Subgrid modelling of MRI-driven turbulence in differentially rotating neutron stars

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Talk based on:
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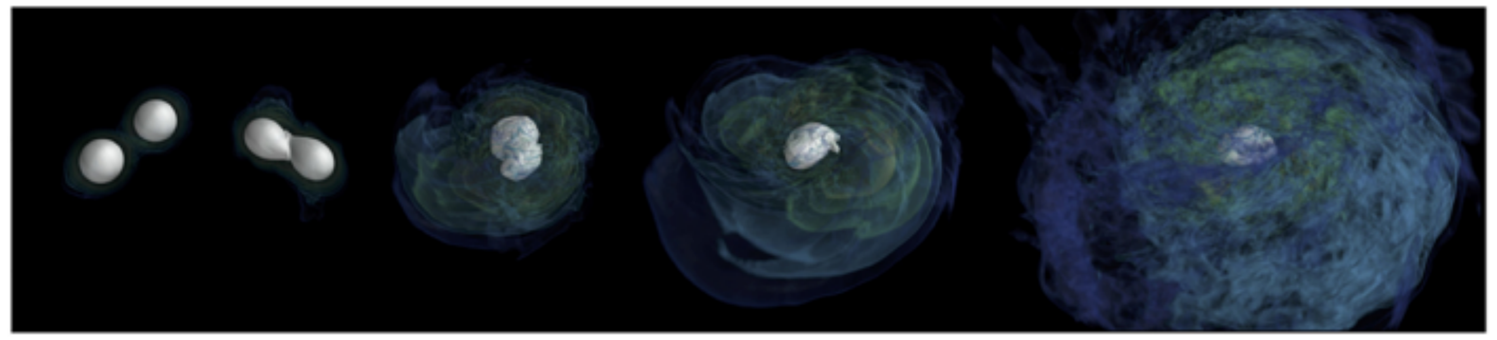
YITP long-term workshop - February 4, 2026



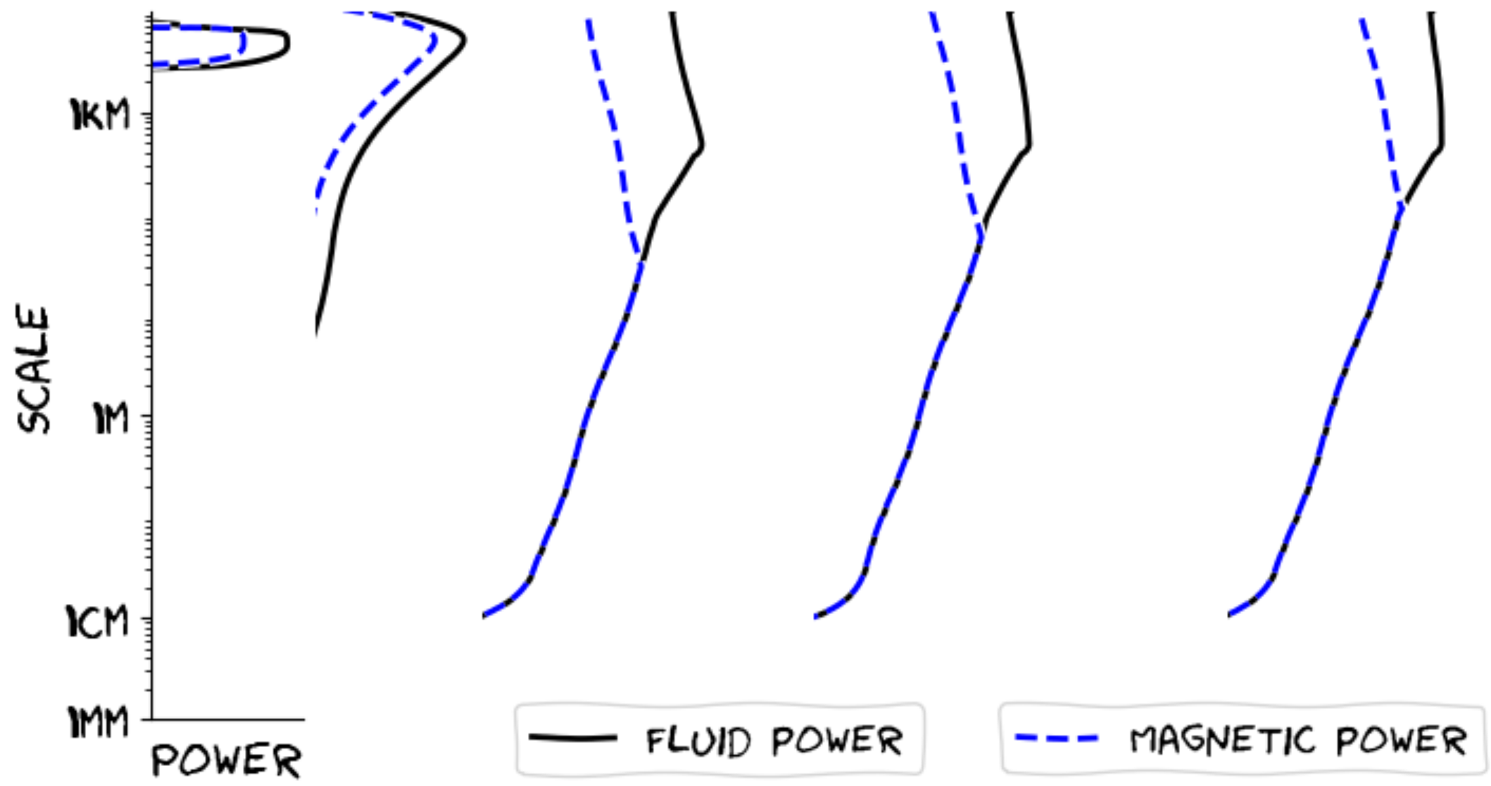
Introduction

MHD turbulence in BNS mergers

In BNS mergers, there can exist transfer of energy across a large range of spatial scales: **inertial range**.

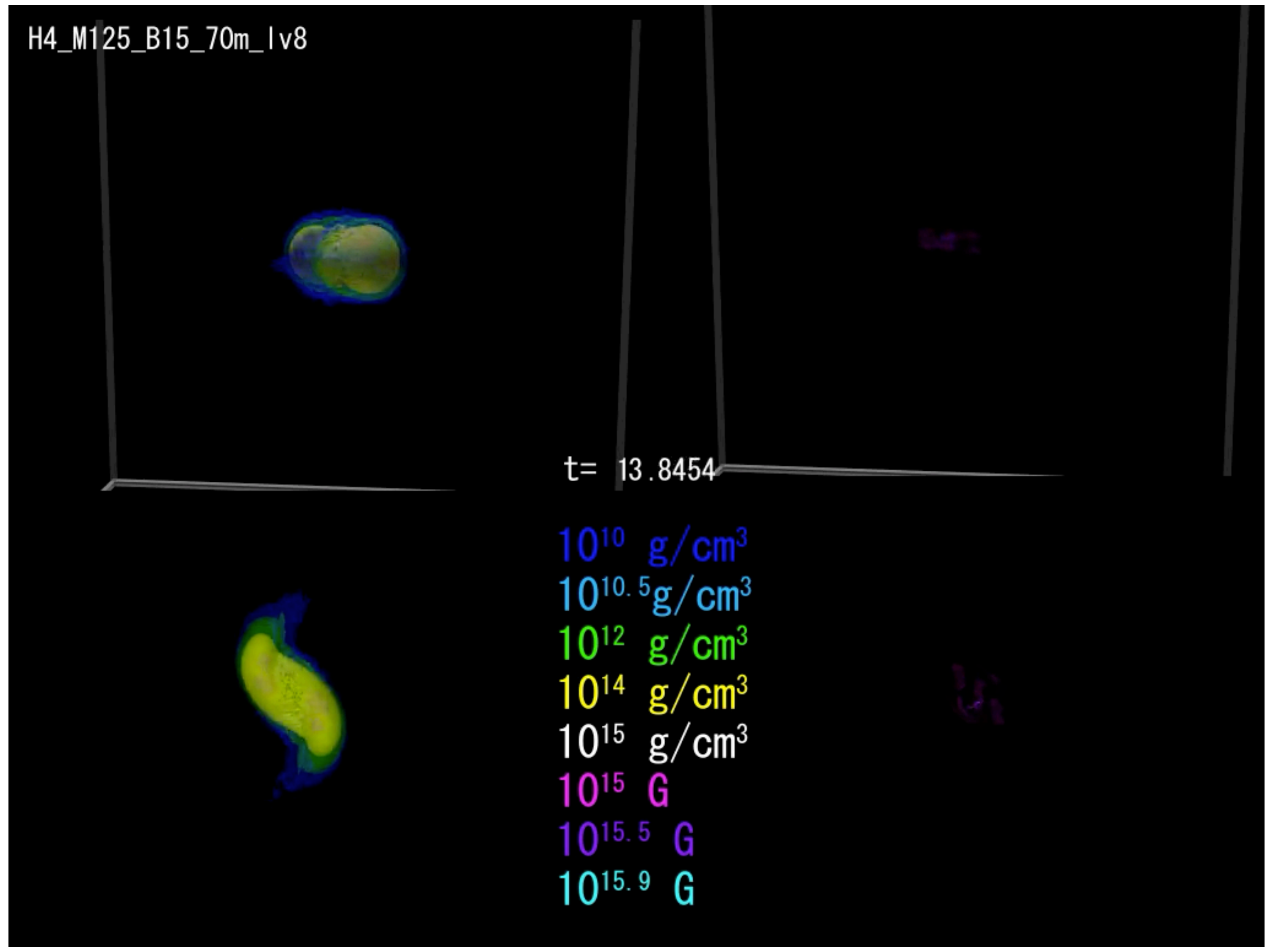
Turbulence can be quantified by the **Reynolds number**. In BNS mergers, $\mathrm{Re} \sim 10^{15} - 10^{16}$.

There exist several **MHD instabilities** that generate turbulence:



E. M. Gutiérrez talk later!

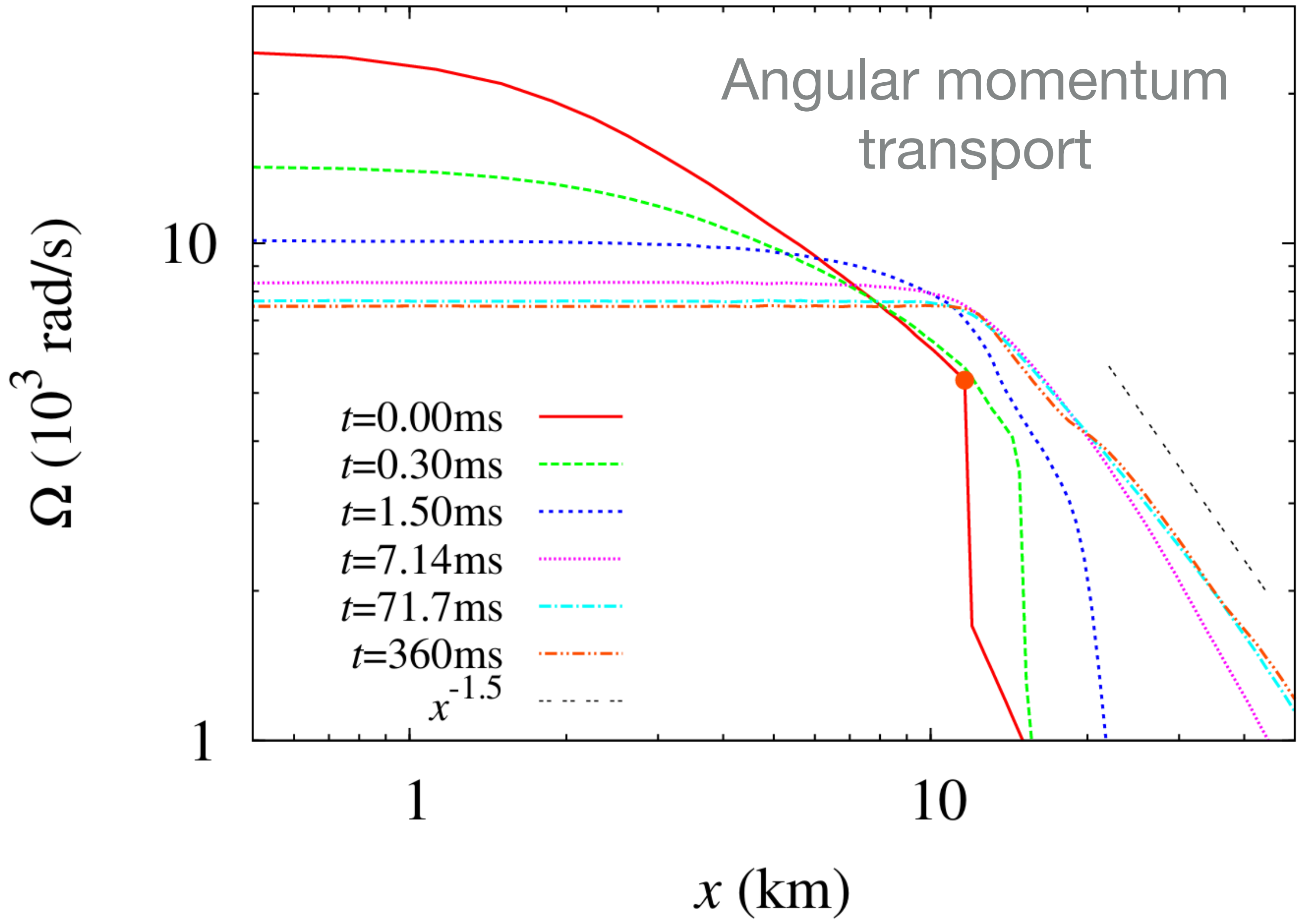
Kelvin-Helmholtz instability



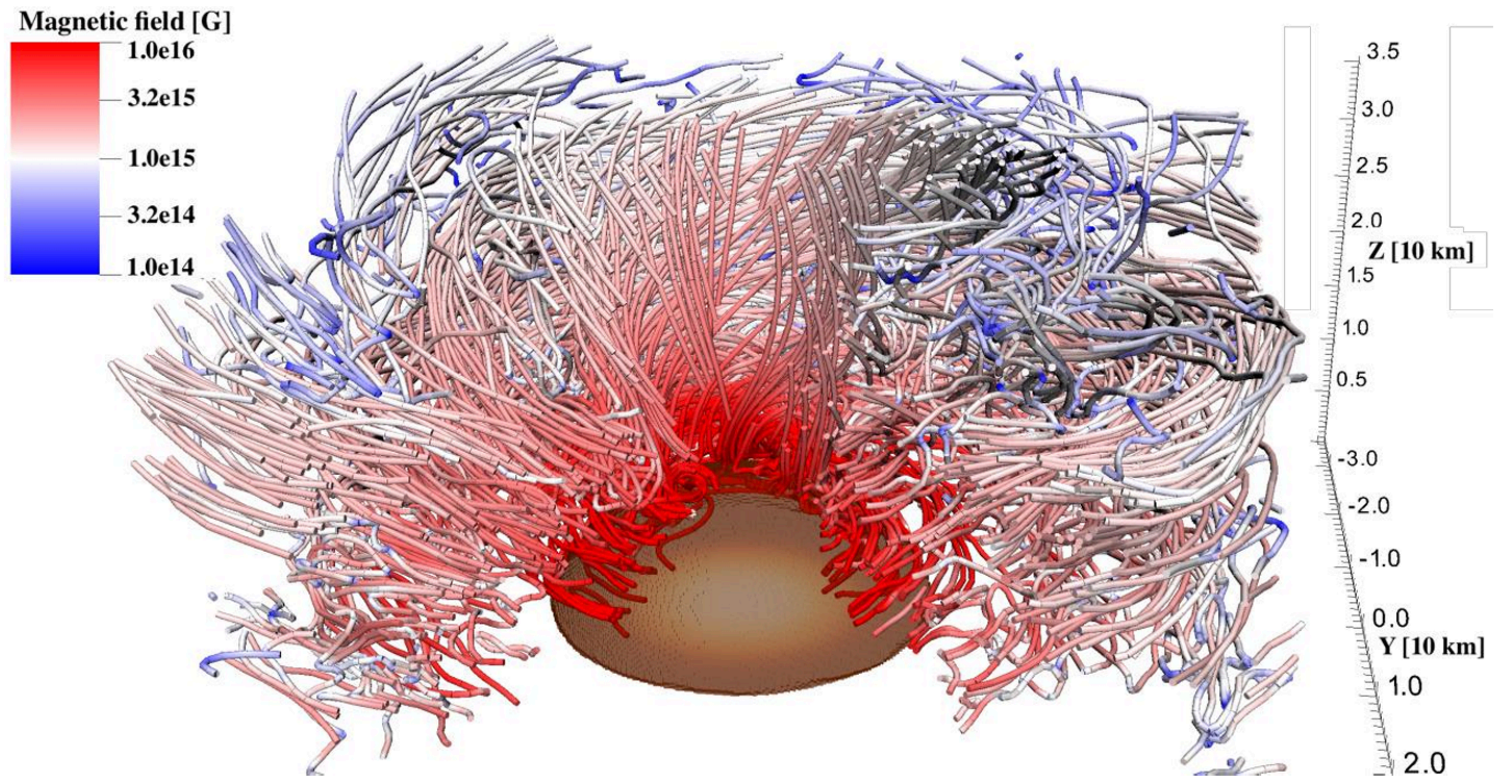
Credit: Kiuchi et al. (2018)

Magnetorotational instability

A. Reboul-Salze talk later!
K. Hayashi talk!
M. Müller's poster!



Credit: Shibata et al. (2017)

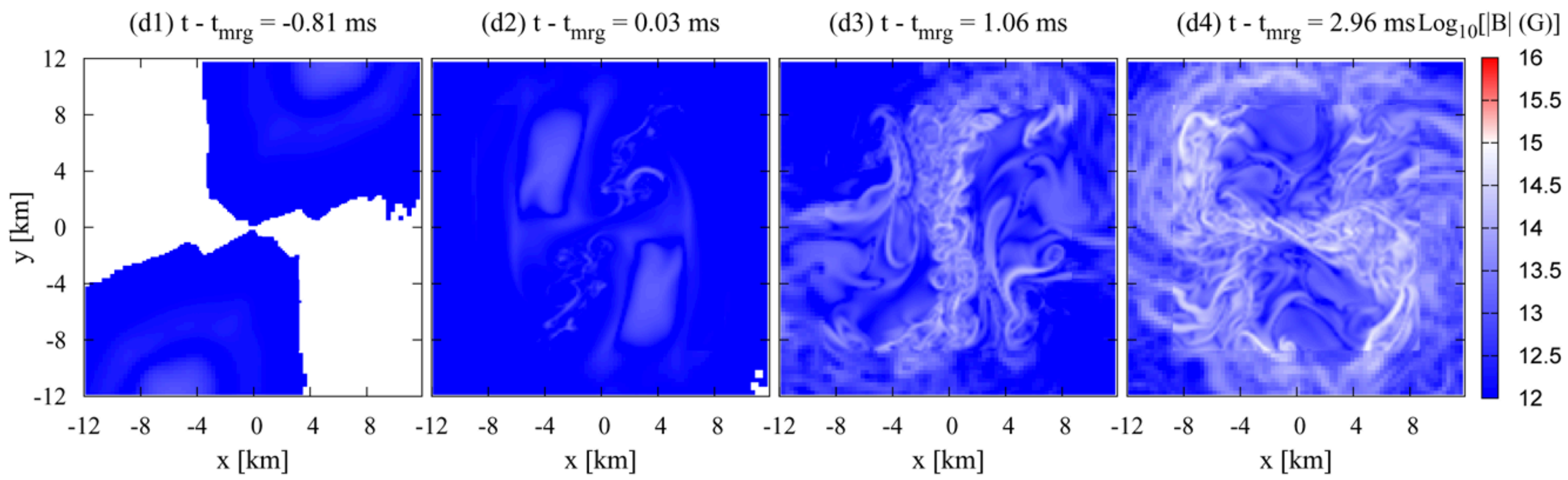


Large-scale magnetic field amplification

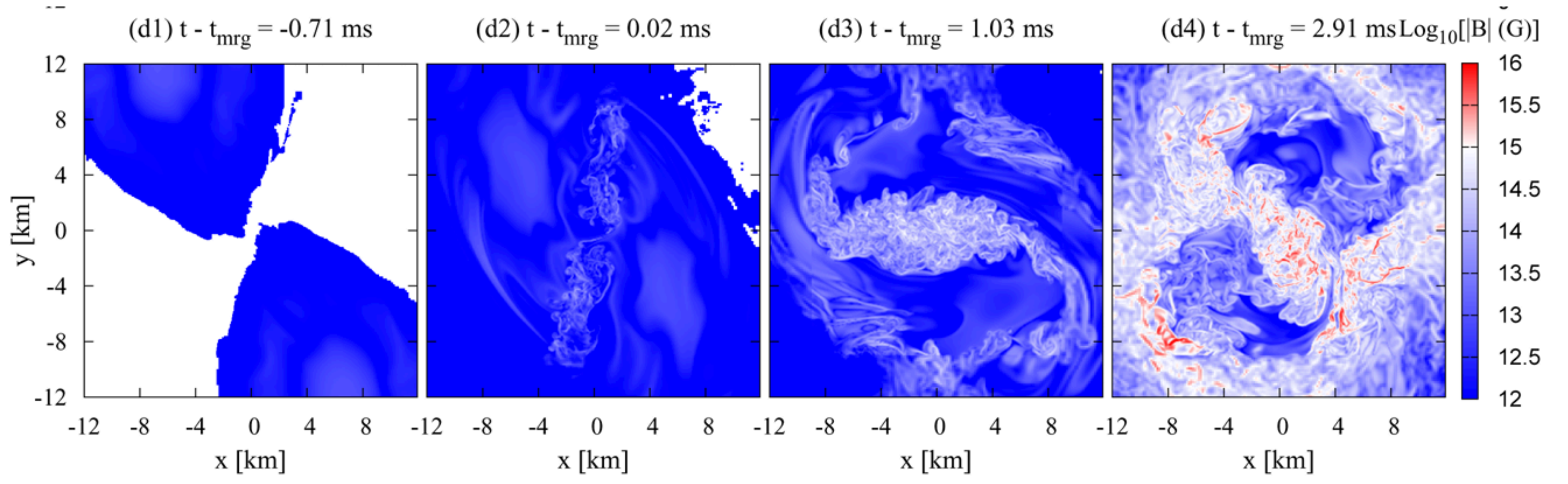
Credit: Kiuchi et al. (2024)

Introduction

Turbulence in BNS mergers: A matter of accuracy



$\Delta x(l_{max}) = 37.5\mathrm{m}$



$\Delta x(l_{max}) = 17.5\mathrm{m}$

Credit: Kiuchi et al. (2015)

Subset of possible solutions

- Improve the numerical methods: higher order schemes in finite difference, finite volume and/or finite element.
- Improve the resolution: need of exascale infrastructure, efficient adaptive mesh refinement and use of GPUs. ⟶ C. Musolino talk!
- Improve the solution modelling: large-eddy simulations.

Introduction

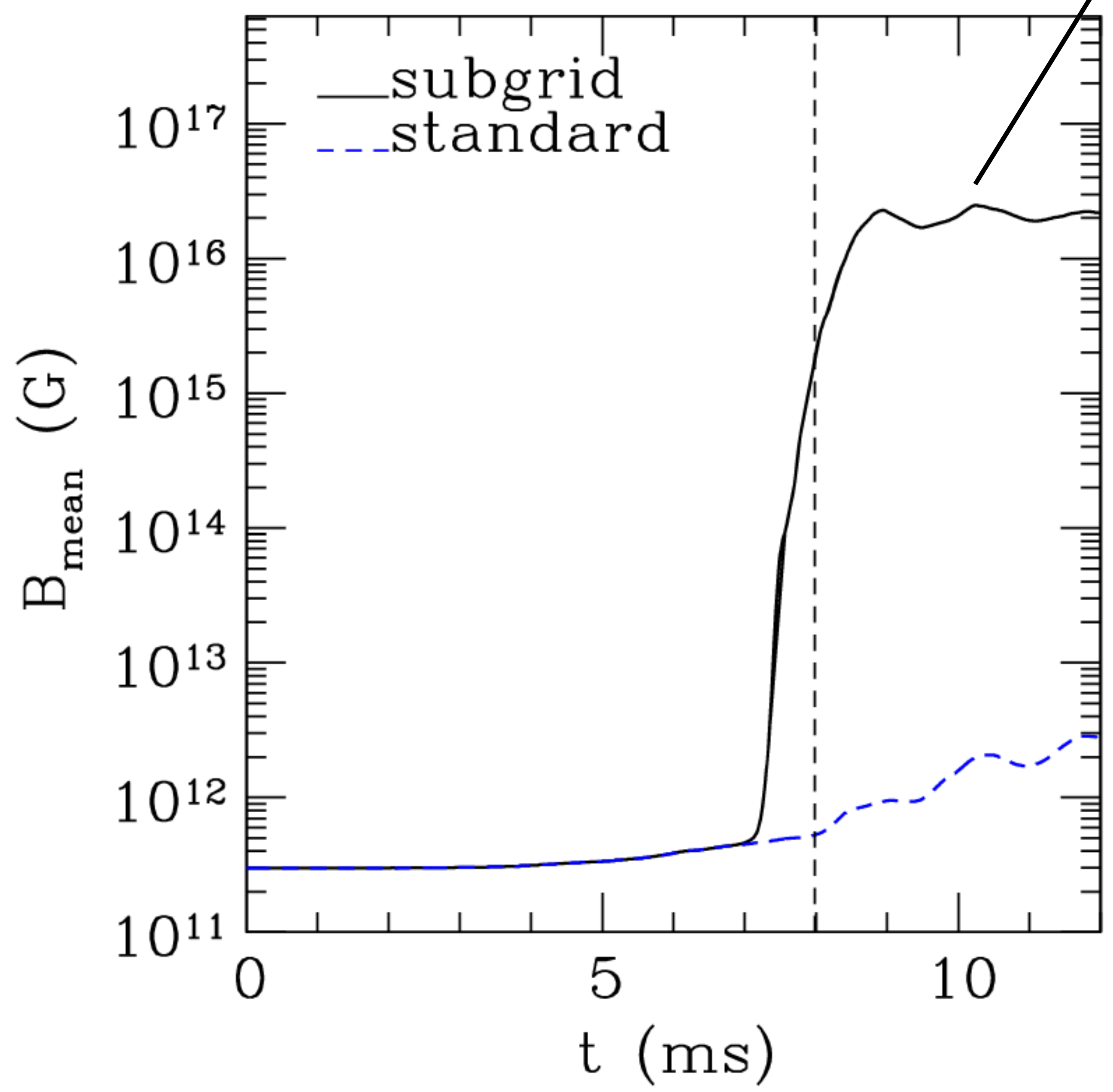
An alternative: large-eddy simulations (LES)

LES aim to describe the small-scale physics with the use of **subgrid models**.

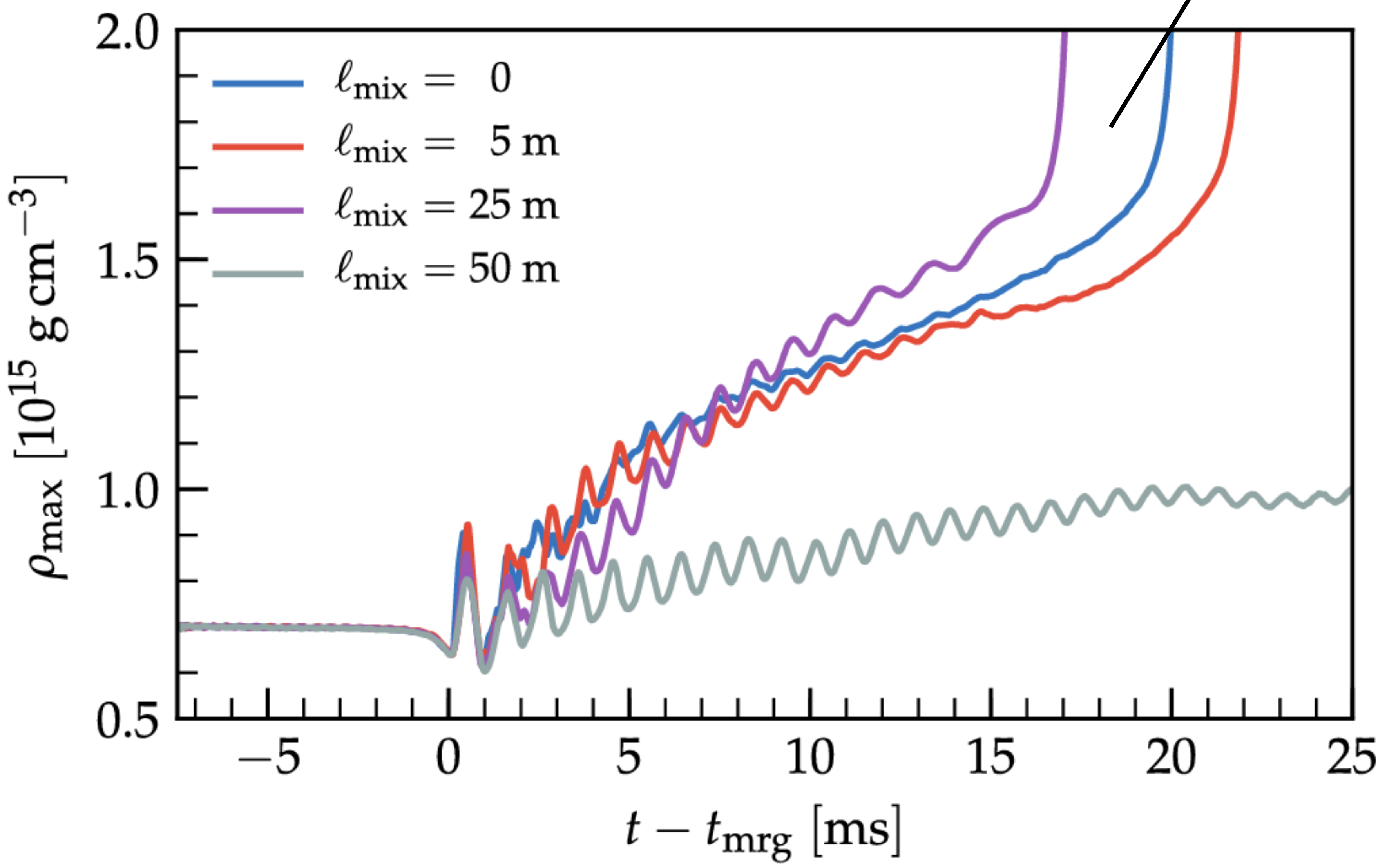
- Employ **modest** resolution.
- Used in **thermonuclear combustion in WDs** (Reinecke+ 2002), **star formation** in galaxy simulations (Joung+ 2009), **cosmological structure** formation (Maier+ 2009), etc.
- Already applied to **BNS simulations** (Giacomazzo+ 2015, Radice 2017, Palenzuela+ 2022, …).

Effective magnetic field amplification

Credit: Giacomazzo et al. (2015)

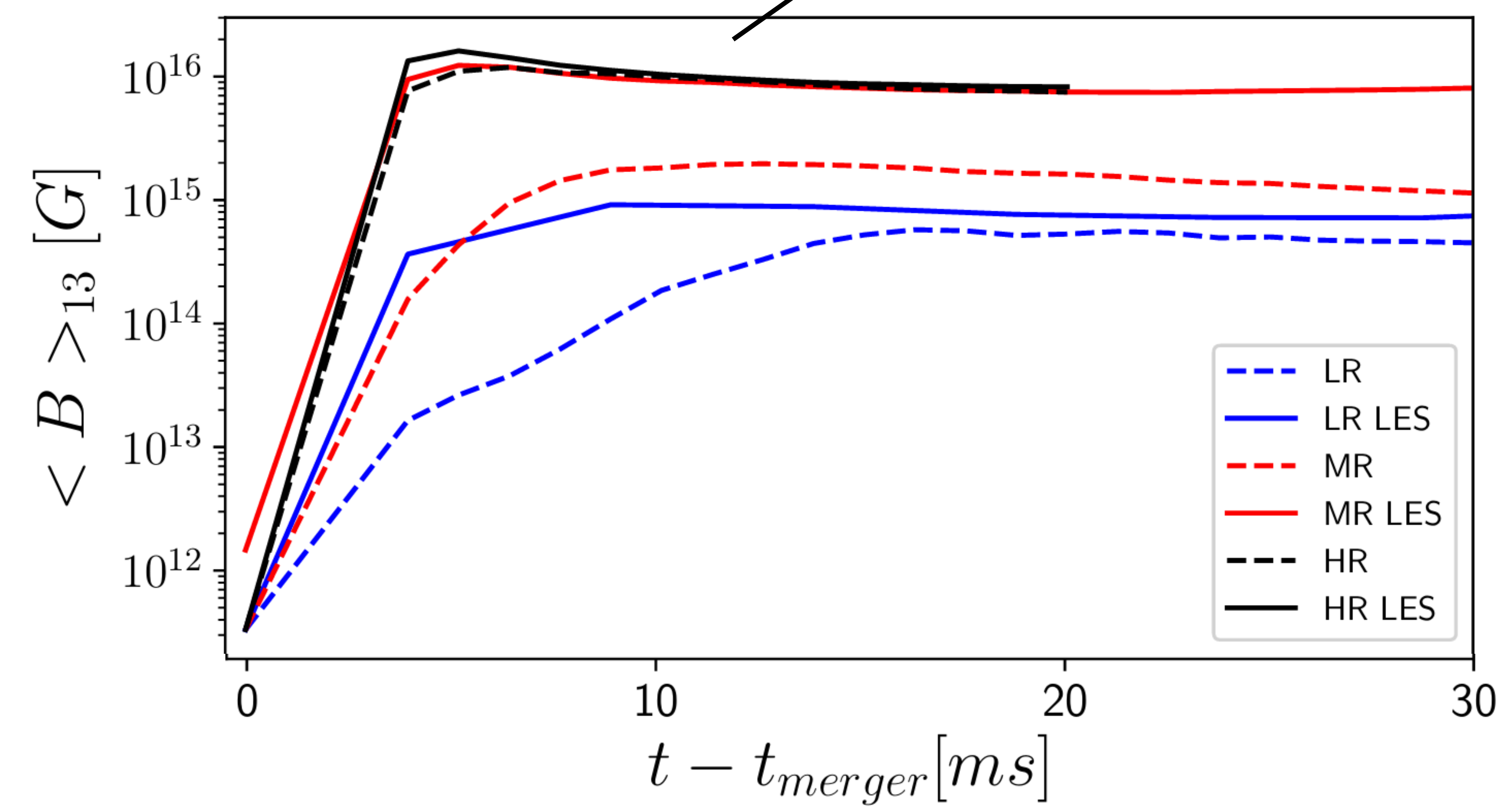


Different collapse time using effective turbulent viscosity



Credit: Radice (2017)

Convergence achieved with subgrid closure



Credit: Palenzuela et al. (2022)

Subgrid modelling of MHD turbulence

Foundations of mean-field MHD

Krause & Raedler (1980)

Let's separate between **resolved and unresolved scales** → $A = \bar{A} + A'$

- Slowly varying part → $\bar{A} = \int G(x) A(t, x) \mathrm{d}x$
- Rapidly fluctuating residual → A'

Products of turbulent components will arise **new terms in the nonlinear MHD equations**:

- Ideal MHD → (Neglecting gravity and nuclear reactions) $\partial_t F^0 + \partial_j F^j = 0$
- Mean-field MHD → $\partial_t \tilde{F}^0 + \partial_j \tilde{F}^j = S\left(\bar{M}_{ij}, \bar{R}_{ij}, \bar{F}_{ij}\right)$

In terms of resolved fields

Turbulent stress tensors

$$M_{ij} = B_i' B_j'$$
$$R_{ij} = v_i' v_j'$$
$$F_{ij} = v_i' B_j' - v_j' B_i'$$

We need to model these quantities

The MInIT subgrid model

A new sub-grid model for MHD turbulence. I. Magnetorotational instability.
MMT et al. 2022. MNRAS.

A new sub-grid model for MHD turbulence. II. Kelvin-Helmholtz instability.
MMT et al. 2024. MNRAS.

The MHD-Instability-Induced-Turbulence (MInIT) model

Based on the evolution of the **turbulent kinetic energy density.**

Partial differential hyperbolic **evolution equation**:

$$\partial_t e_{\text{turb}} + \partial_i(\bar{v}^i e_{\text{turb}}) = S^{\text{turb}}$$

The **source terms** of this equation will depend **on the instability** we are dealing with.

The **turbulent stress tensors** are **proportional** to this turbulent kinetic energy density.

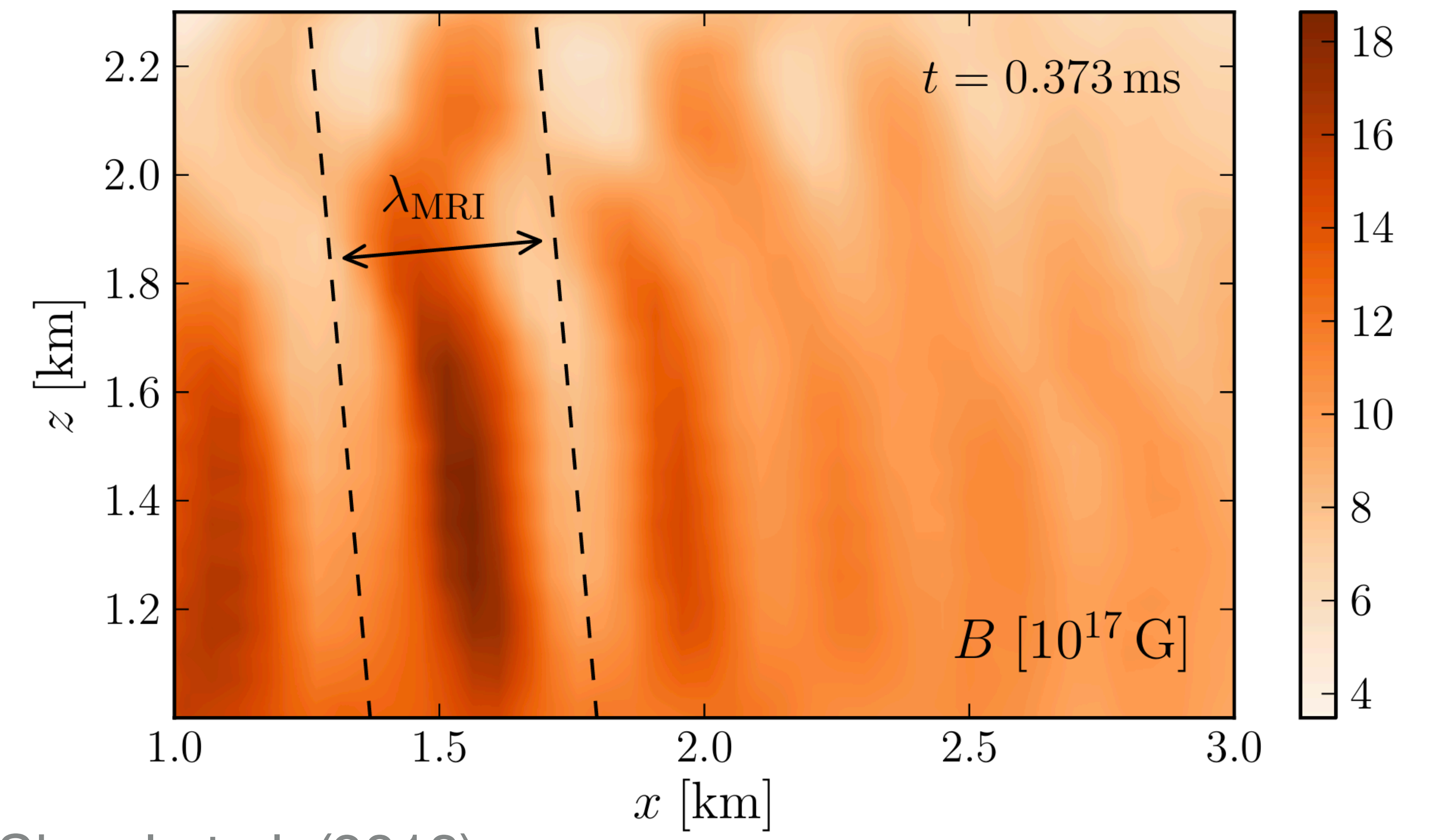
The MInIT model for the MRI

The magnetorotational instability (MRI)

Velikov (1958), Chandrasekhar (1960), Balbus & Hawley (1991), Goodman & Xu (1994)

Instability of **differentially rotating magnetised** fluids. It can be active in:

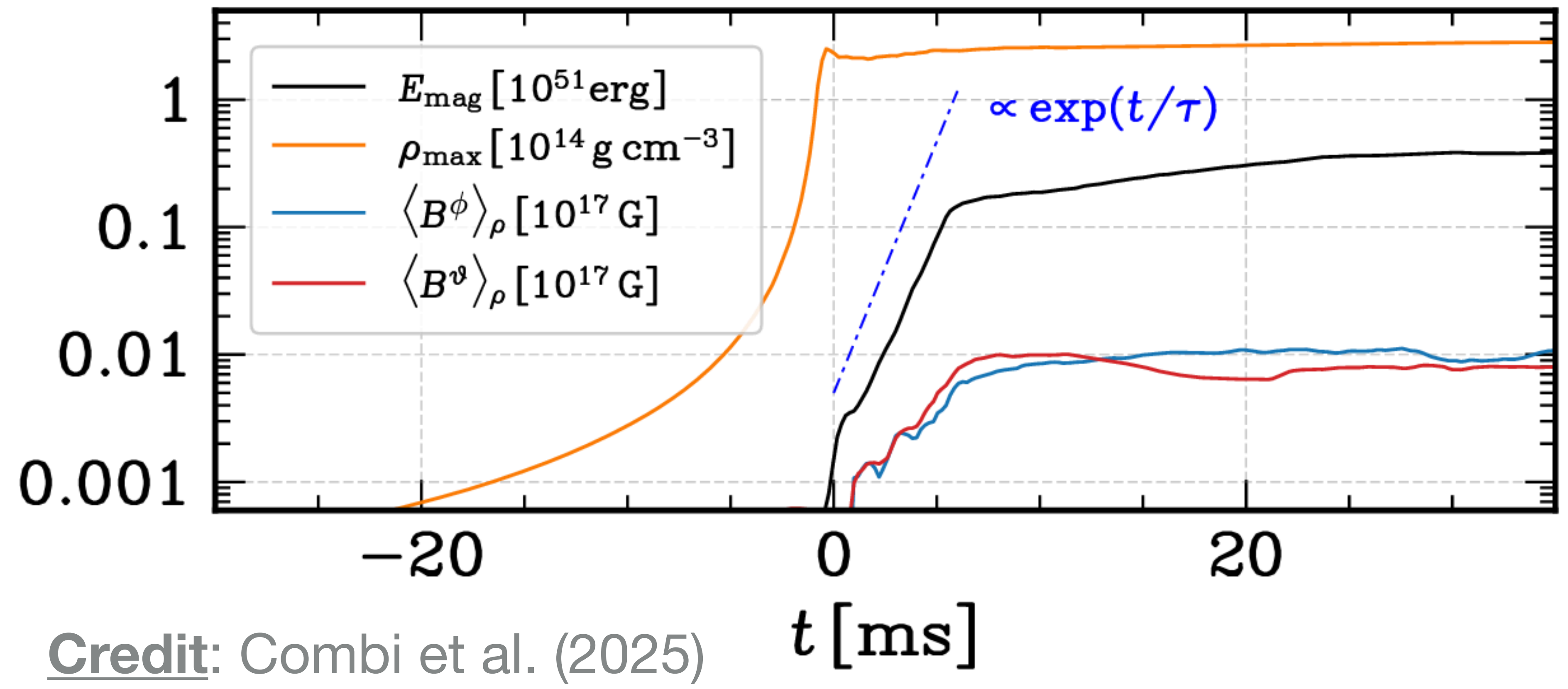
- HMNS remnants: Siegel+ (2013), Most+ (2023), Kiuchi+ (2018, 2024), Neuweiler+ (2025), Rainho+ (incl. MMT, 2025).
- Protoneutron stars: Rembiasz+ (2016a,b), Reboul-Salze+ (2021, 2022), Combi+ (2025). ⟶ in AIC of WDs ⟶ T. Kuroda talk!
- Accretion discs: Sano+ (2004), Sorathia+ (2012).



Credit: Siegel et al. (2013)

- Instability criterion ⟶ $\partial_r \Omega^2 < 0$
- Growth rate ⟶ $\gamma_{\mathrm{MRI}} \propto \Omega$
- Formation of axisymmetric modes of size:

$$\lambda_{\mathrm{MRI}} \approx 0.21\mathrm{km}\left(\frac{\Omega}{1000\mathrm{rad/s}}\right)^{-1}\left(\frac{B_z}{10^{14}\mathrm{G}}\right)\left(\frac{\rho}{10^{15}\mathrm{g/cm^3}}\right)^{-1/2}$$



Credit: Combi et al. (2025)

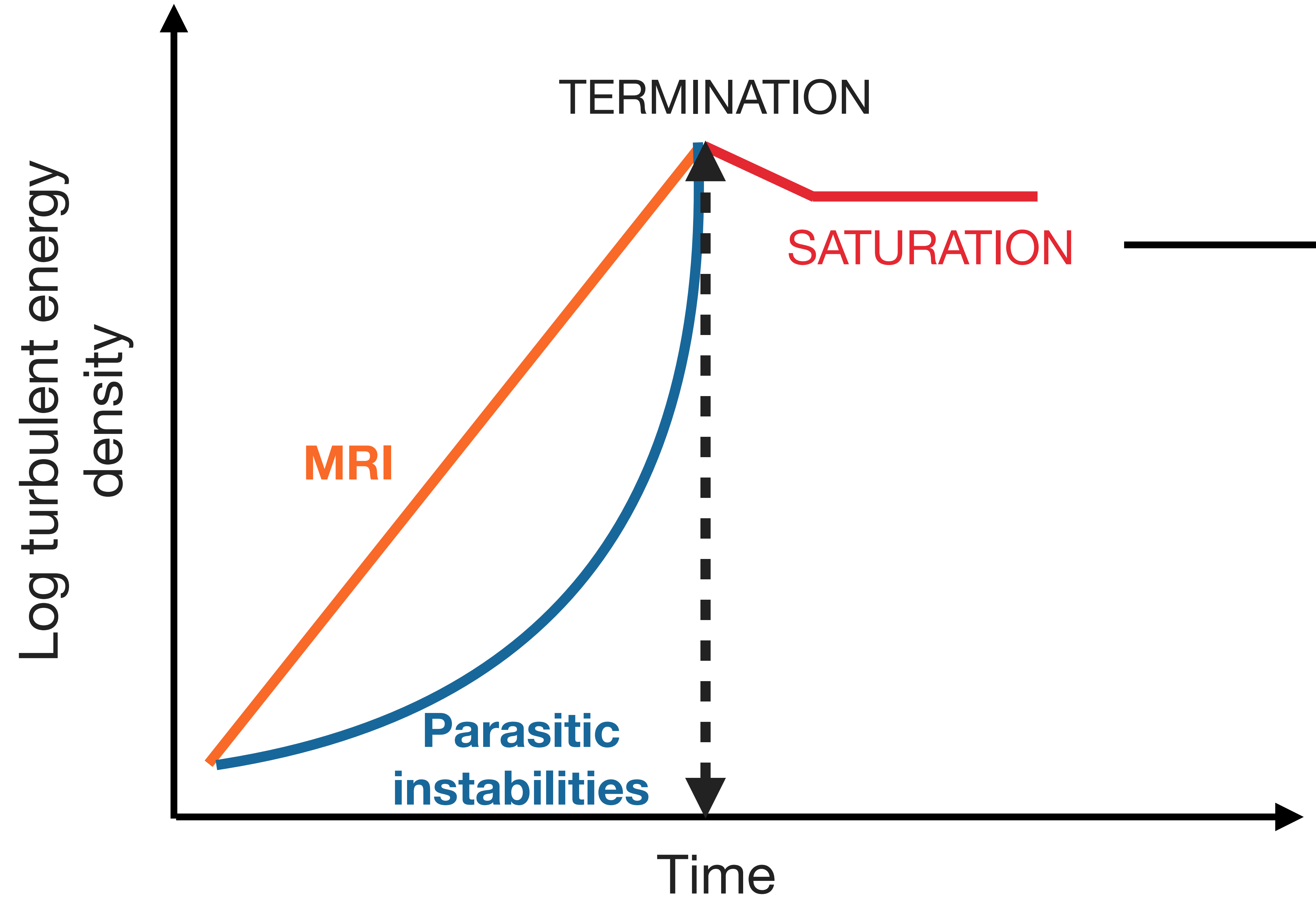
The MInIT model for the MRI

MRI saturation: parasitic instabilities

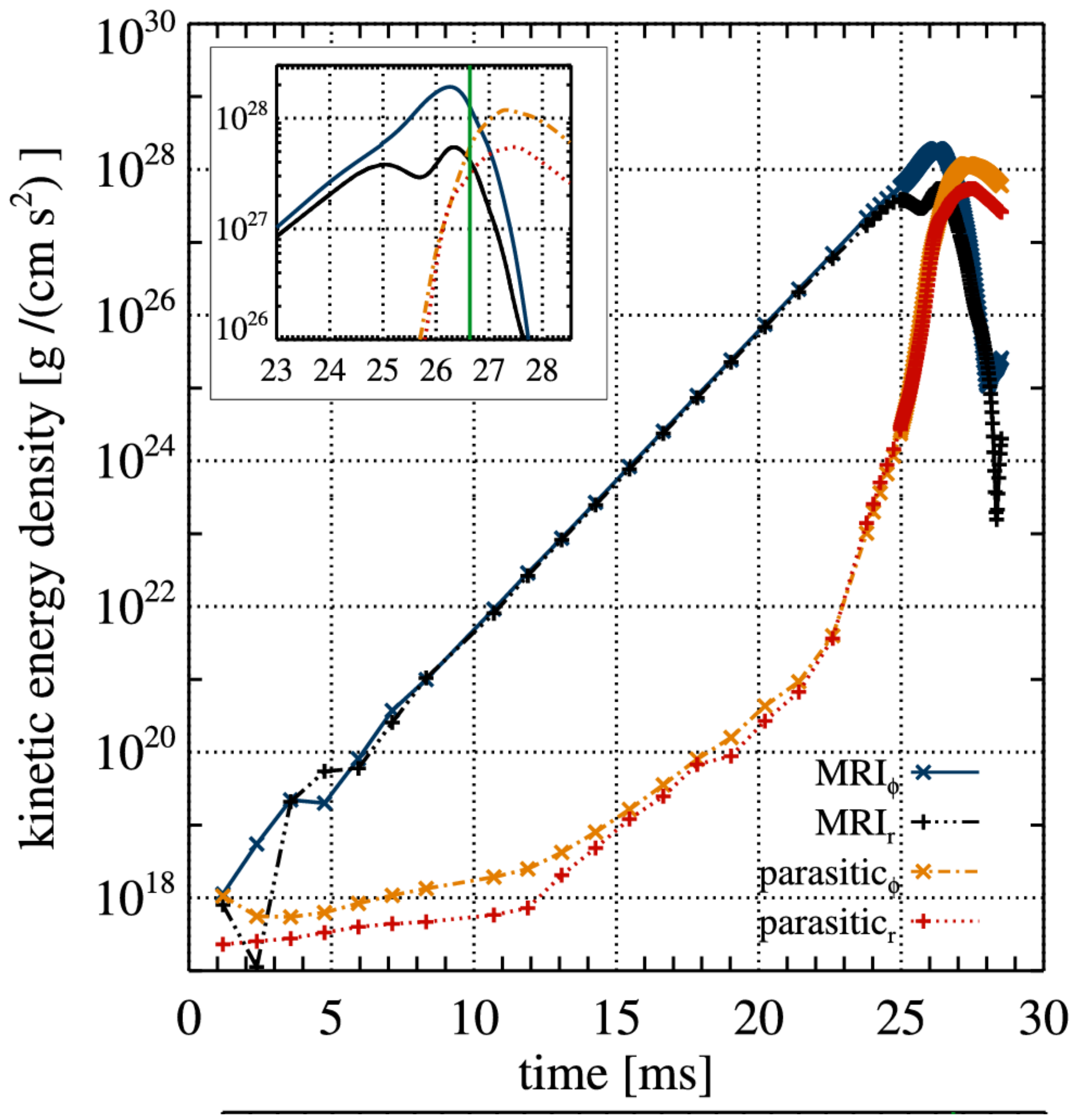
Goodman & Xu (1994), Latter et al. (2009,2010), Pessah (2010), **MMT** & Pessah (2025)

The nonlinear MRI modes are unstable to **parasitic instabilities** → Secondary instabilities that feed off the MRI

In **ideal MHD**: driven by velocity shear → **Kelvin-Helmholtz** type



→ **Sustained turbulence**



Credit: Rembiasz et al. (2016b).

The MInIT model for the MRI

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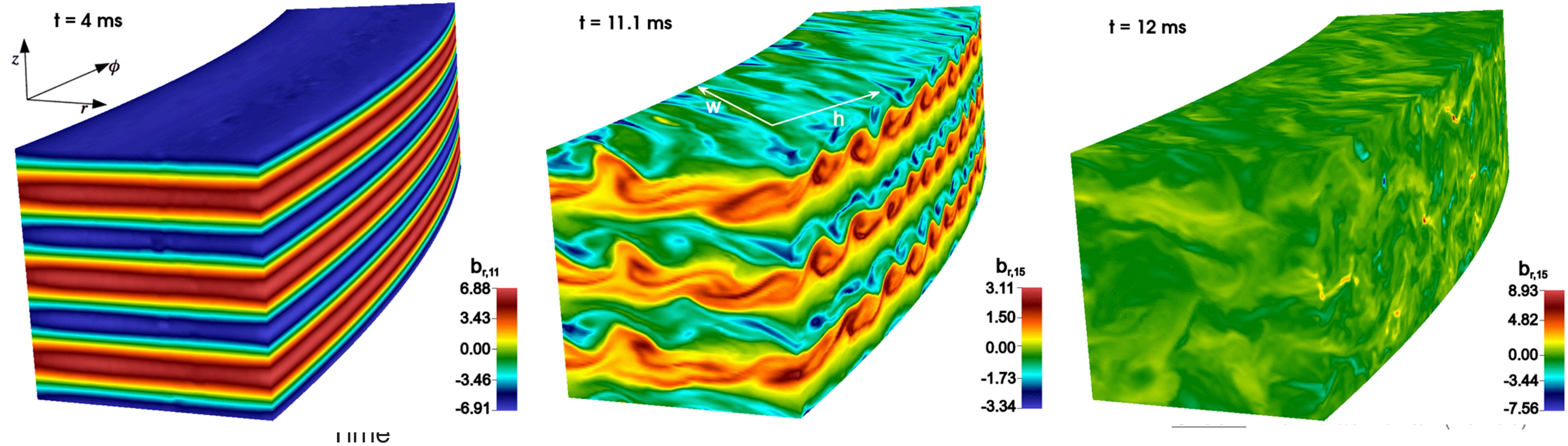

Credit: Rembiasz et al. (2016a).

The MInIT model for the MRI

A new sub-grid model for MHD turbulence. I. Magnetorotational instability. MMT et al. 2022. MNRAS.

Two evolution equations →

$$\partial_t e_{\rm MRI} + \partial_i(\bar{v}^i e_{\rm MRI}) = 2\gamma_{\rm MRI} e_{\rm MRI} - 2\gamma_{\rm PI} e_{\rm PI}$$
$$\partial_t e_{\rm PI} + \partial_i(\bar{v}^i e_{\rm PI}) = 2\gamma_{\rm PI} e_{\rm PI} - S_{\rm TD}$$

Turbulent kinetic energy densities

- Growth rates →

$$\gamma_{\rm MRI} = \frac{q}{2}\Omega \quad \Big| \quad \gamma_{\rm PI} = 0.27\sqrt{1 - \frac{(2-q)^2}{4}}\,\frac{\Omega}{\bar{B}_z}\sqrt{2e_{\rm MRI}}$$

Shear factor

$$q = -\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\ln\varpi}$$

- Turbulent dissipative term →

$$S_{\rm TD} = C\frac{e_{\rm PI}^{3/2}}{\sqrt{\rho}\lambda} \longrightarrow \lambda = \min[\lambda_{\rm MRI}, \Delta_f]$$

Kolmogorov-turbulence model
(Landau & Lifshitz 1987)

- Closure relation →

$$\bar{M}_{ij}(t,\boldsymbol{x}) = \alpha_{ij}^{\rm MRI} e_{\rm MRI}(t,\boldsymbol{x}) + \alpha_{ij}^{\rm PI} e_{\rm PI}(t,\boldsymbol{x})$$
$$\bar{R}_{ij}(t,\boldsymbol{x}) = \frac{1}{\rho(t,\boldsymbol{x})}\left(\beta_{ij}^{\rm MRI} e_{\rm MRI}(t,\boldsymbol{x}) + \beta_{ij}^{\rm PI} e_{\rm PI}(t,\boldsymbol{x})\right)$$
$$\bar{F}_{ij}(t,\boldsymbol{x}) = \frac{\gamma_{ij}^{\rm PI}}{\sqrt{\rho(t,\boldsymbol{x})}} e_{\rm PI}(t,\boldsymbol{x})$$

MRI coefficients: known from analytical studies

PI coefficients: calibrated with local box simulations

Application to global simulations

Subgrid modelling of MRI-driven turbulence in differentially rotating neutron stars. MMT et al. 2025. MNRAS.

Global Newtonian simulations

Label	$\rho_{\max}$ (10^{14} g cm^{-3})	$R_{\rm eq}$ (km)	Ω_c (s^{-1})	J (10^{48} g cm^2 s^{-1})	$\bar{b}_0$ (10^{13} G)
R1B3	7.90	18.53	2887.85	3.12	7.00
R2B3	7.80	18.58	4922.76	5.32	7.00
R3B3	7.61	18.70	7500.03	8.12	7.00
R3B5	7.61	18.70	7500.03	8.12	10.00
R3B4	7.61	18.70	7500.03	8.12	8.50
R3B2	7.61	18.70	7500.03	8.12	5.00
R3B1	7.61	18.70	7500.03	8.12	3.50

- **AENUS** code (Obergaulinger 2008).
- ***j*-constant** rotation profile. .
- Purely **poloidal** magnetic field confined inside the star.
- Axial **symmetry**.

- We only add **subgrid terms** to the **momentum equation**, allowing for angular momentum transport.

$r\phi$ component → Angular momentum stress

$$\partial_t \bar{p}^i + \nabla_j \left[\bar{\rho}\bar{v}^i\bar{v}^j + (\bar{P}_\star + \mathrm{Tr}\bar{\boldsymbol{M}})\delta^{ij} - \bar{b}^i\bar{b}^j + \boxed{\bar{\rho}\bar{R}^{ij} - \bar{M}^{ij}}\right] = \bar{f}^i$$

Application to global simulations

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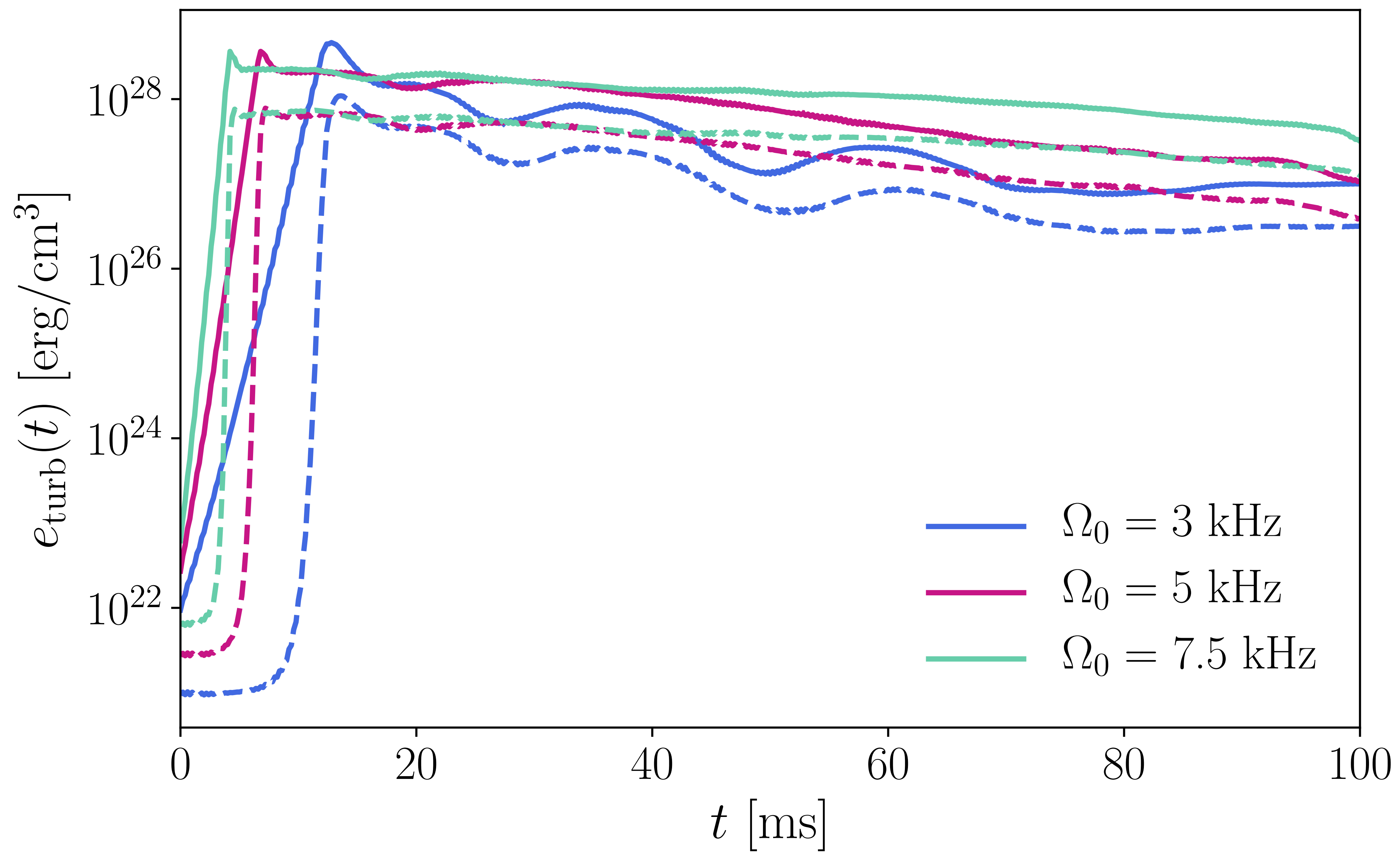
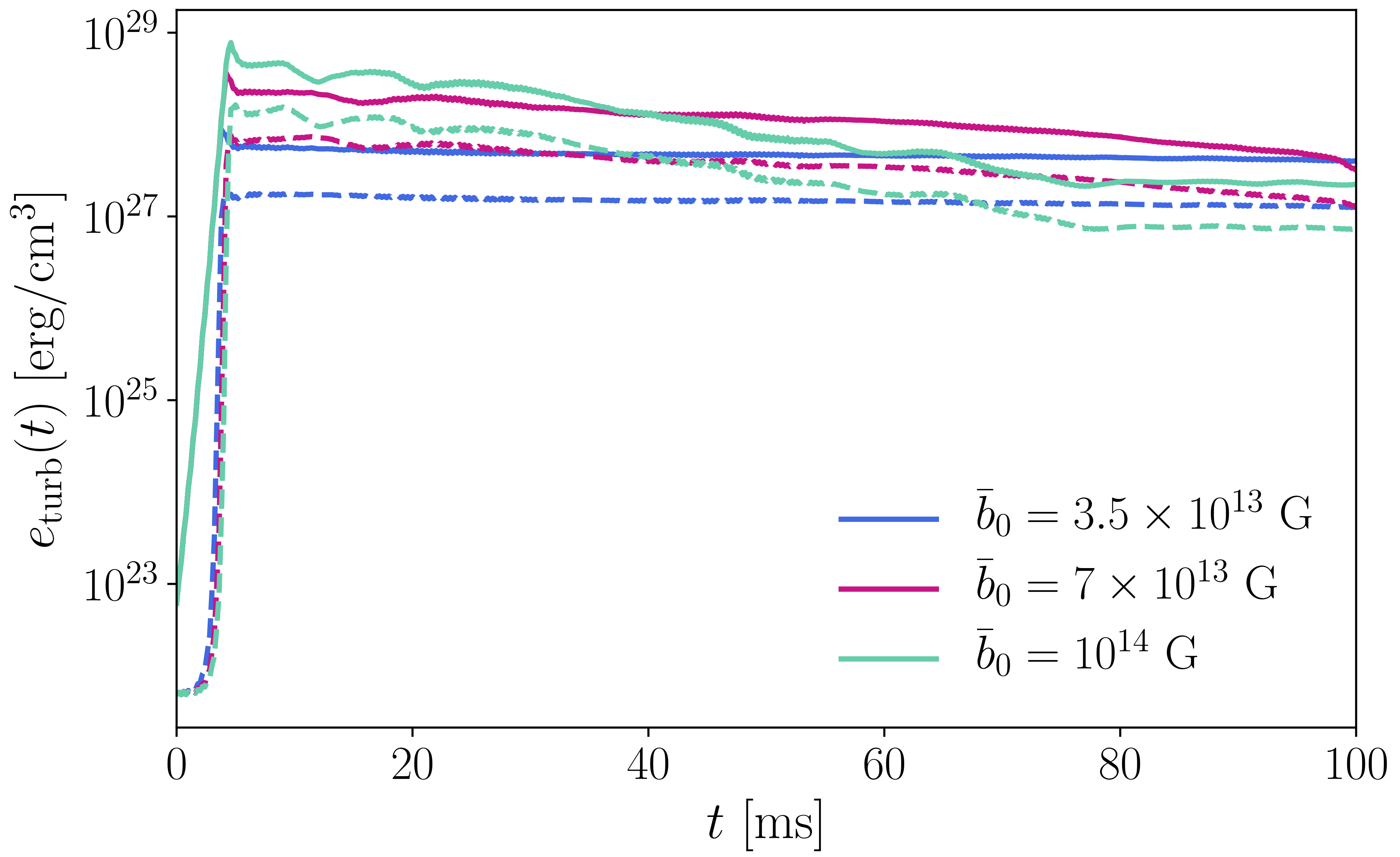
Turbulent energy densities



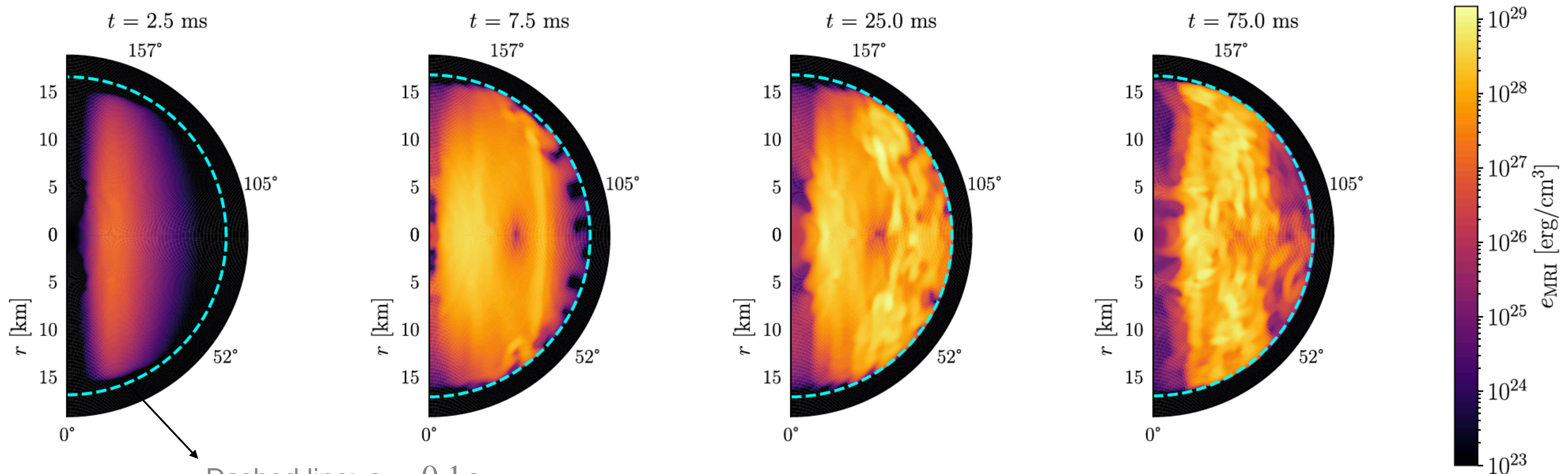


- For **different** rotation **frequencies**, the **saturation time** will be **shorter** as the rotation **frequency increases**. **Same** saturation **amplitude** regardless the value of Ω_0.
- For **different** initial magnetic **field strengths**, the **growth rate** stays the **same**. **Saturation amplitude larger** for **larger** magnetic field **strengths**.
- After $t \sim 20$ ms, **decay** due to angular momentum **transport** in inner stellar region.

Application to global simulations

Subgrid modelling of MRI-driven turbulence in differentially rotating neutron stars. MMT et al. 2025. MNRAS.

Turbulent energy densities



Dashed line: $\rho = 0.1\rho_{\rm max}$

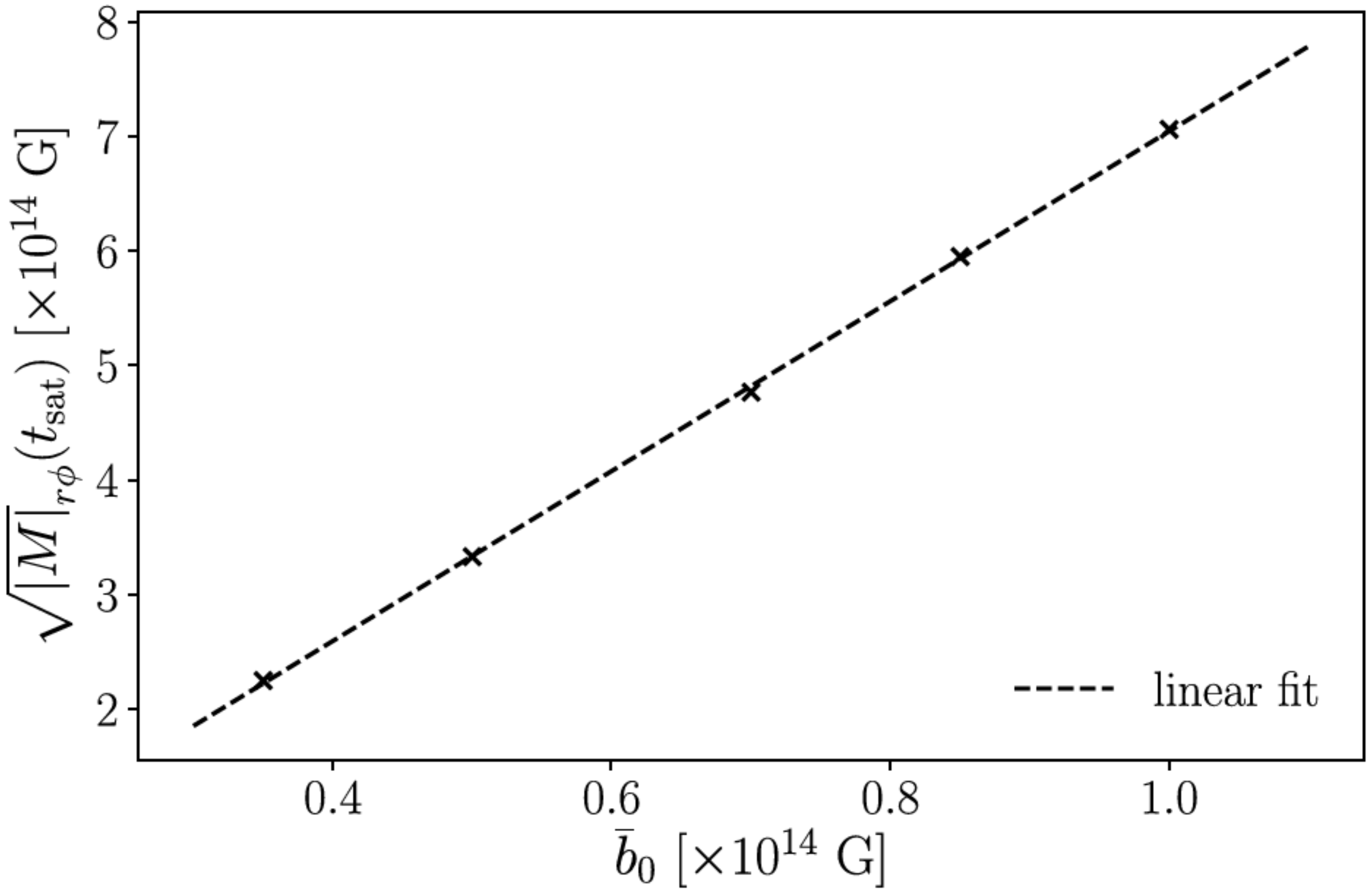
- **Exponential increase** by several orders of magnitude in **all the stellar interior**.
- After several tens of milliseconds, **decay** due to angular momentum **transport** in the inner region.

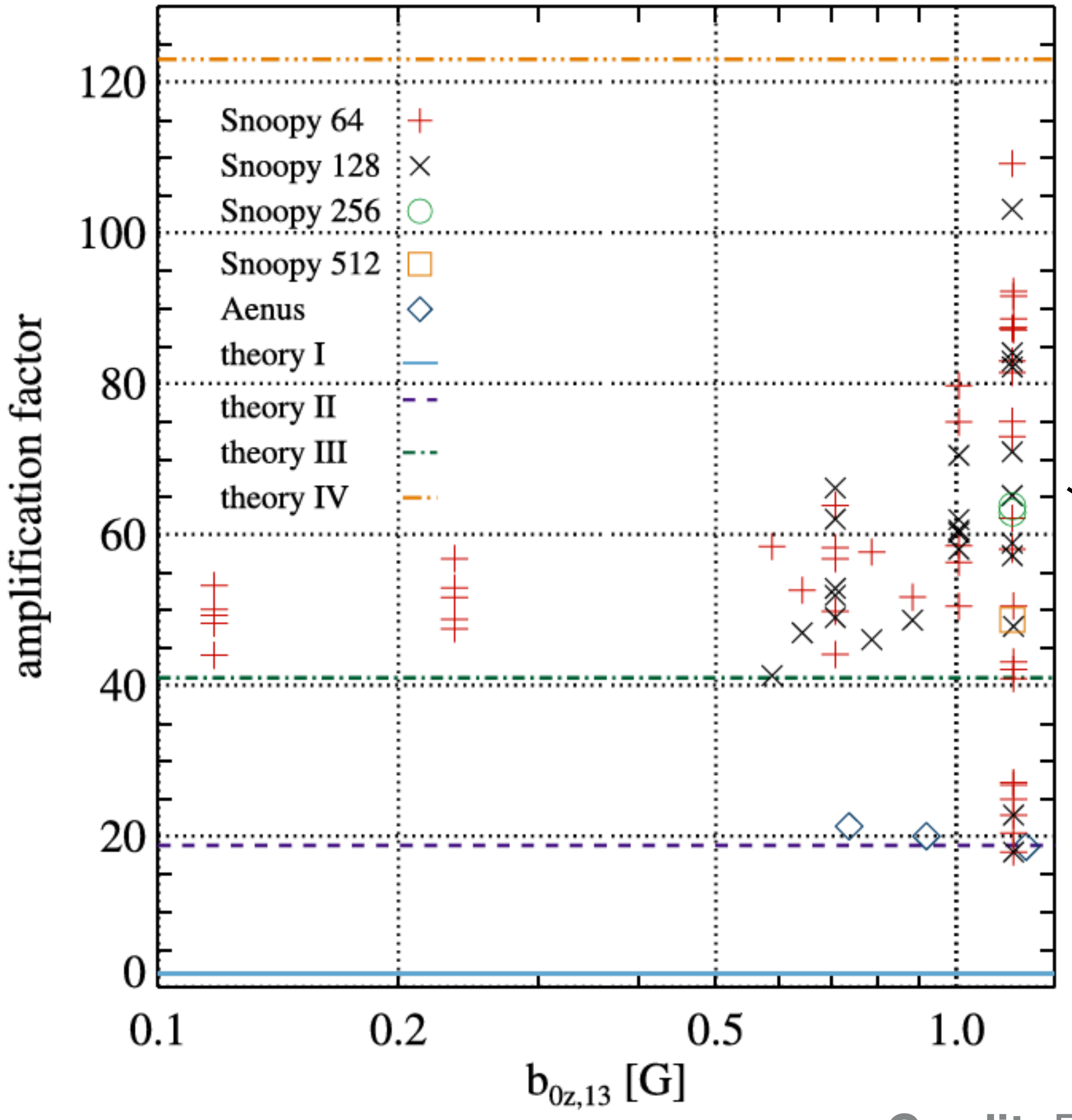
Application to global simulations

Subgrid modelling of MRI-driven turbulence in differentially rotating neutron stars. MMT et al. 2025. MNRAS.

Turbulent energy densities

- The **saturation amplitude grows linearly** with the initial poloidal **magnetic field amplitude**.
- This is in **agreement** with previous results, e.g. Rembiasz et al (2016b).





$$\mathscr{A} \equiv \frac{\sqrt{\bar{M}_{xy}^{\rm sat}}}{\bar{b}_0}$$

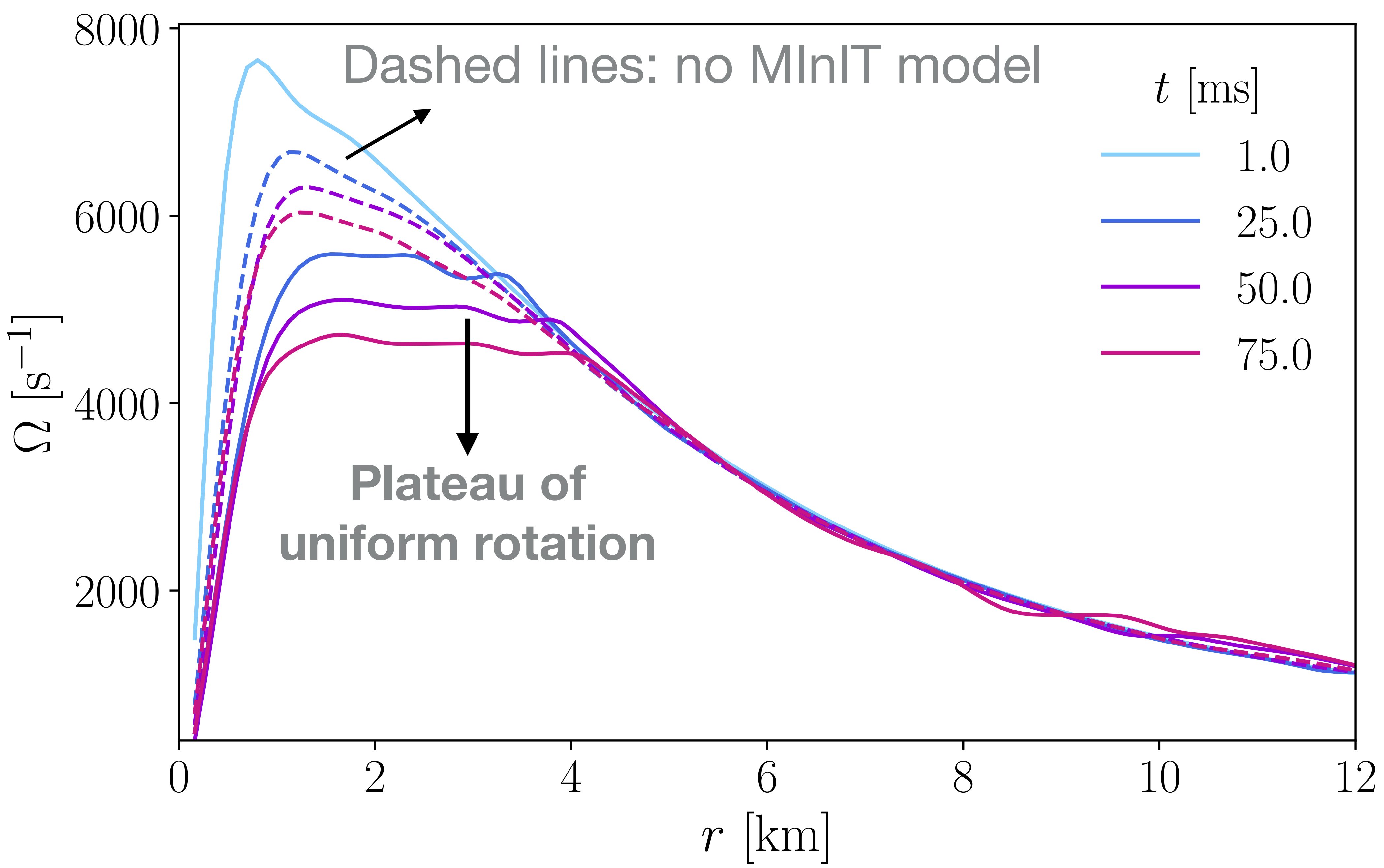
Large scattering due to different initial amplitudes for perturbations

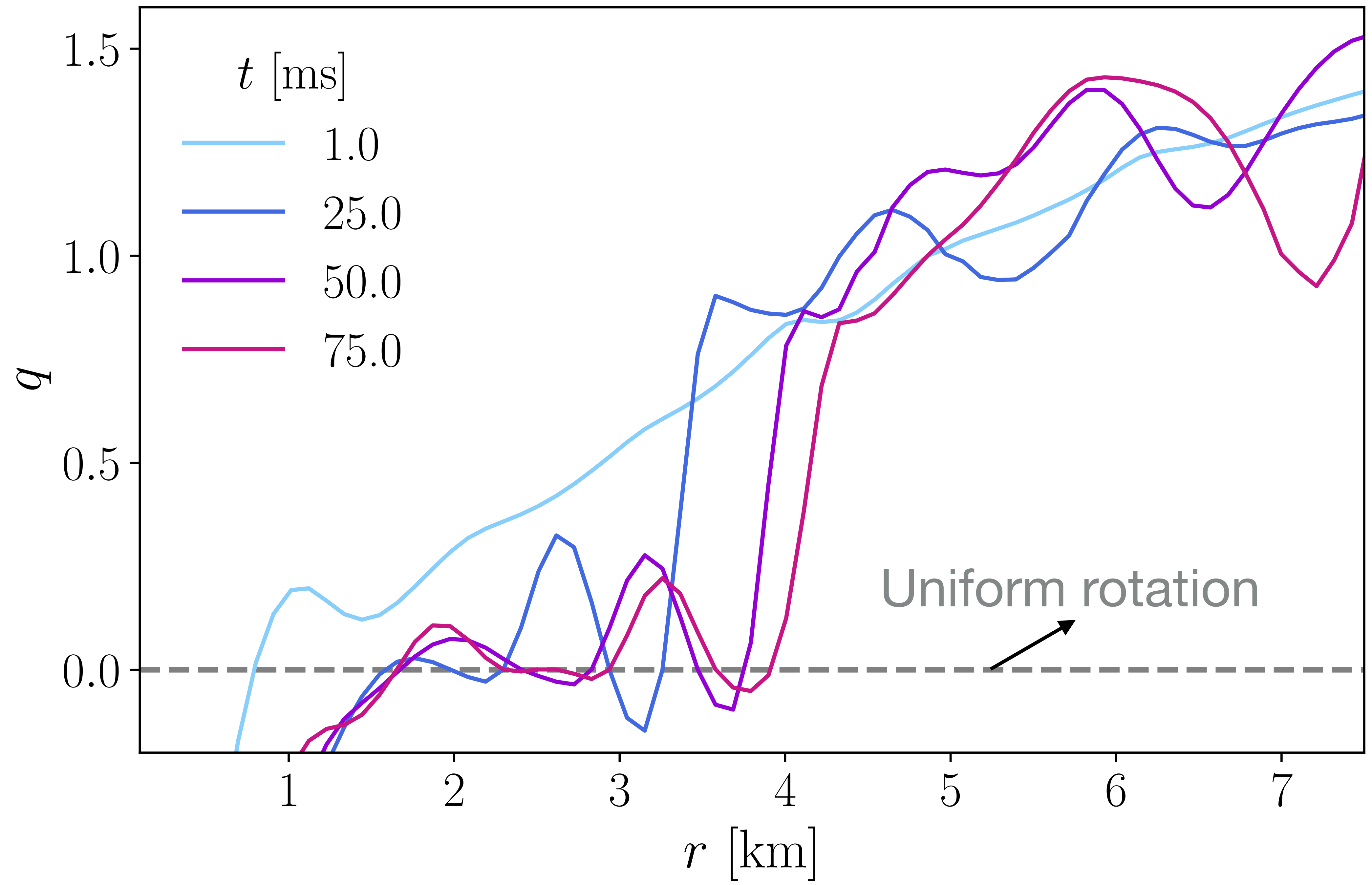
Credit: Rembiasz et al. (2016b)

Application to global simulations

Subgrid modelling of MRI-driven turbulence in differentially rotating neutron stars. MMT et al. 2025. MNRAS.

Radial profiles





- The MRI leads to angular momentum **transport** radially **outwards**. Differential rotation is lost in the inner regions.
- Shear parameter q goes to zero as angular momentum is transported → decay of the instability.

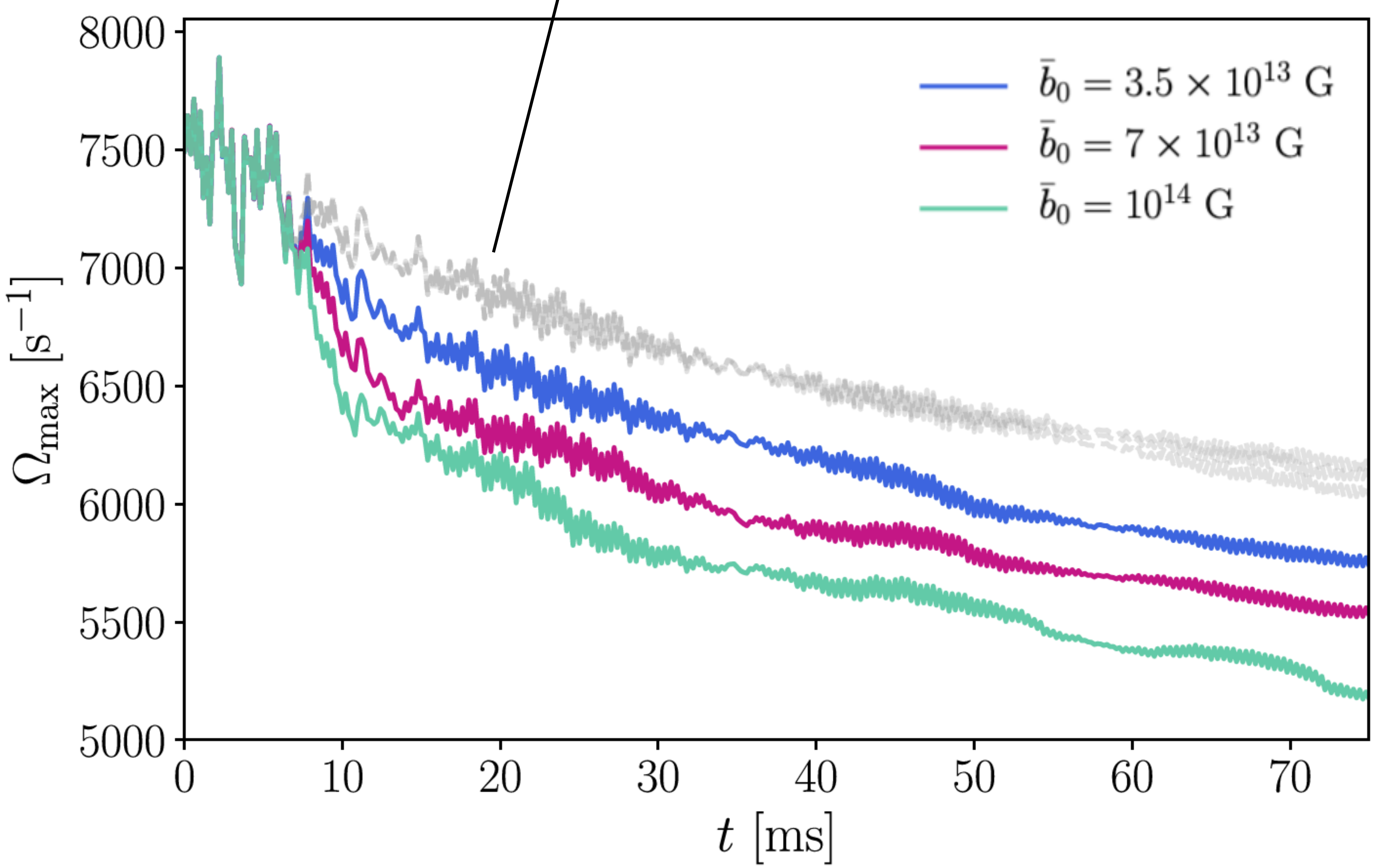
Application to global simulations

Subgrid modelling of MRI-driven turbulence in differentially rotating neutron stars. MMT et al. 2025. MNRAS.

Angular frequency evolution

- Simulations with a **larger** initial poloidal **field** result in a **more efficient** angular momentum **transport.**
- Central rotation frequency decreases faster.

Grey lines: no MInIT model



Conclusions and future work

- **Subgrid models** are an alternative to the use of prohibitively large resolution and expensive numerical methods in simulations of magnetohydrodynamical turbulence.
- **Large-eddy simulations** are currently being performed in the context of **neutron star mergers**.
- Some **promising results**: the MInIT model is able to reproduce the turbulent angular momentum transport due to the MRI.
- Other large-eddy simulations can reproduce the turbulent magnetic field amplification.

- Model extension to study **MRI dynamo** (large-scale magnetic field amplification).
- Develop a **covariant approach** to apply the model in full GR simulations.
- Application to **other scenarios**: core-collapse SNe, accretion discs around BHs, white-dwarf mergers.

THANK YOU

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