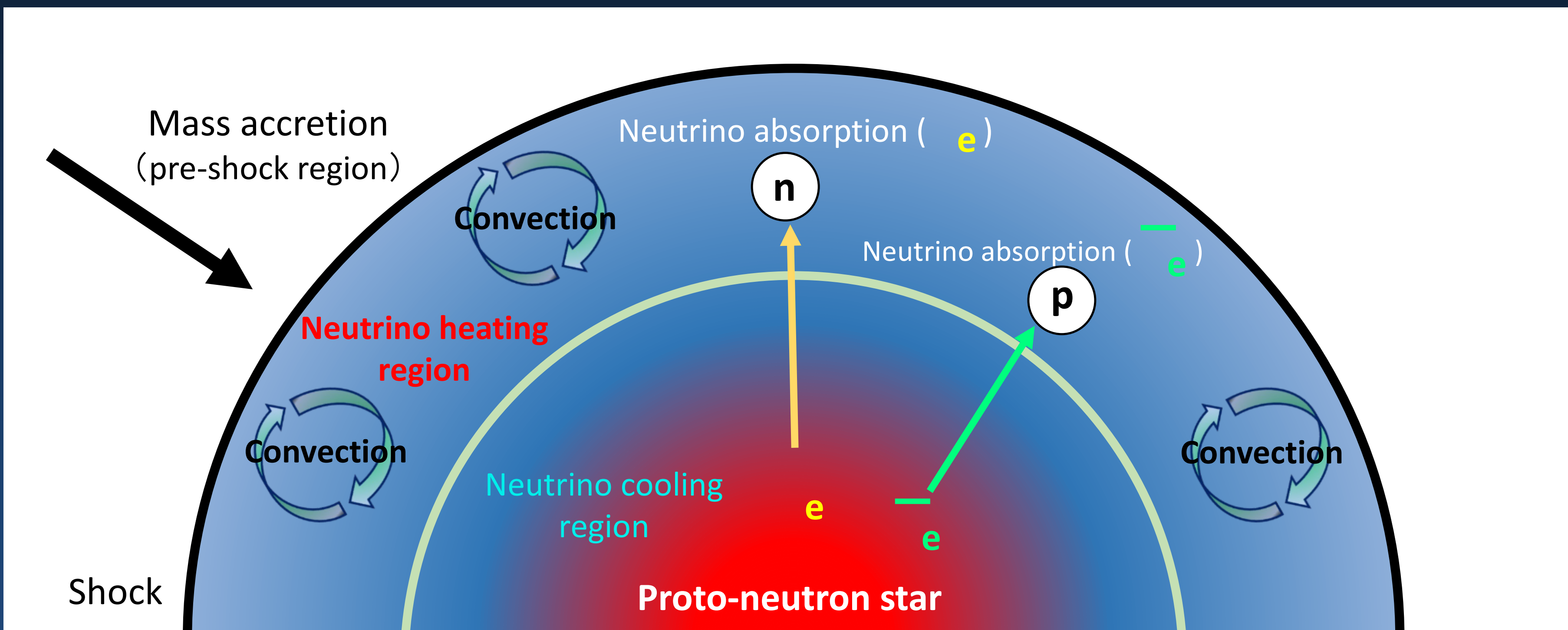
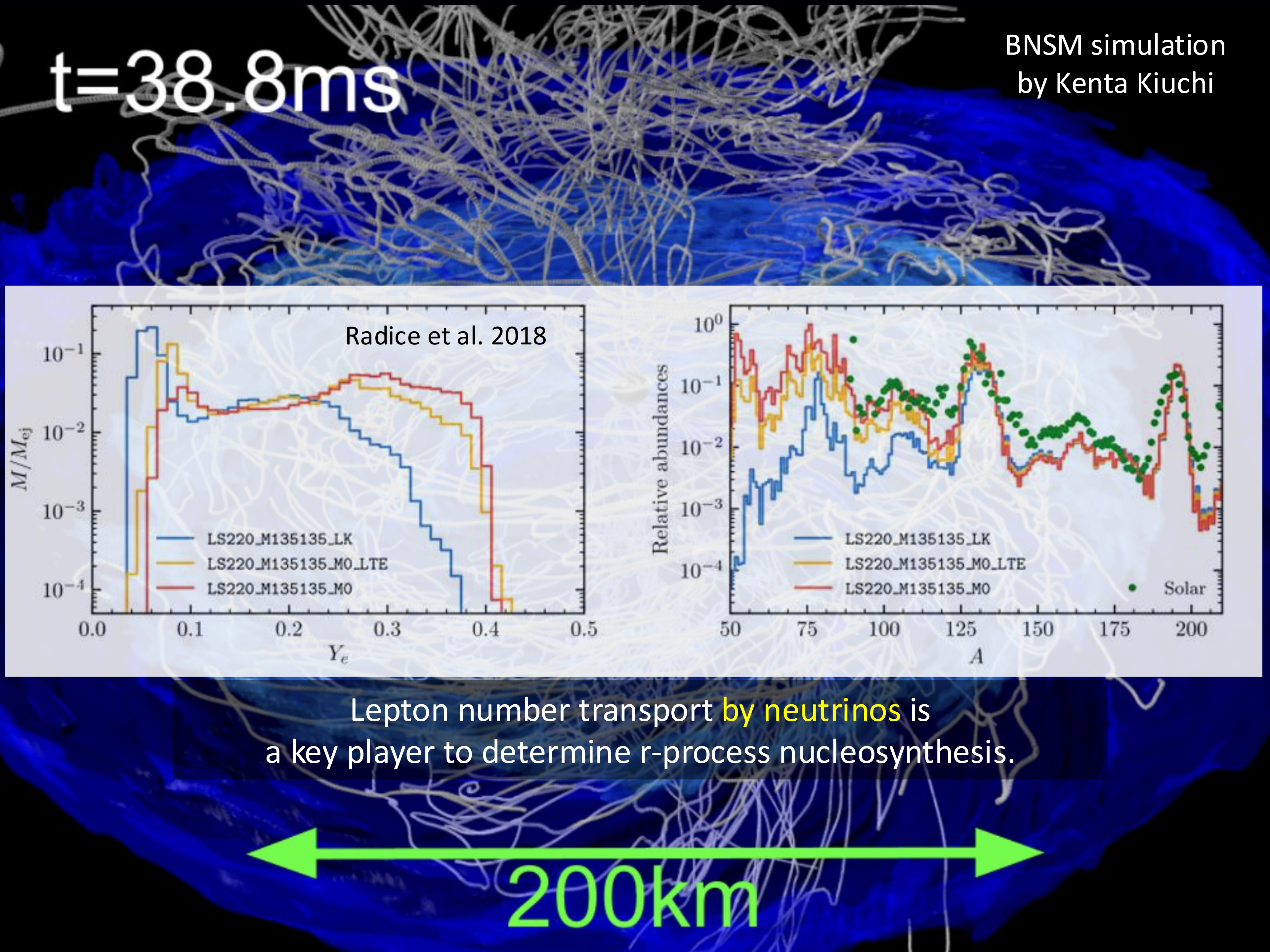
Access to a google document

Discussion session
(Neutrino oscillations)

Moderators: Meng-Ru Wu and Hiroki Nagakura

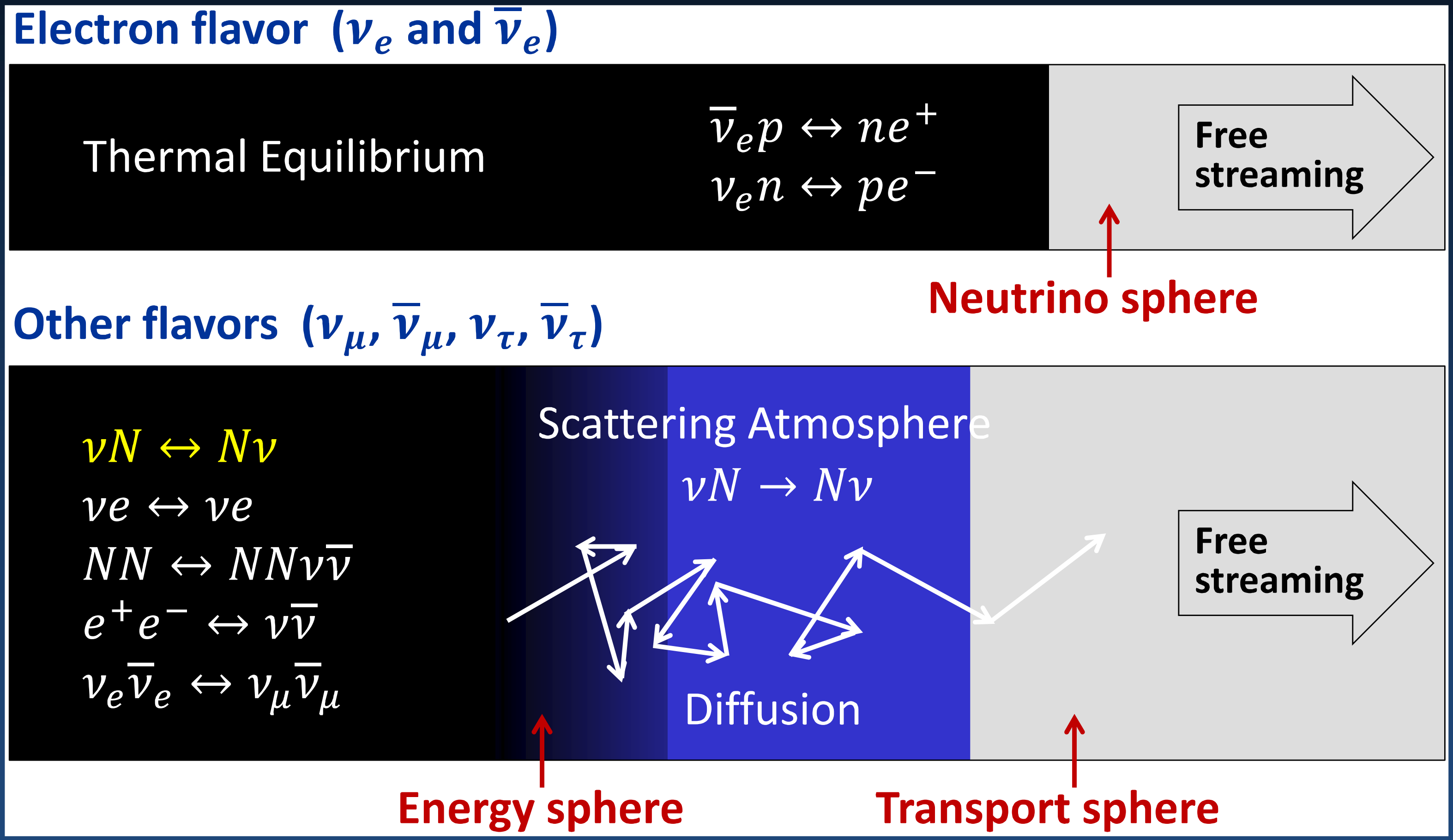
CCSN dynamics hinges on neutrino energy spectrum, angular distributions, and their flavor dependent structures




t=38.8ms
BNSM simulation
by Kenta Kiuchi
Radice et al. 2018
M/M_{ej}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.0
0.1
0.2
0.3
0.4
0.5
Y_e
LS220_M135135_LK
LS220_M135135_M0_LTE
LS220_M135135_M0
Relative abundances
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
50
75
100
125
150
175
200
A
LS220_M135135_LK
LS220_M135135_M0_LTE
LS220_M135135_M0
Solar
Lepton number transport by neutrinos is
a key player to determine r-process nucleosynthesis.
200km

A kinetic framework is essential for modeling of neutrino radiation field

Figure by Thomas Janka 2017



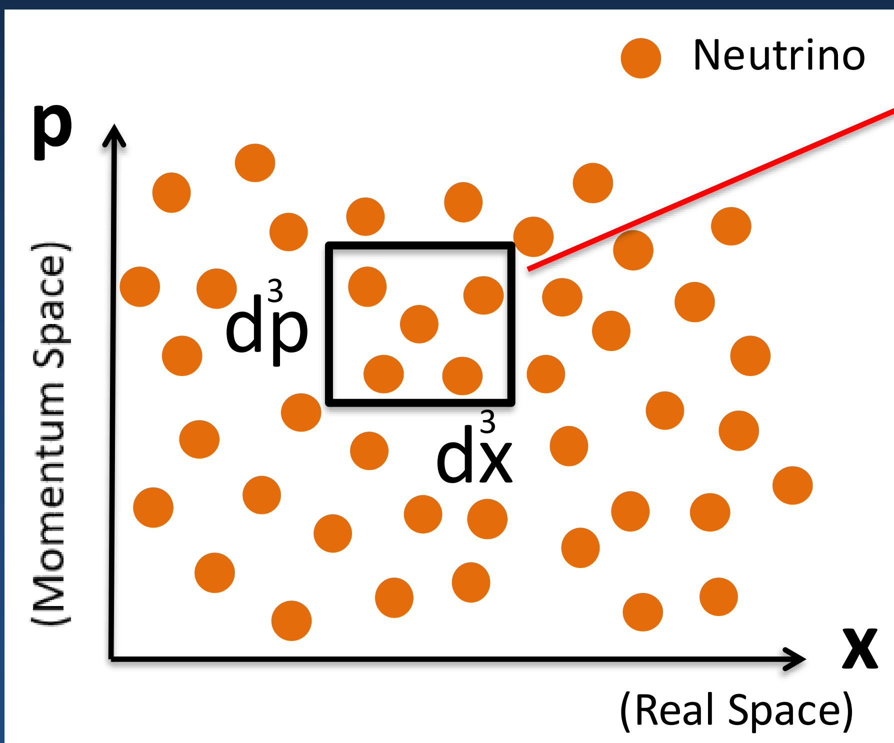
Optically thick ⟷ Optically thin

Boltzmann neutrino transport

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = \left(\frac{\delta f}{\delta \tau}\right)_{\mathrm{col}},$$

(Time evolution + Advection Term) (Collision Term)

6-dimensional phase space



$$dN = f(t, \boldsymbol{p}, \boldsymbol{x})\, d^3p\, d^3x$$

Conservative form of GR Boltzmann eq.

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\,\nu^{-1}p^\alpha f)}{\partial x^\alpha}\bigg|_{q_{(i)}} + \frac{1}{\nu^2}\frac{\partial}{\partial \nu}\left(-\nu f p^\alpha p_\beta \nabla_\alpha e^\beta_{(0)}\right)$$
$$+ \frac{1}{\sin\bar\theta}\frac{\partial}{\partial\bar\theta}\left(\nu^{-2}\sin\bar\theta f \sum_{j=1}^{3} p^\alpha p_\beta \nabla_\alpha e^\beta_{(j)} \frac{\partial \ell_{(j)}}{\partial \bar\theta}\right)$$
$$+ \frac{1}{\sin^2\bar\theta}\frac{\partial}{\partial\bar\varphi}\left(\nu^{-2} f \sum_{j=2}^{3} p^\alpha p_\beta \nabla_\alpha e^\beta_{(j)} \frac{\partial \ell_{(j)}}{\partial \bar\varphi}\right) = S_{\mathrm{rad}},$$

Shibata, Nagakura, Sekiguchi, and Yamada (2014)

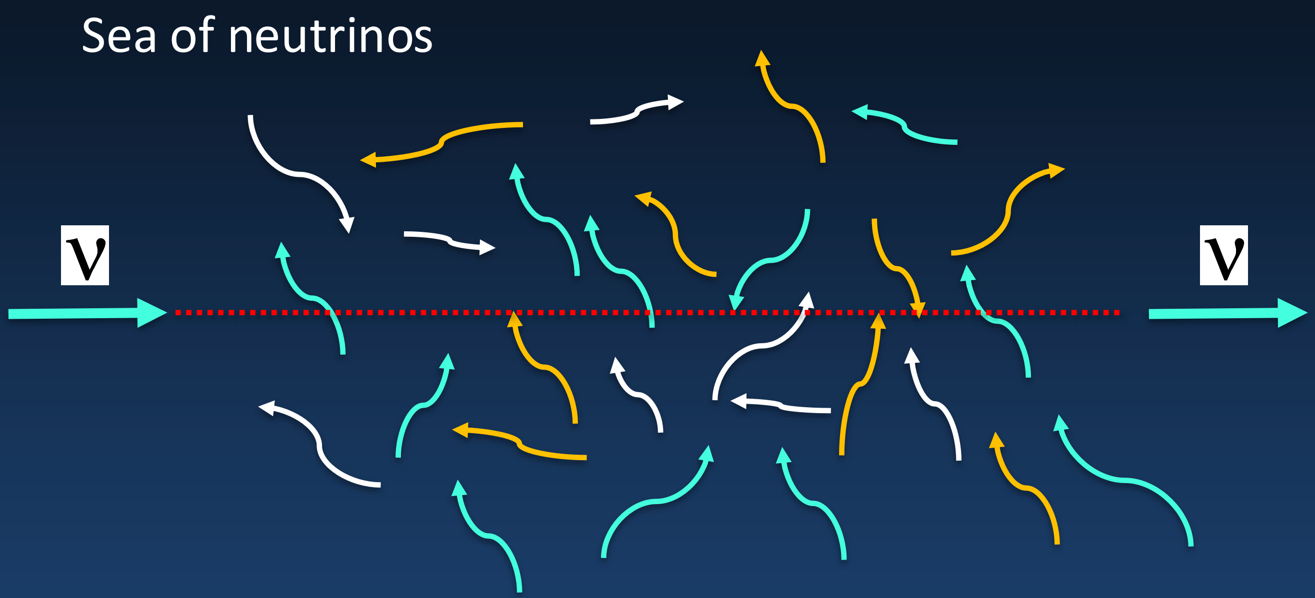
Various Approximate approaches solving neutrino transport

- Gray (energy-integrated) neutrino transport
- Diffusion approximation (e.g., MGFLD)
- Leakage method
- Two-moment method
- Ray-by-Ray approximation
- Isotropic Diffusion Source Approximation (IDSA)

CCSN simulations

Neutrino oscillation induced by self-interactions

Pantalone 1992



- This is a non-equilibrium quantum many-body problem
- Neutrino dispersion relations are modified by refractive effects, analogous to the MSW effect in matter
(Refractive effect is essentially analogous to those for photons in a medium as well)
- Because neutrino flavor (or mass) eigenstates span multiple states (at least three), neutrino self-interactions can enhance flavor coherence

Rich flavor-conversion phenomena driven by neutrino self-interactions

Vacuum: $\omega = \frac{\Delta m^2}{2E_\nu}$,
Matter: $\lambda = \sqrt{2} G_F n_e$,
Self-int: $\mu = \sqrt{2} G_F n_\nu$,

- Slow-mode (Duan et al. 2010)

- Energy-dependent flavor conversion occurs.
- The frequency of the flavor conversion is proportional to $\sqrt{\omega\mu}$

- Fast-mode (FFC) (Sawyer 2005)

- Collective neutrino oscillation in the limit of ω → 0.
- The frequency of the flavor conversion is proportional to μ
- Anisotropy of neutrino angular distributions drives the fast flavor-conversion.

- Collisional flavor instability (CFI) (Johns 2021)

- Asymmetries of matter interactions between neutrinos and anti-neutrinos drive flavor conversion.

$$\mathrm{Im}\,\boxtimes = \pm \frac{\Gamma - \bar{\Gamma}}{2}\, \mathrm{p}\, \frac{\mu S}{(\mu D)^2 + 4!\, \mu S} - \frac{\Gamma + \bar{\Gamma}}{2}$$

Γ: Matter-interaction rate

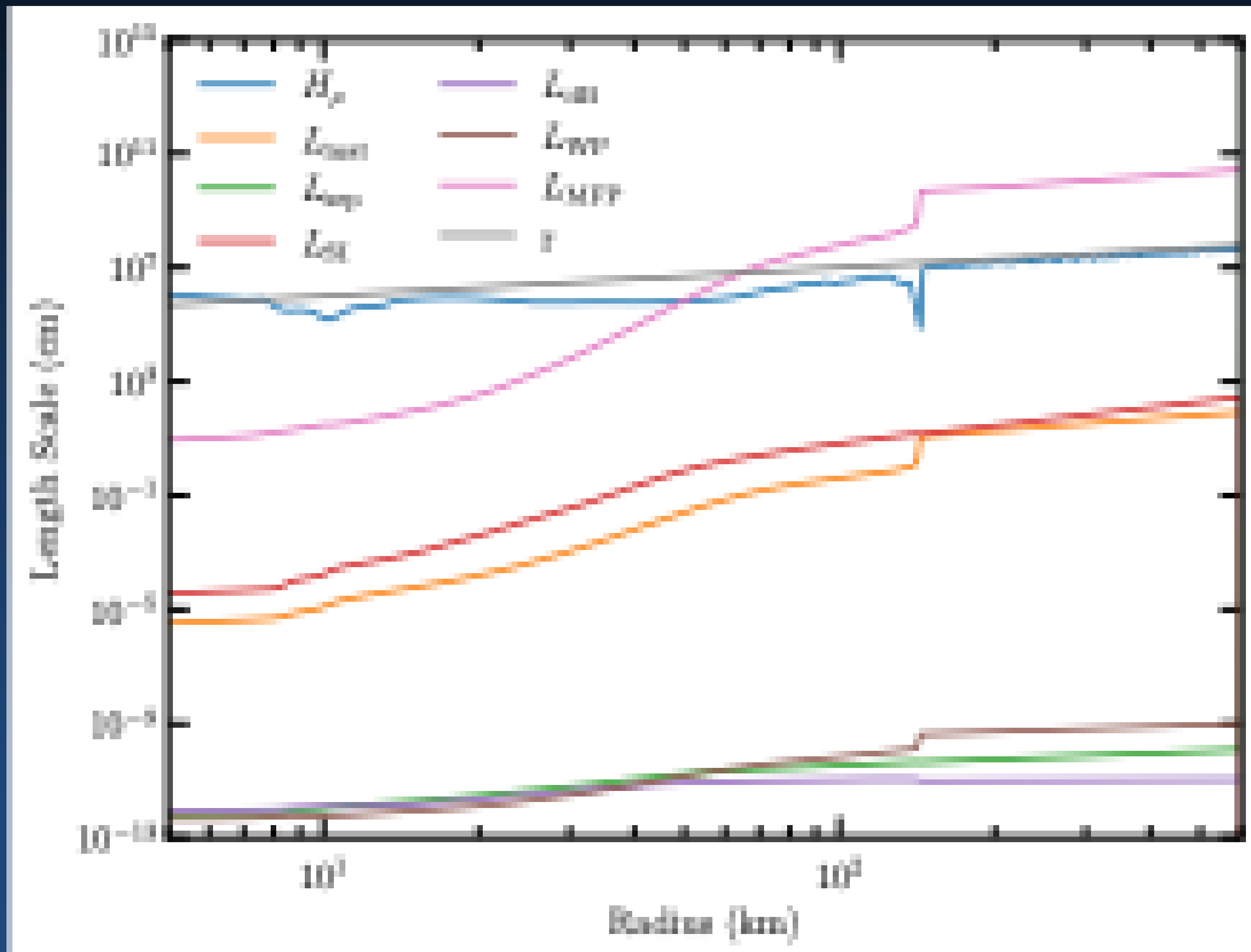
- Matter-neutrino resonance (Malkus et al. 2012)

- The resonance potentially occur in BNSM/Collapsar environment (but not in CCSN).
- Essentially the same mechanism as MSW resonance.

$$|\lambda + \mu| \sim |\omega|$$

Huge scale discrepancy between astrophysical phenomena and neutrino flavor conversions

Johns et al. 2025



Towards self-consistent CCSN and BNSM simulations with neutrino flavor conversions

Phenomenological approach:

Li and Siegel 2021, Just et al. 2022, Fernandez et al. 2022, Ehring et al. 2023

Radiation-hydrodynamic simulations with classical neutrino transport

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = \left(\frac{\delta f}{\delta \tau}\right)_{col},$$

Full Boltzmann transport

or

$$\partial_t(\sqrt{\gamma}E) + \partial_j[\sqrt{\gamma}(\alpha F^j - \beta^j E)] = \alpha\sqrt{\gamma}[P^{ij}K_{ij} - F^j\partial_j \ln\alpha - S^\alpha n_\alpha],$$

$$\partial_t(\sqrt{\gamma}F_i) + \partial_j[\sqrt{\gamma}(\alpha P_i{}^j - \beta^j F_i)] = \sqrt{\gamma}\left[-E\partial_i\alpha + F_k\partial_i\beta^k + \frac{\alpha}{2}P^{jk}\partial_i\gamma_{jk} + \alpha S^\alpha\gamma_{i\alpha}\right],$$

Approximate method (e.g., two-moment method)

+

Flavor instability criterion

Approximate stability analysis

or

adhock criterion

Flavor mixing is incorporated through parameterized prescriptions

$$L_{\nu_e}^{\rm osc} = (1 - a_{\rm osc})L_{\nu_e}^* + a_{\rm osc}L_{\nu_x}$$

$$L_{\bar\nu_e}^{\rm osc} = (1 - b_{\rm osc})L_{\bar\nu_e}^* + b_{\rm osc}L_{\bar\nu_x}.$$

Fernandez et al. 2022

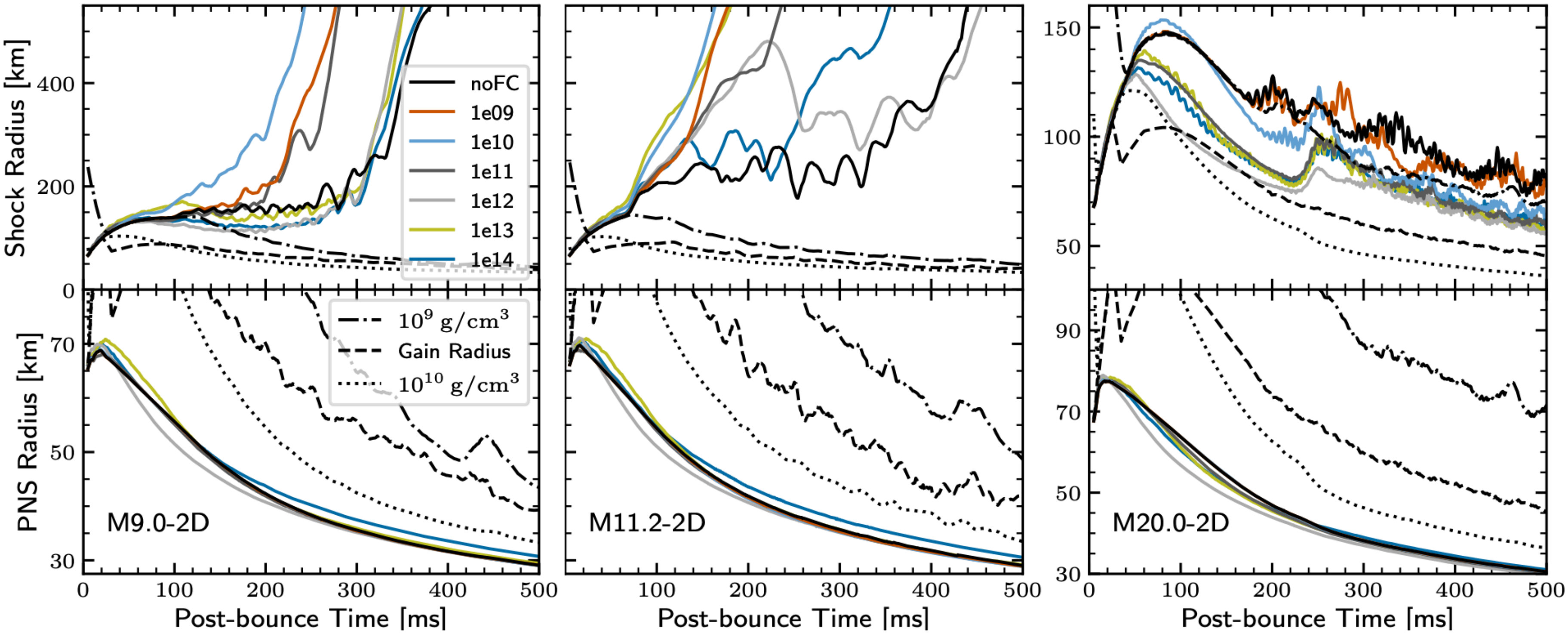
Providing **qualitative insights** into how flavor conversions influence CCSNe/BNSMs.

See also talks by Jakob Ehring and Oliver Just

CCSN simulation in Ehring et al. PRL 2023

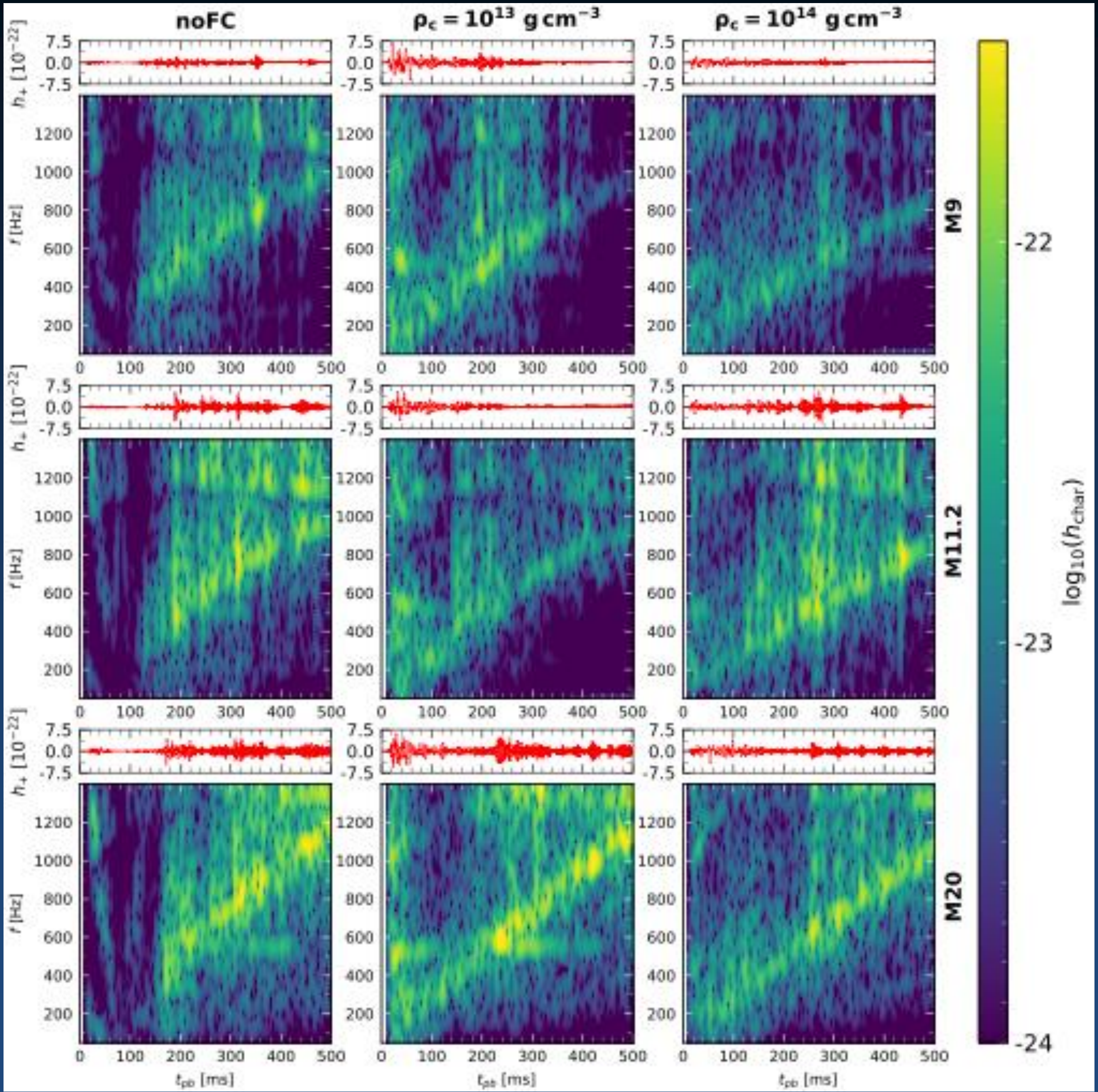
Slide 1 by Jakob Ehring

Shock radius

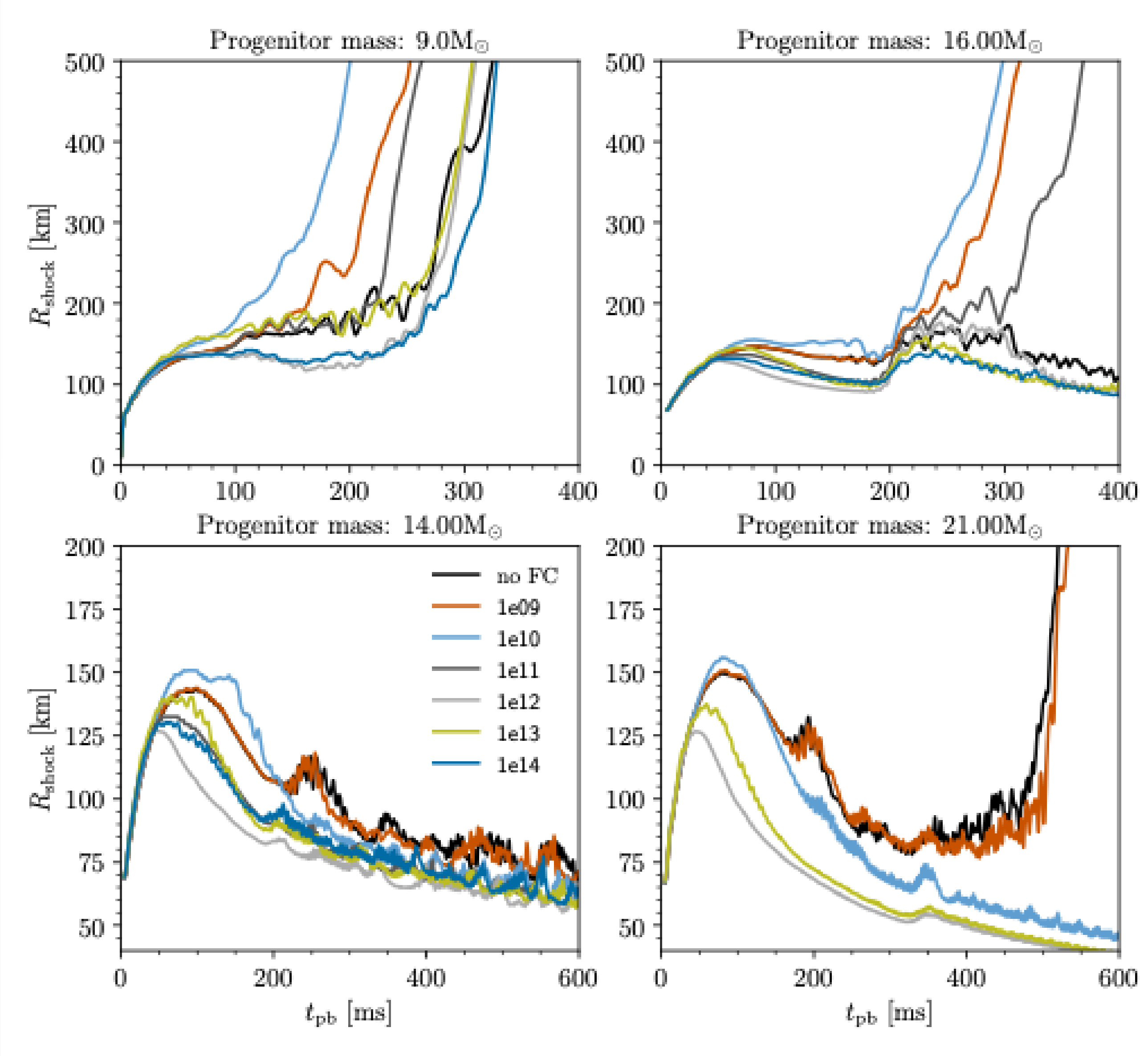


CCSN simulation in Ehring et al. PRL 2026

Slide 2 by Jakob Ehring

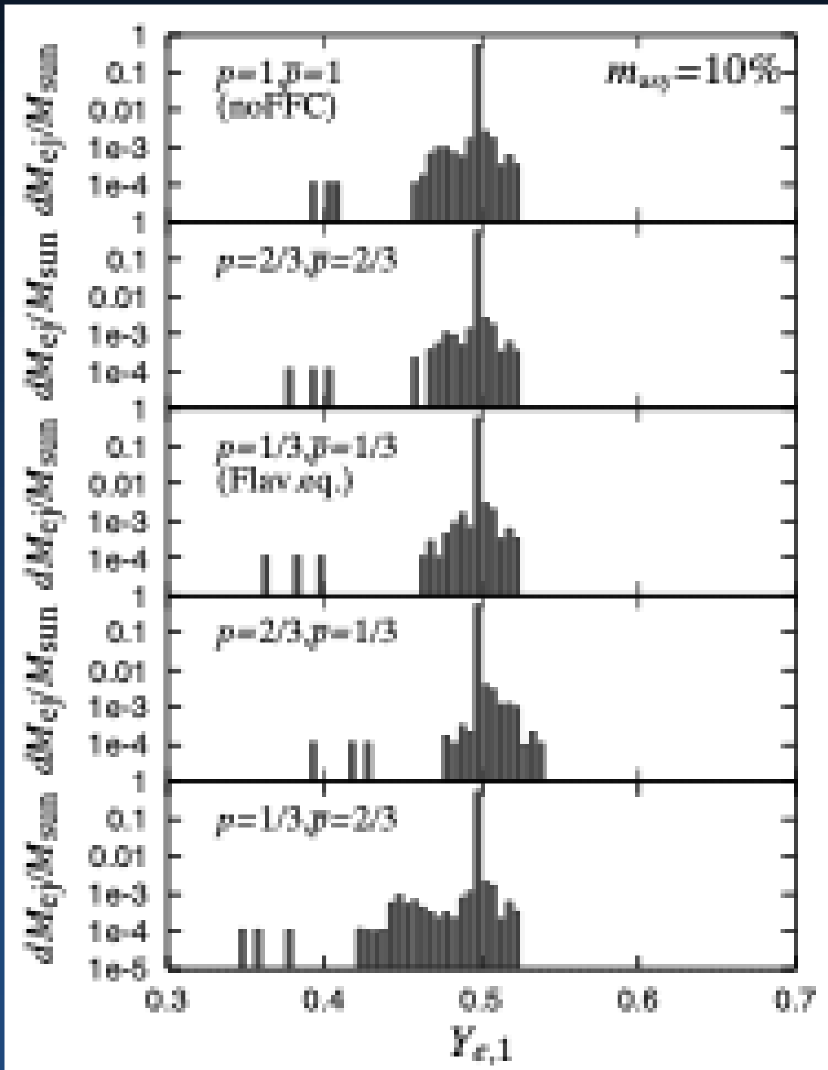


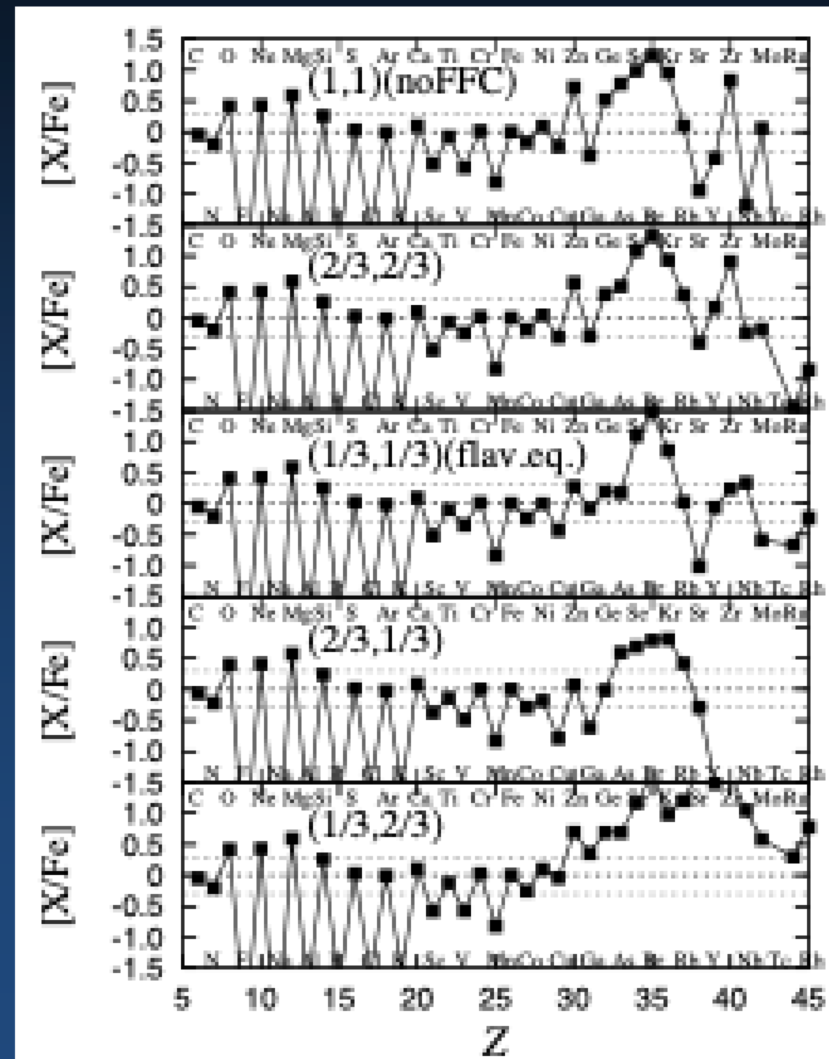
Slide 3 by Jakob Ehring



Fast flavor conversions can change ejecta composition (CCSN)

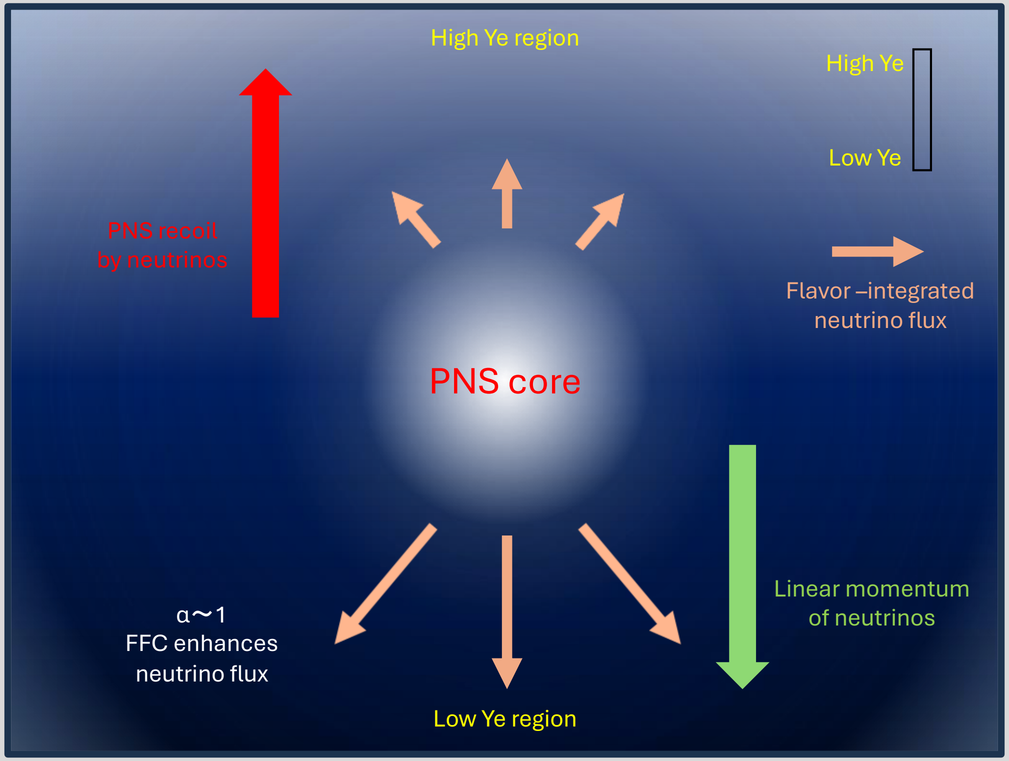
Fujimoto and Nagakura 2023

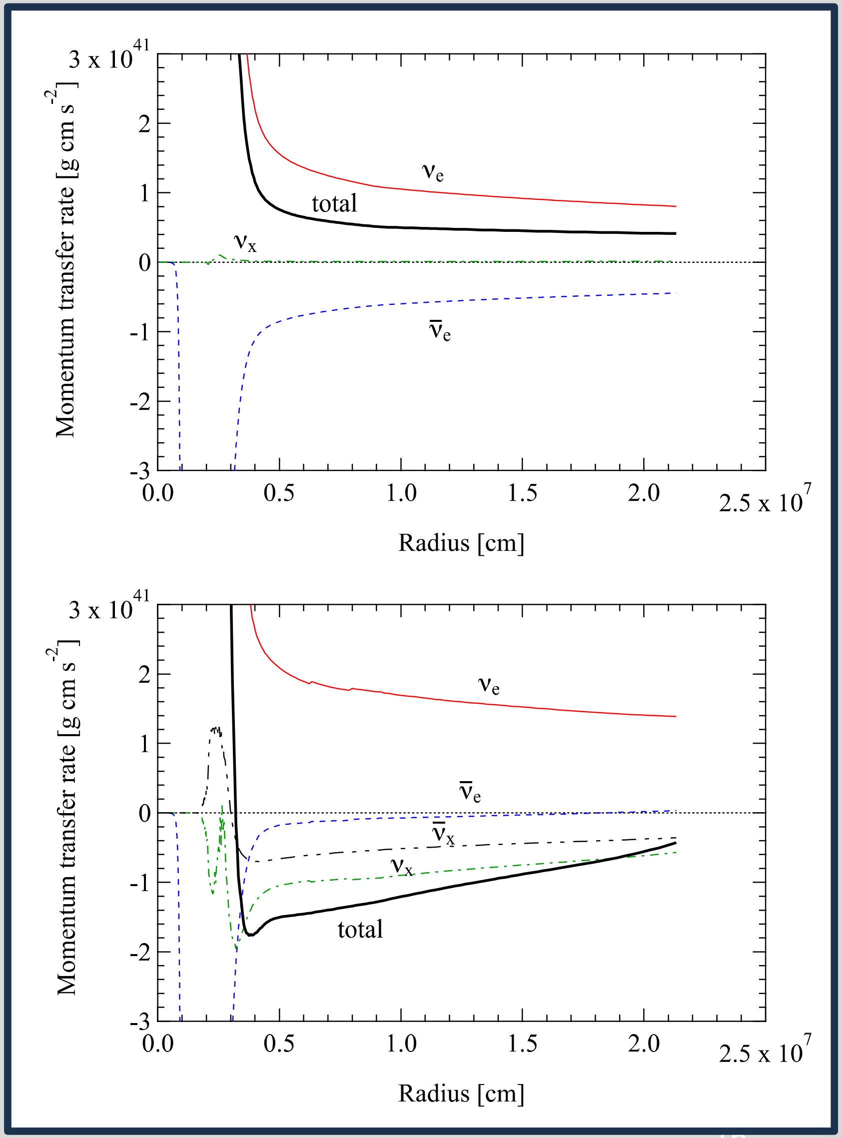




- Neutron star kick powered by neutrino flavor conversions

Nagakura and Sumiyoshi 2024





A representative example of BNSM simulation with FFC

Slide by Oliver

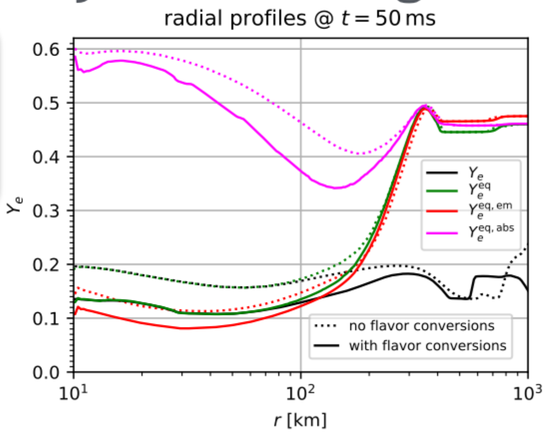
Just et al. 2022

Impact of (effective) flavor mixing in neutrino-cooled disks

Just, Abbar, Wu, Tamborra, Janka, Capozzi PRD 105 (2022)

effective flavor equipartition e.g. like:

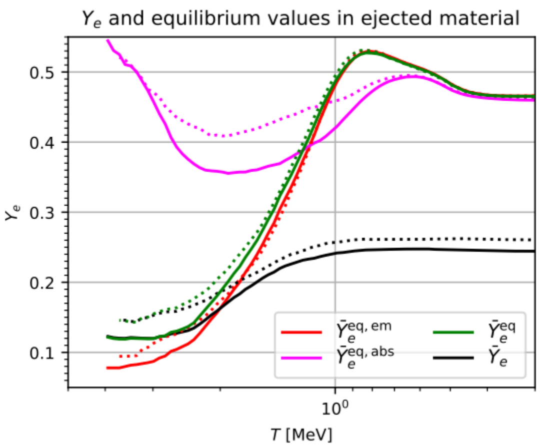
$$n_\nu = \frac{1}{6}\left(n^0_{\nu_e,q} + n^0_{\bar{\nu}_e,q} + 2n^0_{\nu_x,q} + 2n^0_{\bar{\nu}_x,q}\right)$$



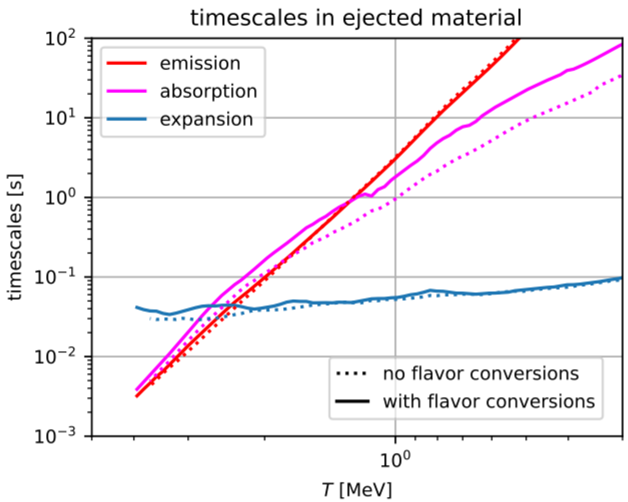
Two main effects:

1) enhanced neutrino cooling rates lead to higher electron degeneracy —> lower value of emission-equilibrium electron fraction $Y_e^{\mathrm{eq,em}}$

$$(\lambda_{e^+} + \lambda_{\nu_e})Y_n - (\lambda_{e^-} + \lambda_{\bar{\nu}_e})Y_p\Big|_{\rho,T,Y_e^{eq}} = 0,$$



2) smaller abundance of electron-type neutrinos —> reduced absorption rates

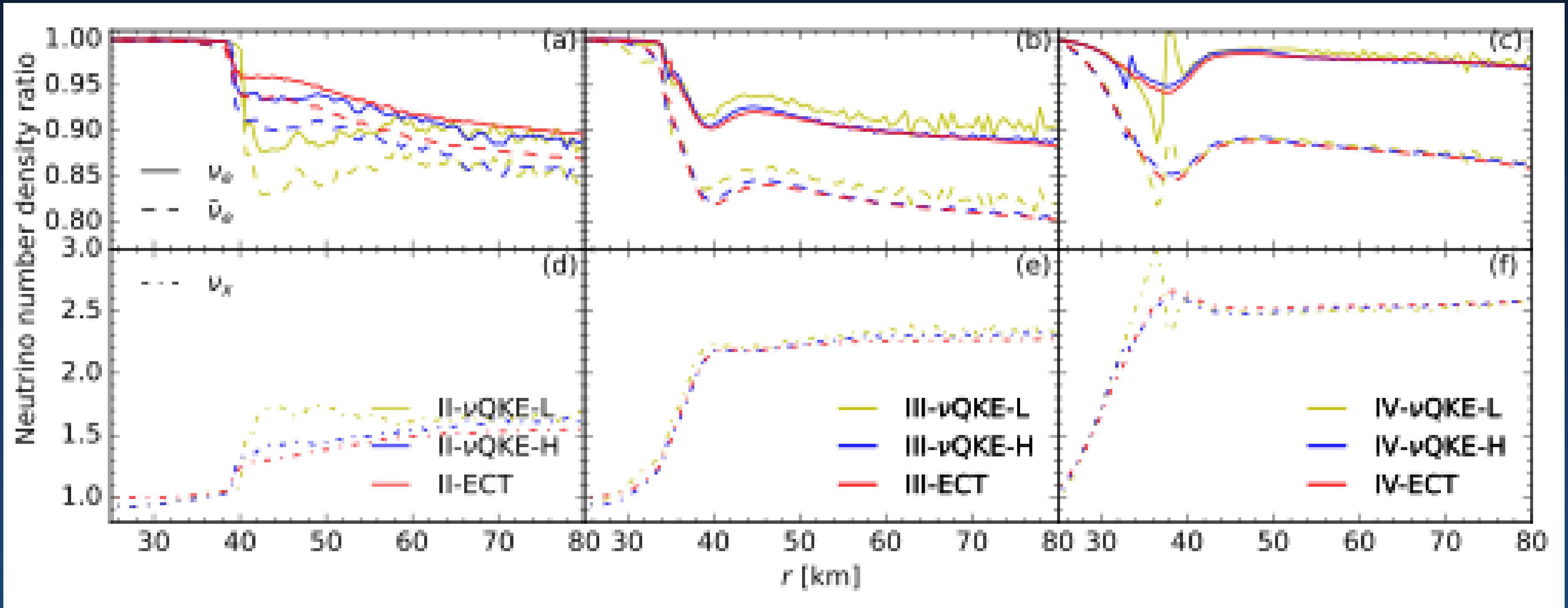


$$\tau_\beta = \frac{1}{Y_p(\lambda_{e^-} + \lambda_{\bar{\nu}_e}) + Y_n(\lambda_{e^+} + \lambda_{\nu_e})}$$

QKE-motivated subgrid model

Effective classical transport

Xiong et al. 2025



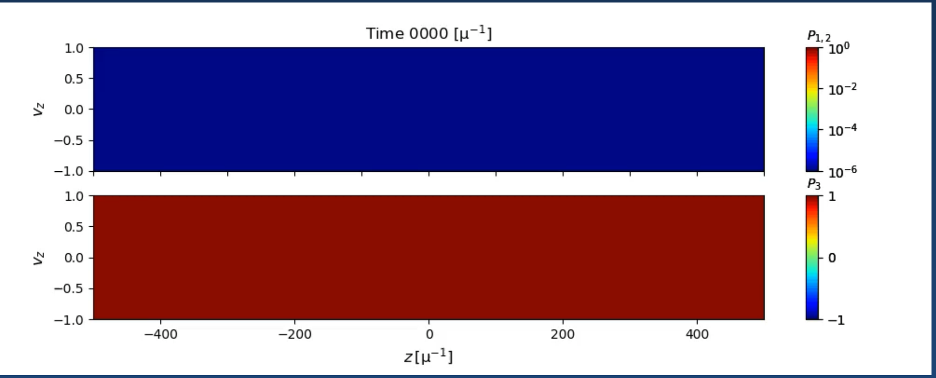
BGK subgrid model

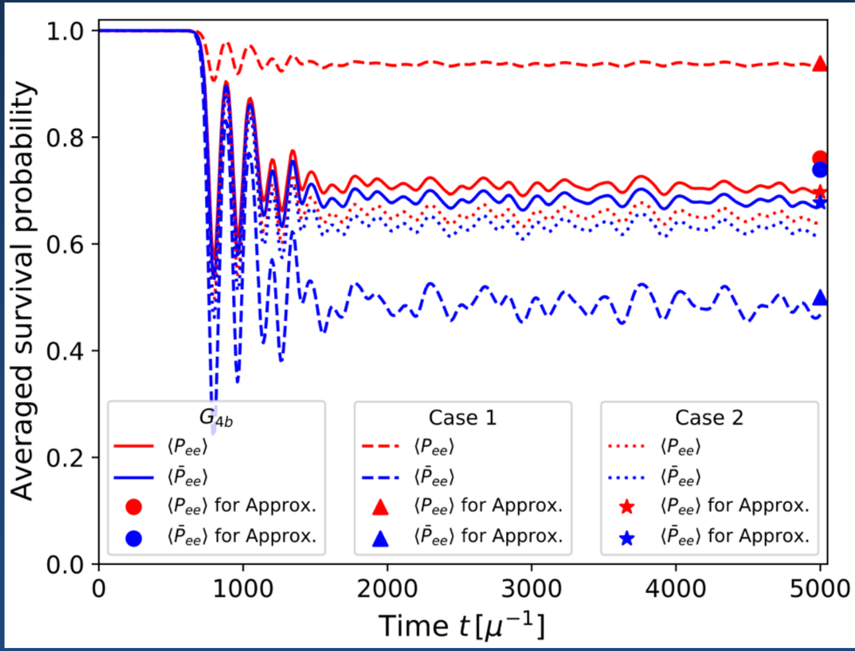
Nagakura et al. 2024

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + i p^\mu n_\mu [H, f]$$: Full QKE

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + p^\mu n_\mu \frac{1}{\tau_a}(f - f^a)$$: Relaxation-time approximation

Analytic approach based on QKE local simulations (FFC)



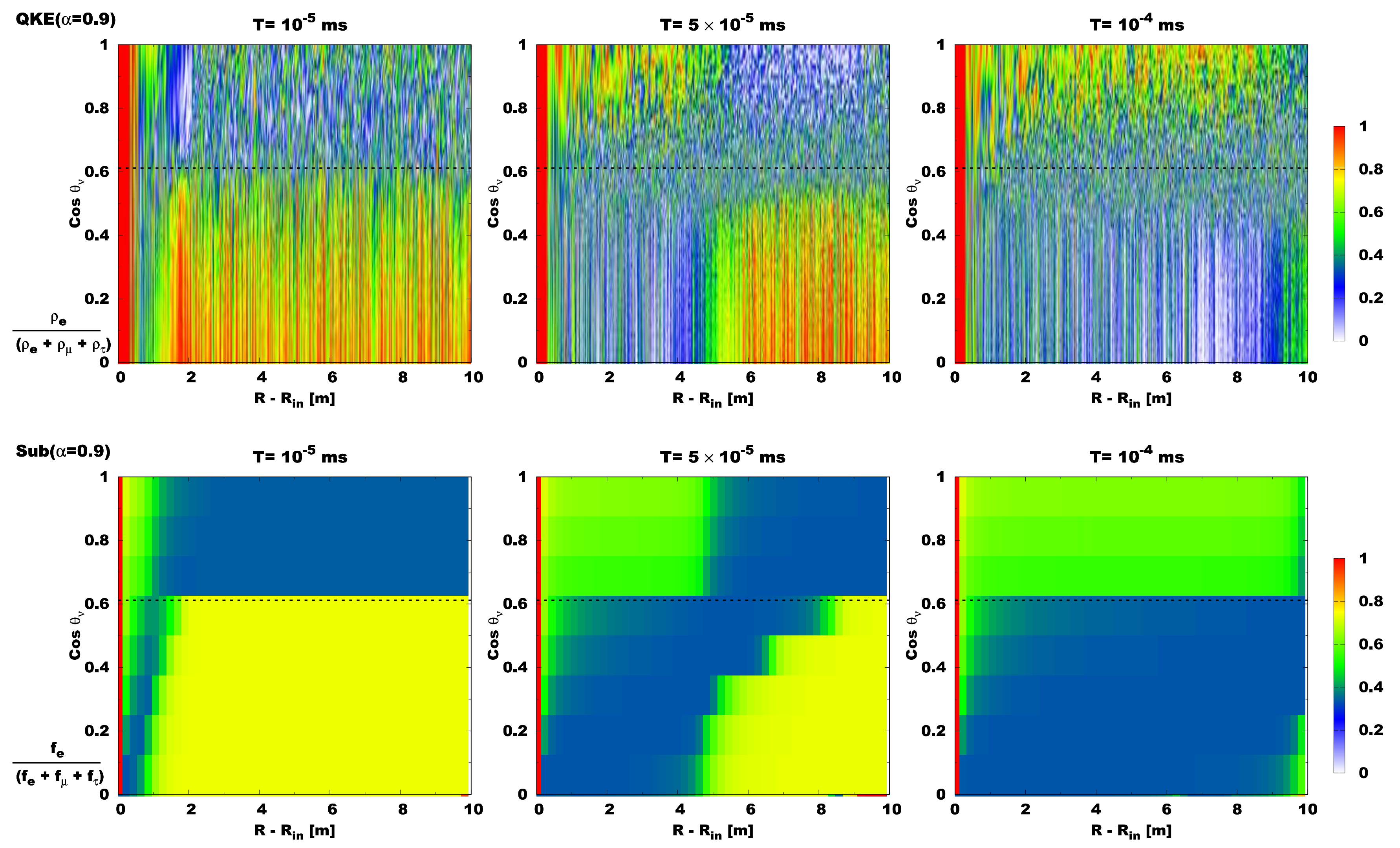


Asymptotic state of FFC can be estimated "analytically" by conservation laws and stability condition

Zaizen and Nagakura 2022

QKE-informed effective approach: BGK subgrid model

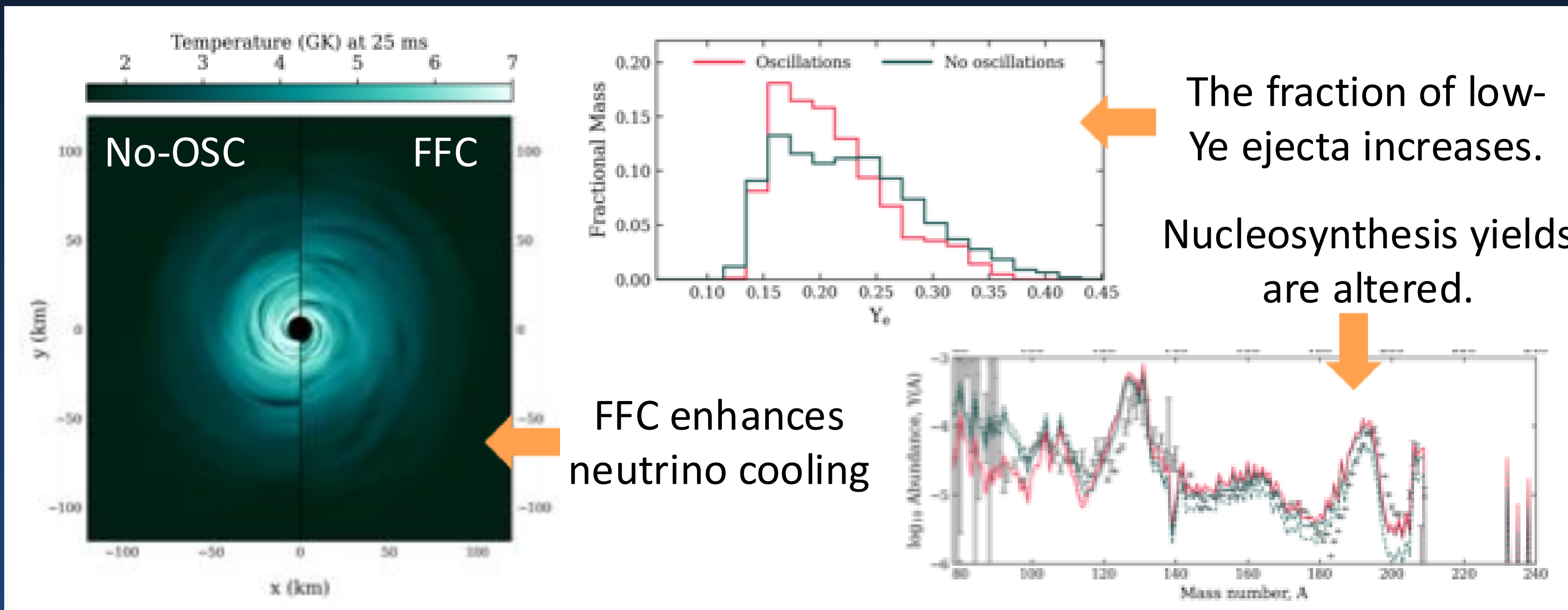
Nagakura et al. 2024



CCSN and BNSM simulations incorporating neutrino flavor conversion based on BGK subgrid model

√ BNSM

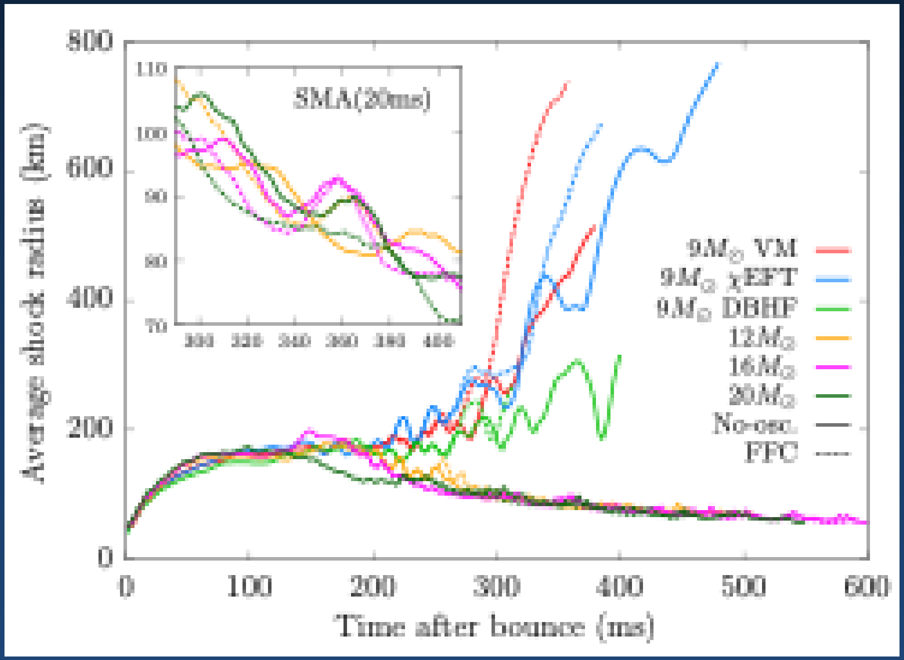
Lund et al. 2025

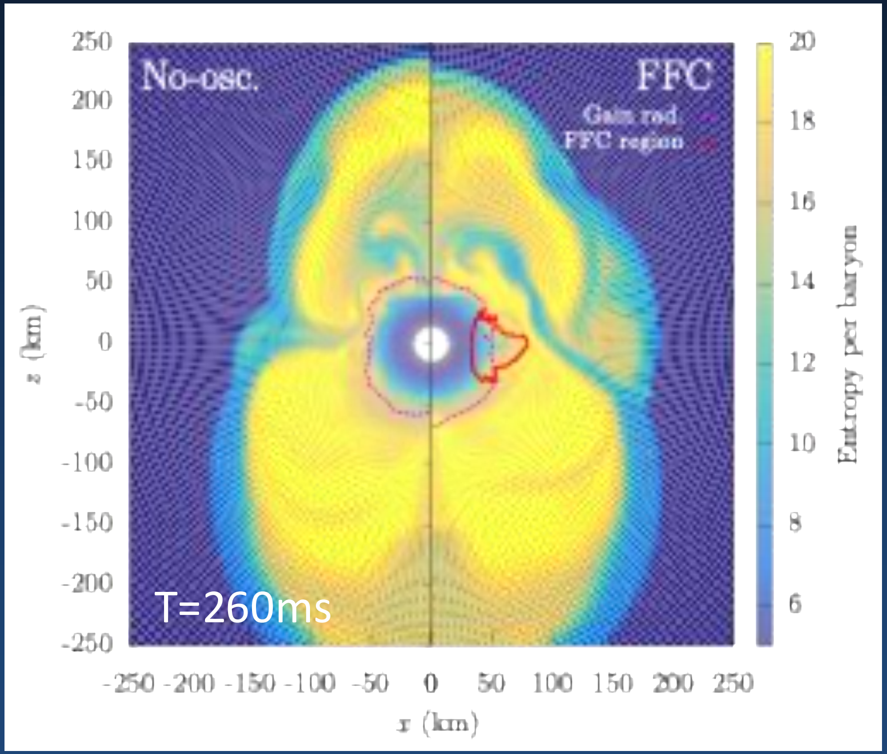


See also Kyohei Kawaguchi's talk and Yi Qiu's talk

CCSN General Relativistic multi-dimensional radiation-hydro simulations with multi-angle neutrino transport with BGK-FFC

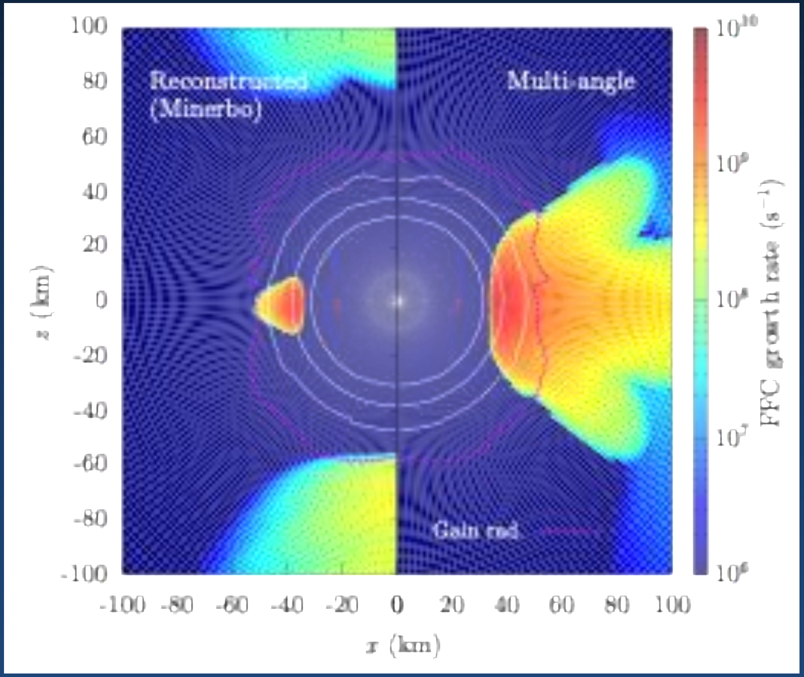
Akaho, Nagakura et al. (arXiv:2601.08269)

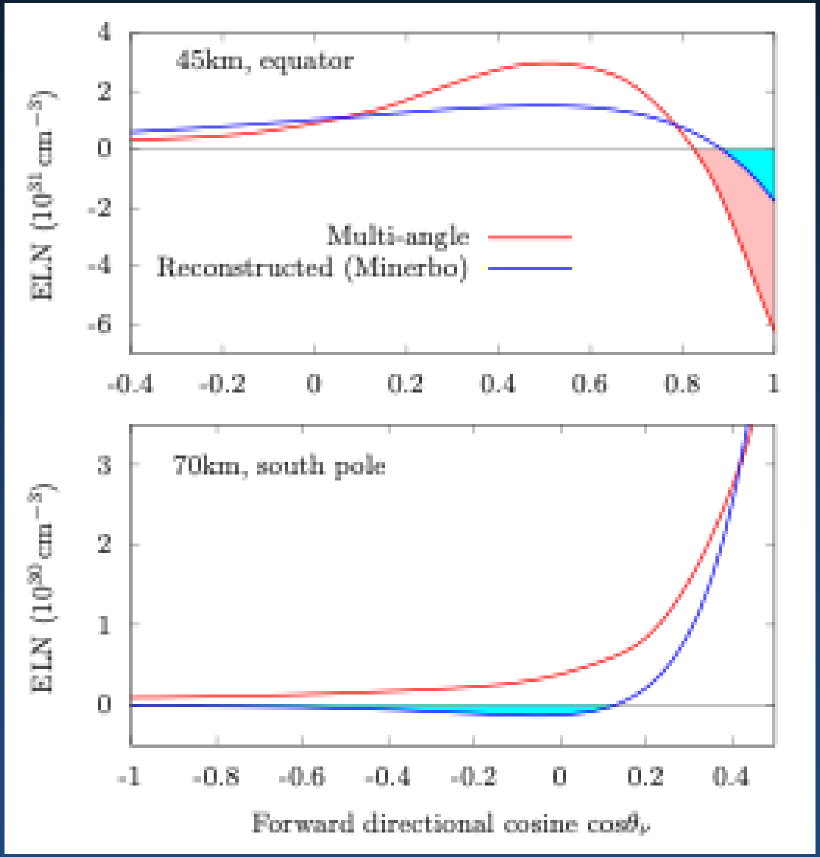




CCSN General Relativistic multi-dimensional radiation-hydro simulations with multi-angle neutrino transport with BGK-FFC

Akaho, Nagakura et al. (arXiv:2601.08269)





Accurate modeling of FFCs in CCSN simulations requires multi-angle neutrino transport