

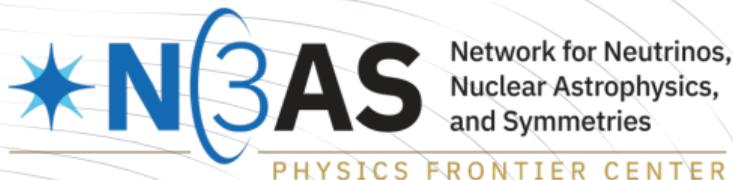
Coupled GR Neutrino-Hydrodynamics with Nuclear Burning in the Gmunu Code

YITP long-term workshop

4 Feb 2026

Patrick Chi-Kit CHEONG

N3AS postdoc fellow
patrick.cheong@berkeley.edu





Outline

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Why?

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- ▶ Nuclear burning gives/take energies
- ▶ change compositions
- ▶ resulting very different observational features
- ▶ What if the system is compact, relativistic, magnetised, ν /radiations, hot/dense enough?
- ▶ e.g. CCSNe, NS merger, ... etc

Intro

EoS

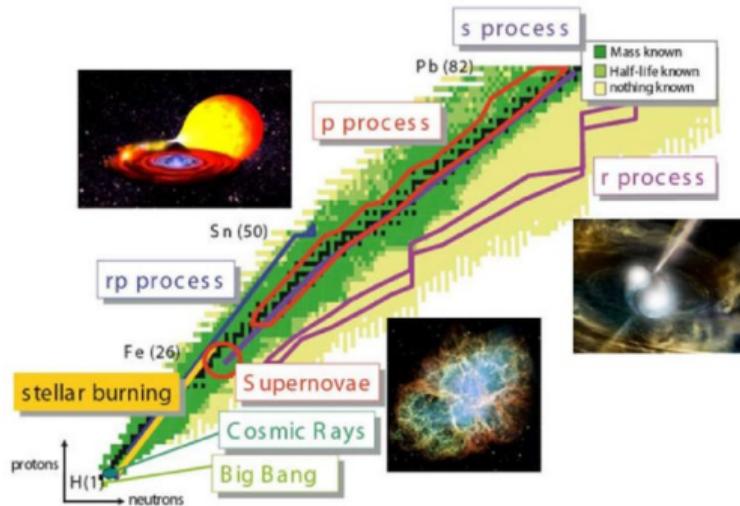
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Credit: Senger



How?

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Summary

- ▶ Do it separately
- ▶ GRMHD + post-processing nuclear simulations
- ▶ Temperature drops below 5 GK very quickly
- ▶ Burning feedback to ejecta, change shapes, velocity, compositions
- ▶ Burning codes usually Newtonian (but with very good networks!) (e.g. Navó+2023)
- ▶ GR + Nuclear burning (e.g. Montero+2012, Uchida+2017,2019, Fujibayashi+2025)



Goal: Try to do better

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Summary

- ▶ Dynamically couple GR ν MHD with a user-specified nuclear reactions
- ▶ Impossible to replace post-processing work
- ▶ Getting the energy feedback and compositions approximately OK, deliver better snapshots
- ▶ see arXiv: 2510.12978



Gmunu: A code for generic astrophysical simulations

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Gmunu (General-relativistic **mult**igrid **num**erical solver) [1, 2, 3, 4]

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Physics modules

- ▶ Consternated-evolution scheme for Einstein equation
 - ▶ Conformally flat condition (CFC)
- ▶ GRMHD
 - ▶ ideal/(resistive + dynamo)
 - ▶ constrained transport
- ▶ Radiative transfer
 - ▶ Two-moment scheme (grey/multi-group)
- ▶ Weakhub [5]: ν microphysics lib
- ▶ **Nuclear burning network**

Numerical features

- ▶ Block-based Adaptive Mesh Refinement (AMR) (provided by MPI-AMRVAC)
- ▶ Parallelised with MPI (provided by MPI-AMRVAC)
- ▶ Multi-dimensional (1-3D)
- ▶ Curvilinear geometries
 - ▶ Cartesian
 - ▶ Cylindrical
 - ▶ Spherical

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Nuclear EoS (finite temperature)

- ▶ above nuclear saturation density
- ▶ We are not so sure which is the best (e.g. LS220, SFHo, DD2, etc)
- ▶ Assume NSE, hence functions of ρ, T, Y_e
- ▶ low density (below saturation) part usually bridged with stellar EoS by assuming NSE

Stellar EoS

- ▶ ideal gas ions + radiation + electron, in LTE
- ▶ less uncertain and works well at low density/temperature
- ▶ Do not assume NSE, so functions of ρ, T, X_l
- ▶ usually the Helmholtz EoS (Timmes+2000), where electron part is tabulated



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However, the specific energy offset for ϵ is NOT arbitrary in GR Hydro, rest-mass should be excluded:

$$\epsilon = \epsilon_{\text{th}}(\rho, T, \{X_l\}) + \frac{1}{m_{\text{u}}} \sum_l \Delta m_l Y_l, \quad (1)$$

where ϵ_{th} is provided by `helmeos`, $Y_l = X_l/A_l$ is the molar abundances, and

$$\Delta m_l = m_l - A_l m_{\text{u}} \quad (2)$$

is the excessive mass of the l -th species with the atomic mass m_l .

Note:

- ▶ Since Δm_l is included, any composition changes will also change ϵ
- ▶ nuclear energy generations are intrinsically handled



If we encounter both regimes

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THE ASTROPHYSICAL JOURNAL, 951:112 (15pp), 2023 July 10

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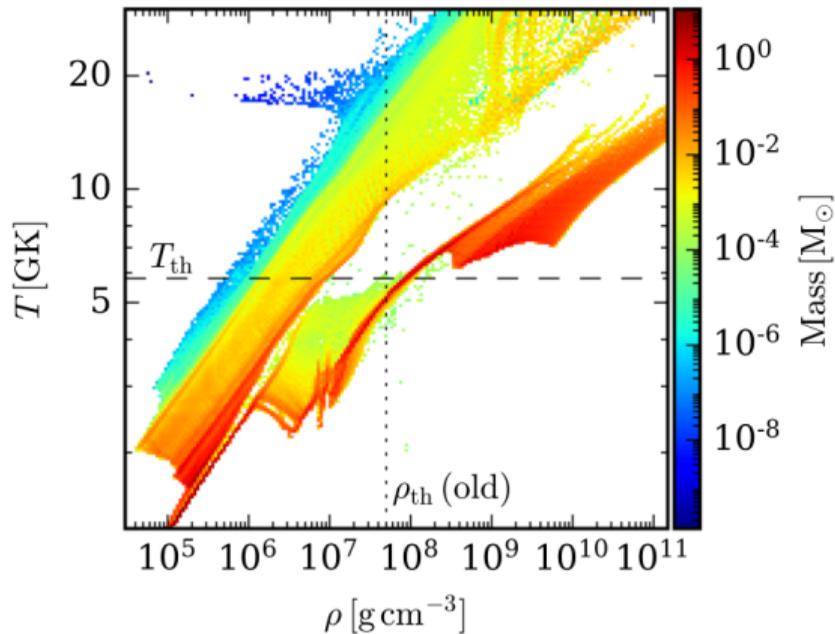
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EoS bridging via T

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$$\left\{ \begin{array}{ll} \text{Nuclear EoS} & \text{if } T > 5.8 \text{ GK, Enforce NSE (don't care } X_l \text{ and stick to } Y_e), \\ \text{interpolation} & \text{if } 5 \text{ GK} < T < 5.8 \text{ GK, solve NSE when } Y_e \text{ and } Y_e(X_l) \text{ do not match,} \\ \text{Stellar EoS} & \text{if } T < 5 \text{ GK, use } X_l \text{ and overwrite } Y_e \end{array} \right. \quad (3)$$

Note:

- ▶ X_l and Y_e are both properly initialised, and evolved everywhere
- ▶ $T > 5 \text{ GK}$ assume NSE
- ▶ The EoSs start to be very similar at $\rho \lesssim 10^7 \text{ g} \cdot \text{cm}^{-3}$.
- ▶ ϵ in the nuclear EoS is usually insensitive to temperature when $T \lesssim 1 \text{ MeV} \approx 11.6 \text{ GK}$.
- ▶ solutions of $\epsilon \rightarrow T$ could exist in both EoSs, you will need to pick a side
- ▶ This is problem dependent



Results

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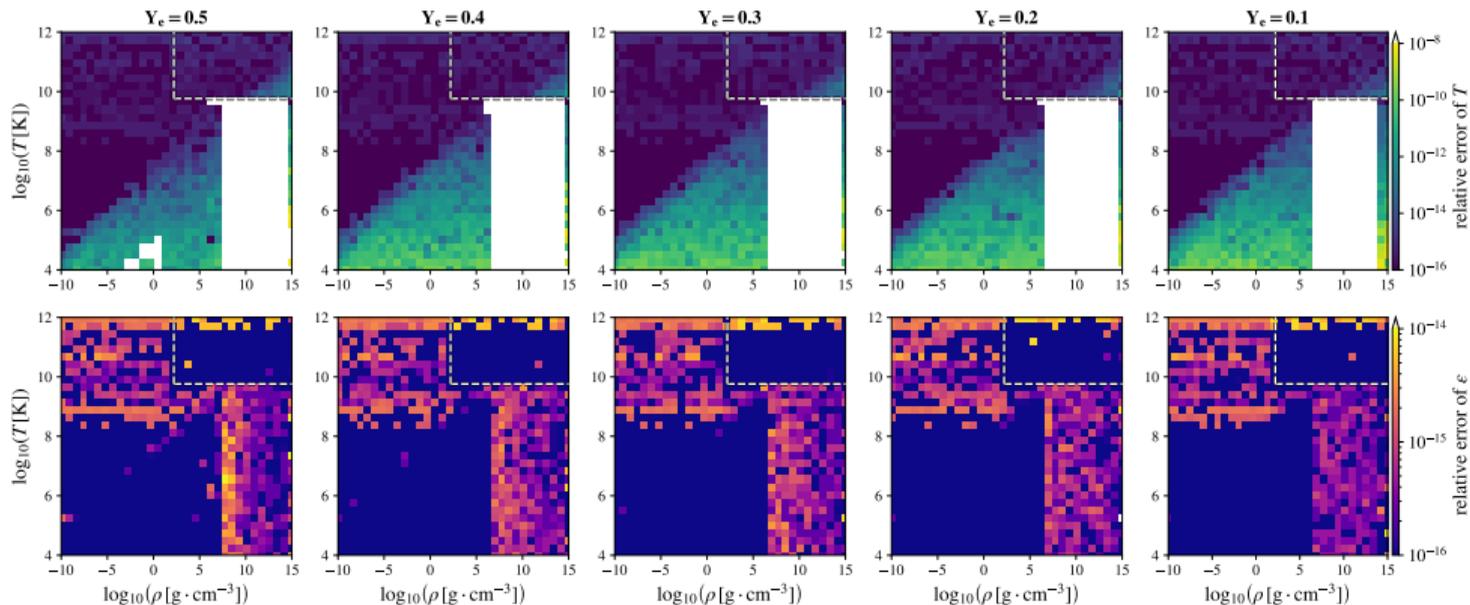
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Nuclear Burning

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$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \rho X_l u^\mu) = A_l m_u \dot{n}_{\text{nuc}} \quad (4)$$

- ▶ the stiff source term is related to \dot{X}_l , given by network.
- ▶ the burning timescale could be compatible or even shorter than hydro, implicit-explicit (IMEX) Runge-Kutta is used
- ▶ (IMEX is also used for neutrino transport)
- ▶ Our solver is Newton-Raphson-based



Burning limiter

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Summary

- ▶ Shocks are typically under resolved
- ▶ Better to apply the burning to the materials that are completely shocked
- ▶ avoid unphysical energy generation, and make the results more ‘converged’ across different codes

$$\frac{|\nabla P \cdot \vec{v}|}{P |v|} > f_{\text{shock}}, \quad \nabla \cdot \vec{v} < 0, \quad (5)$$

$f_{\text{shock}} = 2/3$. slope-limited derivatives are ideal.



Burning limiter

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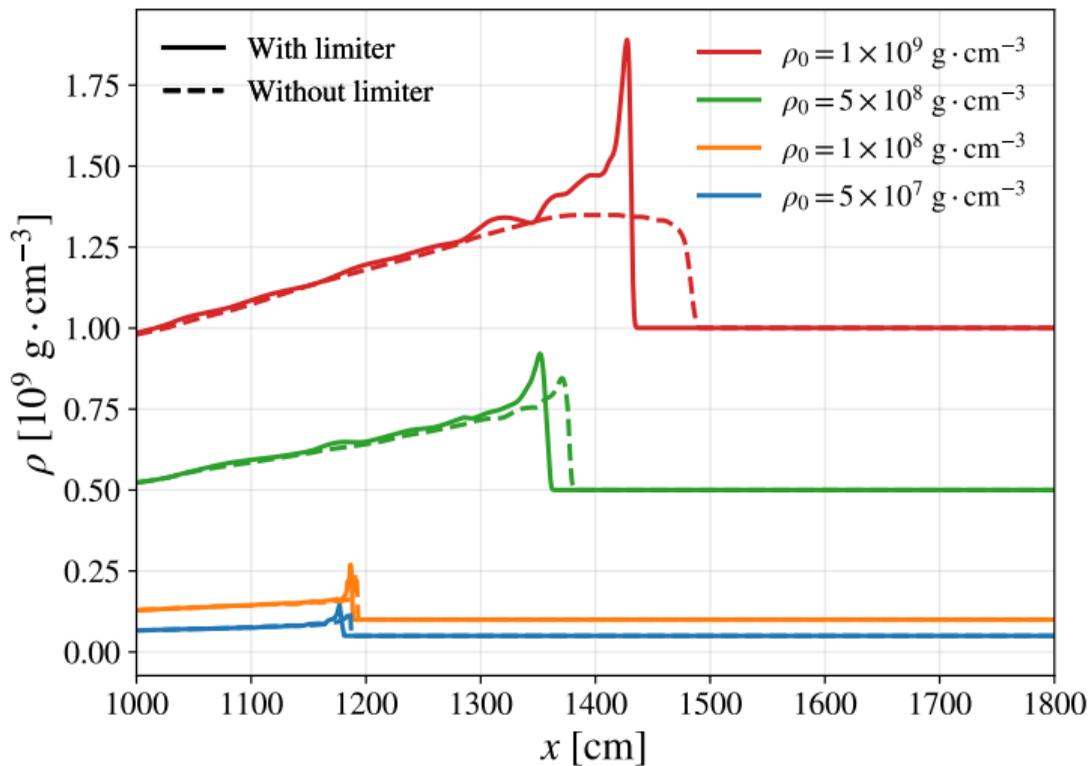
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NSE solver

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- ▶ Useful for EoS bridging and initialisation
- ▶ various ways to do it, typically a 2D root finding problem
- ▶ maybe no need to be super accurate in dynamical simulations

$$\sum_l X_l \left(\mu_p^{\text{kin}}, \mu_n^{\text{kin}} \right) = 1,$$
$$\sum_l \frac{Z_l}{A_l} X_l \left(\mu_p^{\text{kin}}, \mu_n^{\text{kin}} \right) = Y_e, \tag{6}$$

solve μ_p^{kin} and μ_n^{kin} .

$$X_l \left(\mu_p^{\text{kin}}, \mu_n^{\text{kin}} \right) = \frac{m_l}{\rho} g_l \left(\frac{2\pi m_l k_B T}{h^2} \right)^{3/2}$$
$$\times \exp \left(\frac{Z_l \mu_p^{\text{kin}} + N_l \mu_n^{\text{kin}} + Q_l - \mu_l^{\text{C}} + Z_l \mu_p^{\text{C}}}{k_B T} \right), \tag{7}$$

g_l is the nuclear partition functions of nucleus l , which is in general temperature dependent. We simply use 2.



Nuclear Network

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For the moment: Timmes+ <https://cococubed.com/>, considering pynucastro

- ▶ 7-isotope α -chain ^4He , ^{12}C , ^{16}O , ^{20}Ne , ^{24}Mg , ^{28}Si , and ^{56}Ni .
- ▶ 13-isotope α -chain ^4He , ^{12}C , ^{16}O , ^{20}Ne , ^{24}Mg , ^{28}Si , ^{32}S , ^{36}Ar , ^{40}Ca , ^{44}Ti , ^{48}Cr , ^{52}Fe , and ^{56}Ni .
- ▶ 19-isotope n, p, ^1H , ^3He , ^4He , ^{12}C , ^{14}N , ^{16}O , ^{20}Ne , ^{24}Mg , ^{28}Si , ^{32}S , ^{36}Ar , ^{40}Ca , ^{44}Ti , ^{48}Cr , ^{52}Fe , ^{54}Fe , ^{56}Ni .
- ▶ 21-isotope n, p, ^1H , ^3He , ^4He , ^{12}C , ^{14}N , ^{16}O , ^{20}Ne , ^{24}Mg , ^{28}Si , ^{32}S , ^{36}Ar , ^{40}Ca , ^{44}Ti , ^{48}Cr , ^{56}Cr , ^{52}Fe , ^{54}Fe , ^{56}Fe , ^{56}Ni .
- ▶ pynucastro



Burning test

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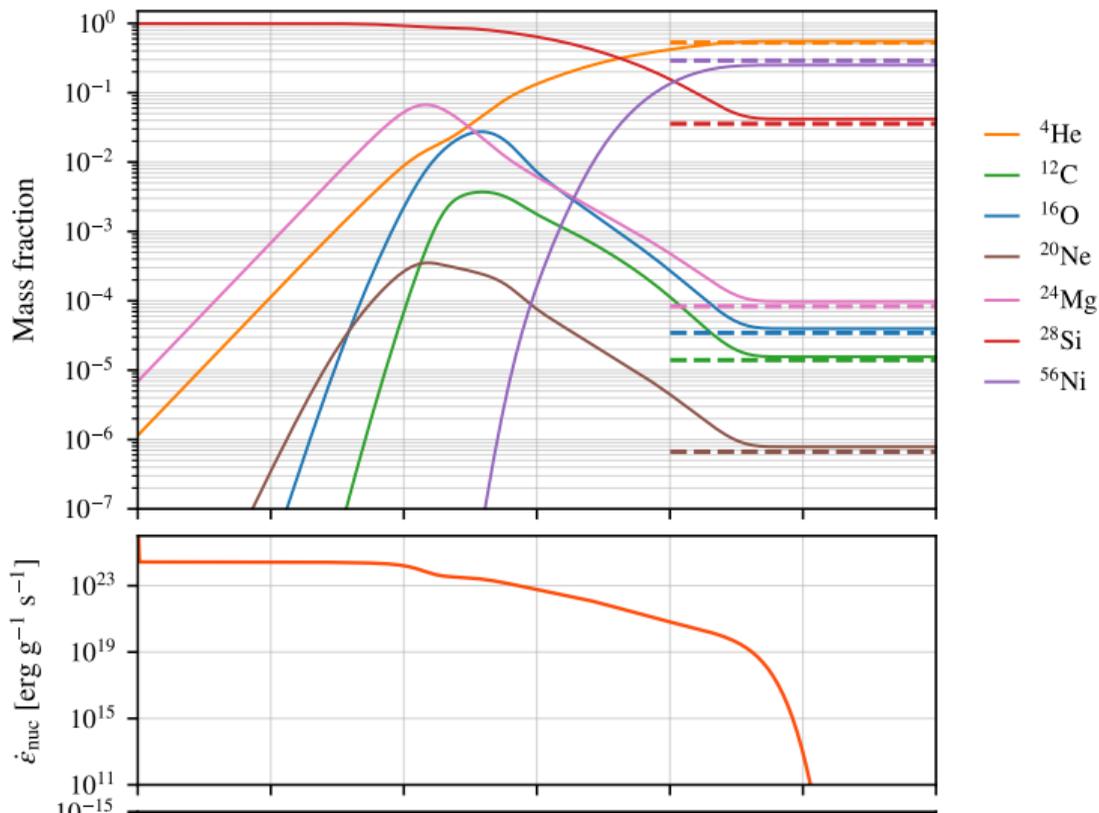
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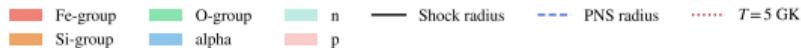
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Results: CCSNe

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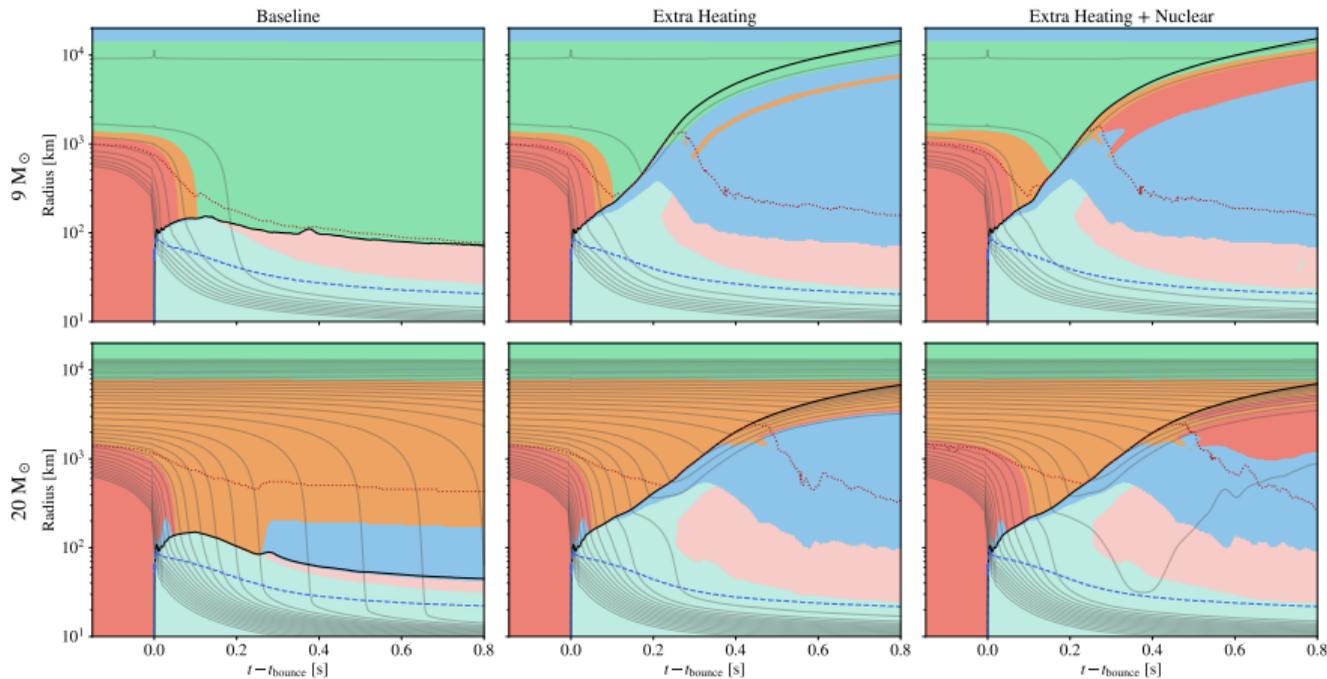
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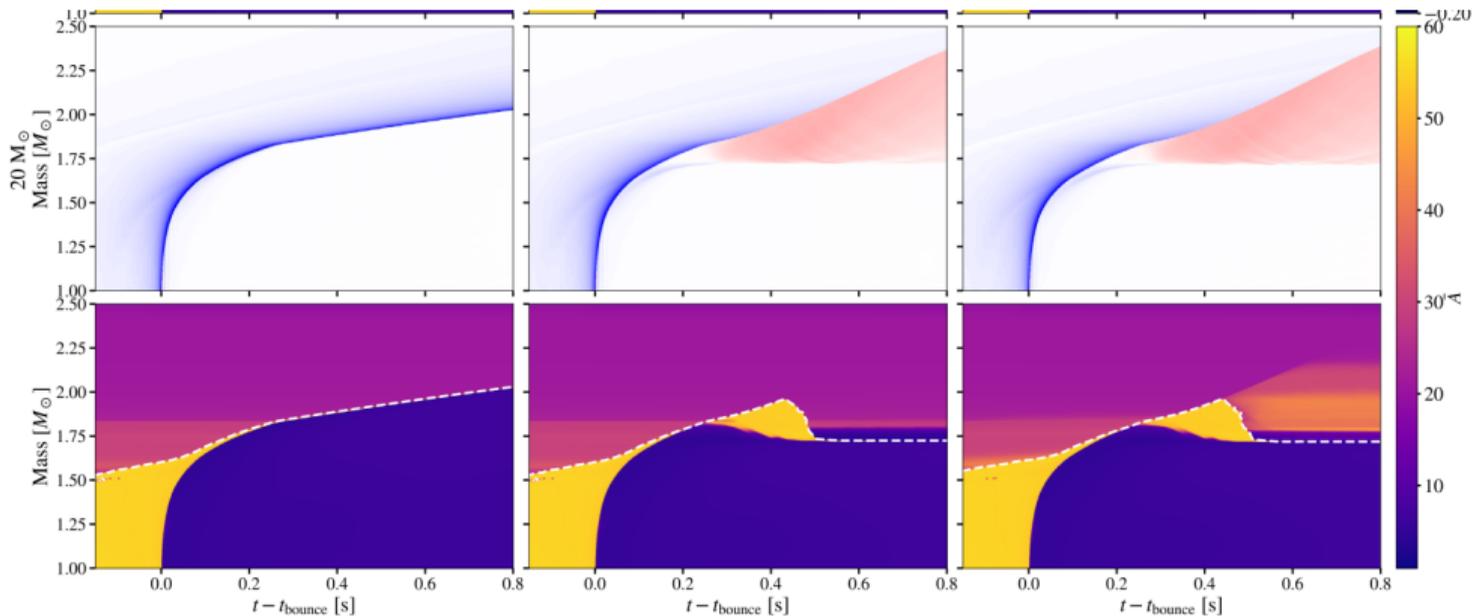
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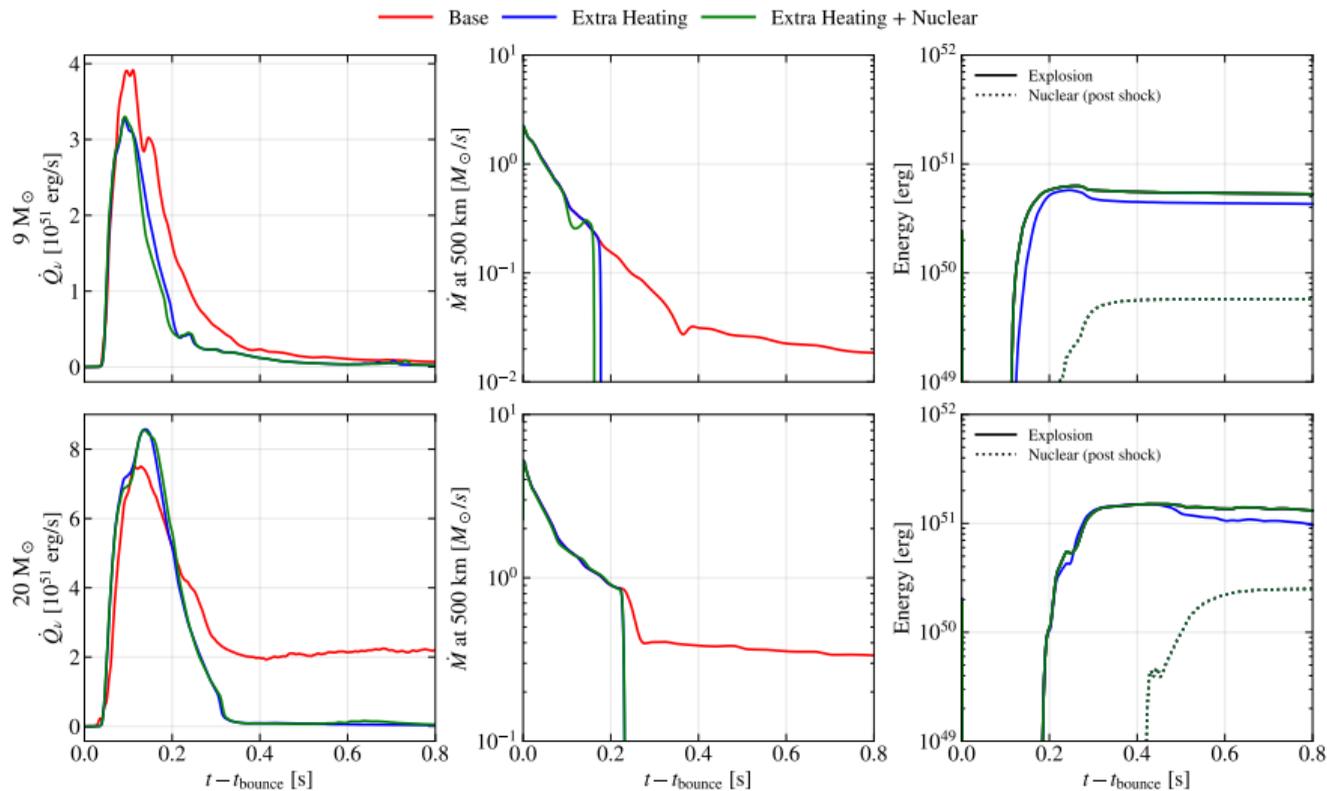
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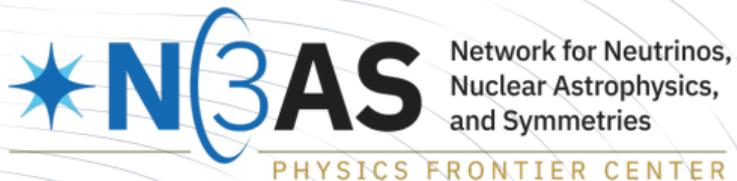
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- ▶ Coupled nuclear burning to `Gmunu`, a step forward to couple all relevant physics self-consistently
- ▶ Considering more generic network couplings (e.g. `pynucastro`)
- ▶ Apply to NS merger problems (adaptive network?)
- ▶ Deal with "grey" zones between neutrinos and nuclear burning

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Thank you for your attention.
Q & A





Results

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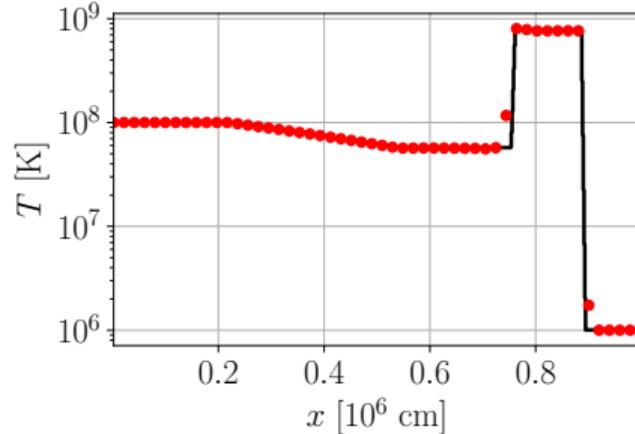
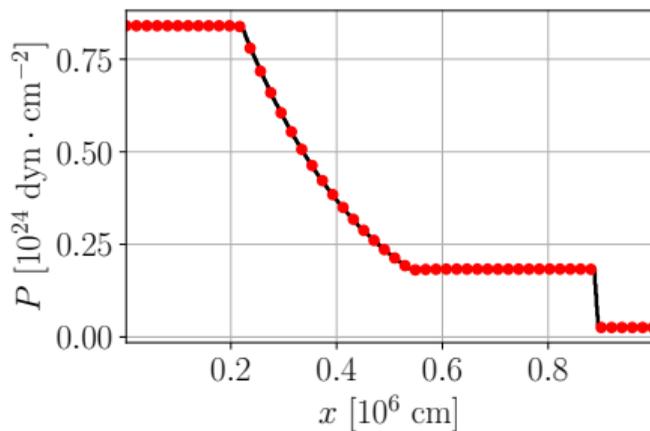
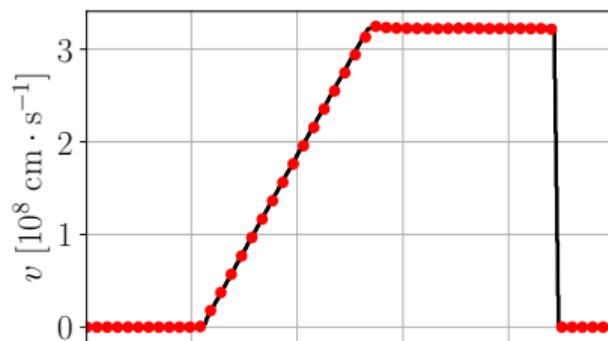
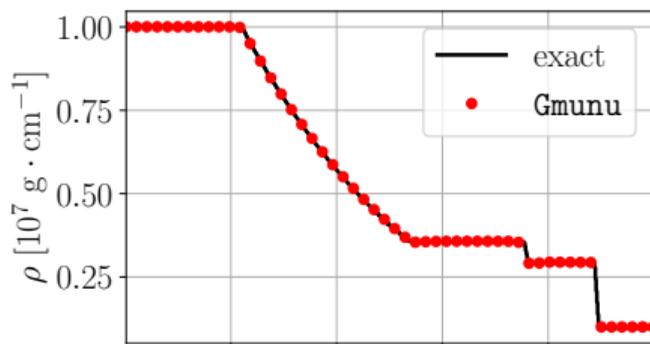
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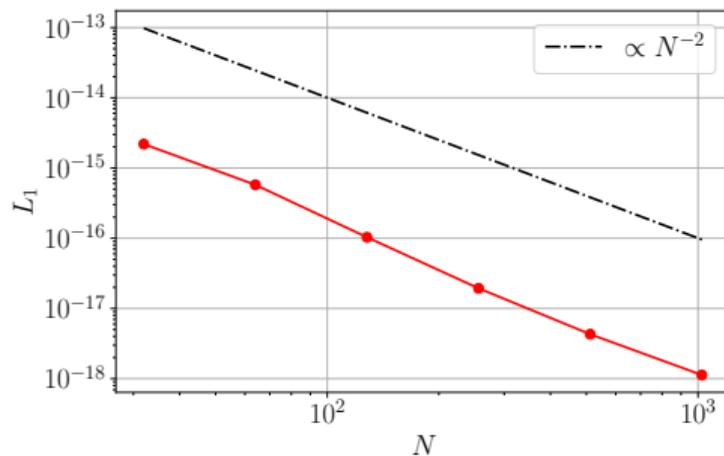
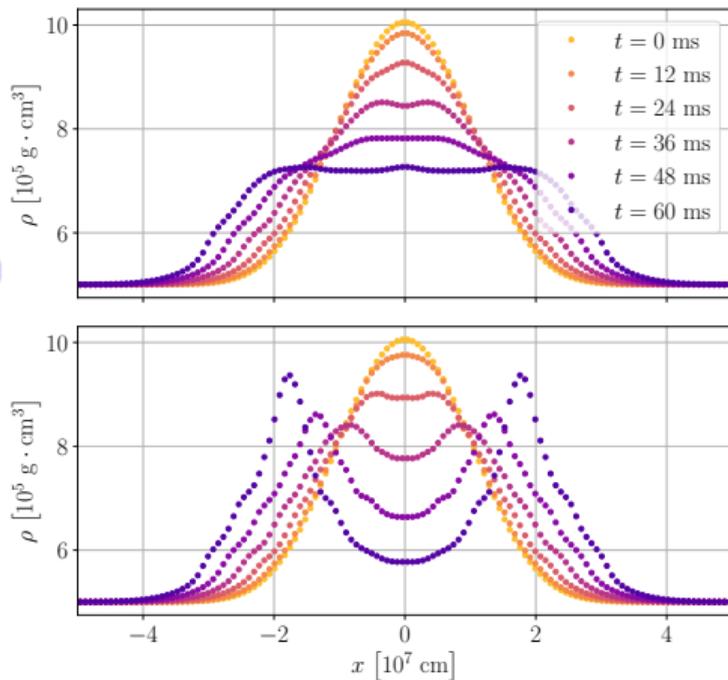
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- [1] P. C.-K. Cheong, L.-M. Lin, and T. G. F. Li, “Gmunu: toward multigrid based Einstein field equations solver for general-relativistic hydrodynamics simulations,” *Classical and Quantum Gravity*, vol. 37, p. 145015, July 2020.
- [2] P. C.-K. Cheong, A. T.-L. Lam, H. H.-Y. Ng, and T. G. F. Li, “Gmunu: paralleled, grid-adaptive, general-relativistic magnetohydrodynamics in curvilinear geometries in dynamical space-times,” *MNRAS*, vol. 508, pp. 2279–2301, Dec. 2021.
- [3] P. C.-K. Cheong, D. Y. T. Pong, A. K. L. Yip, and T. G. F. Li, “An Extension of Gmunu: General-relativistic Resistive Magnetohydrodynamics Based on Staggered-meshed Constrained Transport with Elliptic Cleaning,” *ApJS*, vol. 261, p. 22, Aug. 2022.
- [4] P. C.-K. Cheong, H. H.-Y. Ng, A. T.-L. Lam, and T. G. F. Li, “General-relativistic Radiation Transport Scheme in Gmunu. I. Implementation of Two-moment-based Multifrequency Radiative Transfer and Code Tests,” *ApJS*, vol. 267, p. 38, Aug. 2023.
- [5] H. H.-Y. Ng, P. C.-K. Cheong, A. T.-L. Lam, and T. G. F. Li, “General-relativistic radiation transport scheme in gmunu ii: Implementation of novel microphysical library for neutrino radiation – weakhub,” 2023.