Tidal resonance

Nonlinearities revealed by numerical relativity

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Based on arXiv: 2411.16850 and preliminary results

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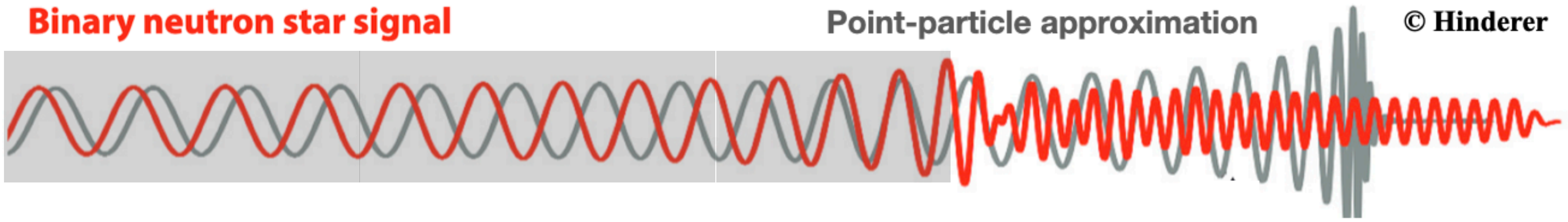
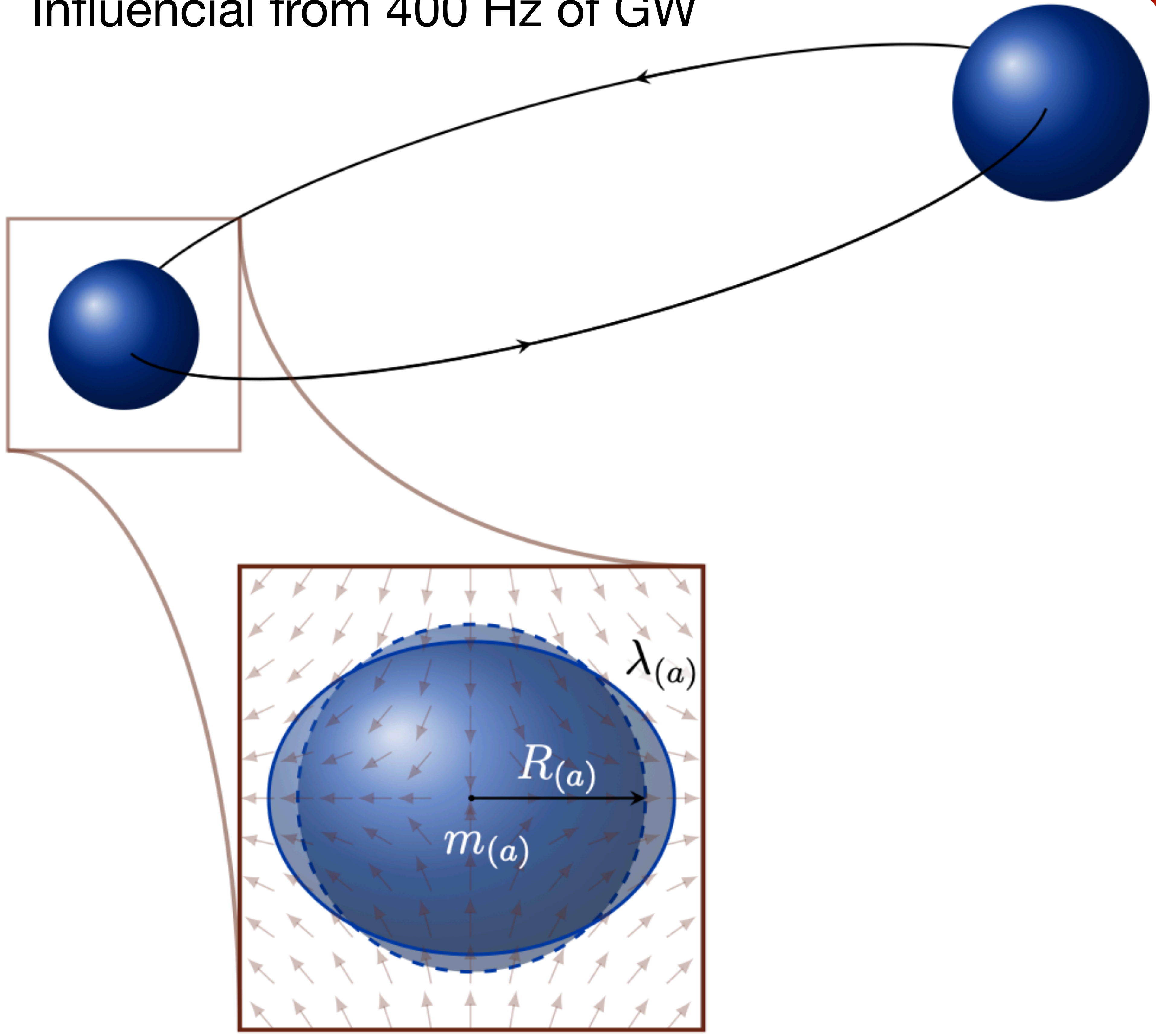
UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

What have been modeled

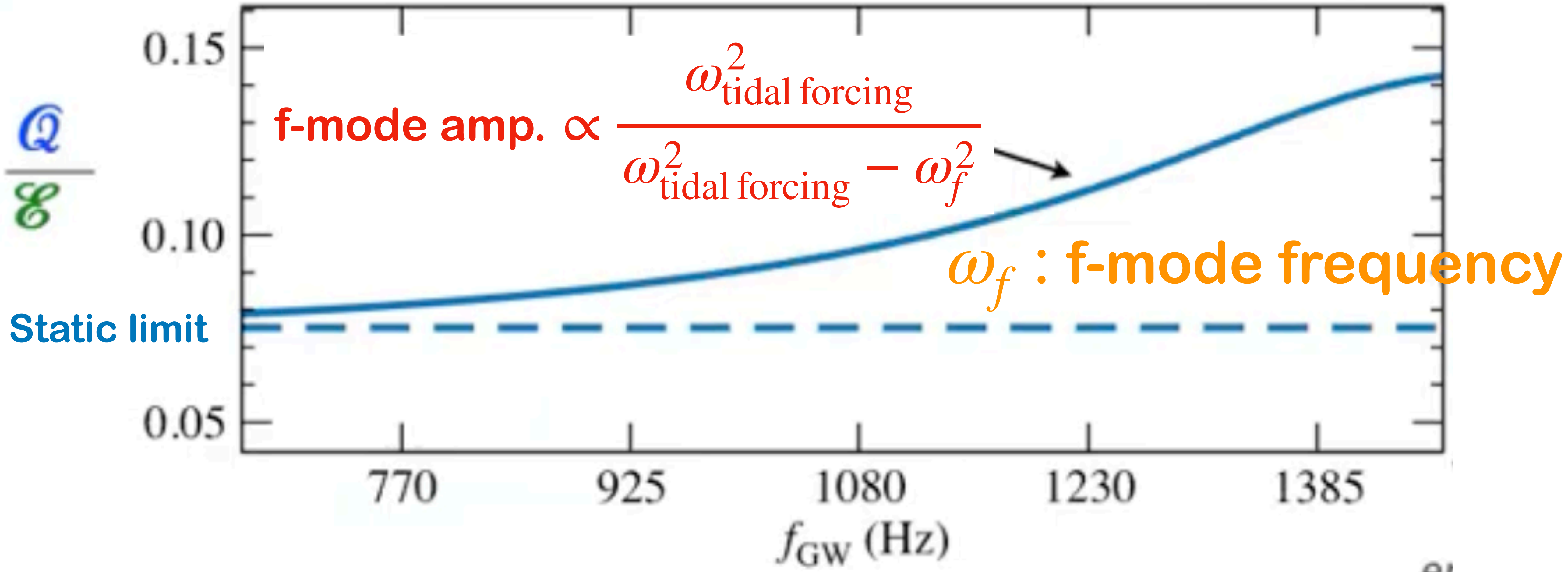


Influencial from 400 Hz of GW

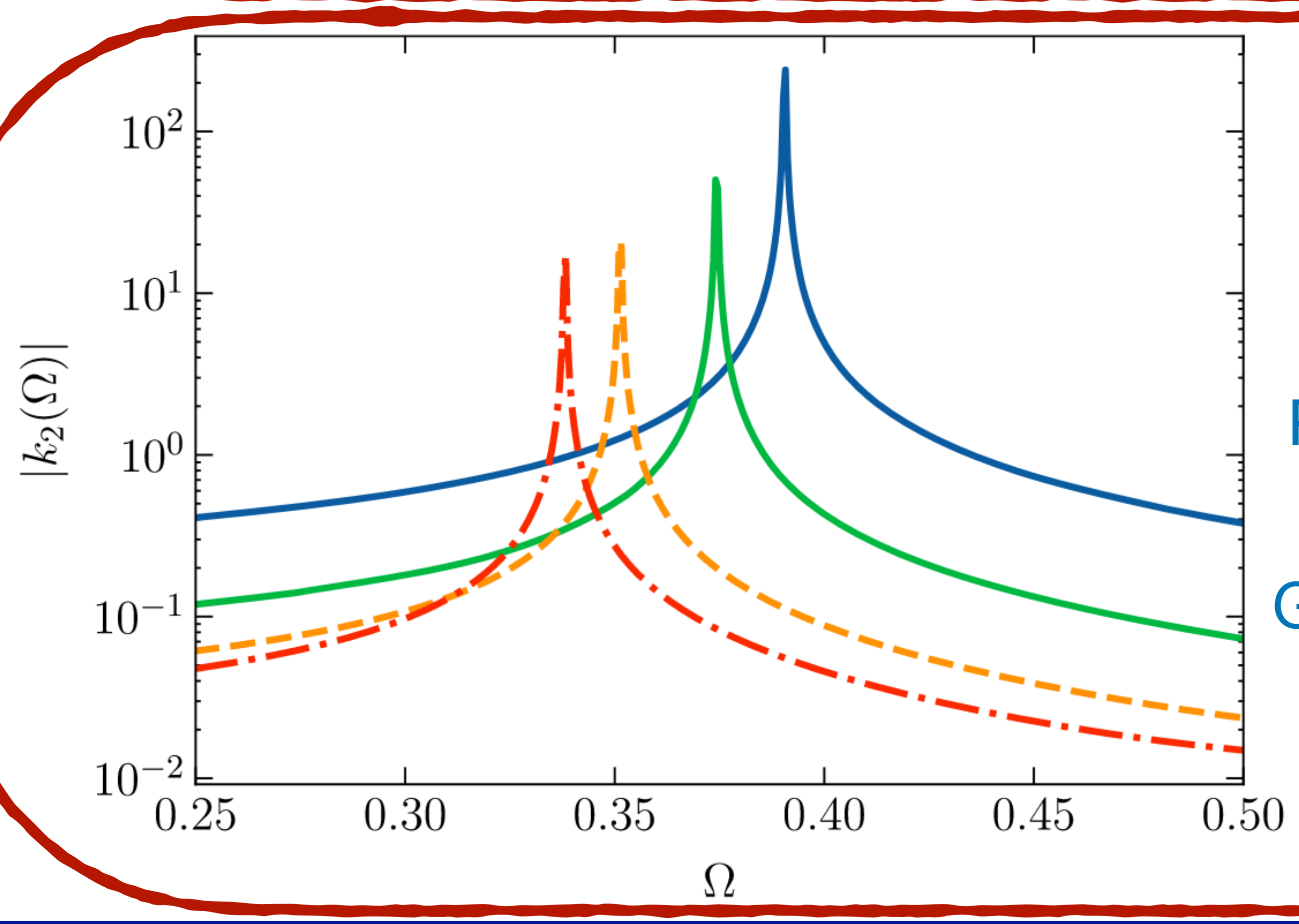


Mandal et al., JHEP 11 (2023), 067

Induced mass quadrupole to tidal field



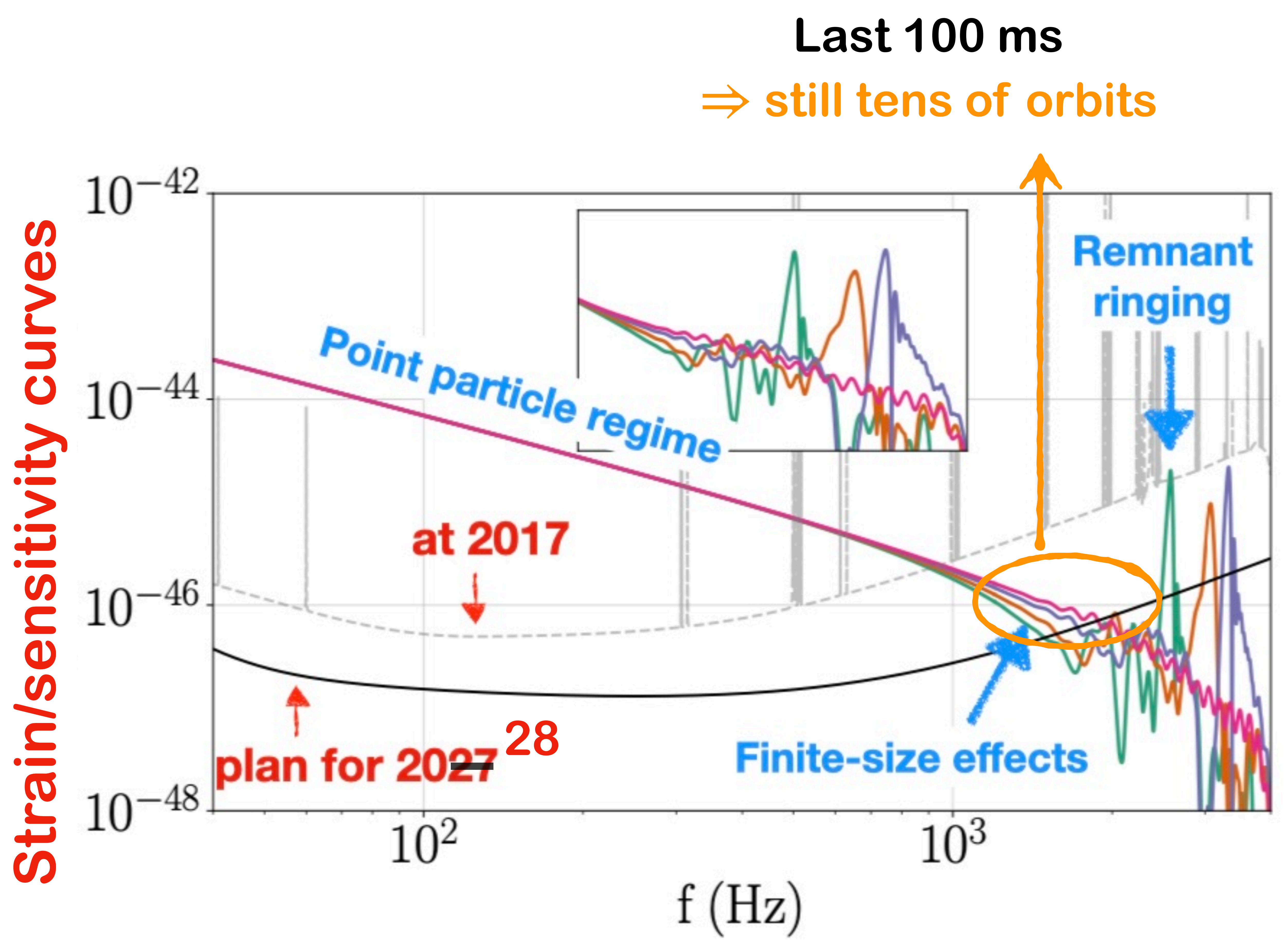
Hinderer et al., Phys.Rev.Lett. 116 (2016) 18



Hegade et al., PRD 109 (2024), 104064
Pnigouras et al., MNRAS 542, 1375 (2025)

Gamba & Bernuzzi, PRD 107 (2023) 4, 044014
Ma + (incl **HJK**), in prep
see Gao's talk tommorrow

What await to be modeled



Chatziioannou et al., arXiv:2407.11153

A minimal list for the nonlinear effect

- Large amplitude oscillation leads to **anharmonic** oscillators [Kwon +, 2410.03831, 2503.11837; Pitre & Poisson, 2506.08722]
- Tidal field torques the star inhomogeously; **tidal-spin** effect [Yu & Lau, PRD ('25); Reboul-Salze +(incl **HJK**), A&A ('25)]
- Whether resonance hits its saturation before passing the resonance window

Signals $\sim 100\,\mathrm{ms}$ before merger = even stronger gravity + dynamical matter imprints

Numerical Relativity is the ultimate tool!

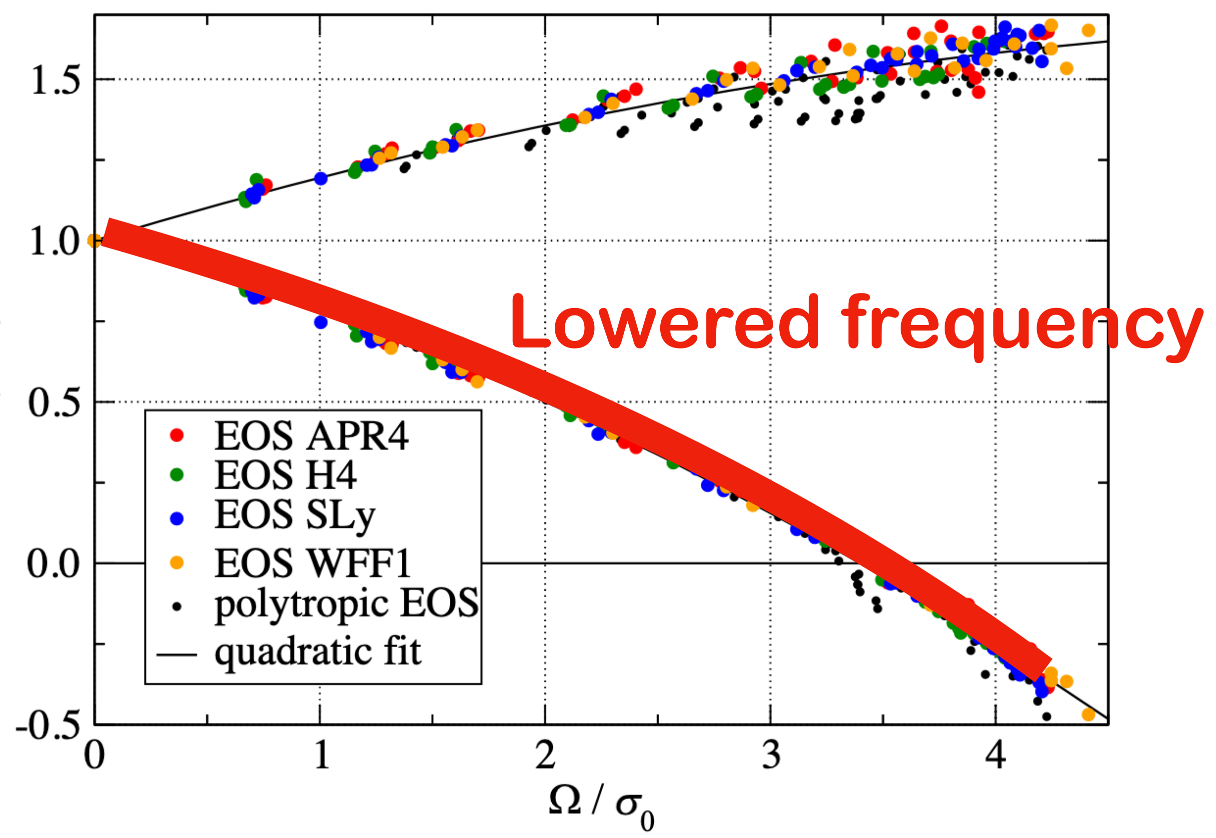
I. Nonlinear effects revealed

The first simulation of f-mode resonance

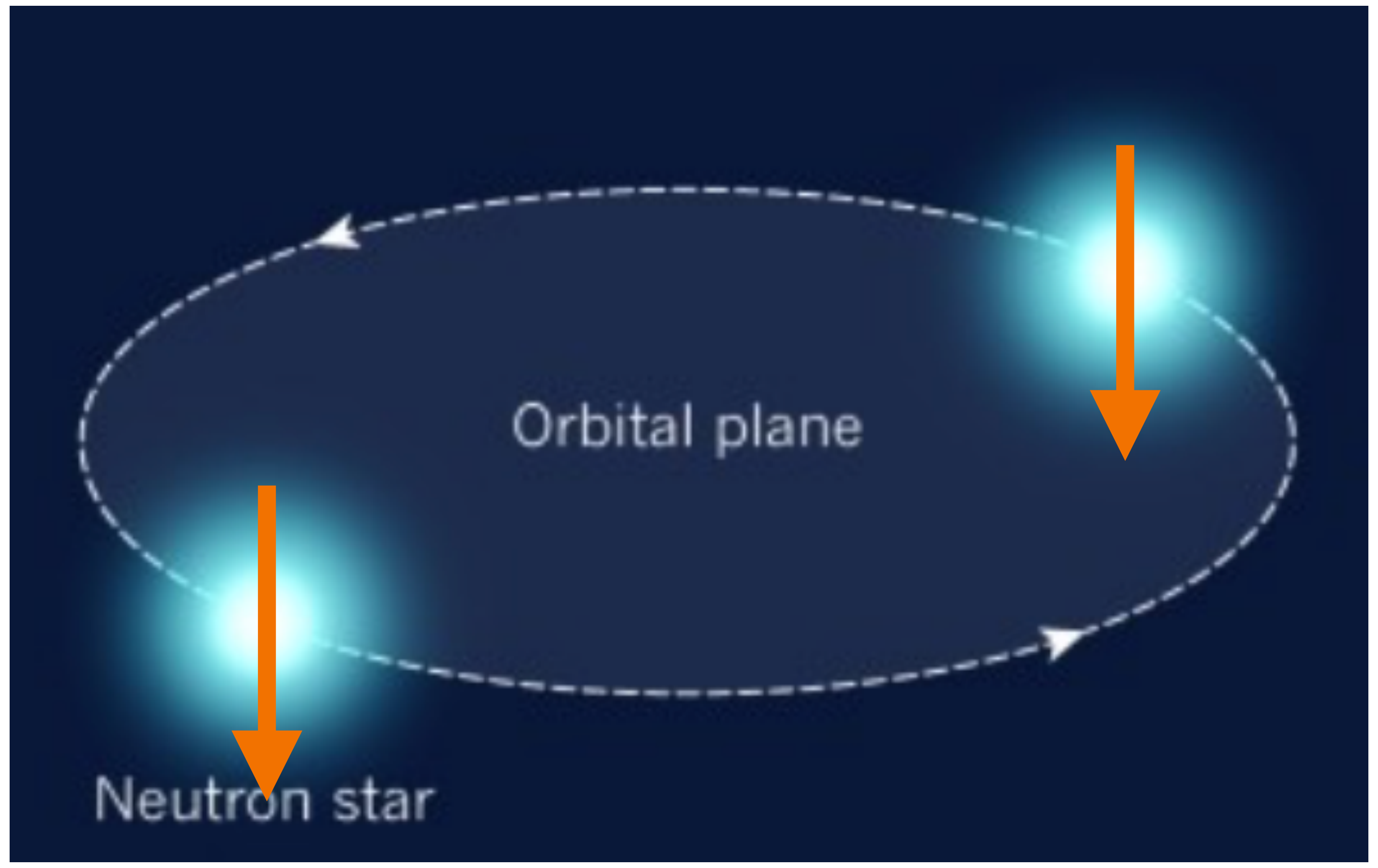
(The mode **moving against spin** has lower frequency)

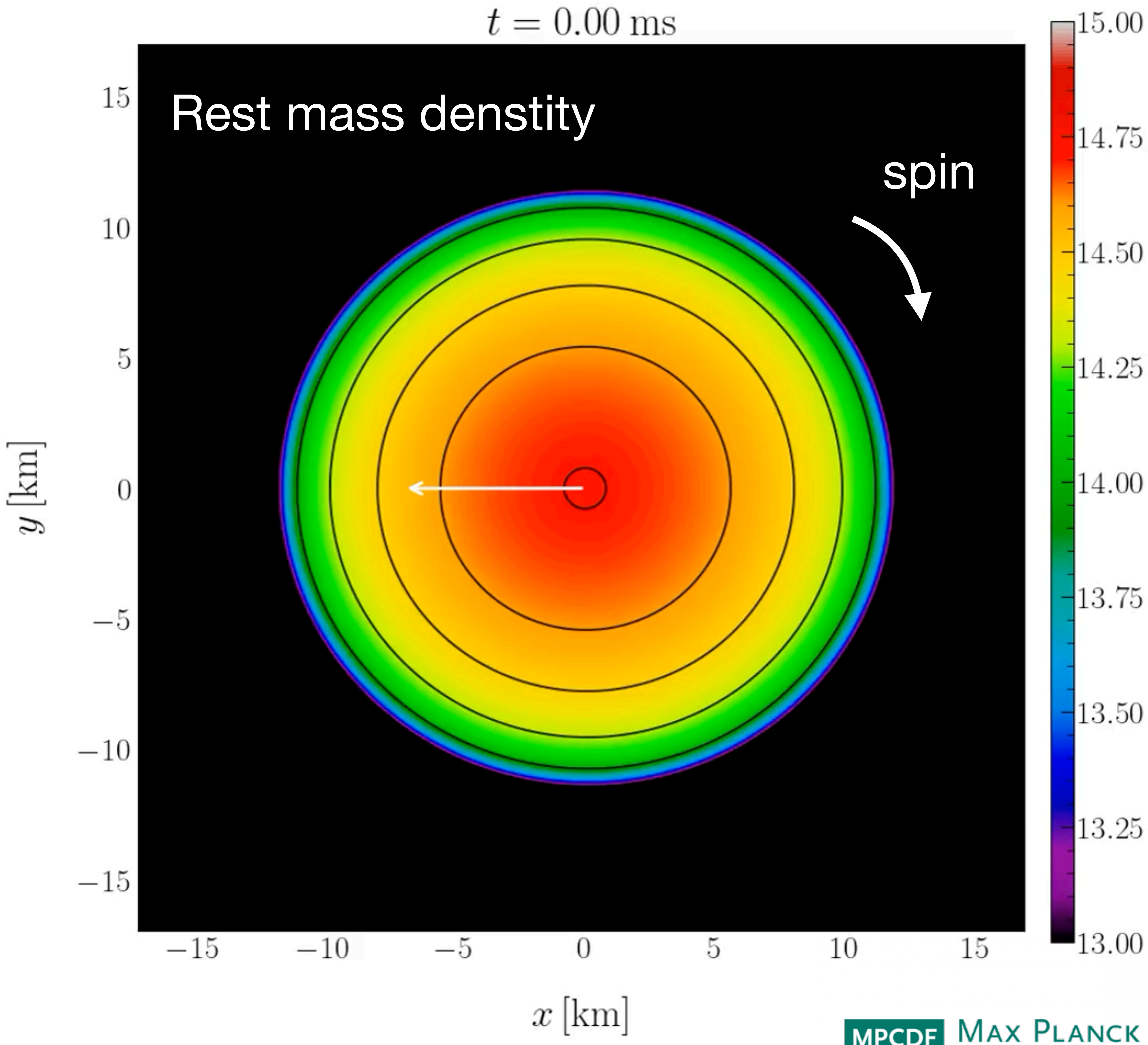
+

(The mode **moving along the tidal field** is enhanced)



[Krueger & Kokkotas, PRL (2020)]





10 levels of mesh refinement, 1M cpu hours, more than 15 orbits

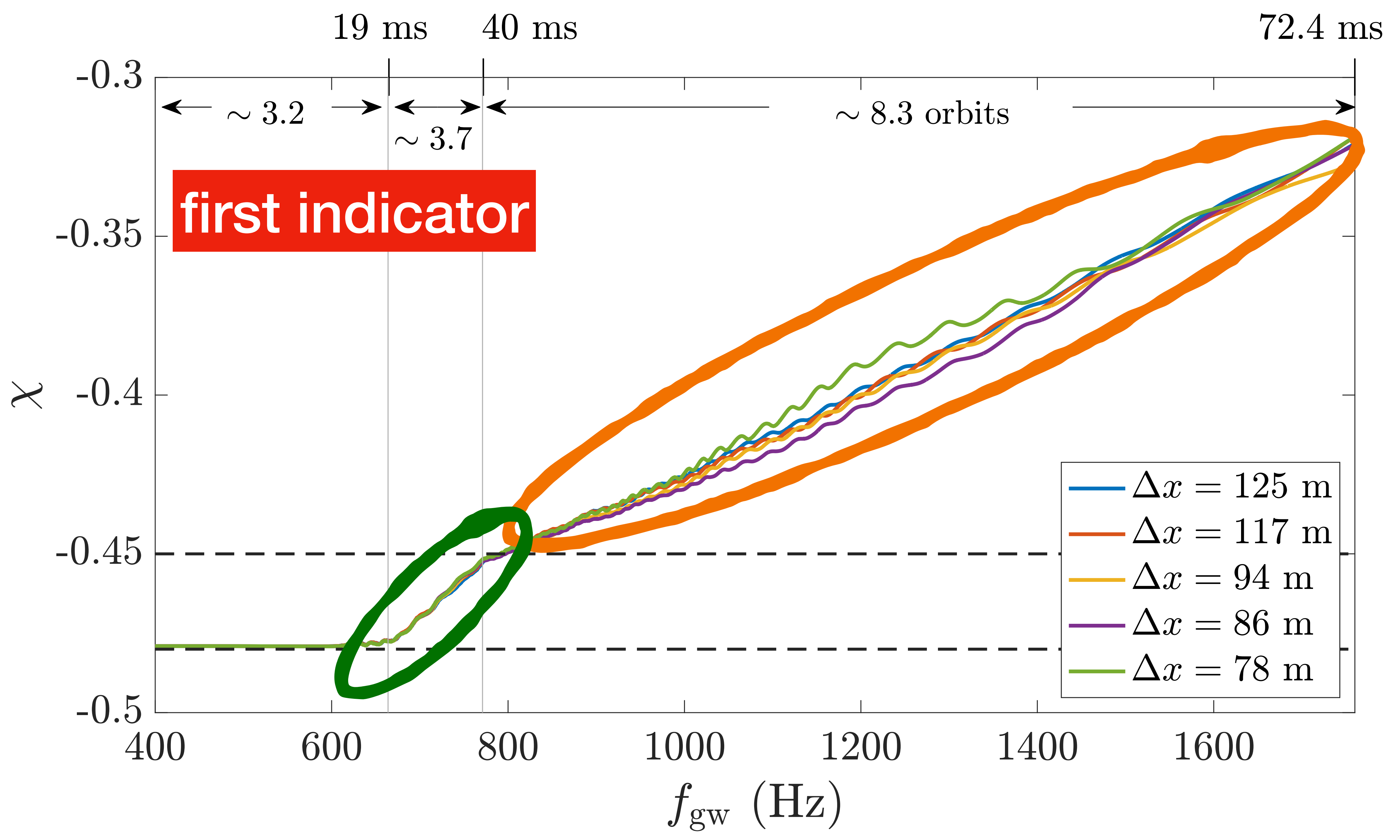
MPCDF Max Planck Computing & Data Facility

Considerable back-reaction to the NS

Dimensionless spin parameter cJ/GM^2

— negative: counter-rotate

- **Tidal-spin effect**: tapping orbital angular momentum that differentially whirls the star [Yu & Lau, PRD ('25); Reboul-Salze (incl **HJK**) +, A&A ('25)]



- **Anharmonic oscillator**: Nonlinear self-interaction of f-mode **blue shifts** its response frequency [Kwon +, 2410.03831, 2503.11837; Pitre & Poisson, 2506.08722]

- As a result, lock the tidal pattern with the orbit ⇒ **maximize torquing**

Two phase spaces for **hidden structure:**

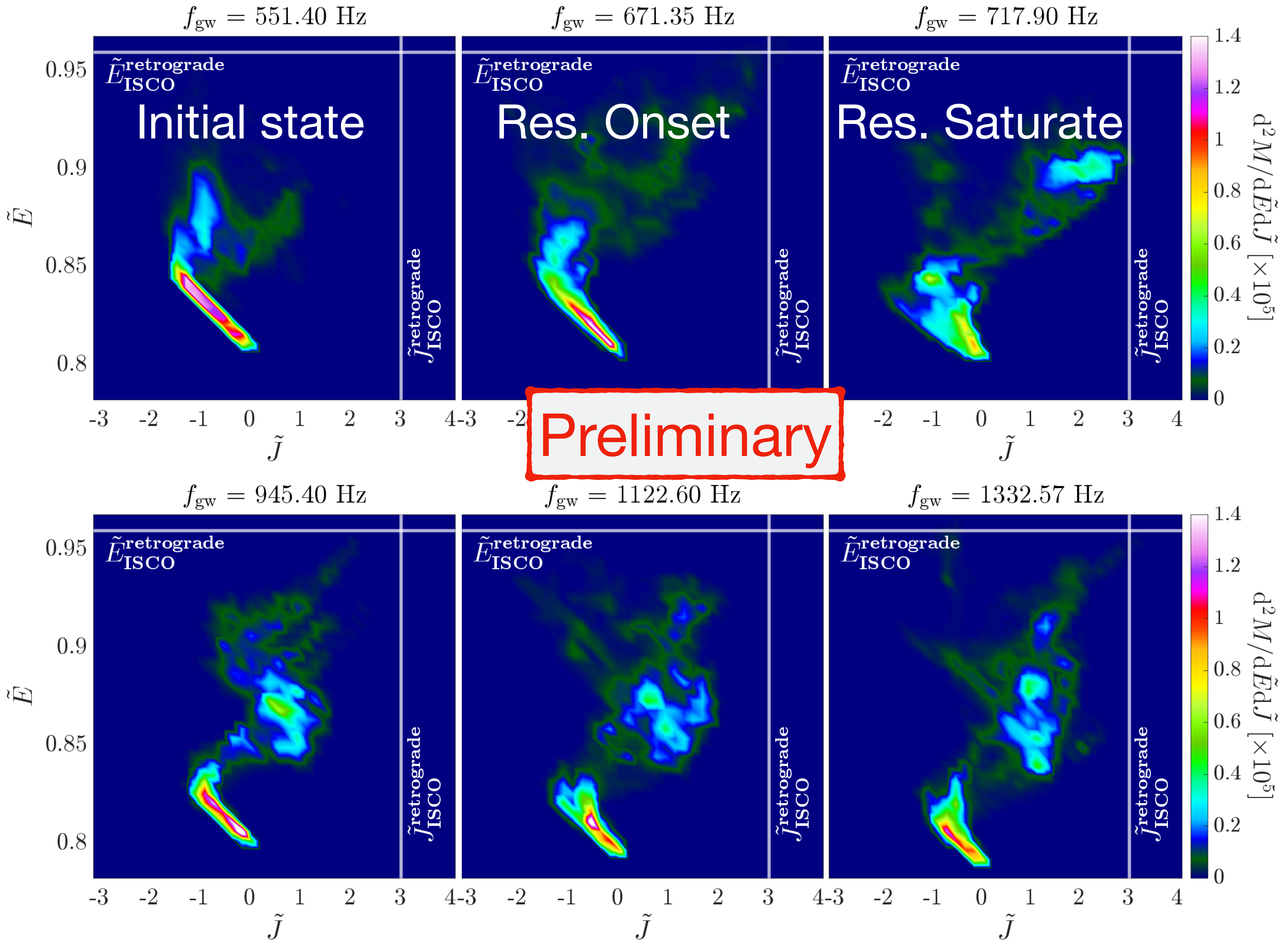
- $\mathcal{S}_1 = (\tilde{J}, \tilde{E})$: specific angular momentum and enegy
- $\mathcal{S}_2 = (F, \Omega)$: $F = u^t u_\phi$ (i.e., rotational law)

Shown for the first time by using NR

Mass-distribution in $\mathscr{S}_1$

Tidal saturation: $f-$mode motion reaches the ISCO from inside to saturate resonance

Post resonance evolution ⇒
(Qualitatively unchanged)



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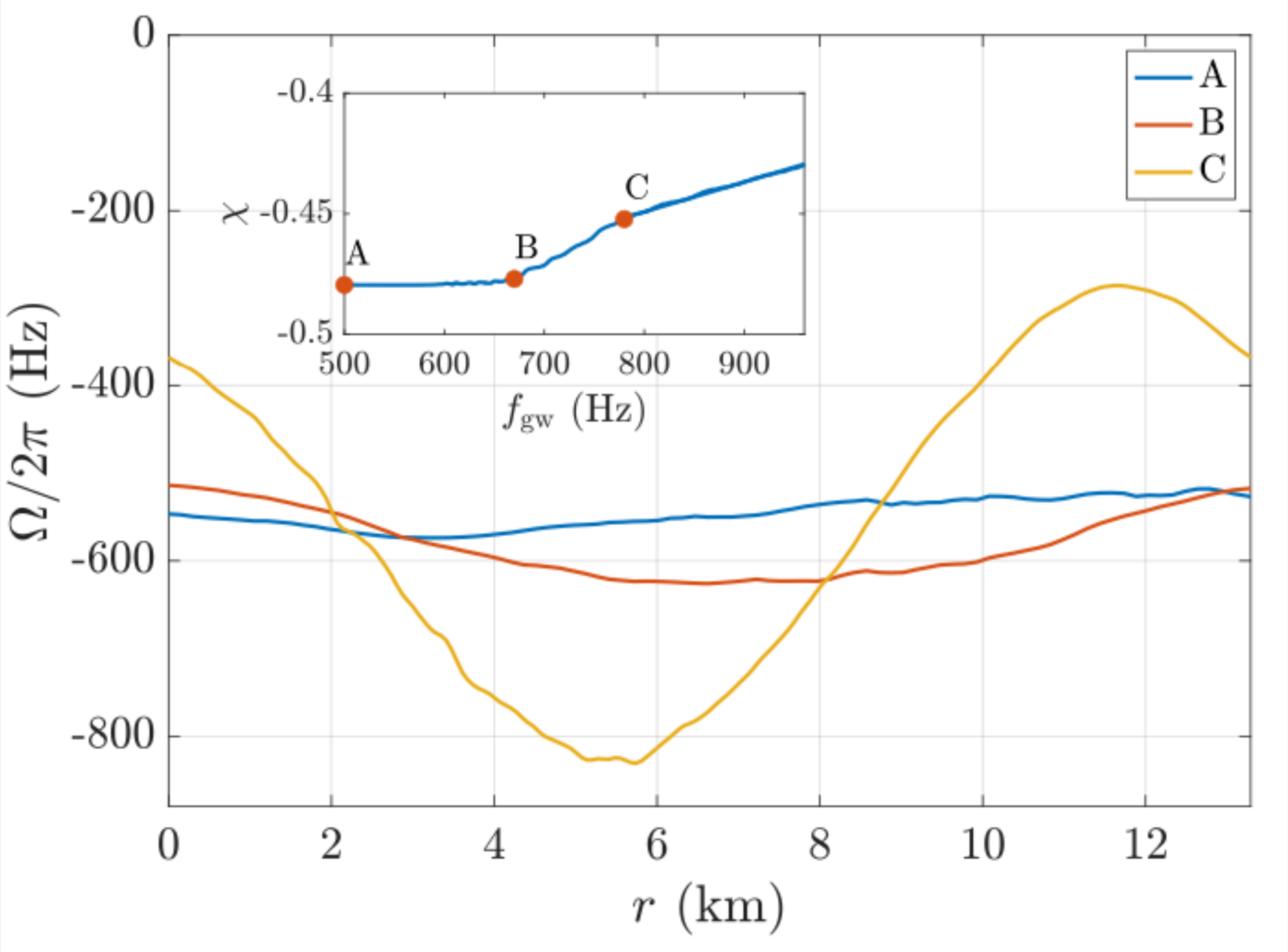
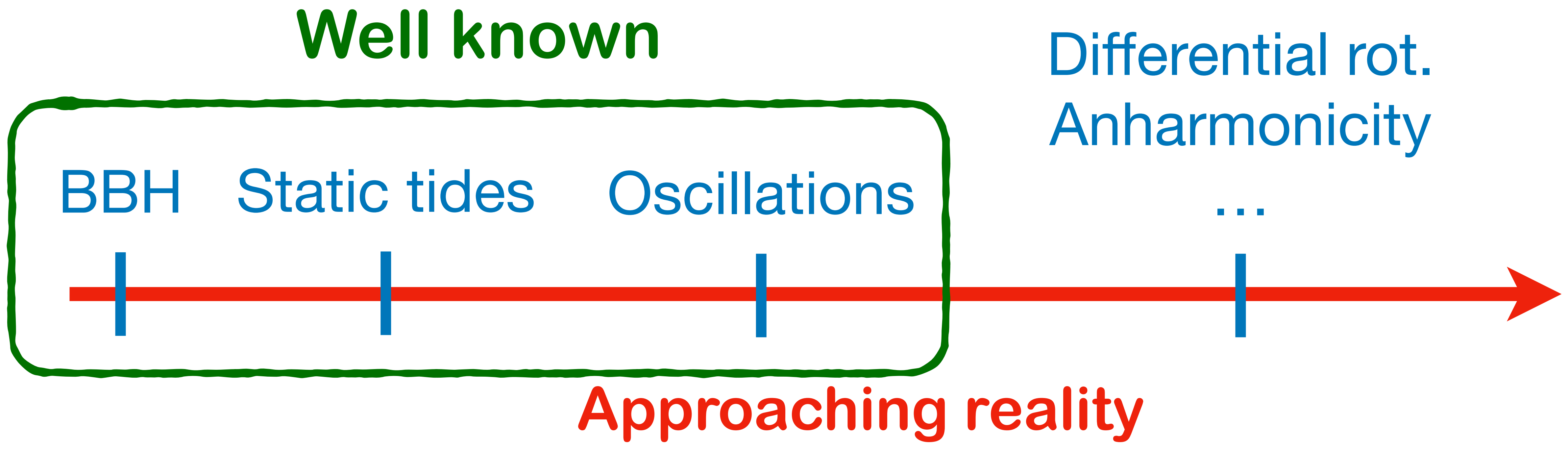
Induced differential rotation

Considerable differential rotation



Required more E and J than rigid rotation



How important are all these nonlinear effects?

II. Huge impact on waveform

Implication/Complication on waveform — I

* Phase acceleration $Q_w := \frac{\omega^2}{\dot{\omega}}$

* Dephasing from TEOBResumS

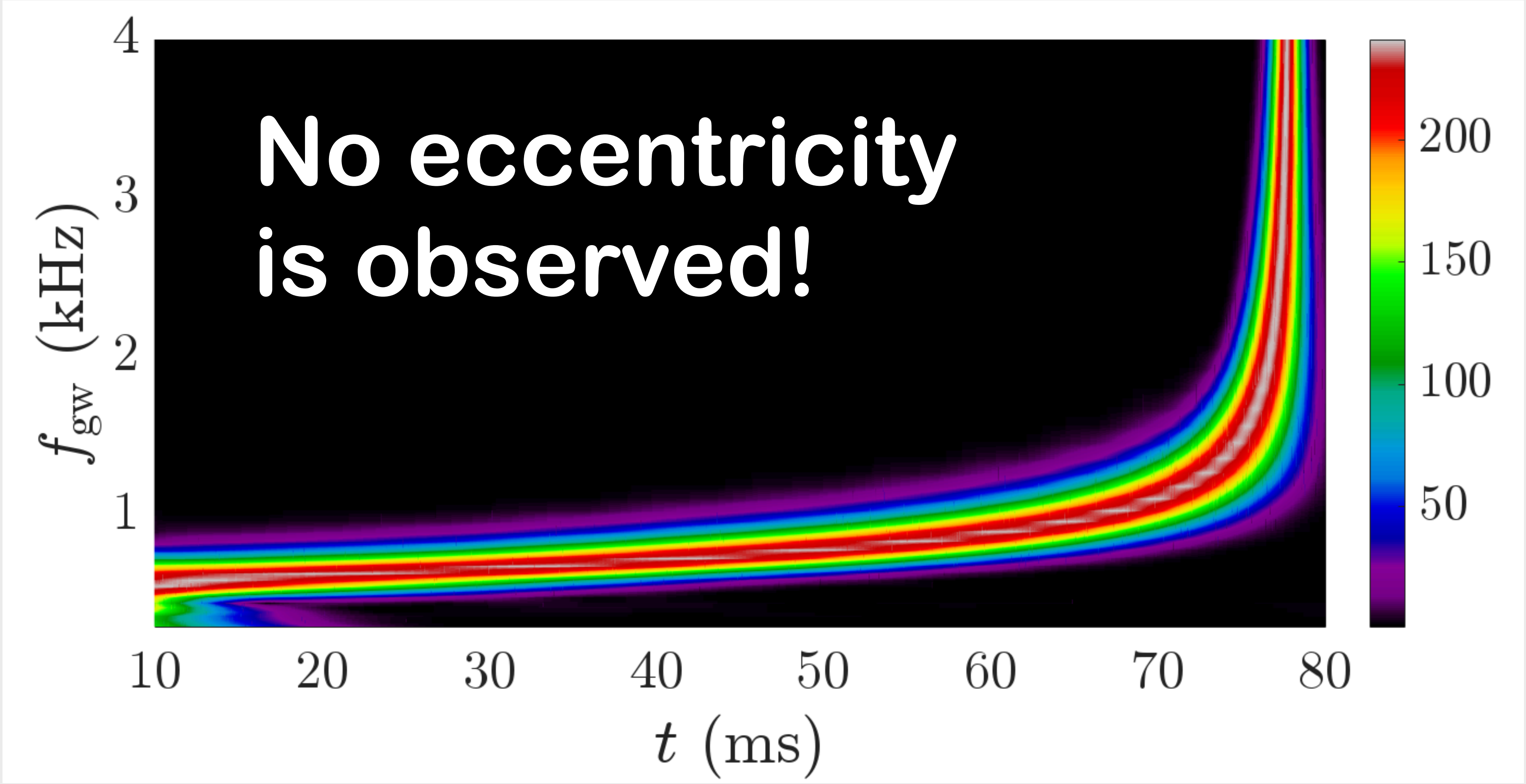
$$\delta\Psi_{\rm gw} = \int \Delta Q_w \, {\rm d}(\ln \omega_{\rm gw}) \lesssim 6.4 \text{ cycles}$$

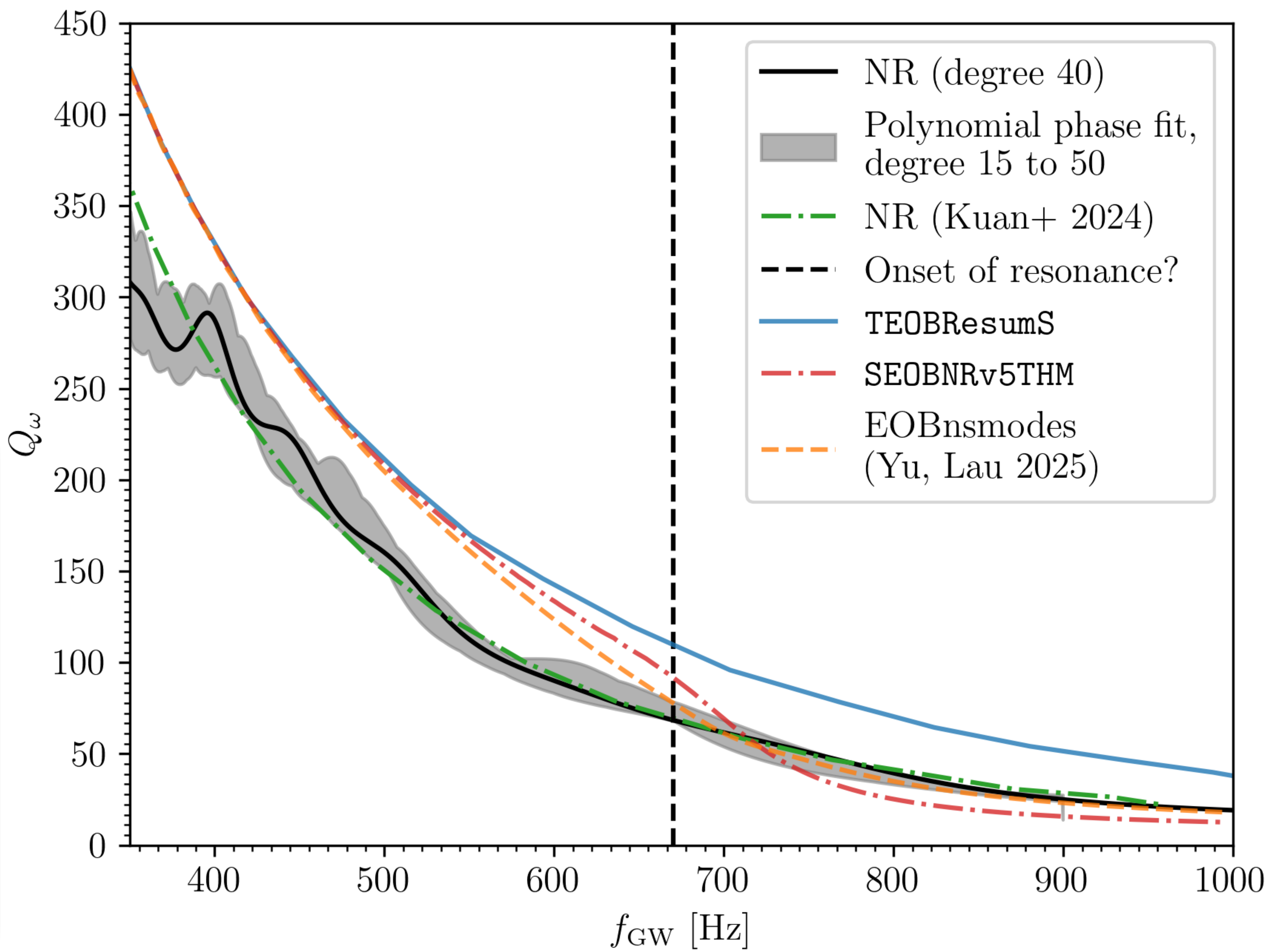
* Two helpful observations:

1. E and J adiabatically transferred

- The thermal law for BNSs obeyed

Spectrogram



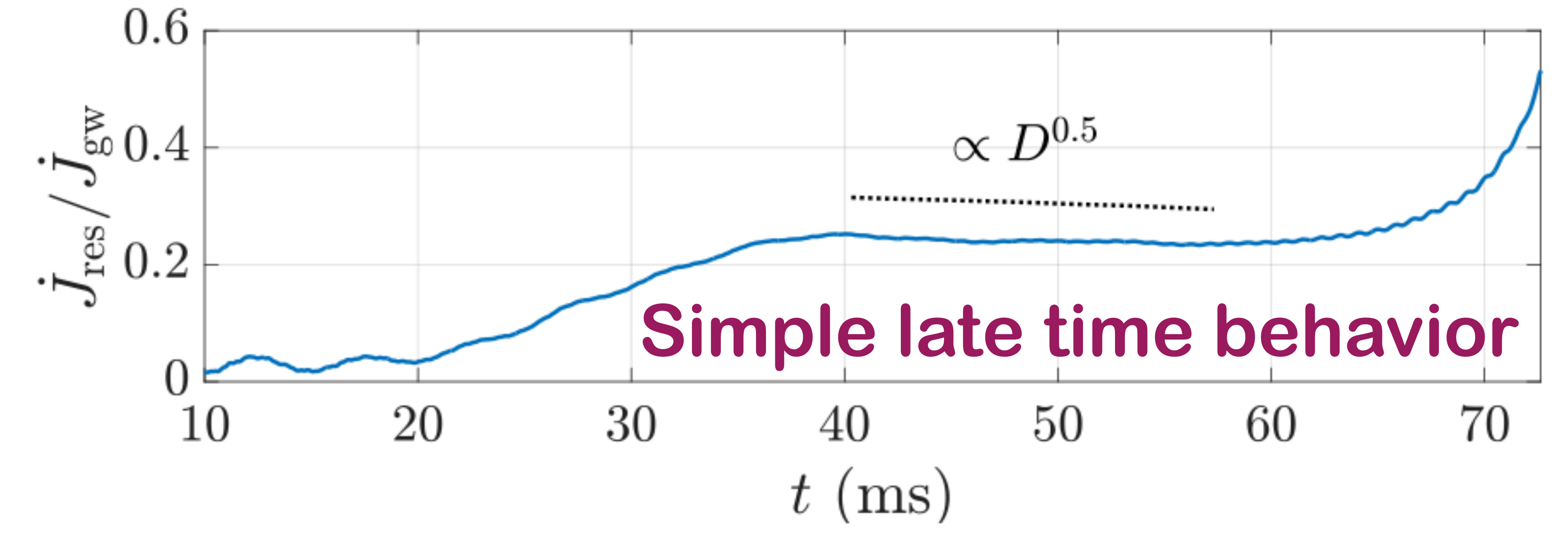


Credit: Marcus Haberland (AEI)

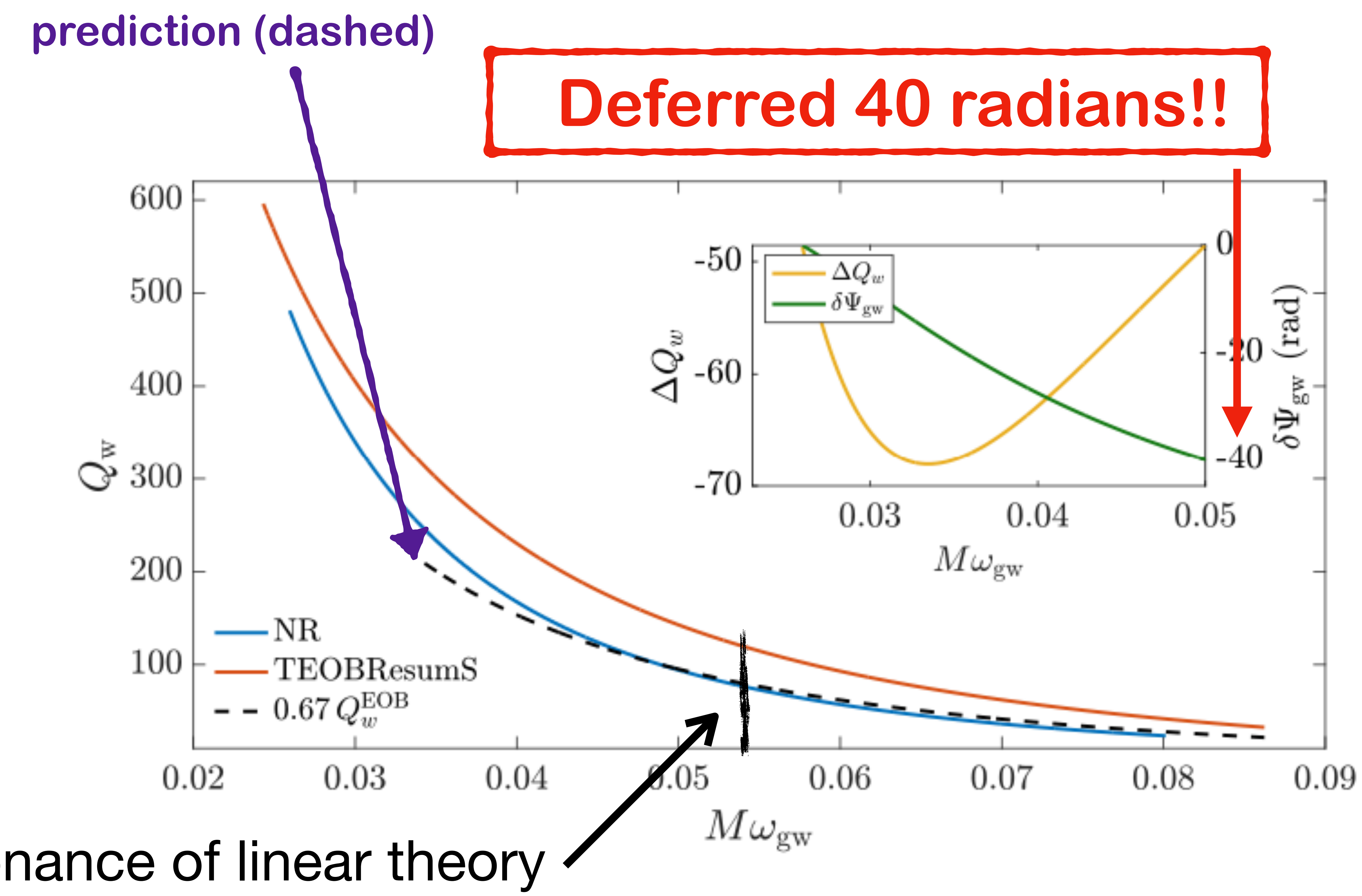
Implication/Complication on waveform — II

2. Coherent dissipation as GW

- Rate of angular moment loss via GW: $\dot{J}_{\rm gw} \propto D^{-3.5}$
- Rate of angular moment to fuel f-mode: $\dot{J}_{\rm res} \propto D^{-3}$



- ☆ Late-time ratio $\dot{J}_{\rm res}/\dot{J}_{\rm gw} \simeq 0.25$
- ☆ $\dot{E}_{\rm res}$ of two NSs = $50\,\%$ more dissipation
- ☆ Balance eq. ⇒ **reduced** $0.67\,Q_w$



III. Toward next-gen waveform modeling

Shown for the first time by using NR

Toward a phenomelogical rotational law

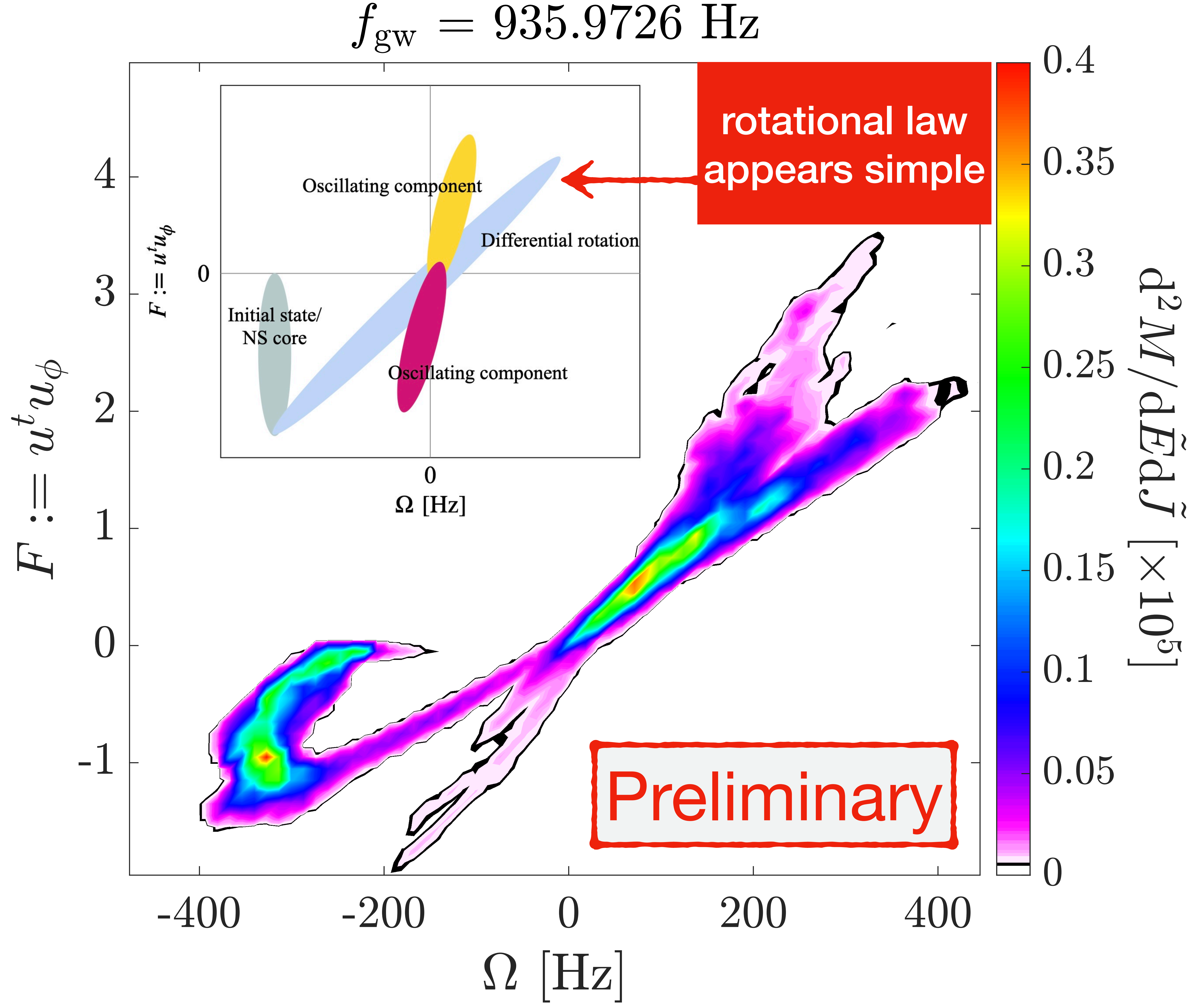
Mass distribution on $\mathcal{S}_2$ uncovers

- A component of differential rotation
- Two oscillating patches for dynamical tides

How do these help modeling?

- Modified const-j law
- f-mode contribution extractable

In post-resonance epoch



Summary

- High-precision GRHD simulations are now capable of resolving the tidal resonance effect: to cover the final patch of BNS parameter space!!
- A need to improve waveform modeling is highlighted
  — A naive way to modify the EOB here is for sure not realistic, but nevertheless indicates that an analytic model including resonance is possible
- Nonlinearities should be phenomenologically modeled for better fidelity of waveform models
  — Minimal list: differential rotation and anharmonicity
  — Useful info: a simple rotation law

Thank you