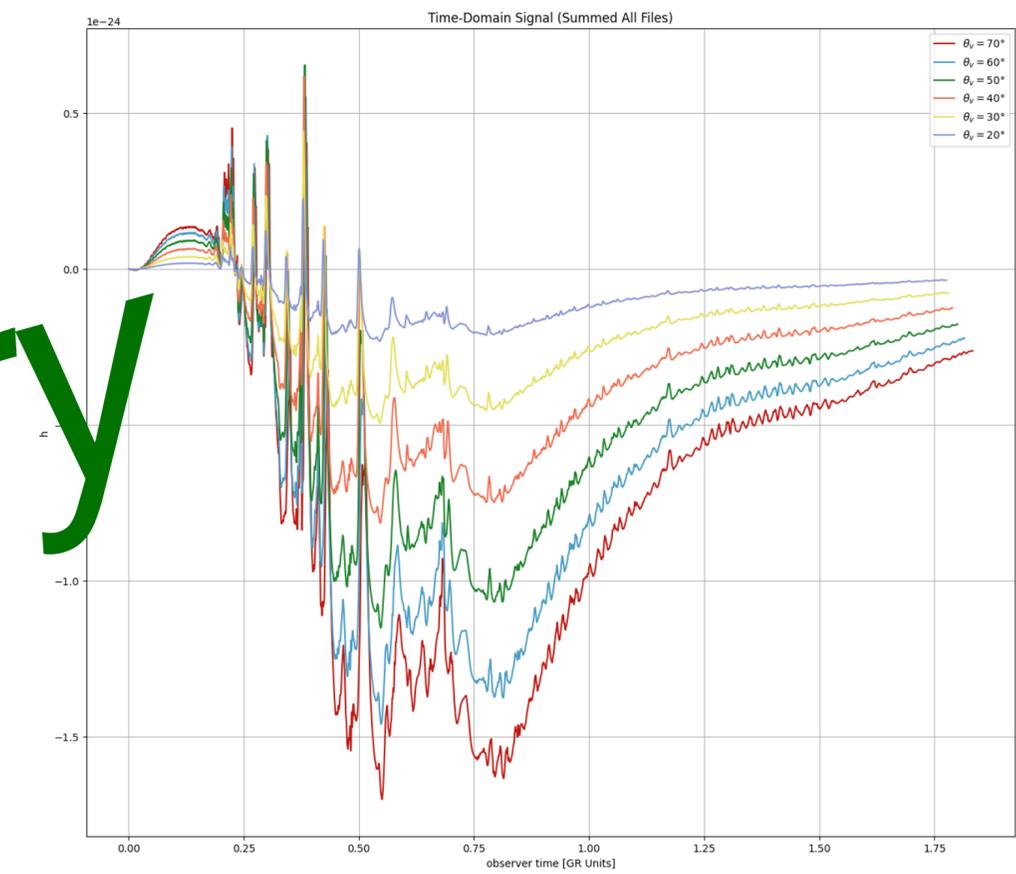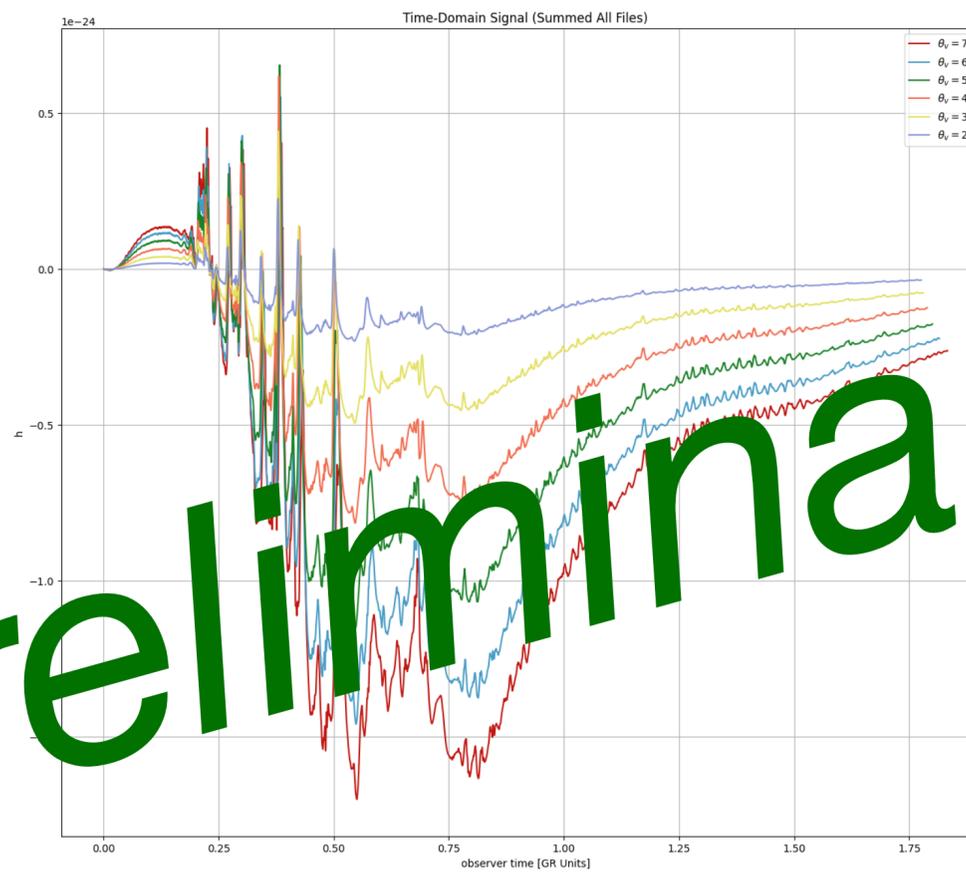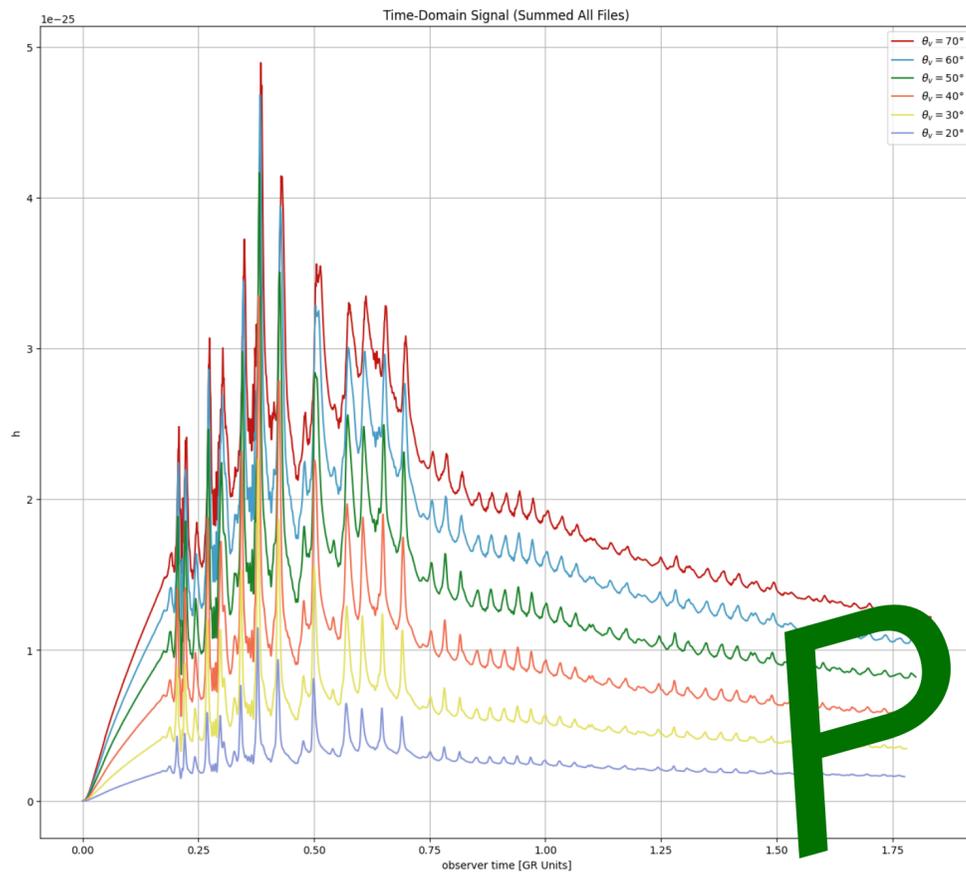
A test on jet simulation (Zuriel, Ofengeim, Bromberg, Piran)

$$h \sim \frac{GE}{c^4 d}$$

Jet-Gravitational waves
A test on jet simulation (Zuriel, Ofengeim, Bromberg, Piran)

$$h \sim \frac{GE}{c^4 d}$$

A (not comic) Break S301 replacing S2

# A (not comic) Break S301 replacing S2

Tidal forces - or lack of…

- *Gravity* discovered **S301** the (so far) **nearest star to SGR A**

- Pericenter 280 r_g

- $v_{pericenter}$ ~25,000 km/sec

- e=0.9825

- 8.7 yr orbit

Abd El Dayen et al., 2026 (Gravity collaboration)+
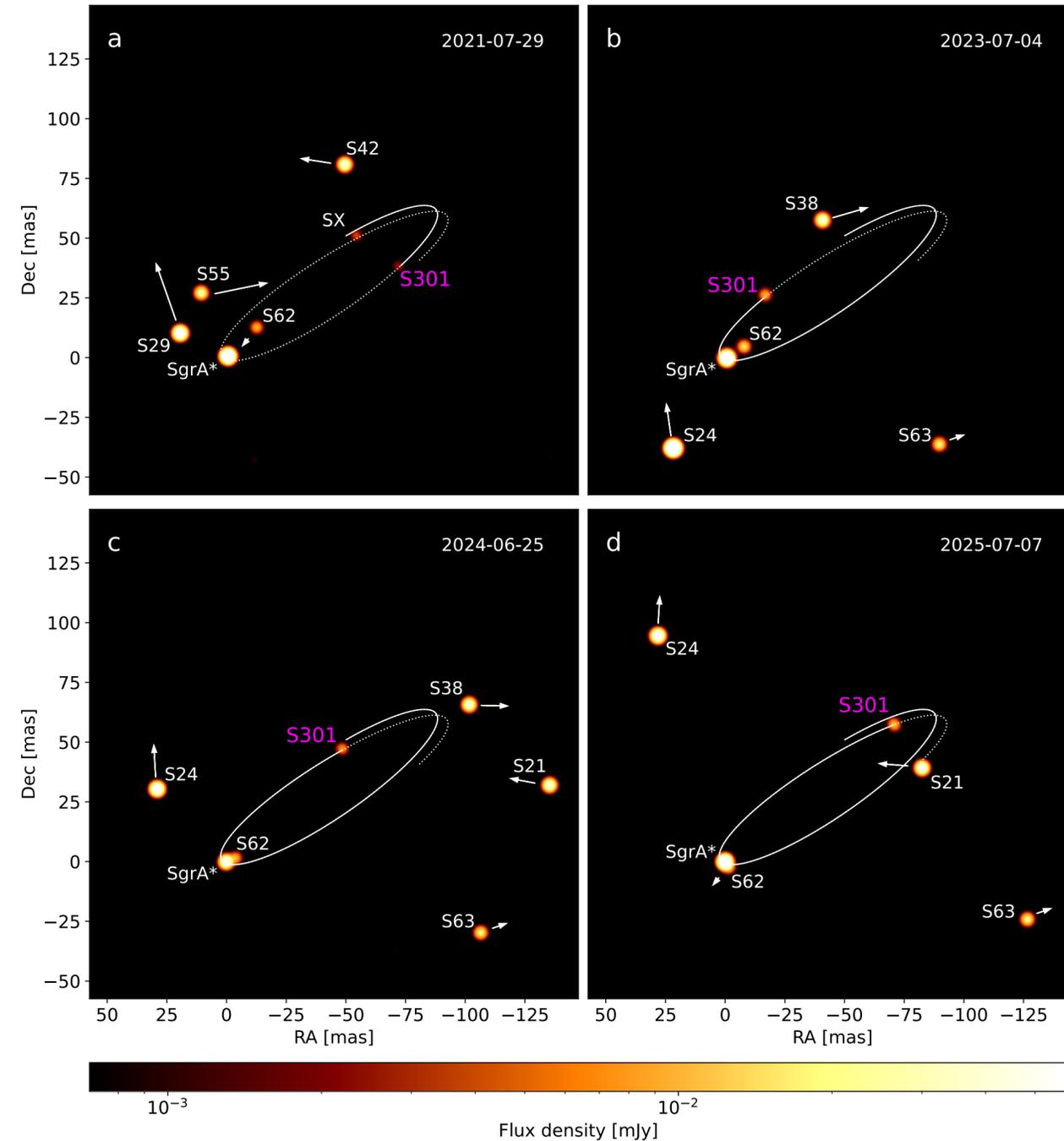
S 301 - Measuring SGR A's Kerr parameter !

Tidal forces - or lack of…

- 280 r_g > 4 R_T ~ 100 r_g is too far even for partial tidal disruption 🙁

- It is even too far for significant tidal effects on the orbits 🙁
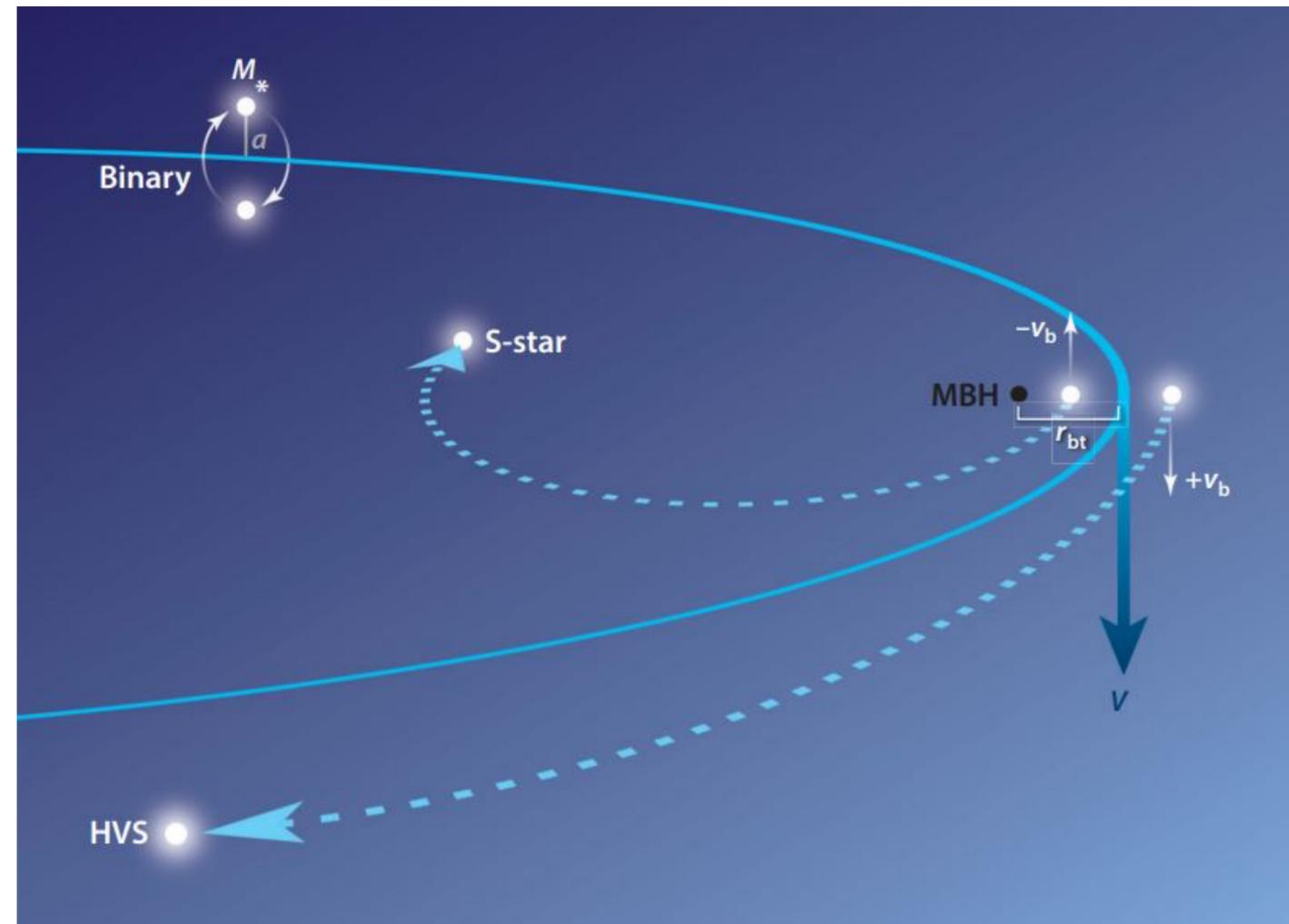
- We can measure the Kerr parameter 😊

Abd El Dayen et al., 2026 (Gravity collaboration)+
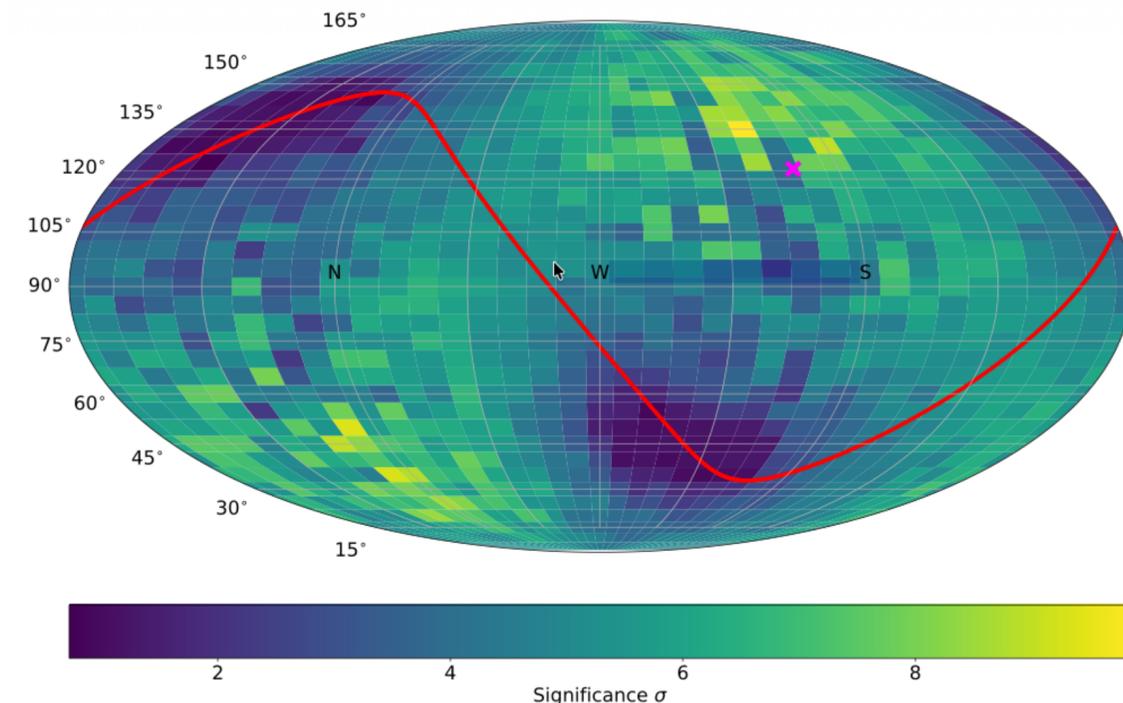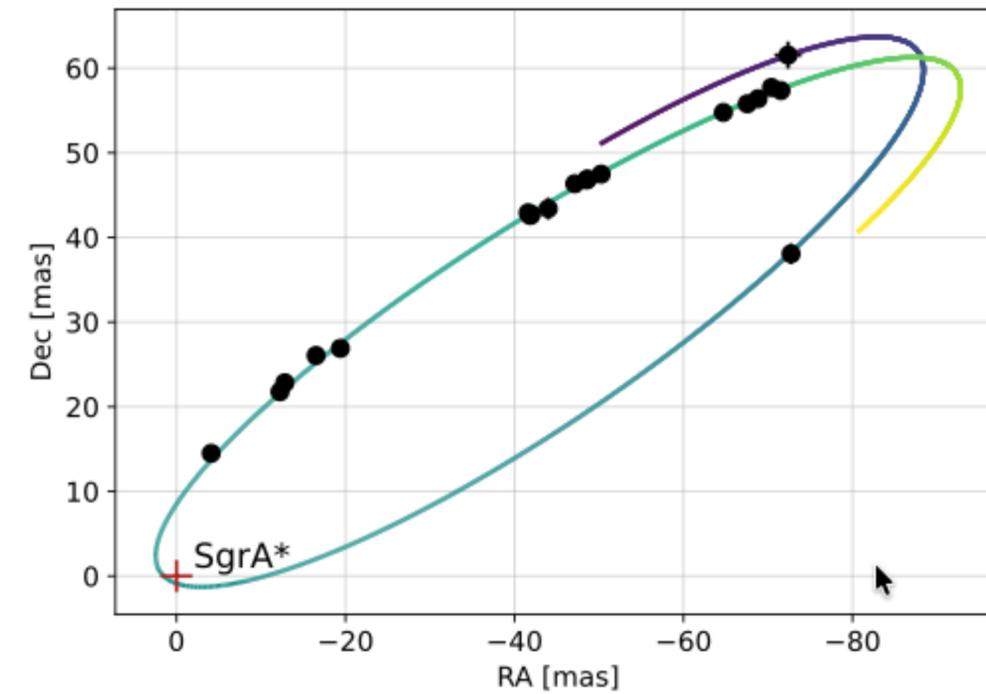
The Origin of S 301
Hills Mechanism - a tidal disruption of a Binary

- Most likely formed by tidal disruption of a binary.

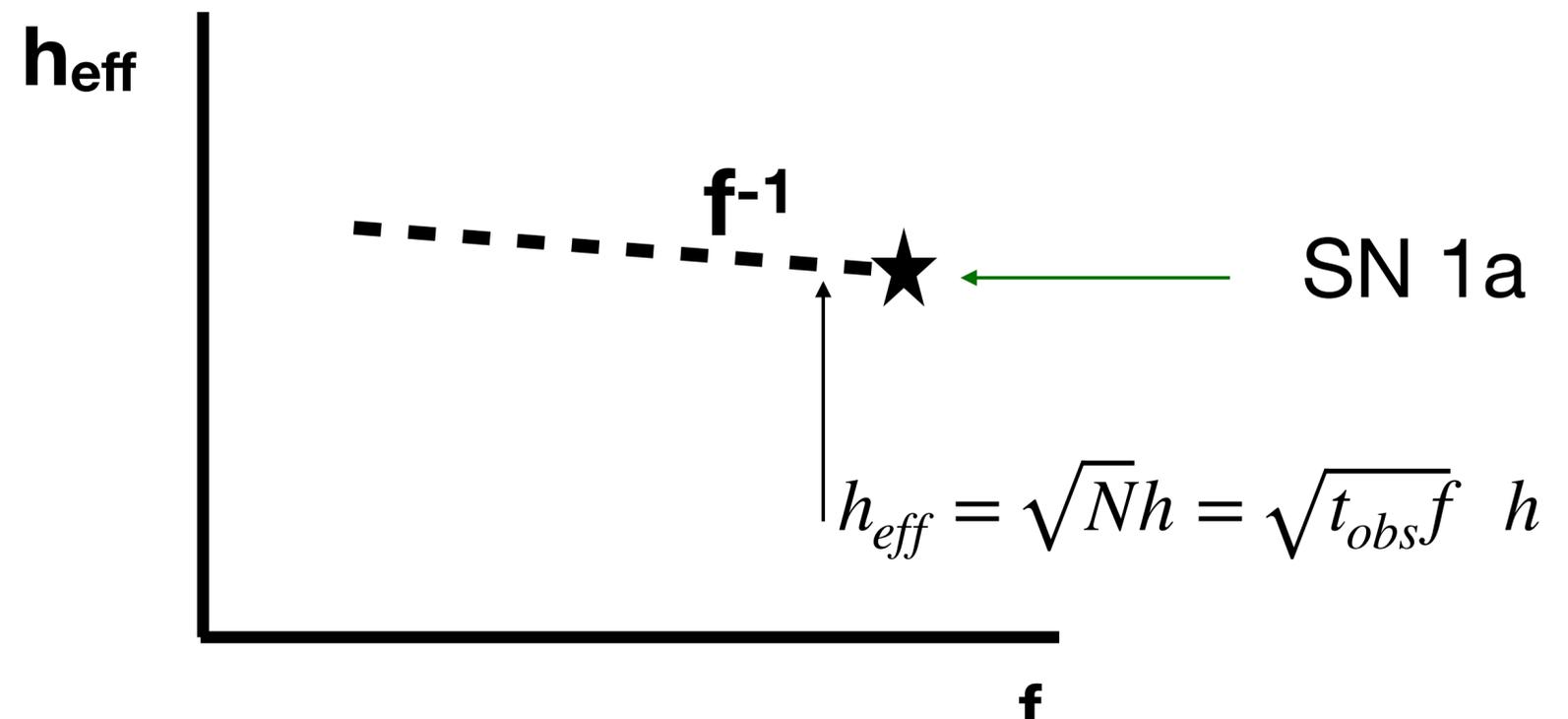- The companion is a hypervelocity star escaping from the galactic center.

- $\epsilon_{cap} \approx 1 - (\dfrac{M_{bin}}{M_{BH}})^{1/3}$

Binary White Dwarf
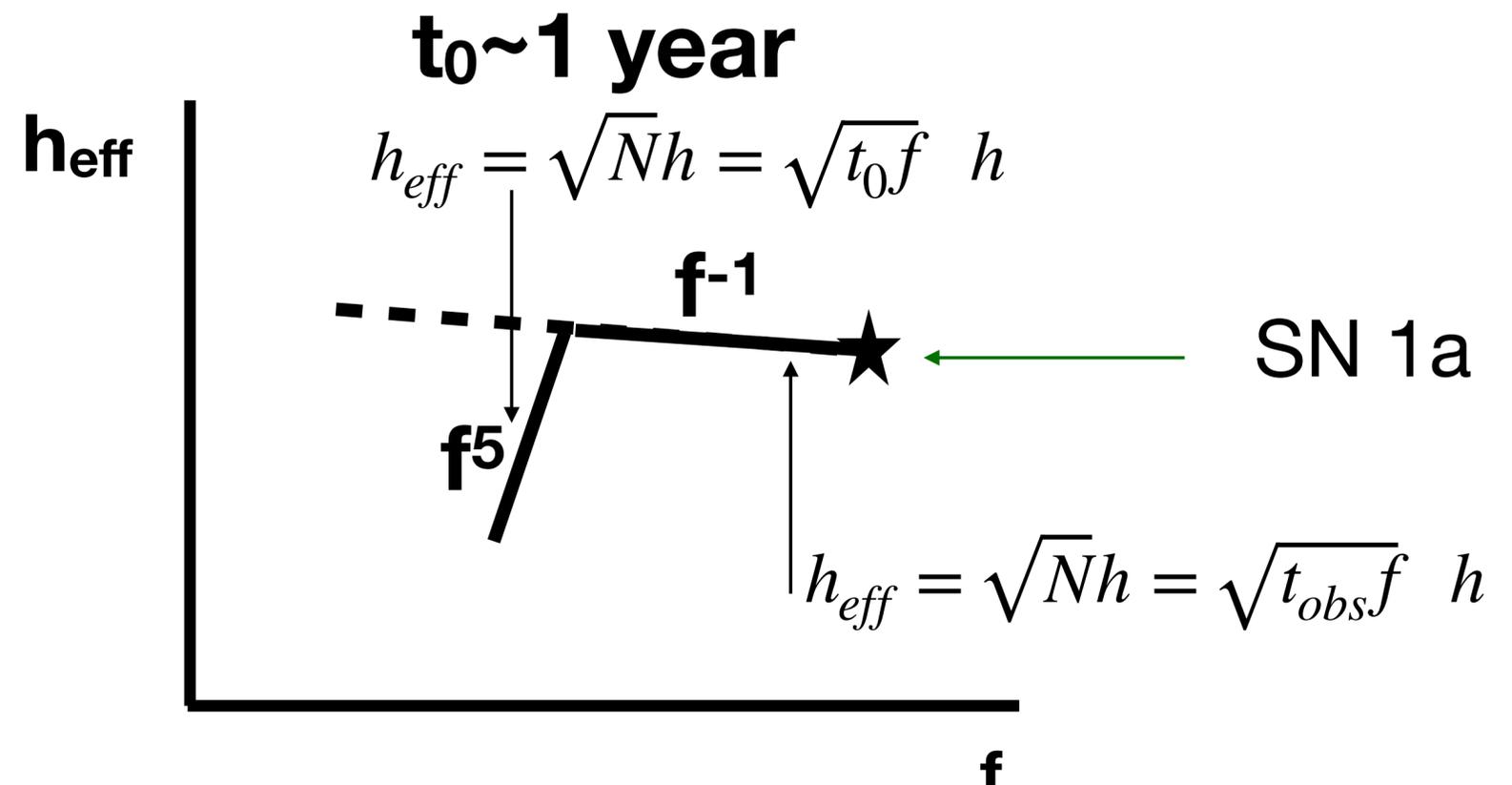## An open frontier - sources of LISA GWs and Ia SNe?

- Classical LISA sources

- Progenitors of SNe 1a

- These curves are based on knowledge of the chirping signal.

$$h_{eff} = \sqrt{N}h = \sqrt{t_{obs}f} \ \ h$$

SN 1a

Binary White Dwarf
An open frontier - sources of LISA GWs and Ia SNe?

- Classical LISA sources

- Progenitors of SNe 1a

- These curves are based on knowledge of the chirping signal.

t$_0$~1 year

h_{eff}

$$h_{eff} = \sqrt{N}h = \sqrt{t_0 f} \ \ h$$

f^{-1}

f^5

SN 1a

$$h_{eff} = \sqrt{N}h = \sqrt{t_{obs}f} \ \ h$$

f

Binary White Dwarf
## An open frontier - sources of LISA GWs and Ia SNe?

- Classical LISA sources

- Progenitors of SNe 1a

- These curves are based on knowledge of the chirping signal.

- But…
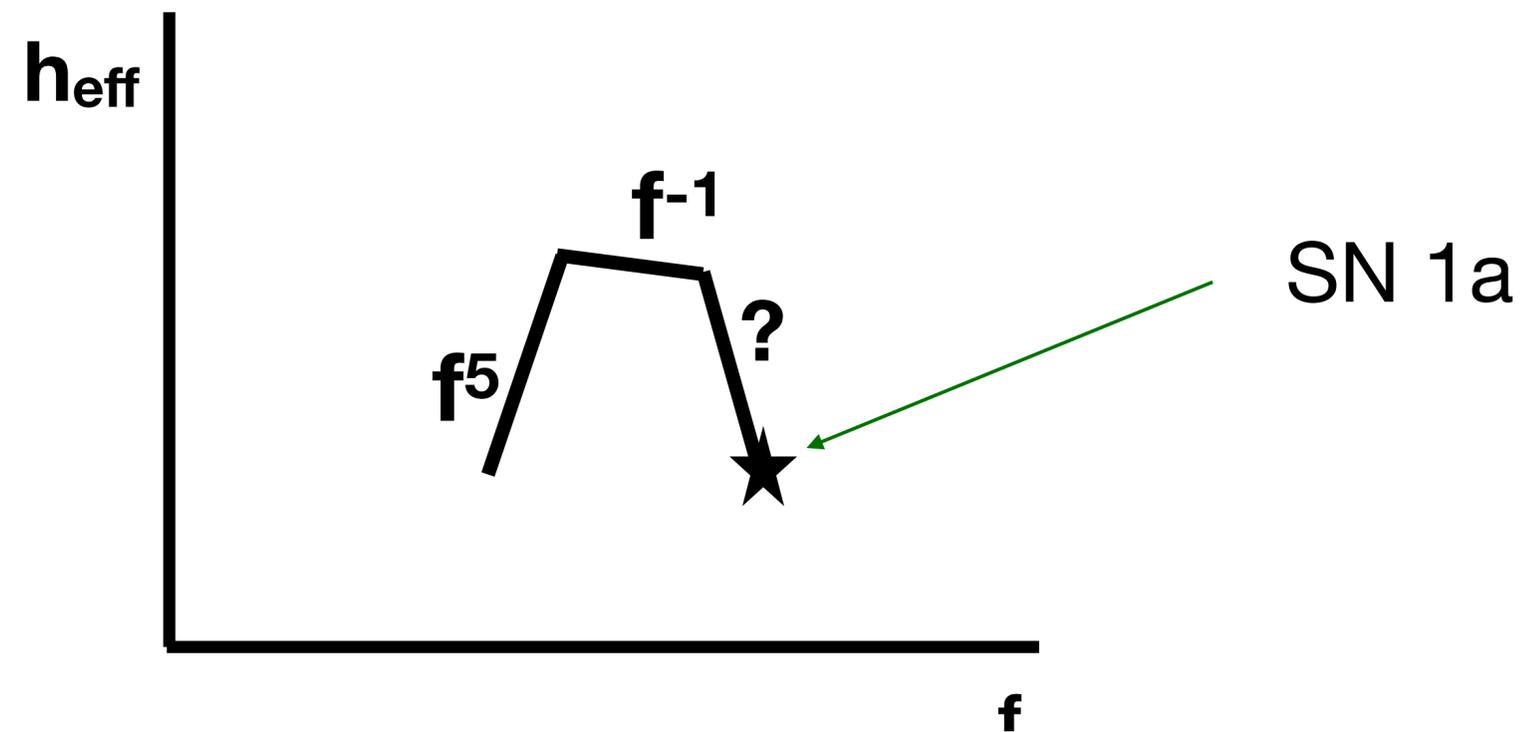
Tidal interaction in binary WDs
## An (almost) Netonian problem with relativistic aspects and implications

- Tidal interaction between the two stars begin to dominate the GW emission long before the merger.

- The effects of this interaction must be well understood in order to understand the initial values for the merger itself.

- We must understand this phase to estimate and detect the (strongest) GW BWD signals just before merger.

- The numerical issue is the huge difference in time scales.
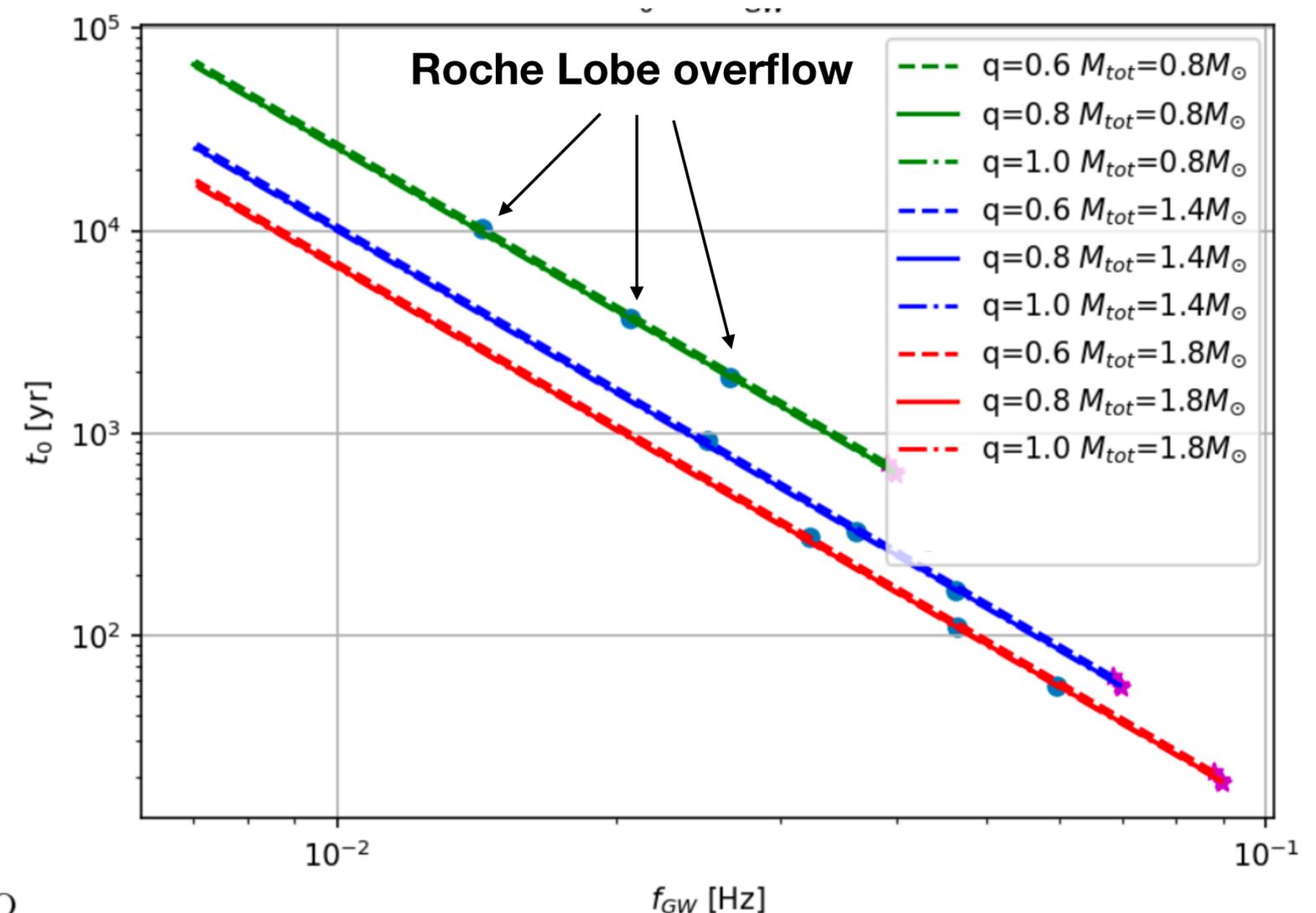
Stellar Black hole - WD disruption
If you want to make it relativistic replace one WD by a BH (or NS).

- Sources of very long GRBs (e.g. Lloyd- Ronning et al., 2024)?

- For a massive WD and a light BH, the disruption will happen very close to the collision (in space - not in time).
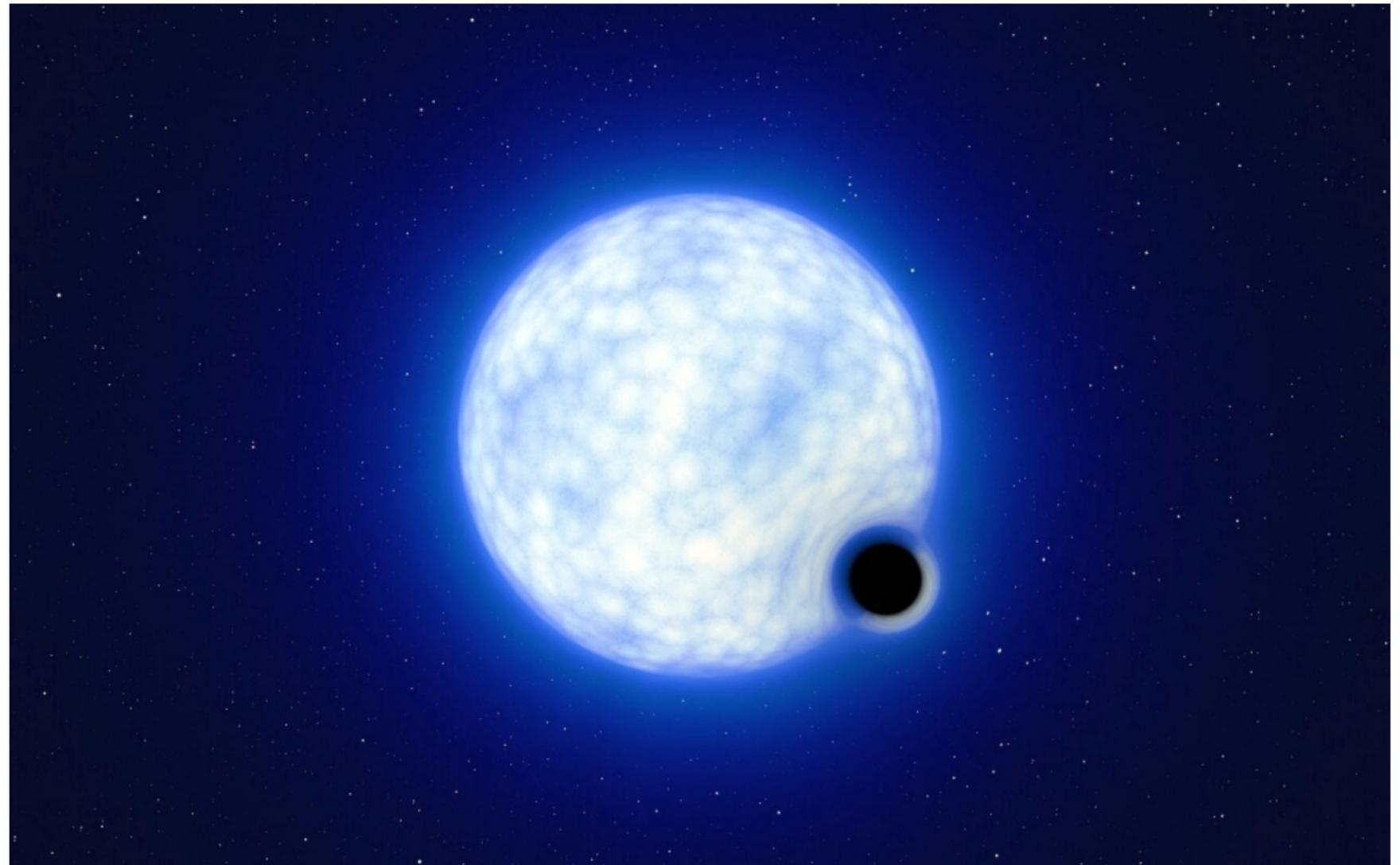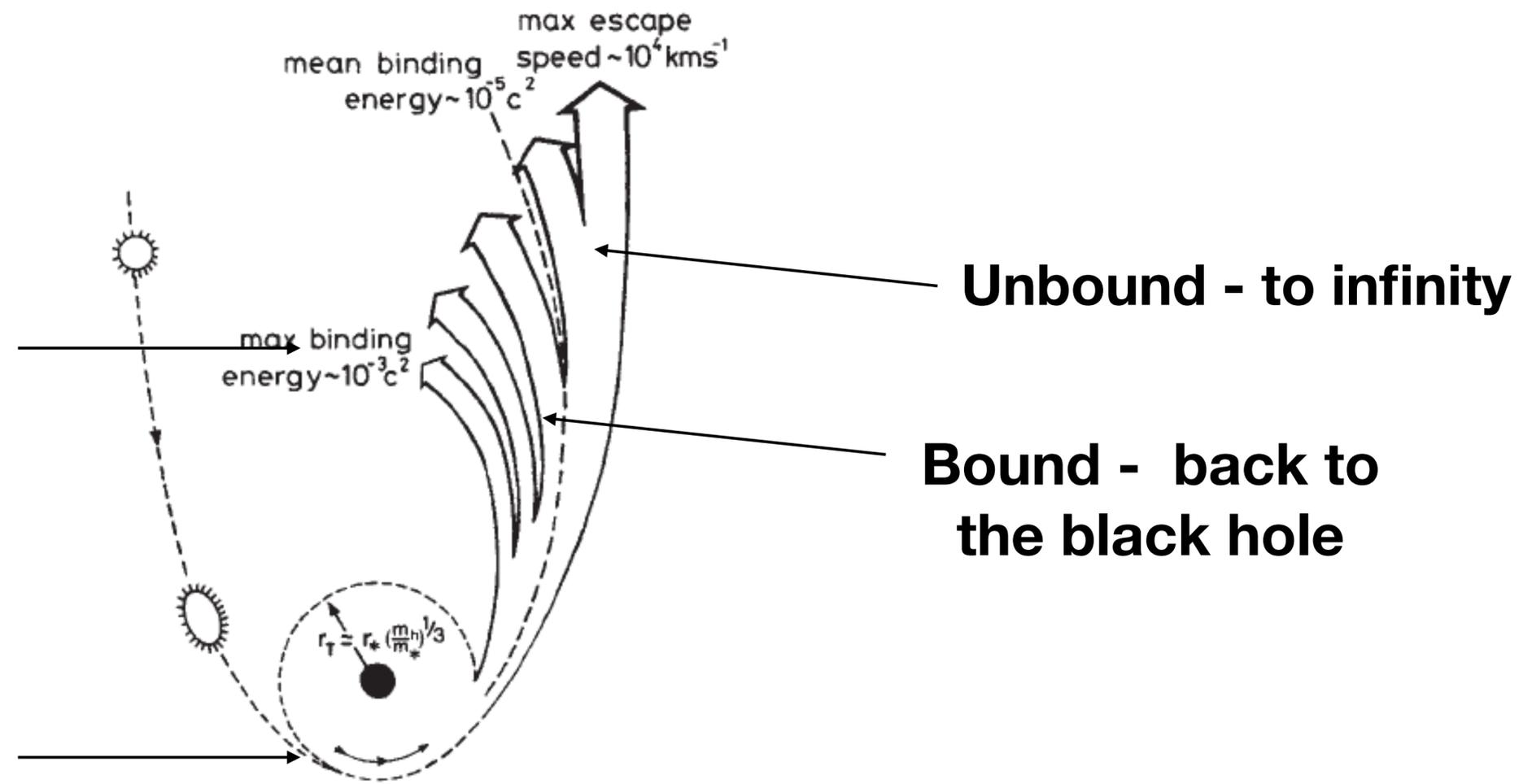
Tidal Disruption
A (partially) open frontier

- The classical picture

$$\Rightarrow a_0 \sim 1000 \, r_g$$
$$t_0 \sim 1 \text{ month}$$

$$\Rightarrow r_t \sim 25 \, r_g$$

max escape speed $\sim 10^4 \text{kms}^{-1}$

mean binding energy $\sim 10^{-5} c^2$

max binding energy $\sim 10^{-3} c^2$

Unbound - to infinity

Bound - back to the black hole

$$r_T = r_* \left(\frac{m_h}{m_*}\right)^{1/3}$$

Rees 1988

What happen when the mass returns
The Classical picture

Expected

- T ~ 500,000 K

- R ~ 25 r_g

- v > 50000 km/sec

- E ~ 10^{53} erg

- **A compact disk of size ~ the tidal radius ~25 r_g**

Optical/UV TDEs

Observations

Observed

- T ~ 50,000 K

- R ~ 500 r_g

- v ~ 5000 km/sec
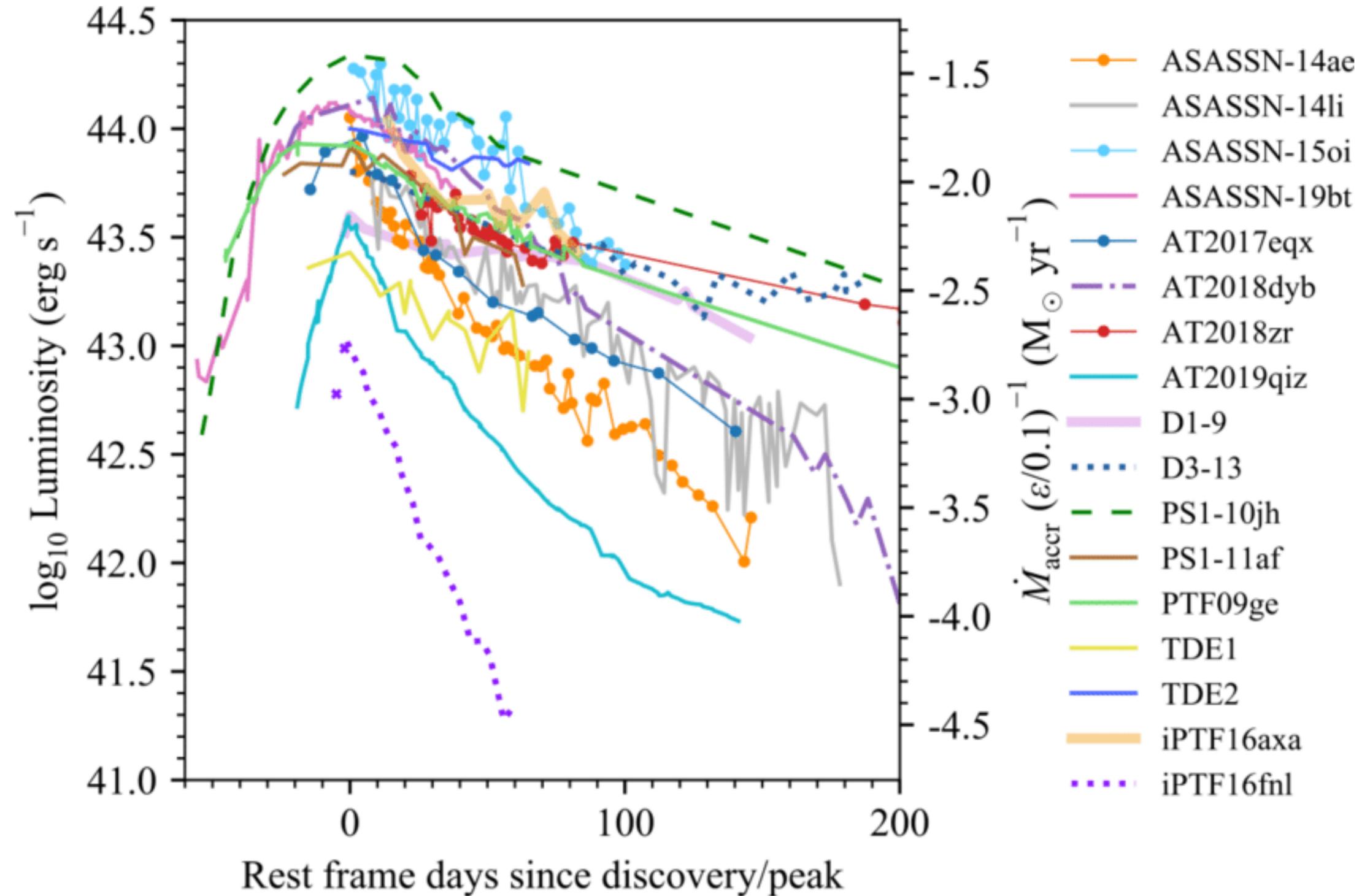
- E ~ 10^{51} erg

Optical/UV TDEs

Observations vs expectations

Observed

- T ~ 50,000 K

- R ~ 500 r_g

- v ~ 5000 km/sec

- E ~ 10^{51} erg

Expected

- T ~ 500,000 K

- R ~ 25 r_g

- v > 50000 km/sec

- E ~ 10^{53} erg

Where has the energy gone?

Early simulation
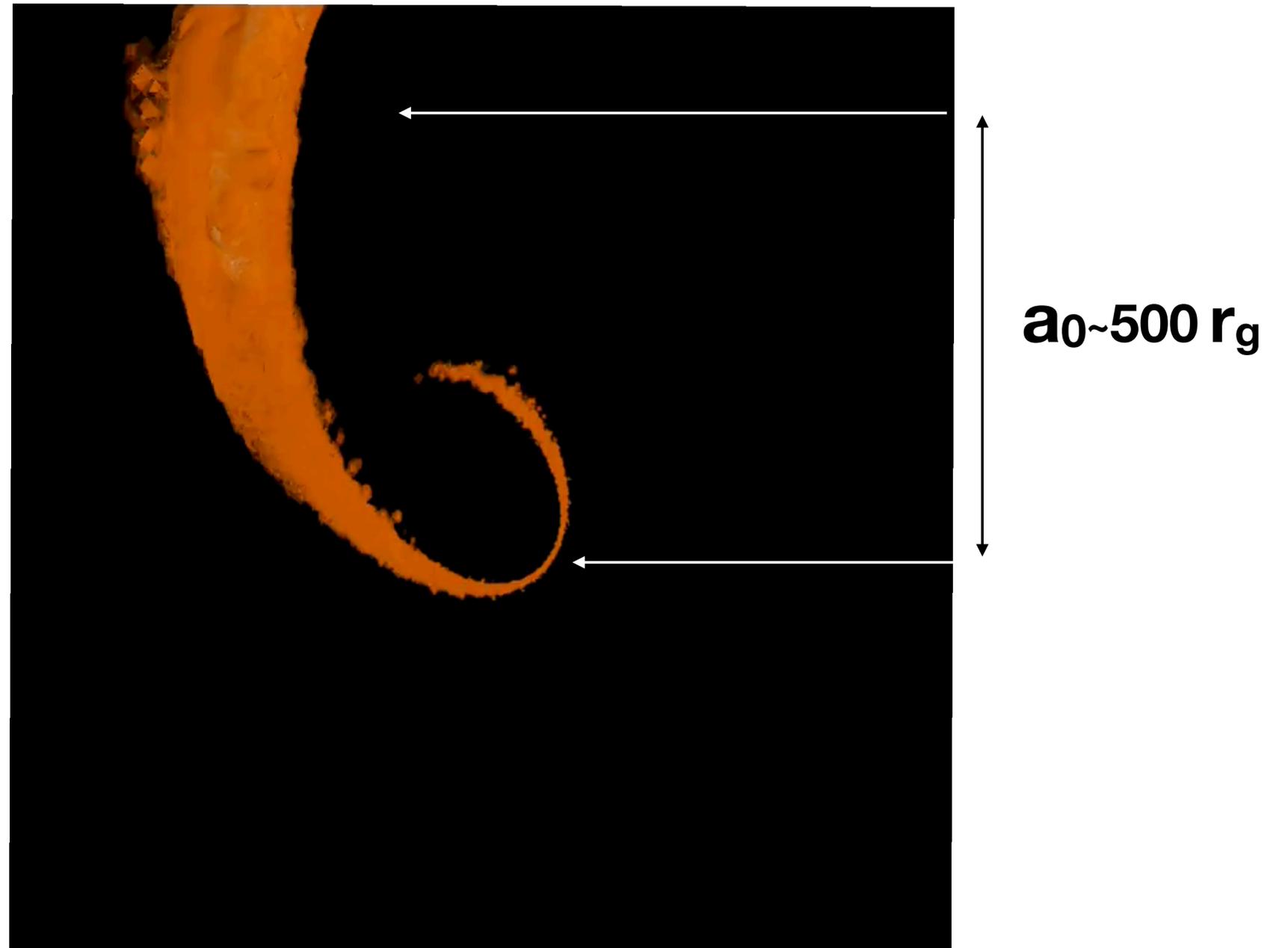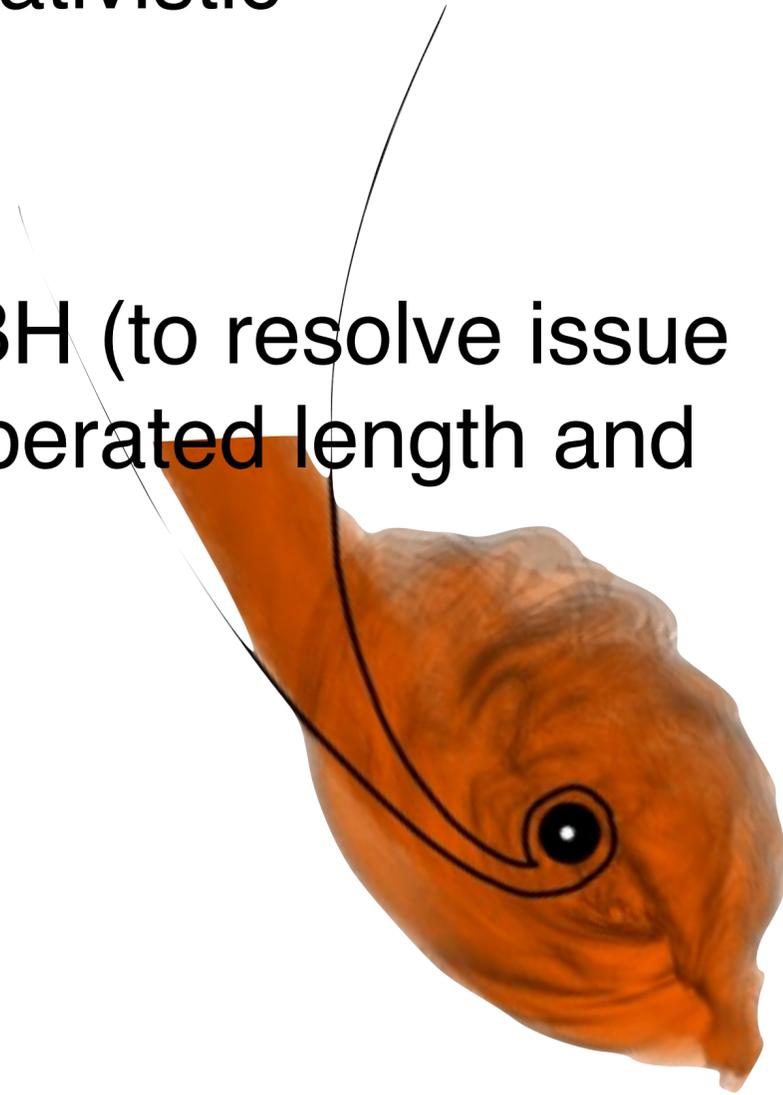## No Disk

- SPH

- Newtonian

- WD on IMBH

WD–BH encounter
****

masses (sol.)	0.2 (WD) & 1000 (BH)
in. separation	50 (in 1.E9 cm)
hydrodynamics	SPH (4 030 000 particles)
EOS, gravity	Helmholtz, N
nucl. burning	red. QSE–network (Hix 98)
simul. time	5.4 min
color coded	column density
penet. factor	12

coding, simulation, visualisation: S. Rosswog

S. Rosswog 2006

Partially relativistic simulation

No disk - Big Cloud ; Shocks

- Partially relativistic

- Full Hydro

- WD on IMBH (to resolve issue of widely seperated length and time scales).
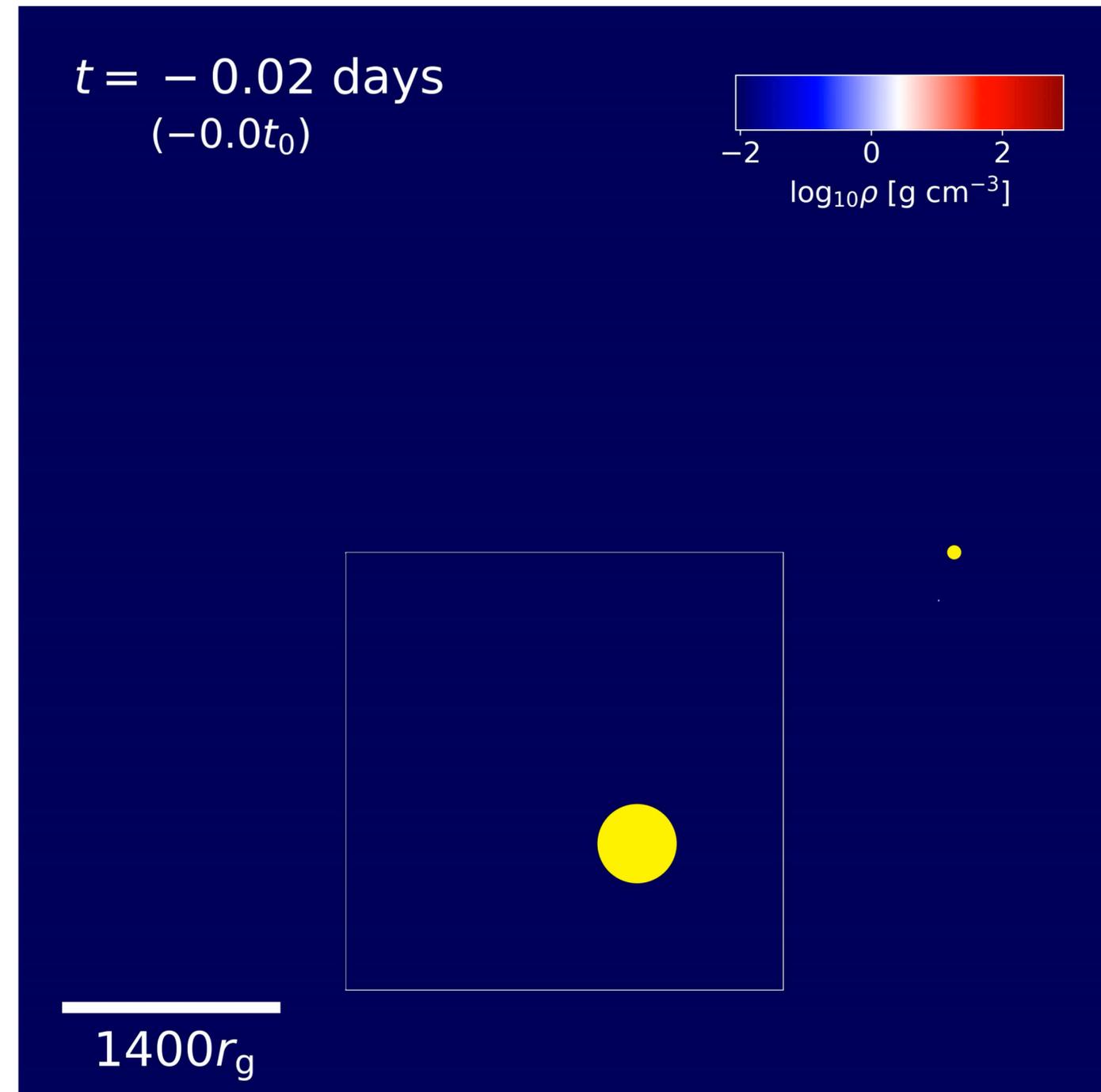
$a_0 \sim 500\ r_g$

Shiokawa, Krolik, TP, 15
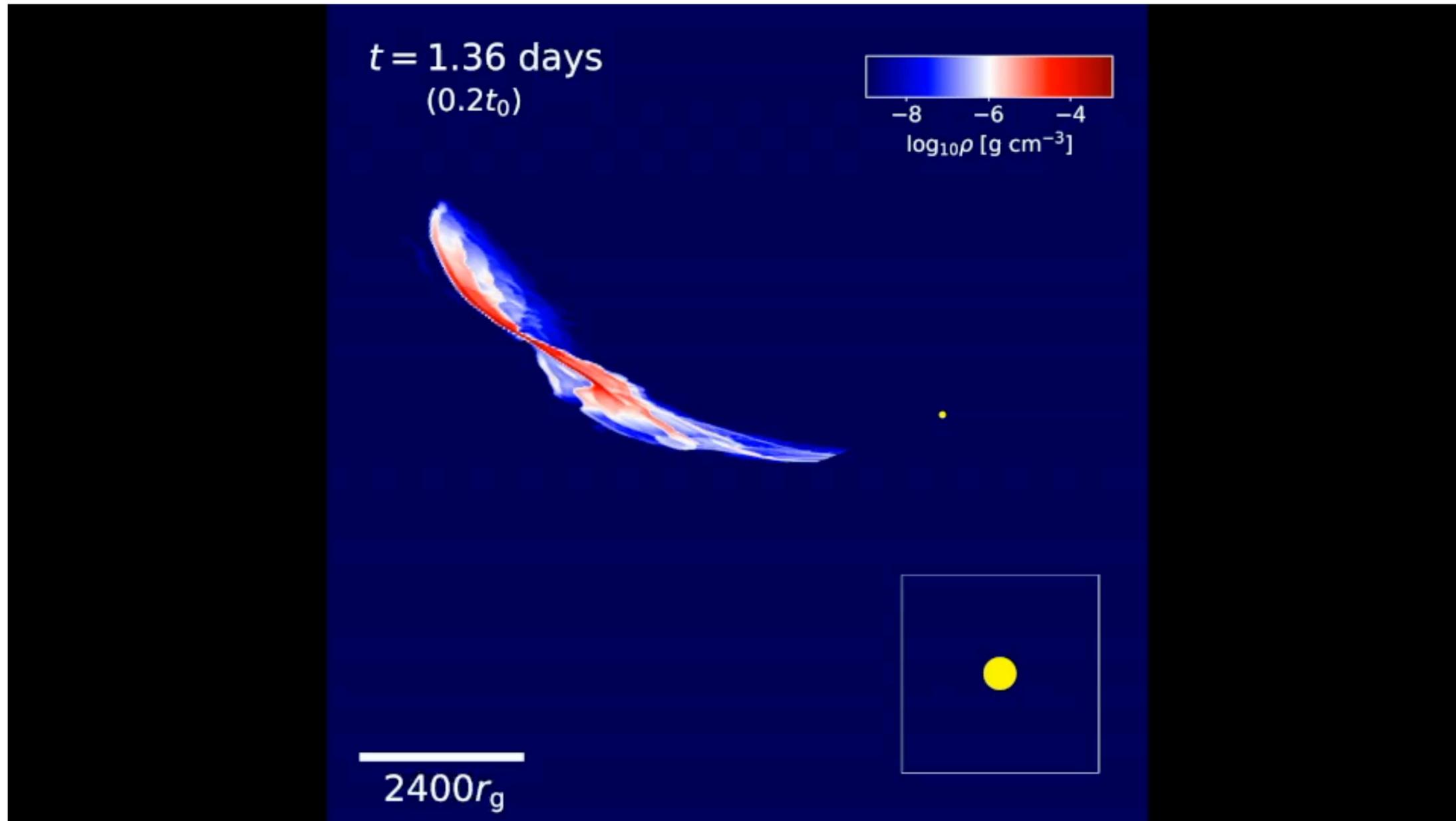
Current fully relativistic simulation

No disk - Big Cloud ; Shocks

- Fully relativistic

- Full Hydro (HARM3D)

- Realistic initial conditions (MESA star) on 10^5 M$_\odot$ black hole.

$t = -0.02$ days

$(-0.0t_0)$

$\log_{10}\rho$ [g cm^{-3}]

$1400r_g$

Ryu, Krolik, TP, Noble, Avara - 2023

Current fully relativistic simulation
No disk - Big Cloud ; Shocks

Ryu, Krolik, TP, Noble, Avara - 2023

BH GR + local gravity = multi patch

Stellar gravity on a patch

$$g_{\mu\nu} \simeq \tilde{g}_{\mu\nu} + h_{\mu\nu}^{\mathrm{sg}}$$

$$h_{00}^{\mathrm{sg}} = -2\Phi_{\mathrm{sg}}c^{-2},$$

$$h_{0i}^{\mathrm{sg}} = h_{i0}^{\mathrm{sg}} = 0,$$

$$h_{ij}^{\mathrm{sg}} = 0,$$

$$e_{(0)}^{\mu} = (1/\sqrt{-\tilde{g}_{00}}, 0, 0, 0)$$

$$\frac{de_{(a)}^{\mu}}{d\tau} + \Gamma_{\alpha\beta}^{\mu}e_{(a)}^{\alpha}e_{(0)}^{\beta} = 0$$
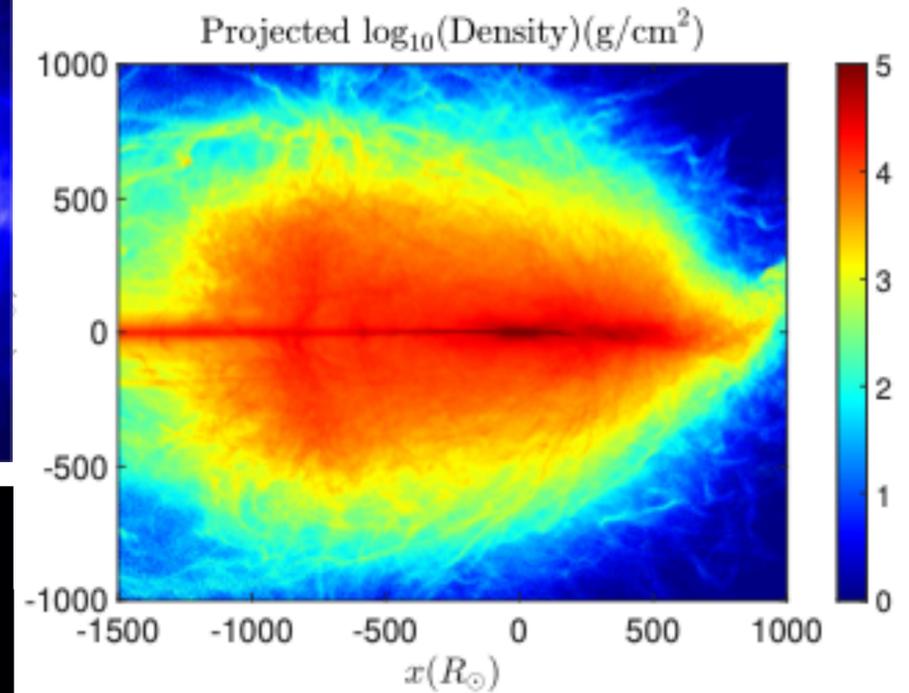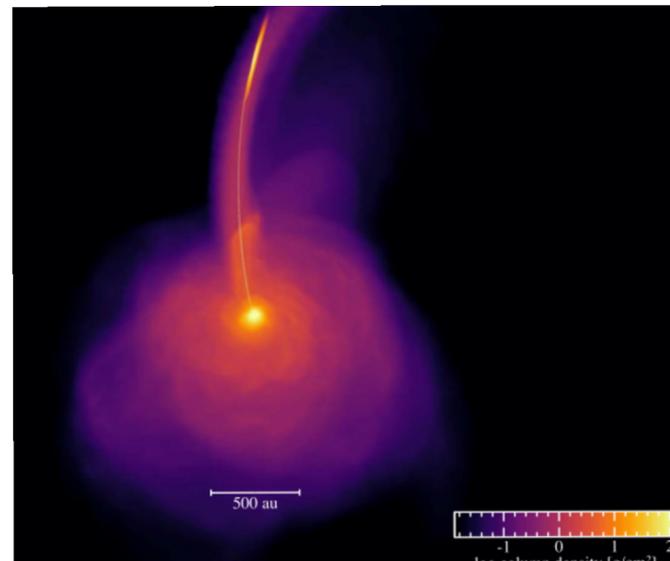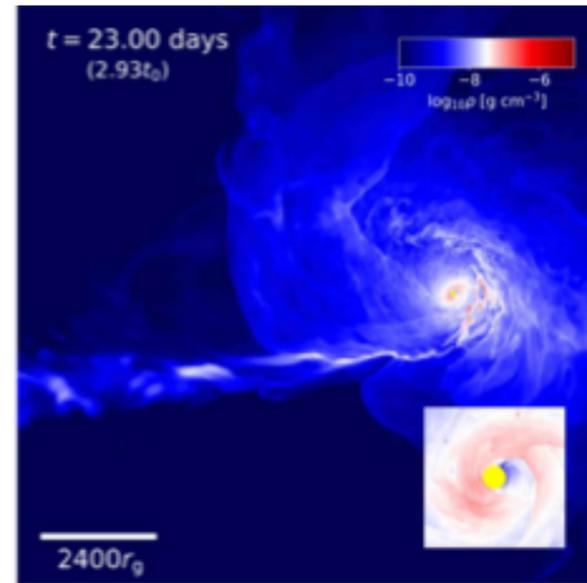
So far only in Schartzschild geometry

$10\,r_{\mathrm{t}}$ $5\,r_{\mathrm{t}}$ $1\,r_{\mathrm{t}}$

Direction
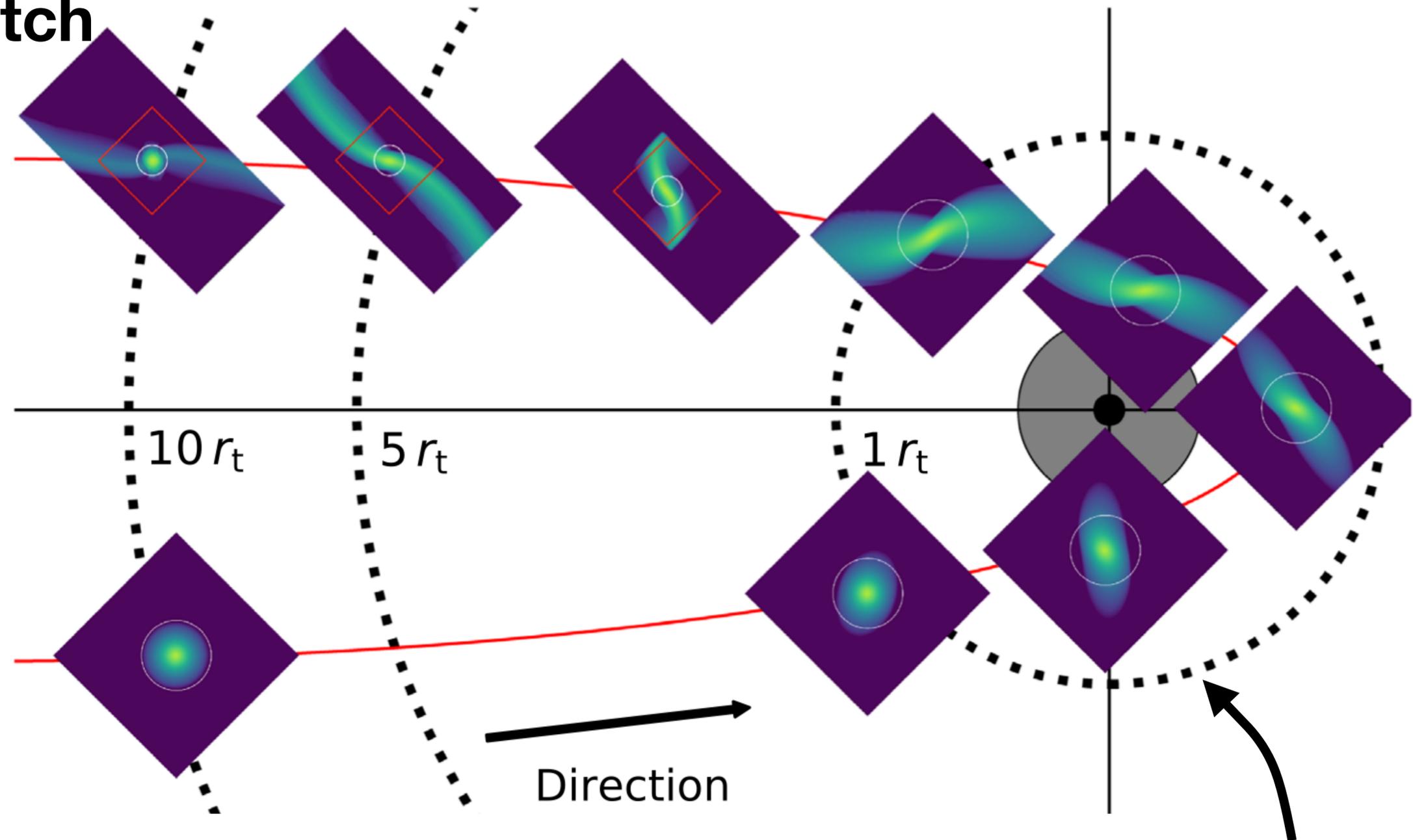
Ryu et al., 2020

The tidal radius $r_{\mathrm{t}} \sim 25\,r_{\mathrm{g}}$

A concordance Picture

Different codes - same result

- **Ryu et al., 2023** (Full GR)

- **Steinberg and Stone 2024** (Shorter, includes radiation transfer)

- **Price et al., 2024** (SPH up to ~9 t_0)

- **Abolmasov et al., 2025** (Full GRMHD, small SMBH, magnetized)
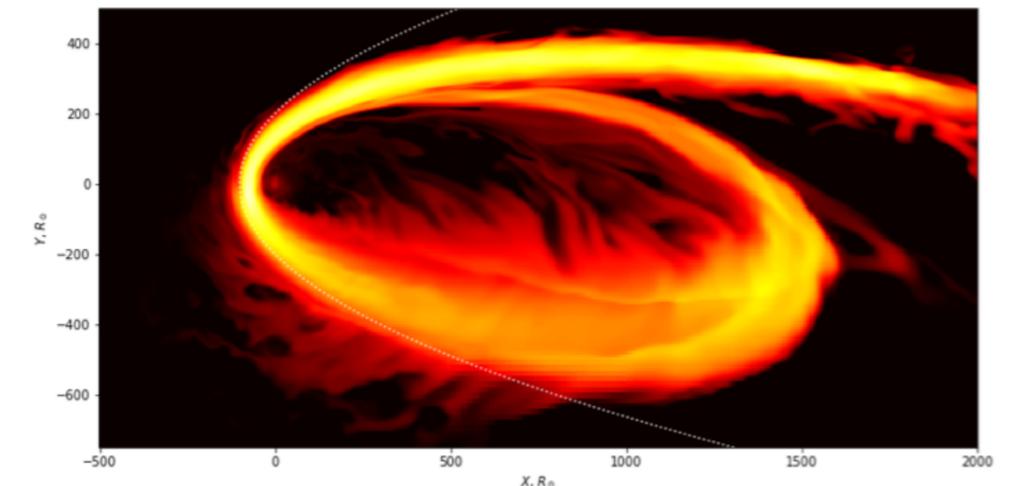
TDE population - another side remark

A luminosity (M*,MBH) model following these results

- These numerical results lead naturally to a simple analytic physical model for the luminosity as a function of the stellar mass and the black hole mass (Krolik, Piran, Ryu 2025).

- The model fits nicely (with no free parameters) the observations (Piran, Krolik, Ryu, 2026)
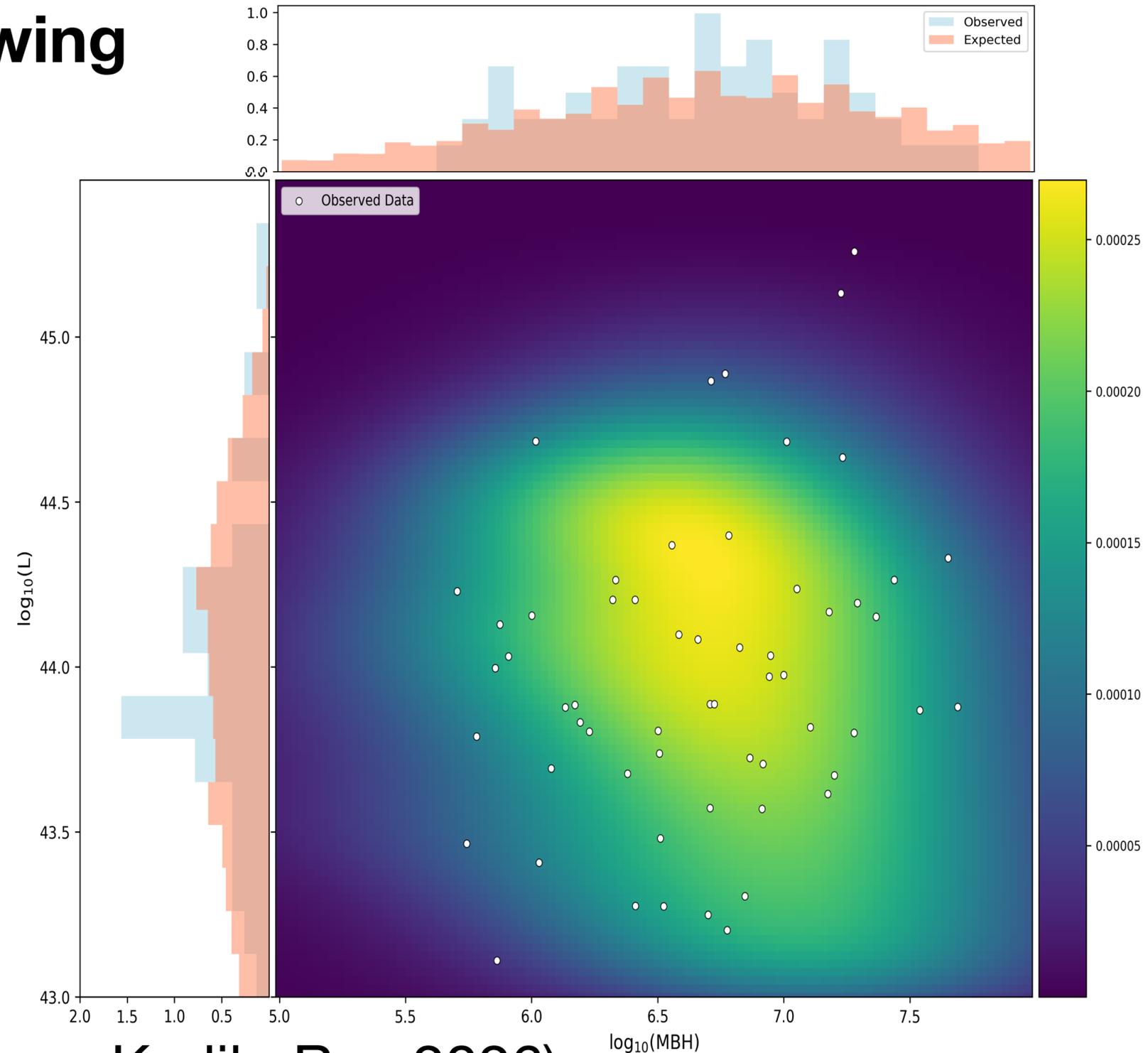
Piran, Krolik, Ryu 2026)

Deeper encounters a TDE at $r_p \sim 10\, r_g$

Some like it more relativistic

- Apsidal precession and other GR effects become stronger as r_p decreases.

- This is common, as in TDEs, the cross-section is linear in r_p.

- For massive ($M_{BH} > 10^8\, M_\odot$) this is the "typical case".

- We see initially strong "nozzle shocks" but they weaken.

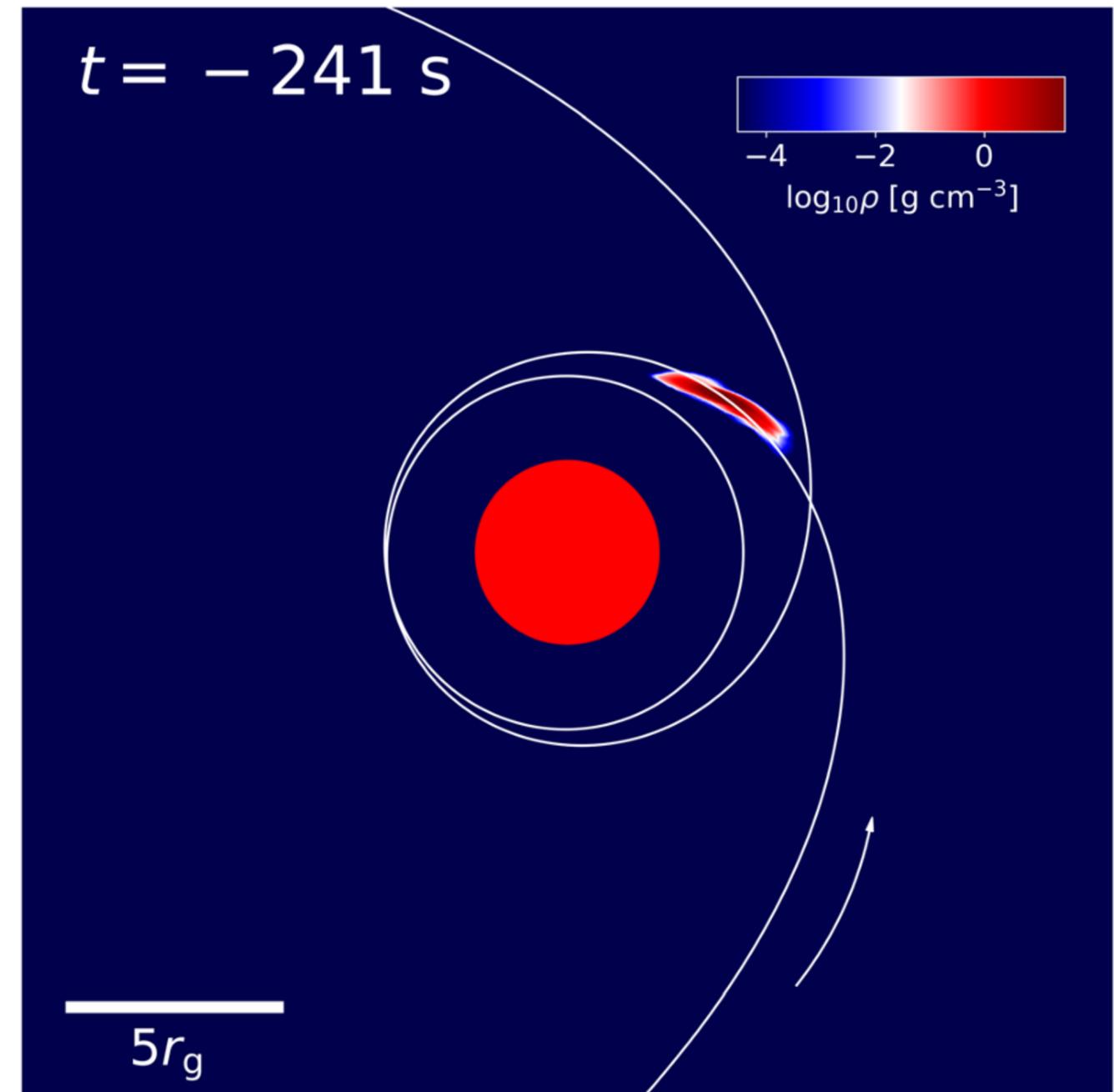- Apsidal precission is there but its effect is not very strong.
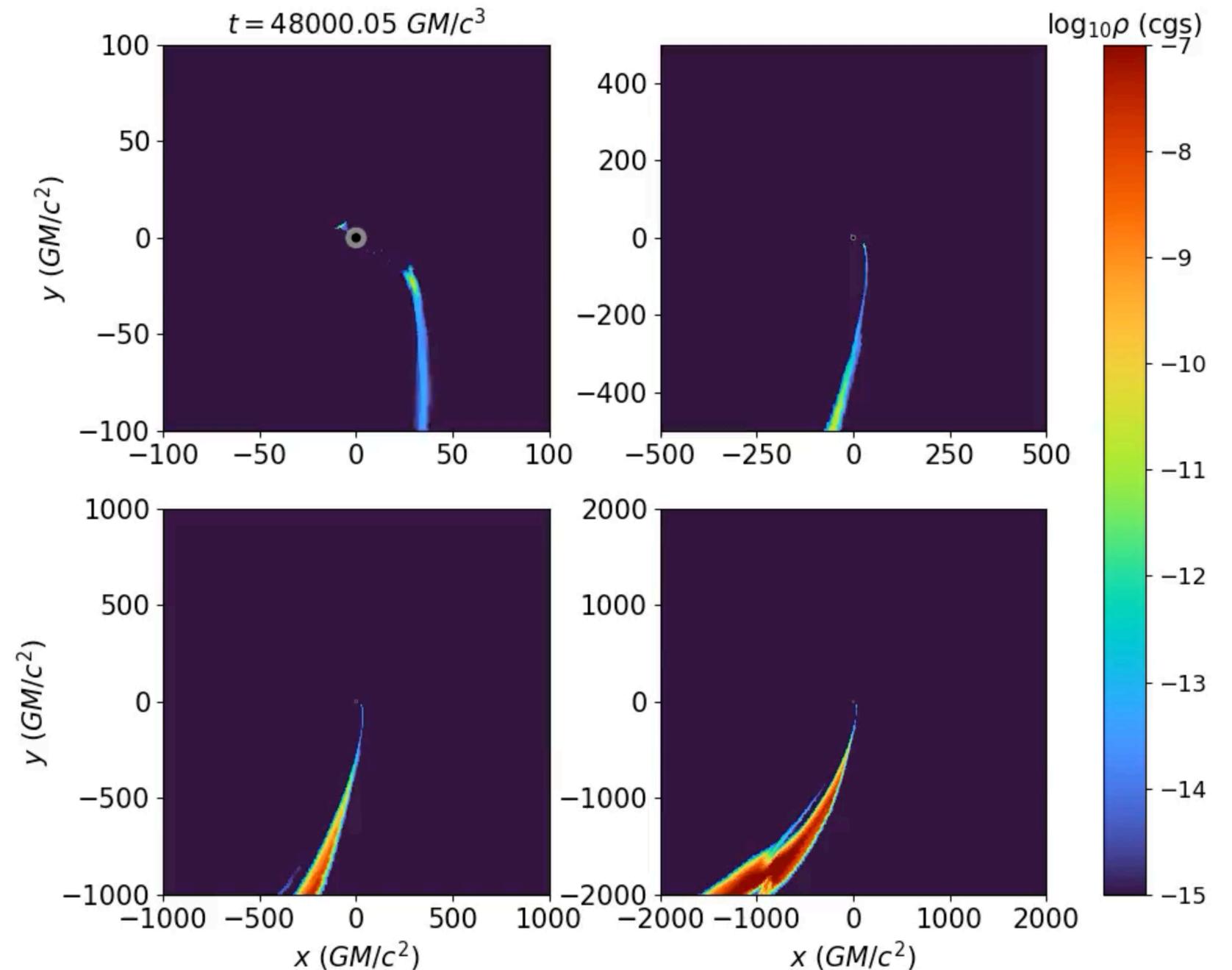
Chan, Ryu, Krolik, Piran, 2026

Extreme TDEs: $r_p \sim$ 4-6 r_g

Some like it even more relativistic

- TDEs are very different when the star's original orbit passes very close to r_{mb} (4 r_g for sch).

- This is common, as in TDEs, the cross-section is linear in r_p.

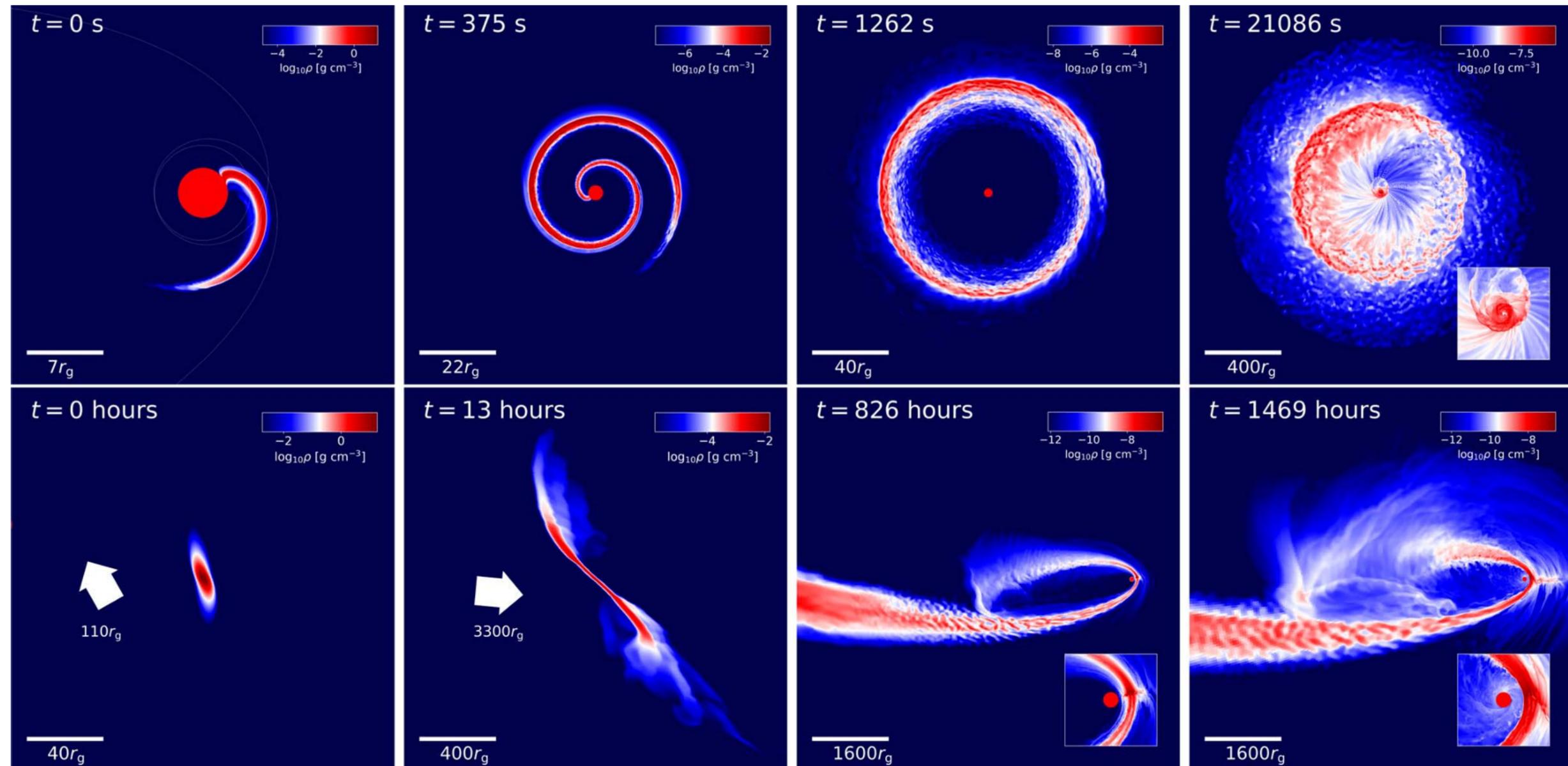- For massive ($M_{BH} > 10^8$ M$_\odot$ this is the "typical case".

Ryu, Krolik, Piran, 2023

Extreme TDEs: $r_p \sim 4\text{-}6\ r_g$
Some like it even more relativistic

- Extreme TDE evolution is very different from the regular one.

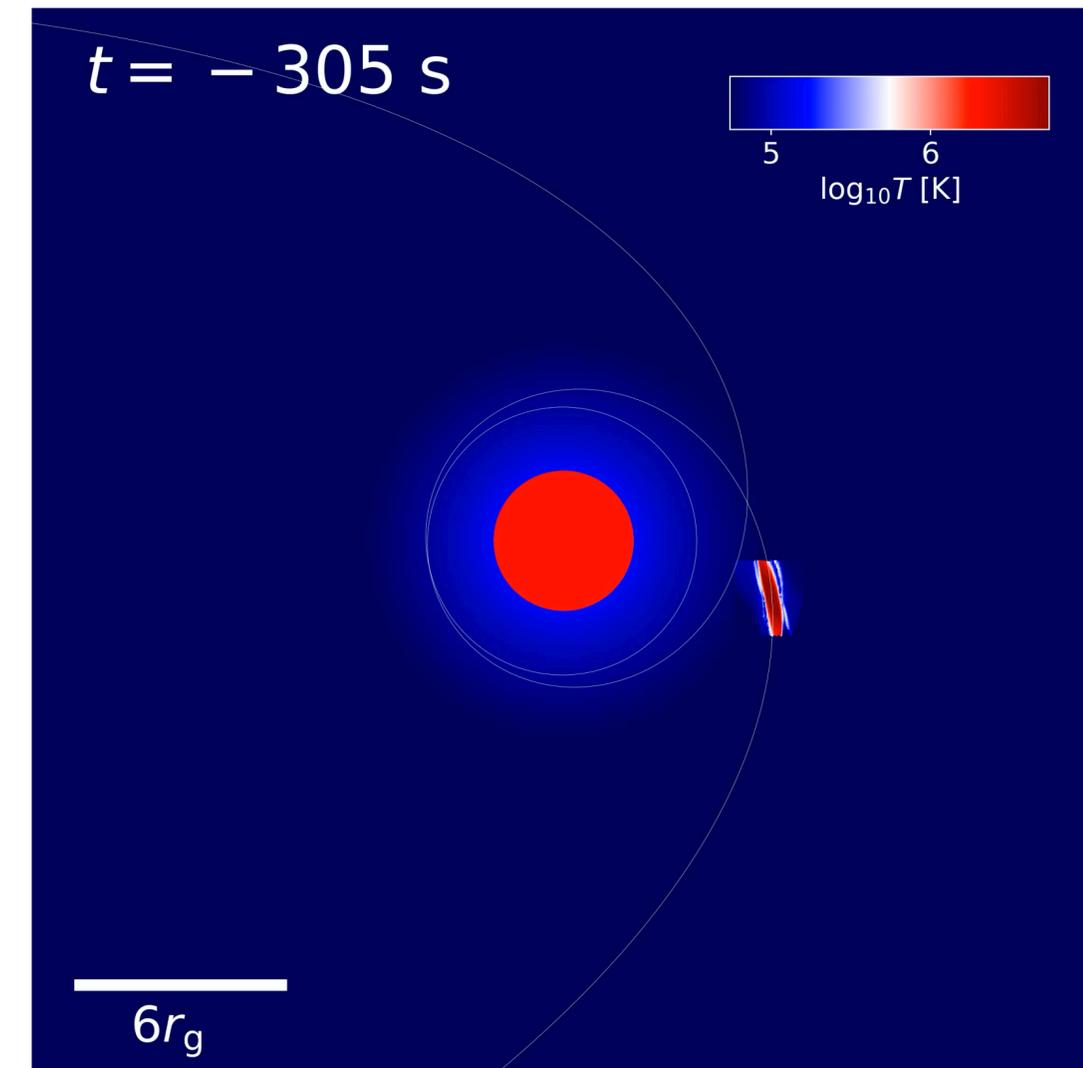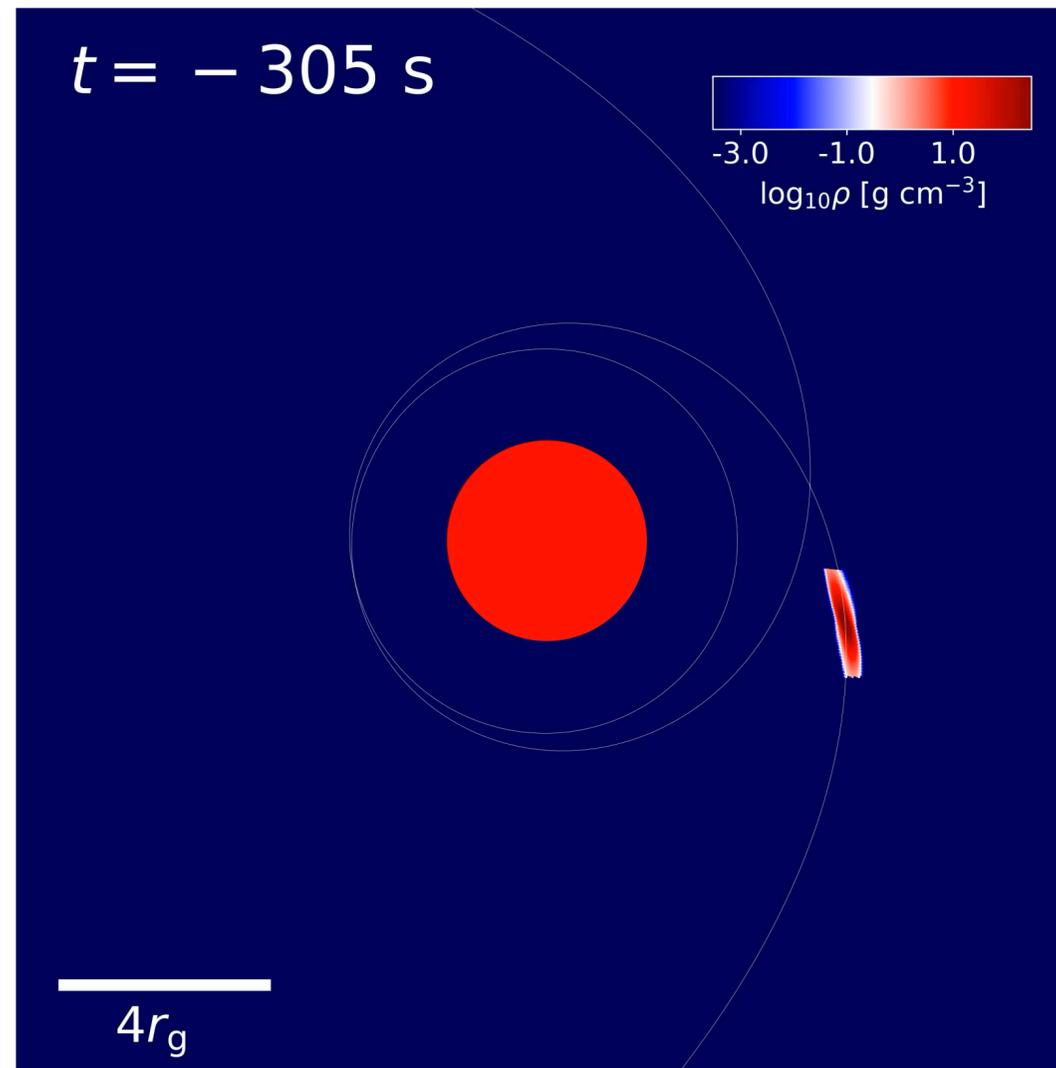- For massive ($M_{BH} > 10^8$ $M_\odot$ this is the "typical case".

Ryu, Krolik, Piran, 2023

Extreme TDEs: $r_p \sim$ 4-6 r_g
Some like it even more relativistic

- Extreme TDE evolution is very different from the regular one.

- For massive ($M_{BH} > 10^8$ $M_\odot$ this is the "typical case".

$t = -305$ s

$log_{10}\rho$ [g cm^{-3}]

-3.0   -1.0   1.0

$4r_g$

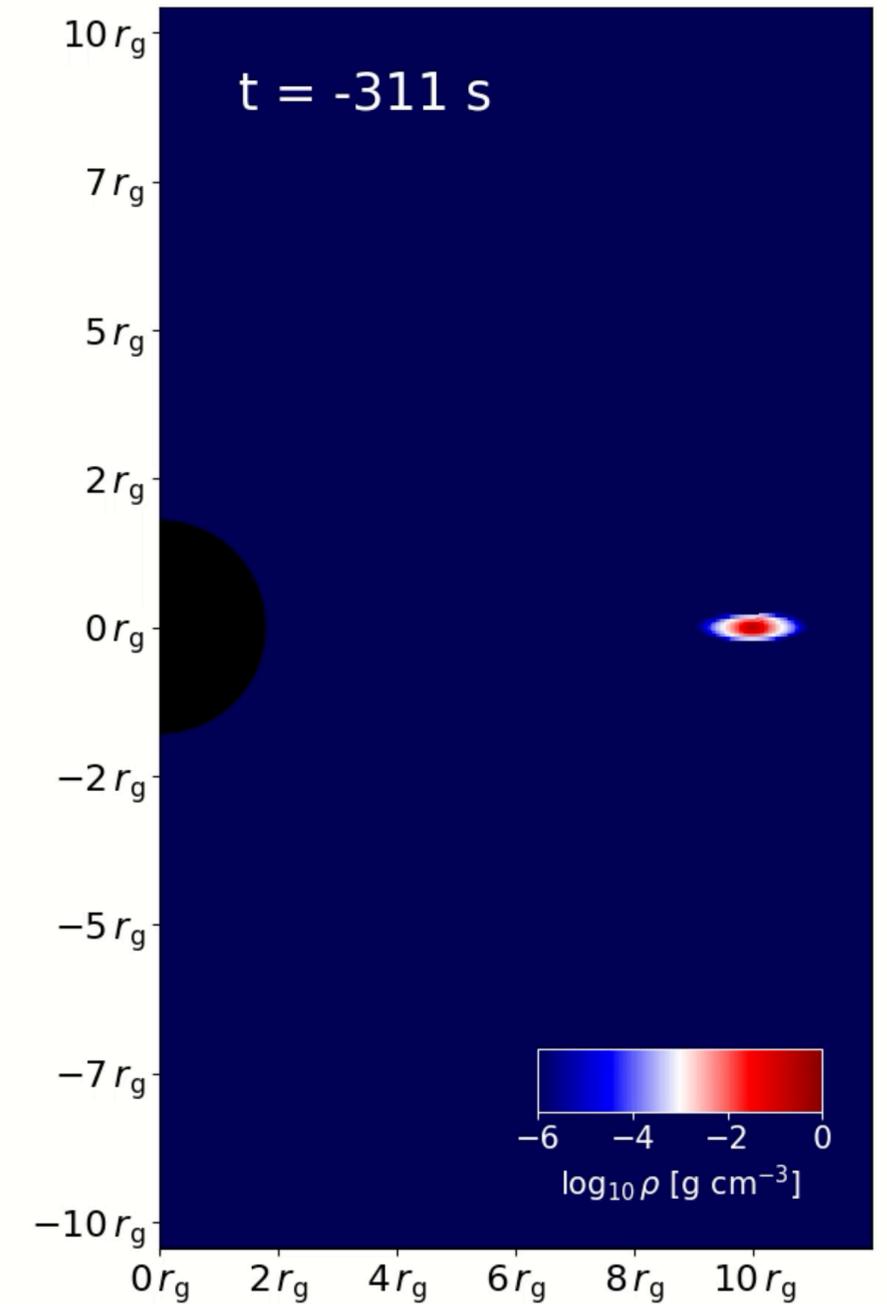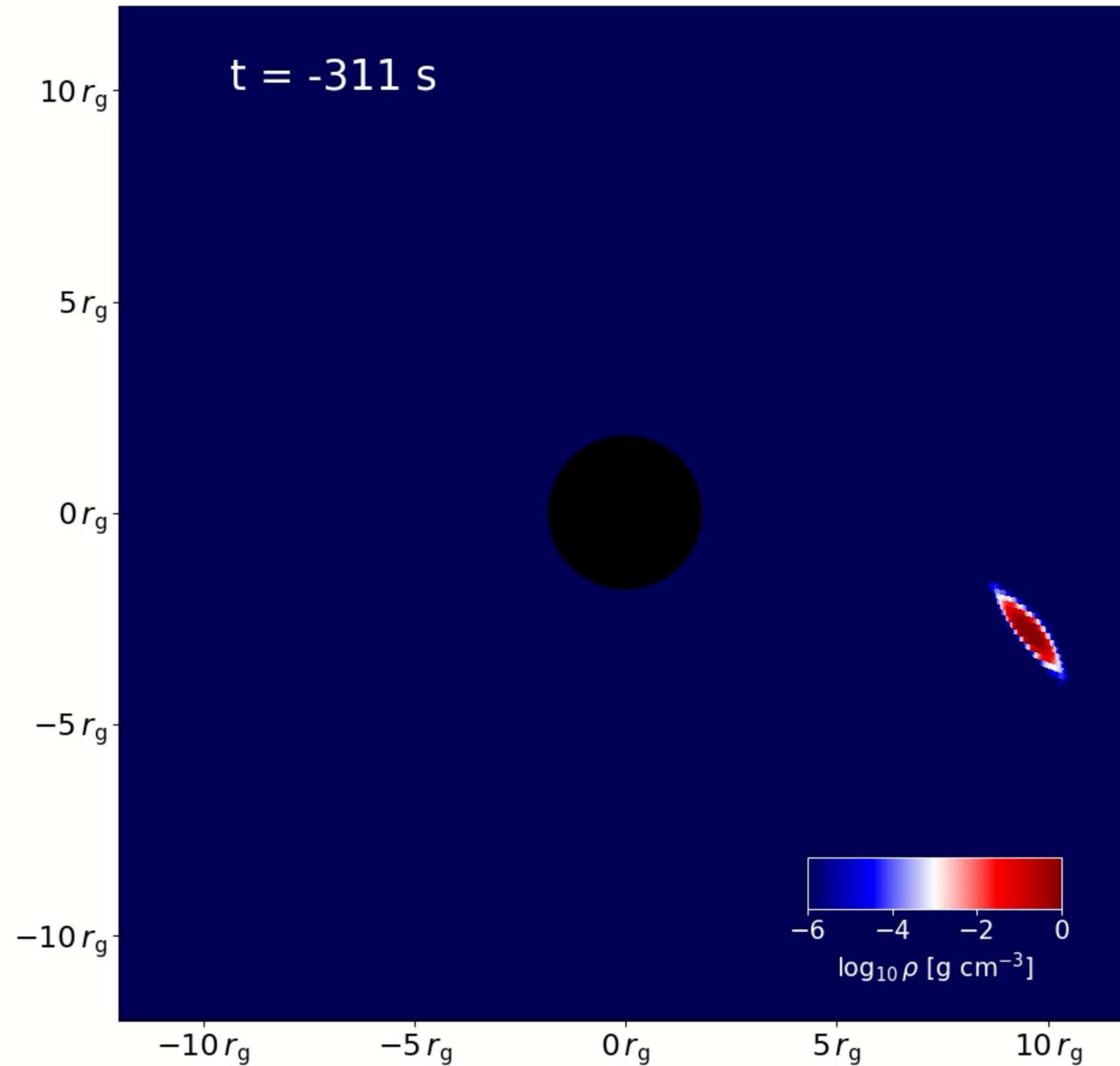$t = -305$ s

$log_{10}T$ [K]

5   6

$6r_g$

Ryu, Krolik, Piran, 2023

Extreme TDEs: $r_p \sim$ 4-6 r_g
Extrme TDE around Kerr black hole

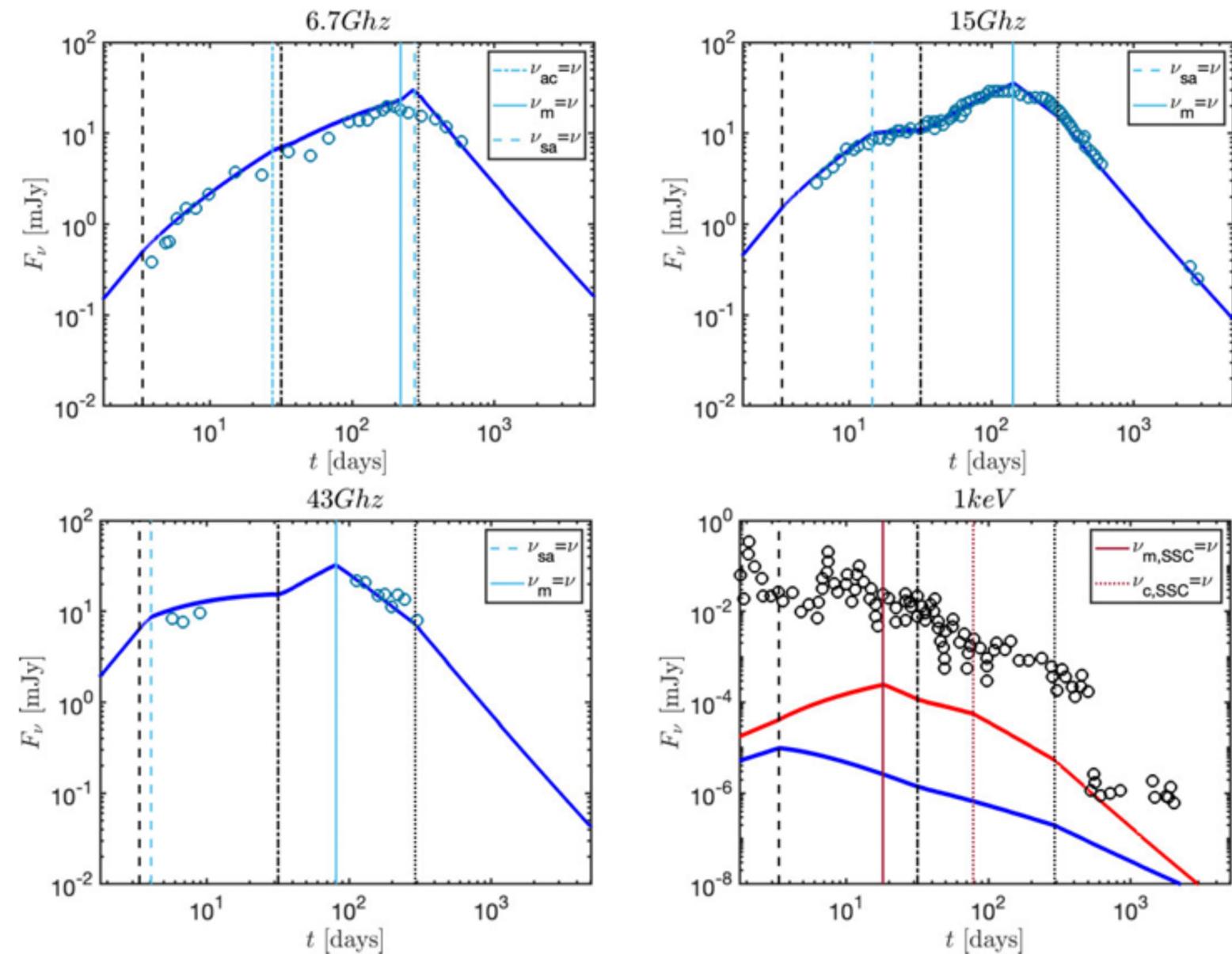- Extreme TDes around Kerr are even more extre as r_{mb} can be smaller.

Aharoni et al., 2026

Some TDEs harbor relativistic jet

SW-1644 was initially identified as GRB but it was a TDE

- SW-1644 was a TDE that revealed a very powerful (> 10^{51} erg) jet.

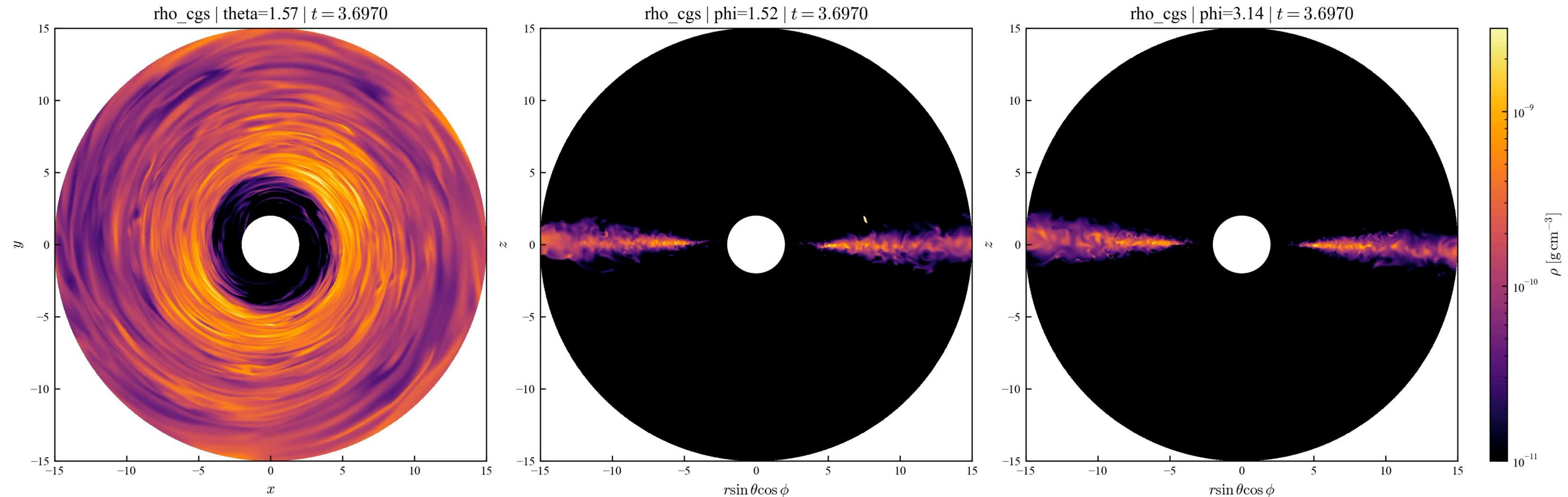- An intersting interpretation is that the jet was viewed off-axis (Matsumoto and Piran 2022)

Beniamini, Piran, Matsumoto 2023

Some TDEs harbor relativistic jet
Jet formation in TDEs is an open puzzle

- A TDE stream hitting a realistic magnetized disk. Will this lead to a jet?

Tripo, Jiang, … 2026 in preparation

from Misao Sasaki:

a few quotes from Masters...

T Nakamura

If physically correct, the essence should be describable within three lines.

If necessary, we must do it by any means, even by a brute force attack.