General relativistic viscous hydrodynamics simulations for BH–torus systems incorporating Monte Carlo-based full Boltzmann neutrino transport

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Based on Kawaguchi, Fujibayashi, Shibata, PRD, (2023, 2025a, 2025b) + in prep.

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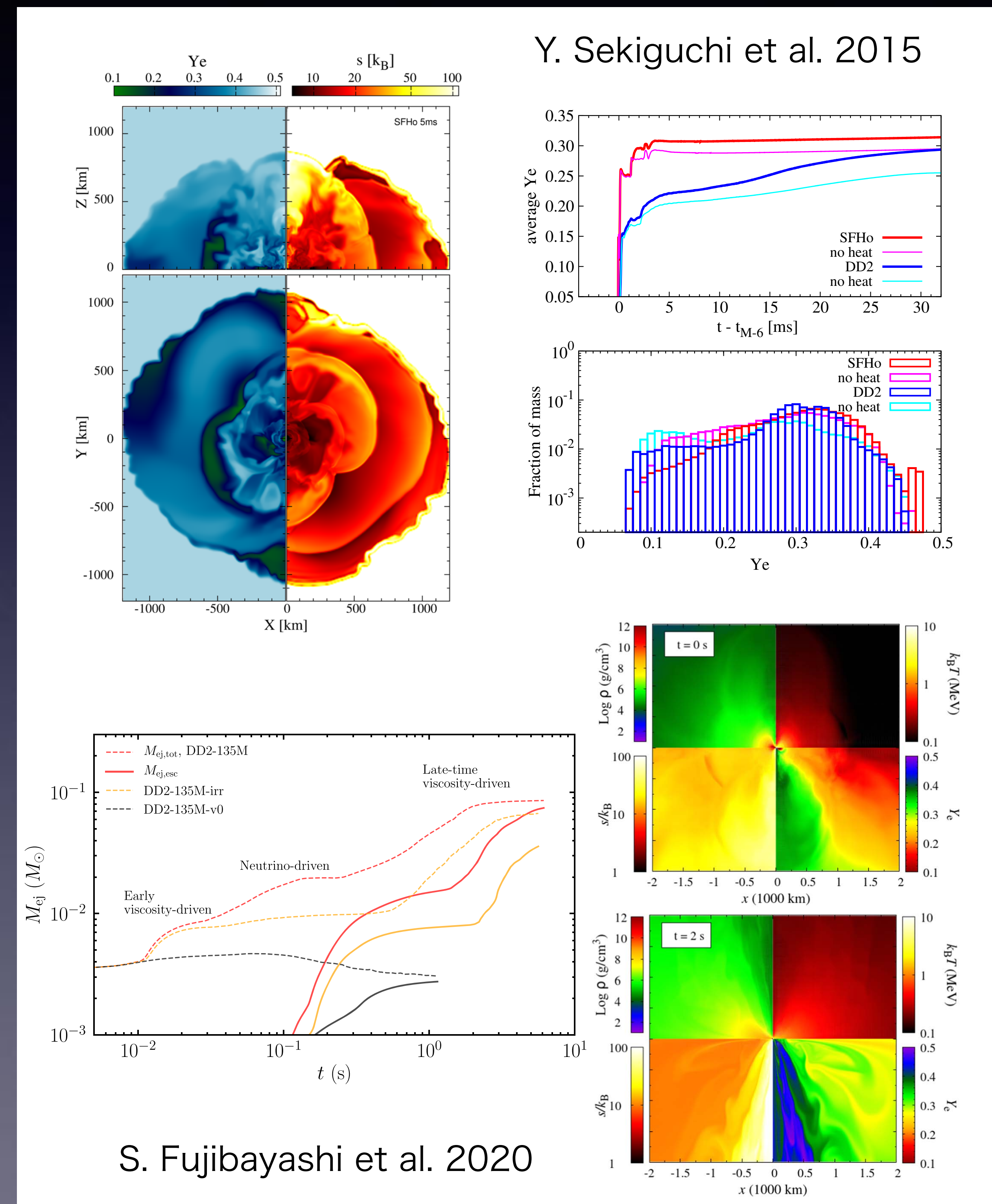
Neutrino-matter interaction

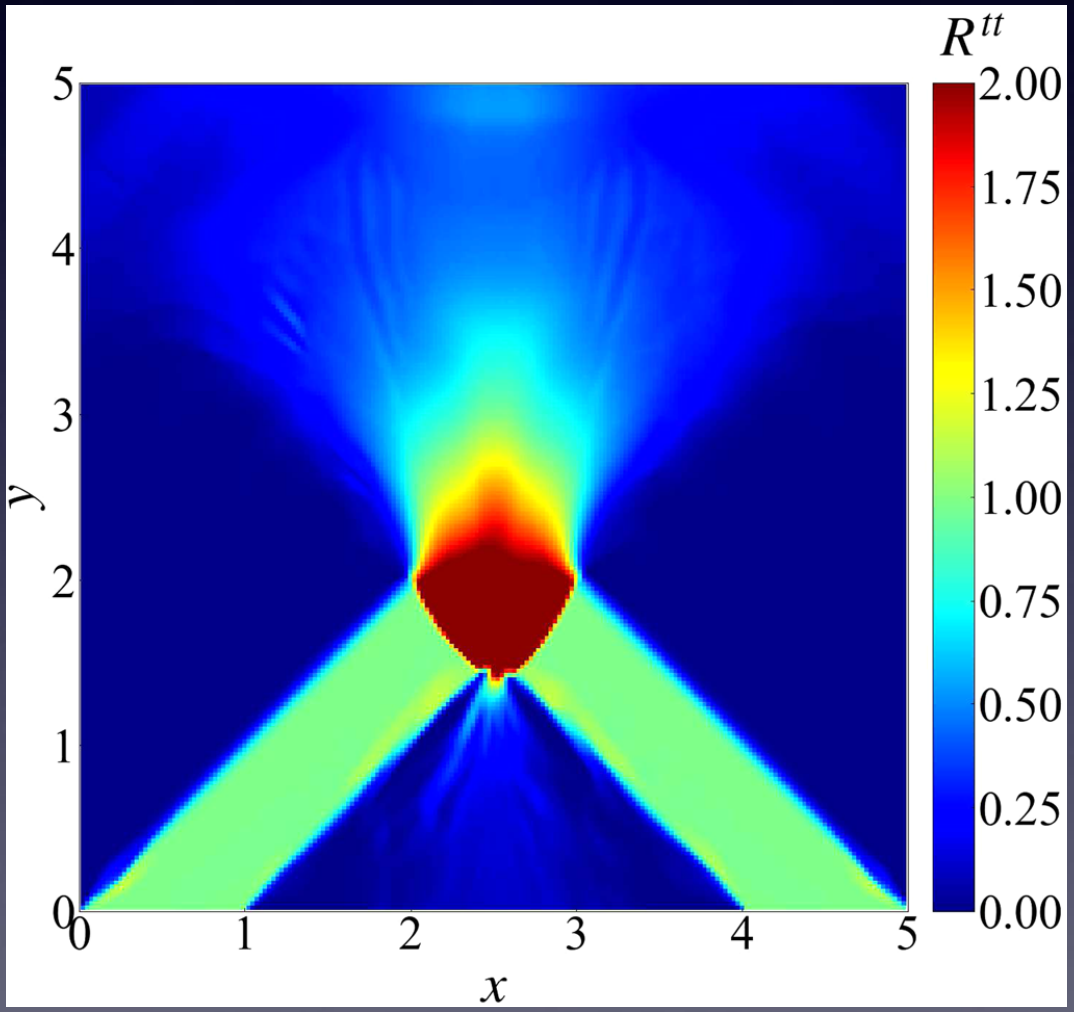
- Neutrino-matter interaction plays an important role in the merger/post-merger phase of a BNS merger:
  - Determines the thermodynamical property of the remnant NS and disk
  - Characterizes the nucleosynthesis in the outflow
  - Possible mechanism for launching a relativistic outflow / jet (pair-annihilation)
- **The moment formalism M1(M0) method** is used for the recent merger simulations to account for neutrino transport

(e.g., K. Thorne 1981, M. Shibata et al. 2011, McKinney et al. 2014, Sadowski et al. 2014, , Y. Sekiguchi et al. 2015, 2016, Takahashi et al. 2016, F. Foucart et al. 2015, Just et al. 2015, D. Radice et al. 2016, Kuroda et al.2016, D. Radice et al. 2022, P. Cheong et al. 2024 , F. Schianchi et al. 2024)

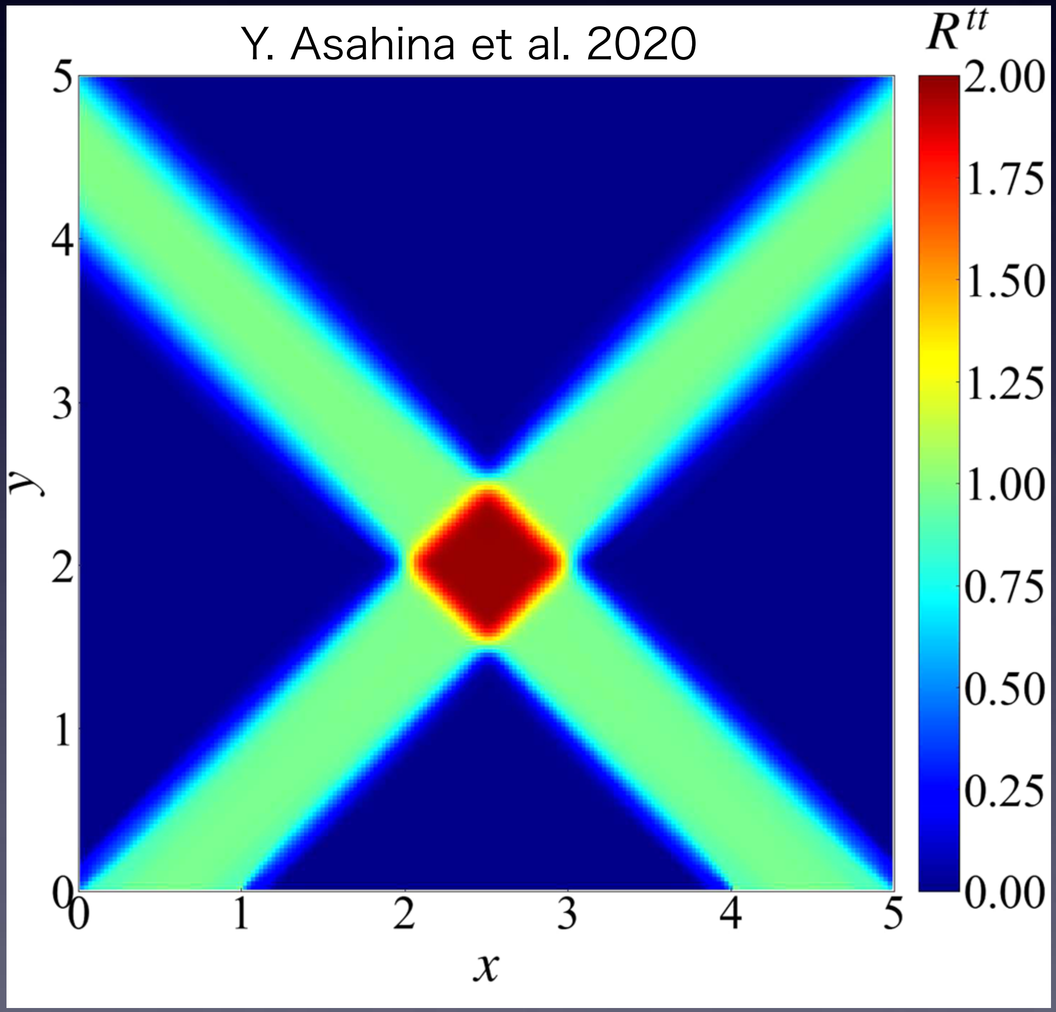


Limitation of M1 method

M1-method



Full Boltzmann (grid-based) method



M1 method is not always guaranteed to provide physically correct results.

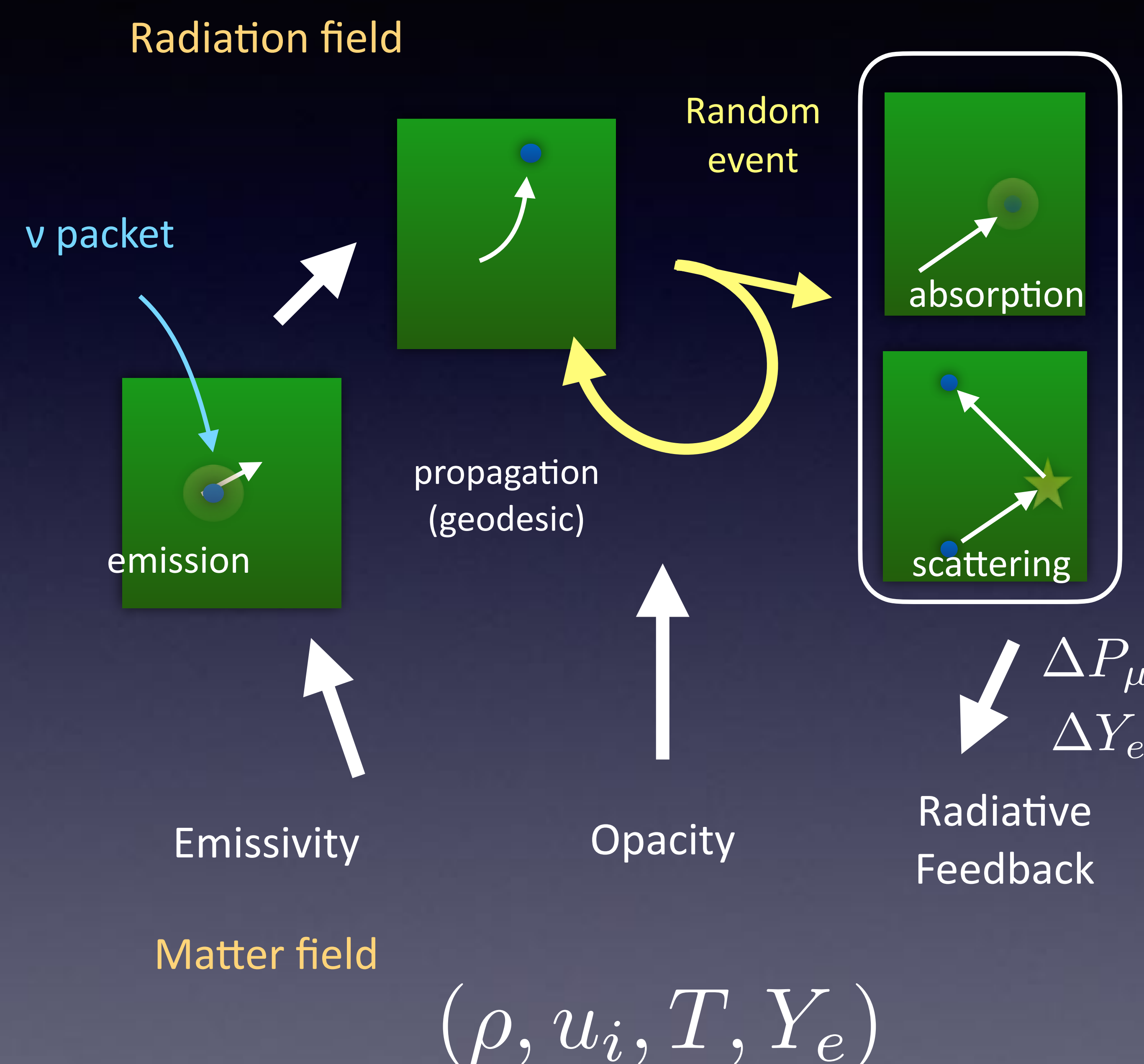
(see, e.g., H. Nagakura et al. 2017 & Y. Asahina et al. 2020 for grid-based full-Boltzmann method in GR)

Monte-Carlo (MC) based Radiation Hydrodynamics

- *Monte-Carlo radiative transfer:*
  numerical technique that statistically simulates the scattering, absorption, and propagation of to model radiative transfer
  (photons: e.g., Lucy 1999, Kasen+ 2006, Sim+ 2007, Jerkstrand+ 2011, Tanaka & Hotokezaka 2013, Kerzendorf+2014, Wollaeger+2014, KK+2018, Bulla 2019)

- Advantage:
  - Straightforward incorporation of complicated *energy* and *angular* dependences
  - Converges to the solution of the Boltzmann eq. in large #MC packet
    (no need of auxiliary closure relation)

- Disadvantage:



We develop a MC based GRνRHD code for **axisymmetric systems**
to study **the long-term (~1s)** post-merger dynamics of neutron star binary mergers

(∝ 1/√N)

Foucart et al. 2017, 2018, 2020, 2022, 2023, 2025, KK+2023,2025ab

Numerical Issues

1. **Stiff source term in energy equation**

$$\Delta t_{\rm dyn} \sim \Delta x/c$$

$$\Delta t_{\rm ems,abs} \ll \Delta t_{\rm dyn}$$

$$\Delta t_{\rm ems,abs} \sim (\kappa\rho c)^{-1}$$

→ Implicit Monte-Carlo Method prescription (Fleck & Cummings 1971)

2. **Stiff source term in terms lepton number equation**
   / unbalance in $\nu_i/\bar{\nu}_i$ number changes during pair process
   ($\dot{n}^{\rm pair}_{\nu_i} = \dot{n}^{\rm pair}_{\bar{\nu}_i}$ is not guaranteed if $\nu_i/\bar{\nu}_i$ are evolved separately)

→ Numerical limiter / correction

3. **Matter-radiation coupling**

Operator splitting for matter/radiation sectors: 1st order time convergence

→ Higher order time integration scheme

Application

Kawaguchi, Fujibayashi, Shibata, PRD, (2025a, 2025b)

Method / Setup / Model

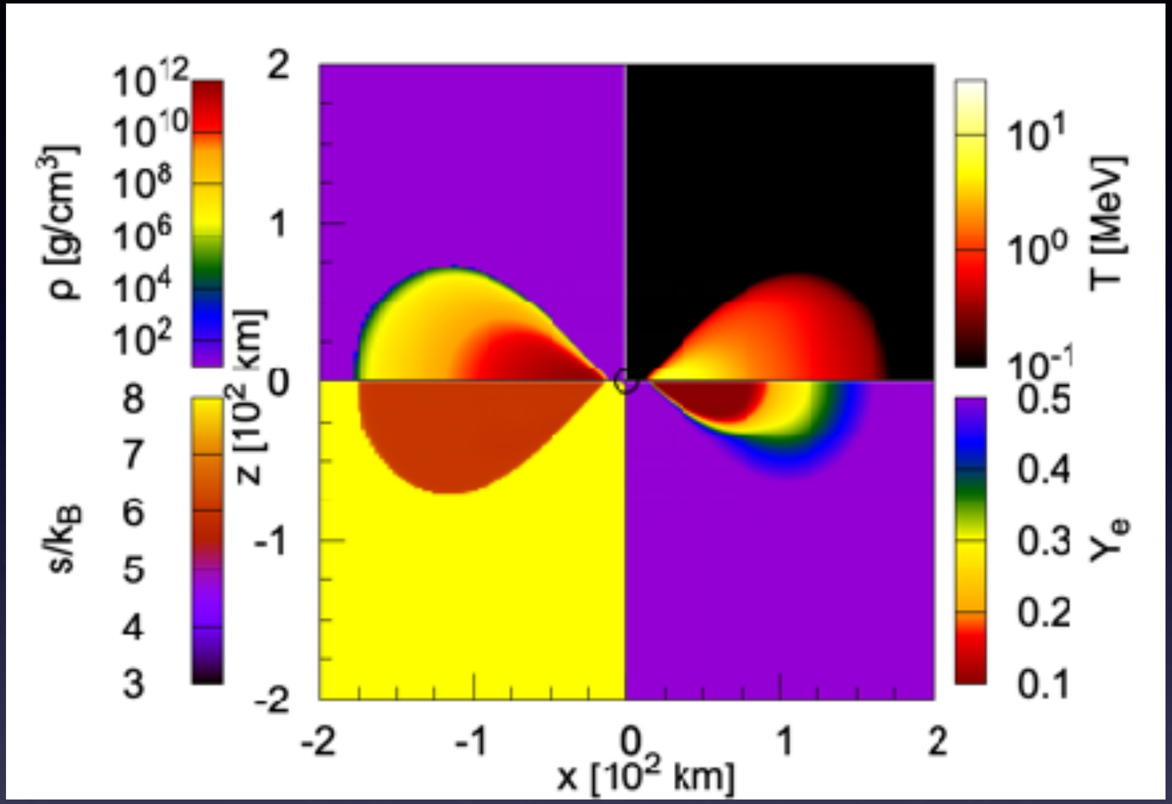
- ***Axisymmetry / Equatorial plane symmetry***
- ***Fixed spacetime*** (Kerr-Schild coordinate)
- Matter field
  GR viscous-hydrodynamics
  (Shibata+ 2017, KK+ 2025a)
- Radiation field $(\nu_e, \bar{\nu}_e, \nu_x)$
  our 2nd order MC scheme
  Multi-species Implicit MC prescription
  \+ numerical limiters for stable / correct evolution
  (KK+ 2023, 2025a)

- Equation of state
  - non-relativistic proton / neutron / α
  - photon
  - relativistic electrons/positrons

- Neutrino interaction (m_e neglected)
  - $\nu_e + n \rightleftharpoons e^- + p$ $\quad \bar{\nu}_e + p \rightleftharpoons e^+ + n$
  - $\nu_i + p \to \nu_i + p$ $\quad \nu_i + n \to \nu_i + n$
  - $\nu_i + \bar{\nu}_i \rightleftharpoons e^- + e^+$

w/ full distribution function information

Initial profile for a BH-tours system (fiducial model)



Initial profiles: quasi-equilibrium configurations

Model parameters:

$M_{\rm BH}\,[M_\odot] = \mathbf{3},\ 6$ *__Bold__: fiducial setup

$\chi_{\rm BH} = 0,\ \mathbf{0.8},\ 0.95$

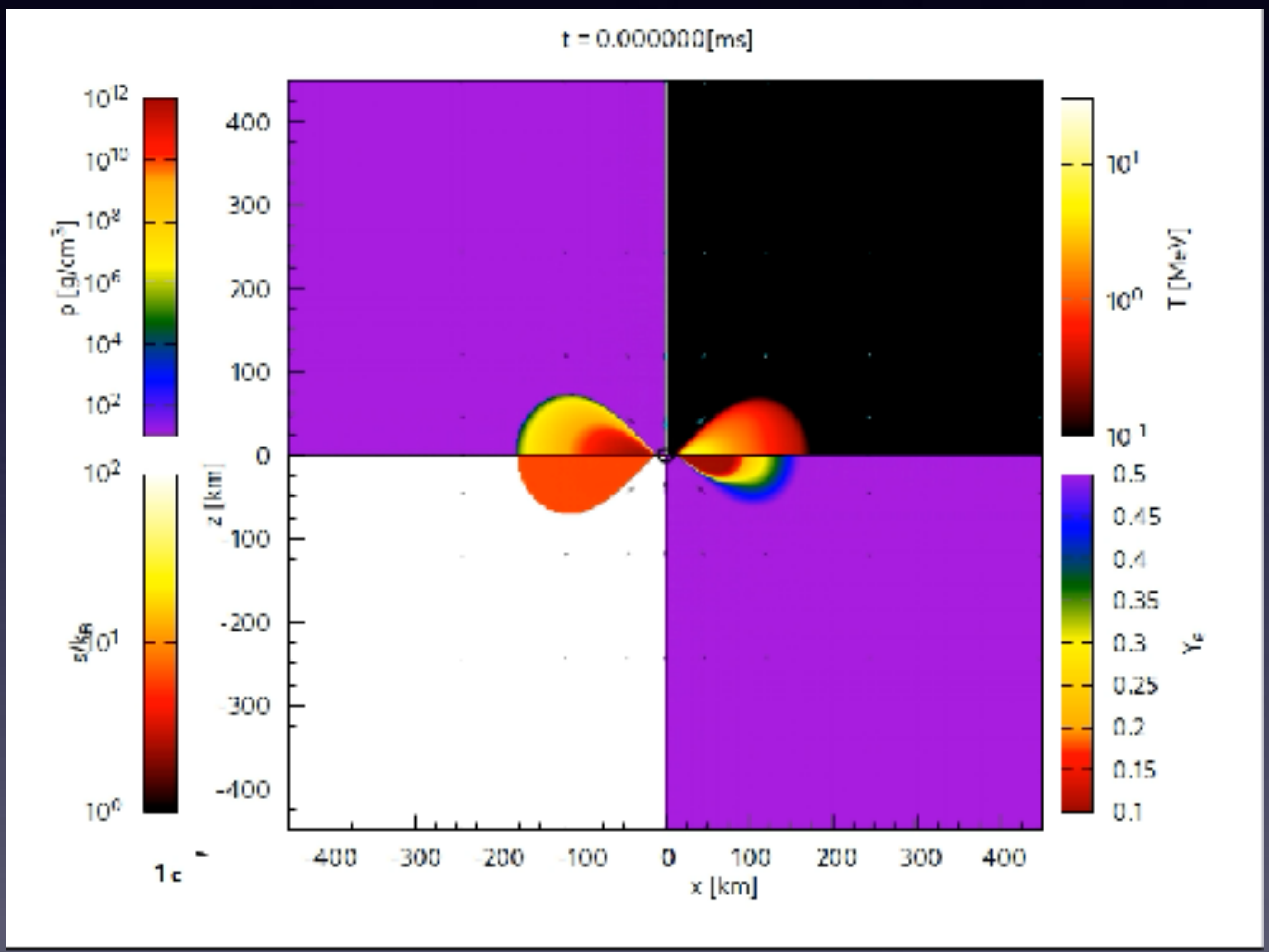
$M_{\rm torus}\,[M_\odot] = \mathbf{0.1},\ 0.3$

$n\ (l \propto \Omega^{-n}) = 1/10,\ \mathbf{1/7},\ 1/5$

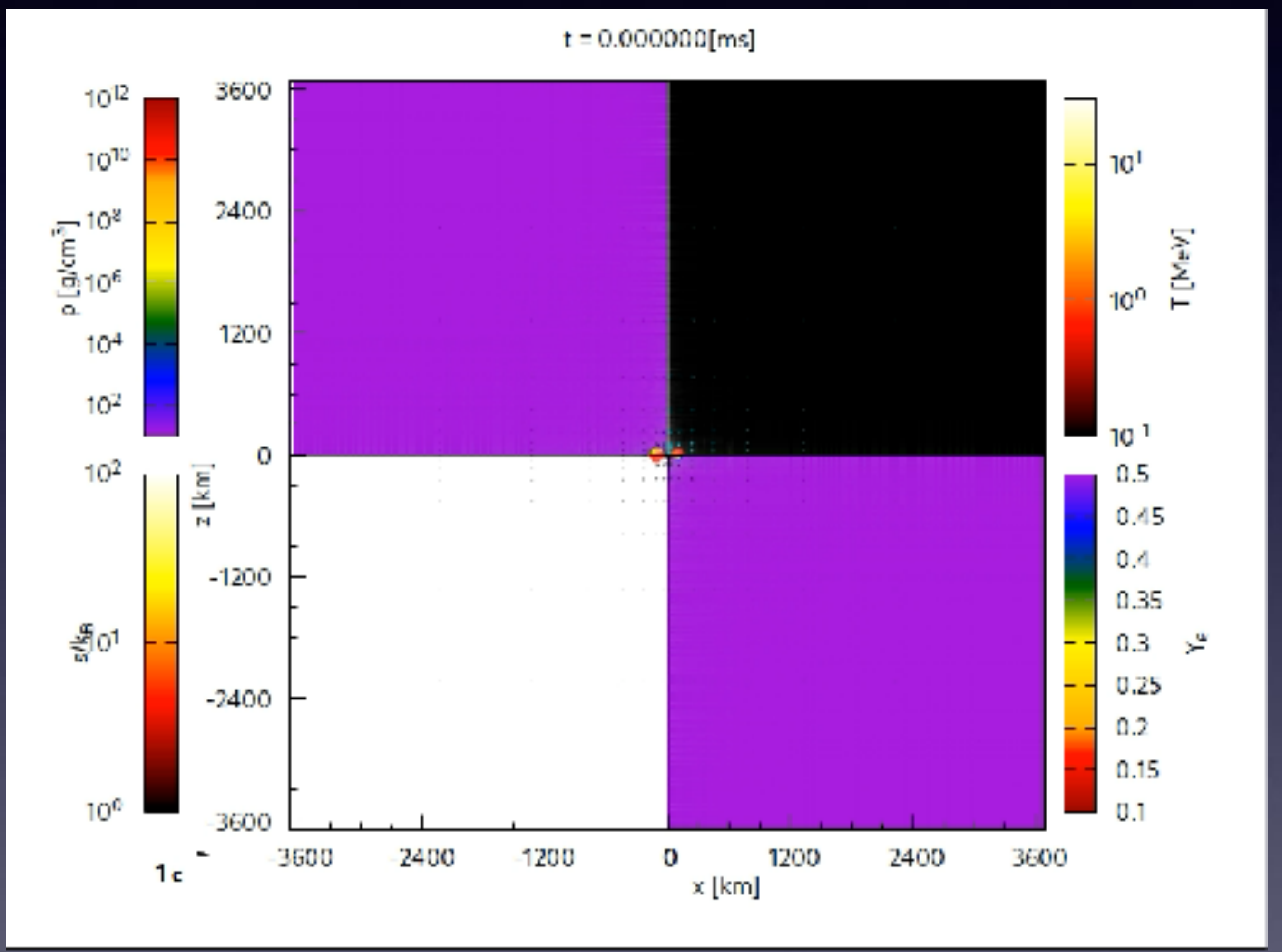
$\alpha_{\rm vis} = 0.02,\ \mathbf{0.05},\ 0.1,\ 0.15$

Results (fiducial model)

Close-up



Zoom out

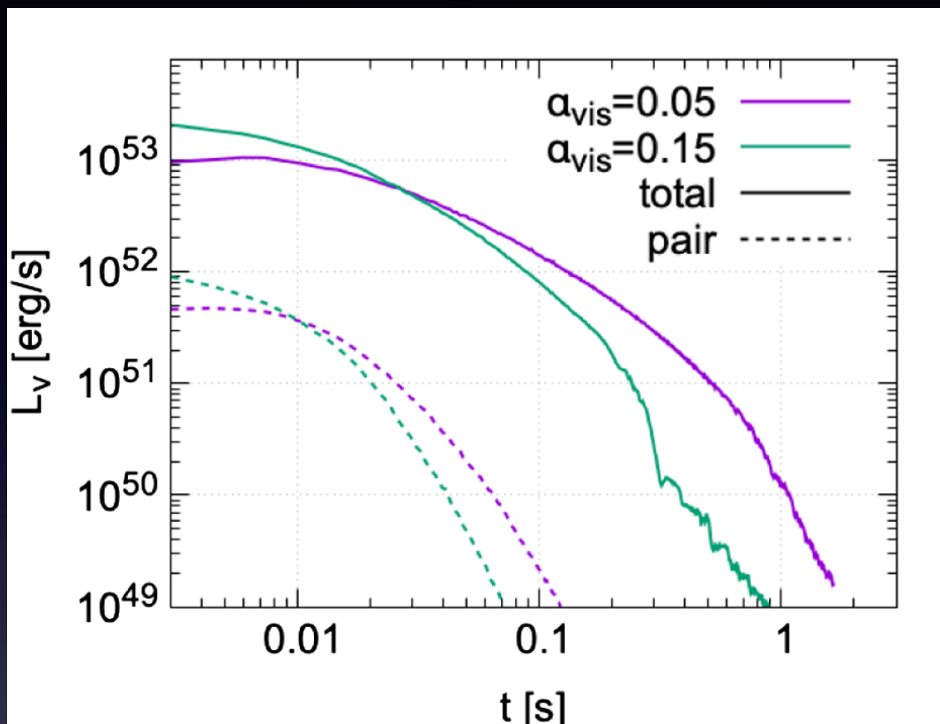


$$M_{\rm BH} = 3\,M_\odot,\ \chi_{\rm BH} = 0.8,\ M_{\rm torus} = 0.1\,M_\odot,\ l \propto \Omega^{-1/7},\ \alpha_{\rm vis} = 0.05$$

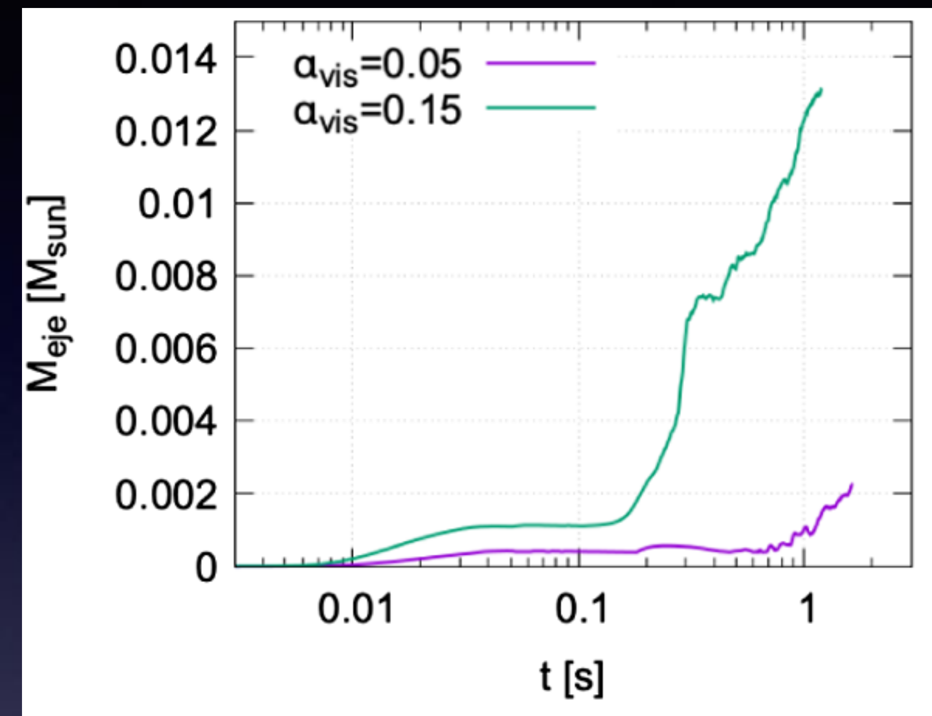
- Mass accretion rate/neutrino luminosity/mass ejection consistent with previous studies (e.g., Fujibayashi+2020)

BH-torus: Result

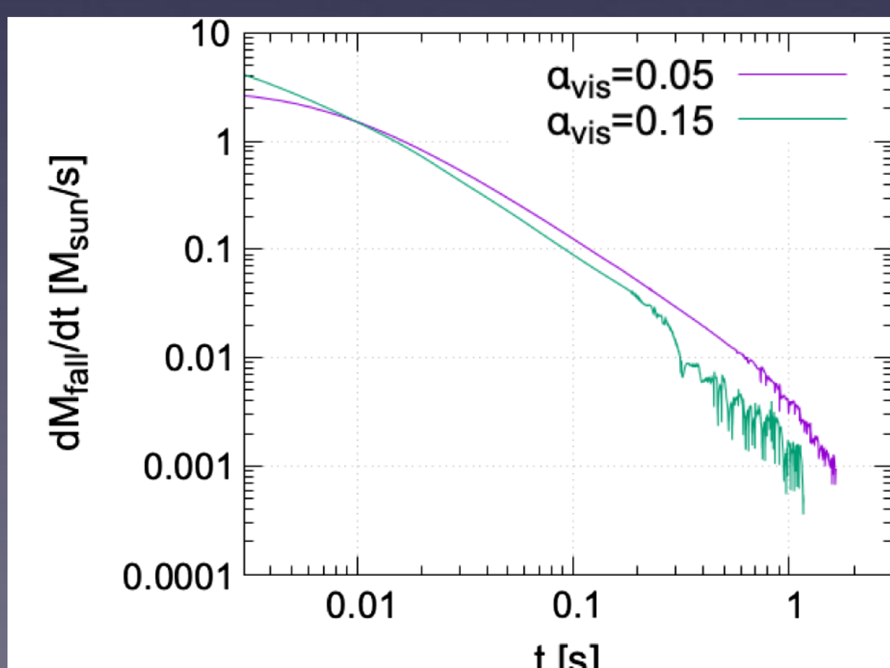
Total neutrino luminosity / pair annihilation rate



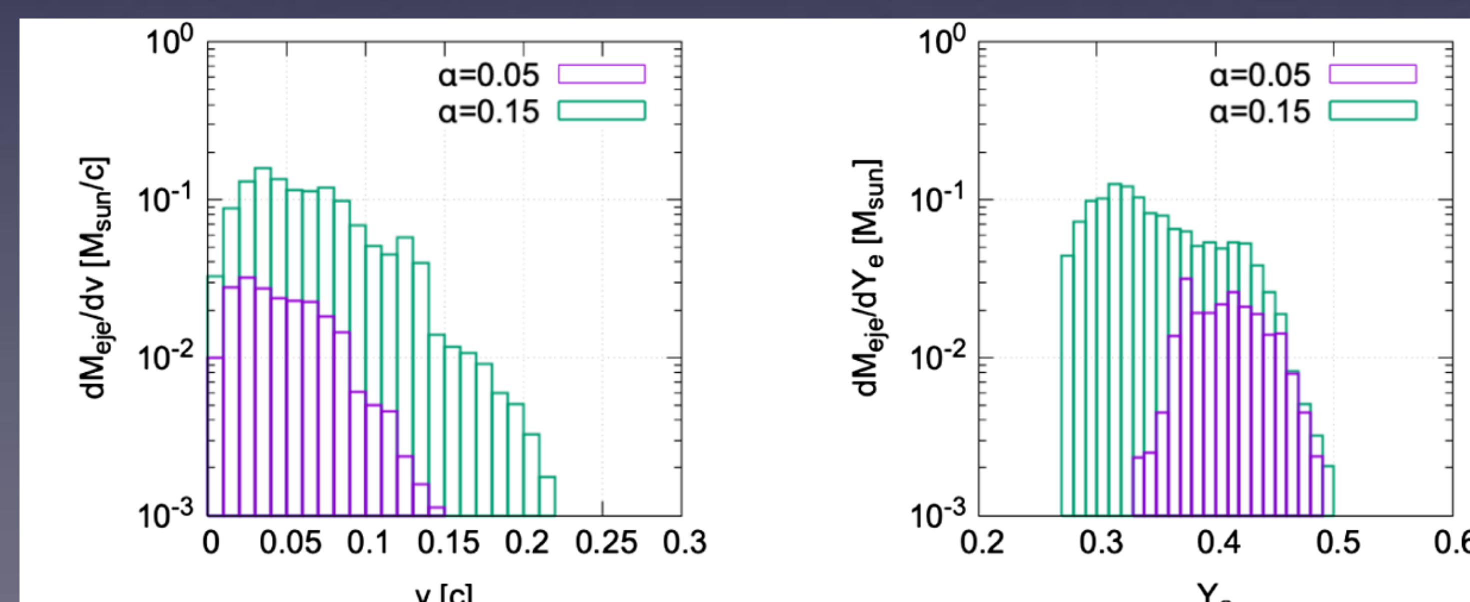
Ejecta mass evolution (>2000km)



Mass accretion rate



Ejecta velocity & Ye



Broadly in agreement with the previous studies (e.g., Richers et al. 2015, Just et al. 2016, Fujibayashi et al. 2020ab, Miller et al. 2020)

Pair process kernel

Moment(-like) decomposition model for pair process kernel

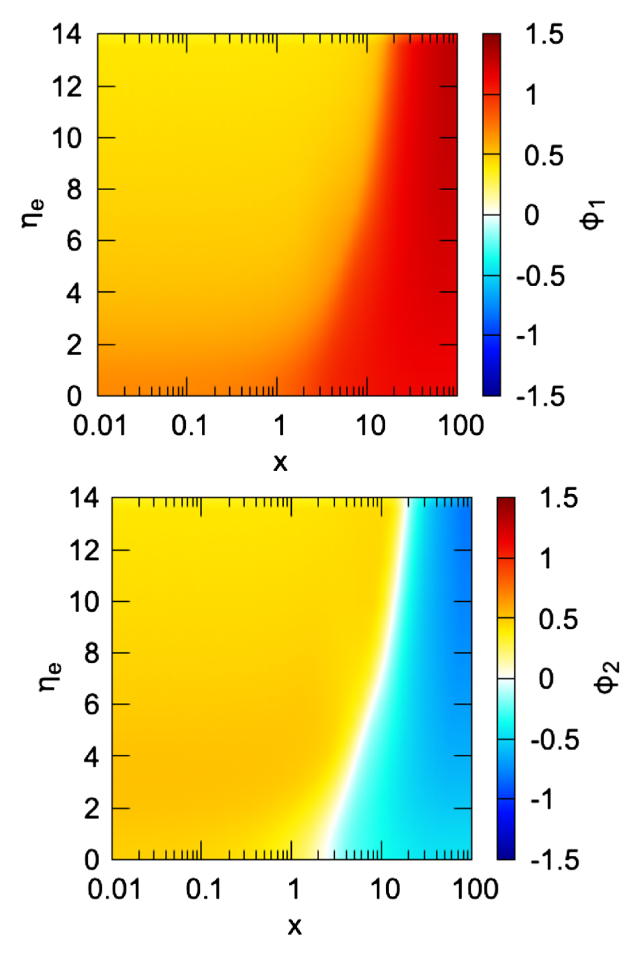
Bruenn 1985

$$R^{\mathrm{ann}}(\omega,\bar{\omega},\mu) \approx \frac{2G_{\mathrm{F}}^2}{3\pi(\hbar c)^4\omega\bar{\omega}}(C_V^2+C_A^2)q_\mu q_\nu \bar{q}^\mu \bar{q}^\nu \times [\phi_1(x)\phi_1(\bar{x}) - \phi_2(x)\phi_2(\bar{x})]$$

Average Fitting Error : $\lesssim 10\,\%$

$x=\omega/k_{\mathrm{B}}T,\ \bar{x}=\bar{\omega}/k_{\mathrm{B}}T \qquad m_e c^2 \ll k_{\mathrm{B}}T, \omega_{\nu_i}$

Fitting functions (ϕ_1 and ϕ_2)



→Pair annihilation rate (effective absorption rate) for single neutrino

$$\alpha^{\mathrm{pair}}(\mathbf{Q}) \approx \frac{2G_{\mathrm{F}}^2}{3\pi(\hbar c)^4}(C_V^2+C_A^2)\left[\phi_1(x)\bar{\Phi}_1^{\mu\nu} - \phi_2(x)\bar{\Phi}_2^{\mu\nu}\right]\frac{q_\mu q_\nu}{\omega}$$

$$\bar{\Phi}_i^{\mu\nu} = \int d^3\bar{\mathbf{Q}}\bar{f}(\bar{\mathbf{Q}})\phi_i(\bar{x})\frac{\bar{q}^\mu\bar{q}^\nu}{\bar{\omega}}.$$

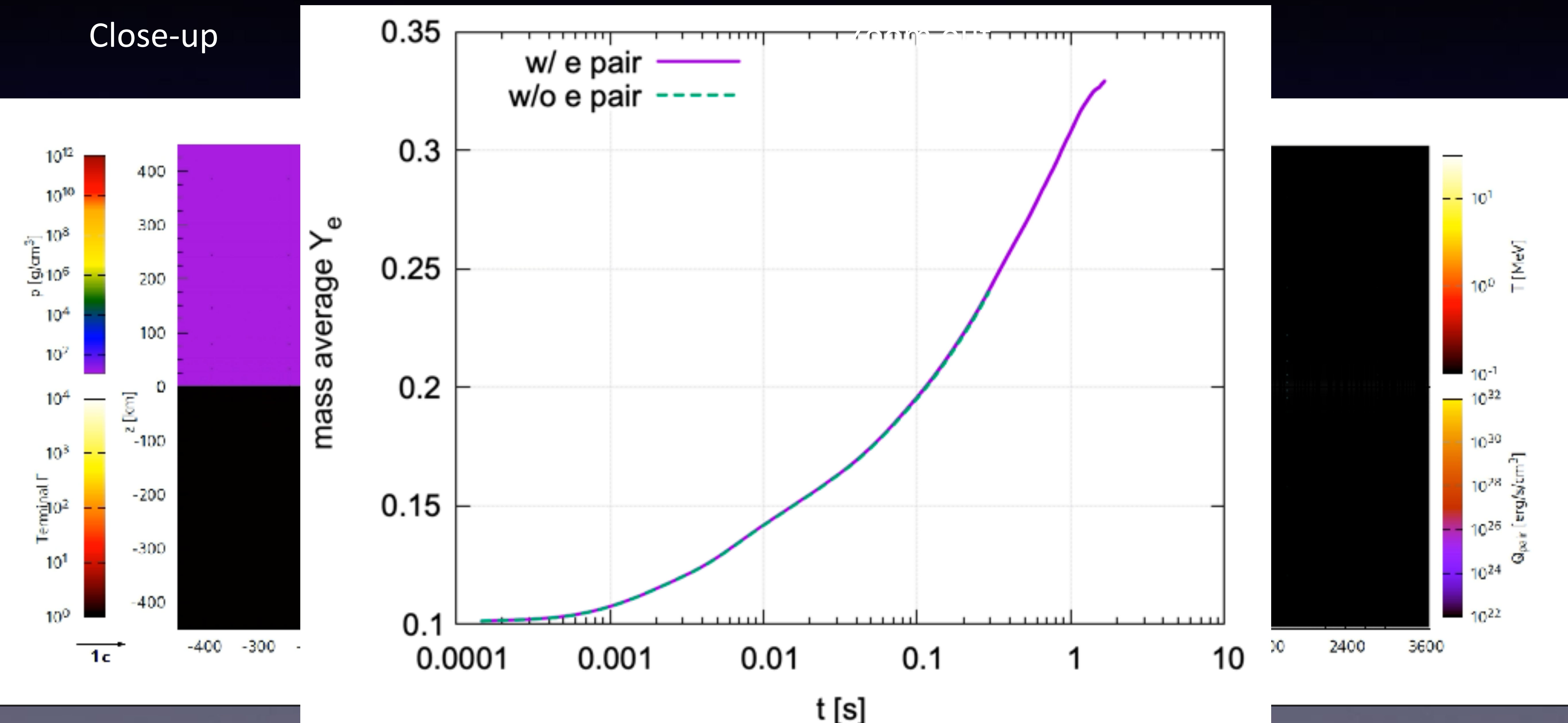
Target neutrino information

Accurate calculation of neutrino pair annihilation rate employing **full distribution function information** by introducing approximate moment(-like) decomposition of the interaction kernel

*Reduces to the exact expression in non-degenerate and high ν-energy limit

Pair annihilation / Relativistic outflows

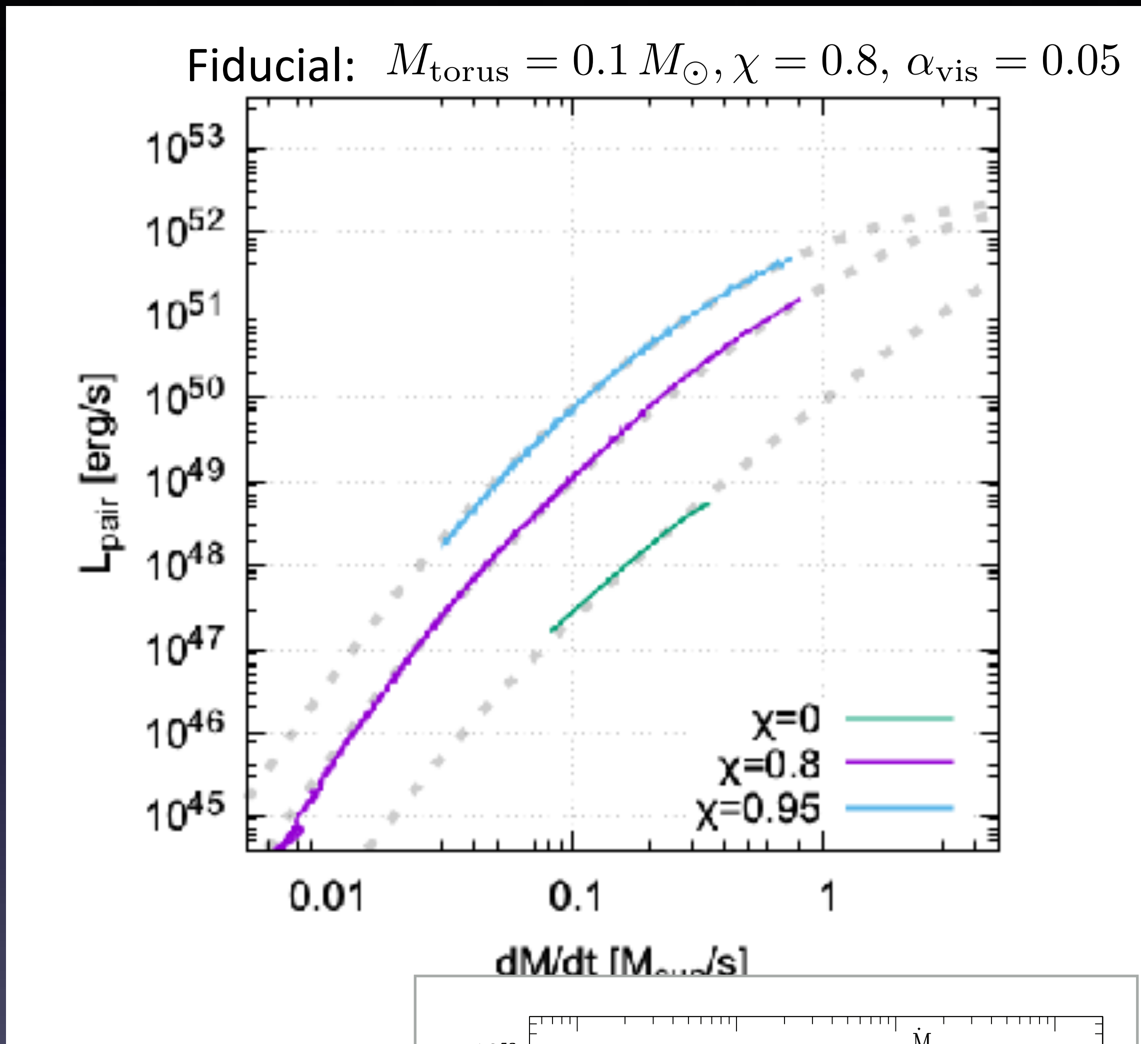
Close-up

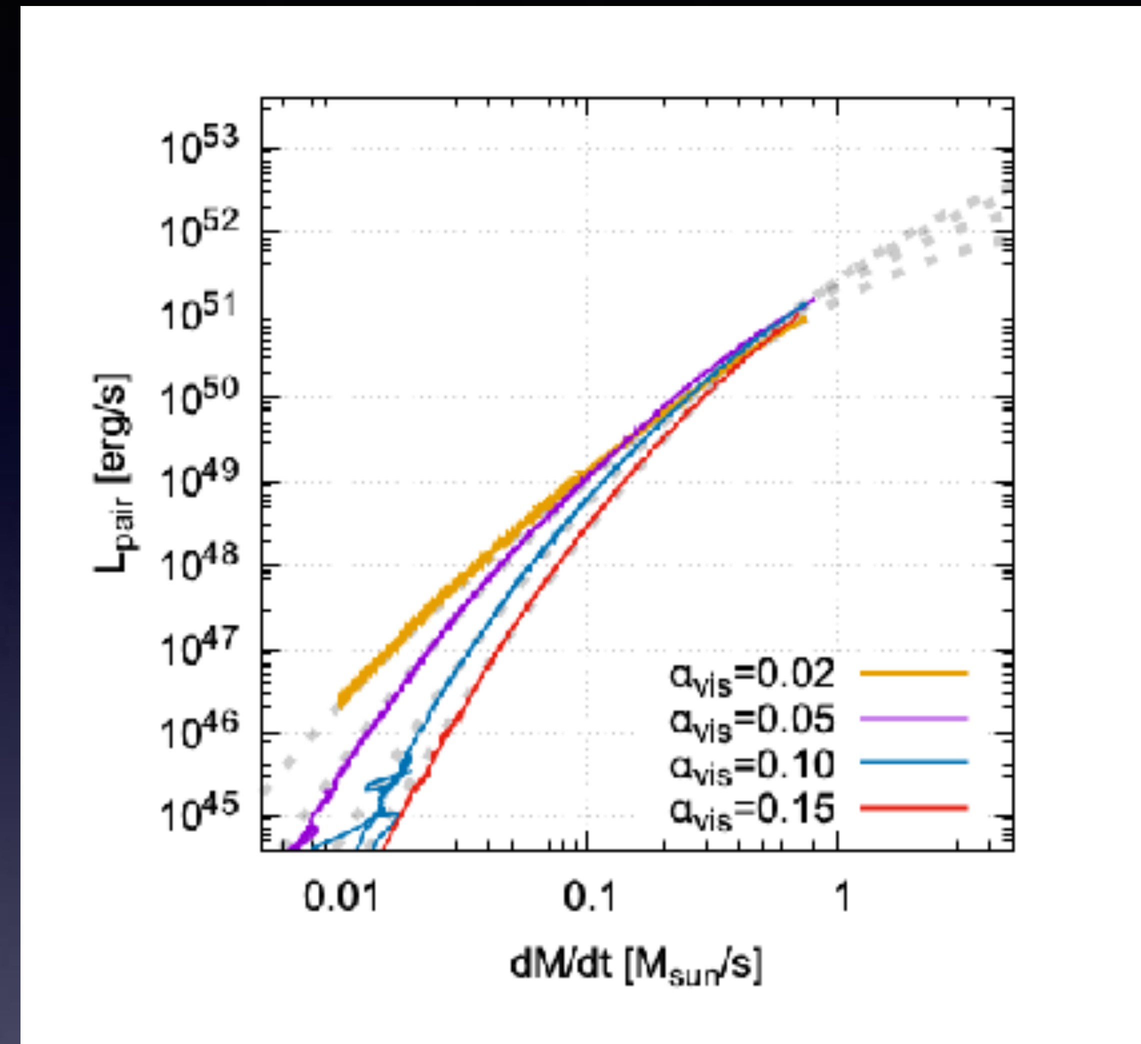


Relativistic outflow is launched in the pole region up to ~200 ms
by taking electron-type pair annihilation into account (see also Just et al. 2016)

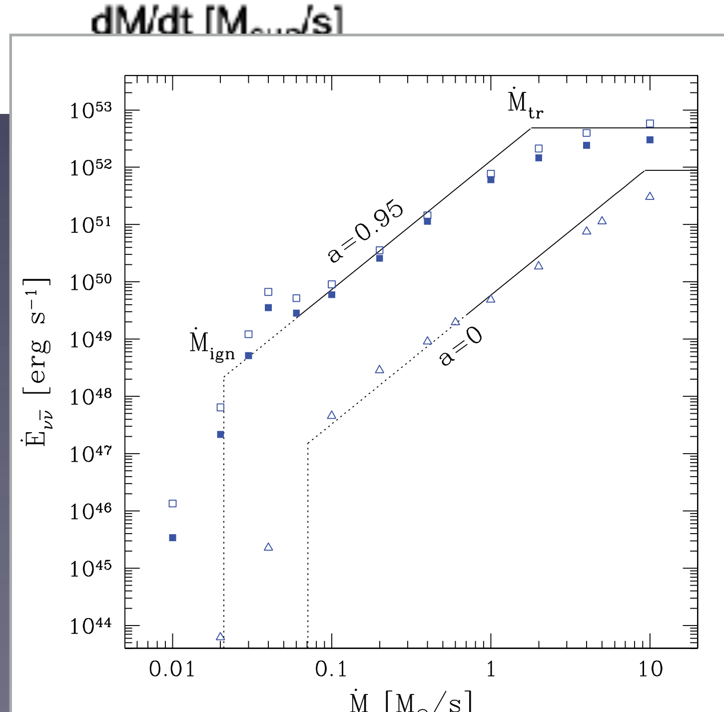
Pair annihilation deposition rate

Total pair annihilation energy deposition rate v.s. Mass accretion rate





cf. previous study (semi-analytical) Zalamea+ 2011



Fitting formula (gray dash curves)

$$L^{\rm fit}_{\rm pair}(\dot{M}) = \frac{L_0}{\left[1+\left(\frac{\dot{M}_1}{\dot{M}}\right)\right]^{5/2}\left[1+\left(\frac{\dot{M}_2}{\dot{M}}\right)\right]^{5/2}}$$

$$L_0 = 4.2\times 10^{52}\,{\rm erg/s}\,\alpha_{\rm vis,0.05} r_{\rm ms,0.8}^{5/8} M_{\rm BH,3},$$

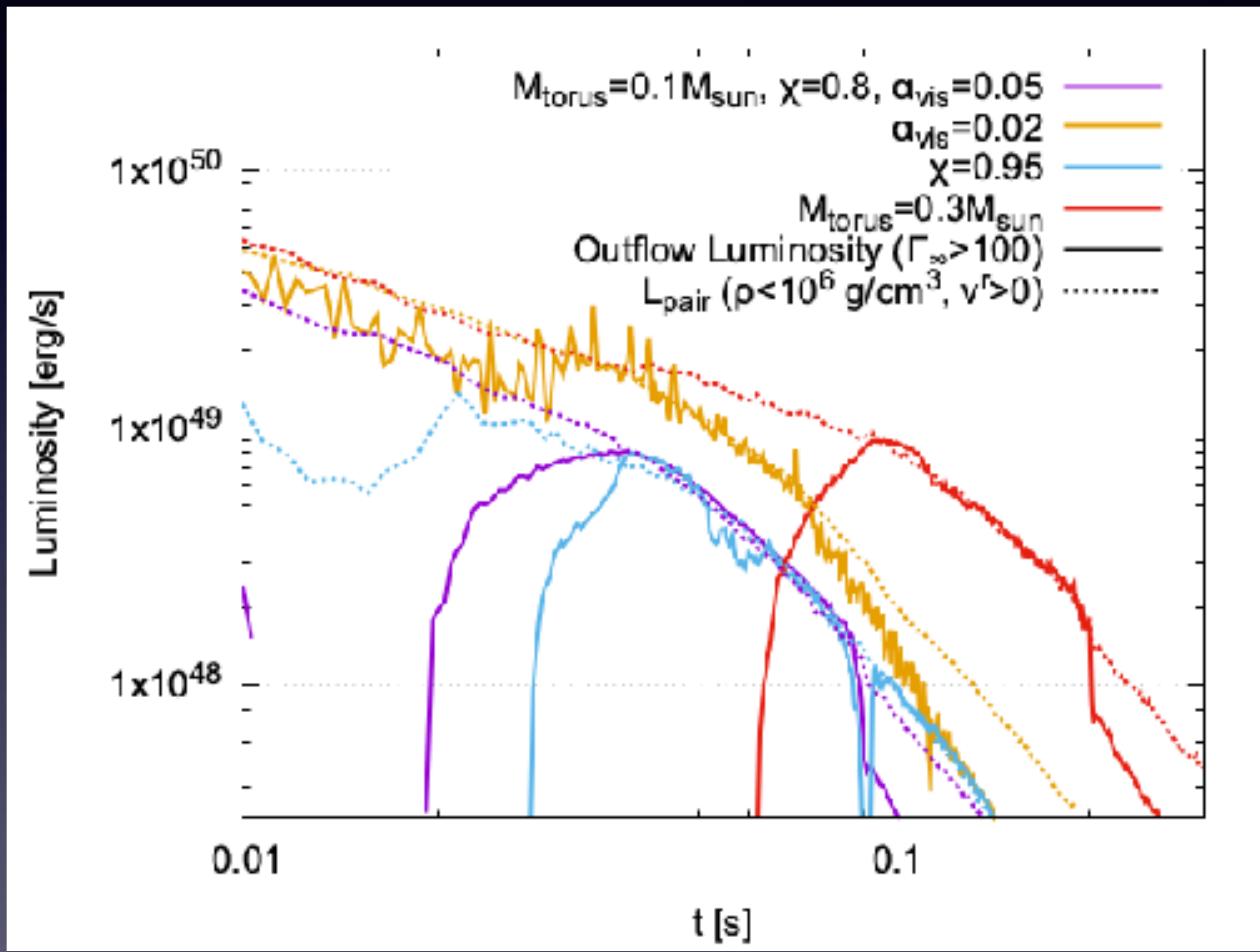
$$\dot{M}_1 = 0.020\,M_\odot/{\rm s}\,\alpha_{\rm vis,0.05}^{5/3} M_{\rm BH,3}^{4/3},$$

$$\dot{M}_2 = 2.2\,M_\odot/{\rm s}\,\alpha_{\rm vis,0.05}^{1/3} r_{\rm ms,0.8}^{12/5} M_{\rm BH,3}.$$

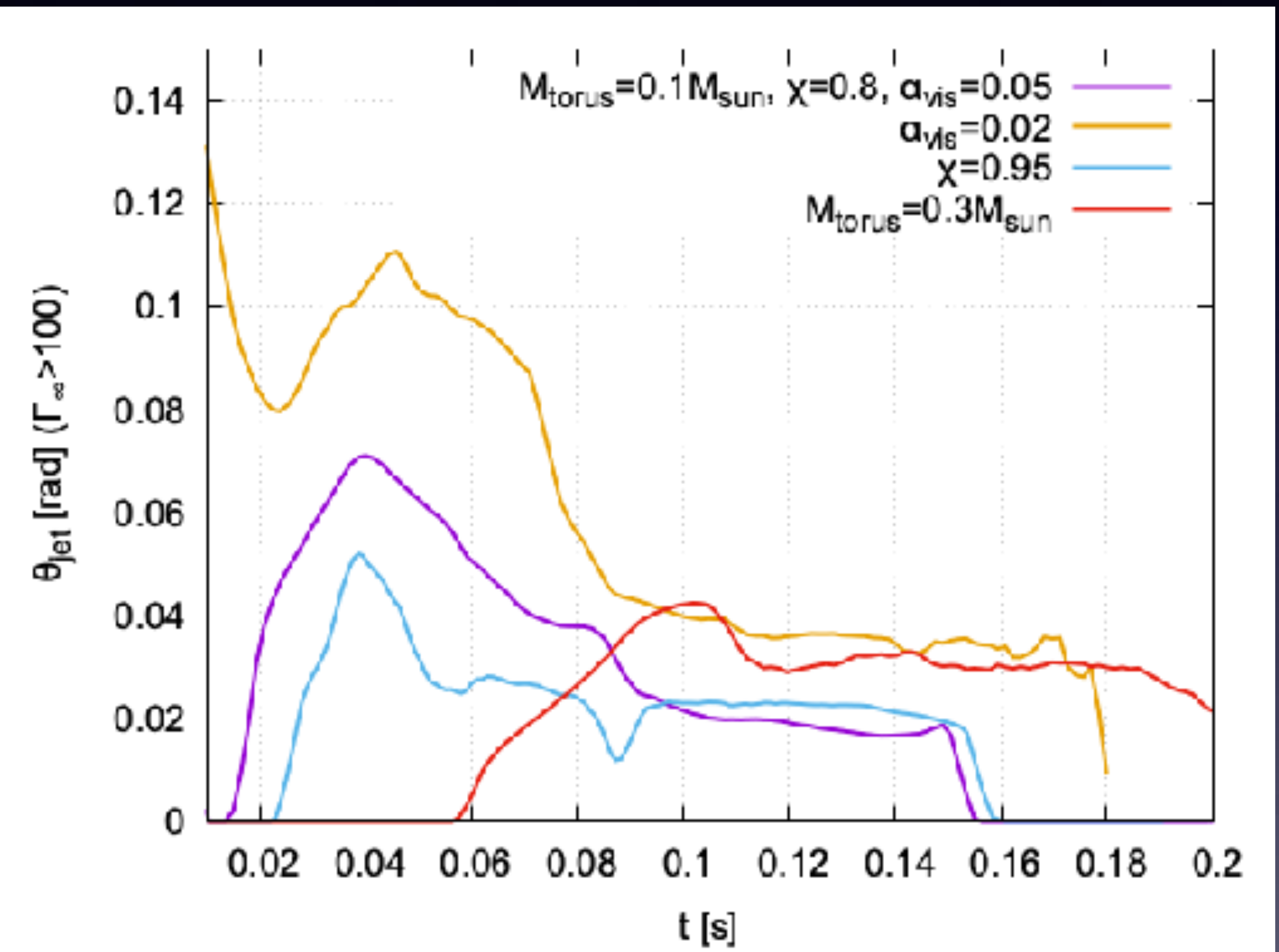
$\dot{M}_1 \sim \dot{M}_{\rm ign}$ cf. Agarwal+ 2025

Relativistic outflow

Relativistic outflow luminosity



Relativistic outflow opening angle



$$E_{\rm jet} \sim 10^{47-48}\,{\rm erg}$$

$$\Delta t_{\rm jet} \approx 0.1 - 0.2\,{\rm s}$$

$$\theta_{\rm jet} \approx 0.02 - 0.2\,{\rm rad}$$

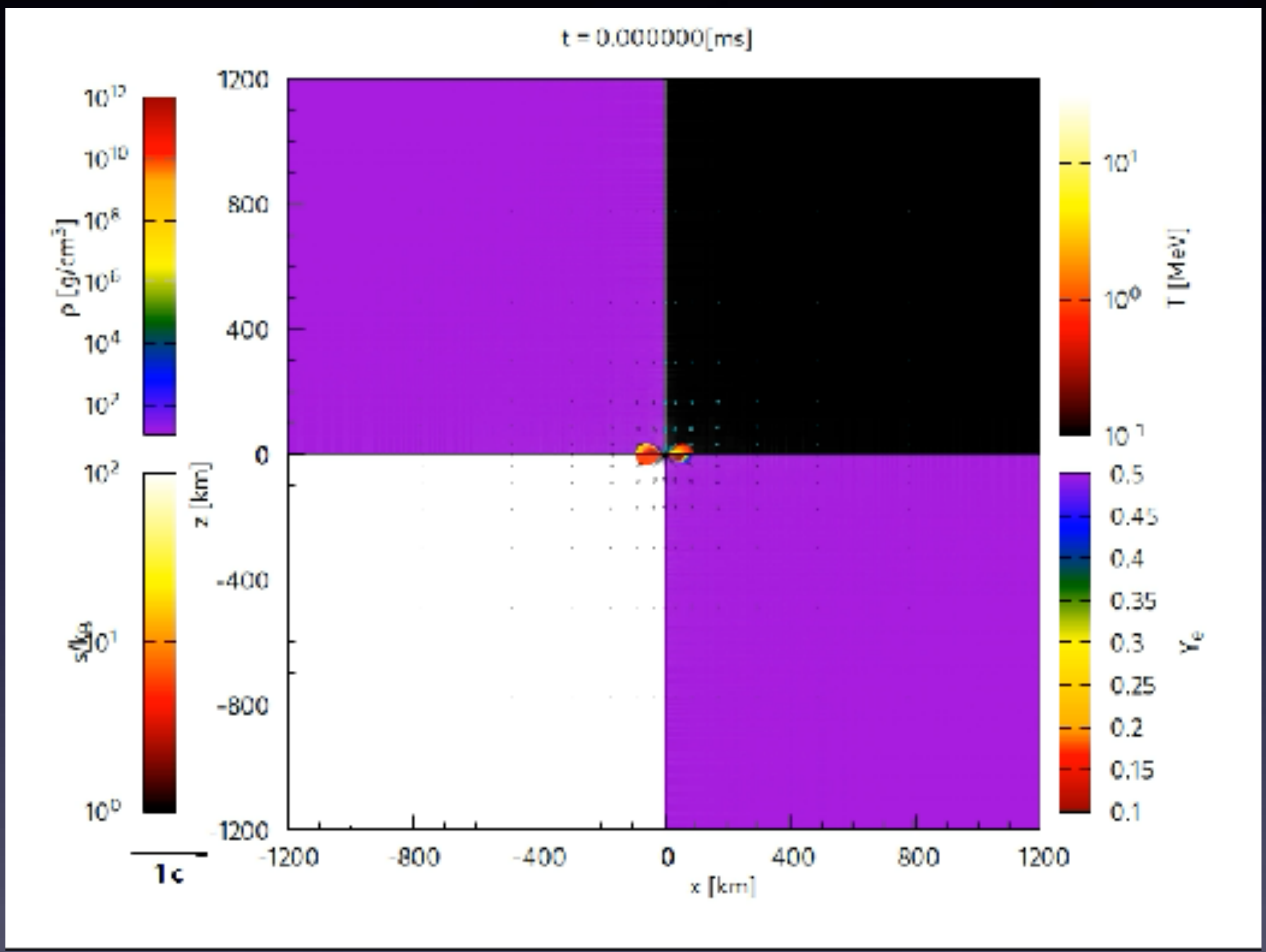
*the first 20ms is excluded for the measurements to avoid the initial relaxation artifact

$$E^{\rm iso}_{\rm jet} \sim 10^{50-51}\,{\rm erg}$$

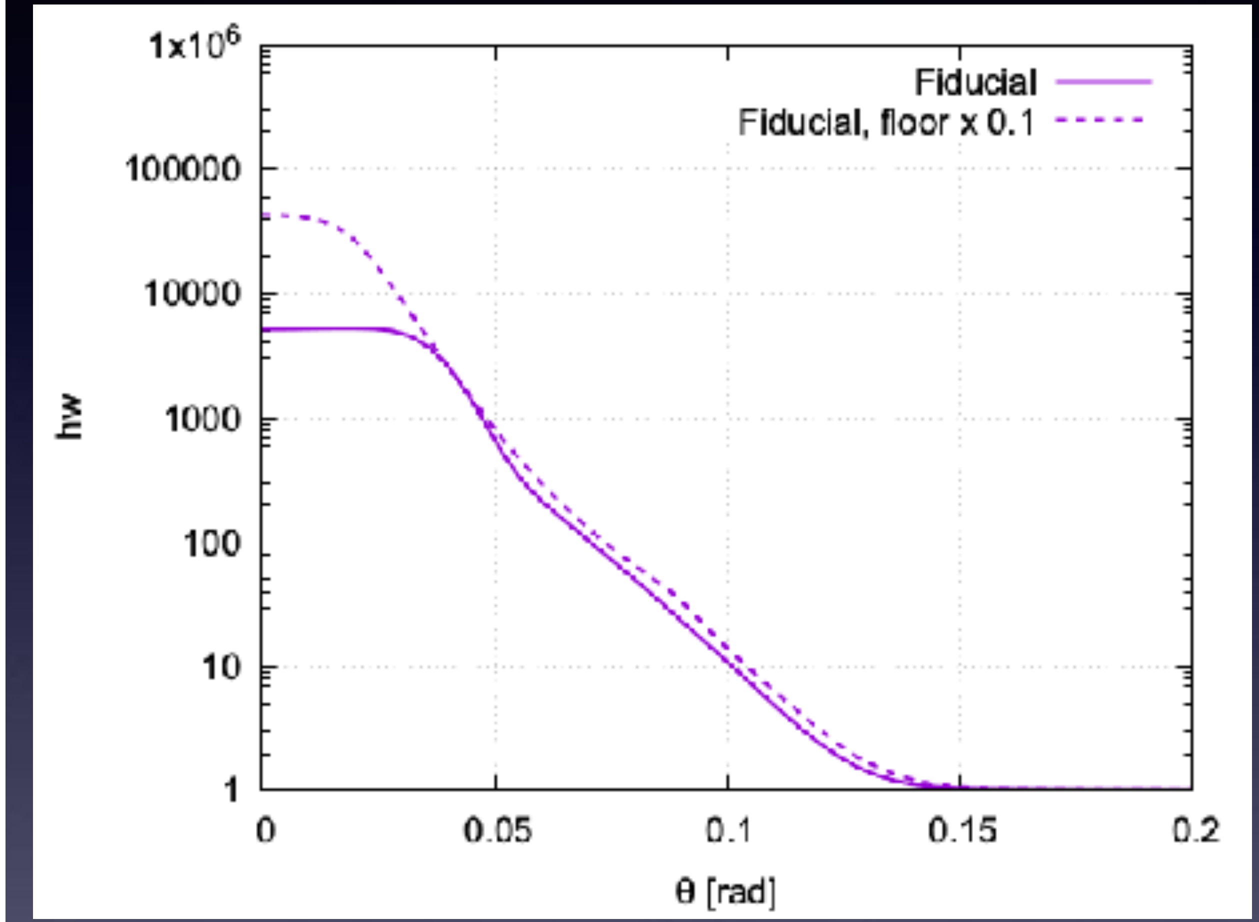
may explain low-luminosity sGRBs / precursors (see also Just+2016)

Caveat: initial data/grid resolution/floor density dependence

Higher BH spin model: $\chi_{\rm BH} = 0.95$



Terminal Lorentz factor @ t=50ms, r=3000km



- Jet collimation is characterized primarily by the matter blown-up during the initial relaxation phase
- ×2 change in E_{jet} by ×2 change in grid resolution (cf. ΔE_ν ~1%, ΔE_{pair}~10%, ΔE_{jet}^{iso}~20%)
- Terminal Lorentz factor is significantly affected by the floor density setup

see also Aloy+2005, Just+2016

FFC in MCRHD code

in prep.

Neutrino Flavor Conversion in NS binary merger systems

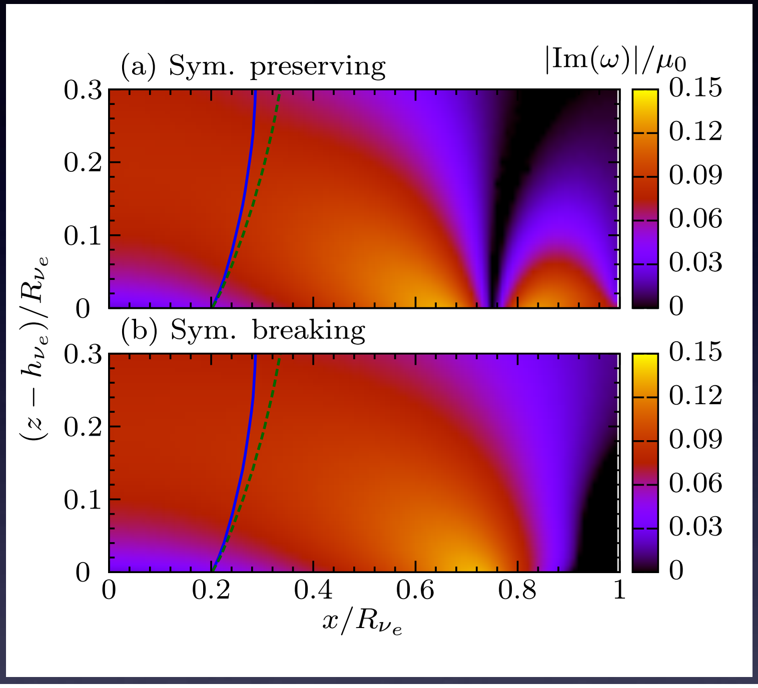
Wu & Tamborra 2017

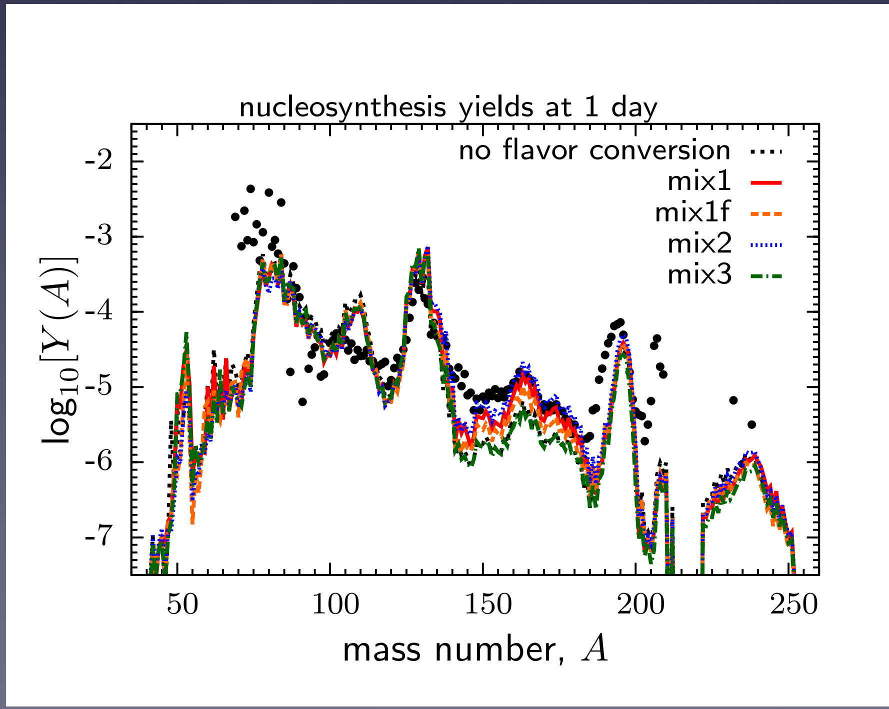
- Neutrino flavor conversion, which is a quantum effect beyond classical transport, being studied in core collapse supernovae (e.g., Sawyer 2005, Wu, Ehring) *and recently more in binary neutron star mergers.* (Wu et al. 2017, Li & Siegel 2021, Just et al. 2022, Fernandez et al. 2022, Richers et al. 2022, Froustey et al. 2024, Nagakura et al. 2025, Lund et al. 2025, Qiu et al. 2025)

- Flavor conversion can alter the thermodynamical property and electron fraction of the ejecta and thus can potentially impact nucleosynthesis outcomes.

- Fast flavor conversion (Sawyer 2005) is triggered by crossings in the angular distributions of the electron and heavy lepton numbers ***(ELN-XLN crossing)*** and depends sensitively on the neutrino distribution functions.

- Solving Quantm kinemtic equation (off-diagonal component in density matrix) + characteristic growth timescale $\sim \mathrm{ns}$, posing a major challenge for numerical simulations.





Just et al. 2022

BGK model on MCRHD

- BGK (Bhatnagar-Gross-Krook) model for νFC:
  a subgrid heuristic method to capture νFC
  (Nagakura et al. 2024)
  → enables us to approximately capture the FFC by skipping detailed conversion process and treating it as a relaxation process with asymptotic state models.

- ***Only classical distribution function information is needed in the BGK model.***

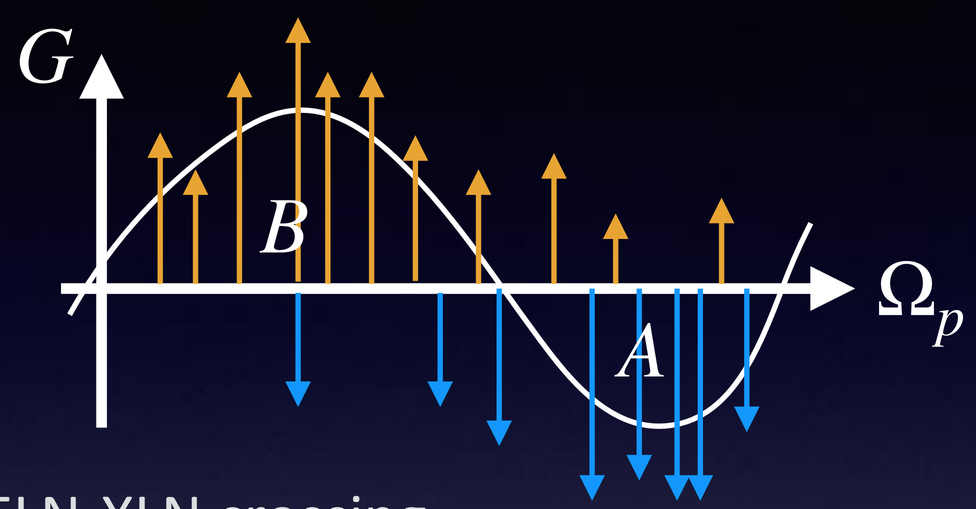
- In the MC method, the neutrinos distribution functions are described in the set of Dirac functions.
  → need to reconstruct as a smooth function of momentum angular space for determine ELN-XLN Crossing
  (See also Lund et al. 2025 for the implemention of the BGK model in MCRHD)

- We reconstruct the neutrino angular distribution functions by the spherical harmonics expansion

$$\frac{dn_{\nu_i}}{d\Omega_p} \approx \sum a_{lm} Y_{lm} \qquad a_{lm} = \int \frac{dn_{\nu_i}}{d\Omega_p} Y^*_{lm} d\Omega_p = \sum w_k Y^*_{lm,k}$$

- Flavor of the MC packets are probabilistically converted based on the time scale and asymptotic state (survival probability) with operator splitting method

Neutrino angular distribution function



ELN-XLN crossing

$$G = 4\pi\left(\frac{dn_{\nu_e}}{d\Omega_p} - \frac{dn_{\bar{\nu}_e}}{d\Omega_p} - \frac{1}{2}\frac{dn_{\nu_x}}{d\Omega_p} + \frac{1}{2}\frac{dn_{\bar{\nu}_x}}{d\Omega_p}\right)$$

$$A = \left|\int_{G<0} G \frac{d\Omega_p}{4\pi}\right| \qquad B = \int_{G>0} G \frac{d\Omega_p}{4\pi}$$

BGK model

$$\frac{df}{dt} = -\frac{1}{\tau_a}(f - f_a)$$

$$\tau_a \approx 2\pi (AB)^{-1/2} \; : \text{(relaxation timescale)}$$

$$f_a = f_a(G, A, B) \; : \text{(aymptotic state)}$$

Numerical Issues

- *1) MC shot noise (Foucart et al. 2025)*

  MC shot noise induces
  an artificial ELN-XLN crossing
  → always plays a role to overestimate the FFI

- *2) Tradeoff between*
  Angular resolution / MC shot noise

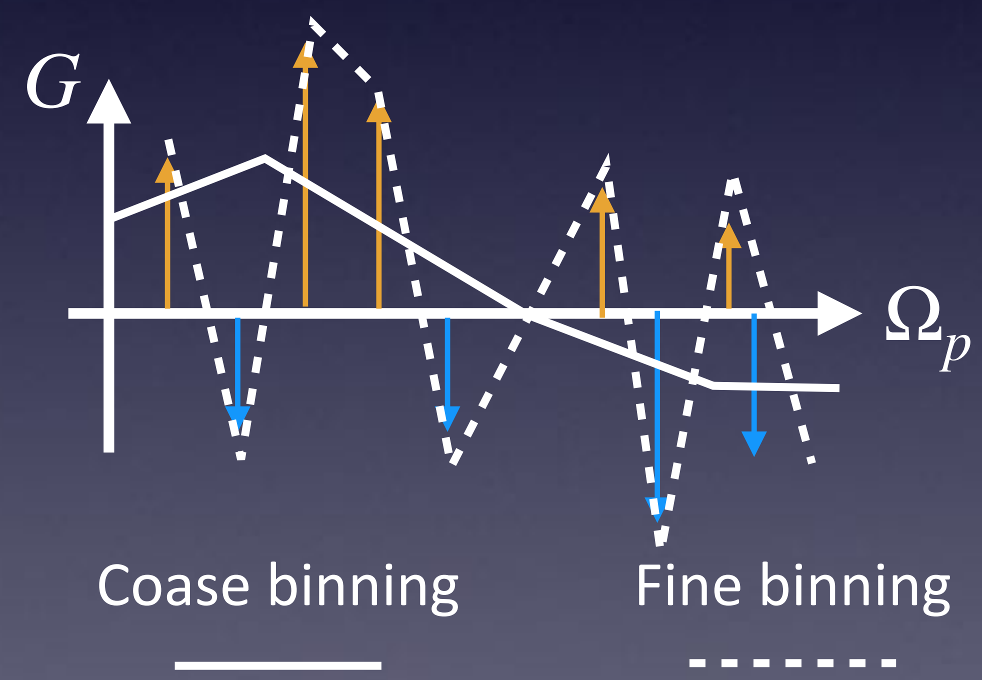
  large angular resolution
  ($\Longleftrightarrow$ consider large l modes)
  → less MC packets per effective angular bin
  → more dominated by MC shot noise

- *3) Conservation of total ELN-XLN crossing*

  total ELN-XLN crossing is conserved during the FFC, but it is not numerical guaranteed for the naive MC packet-based conversions

1)

2)



3)

$$\int G d\Omega_p = \text{const}\,.$$

Prescriptions

- *1) MC shot noise (see also Lund et al. 2025)*

  Set a Binning MC packets in neighbor spatial cells (supercell) + Set threshold on ELN-XLN crossing careful estimating the MC shot noise
  → Effectively increase MC packet number and only employ statistically significant ELN-XLN crossing

- *2) (Effectively) Adaptive angular resolution*

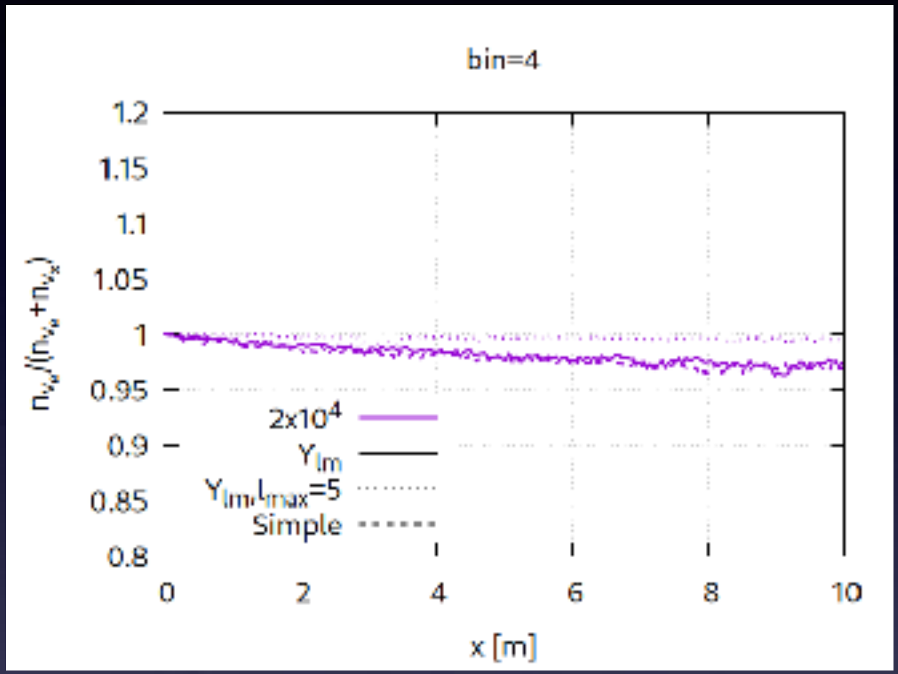
  Apply a filter to suppress (discard) the contribution from the l modes that are dominated by the MC shot noise

- *3) Approximate conservation of the total ELN-XLN crossing*

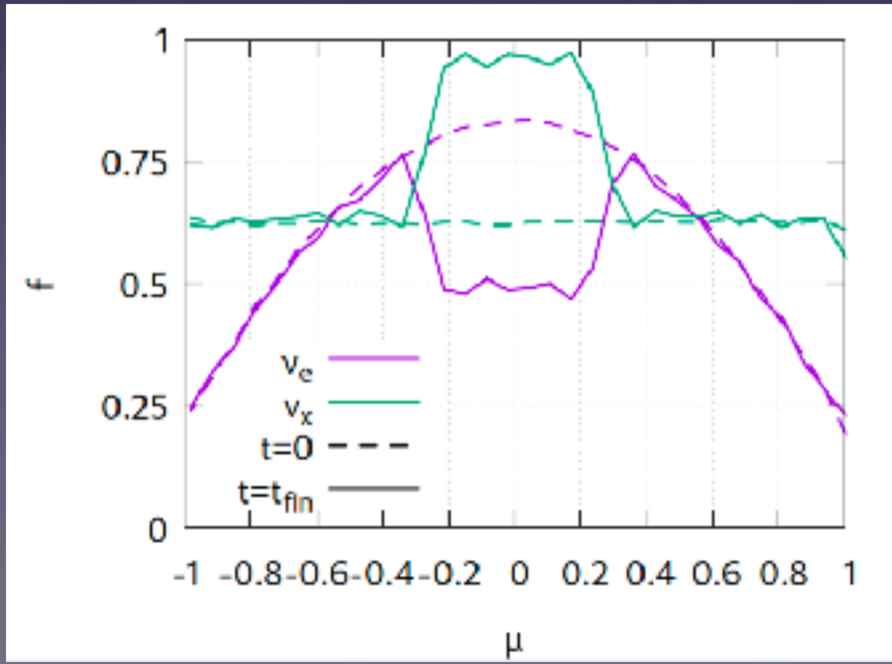
  Introduce a procedure that randomly selects and adjusts which MC packets actually trigger FFC from among the candidates that could cause FFC to minimize violations of total ELN–XLN crossing conservation.

Our scheme tends underestimates the FFI

Test problems (1d & 1 zone)



Nagakura et al. 2025



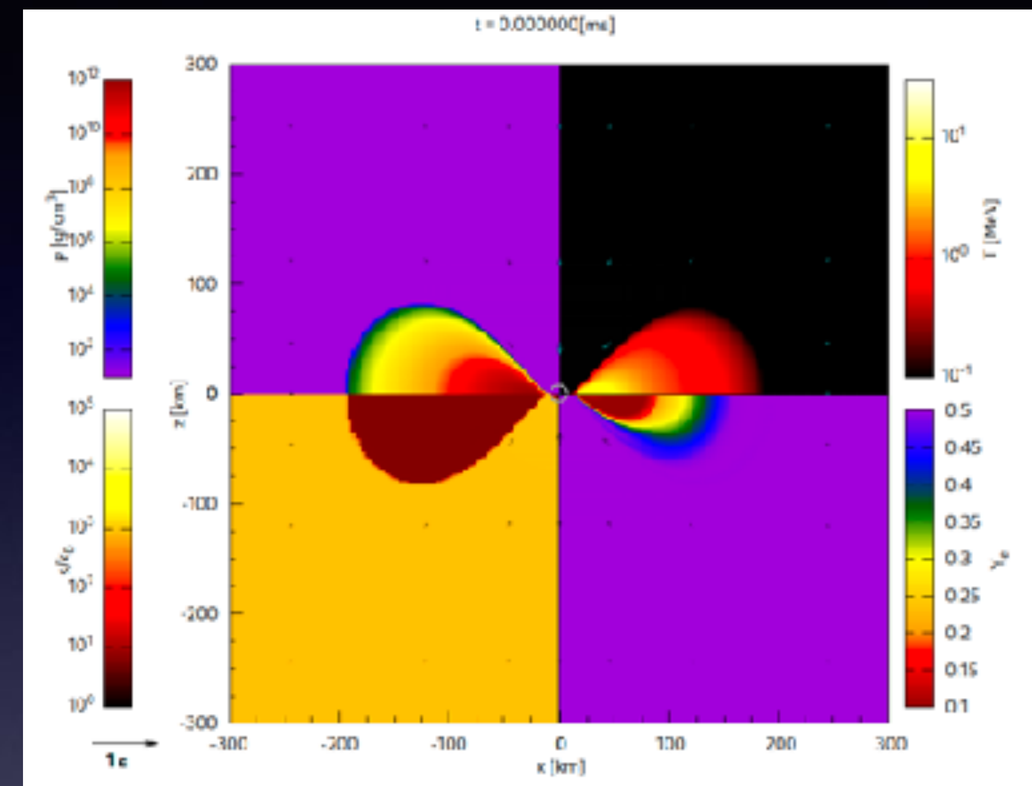
Lund et al. 2025

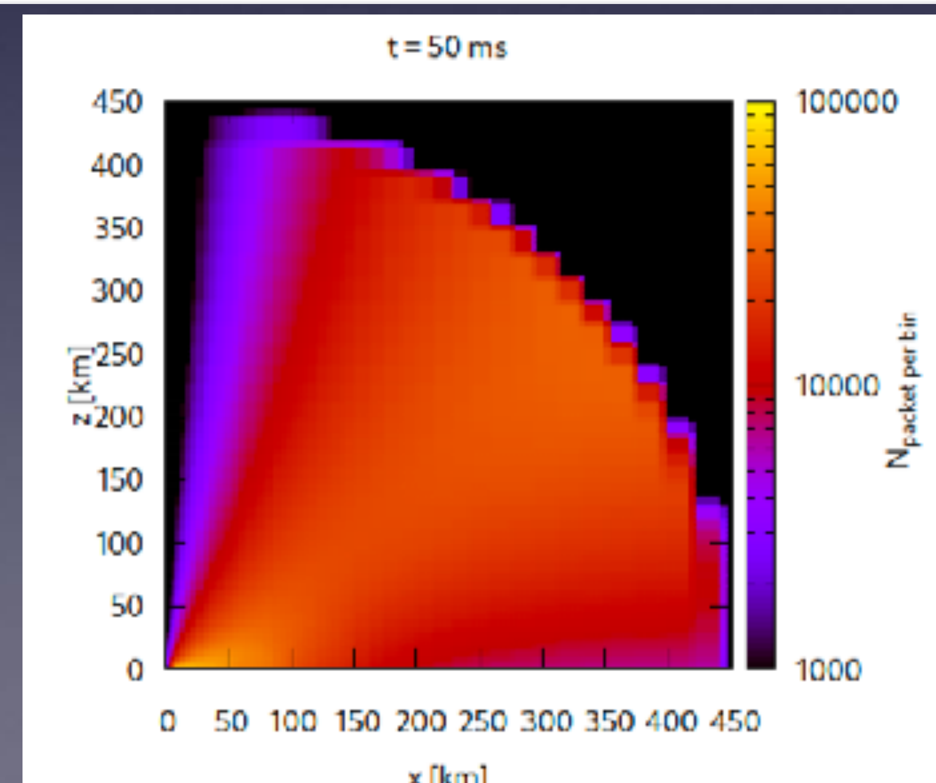
Application to BH torus system

- $M_{\rm BH} = 3\,M_\odot, \chi_{\rm BH} = 0.8$
- $M_{\rm torus} = 0.1\,M_\odot, j \propto \Omega^{-1/5}, \alpha_{\rm vis} = 0.05$
- $(N_x, N_z) = (320{,}320)$
- $(\Delta N_{{\rm bin},x}, \Delta N_{{\rm bin},z}) = (4{,}4)$
- $N_{\rm packet\ per\ cell} \sim 10^3\ (N_{\rm packet\ per\ bin} \sim 10^4 - 10^5)$

- Updated neutrino transport/ microphysics:
  - 3 species → 4 species $(\nu_e, \bar{\nu}_e, \nu_x, \bar{\nu}_x)$
  - EoS: DD2 EoS + Helmholtz EoS
    (Banik et al. 2014, Timmes et al. 2000,
    Fujibayashi et al. 2020)
  - + finite electron mass effects
    in absorption rate and emissivity

Initial profile for a BH-tours system (fiducial model)

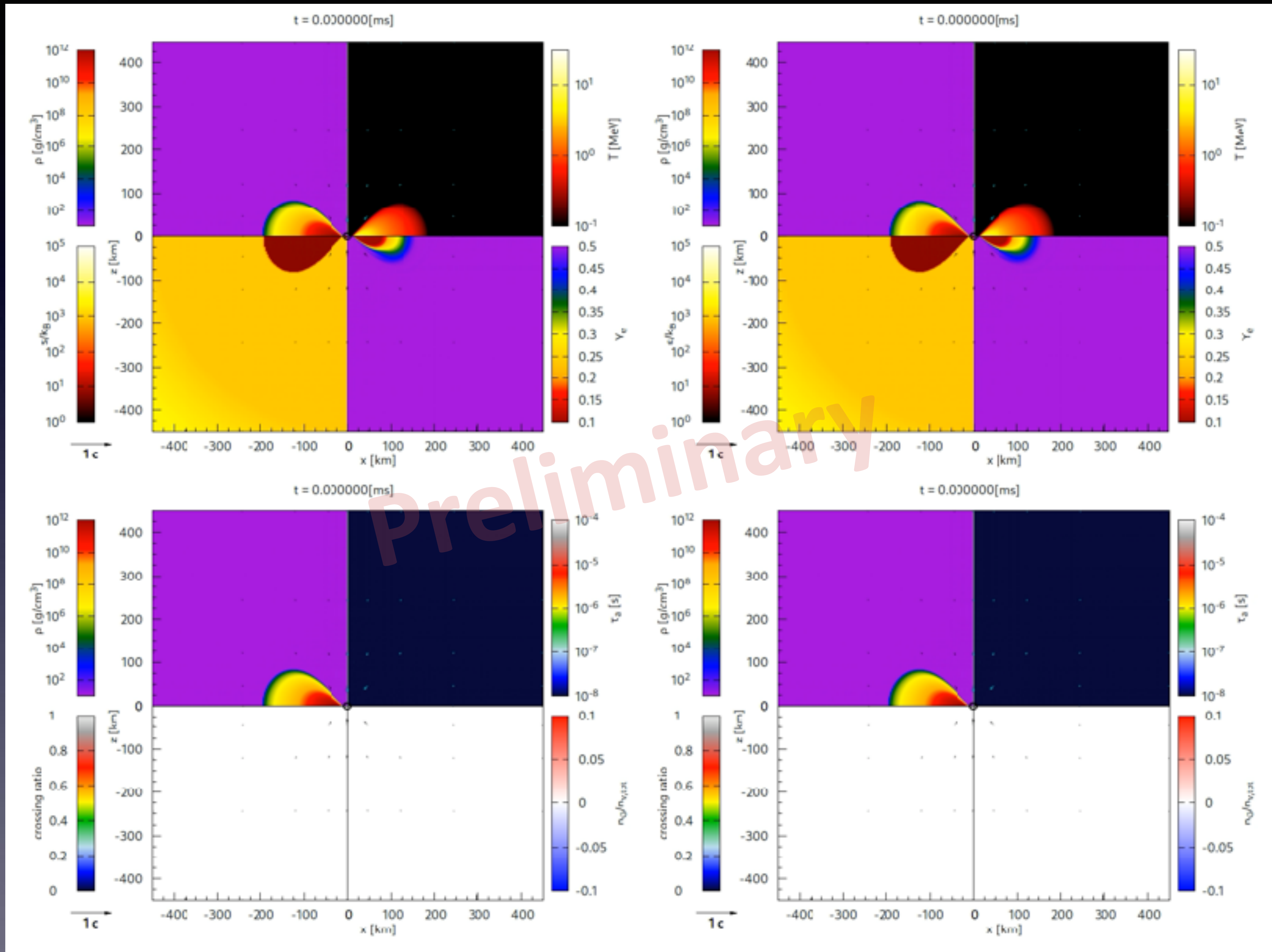




w/o FFC

Animation (up to ~500ms)

w/ FFC



$$\text{Crossing Ratio} = \min\left(\frac{A}{B}, \frac{B}{A}\right)$$

$$n_G = \left[n_{\nu_e} - n_{\bar{\nu}_e} - \frac{1}{2}(n_{\nu_x} - n_{\bar{\nu}_x})\right]$$

Neutrino emission

Broadly Consistent with the results of e.g., Just et al. 2022, Lund et al. 2025

Matter profile

Broadly Consistent with the results of e.g., Just et al. 2022, Lund et al. 2025

*Ejecta is not yet launched significantly (expected for >500ms)

Summary

- We develop an axisymmetric (2D) GR-MCνRHD code to investigate BH-torus systems with various improvements applied our code to BH-torus systems span a wide range of parameters. In particular, neutrino-antineutrino pair annihilation is taken into account for both electron and heavy lepton types, considering non-thermal distribution effects.
- We find that the evolution of the system and the various key quantities (e.g., ν Luminosity, ejecta mass, torus Ye, pair annihilation luminosity) are consistent with the results of the previous studies employing the leakage or M1 schemes (and also those employing the MC method)
- We found a clear relationship between the pair annihilation energy deposition rate and mass accretion rate, consistent with the previous work. We also provide a fitting formula for the pair annihilation energy deposition rate as a function of accretion rate.
- We showed that the pair annihilation leads to the formation of relativistic fireballs and jet launching in most cases for the first time employing a full Boltzmann transport scheme.
  The isotropic-equivalent energies of these outflows reach $\lesssim 10^{50\text{-}51}$ erg with durations $\lesssim 0.2$ s, which may account for low-luminosity sGRBs and GRB precursors.
- We implemented the BGK model for neutrino fast flavor conversion in dynamical and self-consistent manner in time-dependent simulations accounting for the full neutrino distribution function information.
- We so far evolved the BH-torus system up to~500 ms, and find that the disk average Ye can be lowered up to ~0.04 likely due to higher electron degeneracy of matter resulting from enhanced neutrino cooling.
- The simulation will be continued until ~1s -> ejecta properties will also be studied.
- Tasks: realistic ID / accurate jet propagation / NR / MHD / MNS